Higgs Physics from the Lattice

Lecture 2: Triviality and Higgs Mass Upper Bound

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Outline of Lecture Series: Higgs Physics from the Lattice

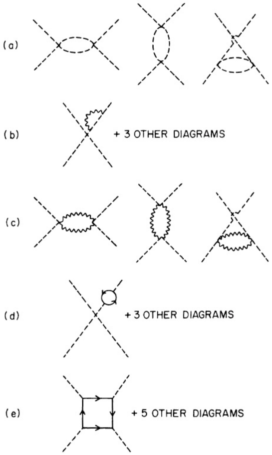
1. **Standard Model Higgs Physics**
   - Outlook for the Higgs Particle
   - Standard Model Review
   - Expectations from the Renormalization Group
   - Nonperturbative Lattice Issues?

2. **Triviality and Higgs Mass Upper Bound**
   - Renormalization Group and Triviality in Lattice Higgs and Yukawa couplings
   - Higgs Upper Bound in 1-component ϕ^4 Lattice Model
   - Higgs Upper Bound in O(4) Lattice Model
   - Strongly Interacting Higgs Sector?
   - Higgs Resonance on the Lattice

3. **Vacuum Instability and Higgs Mass Lower Bound**
   - Vacuum Instability and Triviality in Top-Higgs Yukawa Models
   - Chiral Lattice Fermions
   - Top-Higgs and Top-Higgs-QCD sectors with Chiral Lattice Fermions
   - Higgs mass lower bound
   - Running couplings in 2-loop continuum Renormalization Group

Standard Model Scales: Running Higgs coupling

1-loop Higgs couplings:



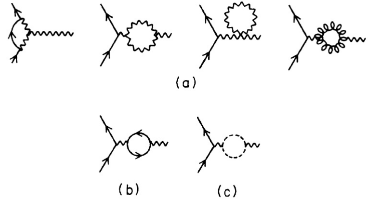
1-loop Feynman diagrams: Higgs boson self-couplings

Running Higgs coupling $\lambda(t)$ is defined as the Higgs 4-point function at scale t=$\log\frac{p}{\mu}$

Higgs beta function: $\beta_\lambda(t) = \frac{d\lambda(t)}{dt}$

Standard Model Scales: Running gauge and Yukawa couplings

1-loop gauge couplings:

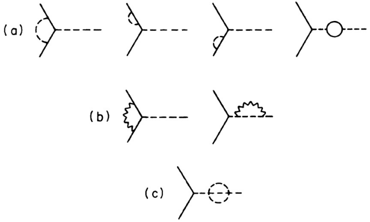


1-loop Feynman diagrams: gauge boson couplings to fermions

Running gauge couplings $g(t), g'(t), g_3(t)$ can be defined as the gauge-fermion 3-point function at scale t=log$\frac{p}{\mu}$

gauge beta functions: $\beta_g(t) = \frac{dg(t)}{dt}$

1-loop Yukawa couplings:



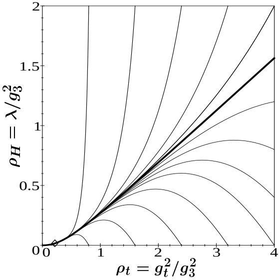
1-loop Feynman diagrams: Higgs boson Yukawa couplings to fermions

Running Top coupling $g_{Top}(t)$ is defined as the Higgs fermion 3-point function at scale t=log$\frac{p}{\mu}$

Top beta function: $\beta_{g_{Top}}(t) = \frac{dg_{Top}(t)}{dt}$

Standard Model Scales: RG Fixed Points and Triviality

running of λ, g_t, g_3



- Top-Higgs sector (1-loop)

  with notation $R = \frac{\lambda}{g_t^2} = \frac{m_H^2}{4m_t^2}$

  $\frac{dg_t^2}{dt} = \frac{9}{16\pi^2} g_t^4$

  $g_t^2 \frac{dR}{dg_t^2} = \frac{1}{3}(8R^2 + R - 2)$

  IR fixed line at $\bar{R} = \frac{1}{16}(\sqrt{65} - 1) = 0.44$

  Trivial fixed point only!

  Is the Landau pole the upper bound?

  Is $\lambda(\Lambda) = 0$ the lower bound?

- Top-Higgs-QCD sector (1-loop)

  Pendleton-Ross fixed point:

  $m_t = \sqrt{\frac{2}{9}} g_3(\mu = m_t) v/\sqrt{2} \approx 95\ GeV$

  $m_H = \sqrt{\frac{(\sqrt{689} - 25)}{72}} g_3 \sqrt{2} v \approx 53\ GeV$

Weak gauge couplings and 2-loop destabilize the Pendleton-Ross fixed point

"Landau pole" only in α_1 at $\mu = 10^{41}$GeV with all couplings running?

Landau pole in perturbation theory: Higgs mass upper bound

The RGE in the pure Higgs sector is known to three-loop order in the $\overline{\mathrm{MS}}$ scheme

$$\beta(\lambda) = \frac{\mathrm{d}\,\lambda}{\mathrm{d}\,t} = \frac{3}{2\pi^2}\lambda^2 - \frac{39}{32\pi^4}\lambda^3 + \frac{7176 + 4032\zeta(3)}{(16\pi^2)^3}\lambda^4.$$

The RGE exhibits an IR attractive fixed point λ=0 (perturbative "triviality") with general solution in one loop order

$$\lambda(\mu) = \frac{1}{\frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} ln\frac{\Lambda}{\mu}} \quad , \qquad \lambda(\Lambda) = \frac{\lambda(\mu)}{1 - \frac{3}{2\pi^2}\lambda(\mu) ln\frac{\Lambda}{\mu}} \quad .$$

Increasing $\lambda(\mu)$ at fixed $\lambda(\Lambda)$, the Landau pole is hit at $\frac{3}{2\pi^2}\lambda(\mu) ln\frac{\Lambda}{\mu} = 1$ with the naive upper bound $\boxed{\lambda(\mu) < \frac{2\pi^2}{3 ln\frac{\Lambda}{\mu}}}$ which is related to the upper bound on the Higgs mass $\boxed{\lambda = \frac{m_H^2}{v^2}\ .}$

When we try to increase $\lambda(\mu)$ at fixed $\lambda(\Lambda)$ beyond the Landau pole limit, something else will happen which will reveal the intrinsic non-removable cutoff in the theory. This will be illustrated in the large N limit of the Higgs-Yukawa fermion model.

