Fundamental constants and electroweak phenomenology from the lattice

Lecture II: quark masses

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@ INT summer school 2007,

Seattle, August 2007.

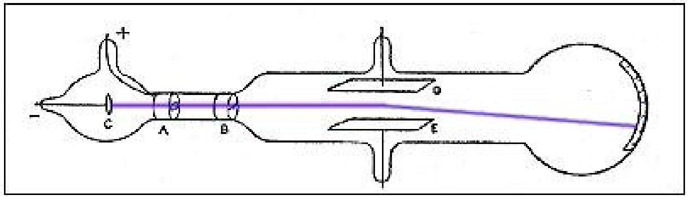
II. Quark masses

1. How to define
   - Pole mass; running mass
2. Heavy quark masses: continuum extraction
   - Quarkonium sum rules
   - B meson semileptonic decays
3. Lattice calculation: basic strategy
   - Input choices for heavy and light quarks
4. Lattice calculation: case study for heavy quark masses
   - Perturbative and non-perturbative matchings
   - Bottom and charm quarks

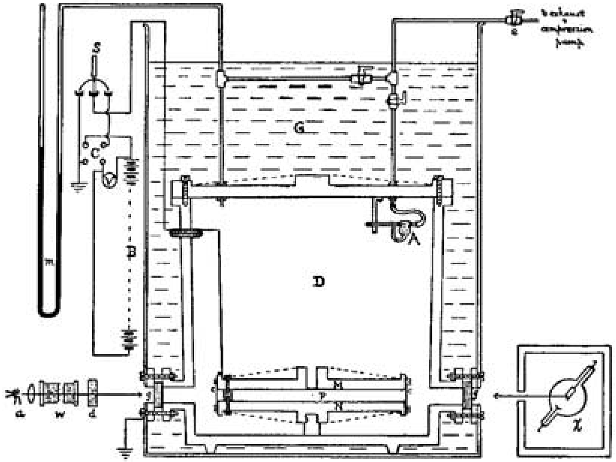
II. Quark masses
1. How to define

Measuring the particle mass

J. J. Thomson (1897)



R. Millikan (1913)



- In QED (electron mass)
  - Measure the drift of the particle in electric/magnetic fields → e/*m*

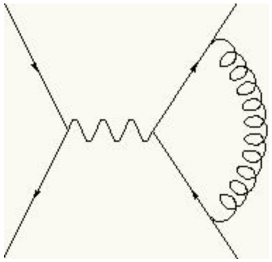
$$m\dot{v} = e[E + v \times B]$$

  - Together with the measurement of e (the coupling constant), the mass *m* is obtained.

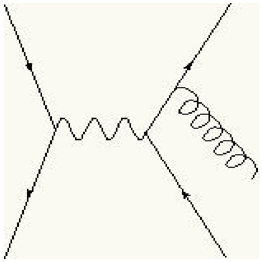
In QCD, quarks are confined…

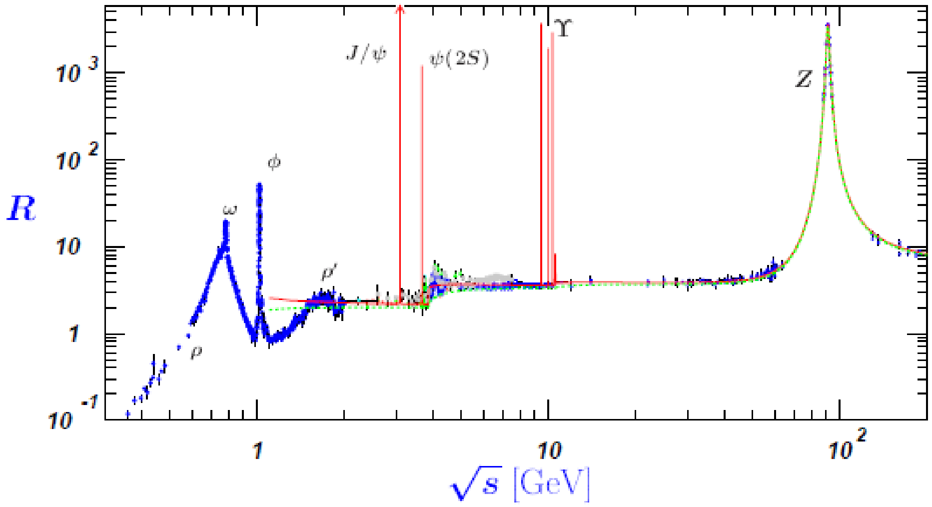
- No direct way to measure its mass alone.
- You may consider a perturbative process like $e^+e^- \rightarrow$hadrons. But the light quark mass is negligible, thus no sensitivity.
  - Even for heavy quarks, sensitivity is lost in the region where PT can safely be used.

e+



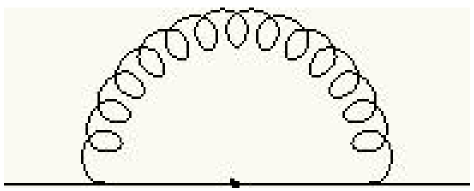
e-





No way to measure? See Sec 2. Let us consider its definitions first.

