Higgs Physics from the Lattice

Lecture 1: Standard Model Higgs Physics

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University of California, San Diego

INT Summer School on ”Lattice QCD and its applications”
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Outline of Lecture Series: Higgs Physics from the Lattice

1. **Standard Model Higgs Physics**
    - Outlook for the Higgs Particle
    - Standard Model Review
    - Expectations from the Renormalization Group
    - Nonperturbative Lattice Issues?

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   - Renormalization Group and Triviality in Lattice Higgs and Yukawa couplings
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3. **Vacuum Instability and Higgs Mass Lower Bound**
   - Vacuum Instability and Triviality in Top-Higgs Yukawa Models
   - Chiral Lattice Fermions
   - Top-Higgs and Top-Higgs-QCD sectors with Chiral Lattice Fermions
   - Higgs mass lower bound
   - Running couplings in 2-loop continuum Renormalization Group

The New York Times

Tuesday, May 15, 2007

A Giant Takes On Physics' Biggest Questions

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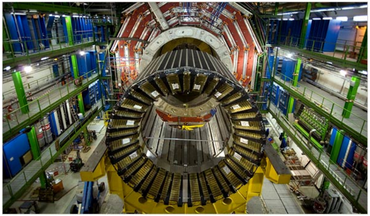

"The physicists are scrambling like Spiderman over this assembly, appropriately named Atlas, getting ready to see the universe born again. According to the Standard Model, the Higgs can have only a limited range of masses without severe damage to the universe. If it is too light, the universe will decay. If it is too heavy, the universe would have blown up already...

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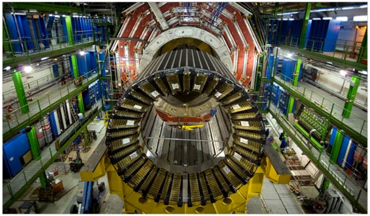

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1. Planck scale → magic value connection

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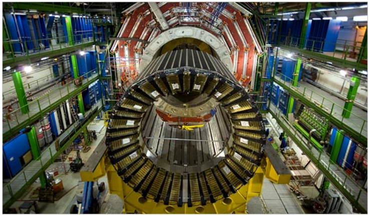

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2. **What if UV completion in TeV range?**

Some comments on the lectures

1. **Cutoff plays a very different role in Standard Model Higgs Physics**
   - Not like QCD where cutoff is your enemy
   - In Standard Model cutoff represents the threshold of new physics
   - New physics is not hypercubic, cutoff is your enemy AND friend
   - Is there a way to do this right on the lattice?

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4. **Lectures, Notations**
   - Level between university lecture course and review talks
   - Notations are entirely uniform throughout the lectures
   - $\pm\frac{1}{2}m^2\phi^2$, $m^2 \to \mu^2$, and $\frac{\lambda}{4!}\phi^4$ are examples (and should not confuse)

Standard Model: Outlook for the Higgs particle

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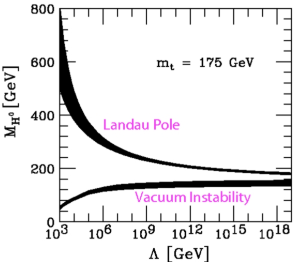

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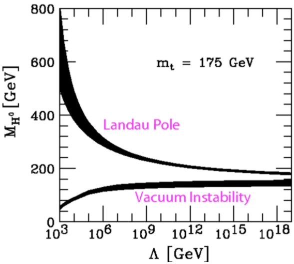

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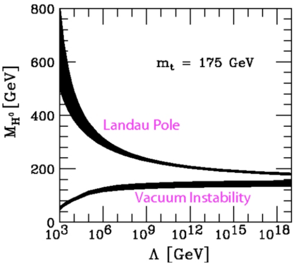

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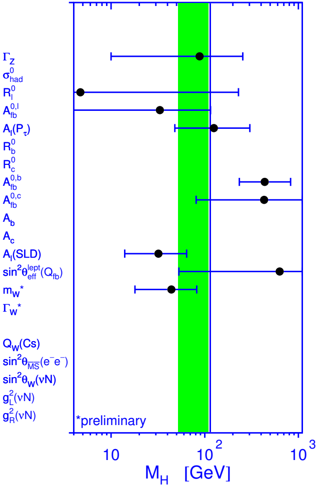
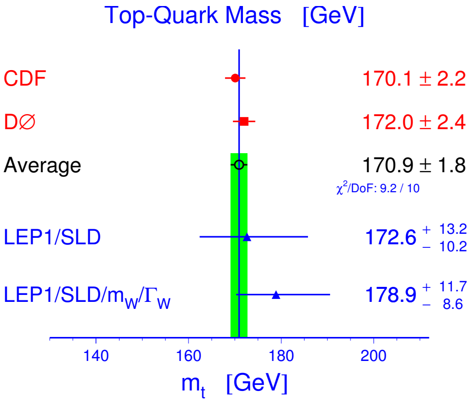
Should this PDG figure be reworked nonperturbatively? for low cutoff Λ

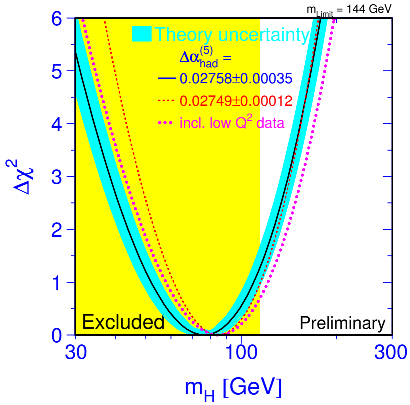
Effects of new Higgs physics from UV completion?