Functional integral saddle point: Large $\mathrm{N_F}$ limit of Top-Higgs model

The Lagrangian density of the continuum model after rescaling the coupling constants:

$$\mathcal{L} = \frac{1}{2}\phi\left(-\Box + m^2)\phi\right) + \bar{\psi}_i\gamma_\mu\partial_\mu\psi_i - \frac{y}{\sqrt{N_F}}\bar{\psi}_i\psi_i\phi + \frac{1}{24}\frac{\lambda}{N_F}\phi^4\ , \quad i = 1, 2, ...N_F\ .$$

With rescaling of the scalar field $\phi \rightarrow \sqrt{N_F}$ and integrating out the fermion fields, the partition function is given by

$$Z = \int \mathcal{D}\phi \exp\left[-N_F\left(-\mathrm{Tr}\ln(\gamma_\mu\partial_\mu - y\phi) + \int d^4x[\frac{1}{2}\phi(-\Box + m^2)\phi + \frac{1}{24}\lambda\phi^4]\right)\right]\ .$$

The $\mathrm{N_F} \rightarrow \infty$ limit is a saddle point for the functional integral. The solution of the saddle point equations is equivalent to summing all Feyman diagrams with leading fermion bubbles which are proportional to $\mathrm{N_F}$.

Renormalization: Large $\mathrm{N_F}$ limit of Top-Higgs model

- We first consider a Higgs-Yukawa model of a single real scalar field coupled to N_F massless fermions in the exact $N_F \to \infty$ limit

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- Bare Lagrangian of the Higgs-Yukawa theory in Euclidean space-time is defined by

$$\mathcal{L} = \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{24}\lambda_0\phi_0^4 + \frac{1}{2}\left(\partial_\mu\phi_0\right)^2 + \bar{\psi}_0^a\left(\gamma_\mu\partial_\mu + y_0\phi_0\right)\psi_0^a,$$

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- where $a = 1, ..., N_F$ sums over the degenerate fermion flavors and the subscript 0 denotes bare quantities.
- We rewrite this as

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} m_0^2 Z_\phi \phi^2 + \frac{1}{24} \lambda_0 Z_\phi^2 \phi^4 + \frac{1}{2} Z_\phi \left(\partial_\mu \phi \right)^2 + Z_\psi \bar{\psi}^a \left(\gamma_\mu \partial_\mu + y_0 \sqrt{Z_\phi} \phi \right) \psi^a \\
&= \frac{1}{2} (m^2 + \delta m^2) \phi^2 + \frac{1}{24} (\lambda + \delta\lambda) \phi^4 + \frac{1}{2} (1 + \delta z_\phi) \left(\partial_\mu \phi \right)^2 \\
&\quad + (1 + \delta z_\psi) \bar{\psi}^a \gamma_\mu \partial_\mu \psi^a + \bar{\psi}^a (y + \delta y) \phi \psi^a,
\end{aligned}$$

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&= \frac{1}{2}(m^2+\delta m^2)\phi^2 + \frac{1}{24}(\lambda+\delta\lambda)\phi^4 + \frac{1}{2}(1+\delta z_\phi)\left(\partial_\mu\phi\right)^2 \\
&\quad +(1+\delta z_\psi)\bar{\psi}^a\gamma_\mu\partial_\mu\psi^a + \bar{\psi}^a(y+\delta y)\phi\psi^a,
\end{aligned}$$

- with wavefunction renormalization factors, renormalized parameters, and counterterms

$$Z_\phi = 1+\delta z_\phi, \quad Z_\psi = 1+\delta z_\psi\ ,$$

$$m_0^2 Z_\phi = m^2+\delta m^2, \qquad \lambda_0 Z_\phi^2 = \lambda+\delta\lambda, \qquad Z_\psi\sqrt{Z_\phi}y_0 = y+\delta y\ .$$

Renormalization: Large N_F limit of Top-Higgs model

- In the limit where N_F becomes large, all Feynman diagrams with Higgs loops are suppressed relative to those with fermion loops. Hence, two of the counterterms vanish,

Renormalization conditions (1,2)

$$\delta y = 0, \qquad \delta z_\psi = 0$$

as there are no radiative corrections to the fermion propagator or to the Higgs-fermion coupling.

- To maintain tree level relation $m^2 + \frac{1}{6}\lambda v^2 = 0$ to all orders:

Renormalization condition (3)

$$\delta m^2 + \frac{1}{6}\delta\lambda v^2 - 4N_F y^2 \int_k \frac{1}{k^2 + y^2 v^2} = 0$$

Renormalization: Large N_F limit of Top-Higgs model

- In the large N_F limit, the inverse propagator of the Higgs fluctuation $\varphi = \phi - v$ is

$$
\begin{aligned}
G^{-1}_{\varphi\varphi}(p^2) &= p^2 + m^2 + \frac{1}{2}\lambda v^2 + p^2 \delta z_\phi + \delta m^2 + \frac{1}{2}\delta\lambda v^2 - \Sigma(p^2) \\
\Sigma(p^2) &= -4N_F y^2 \int_k \frac{y^2 v^2 - k.(k-p)}{(k^2 + y^2 v^2)((k-p)^2 + y^2 v^2)}
\end{aligned}
$$

We impose the condition $G^{-1}_{\varphi\varphi}(p^2 \to 0) = p^2 + m_H^2$, which separates into two renormalization conditions:

Renormalization condition (4)

$$\delta m^2 + \frac{1}{2}\delta\lambda v^2 - \Sigma(p^2 = 0) = 0$$

Renormalization condition (5)

$$\delta z_\phi - \left.\frac{d\Sigma(p^2)}{dp^2}\right|_{p^2=0} = 0$$

Counterterms: Large N_F limit of Top-Higgs model

- Renormalization condition $\delta m^2 + \frac{1}{2}\delta\lambda v^2 - \Sigma(p^2 = 0) = 0$ maintains the tree-level relation $m_H^2 = m^2 + \frac{1}{2}\lambda v^2 = \frac{1}{3}\lambda v^2$ exactly. The counterterms precisely cancel all the finite and infinite radiative contributions.
- The Higgs mass defined as the zero-momentum piece of $G_{\varphi\varphi}^{-1}$ is identical to the curvature $U''_{\text{eff}}(v)$. True physical mass given by the pole of the propagator and the renormalized masses m_H and m can be related in perturbation theory.
- Renormalization conditions (3) and (4) yield

$$\begin{aligned} \delta m^2 &= 4N_F y^2 \int_k \frac{k^2 + 2y^2v^2}{(k^2 + y^2v^2)^2} \,, \\ \delta\lambda &= -24N_F y^4 \int_k \frac{1}{(k^2 + y^2v^2)^2} \,. \end{aligned}$$

  The closed form for δz_ϕ is more complicated and less illuminating.
- To demonstrate triviality , we use some finite cutoff in the momentum integrals and examine what occurs as this cutoff is removed. We will use a simple hard-momentum cutoff $|k| \leq \Lambda$. Exactly the same conclusions would be reached using instead e.g. Pauli-Villars regularization.

Counterterms: Large N_F limit of Top-Higgs model

- The non-zero counterterms are

$$\begin{aligned}
\delta m^2 &= \frac{N_F y^2}{2\pi^2}\left[\frac{1}{2}\Lambda^2 + \frac{y^4 v^4}{2(\Lambda^2 + y^2 v^2)} - \frac{1}{2} y^2 v^2\right] \\
\delta\lambda &= -\frac{3N_F y^4}{\pi^2}\left[\frac{y^2 v^2}{2(\Lambda^2 + y^2 v^2)} - \frac{1}{2} - \frac{1}{2}\ln\left(\frac{y^2 v^2}{\Lambda^2 + y^2 v^2}\right)\right] \\
\delta z_\phi &= -\frac{N_F y^2}{2\pi^2}\left[\frac{1}{4}\ln\left(\frac{y^2 v^2 + \Lambda^2}{y^2 v^2}\right) + \frac{-5\Lambda^4 - 3\Lambda^2 y^2 v^2}{12(\Lambda^2 + y^2 v^2)^2}\right].
\end{aligned}$$

- In the large N_F limit, the fermion inverse propagator receives no radiative correction, $G^{-1}_{\psi\psi}(p) = p_\mu\gamma_\mu + yv$, so we identify the fermion mass as $m_T = yv$.
- Because both δy and δz_ψ vanish, we can substitute$y^2 = Z_\phi y_0^2$

$$Z_\phi = \left[1 + \frac{N_F y_0^2}{8\pi^2}\left(\ln\left[\frac{\Lambda^2}{m_T^2}\right] - \frac{5}{3}\right)\right]^{-1} .$$

For any finite bare Yukawa coupling y_0, the Higgs wavefunction renormalization factor Z_ϕ vanishes logarithmically as the cutoff is removed, $m_T/\Lambda \to 0$.

This same logarithmic behavior will appear in all of the renormalized couplings, for any choice of bare couplings. Triviality: a finite cutoff must be kept to maintain non-zero interactions.

Triviality: Large $\mathrm{N_F}$ limit of Top-Higgs model

Explicitly, the renormalized Yukawa coupling is

$$
\begin{aligned}
y^2 &= y_0^2 Z_\phi = y_0^2 \left[1 + \frac{N_F y_0^2}{8\pi^2}\left(\ln\left[\frac{\Lambda^2}{m_T^2}\right] - \frac{5}{3}\right)\right]^{-1} \\
&\to \left[\frac{N_F}{8\pi^2}\ln\frac{\Lambda^2}{m_T^2}\right]^{-1}, \qquad \text{as } \frac{m_T}{\Lambda} \to 0.
\end{aligned}
$$

For the renormalized Higgs coupling, we have

$$
\begin{aligned}
\lambda &= \lambda_0 Z_\phi^2 - \delta\lambda = \lambda_0 Z_\phi^2 + \frac{3N_F y^4}{\pi^2}\left[\frac{m_T^2}{2(\Lambda^2 + m_T^2)} - \frac{1}{2} - \frac{1}{2}\ln\left(\frac{m_T^2}{\Lambda^2 + m_T^2}\right)\right] \\
&\to Z_\phi^2\left[\lambda_0 + \frac{3N_F y_0^4}{\pi^2}\left(-\frac{1}{2} - \frac{1}{2}\ln\frac{m_T^2}{\Lambda^2}\right)\right] \\
&\to 12\left[\frac{N_F}{8\pi^2}\ln\frac{\Lambda^2}{m_T^2}\right]^{-1}, \qquad \text{as } \frac{m_T}{\Lambda} \to 0.
\end{aligned}
$$

Triviality: Large N_F limit of Top-Higgs model

The slow logarithmic vanishing of y and λ enables a relatively large separation of cutoff and physical scales and still maintain significant interactions. Such a theory can in some limited sense be considered physical, if the cutoff effects are sufficiently small.