Quark mass in perturbation theory



- Appears in the QCD lagrangian

$$L_{QCD} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}(G^a_{\mu\nu})^2$$

- Can be renormalized perturbatively

$$S(p) = \frac{i}{\not{p} - m + \Sigma(p)}$$

The pole of the dressed propagator gives a definition of quark mass (pole mass)

$$\not{p} - m_{pole} - \Sigma(p, m_{pole})\,|_{p^2 = m^2_{pole}} = 0$$

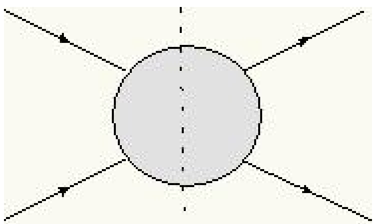
- Infrared finite
- Gauge independent
- Renormalization scale independent

Kronfeld, PRD58, 051501 (1998)

Pole mass

$$\not{p} - m_{pole} - \Sigma(p, m_{pole})|_{p^2=m_{pole}^2} = 0$$

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to 2\pi i \delta(p^2 - m^2)$$



- Well-defined perturbatively, but should not exist non-perturbatively.
- If it exists, quarks must appear as asymptotic states.
  - The pole of the propagator implies a branch cut in scattering amplitudes. Optical theorem relates it to the asymptotic state.

Another mass definition

- Consider the quark self-energy (at one-loop)

$$S(p) = \frac{i}{\not{p} - m + \Sigma(p)} = \frac{i[1+\Sigma_2(p,m)]}{\not{p} - m[1+\Sigma_1(p,m)]}$$

$$\Sigma_1(p,m) = C_F \frac{\alpha_s}{2\pi}\left\{\frac{3}{\bar{\varepsilon}} + \frac{5}{2} - \frac{3}{2}\ln\frac{m^2-p^2}{\mu^2} - \frac{1}{2}\frac{m^2}{p^2}\left[1-\left(4-\frac{m^2}{p^2}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right]\right\}$$

  - Renormalize by simply chop off the divergent term
    $$\frac{1}{\bar{\varepsilon}} = \frac{1}{\varepsilon} + \frac{1}{2}(\ln 4\pi + \gamma)$$
    which gives the definition of the MSbar mass.
  - This mass must depend on μ to make the physical amplitude independent on μ; thus the "running" quark mass m(μ).
  - Relation to the pole mass is easily obtained:

$$m_{pole} = m(\mu)\left\{1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} + \ln\frac{\mu^2}{m^2}\right) + \ldots\right\}$$

Running quark mass

- Like the coupling constant, the quark mass *runs*.

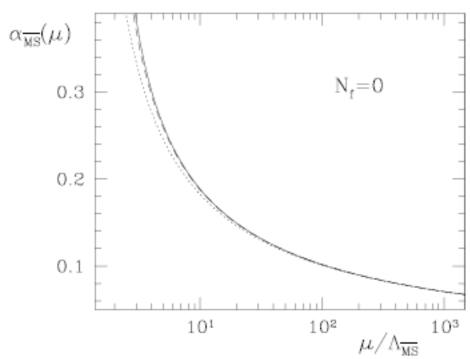
$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \beta(\alpha_s);$$

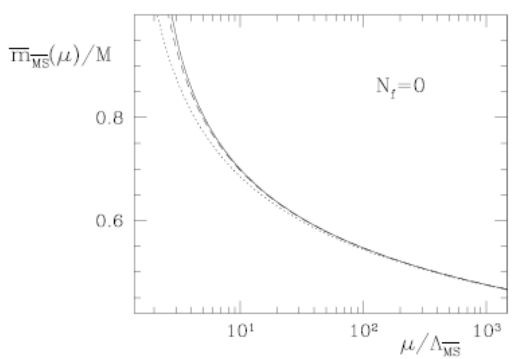
$$\mu^2 \frac{\partial m}{\partial \mu^2} = -\gamma_m(\alpha_s)$$

$$m(\mu) = \hat{m}\,[\alpha_s(\mu)]^{\gamma_1/\beta_0} \exp\left[-\int_0^{\alpha_s(\mu)} d\alpha'_s \left(\frac{\gamma_m(\alpha'_s)}{\beta(\alpha'_s)} - \frac{\gamma_1}{\beta_0 \alpha'_s} \right) \right]$$

  - An integration constant is introduced: renormalization group invariant mass $\hat{m}$

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)} + \dots \right]$$





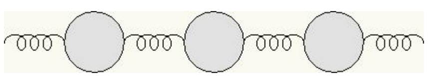
Plots from
ALPHA, NPB544,669(1999)

Problem of the pole mass

- Perturbative series is divergent:

$$\frac{m_{pole}}{m(m)} = 1 + \frac{\alpha_s}{\pi}\frac{4}{3} + \left(\frac{\alpha_s}{\pi}\right)^2 (13.4 - 1.0 N_f) + \left(\frac{\alpha_s}{\pi}\right)^3 (190.6 - 26.7 N_f + 0.7 N_f^2) + \ldots$$

  - In fact, the coefficient diverges as $\beta_0 n!$, if one sums up a leading log diagrams (large β_0 limit)



$$\sim \int \frac{dk^2}{k^2} f(k^2)[\beta_0 \alpha_s(\mu) \ln(k^2/\mu^2)]^n \sim (\beta_0 \alpha_s)^n n!$$

  - Summation of the divergent series has an ambiguity of order

$$\exp\left[\frac{1}{\beta_0 \alpha_s(\mu)}\right] \sim \frac{\Lambda_{QCD}}{\mu}$$

  called a renormalon ambiguity, which leads to an $O(\Lambda_{QCD})$ ambiguity of the pole mass.

