Fundamental constants and electroweak phenomenology from the lattice

Lecture I: strong coupling constant

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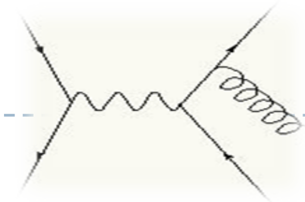
QCD, the theory of strong interaction

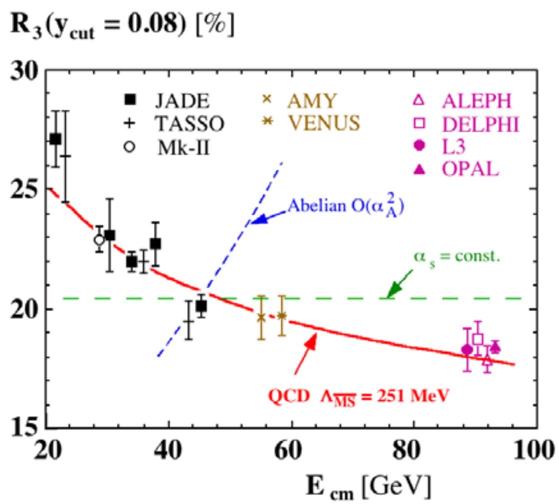
- We know that QCD is *the* theory of strong interaction. Any motivation for further study…?

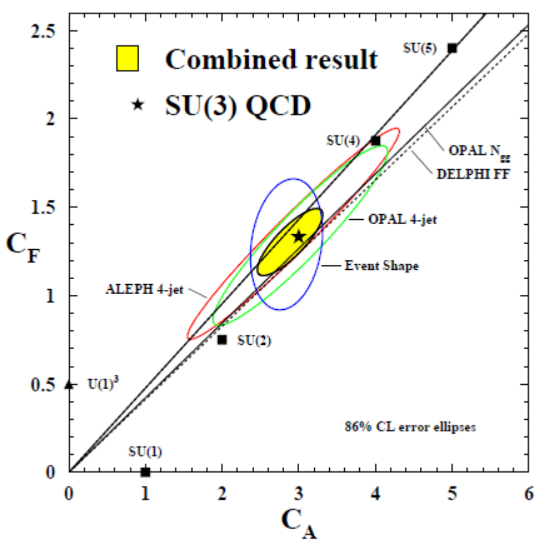
  As a nuclear theorist:
  - want to know the properties of hadrons and nuclei, hopefully from the first principles

  As a particle theorist:
  - want to solve the (non-SUSY) Yang-Mills theory, anyway
  - want to test QCD including its non-perturbative aspects
  - want to analyze the exp data at LHC; need for the study of more interesting physics, like Higgs and SUSY models
  - want to test the Standard Model more precisely through low energy measurements; hadronic uncertainty is the obstacle

How is QCD tested?





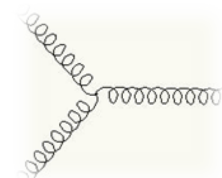


Examples

- 3-jets event rate in the e^+e^- collision

$$R_3 = \frac{\sigma\left(e^+e^- \to 3\text{- jets}\right)}{\sigma\left(e^+e^- \to \text{hadrons}\right)} = C_1\alpha_s(\mu^2) + \ldots$$

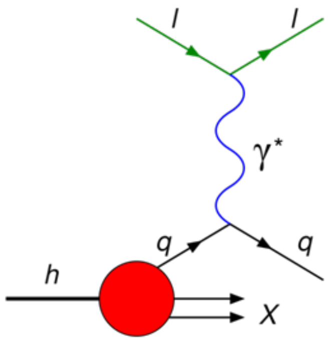
  - Scale dependence of α_s clearly seen
- Including 4-jets
  - Sensitivity to the 3-gluon vertex
  - Can test the group structure

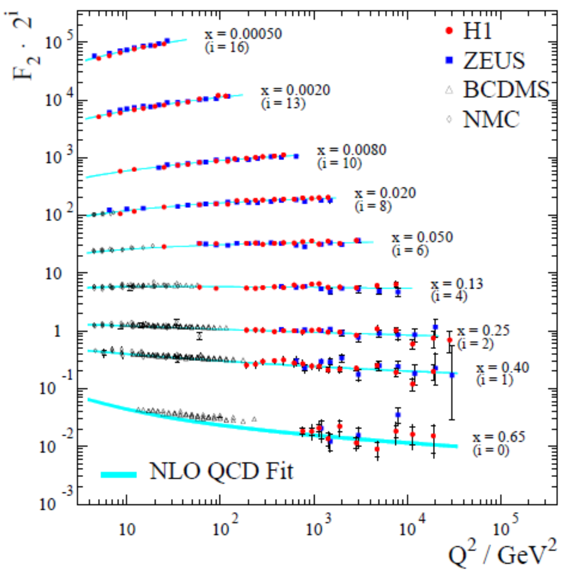


$$C_A = N = 3,\ C_F = \frac{N^2 - 1}{2N} = \frac{4}{3}$$

Plots from Bethke, Prog Part Nucl Phys 58 (2007) 351.

More tests of QCD





- Deep inelastic scattering

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1+(1+y)^2)F_1 + \frac{1-y}{x}(F_2 - 2xF_1)\right]$$

  - Structure function (or parton density) F_i; their Q^2 dependence is from the QCD loop effects.

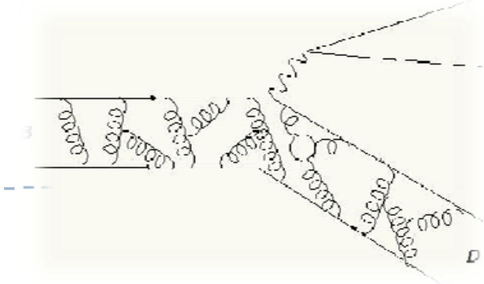
At the perturbative level, QCD describes various exp to a good precision.

Non-perturbative test?

Look at the quantities which can be determined from different inputs: perturbative and non-perturbative

- Strong coupling constant
  - High energy scattering + perturbation theory
  - Low energy spectrum + lattice
- Heavy quark masses
  - Quarkonium spectral sum rule (mostly perturbative)
  - Low-lying spectrum + lattice
  - From heavy-light systems + lattice

Hadronic uncertainty



Not just testing QCD:

- Flavor physics
  - Extract fundamental constants (CKM matrix elements) from physical processes; Search for new physics effects: Many examples will appear in this lecture
- Processes involving quarks are always contaminated by hadronic uncertainty (= non-perturbative QCD effects). What to do?
  - Look for processes which are perturbative
  - Look for processes for which some symmetry helps to eliminate the uncertainty
  - Calculate them on the lattice

Contents

I. Strong coupling constant

1. How to define
   - Running coupling
   - Scheme dependence; MSbar, one's favorite choice
   - Experimental measurements
2. Lattice calculation: scale setting
   - Basic steps: scale setting + scheme conversion
3. Lattice calculation: coupling conversion
   - Lattice perturbation theory
   - Scheme conversion through heavy quark potential
4. Recent lattice calculations
   - HPQCD, …

II. Quark masses

1. How to define
   - Pole mass; running mass
2. Heavy quark masses: continuum extraction
   - Quarkonium sum rules
   - B meson semileptonic decays
3. Lattice calculation: basic strategy
   - Input choices for heavy and light quarks
4. Lattice calculation: case study for heavy quark masses
   - Perturbative and non-perturbative matchings
   - Bottom and charm quarks