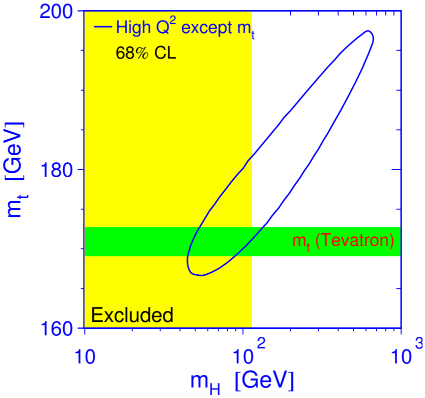
Standard Model: Separate Higgs and Top mass fits





Standard Model: Correlated Higgs and Top mass fits

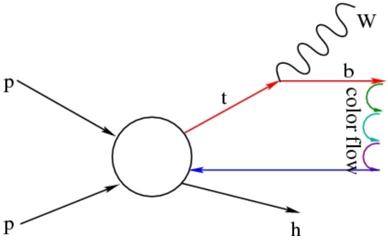




Significant Higgs 1-loop corrections growing like the logarithm of m_H affect 6 amplitudes with external vector bosons only

Two each for 2,3,4 point functions. Only 2-point function is inderictly accessible experimentally today.

Standard Model: Top quark pole mass on the lattice



- Top quark capture in BW channel
- Experimentalists measure the pole mass
- Color backflow correction (1-2 GeV uncertainty?)
- Lattice work is in small finite box $L \cdot \Lambda_{QCD} \ll 1$
- Running $g_{QCD}(L)$ is close to M_{Top} scale in small QCD world (L large for Top-Higgs dynamics)
- In lattice simulation measure Top quark pole mass or propagator mass

Standard Model: Three families of quarks and leptons

- Standard Model is gauge theory with symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ of strong, weak and electromagnetic interactions
- Forces are exchanged via spin-1 gauge fields: eight massless gluons, one massless photon, and three massive weak bosons, $W^{\pm}$ and Z

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- Leptons and quarks are organized in a three-fold family structure:

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

- with $SU(2)_L$ doublets and singlets in each family (quark in three colours):

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , \; l_R^- , \; q_{uR} , \; q_{dR}$$

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- Spontaneous symmetry breaking in vacuum from electroweak gauge group to electromagnetic gauge group generates Higgs mechanism:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\text{SSB}} SU(3)_c \otimes U(1)_{\text{QED}}$$

Standard Model: $\mathrm{SU(3)_c}$ and QCD

- Free quark Lagrangian $\boxed{\mathcal{L}_0 = \sum_f \bar{q}_f \left(i\gamma^\mu \partial_\mu - m_f\right) q_f}$ is invariant under arbitrary *global* $SU(3)_c$ transformations $q_f^\alpha \longrightarrow (q_f^\alpha)' = U^\alpha{}_\beta \, q_f^\beta$
- Require now the Lagrangian to be also invariant under *local* $SU(3)_c$ transformations, $\theta_a = \theta_a(x)$ (generalization of QED)
- We need to change the quark derivatives by covariant objects with eight independent gauge parameters and eight different gauge bosons $G_a^\mu(x)$

$$D^\mu q_f \equiv \left[\partial^\mu + i g_s \frac{\lambda^a}{2} G_a^\mu(x)\right] q_f \equiv \left[\partial^\mu + i g_s G^\mu(x)\right] q_f$$

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- To build gauge-invariant kinetic term for gluon fields, we introduce the corresponding field strengths:

$$G_a^{\mu\nu}(x) = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu$$

Taking the proper normalization for the gluon kinetic term, we get the $SU(3)_c$ invariant Lagrangian of Quantum Chromodynamics:

$$\mathcal{L}_{\text{QCD}} \equiv -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_f \bar{q}_f \left(i\gamma^\mu D_\mu - m_f\right) q_f$$

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detailed form of the Lagrangian:

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& - g_s\, G_a^\mu \sum_f \bar{q}_f^\alpha\, \gamma_\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^\beta \\
& + \frac{g_s}{2} f^{abc}\,(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu)\, G_\mu^b\, G_\nu^c - \frac{g_s^2}{4} f^{abc} f_{ade}\, G_b^\mu\, G_c^\nu\, G_\mu^d\, G_\nu^e\,.
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- First line in Lagrangian: kinetic terms $\Longrightarrow$ propagators
- Second line: color interaction between quarks and gluons with $SU(3)_c$ matrices λ^a
- Third line: $G_a^{\mu\nu} G_{\mu\nu}^a$ term generates cubic and quartic gluon self-interactions (same coupling g_s appears in all parts of the Lagrangian)

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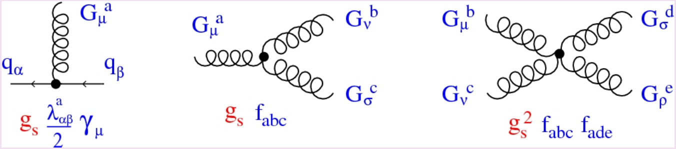

