Superfluidity in the inner crust of neutron stars

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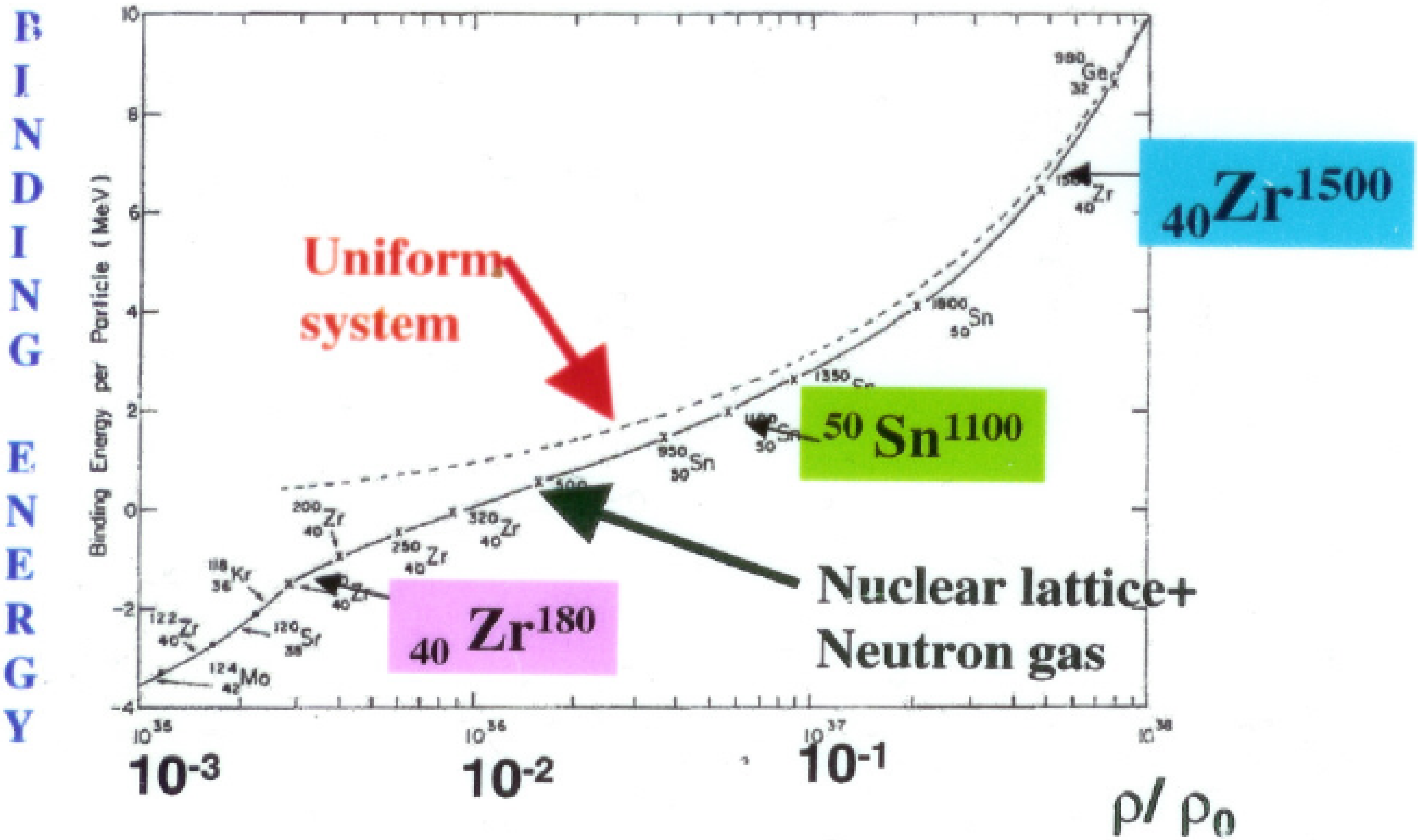
What is the role of the nuclear clusters on pairing correlations in the crust?