III. Chiral dynamics and light quark masses

1. Chiral symmetry breaking and quark masses
   - GMOR relation
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2. Lattice calculation of light quark masses
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   - Perturbative and non-perturbative matchings
3. Pion loop effects
   - Chiral log effects on chiral extrapolations
   - Quark masses and pion/kaon decay constants
   - Pion form factor and general strategy

IV. CKM phenomenology: at tree level

1. Quark flavor physics
   - Flavor changing interactions; from W-exchange to four-fermi interactions; FCNC
   - Quark mixings: the CKM matrix, unitarity triangle
2. Vus, the Cabibbo angle
   - Flavor SU(3) breaking: one-loop ChPT and higher order corrections; Lattice calculation
3. Vcb
   - Inclusive and exclusive semi-leptonic decays
   - Heavy quark symmetry; lattice calculation
4. Vub
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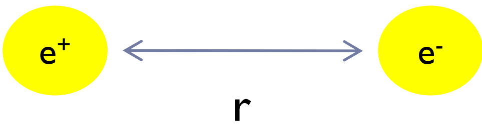
   - Lattice calculation for exclusive processes

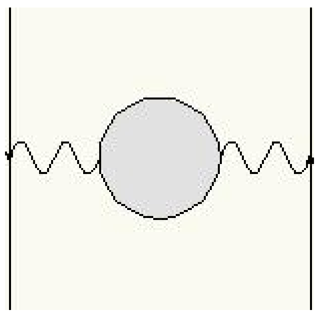
V. CKM phenomenology: at loop level

1. Kaon mixing
   - Indirect and direct CP violations
   - Lattice calculation of B_K
   - ε'/ε, the grand challenge for the lattice
2. B meson mixings
   - Lattice calculation, extraction of Vtd, Vts
3. Phenomenology of B meson decays
   - Many interesting decay modes: a few examples
   - Further opportunities for lattice QCD
4. Other applications
   - Muon g-2, neutron electric dipole moment, …

I. Strong coupling constant
1. How to define

Defining the coupling constant





$$V(q^2) = \frac{\alpha_{\text{eff}}(q^2)}{q^2}$$

- In QED:
  - Measure the force between two test charges, then α is easily extracted.

$$F(r) = \frac{\alpha}{r^2}, \qquad \alpha = \frac{e^2}{4\pi}$$

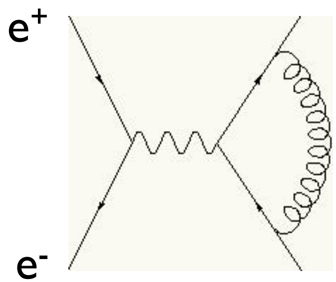
  - Note: running coupling
    - QED coupling constant depends on the scale,

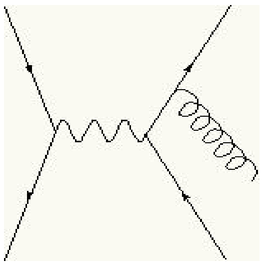
$$\alpha_{\text{eff}}(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\left\{\log\left(\frac{-q^2}{m^2}\right) - \frac{5}{3}\right\}}$$

but the infrared limit is regularized by the electron mass.

$$V(r) = -\frac{\alpha}{r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2mr}}{(mr)^{3/2}} + \ldots\right)$$

In QCD, what to do?





- Quarks are confined; no way to put test charges.
  - Well, you may consider an Gedanken-experiment, but not possible in practice.
- Consider, instead, an experiment like $e^+e^- \to$hadrons

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= 3\sum_q Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)\right\}$$

α_s is obtained by solving this eq.

Ultraviolet divergences

$$K_{QCD} = 1 + \frac{\alpha_s(\mu)}{\pi} + C_2\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + C_3\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + \ldots$$

- Beyond the leading order, the UV divergence must be renormalized.
  - A renormalization scheme must be specified. A popular choice: the modified minimal subtraction MSbar
    - With the dimensional regularization (ε=4-D), subtract

$$\frac{2}{\varepsilon} - \gamma_E + \ln(4\pi)$$

  - Once you decide to use it, you must stick to using it!
  - In other words the α_s thus extracted must be understood in this particular choice of the renormalization scheme.

Scheme dependence

- Any physical quantity should not depend on the choice of the renormalization scheme.

$$K_{QCD} = 1 + \frac{\alpha_s^{(I)}(\mu)}{\pi} + C^{(I)}\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s^{(I)}(\mu)}{\pi}\right)^2 + \dots$$
$$= 1 + \frac{\alpha_s^{(II)}(\mu)}{\pi} + C^{(II)}\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s^{(II)}(\mu)}{\pi}\right)^2 + \dots$$

One can read off the relation between the two schemes.

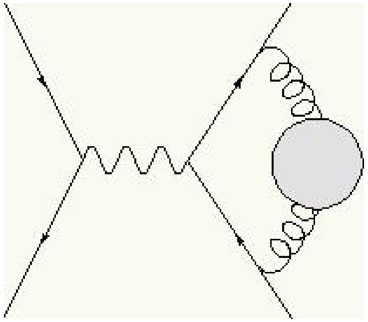
$$\alpha_s^{(II)}(\mu) = \alpha_s^{(I)}(\mu)\left\{1 + \frac{C^{(I)} - C^{(II)}}{\pi}\alpha_s + \dots\right\}$$

This is related to the ratio of the Λ parameters

$$\frac{\Lambda^{(I)}}{\Lambda^{(II)}} = \exp\left(-\frac{2\pi}{\beta_0}\left(\frac{1}{\alpha_s^{(I)}(\mu)} - \frac{1}{\alpha_s^{(II)}(\mu)}\right)\right) = \exp\left(-\frac{2(C^{(I)} - C^{(II)})}{\beta_0}\right)$$

Renormalization scale

$$K_{QCD} = 1 + \frac{\alpha_s(\mu)}{\pi} + C_2\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + C_3\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^3 + \ldots$$



Due to the renormalization…

- A renormalization scale μ is involved. A good choice is $\mu^2=s$ to minimize the perturbative coefficients due to possible large logs

$$\beta_0 \ln \frac{s}{\mu^2}$$

  which can be identified as a running coupling effect.

- If we change μ consistently (in C_i and α_s), then the physics result must be unchanged up to neglected higher order corrections.