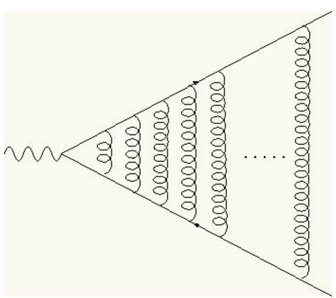
- Pole mass cannot be determined beyond the $O(\Lambda_{QCD})$ level; consistent with the quark confinement.

- One must use some perturbative definition to quote the quark mass.
  - MSbar is a preferred choice.
  - Other definitions are also used, especially in the analysis of quarkonium. Conversion is possible (no dangerous ambiguity).

II. Quark masses
2. Heavy quark masses (continuum)

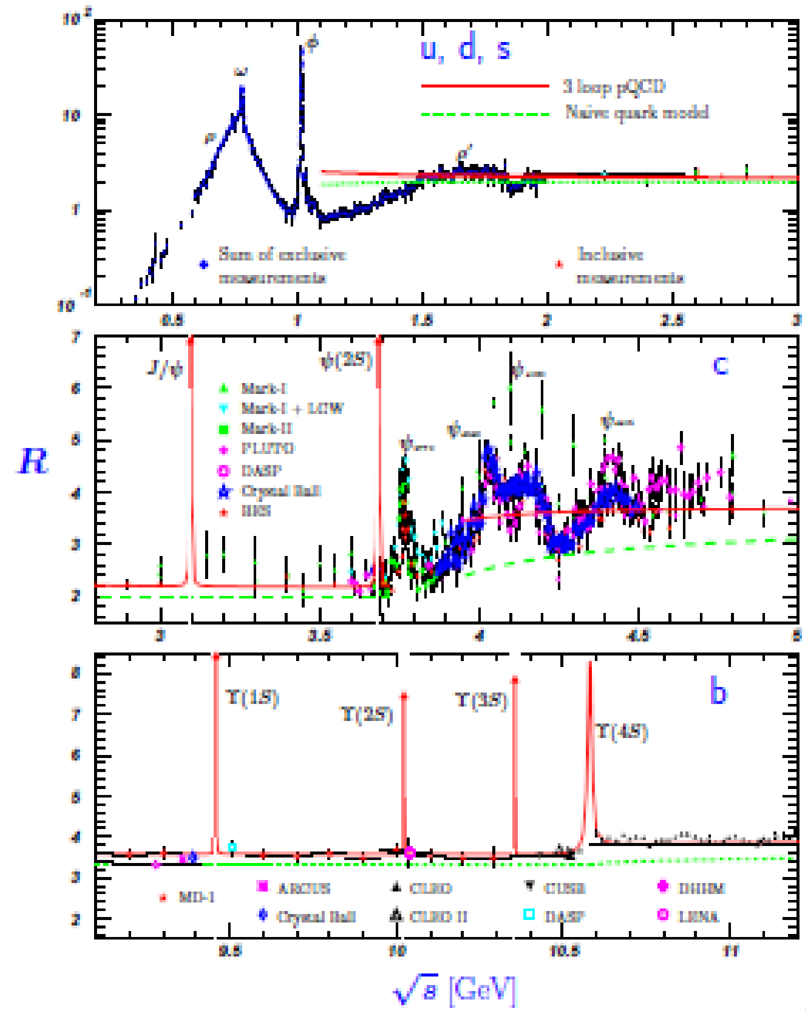
Heavy quark mass (mainly bottom)

- Use the pair production $e^+e^- \to bb$
- Only the resonance region is sensitive to the quark mass; but at the same time non-perturbative.



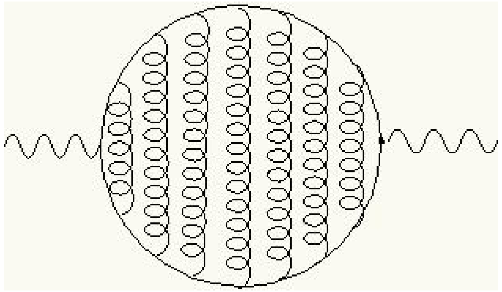
$$\sim \left(\frac{\alpha_s}{v}\right)^n$$

  - $\alpha_s \sim v$: must be resummed.



Smearing

Poggio, Quinn, Weinberg, PRD13, 1958 (1976)



- Consider a smeared R ratio:

$$\bar{R}(s,\Delta) \equiv \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s-s')^2+\Delta^2}$$
$$= \frac{1}{2\pi i}\int_0^\infty ds' R(s')\left(\frac{1}{s-s'+i\Delta} - \frac{1}{s-s'-i\Delta}\right)$$
$$= \frac{1}{2i}\left[\Pi(s+i\Delta) - \Pi(s-i\Delta)\right]$$

$$\mathrm{Im}\,\Pi(s) \propto R(s) = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

instead of ImΠ(s).

- Imaginary momentum flows into the loop; can avoid the threshold singularity, which leads to the $(\alpha_s/v)^n$ resumation.
- Δ must be larger than Λ_{QCD} in order to avoid non-perturbative physics.