The ratio of the Higgs and fermion masses in the large N_F is

$$\frac{m_H^2}{m_T^2} = \frac{\lambda v^2}{3y^2 v^2} = \frac{\lambda}{3y^2} \rightarrow 4, \qquad \text{as } \frac{m_T}{\Lambda} \rightarrow 0.$$

Although completely unphysical, we can also consider the limit $m_T/\Lambda \gg 1$, where the cutoff is much below the physical scale. From Equation , we see this gives

$$\delta\lambda = 0, \qquad \delta z_\phi = 0,$$

and hence $Z_\phi \rightarrow 1$. In this limit, the connection between bare and renormalized parameters is simply

$$\lambda = \lambda_0, \qquad y = y_0.$$

This result is not surprising: deep in the cutoff regime, we simply have the bare theory, with no separation into renormalized parameters and their counterterms. This will be relevant when we discuss the Landau pole and the vacuum instability.

Renormalization Group flow: Large N_F limit of Top-Higgs model

The physical properties of the theory, at finite cutoff Λ are fixed by the choice of a complete set of bare parameters. Using the explicit cutoff dependence of y and λ, we can calculate the Callan-Symanzik flow of the renormalized couplings inn the $m_T/\Lambda \ll 1$ limit

$$\beta_y(y,\lambda) = \Lambda \frac{dy^2}{d\Lambda} = -y_0^2 Z_\phi^{-2} \frac{N_F y_0^2}{4\pi^2} = -\frac{N_F y^4}{4\pi^2}$$

$$\beta_\lambda(y,\lambda) = \Lambda \frac{d\lambda}{d\Lambda} = \frac{1}{16\pi^2}\left[-8N_F\lambda y^2 + 48 N_F y^4\right]$$

This is *exactly* the same RG flow one would calculate in the large N_F limit using e.g. dimensional regularization, where the cutoff simply does not appear and the renormalized couplings flow with the arbitrary renormalization scale μ. The overall signs of the β functions would be opposite: increasing Λ corresponds to decreasing μ).

We should expect this: when the cutoff is far above the physical scales, the finite cutoff effects are negligible and we must reproduce the unique cutoff-independent β functions.

However, as the cutoff is reduced and $\boxed{m_T/\Lambda}$ increases, this cannot continue to hold indefinitely, as the renormalized couplings must eventually flow to the bare ones!

Renormalization Group flow: Large $\mathbf{N_F}$ limit of Top-Higgs model

Let us demonstrate an explicit example of the Callan-Symanzik RG flow. In the large N_F limit, $m_T = yv = y_0 v_0$. The bare vev is determined by the minimum of the bare effective potential

$$U_{\mathrm{eff},0} = \frac{1}{2} m_0^2 \phi_0^2 + \frac{1}{24} \lambda_0 \phi_0^4 - 2N_F \int_k \ln\left[1 + y_0^2 \phi_0^2 / k^2\right].$$

Using a hard-momentum cutoff, this gives

$$m_0^2 + \frac{1}{6} \lambda_0 v_0^2 - \frac{N_F y_0^2}{2\pi^2} \left[\frac{1}{2}\Lambda^2 + \frac{1}{2} y_0^2 v_0^2 \ln\left(\frac{y_0^2 v_0^2}{\Lambda^2 + y_0^2 v_0^2}\right)\right] = 0.$$

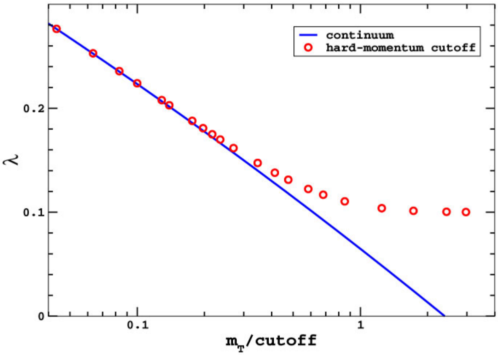
We express all dimensionful quantities in units of the cutoff Λ. We pick some fixed values for λ_0 and y_0. Varying the value of m_0^2/Λ^2 changes the solution v_0/Λ of Equation and hence the ratio m_T/Λ. As we said, choosing the values of the bare parameters completely determines everything in the theory. For example, to attain a very small value of m_T/Λ requires m_0^2/Λ to be **precisely fine tuned!** **This is the origin of the so-called fine-tuning problem.**

The critical surface, where $v_0/\Lambda = 0$, is the transition line

$$\frac{m_0^2}{\Lambda^2} - \frac{N_F y_0^2}{4\pi^2} = 0\,,$$

with all counterterms and renormalized parameters expressed in terms of λ_0, y_0, m_0^2 and v_0.

Renormalization Group flow: Large N_F limit of Top-Higgs model



running Higgs coupling (Holland,JK)

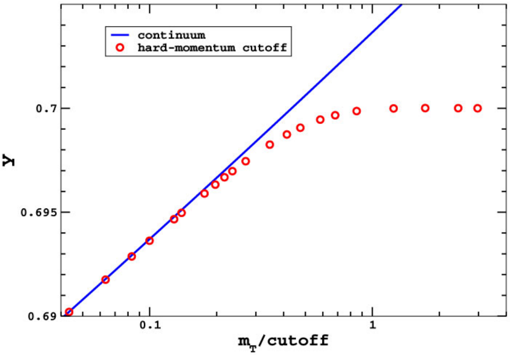
All of the counterterms and renormalized parameters can be expressed in terms of λ_0, y_0, m_0^2 and v_0.

We make an arbitrary choice $\lambda_0 = 0.1$, $y_0 = 0.7$ and vary the value of m_0^2/Λ^2 to explore the range $10^{-13} < m_T/\Lambda < 10^2$.

When the cutoff is high, the exact RG flow is exactly the same as if the cutoff had been completely removed and follows precisely the continuum form of Equation.

However, as the cutoff is reduced, the exact RG flow eventually breaks away from the continuum form and reaches a plateau at the value of the bare coupling.