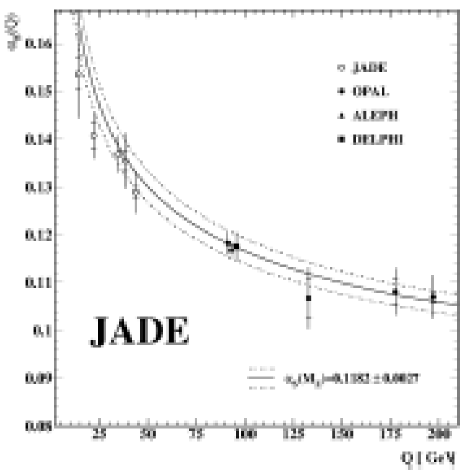
Running coupling

- In other words, the *running* coupling constant is introduced such that the observable is independent of μ.

$$\frac{dK_{QCD}}{d\mu} = \frac{d}{d\mu}\left[1+\frac{\alpha_s(\mu)}{\pi}+C_2\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2+C_3\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}\right)^3+\dots\right]=0$$

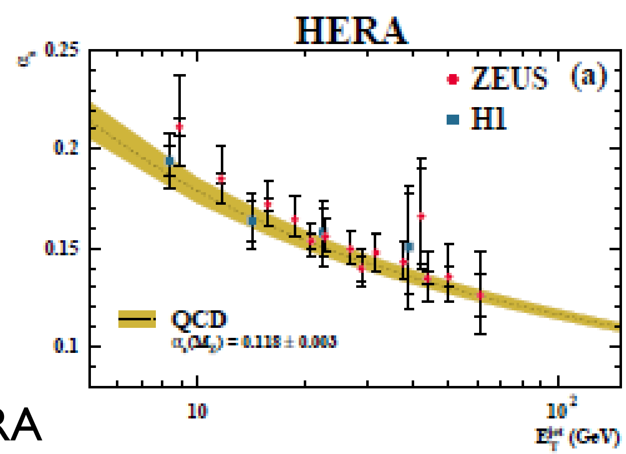
It leads to

$$\alpha_s(\mu)=\frac{4\pi}{\beta_0\ln(\mu^2/\Lambda^2)}\left[1-\frac{2\beta_1}{\beta_0^2}\frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)}+\dots\right]$$



e+e- four-jets

DIS at HERA



Unambiguous definition

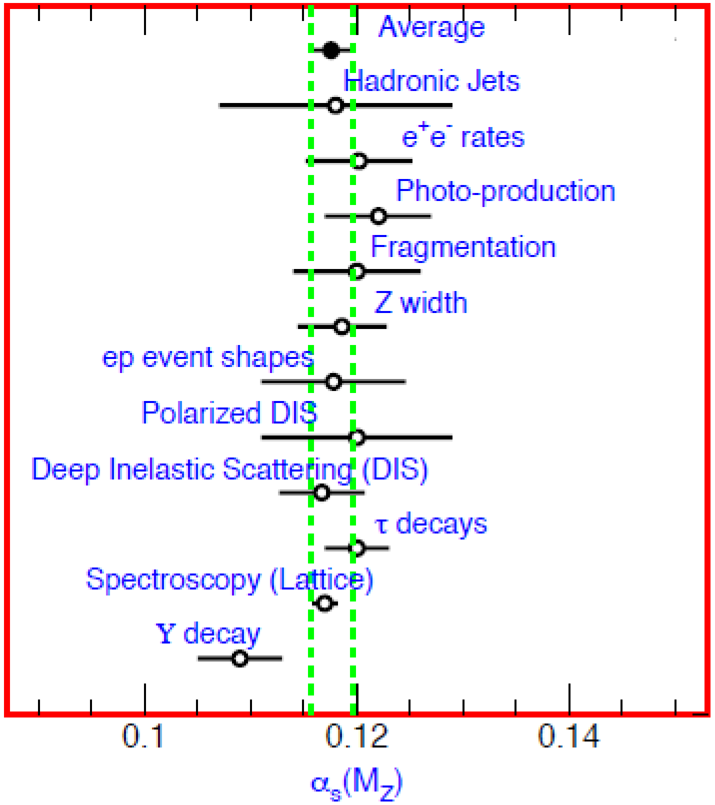
The definition relies on perturbation theory.

When you quote a value of α_s, you must specify

- Renormalization scheme:
  - e.g. MSbar
- Renormalization scale:
  - e.g. $\mu=M_Z$
- Number of flavors:
  - e.g. Nf=5
- Order of the truncation:
  - e.g. three loop

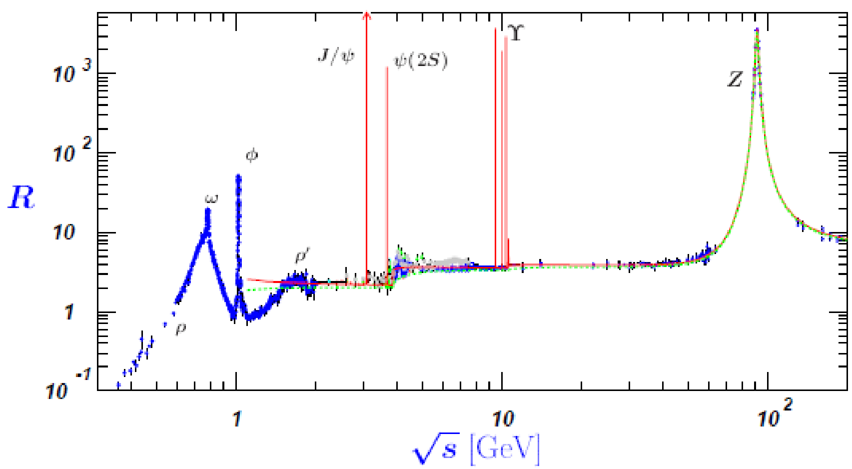
These are the common choices.