- Mean field level : spatial dependence of pairing, thermal effects, vortices
(cf. talks by Barranco, Sandulescu)

- Beyond mean field: induced interaction

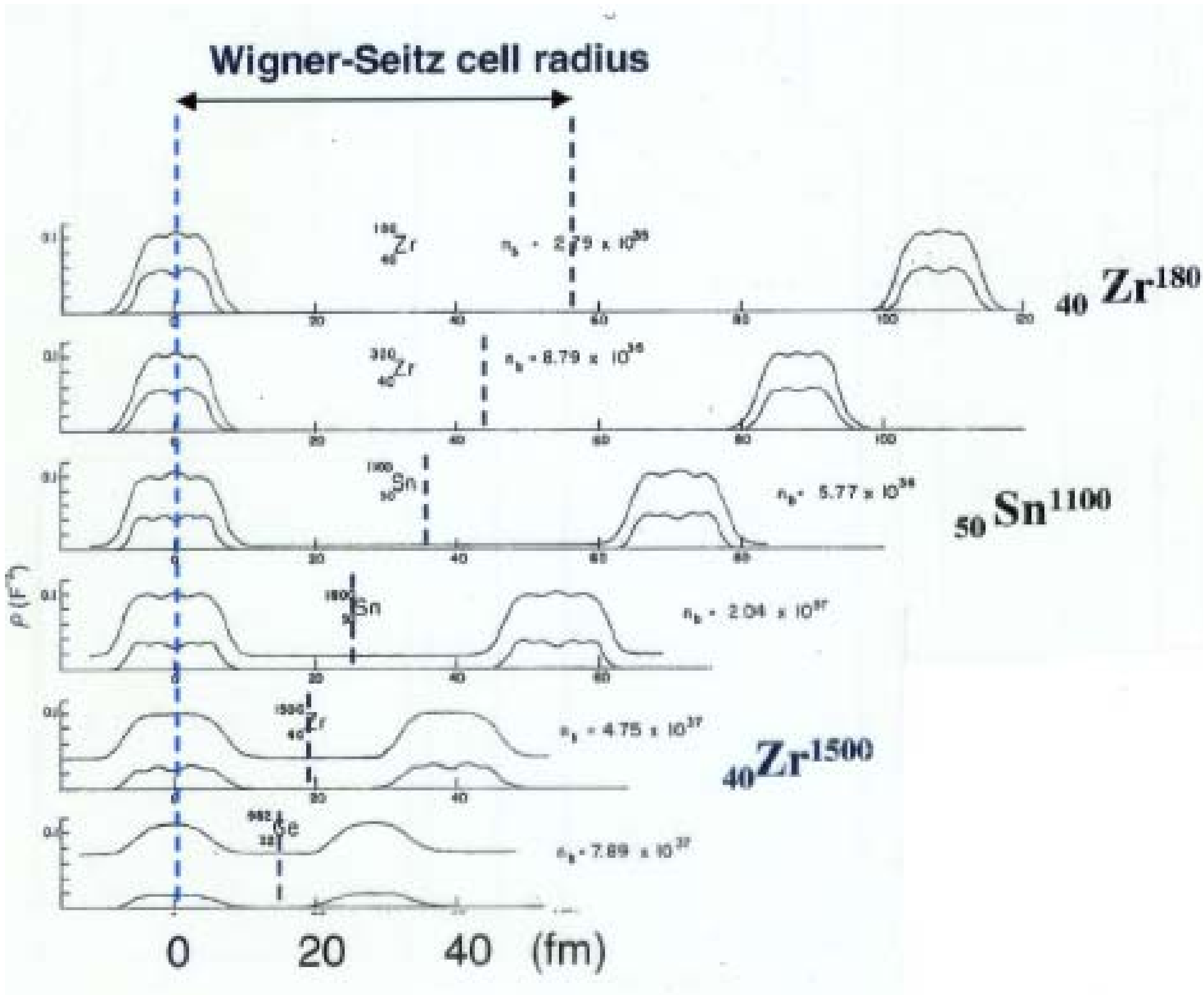
Assumption: Wigner-Seitz approximation (Negele-Vautherin results)
(cf. talks by Baldo, Margueron)