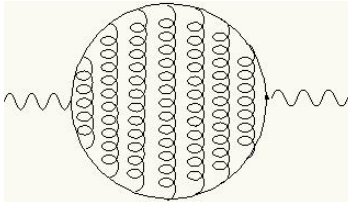
Quark-hadron duality

Sum rule

Use the analytic properties

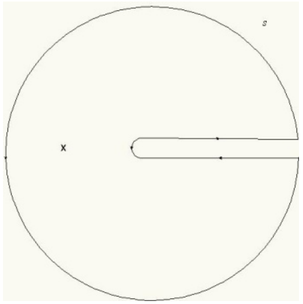
- Vacuum polarization

$$(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2) = -i\int d^4x e^{iqx}\langle 0|\mathrm{T}V_\mu(x)V_\nu(0)|0\rangle$$



- Optical theorem

$$\Pi(z) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{z-s}$$



- Moments

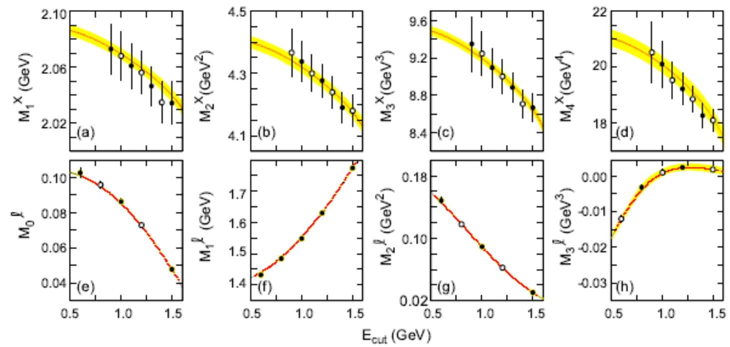
$$\frac{1}{n!}\left(\frac{d\Pi(z)}{dz^n}\right)_{z=0} = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s^{n+1}}$$

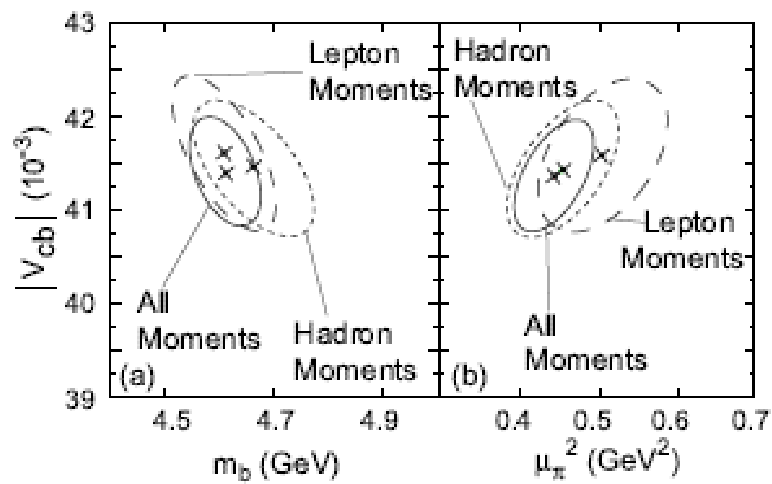
  - LHS: perturbatively calculable using OPE for sufficiently small n.
  - RHS: use experimental inputs R(s)~ImΠ(s); integral naturally smeared over large range of s, if n not too large.
  - Continuum more suppressed for larger n; more sensitivity to m_b.

B meson semileptonic decays

- From inclusive semileptonic B meson decays $B \to X_c l \nu$, m_b (and m_c) can be determined together with $|V_{cb}|$.
  - Use the heavy quark expansion; smearing corresponds to the integral over final states.
  - Measurements of hadronic mass (Xc) and lepton energy (l) moments.
  - Detailed discussion will be given in Part IV.

BaBar, PRL93, 011803 (2004)



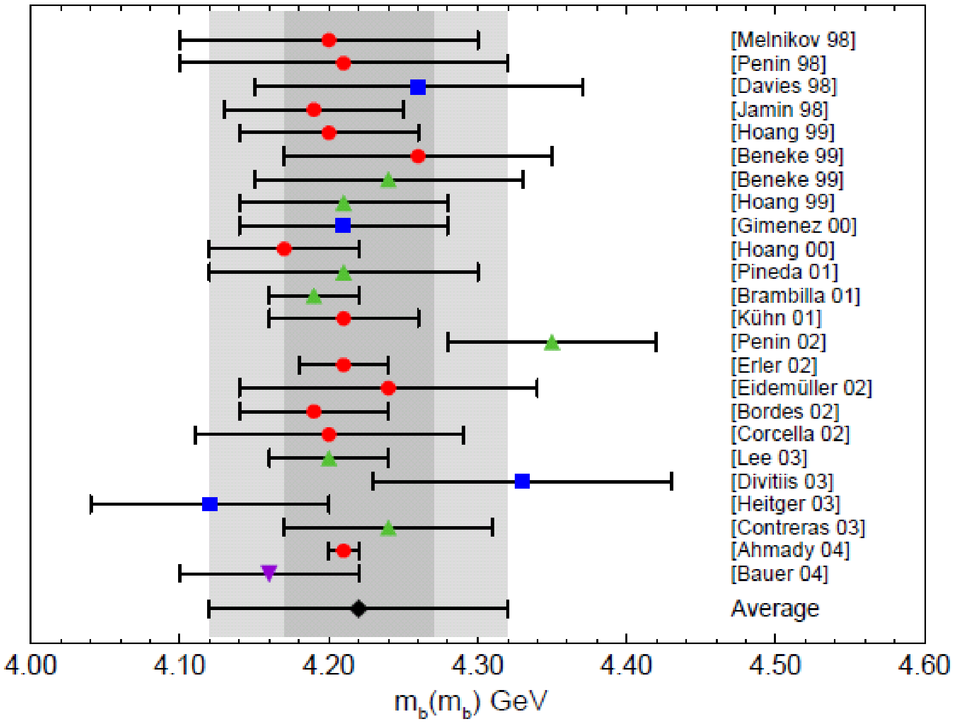


b quark mass

- Compilation of many results are found in the PDG review; an average
  $$\overline{m}_b(\overline{m}_b) = 4.20 \pm 0.07\ \mathrm{GeV}$$
  excluding lattice.