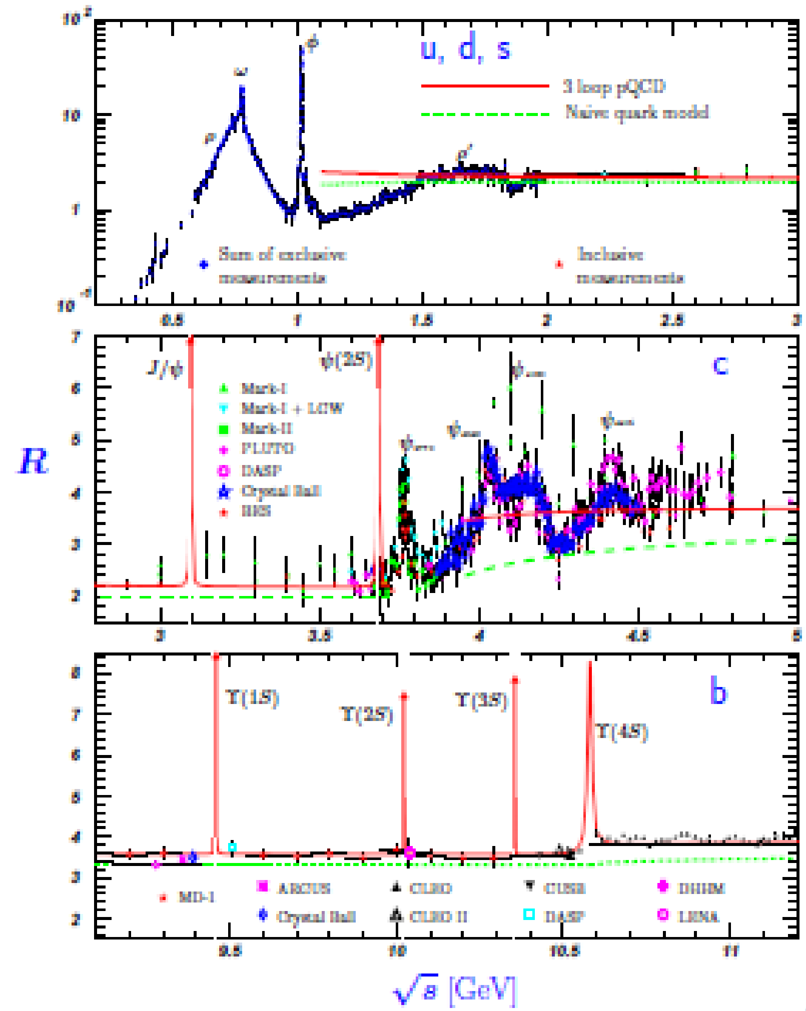
PDG 2006

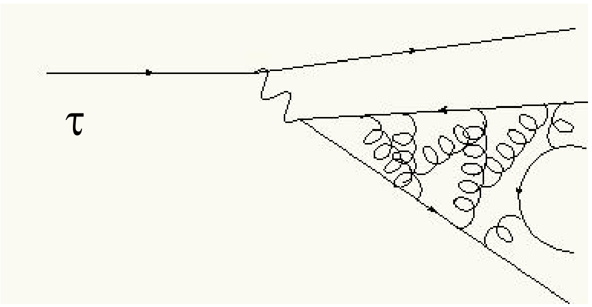


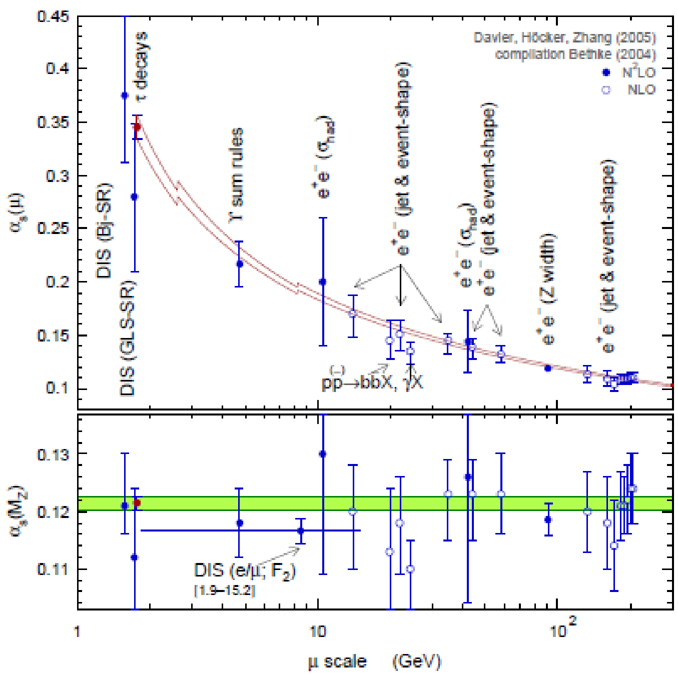
Some experimental measurements

- e^+e^- annihilation
  - Beautiful agreements
  - One must avoid the resonance regions (light hadrons, charm, bottom)









- # Hadronic τ decays
  - Looks similar to the e+e- annihilation.
  - Scale is much lower
  - contains non-perturbative contribution; evaluated using OPE

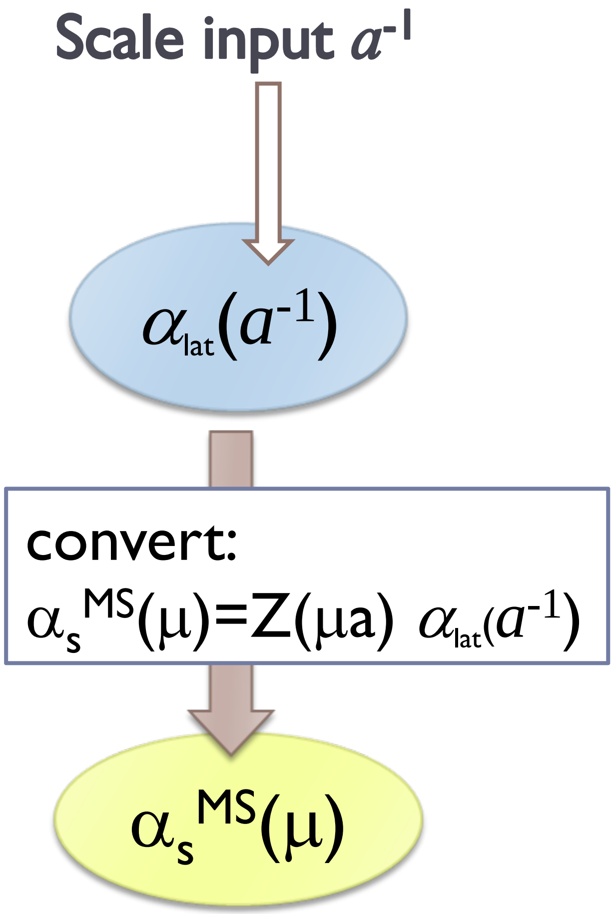
$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left\{\left(1+\frac{2s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right\}$$

$$= R_0\left[1+\frac{\alpha_s(m_\tau)}{\pi}+\ldots+c\,\frac{\langle \alpha_s GG\rangle}{m_\tau^4}+c'\,\frac{\langle m\bar{q}q\rangle}{m_\tau^4}+\ldots\right]$$

  - Nevertheless, final precision is very good; subject to test with other non-perturbative techniques.

I. Strong coupling constant
2. Lattice calculation: scale setting

The basic strategy



… Very simple

1. Choose a set of lattice parameters: $\beta=6/g_{lat}^2$, m_q
2. Determine the lattice spacing a with some physical input; it gives you a relation $\alpha_{lat}(a^{-1})$
3. Convert the bare lattice coupling $\alpha_{lat}(a^{-1})$ to $\alpha_s^{MS}(\mu)$
4. Run to your favorite scale, e.g. $\mu=M_Z$.

Scale setting

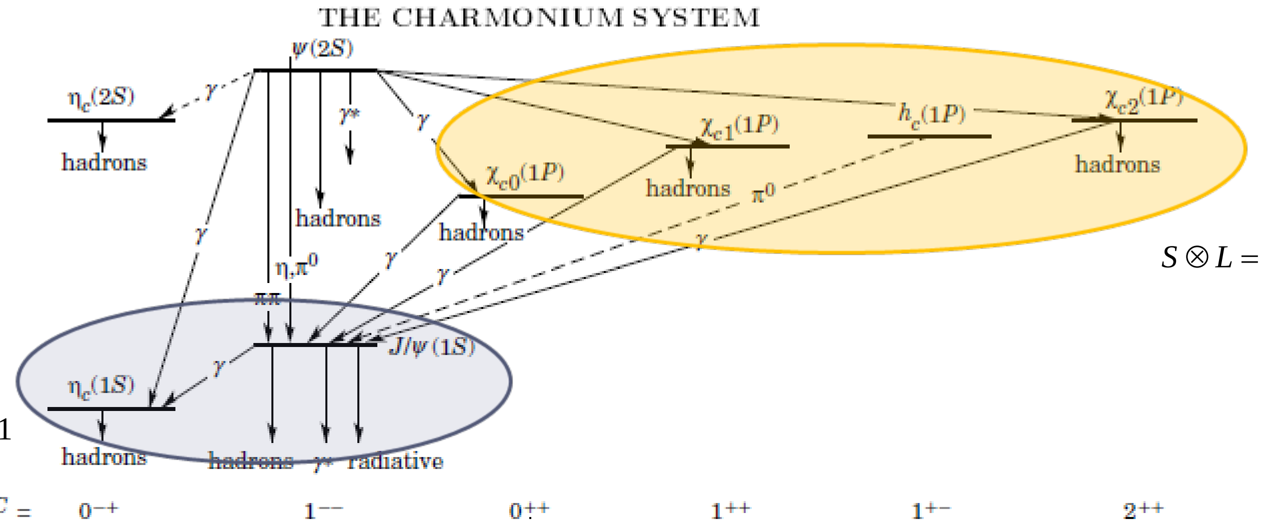
In any lattice QCD calculation you need a scale input. What is the best choice (reliable, stable, easy to calculate)?