Renormalization Group flow: Large N_F limit of Top-Higgs model



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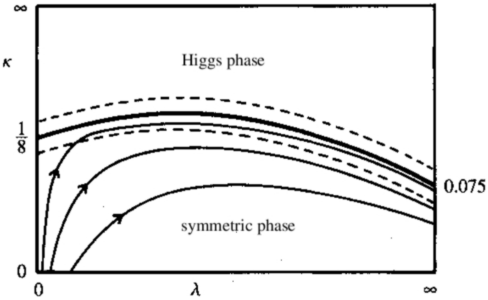
Higgs Mass Upper Bound: 1-component Higgs field

1-component lattice ϕ^4 model is characterized by two bare parameters κ, λ with lattice action

$$S = \sum_x \left[-2\kappa \sum_{\mu=1}^{4} \phi(x)\phi(x+\hat{\mu}) + \phi(x)^2 + \lambda(\phi(x)^2 - 1)^2 \right].$$

There are two phases separated by a line of critical points $\kappa = \kappa_c(\lambda)$. For $\kappa > \kappa_c(\lambda)$ the symmetry is spontaneously broken and the bare field $\phi(x)$ has a non-vanishing expectation value v.



Higgs Mass Upper Bound: 1-component Higgs field

Study the vacuum expectation value and the connected two-point function

$$G(x) = \langle \phi(x)\phi(0) \rangle_c = \langle \phi(x)\phi(0) \rangle - v^2 \,.$$

A renormalized mass m_r and a field wave function renormalization constant Z_r are defined through the behavior of Fourier transform of $G(x)$ for small momenta:

$$\tilde{G}(k)^{-1} = Z_r^{-1} \left\{ m_r^2 + k^2 + O(k^4) \right\} \,.$$

We also define the normalization constant associated with the canonical bare field through

$$\hat{Z}_r = 2\kappa Z_r = 2\kappa m_r^2 \chi \,.$$

In the framework of perturbation theory correlation functions of the multiplicatively renormalized field

$$\phi_r(x) = Z_r^{-1/2} \phi(x) \,,$$

have at all orders finite continuum limits after mass and coupling renormalization are taken into account. Correspondingly a renormalized vacuum expectation value is defined through $v_r = v Z_r^{-1/2}$.

Finally a particular renormalized coupling is defined by

$$g_r \equiv 3m_r^2 / v_r^2 = 3m_r^4 \chi / v^2 \,.$$

Higgs Mass Upper Bound: 1-component Higgs field

The renormalization group equations predict that the mass and vacuum expectation value go to zero according to

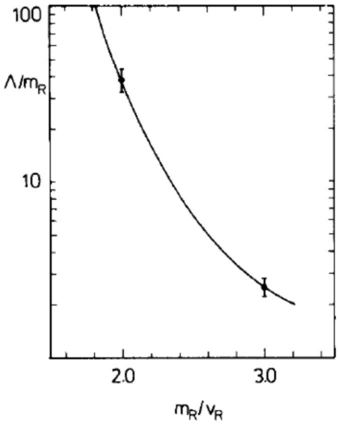
$$\begin{aligned} m_r &\approx \tau^{1/2}|ln(\tau)|^{-1/6}\,, \\ v_r &\approx \tau^{1/2}|ln(\tau)|^{1/3}\,, \end{aligned}$$

for $\tau = \kappa/\kappa_c - 1 \to 0$, and correspondingly the renormalized coupling is predicted to go to zero logarithmically which is the expression of triviality.
The critical behavior in the broken phase is conveniently expressed in terms of three integration constants C_i', $i = 1, 2, 3$ appearing in the critical behaviors:

$$\begin{aligned} m_r &= C_1'(\beta_1 g_r)^{17/27} e^{-1/\beta_1 g_r}\,\{1 + O(g_r)\}\,, \quad \beta_1 = \frac{3}{16\pi^2}\,, \\ Z_r &= C_2'\left\{1 - \frac{7}{36}\frac{g_r}{16\pi^2} + O(g_r^2)\right\}, \\ \kappa - \kappa_c &= \frac{1}{2}C_3' m_r^2 g_r^{-1/3}\,\{1 + O(g_r)\}\,. \end{aligned}$$

These constants were estimated by relating them to the corresponding constants C_i in the symmetric phase. These in turn were computed by integrating the renormalization group equations with initial data on the line $\kappa = 0.95\kappa_c(\lambda)$ obtained from high temperature expansions.

Higgs Mass Upper Bound: 1-component Higgs field



One massive Higgs particle

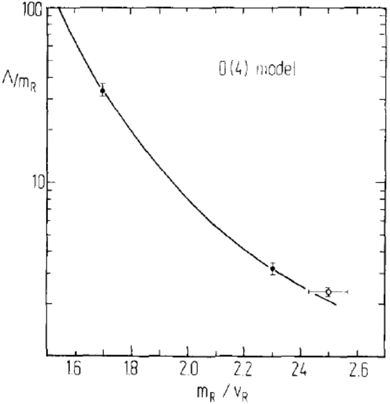
analytic results: Lüscher, Weisz

lattice simulations: JK,Lin,Shen

$$\boxed{\frac{m_R}{v} \approx 3.2 \text{ at } am_R = 0.5}$$

How far can we lower the cutoff?

Higgs Mass Upper Bound: 4-component O(4) Higgs model



One massive Higgs particle +
3 Goldstone particles

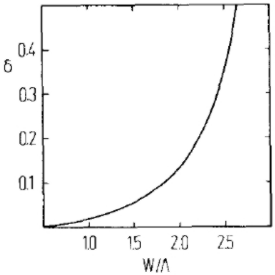
analytic results: Lüscher, Weisz

lattice simulations: JK,Lin,Shen

$$\boxed{\frac{m_R}{v} \approx 2.6 \text{ at } am_R = 0.5}$$

How far can we lower the cutoff?

Higgs Mass Upper Bound: 4-component O(4) Higgs model



δ measures cutoff effects in Goldstone-Goldstone scattering

Lüscher, Weisz

Ad hoc and connected with lattice artifacts

But threshold of new physics is in the continuum!