  Can lattice calculation compete with this? How?

QWG hep-ph/0412158



Red circles: sum rule; Green triangle: Υ 1S; Purple triangle: semileptonic; Blue squares: lattice

II. Quark masses
3. Lattice calculation: basic strategy

Inputs

In general, lattice QCD simulation requires inputs for

- Lattice scale $1/a \Rightarrow$ determines $\alpha_s(1/a)$
  - Inputs discussed in Part I.
- Quark masses for each flavor
  - up and down quarks m_{ud} (often assumed to be degenerate)
    - from pseudo-scalar meson mass m_π, good sensitivity because $m_\pi^2 \sim m_{ud}$.
  - Strange quark m_s
    - from m_K, for the same reason.
  - Charm quark m_c
    - either from D (heavy-light) or J/ψ (heavy-heavy) mass
  - Bottom quark m_b
    - either from B (heavy-light) or Υ (heavy-heavy) mass

Up, down, strange

- Gell-Mann-Oakes-Renner (GMOR) relation (1968)
  - At the leading order in m_q

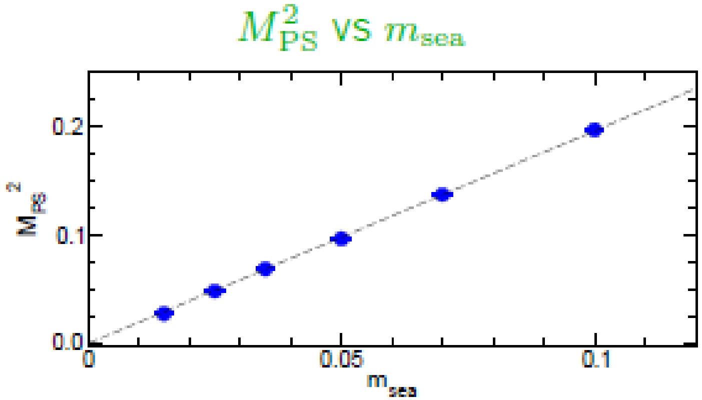
$$(m_u + m_d)\langle \bar{q}q \rangle \cong -f_\pi^2 m_\pi^2$$

$$(m_u + m_s)\langle \bar{q}q \rangle \cong -f_K^2 m_K^2$$

  - The slope yields

$$B_0 = \frac{-\langle \bar{q}q \rangle}{f_\pi^2}$$

  - Lattice calculation gives B_0, then m_q is obtained with an input of the experimental value of m_π or m_K.
  - m_q is in the lattice regularization; need a matching to obtain the MSbar value.



JLQCD (2006)
Dynamical overlap

Beyond LO, much more complicated; see Lecture III.

Charm and bottom

- Heavy-light (D, B)

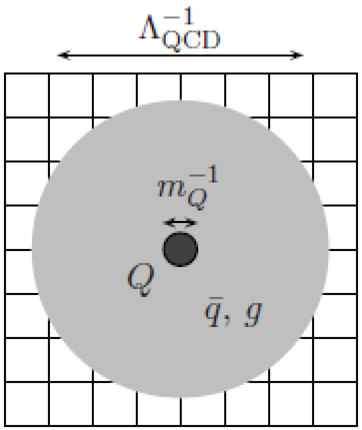
$$m_H = m_Q + E_{Q\bar{q}}$$

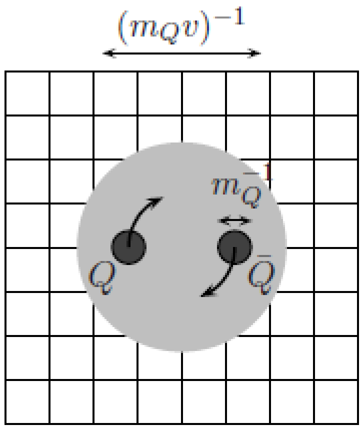
  - E_{Qq} denotes a binding energy.
  - Simply calculate the meson mass; tune m_Q until m_H reproduces the experimental value.
  - Calculate E_{Qq}, whose m_Q dependence is subleading. Then, m_H-E_{Qq} gives m_Q. (Heavy Quark Symmetry)

- Heavy-heavy (J/ψ, Υ)

$$m_H = m_Q + m_{\bar{Q}} + E_{Q\bar{Q}}$$

  - E_{QQ} denotes a binding energy.
  - Binding energy crucially depends on m_Q.





Conversion

convert:
$m^{MS}(\mu)=Z_m(\mu a)m^{lat}(a^{-1})$

- Once the bare quark mass is fixed on the lattice, it must be converted to the continuum definition, because the pole mass is not adequate.
  - Just like the conversion of the coupling constant.
  - May use the perturbation theory (Use the renormalized coupling!). But, in most cases, known only at the one-loop level. (Exceptions are HQET, Asqtad, stochastic PT(?).)