- ρ meson mass:
  - Standard choice in the past. But a decaying particle with a large width. No way to control the m_q dependence near and below the $\pi\pi$ threshold.
- Pion decay constant (or K)
  - Stable particle. Not difficult to calculate. Need controlled chiral extrapolation. Matching of A_μ should be done non-perturbatively.
- string tension (or r_0):
  - Another popular choice. Very easy to calculate. But not a directly measurable quantity. Need to involve a potential model for quarkonium spectrum.

Can be any other physical quantity; must agree among them.

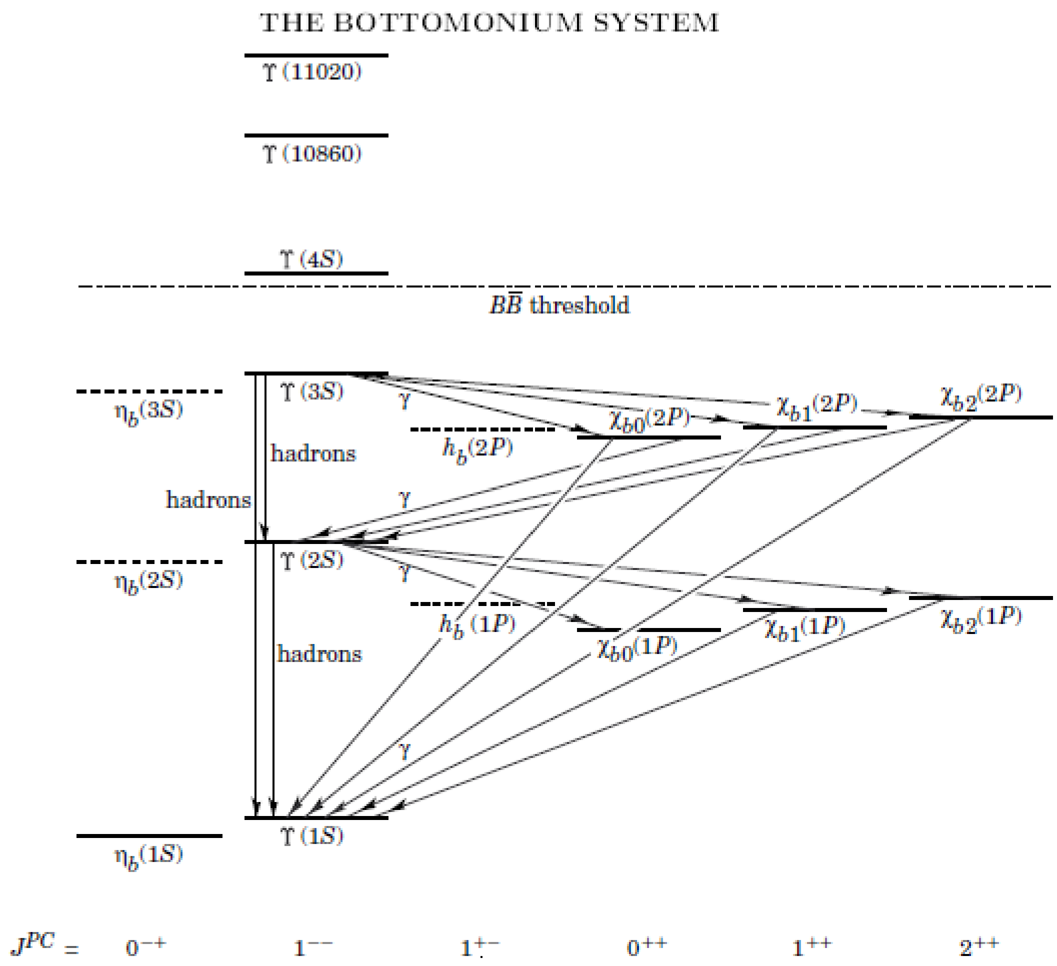
Quarkonium spectrum

- Charmonium, or bottomonium, spectrum is useful, because,
  - Low-lying spectrum experimentally very well known.
  - System is non-relativistic. Potential model works reasonably well. Can easily trace systematic errors.



$$S \otimes L = 1 \otimes 1 = 0 \oplus 1 \oplus 2$$
$$0 \otimes 1 = 1$$

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$


THE BOTTOMONIUM SYSTEM
ϒ(11020)
ϒ(10860)
ϒ(4S)
$B\bar{B}$ threshold
$\eta_b(3S)$
ϒ(3S)
γ
$\chi_{b0}(2P)$
$\chi_{b1}(2P)$
$\chi_{b2}(2P)$
$h_b(2P)$
hadrons
hadrons
$\eta_b(2S)$
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$h_b(1P)$
$\chi_{b0}(1P)$
$\chi_{b1}(1P)$
$\chi_{b2}(1P)$
hadrons
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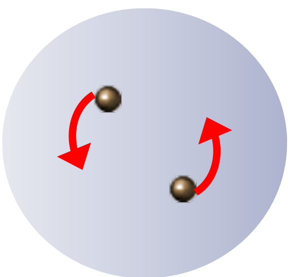
J^{PC} = 0^{-+} 1^{--} 1^{+-} 0^{++} 1^{++} 2^{++}

Non-relativistic dynamics



$$\frac{\langle p^2 \rangle}{2m_Q} \sim \frac{4}{3}\left\langle \frac{\alpha_s}{r} \right\rangle$$

$$v = \frac{\langle p \rangle}{m_Q} \sim \alpha_s$$

- Non-relativistic expansion

$$L_Q = Q^+\left[iD_0 + \frac{D^2}{2m_Q} + \frac{\sigma \cdot B}{2m_Q} + \frac{D\cdot E - E\cdot D}{8m_Q^2} + \frac{\sigma\cdot(D\times E - E\times D)}{8m_Q^2} + \frac{(D^2)^2}{8m_Q^3} + \ldots \right] Q$$

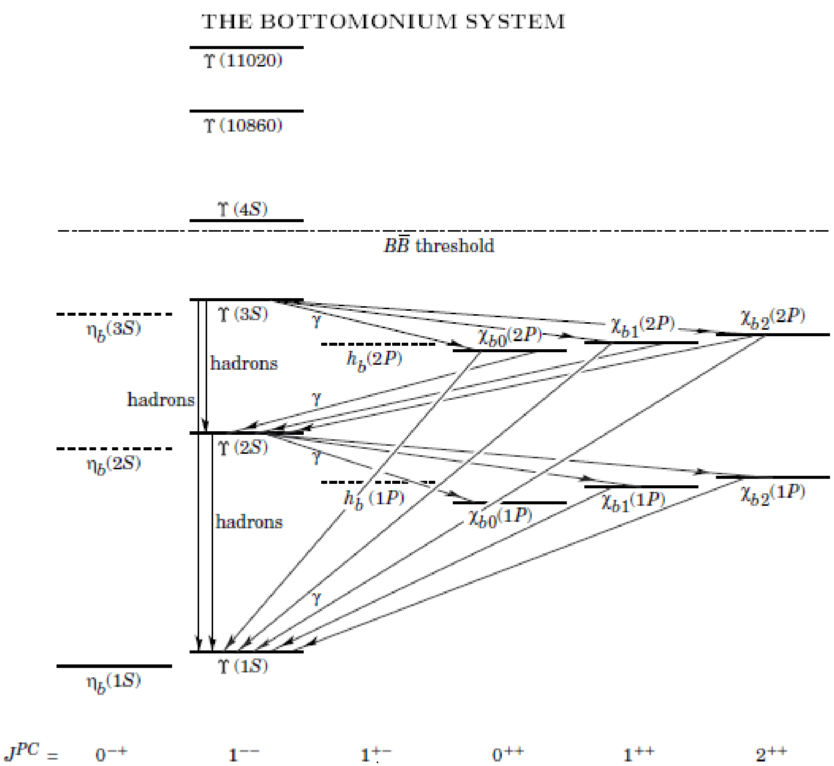
$\sim m_Q v^2$ $\qquad\qquad$ $\sim m_Q v^4$