$\boxed{\textbf{What do we do when cutoff is low?}}$

Continuum Wilsonian RG

UV Completion

unknon new physics

UV Completion

unknon new physics

Below new scale M integrated UV completion is represented by non-local $\mathcal{L}_{\rm eff}$ with all higher dimensional operators,

$$\frac{1}{M^2}\phi\Box^2\phi,\ \frac{1}{M^4}\phi\Box^3\phi,\ \frac{\lambda_6}{M^2}\phi^6,\ \ldots$$

Propagator $\frac{K(p^2/M^2)}{p^2+M^2}$ with analytic K thins out UV completion with exponential damping

---

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Narrow definition of Standard Model: only relevant and marginal operators at scale M

Higgs Physics and the Lattice

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Lattice Wilsonian RG

Regulate with lattice at scale $\Lambda = \pi/a$
$\mathcal{L}_{\text{lattice}}$ has all higher dimensional operators

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$\boxed{\text{Scale M missing?}}$

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Scale M missing?

Possible to insert intermediate continuum scale M with $\mathcal{L}_{\text{eff}}$ to include

$$\frac{1}{M^2}\phi\Box^2\phi,\ \frac{1}{M^4}\phi\Box^3\phi,\ \frac{\lambda_6}{M^2}\phi^6,\ \ldots$$

to represent new degreese of freedom above M

or, Lee-Wick and other UV completions

which exist above scale M (not effective theories!)

At the physical Higgs scale $v = 250\ GeV$ only relevant and marginal operators survive
Only $\frac{1}{2}m_H^2\phi^2$ and $\lambda\phi^4$ terms in $V_{\text{Higgs}}(\phi)$, in addition to $(\nabla\phi)^2$ operator
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Example for UV Completion: Higer derivative (Lee-Wick) Higgs sector

Jansen,JK,Liu, Phys. Lett. B309 (1993) p.119 and p.127

(Model was recently reintroduced by Grinstein, O'Connell, Wise in arXiv:0704.1845)

- Represent the Higgs doublet with four real components ϕ^a which transform in the vector representation of $O(4)$ and include new higher derivative terms in the kinetic part of the $O(4)$ Higgs Lagrangian,

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{\cos(2\Theta)}{M^2}\,\Box\phi^a\,\Box\phi^a + \frac{1}{2M^4}\,\Box\partial_\mu\phi^a\,\Box\partial^\mu\phi^a - V(\phi^a\phi^a)$$

- The Higgs potential is $V(\phi^a\phi^a) = -\frac{1}{2}\mu^2\phi^a\phi^a + \lambda(\phi^a\phi^a)^2$.
- The higher derivative terms of the Lagrangian lead to complex conjugate ghost pairs in the spectrum of the Hamilton operator
- Complex conjugate pairs of energy eigenvalues and the related complex pole pairs in the propagator are parametrized by $\mathcal{M} = Me^{\pm i\Theta}$. Choice $\Theta = \pi/4$ simplifies.
- The absolute value M of the complex ghost mass $\mathcal{M}$ will be set on the TeV scale
- **Unitary S-matrix, macroscopic causality, Lorent invariance?**

$\lambda > 0$ asymptotically !

Vacuum instability ?

Example for UV Completion: Gauged Lee-Wick extension

- Higher derivative Yang-Mills gauge Lagrangian for the $SU(2)_L \times U(1)$ weak gauge fields W_μ, B_μ follows similar construction with covariant derivative $D^{ab}_\mu = \delta^{ab}\partial_\mu + gf^{abc}W^c_\mu$,

$$\mathcal{L}_{\mathrm{W}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4M^4}D^2 G^a_{\mu\nu}D^2 G^{a\mu\nu} \,,$$

- $\mathcal{L}_{\mathrm{W}}$ is superrenormalizable but not finite.
- Full gauged Higgs sector is described by the Lagrangian $\mathcal{L} = \mathcal{L}_W + \mathcal{L}_B + \mathcal{L}_{\mathrm{Higgs}}$,

$$\mathcal{L}_{Higgs} = (D_\mu\Phi)^\dagger D^\mu\Phi + \frac{1}{2M^4}(D_\mu D^\dagger D\Phi)^\dagger (D_\mu D^\dagger D\Phi) - V(\Phi^\dagger\Phi)$$

- Gauge-covariant derivative is $D_\mu\Phi = \left(\partial_\mu + i\frac{g}{2}\sigma\cdot W_\mu + i\frac{g'}{2}B_\mu\right)\Phi$.
- Similar fermion construction: $\mathcal{L}_{\mathrm{fermion}} = i\overline{\Psi}\not{D}\,\Psi + \frac{i}{2M^4}\overline{\Psi}\ \not{D}^2\ \not{D}\ \not{D}^2\ \Psi$.

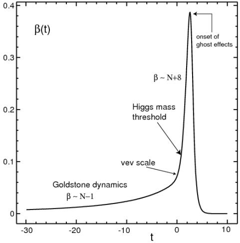
SM particle content is doubled

Logarithmic divergences only

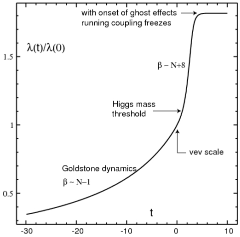
Higher derivative β-function and RG: Lee-Wick UV Completion

Liu's Thesis (1994)

β function:



running Higgs coupling:

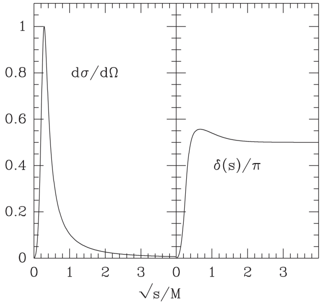


- **Higher derivative Higgs sector is finite field theory**
- **Mass dependent $\beta(t)$-function vanishes asymptotically**
- **Grows logarithmically in gauged Higgs sector**
- **Running Higgs coupling $\lambda(t)$ freezes asymptotically**
- **The fixed line of allowed Higgs couplings must be positive!**
- **Vacuum instability?**
  Higgs mass lower bound from $\lambda(\infty) > 0$

S-matrix, Unitarity, and Causality: Lee-Wick UV Completion

Liu,Jansen,JK
Nucl.Phys. B34 (1994) p.635

cross section phase shift



m_H=1 TeV, M = 3.6 TeV

v/M=0.07, m_H/M = 0.28

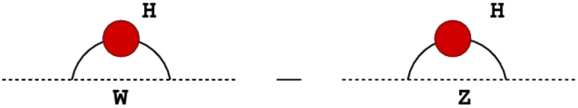
- **Equivalence theorem, Goldstone scattering**
- **Higgs mass upper bound relaxed**
- **m_H=1 TeV, or higher,** but ρ-parameter and other Electroweak precision?
- **Phase shift reveals ghost**, microscopic time advancement, only $\pi/2$ jump in phase shift

What about the ρ-parameter?