    Ex). O(a)-improved Wilson fermion

$$m^{MS}(\mu=a^{-1})=[1+2.05\alpha_s+\ldots]m^{lat}(a^{-1})$$

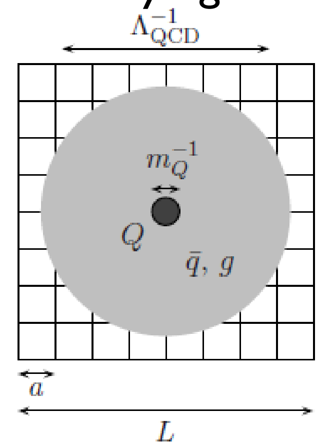
  - Non-perturbative renormalization is desirable.

II. Quark masses
4. Case study: heavy quark mass

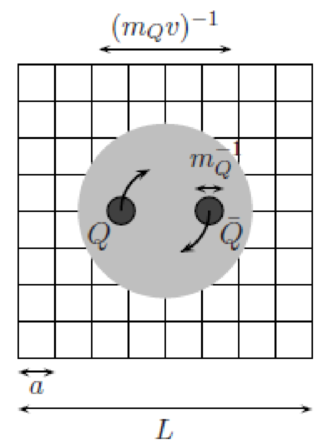
Heavy quark on the lattice

- Additional complication for heavy quarks:
  - Compton wave length ~1/m_Q is comparable to or shorter than the lattice spacing.
  - But, such a short distance scale is irrelevant to the bound state dynamics.
- Construction of effective theories
  - HQET: the leading order in 1/m_Q expansion.
    - Used for heavy-light.
    - Higher orders may be included as operator insertions, or in the Lagrangian as in NRQCD.
  - NRQCD: expansion in v~α_s.
    - Used for heavy-heavy.

heavy-light



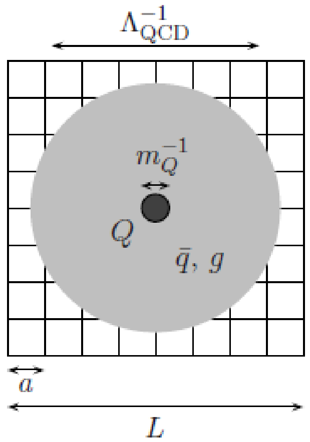
heavy-heavy



Dedicated lectures by Kronfeld

Heavy Quark Effective Theory (HQET)

- Write the momentum of heavy quark as $p = m_Q v + k$
  - v : four-velocity of the heavy quark.
  - k: residual momentum



- Heavy quark mass limit:
  - propagator

$$i\frac{\not{p}+m_Q}{p^2-m_Q^2+i\varepsilon} = i\frac{m_Q\not{v}+m_Q+\not{k}}{2m_Q v\cdot k+k^2+i\varepsilon} \rightarrow i\frac{1+\not{v}}{2}\frac{1}{v\cdot k+i\varepsilon}$$

  - Lagrangian

$$L_Q = \overline{Q}_v(iv\cdot D)Q_v; \quad Q(x) = e^{-im_Q v\cdot x}Q_v(x)$$

Georgi (1990), Eichten-Hill (1990)

  - Heavy quark mass drops out from the dynamics = Heavy Quark Symmetry

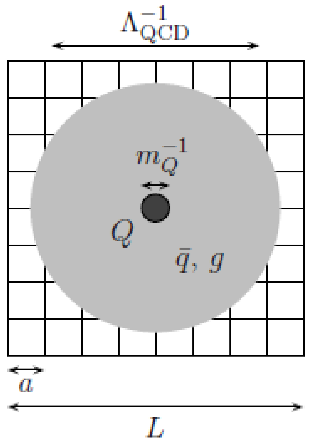
Isgur-Wise (1989)

HQET on the lattice

- Discretize the HQET lagrangian
  - Assuming $v^\mu=(1,0)$: rest frame of the heavy quark

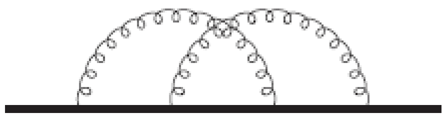
$$S_Q=\sum_x Q^+(x)[1-U_4^+(x-\hat{4})]Q(x-\hat{4})$$

  - Heavy quark propagator becomes a static color source.
  - Heavy-light meson mass: $m_H=m_Q+E_{Q\bar{q}}$
    Calculate E_{Qq}, then, m_H-E_{Qq} gives m_Q up to Λ_{QCD}/m_Q corrections.



Conversion at two-loop

Martinelli-Sachrajda, NPB559, 429 (1999).



- Conversion to the MSbar scheme done to two-loop (static theory is simple!)