The inner crust: coexistence of a Coulomb lattice of finite nuclei with a sea of free neutrons



J. Negele, D. Vautherin
Nucl. Phys. A207 (1974) 298

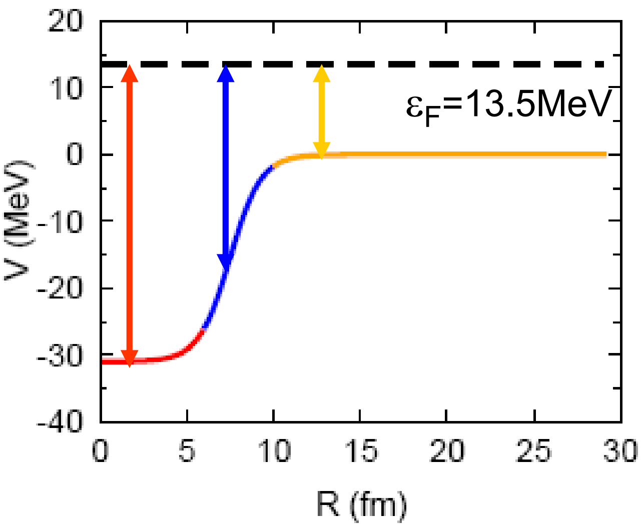
M. Baldo et al
Nucl. Phys. A750 (2005) 409