Standard Model: $\mathbf{SU(2)_L \otimes U(1)_Y}$ sector

- The symmetry group to be gauged is $G \equiv SU(2)_L \otimes U(1)_Y$
  L refers to left-handed fields and Y is the weak hypercharge
- For simplicity, single family of quarks

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L , \qquad \psi_2(x) = u_R , \qquad \psi_3(x) = d_R$$

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- As in QCD case, we start from the free Lagrangian:

$$\mathcal{L}_0 = i\,\bar{u}(x)\,\gamma^\mu\,\partial_\mu u(x) + i\,\bar{d}(x)\,\gamma^\mu\,\partial_\mu d(x) = \sum_{j=1}^{3} i\,\overline{\psi}_j(x)\,\gamma^\mu\,\partial_\mu \psi_j(x)$$

Standard Model: $\mathrm{SU(2)_L \otimes U(1)_Y}$ sector

- $\mathcal{L}_0$ is invariant under global G transformations in flavour space:

$$\begin{aligned}
\psi_1(x) &\xrightarrow{G} \psi_1'(x) \equiv \exp\{iy_1\beta\}\, U_L\, \psi_1(x)\,,\\
\psi_2(x) &\xrightarrow{G} \psi_2'(x) \equiv \exp\{iy_2\beta\}\, \psi_2(x)\,,\\
\psi_3(x) &\xrightarrow{G} \psi_3'(x) \equiv \exp\{iy_3\beta\}\, \psi_3(x)
\end{aligned}$$

- The $SU(2)_L$ transformation

$$U_L \equiv \exp\left\{i\,\frac{\sigma_i}{2}\,\alpha^i\right\} \qquad (i = 1,2,3)$$

  only acts on the doublet field ψ_1
- The parameters y_i are called hypercharges, since the $U(1)_Y$ phase transformation is analogous QED
- The matrix transformation U_L is non-Abelian as in QCD
- Notice that we have not included a mass term because it would mix the left- and right-handed fields spoiling symmetry considerations

Standard Model: $\mathrm{SU(2)_L \otimes U(1)_Y}$ sector

- We can now require the Lagrangian to be also invariant under local $SU(2)_L \otimes U(1)_Y$ gauge transformations with $\alpha^i = \alpha^i(x)$ and $\beta = \beta(x)$
- In order to satisfy this symmetry requirement, we need to change the fermion derivatives by covariant objects and for four gauge parameters, $\alpha^i(x)$ and $\beta(x)$, four different gauge bosons are needed:

$$
\begin{aligned}
D_\mu \psi_1(x) &\equiv \left[\partial_\mu + i\, g\, \widetilde{W}_\mu(x) + i\, g'\, y_1\, B_\mu(x)\right] \psi_1(x)\,, \\
D_\mu \psi_2(x) &\equiv \left[\partial_\mu + i\, g'\, y_2\, B_\mu(x)\right] \psi_2(x)\,, \\
D_\mu \psi_3(x) &\equiv \left[\partial_\mu + i\, g'\, y_3\, B_\mu(x)\right] \psi_3(x)
\end{aligned}
$$

where $\widetilde{W}_\mu(x) \equiv \frac{\sigma_i}{2} W^i_\mu(x)$ is $SU(2)_L$ matrix field

- Thus we have the correct number of gauge fields to describe the $W^\pm$, Z and γ gauge bosons

$\mathcal{L} = \sum_{j=1}^{3} i\, \overline{\psi}_j(x)\, \gamma^\mu D_\mu \psi_j(x)$ is invariant under local G transformations

Standard Model: $\mathbf{SU(2)_L \otimes U(1)_Y}$ sector

In order to build the gauge-invariant kinetic term for the electroweak gauge fields, we introduce the corresponding field strengths:

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu , \qquad W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\epsilon^{ijk}\,W^j_\mu\,W^k_\nu , \qquad \widetilde{W}_{\mu\nu} \equiv \frac{\sigma_i}{2}\,W^i_{\mu\nu}$$

$B_{\mu\nu}$ remains invariant under G transformations, while $\widetilde{W}_{\mu\nu}$ transforms covariantly:

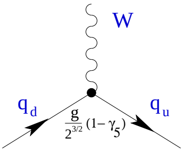
$$B_{\mu\nu} \xrightarrow{G} B_{\mu\nu} , \qquad \widetilde{W}_{\mu\nu} \xrightarrow{G} U_L\,\widetilde{W}_{\mu\nu}\,U_L^\dagger$$

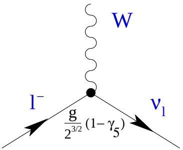
Properly normalized kinetic Lagrangian is given by

$$\mathcal{L}_{\text{Kinetic}} = -\frac{1}{4}\,B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{2}\,\mathrm{Tr}\left[\widetilde{W}_{\mu\nu}\,\widetilde{W}^{\mu\nu}\right] = -\frac{1}{4}\,B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{4}\,W^i_{\mu\nu}\,W_i^{\mu\nu} .$$

- Since the field strengths $W^i_{\mu\nu}$ contain a quadratic piece, the Lagrangian $\mathcal{L}_{\text{Kinetic}}$ gives rise to cubic and quartic self-interactions among the gauge fields
- The strength of these interactions is given by the same $SU(2)_L$ coupling g which appears in the fermionic piece of the Lagrangian
- The $SU(2)_L \otimes U(1)_Y$ Lagrangian only contains massless fields