The ρ-parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 + \frac{Z^{(+)}}{Z^{(0)}}$$

$\cos\theta_W$ is determined independently in high precision lepton processes. The 1-loop vacuum polarization operator will shift the physical pole locations for the weak gauge bosons:



$$\rho - 1|_{\text{Higgs}} = \frac{\Pi_W^H}{M_{W,tree}^2} - \frac{\Pi_Z^H}{M_{Z,tree}^2}$$

$$= -\frac{3}{4} g'^2 \int_{k^2<\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\Sigma_H(k^2)}{(k^2 + M_{W,tree}^2)(k^2 + M_{Z,tree}^2)(k^2 + \Sigma_H^2(k^2))}$$

Heavy Higgs with acceptable ρ-parameter would be a broad resonance

How to calculate Higgs resonance on the lattice?

Higgs resonance: Finite width in finite volume?

- Energy spectrum of two-particle states in a finite box with periodic boundary conditions
  $\Longrightarrow$ elastic scattering phase shifts in infinite volume

Higgs resonance: Finite width in finite volume?

- Energy spectrum of two-particle states in a finite box with periodic boundary conditions $\Longrightarrow$ elastic scattering phase shifts in infinite volume
- Two-particle energy levels are calculable by Monte Carlo techniques $\Longrightarrow$ extract phase shifts from numerical simulations on finite lattices

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- Infinite bare λ limit $\Longrightarrow$ 4-dimensional $O(4)$ non-linear σ-model in broken phase with unstable Higgs particle
- Small Goldstone mass is required by the method $\Longrightarrow$ add external source term to the lattice action of the 4-dimensional $O(4)$ non-linear σ-model:

$$S = -2\kappa \sum_x \sum_{\mu=1}^{4} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha + J \sum_x \phi_x^4$$

The scalar field is represented as a four component vector ϕ_x^α of unit length: $\phi_x^\alpha \phi_x^\alpha = 1$

Two Goldstone bosons ("pions") of mass m_π with zero total momentum in a cubic box of size L^3 in the elastic region are characterised by centre-of-mass energies W or momenta $\vec{k}$ defined through $W = 2\sqrt{m_\pi^2 + \vec{k}^2}$, $k = |\vec{k}|$ with W and k in the ranges:

$2m_\pi < W < 4m_\pi \quad \Leftrightarrow \quad 0 < k/m_\pi < \sqrt{3}$

They are classified according to irreducible representations of the cubic group. Their discrete energy spectrum W_j, $j = 0, 1, 2, \ldots$, is related to the scattering phase shifts δ_l with angular momenta l which are allowed by the cubic symmetry of the states

Higgs resonance: Finite width in finite volume?

In the subspace of cubically invariant states only angular momenta $l = 0, 4, 6, \ldots$ contribute. Due to the short-range interaction it seems reasonable to neglect all $l \neq 0$. Then an energy value W_j belongs to the two-particle spectrum, if the corresponding momentum $k_j = \sqrt{(W_j/2)^2 - m_\pi^2}$ is a solution of

$$\delta_0(k_j) = -\phi\left(\frac{k_j L}{2\pi}\right) + j\pi$$

The continuous function $\phi(q)$ is given by

$$\tan\left(-\phi(q)\right) = \frac{q\pi^{3/2}}{\mathcal{Z}_{00}(1, q^2)} \quad , \quad \phi(0) = 0$$

with $\mathcal{Z}_{00}(1, q^2)$ defined by analytic continuation of

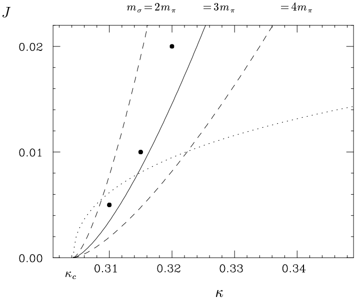
$$\mathcal{Z}_{00}(s, q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbf{Z}^3} \left(\vec{n}^{\,2} - q^2\right)^{-s} .$$

This result holds only for $0 < q^2 < 9$, which is fulfilled in our simulations, and provided finite volume polarisation effects and scaling violations are negligible

In our Monte-Carlo investigation we calculate the momenta k_j corresponding to the two-particle energy spectrum W_j for a given size L, and can then read off the scattering phaseshift $\delta_0(k_j)$ from its defining equation In order to scan the momentum dependence of the phase shift we have to vary the spatial extent L of the lattice

Higgs resonance: Finite width in finite volume?