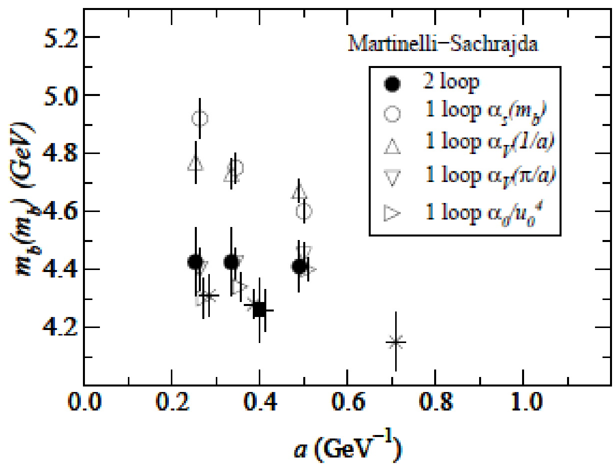
$$\overline{m}_b(\overline{m}_b) = \left[1 - \frac{4}{3}\frac{\alpha_s(\overline{m}_b)}{\pi} - 11.66\left(\frac{\alpha_s(\overline{m}_b)}{\pi}\right)^2 + \ldots\right]$$

Continuum ← Pole

$$\times\left[M_B - E_{\bar{b}q} + 2.117\alpha_s(\overline{m}_b) + (3.7\ln(\overline{m}_b a) - 1.3)\alpha_s^2(\overline{m}_b) + \ldots\right]$$

Pole ← Lattice

- Two-loop essential for stable (and thus precise) determination of m_b.
- Now, known to three-loop (Di Renzo-Scorzato, JHEP 02(2001)020)



Limitation of effective theory

- Obviously, HQET (at LO) ignores the $1/m_Q$ effects.
  - Higher order terms can be included. The leading corrections:

$$H = -\frac{D^2}{2m_Q} - \frac{\sigma \cdot B}{2m_Q}$$

  - The coefficients of terms are constrained by the Lorentz invariance, thus giving $1/2m_Q$.
  - But, in the quantum theory they are renormalized differently, since the Lorentz invariance is violated by the choice of the reference frame v^μ.

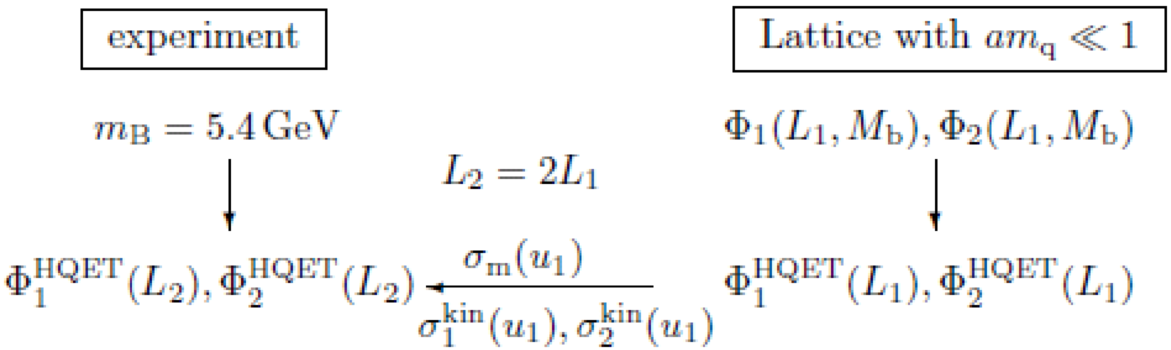
$$H = -\frac{D^2}{2(Z_m m_Q)} - c_B \frac{\sigma \cdot B}{2(Z_m m_Q)}$$

  - The coefficients (Z_m and c_B) must be calculated (non-)perturbatively.
  - The same complication arises at every order of the expansion.

State-of-the-art

Della Morte, Garron, Papinutto, Sommer, JHEP 0701, 007 (2007).

- $1/m_Q$ terms renormalized non-perturbatively.
  - This is non-trivial, because it requires a non-perturbative input.
  - The matching was done on a fine lattice against the usual relativistic fermion in the region ($m_Q a << 1$). The lattice is necessarily small there.
  - Then match onto coarser lattices using the step scaling technique. $\Rightarrow$ Lectures by Sint



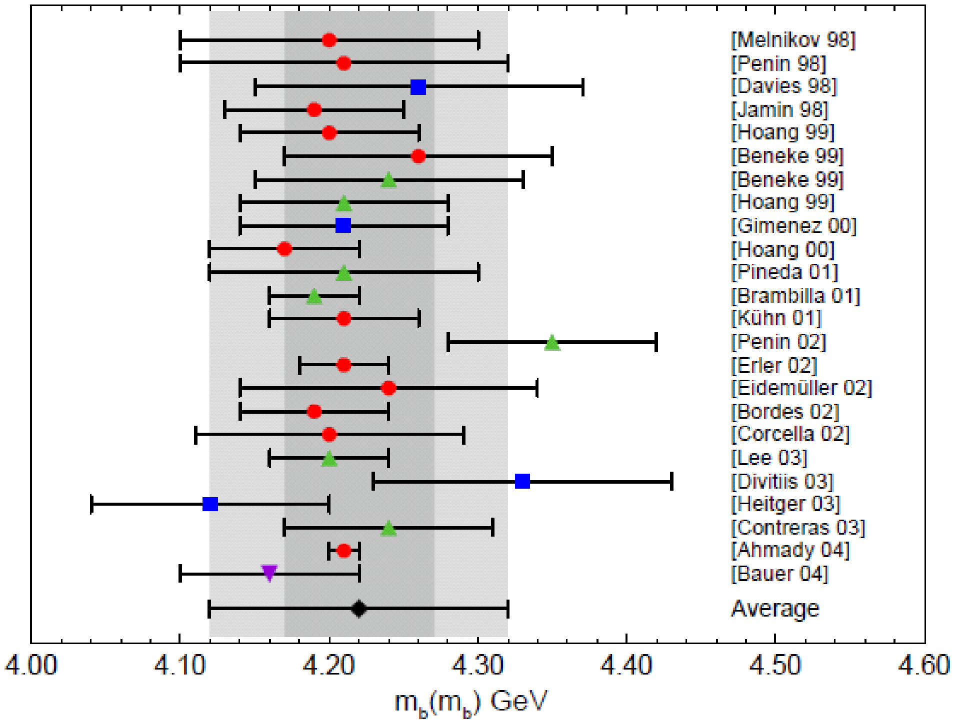
State-of-the-art

- matching to MSbar done non-perturbatively.
  - The result in the quenched theory.