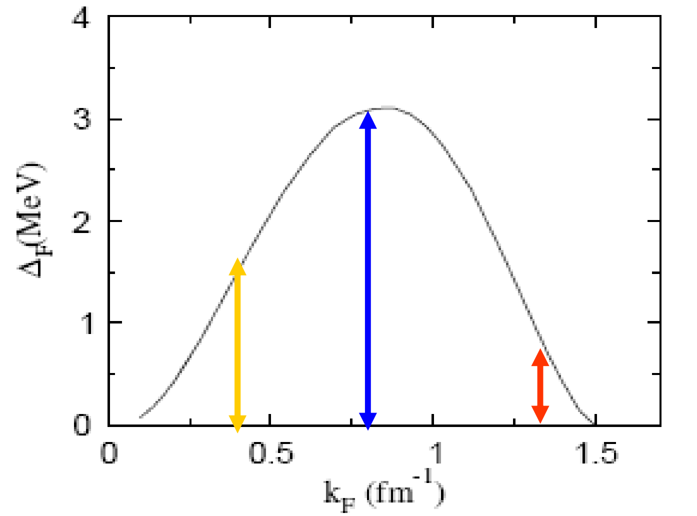
The Negele & Vautherin classical paper

Proximity effects on the pairing field

Potential in the Wigner cell



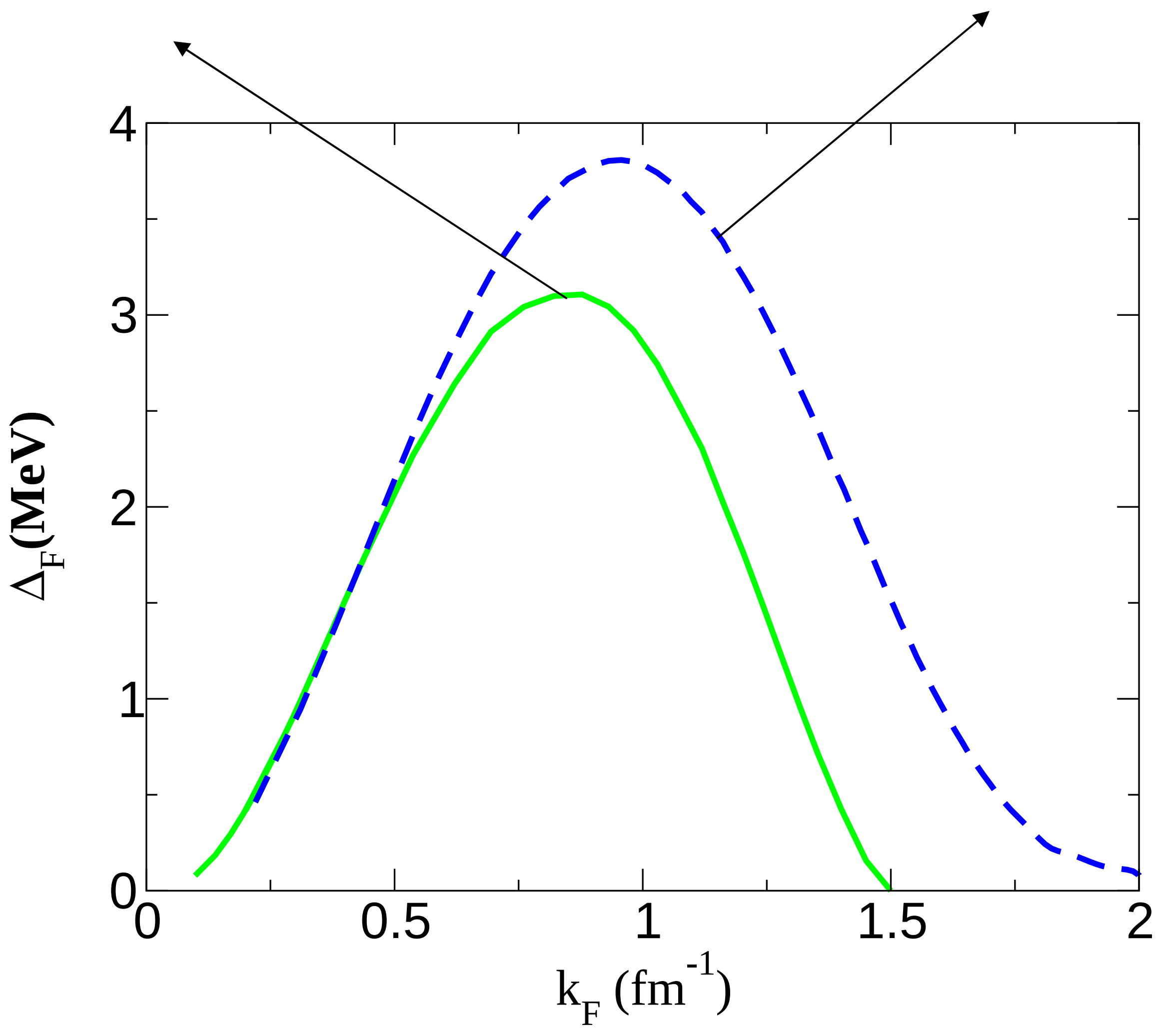
Pairing gap in uniform neutron matter



F. Barranco et al., PLB390 (1997)13
H. Esbensen et al., PRC58(1998)1257
P.M. Pizzochero, F. Barranco,
E. Vigezzi, R.A. Broglia, APJ 569(2002)381

N. Sandulescu et al., Phys. Rev. C70(2004)025801
C. Monrozeau et al., nucl-th/ 0703064

Dependence on the pairing interaction

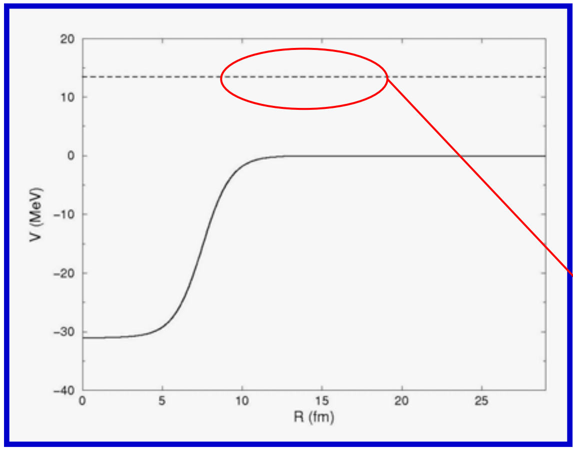


Presence of the cluster

Wigner–Seitz method:

- nucleons bound in a spherical box with radius equal to that suggested by Negele–Vautherin;
- Presence of nucleus accounted by a Wood–Saxon potential in the center of the box.