Standard Model: Charged currents in $SU(2)_L \otimes U(1)_Y$ sector





The Electroweak Lagrangian contains interactions of the fermion fields with the gauge bosons:

$$\mathcal{L} \quad \longrightarrow \quad -g\,\overline{\psi}_1\gamma^\mu \widetilde{W}_\mu \psi_1 \;-\; g'\,B_\mu \sum_{j=1}^{3} y_j\,\overline{\psi}_j\gamma^\mu\psi_j$$

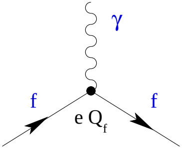
$SU(2)_L$ matrix $\widetilde{W}_\mu \;=\; \frac{\sigma^i}{2}\,W^i_\mu \;=\; \frac{1}{2}\begin{pmatrix} W^3_\mu & \sqrt{2}\,W^\dagger_\mu \\ \sqrt{2}\,W_\mu & -W^3_\mu \end{pmatrix}$ generates charged-current interactions

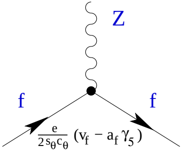
with the boson field $W_\mu \equiv (W^1_\mu + i\,W^2_\mu)/\sqrt{2}$

For a single family of quarks and leptons:

$$\mathcal{L}_{\rm CC} \;=\; -\frac{g}{\sqrt{2}}\left\{W^\dagger_\mu\;[\bar{u}\gamma^\mu(1-\gamma_5)d \;+\; \bar{\nu}_e\gamma^\mu(1-\gamma_5)e] \;+\; \text{h.c.}\right\}$$

Standard Model: Z and γ in $SU(2)_L \otimes U(1)_Y$ sector





Mixing of the neutral gauge fields W_μ^3 and B_μ generates the Z boson and the photon γ:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

The neutral-current Lagrangian is given by

$$\mathcal{L}_{NC} = -\sum_j \bar{\psi}_j \gamma^\mu \left\{ A_\mu \left[g\,\frac{\sigma_3}{2}\,\sin\theta_W + g'\,y_j\,\cos\theta_W \right] + Z_\mu \left[g\,\frac{\sigma_3}{2}\,\cos\theta_W - g'\,y_j\,\sin\theta_W \right] \right\} \psi_j\,.$$

In order to get QED from the A_μ piece, one needs to impose the conditions:

$$g\,\sin\theta_W = g'\,\cos\theta_W = e\,, \qquad Y = Q - T_3\,,$$

where $T_3 \equiv \sigma_3/2$ and Q denotes the electromagnetic charge operator

$$Q_1 \equiv \begin{pmatrix} Q_{u/\nu} & 0 \\ 0 & Q_{d/e} \end{pmatrix}, \qquad Q_2 = Q_{u/\nu}\,, \qquad Q_3 = Q_{d/e}$$

Standard Model: Z and γ in $\mathrm{SU(2)_L \otimes U(1)_Y}$ sector

Fermion hypercharges in terms of their electric charge and weak isospin:

Quarks: $y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} = \frac{1}{6}\,,\qquad y_2 = Q_u = \frac{2}{3}\,,\qquad y_3 = Q_d = -\frac{1}{3}$

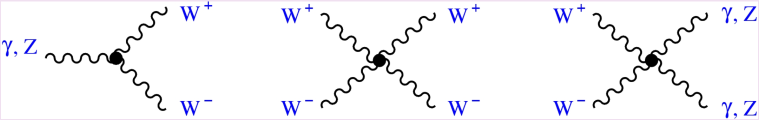
Leptons: $y_1 = Q_\nu - \frac{1}{2} = Q_e + \frac{1}{2} = -\frac{1}{2}\,,\qquad y_2 = Q_\nu = 0\,,\qquad y_3 = Q_e = -1$

Neutral-current Lagrangian can be written as $\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}^Z_{\rm NC}\,,$

$$\mathcal{L}_{\rm QED} = -e\,A_\mu \sum_j \overline{\psi}_j \gamma^\mu Q_j \psi_j \equiv -e\,A_\mu\,J^\mu_{\rm em}, \qquad \mathcal{L}^Z_{\rm NC} = -\frac{e}{\sin\theta_W \cos\theta_W}\,J^\mu_Z\,Z_\mu$$

$$J^\mu_Z \equiv \sum_j \overline{\psi}_j \gamma^\mu \left(\sigma_3 - 2\sin^2\theta_W Q_j\right)\psi_j = J^\mu_3 - 2\sin^2\theta_W\,J^\mu_{\rm em}$$

Weak boson cubic and quartic self-interactions are generated (like in QCD):

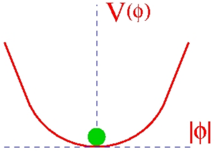


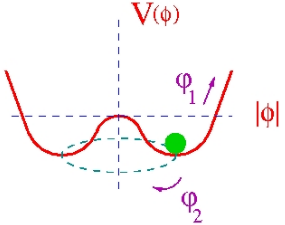
Standard Model: Spontaneous symmetry breaking

Let us consider a complex scalar field $\phi(x)$, with the Lagrangian:

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi) , \qquad V(\phi) = \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$\mathcal{L}$ is invariant under global phase transformations of the scalar field:
$\phi(x) \longrightarrow \phi'(x) \equiv \exp\{i\theta\}\, \phi(x)$





In order to have a ground state the potential should be bounded from below, i.e., $h > 0$

1. $\boldsymbol{\mu^2 > 0}$: The potential has only the trivial minimum $\phi = 0$. It describes a massive scalar particle with mass μ and quartic coupling λ

2. $\boldsymbol{\mu^2 < 0}$: The minimum is obtained for those field configurations satisfying

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} > 0 , \quad V(\phi_0) = -\frac{\lambda}{4} v^4$$

Owing to the $U(1)$ phase-invariance of the Lagrangian, there is an infinite number of degenerate states of minimum energy $\phi_0(x) = \frac{v}{\sqrt{2}} \exp\{i\theta\}$

Standard Model: SSB and Goldstone theorem

By choosing a particular ground state solution, $\theta = 0$ for example, the symmetry gets spontaneously broken. If we parametrize the excitations over the ground state as

$$\phi(x) \equiv \frac{1}{\sqrt{2}} \left[v + \varphi_1(x) + i\,\varphi_2(x) \right] ,$$

where φ_1 and φ_2 are real fields, the potential takes the form

$$V(\phi) = V(\phi_0) - \mu^2 \varphi_1^2 + \lambda\, v\, \varphi_1 \left(\varphi_1^2 + \varphi_2^2 \right) + \frac{\lambda}{4} \left(\varphi_1^2 + \varphi_2^2 \right)^2 .$$

Thus, φ_1 describes a massive state of mass $m_{\varphi_1}^2 = -2\mu^2$, while φ_2 is massless.

Origin of a massless particle when $\mu^2 < 0$: the field φ_2 describes excitations around a flat direction in the potential which do not cost any energy.

The fact that there are massless excitations associated with the SSB mechanism is a completely general result:

Goldstone theorem
If a Lagrangian is invariant under a continuous symmetry group G, but the vacuum is only invariant under a subgroup $H \subset G$, then there must exist as many massless spin-0 particles (Goldstone bosons) as broken generators

Standard Model Higgs Sector: Higgs-Kibble mechanism

The basic building block of the Higgs sector is the $SU(2)_L$ doublet of complex scalar fields

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$$

Gauged Higgs Lagrangian, invariant under local $SU(2)_L \otimes U(1)_Y$:

$$\mathcal{L}_S = \left(D_\mu \phi\right)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 \qquad (\lambda > 0\,,\ \mu^2 < 0)\,,$$

$$D^\mu \phi = \left[\partial^\mu + i\, g\, \widetilde{W}^\mu + i\, g'\, y_\phi\, B^\mu\right] \phi\,, \qquad y_\phi = Q_\phi - T_3 = \frac{1}{2}\,.$$

Scalar hypercharge is fixed by having the correct couplings between $\phi(x)$ and $A^\mu(x)$; i.e., the photon does not couple to $\phi^{(0)}$, and $\phi^{(+)}$ has the right electric charge.
There is a infinite set of degenerate states with minimum energy, satisfying

$$\left|\langle 0|\phi^{(0)}|0\rangle\right| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}\,.$$

Note that we have made explicit the association of the classical ground state with the quantum vacuum. Since the electric charge is a conserved quantity, only the neutral scalar field can acquire a vacuum expectation value.

Standard Model Higgs Sector: Higgs-Kibble mechanism

Once we choose a particular ground state, the $SU(2)_L \otimes U(1)_Y$ symmetry gets spontaneously broken to the electromagnetic subgroup $U(1)_{\rm QED}$, which by construction still remains a true symmetry of the vacuum. According to the Goldstone theorem three unwanted massless states should then appear. Higgs-Kibble mechanism is the answer.

Higgs-Kibble mechanism

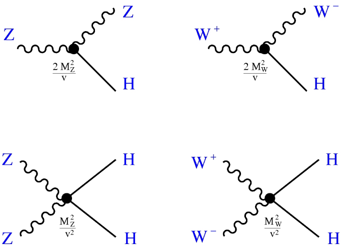
1. Parametrize the complex scalar doublet with four real fields $\theta^i(x)$ and $H(x)$
   $\phi(x) \ = \ \exp\left\{i \, \frac{\sigma_i}{2} \, \theta^i(x)\right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$
2. Local $SU(2)_L$ invariance allows to rotate away any dependence on $\theta^i(x)$.
3. In the physical (unitary) gauge, $\theta^i(x) = 0\,$,

   $$\left(D_\mu\phi\right)^\dagger D^\mu\phi \quad \xrightarrow{\theta^i=0} \quad \frac{1}{2}\,\partial_\mu H \partial^\mu H + (v+H)^2 \left\{\frac{g^2}{4}\, W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}\, Z_\mu Z^\mu\right\},$$

   and the eliminated $\theta^i(x)$ fields are precisely the would-be massless Goldstone bosons associated with the SSB mechanism
4. Through vacuum expectation value of neutral scalar, gauge bosons acquired masses: $M_Z \,\cos\theta_W \ = \ M_W \ = \ \frac{1}{2}\, v\, g$

Standard Model: The Higgs Boson

- After $\mathcal{L}_S$ added to $SU(2)_L \otimes U(1)_Y$ model the total Lagrangian is still gauge invariant which guarantees renormalizability.
- After SSB, three broken generators give rise to three massless Goldstone bosons which, owing to the underlying local gauge symmetry, can be eliminated from the Lagrangian.
- Going to the unitary gauge, we discover that the $W^{\pm}$ and the Z (but not the γ, because $U(1)_{\mathrm{QED}}$ is an unbroken symmetry) have acquired masses.
- Number of degrees of freedom conserved after SSB.