- Expansion in terms of velocity

$$D \sim m_Q v, \quad D_0 \sim m_Q v^2, \quad E \sim m_Q^2 v^3, \quad B \sim m_Q^2 v^4, \ldots$$

- Leading order splittings
  - Radial (1S-2S, …), Orbital (1S-1P, …)
- Higher order corrections due to
  - Hyperfine splitting: $\sigma\cdot B$
  - Fine splitting (spin-orbit): $\sigma\cdot(D\times E)$

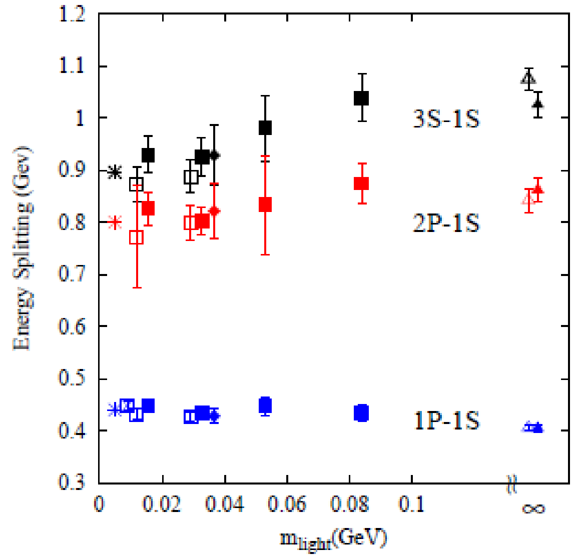
Spin-averaged splittings

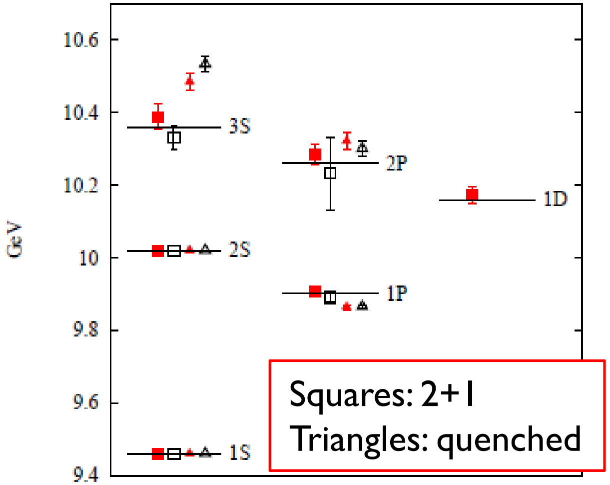


- 1S-1P or 1S-2S
  - S wave: $(m_{0-}+3m_{1-})/4$
  - P wave: $(m_{0+}+3m_{1+}+5m_{2+})/9$
- Insensitive to the details of the heavy quark lagrangian ($\sim v^4$)
- Insensitive to the precise value of m_Q
  - 1S-1P = 458 (c), 450 (b) MeV
  - 1S-2S = 606 (c), 569 (b) MeV

Somewhat accidental, due to a scaling $\sim m_Q \alpha_s^2$.

Recent lattice calc (bottomonium)





HPQCD-UKQCD (Gray et al., PRD72, 094507 (2007))

- On the 2+1-flavor MILC improved-staggered lattices
- Using the NRQCD action (corrected to v^6) for heavy quark.

- Sea quark mass dependence mild.
- Excellent agreement with the experimental values for 1P-1S, 2P-1S, 3S-1S
- Lattice spacing obtained to 2-3% level.

I. Strong coupling constant
3. Lattice calculation: conversion

Conversion

convert:
$\alpha_s^{MS}(\mu)=Z(\mu a)\ \alpha_{lat}(a^{-1})$

- Requires perturbative expansion, but the convergence is bad!

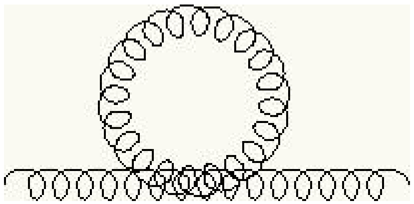
$$Z(\mu a=1)=1+5.9\alpha_{\rm lat}+43.4\alpha_{\rm lat}^2+\ldots \quad \text{at } n_f=0$$

Luscher-Weisz, NPB452, 234 (1995).

  - At β=6, α_{lat}=0.08, then Z=1+0.47+0.28+…
  - Not feasible to achieve an accurate determination,

- This is an example of the more general problem: poor convergence of lattice perturbation, if the bare lattice coupling is used
  - Solution given by Lepage-Mackenzie, PRD48(1993)2250.

Boosted coupling



Tadpole diagram leads to a quadratic divergence

A common choice:

$$u_0^4 \equiv \left\langle \frac{1}{3} \mathrm{Tr} U_{plaq} \right\rangle$$

Boosted coupling:

$$\tilde{g}_{lat}^2 = \frac{g_{lat}^2}{u_0^4}$$

- Correspondence between the lattice and continuum gauge fields

$$U_\mu(x) \equiv e^{iagA_\mu(x)} = 1 + iagA_\mu(x) - \frac{1}{2} a^2 g^2 A_\mu^2(x) + \ldots$$

  - The terms with higher powers of a are not really suppressed much, because of power divergences.

- Replace as $U_\mu(x) \to u_0 \left[1 + iagA_\mu(x) + \ldots \right]$

and use some non-perturbative input for u_0.

- Gauge action can be rewritten as

$$S_g = \sum \frac{1}{g_{lat}^2} \mathrm{Tr}(U_{\mathrm{plaq}} + \mathrm{h.c.}) = \sum \frac{1}{\tilde{g}_{lat}^2 u_0^4} \mathrm{Tr}(U_{\mathrm{plaq}} + \mathrm{h.c.})$$

$$\to \sum \frac{1}{4\tilde{g}_{lat}^2} F_{\mu\nu}^2 + \ldots$$

Prescription

- Reorganize the perturbation series
  - Example: the scheme conversion

$$\alpha_{MS}(\mu=1/a)=\alpha_{\rm lat}+5.9\alpha_{\rm lat}^2+43.4\alpha_{\rm lat}^3+\ldots$$

  - Expand in terms of $\tilde{\alpha}_{\rm lat}=\alpha_{\rm lat}/P$ using

$$P^{\rm pert}=1-4.189\alpha_{\rm lat}+5.355\alpha_{\rm lat}^2+\ldots$$

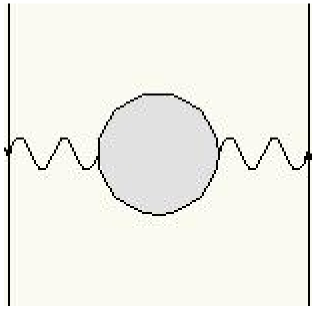
Namely,

$$\alpha_{MS}=P^{\rm pert}\frac{\alpha_{\rm lat}}{P}+5.9\left(P^{\rm pert}\right)^2\left(\frac{\alpha_{\rm lat}}{P}\right)^2+43.4\left(P^{\rm pert}\right)^3+\ldots$$

$$=\frac{\alpha_{\rm lat}}{P}+1.7\times\left(\frac{\alpha_{\rm lat}}{P}\right)^2-11.4\times\left(\frac{\alpha_{\rm lat}}{P}\right)^3+\ldots$$

Convergence of the series is much better when expanded in the boosted coupling.