$$\bar{m}_b(\bar{m}_b) = [4.35(5) - 0.05(3)]\,\mathrm{GeV}$$

HQET $\qquad$ $1/m_b$

  - Compared to the earlier result, 4.41(5)(10) GeV, by Martinelli-Sachrajda in the heavy quark limit (pert matching).
  - PDG value 4.20(7) GeV



Della Morte et al. (2007)

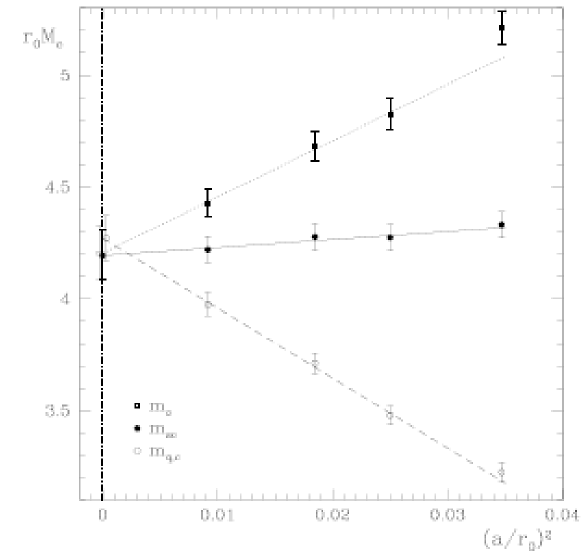
Charm quark

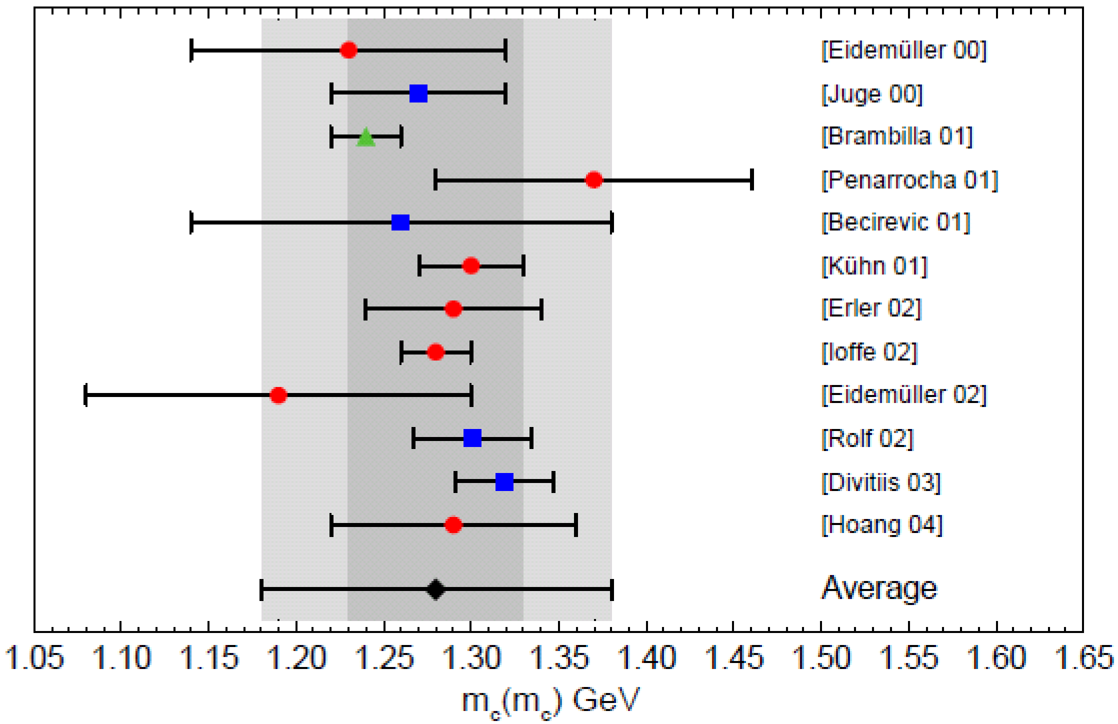
- HQET is of little use. Use the conventional lattice fermion with a as small as possible to avoid the $(am_c)^2$ error. Extrapolation in a will be essential.
- Non-perturbative matching of quark mass is available for the O(a)-improved Wilson fermion. Again obtained with the step scaling technique.
- Quenched calculation (Rolf-Sint, 2002) gave

$$\overline{m}_c(\overline{m}_c) = 1.30(3)\ \text{GeV}$$

  - PDG value 1.25(9) GeV

Rolf-Sint, JHEP 0212, 007 (2002)





Will become competitive if done with dynamical quarks!