The scalar Lagrangian $\mathcal{L}_S$ has introduced a new scalar particle into the model: the Higgs H. In terms of the physical fields (unitary gauge), $\mathcal{L}_S$ takes the form

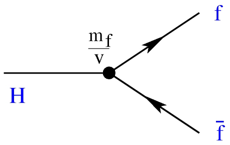
$$\mathcal{L}_S = \frac{1}{4}\lambda v^4 + \mathcal{L}_H + \mathcal{L}_{HG^2} ,$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2} M_H^2 H^2 - \frac{M_H^2}{2v} H^3 - \frac{M_H^2}{8v^2} H^4$$

$$\mathcal{L}_{HG^2} = M_W^2 W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v} H + \frac{H^2}{v^2}\right\} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left\{1 + \frac{2}{v} H + \frac{H^2}{v^2}\right\}$$

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v .$$

Standard Model: Yukawa couplings and fermion masses



A fermionic mass term $\mathcal{L}_m = -m\,\overline{\psi}\psi = -m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$ is not allowed, because it breaks the gauge symmetry. However, since we have introduced an additional scalar doublet into the model, we can write the following gauge-invariant fermion-scalar coupling:

$$\mathcal{L}_Y = -c_1\left(\bar{u},\bar{d}\right)_L\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix}d_R - c_2\left(\bar{u},\bar{d}\right)_L\begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix}u_R - c_3\left(\bar{\nu}_e,\bar{e}\right)_L\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix}e_R + \text{h.c.},$$

where the second term involves the C-conjugate scalar field $\phi^c \equiv i\,\sigma_2\,\phi^*$. In the unitary gauge (after SSB), this Yukawa-type Lagrangian takes the simpler form

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(v+H)\left\{c_1\,\bar{d}d + c_2\,\bar{u}u + c_3\,\bar{e}e\right\}.$$

The SSB mechanism generates fermion masses:

$$m_d = c_1\,\frac{v}{\sqrt{2}}, \qquad m_u = c_2\,\frac{v}{\sqrt{2}}, \qquad m_e = c_3\,\frac{v}{\sqrt{2}}.$$

Yukawa couplings are fixed by the arbitrary fermion masses:

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{m_d\,\bar{d}d + m_u\,\bar{u}u + m_e\,\bar{e}e\right\}.$$

Standard Model: Electroweak parameters

- We have now all the needed ingredients to describe the electroweak interaction within a well-defined Quantum Field Theory.
- Higgs-Kibble mechanism has produced a precise prediction for the $W^{\pm}$ and Z masses, relating them to the vacuum expectation value of the scalar Higgs field:

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}\,, \qquad M_W = 80.398 \pm 0.025 \text{ GeV}\,.$$

- From these experimental numbers, one obtains the electroweak mixing angle

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223\,.$$

- The Fermi coupling

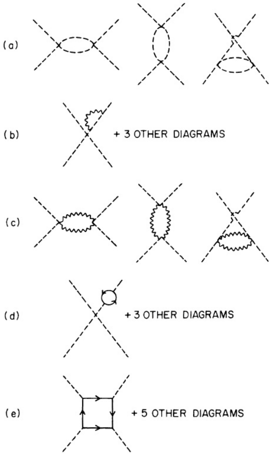
$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} = \frac{4\pi\alpha}{\sin^2\theta_W M_W^2} \equiv 4\sqrt{2}\,G_F\,,$$

gives a direct determination of the electroweak scale,

$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} = 246 \text{ GeV}\,.$$

Standard Model Scales: Running Higgs coupling

1-loop Higgs couplings:



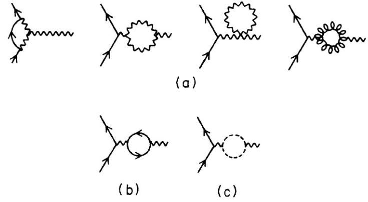
1-loop Feynman diagrams: Higgs boson self-couplings

Running Higgs coupling $\lambda(t)$ is defined as the Higgs 4-point function at scale t=$\log\frac{p}{\mu}$

Higgs beta function: $\beta_\lambda(t) = \frac{d\lambda(t)}{dt}$

Standard Model Scales: Running gauge and Yukawa couplings

1-loop gauge couplings:

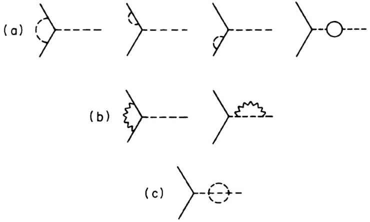


1-loop Feynman diagrams: gauge boson couplings to fermions

Running gauge couplings $g(t), g'(t), g_3(t)$ can be defined as the gauge-fermion 3-point function at scale t=log$\frac{p}{\mu}$

gauge beta functions: $\beta_g(t) = \frac{dg(t)}{dt}$

1-loop Yukawa couplings:



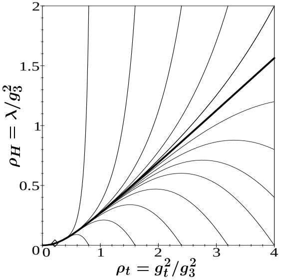
1-loop Feynman diagrams: Higgs boson Yukawa couplings to fermions

Running Top coupling $g_{Top}(t)$ is defined as the Higgs fermion 3-point function at scale t=log$\frac{p}{\mu}$

Top beta function: $\beta_{g_{Top}}(t) = \frac{dg_{Top}(t)}{dt}$

Standard Model Scales: RG Fixed Points and Triviality

running of λ, g_t, g_3



- Top-Higgs sector (1-loop)

  with notation $R = \frac{\lambda}{g_t^2} = \frac{m_H^2}{4m_t^2}$

  $\frac{dg_t^2}{dt} = \frac{9}{16\pi^2} g_t^4$

  $g_t^2 \frac{dR}{dg_t^2} = \frac{1}{3}(8R^2 + R - 2)$

  IR fixed line at $\bar{R} = \frac{1}{16}(\sqrt{65} - 1) = 0.44$

  Trivial fixed point only!

  Is the Landau pole the upper bound?

  Is $\lambda(\Lambda) = 0$ the lower bound?

- Top-Higgs-QCD sector (1-loop)

  Pendleton-Ross fixed point:

  $m_t = \sqrt{\frac{2}{9}} g_3(\mu = m_t) v/\sqrt{2} \approx 95\ GeV$

  $m_H = \sqrt{\frac{(\sqrt{689} - 25}{72}} g_3 \sqrt{2} v \approx 53\ GeV$

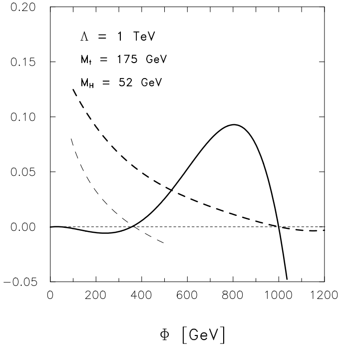
Weak gauge couplings and 2-loop destabilize the Pendleton-Ross fixed point

"Landau pole" only in α_1 at $\mu = 10^{41}\mathrm{GeV}$ with all couplings running?

Standard Model Scales: Vacuum Instability

Casas,Espinoza,Quiros

Effective Higgs potential



- Vacuum instability: m_t destabilizes ground state if m_H below some critical value
- Small running couplings $\lambda(\mu), g_t(\mu), g_3(\mu)$ in RG improved $V_{\text{eff}}(\phi)$
- Tunneling time $\gg$ 14 billion years across barrier is often used to set lower Higgs mass bound (Hall,Ellis,Linde,Sher, ...)
- **Higgs vacuum instability: misrepresentation of triviality?**

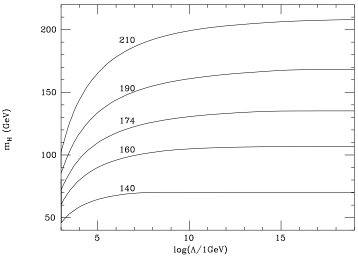
  Tunneling universe from false vacuum?

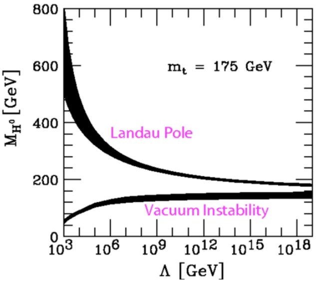
Critical in the argument that $\lambda(\mu)$ turns negative at some scale in the RG evolution of the Standard Model!

Standard Model Scales: Higgs Mass Lower Bound

Altarelli,Isidori and Casas,Espinosa,Quiros

Higgs mass lower bound (m_{top} dependent):



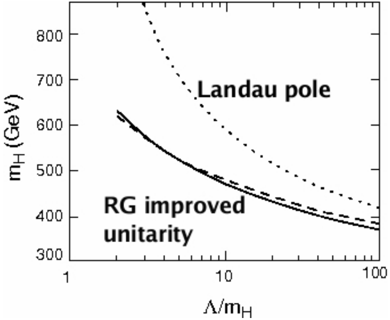


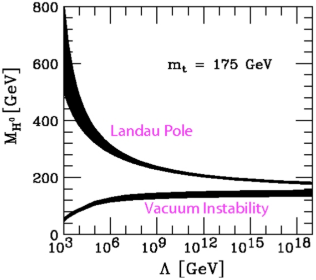
Interpretation of Higgs vacuum instability when intrinsic cutoff?

Requires more precise definition of Standard Model and its extensions

Standard Model Scales: Higgs Mass Upper Bound

Higgs mass upper bound:



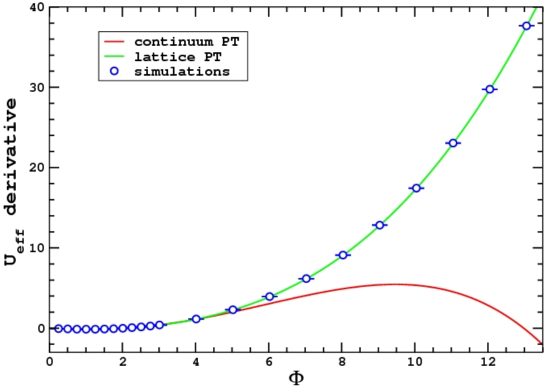


Landau pole and unitarity? Triviality?