Renormalized coupling



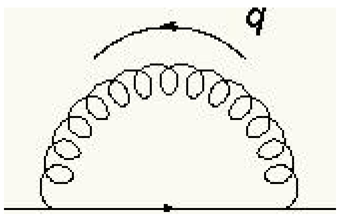
$$V(q) = -C_F \frac{\alpha_V(q)}{q^2}$$

- Another sensible way of defining the coupling constant: use a physical quantity, e.g. heavy quark potential.
  - Potential $V(q)$ defines $\alpha_V(q)$
  - Relation to other definition can be obtained by calculating $V(q)$.
    $$\alpha_V(q=1/a) = \alpha_{lat}\left[1 + 6.706\times\alpha + \ldots\right]$$
  - Note that it is much closer to MSbar.
    $$\alpha_{MS}(q) = \alpha_V(q)\left[1 - 0.822\times\alpha + \ldots\right]$$
  - Can be calculated non-perturbatively on the lattice (in principle). That means, a non-perturbative input.

Coupling determination

- Expansion using the renormalized coupling.
  - "Measure" $\alpha_V(q)$ through, e.g., the plaquete expectation value.

$$-\ln P = \frac{4\pi}{3}\alpha_V(q)\left[1+(4\pi\beta_0\ln(aq)-3.33)\alpha_V+\ldots\right]$$



$$\alpha \quad \rightarrow \quad \alpha(q)$$

  - The "best choice" for the scale q is estimated by an average momentum flow on the gluon line.

$$\ln(q^{*2}) \equiv \frac{\int d^4q\, f(q)\ln(q^2)}{\int d^4q\, f(q)}$$

    - Based on Brodsky-Lepage-Mackenzie, PRD28(1983)228.
    - For the plaquette, gives $q^*=3.40/a$, then

$$-\ln P = \frac{4\pi}{3}\alpha_V(3.40/a)\left[1-1.19\alpha_V+\ldots\right]$$

Conversion, again

- Now, the conversion can be done from α_V to α_{MS}, using the better behaved perturbative expansion.

$$\alpha_{MS}(q) = \alpha_V(q)\left[1 - 0.822\alpha_V(q) - 2.665\alpha^2 + \ldots\right]$$

Peter, PRL78(1997)602.

- Then, the determination of α_s is done up to relative $O(\alpha_s^3)$ corrections at a reference scale q (=3.40/a).

  - All the expressions correspond to the quenched QCD (N_f=0). Similar expressions available for general N_f.
  - Early calculations were done in N_f=0; some theoretical argument and guesstimate used to N_f=2 (or 3). Recent calculations are N_f=2(+1).
  - Numbers depend on the choice of the lattice action.

I. Strong coupling constant
4. Recent lattice calculations

Case study 1: HPQCD

Mason et al., PRL95, 052002 (2005).

- Uses the MILC 2+1 flavor simulations with the improved staggered fermion
  - Fast, U(1) chiral symmetry
  - Taste breaking: light hadron physics are affected, need the SChPT.
  - Heavy quarks less affected, comes from quark loops, which is perturbative except in the threshold region.
  - Rooting issues: not a valid QFT at finite a, probably okay in the continuum limit.
- Scale setting from Bottomonium spectrum
- Conversion to MSbar using automated PT through α_V

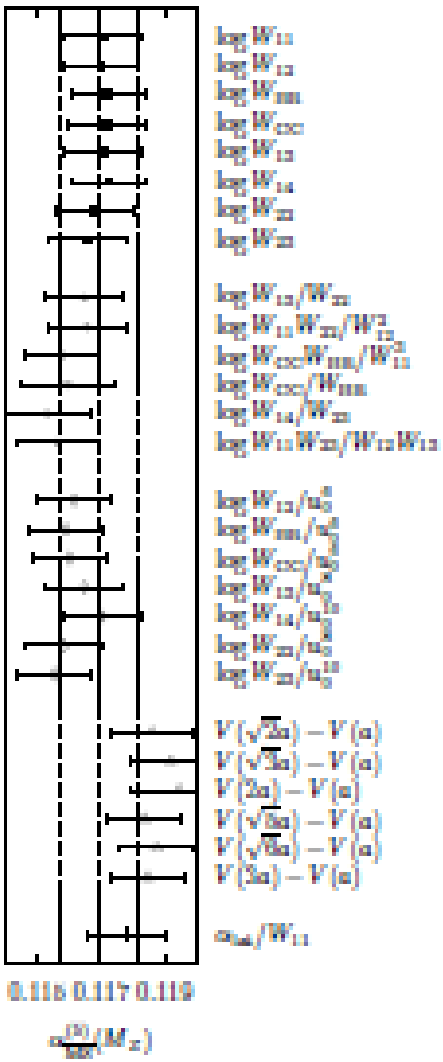
Automated perturbation theory

- Use a highly improved lattice action
  - Better scaling; but very complicated. Writing the Feynman rules is already too hard to do by hand. Need two-loop (or even three-loop) calculations.
  - Automated PT technique was developed (Trottier, Mason).
- Use many short distance quantities for the input of α_V.

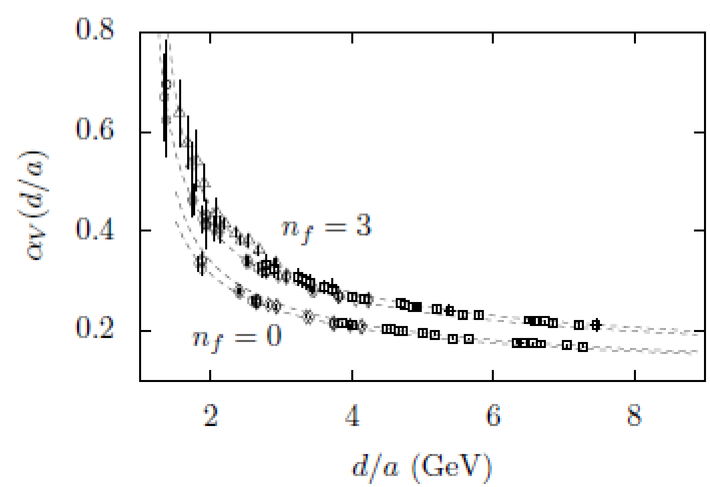
$$
\begin{aligned}
\log W_{11} &= -3.068\,\alpha_V(3.33/a)\,(1-1.068\,\alpha_V \\
&\quad +1.69(4)\,\alpha_V^2 - 5(2)\,\alpha_V^3 - 1(6)\,\alpha_V^4 \cdots) \\
\log W_{12} &= -5.551\,\alpha_V(3.00/a)\,(1-0.858\,\alpha_V \\
&\quad +1.72(4)\,\alpha_V^2 - 5(2)\,\alpha_V^3 - 1(6)\,\alpha_V^4 \cdots)
\end{aligned}
$$

  - PT calculated to α_V^2 ; higher orders are fitted with lattice data at three lattice spacings.