- The two-particle states in our model are also classified according to the remaining $O(3)$-symmetry: They have "isospin" 0, 1, 2. Since we expect the σ-resonance to be in the isospin-0 channel, we restrict ourselves to that case.
- Operator $\mathcal{O}_0(t) = \tilde{\Phi}^4_{\vec{0},t} = L^{-3}\sum_{\vec{x}} \Phi^4_{\vec{x},t}$ for σ-field at zero momentum is included
- Two-particle correlation matrix function
  $C_{ij}(t) = \left\langle \left(\mathcal{O}_i(t) - \mathcal{O}_i(t+1)\right) \mathcal{O}_j(0) \right\rangle \; i, j = 0, 1, 2, \ldots$
- Eigenvalues decaying as $\exp(-W_i t)$
- The set of states is truncated at finite $i = r$ to keep W_r below inelastic threshold



- Lines of constant ratios m_σ/m_π and $m_\pi L$ (dotted line)
- Reflection multi-cluster algorithm for finite external source J
- Set of 3 κ, J configurations with middle point $\kappa = 0.315$, $J = 0.01$ tuned to keep σ-mass in elastic region
- Cylindrical lattices $L^3 \cdot T$ up to sizes of $24^3 \cdot 32$, $32^3 \cdot 40$

Higgs resonance: Finite width in finite volume?

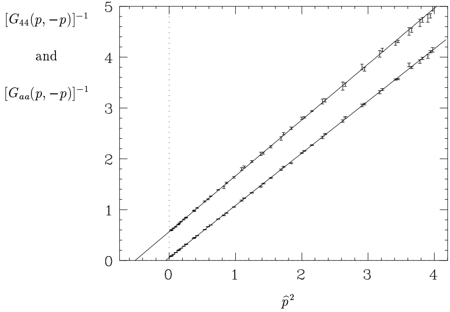
- The pion mass m_π can be measured by a fit to the inverse propagator in four-dimensional momentum space:

$$G_{aa}^{-1}(p,-p) = Z_\pi^{-1}\{m_\pi^2 + \hat{p}^2\},\ \mathrm{a} = 1, 2, 3$$

- For perturbation theory checks we also want m_σ and λ_R,

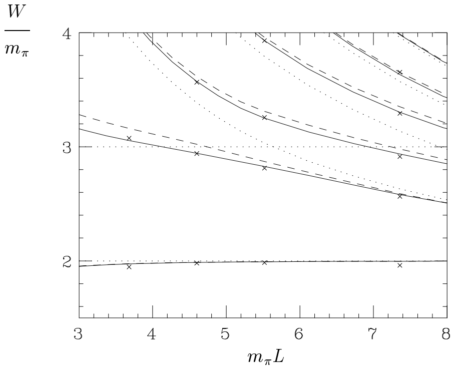
$$\lambda_R = 3Z_\pi \frac{m_\sigma^2 - m_\pi^2}{\Sigma^2}$$

  where Σ is the infinite volume σ field expectation value
- Two-parameter fit to the inverse σ and π propagators in momentum space
- $32^3 \cdot 40$ lattice at $\kappa = 0.315, J = 0.01$



- The negative intercept and slope of the inverse Goldstone propagator $G_{aa}^{-1}(p,-p)$ determine the propagator mass m_π
- The negative intercept and slope of the inverse σ propagator $G_{44}^{-1}(p,-p)$ determine the propagator mass m_σ for the Higgs particle

Higgs resonance: Two-particle states



- Comparison of simulation results (crosses) with perturbative prediction in isospin 0 channel at $\kappa = 0.315, J = 0.01$
- Solid lines depict perturbative prediction (dashed lines perturbative estimates based on propagator masses)
- Dotted lines represent free pion pairs
- The location of the resonance energy m_σ is indicated by dotted horizontal line at $W \approx 3m_\pi$

Higgs resonance: Phase shift in perturbation theory

1. We will use the linear model for the phase shift calculation
   - $S[\phi; j, m^2] = \int d^4x \left[\frac{1}{2} \partial_\mu \phi^\alpha \partial_\mu \phi^\alpha + -\frac{1}{2} m^2 \phi^\alpha \phi^\alpha + \frac{\lambda}{4!} (\phi^\alpha \phi^\alpha)^2 - j\phi^N \right]$
   - with $\alpha = 1, ..., N$ and $N = 4$ in our case.
   - The mass parameter m^2 is negative in broken phase

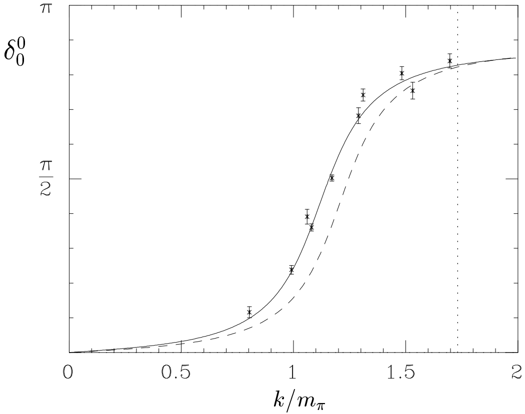
2. Pions have "isospin" index a=1,...N-1 (single pion state isospin is I=1 for N=4)
   - Two-particle state decomposes into I=0,1,2 irreducible representations
   - Partial wave decomposition $T^I = \frac{16\pi W}{k} \sum_{l=0}^{\infty} (2l+1) P_l(cos\vartheta) t_l^I$
   - Bose symmetry requires $t_l^I = 0$ if I+l is odd
   - $t_l^I = \frac{1}{2i}\left(e^{2i\delta_l^I} - 1\right)$ with real phase shifts δ_l^I in elastic region $2m_\pi < W < 4m_\pi$

3. Leading perturbative result:

$$\delta_0^0 = \lambda_R \frac{N-1}{48\pi} \frac{k}{W} \left(1 - \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - W}\right) + \delta_0^2$$

$$\delta_0^2 = \frac{\lambda_R}{96\pi} \frac{m_\sigma^2 - m_\pi^2}{kW} \left(1 + \frac{m_\sigma^2}{2k^2}\right) \ln\left(\frac{4k^2 + m_\sigma^2}{m_\sigma^2}\right) - \frac{\lambda_R}{48\pi} \frac{m_\sigma^2 - m_\pi^2}{kW}$$

Higgs resonance: Phase shift



Göckeler et al.

$am_\sigma = 0.72$

$m_\sigma = 3.07\ m_\pi$

$\Gamma_\sigma = 0.18\ m_\sigma$

Agreement with perturbation theory