Standard Model Scales: Fate of the false Higgs vacuum?

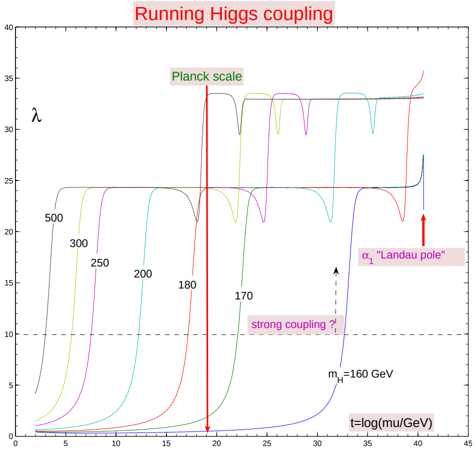
Holland,JK

Lattice effective potential



- Staggered fermion lattice simulations clearly show the absence of vacuum instability
- Lattice applications which are relevent for continuum SM will require chiral fermions (demanding extreme lattice computing)
- Confirmed by large N using other regulators
- Generic behavior: $\phi \geq \Lambda_{\text{cutoff}}$ for unstable range of $V_{\text{eff}}(\phi)$
- What happens when ignoring the cutoff in the running RG equations?

Standard Model Scales: Higgs Mass Upper Bound



Upper bound and running coupling λ

At large Higgs masses growing λ requires strong coupling

Call for nonperturbative calculations

Running gauge couplings are important!

Is large m_H consistent with EW data?

What is the Standard Model beyond PT?

Continuum Wilsonian RG

UV Completion

unknon new physics

UV Completion

unknon new physics

Below new scale M integrated UV completion is represented by non-local $\mathcal{L}_{\rm eff}$ with all higher dimensional operators,

$$\frac{1}{M^2}\phi\Box^2\phi,\ \frac{1}{M^4}\phi\Box^3\phi,\ \frac{\lambda_6}{M^2}\phi^6,\ \ldots$$

Propagator $\frac{K(p^2/M^2)}{p^2+M^2}$ with analytic K thins out UV completion with exponential damping

---

Continuum Wilsonian RG

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unknon new physics

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At the symmetry breaking scale $v = 250\ GeV$ only relevant and marginal operators survive

Only $\frac{1}{2}m_H^2\phi^2$ and $\lambda\phi^4$ terms in $V_{\rm Higgs}(\phi)$, in addition to $(\nabla\phi)^2$ operator

Narrow definition of Standard Model: only relevant and marginal operators at scale M

Standard Model Scales: Higgs Physics and the Lattice

Continuum Wilsonian RG

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unknon new physics

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Narrow definition of Standard Model: only relevant and marginal operators at scale M

Lattice Wilsonian RG

Regulate with lattice at scale $\Lambda = \pi/a$

$\mathcal{L}_{\rm lattice}$ has all higher dimensional operators

$$a^2\phi\Box^2\phi, a^4\phi\Box^4\phi, a^2\lambda_6\phi^6, ...$$

Standard Model Scales: Higgs Physics and the Lattice

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unknon new physics

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Lattice Wilsonian RG

Regulate with lattice at scale $\Lambda = \pi/a$
$\mathcal{L}_{\rm lattice}$ has all higher dimensional operators

$$a^2\phi\Box^2\phi, a^4\phi\Box^4\phi, a^2\lambda_6\phi^6, \ldots$$

At the physical Higgs scale $v = 250\ GeV$ only relevant and marginal operators survive
Only $\frac{1}{2}m_H^2\phi^2$ and $\lambda\phi^4$ terms in $V_{\rm Higgs}(\phi)$, in addition to $(\nabla\phi)^2$ operator
Choice of $\mathcal{L}_{\rm lattice}$ is irrelevant unless crossover phenomenon is required to insert intermidate M scale Two-scale problem for the lattice

Standard Model Scales: Higgs Physics and the Lattice

Continuum Wilsonian RG

UV Completion
unknon new physics

Below new scale M integrated UV completion is represented by non-local $\mathcal{L}_{\text{eff}}$ with all higher dimensional operators,

$$\frac{1}{M^2}\phi\Box^2\phi,\ \frac{1}{M^4}\phi\Box^3\phi,\ \frac{\lambda_6}{M^2}\phi^6,\ \ldots$$

Propagator $\frac{K(p^2/M^2)}{p^2+M^2}$ with analytic K thins out UV completion with exponential damping

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Narrow definition of Standard Model: only relevant and marginal operators at scale M

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Scale M missing?

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Scale M missing?

Possible to insert intermediate continuum scale M with $\mathcal{L}_{\text{eff}}$ to include

$$\frac{1}{M^2}\phi\Box^2\phi,\ \frac{1}{M^4}\phi\Box^3\phi,\ \frac{\lambda_6}{M^2}\phi^6,\ \ldots$$

to represent new degreese of freedom above M

or, Lee-Wick and other UV completions

which exist above scale M (not effective theories!)

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