Simulation results



- Consistency checks
  - With many different (short distance) quantities
  - Obtained at different q*



- Final numbers:

$$\alpha_V^{(3)}(7.5\,\mathrm{GeV}) = 0.2082(40),$$

$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.1170(12).$$

Room for improvement?

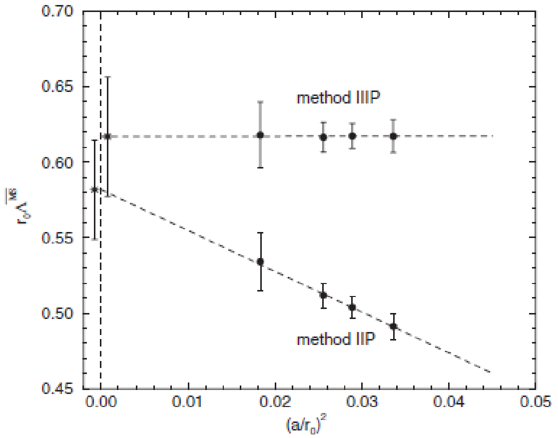
Sources of errors

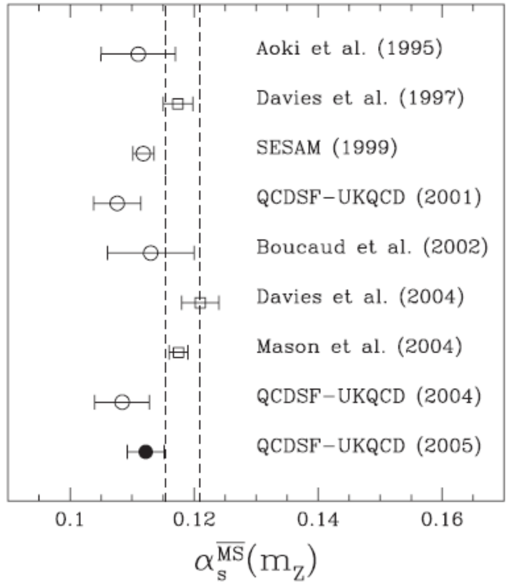
- Lattice spacing (<1% uncertainty)
  - 1.4%-3% depending on the β value.
  - Beyond this level, lattice spacing must be reduced to 0.05 fm. NRQCD may not be used (1/*am* too large).
  - Or, further improve gauge, light quark, NRQCD actions?
- Perturbative expansion (<1% uncertainty)
  - α_V^3 included. Even higher order calculation??

Case study 2: QCDSF-UKQCD

- Uses the non-perturbatively O(*a*)-improved Wilson fermion at N_f=2, combined with N_f=0
  - Four lattice spacings, the smallest *a*=0.07 fm.
- Scale setting from heavy quark potential (r_0=0.467 fm)
  - r_0 is easy to calculate, but not known experimentally.
  - This particular value is from a global fit of nucleon mass in N_f=2 data (CP-PACS, JLQCD, QCDSF, UKQCD).
  - MILC reported r_0=0.467(10) fm from a matching to the bottomonium spectrum.
- Coupling conversion including α_s^3 (NNLO)
  - With the boosted coupling.

QCDSF-UKQCD results





- Continuum limit for $r_0\Lambda$
  - Discretization effect nicely controlled.
- Extrapolation to N_f=3
  - Done by matching the force perturbatively.
  - Error is not really known.
- Final result

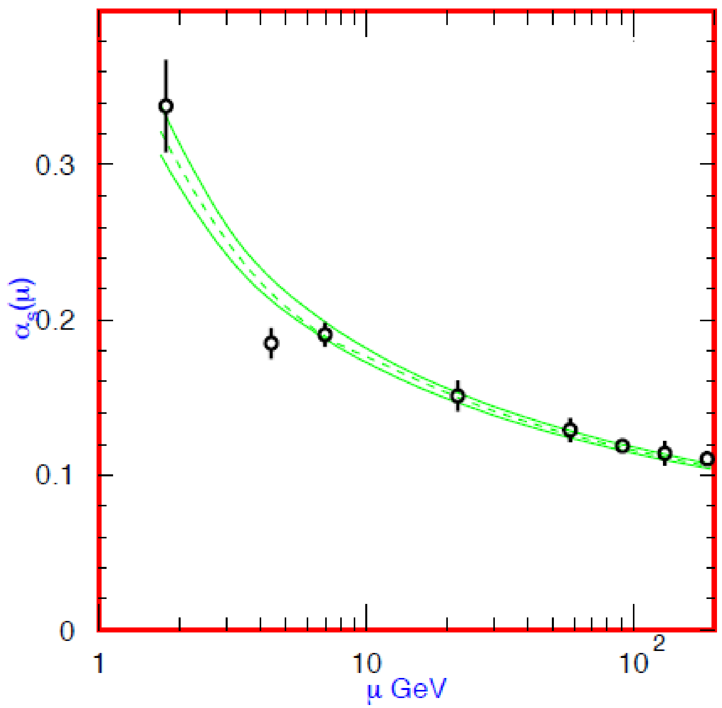
$$\alpha_{MS}^{(5)}(M_Z) = 0.112(1)(2).$$

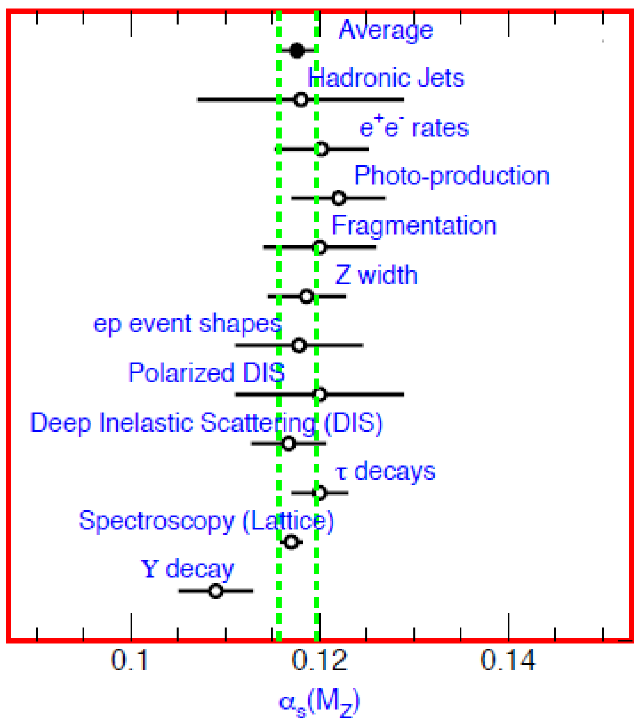
  - About 2σ lower than HPQCD with x2 larger error bar.

Comparison to phenomenological values

- Very nice agreement
  - Mason et al., "the QCD of confinement is the same theory as the QCD of jets"

PDG 2006





Further improvement…?

- Require non-perturbative matching
  - How? MSbar is defined within perturbation theory.
  - Possible by first going to very high scale, say 100 GeV, using non-perturbative running, and then convert to MSbar.
  - Called the *step scaling* (ALPHA collaboration).
  - Fully covered by Sint's lecture.