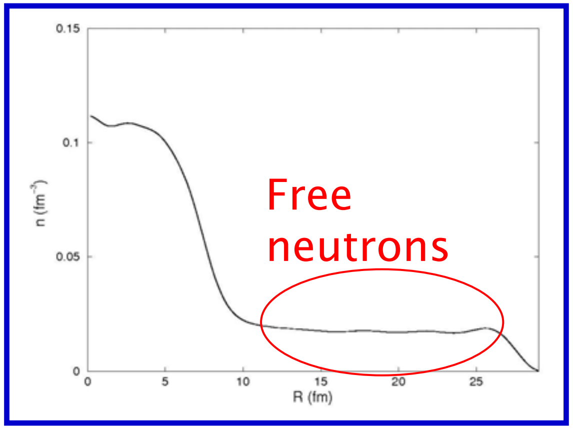
Wood–Saxon potential



Cell with:

- 1750 neutrons
- 110 bound neutrons
- Radius = 28 fm
- EF= 13.5 MeV

Density distribution

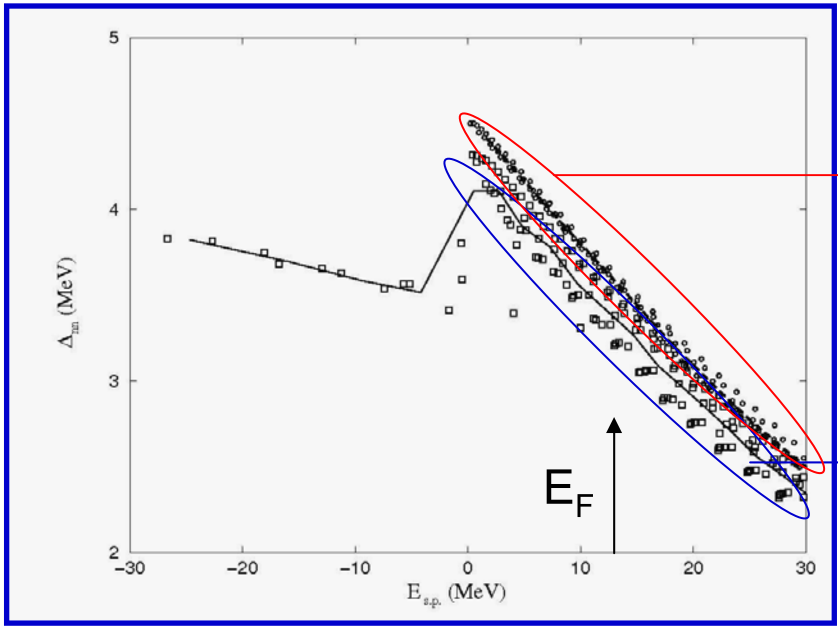


Gap equation

Single-particle wave functions used as a basis for simplified version of Hartree-Fock-Bogoliubov equation

$$\begin{pmatrix} (\epsilon_k - \epsilon_F) & \Delta \\ -\Delta & -(\epsilon_k - \epsilon_F) \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix} \qquad \Delta_{a_1 a_2} = -\frac{1}{2} \sum_{b_1 b_2} \sum_k U^k_{b_1} V^k_{b_2} \langle a_1 \tilde{a}_2 | v(12) | b_1 \tilde{b}_2 \rangle$$

- Simplified because the self-consistency was considered only in the pairing channel.



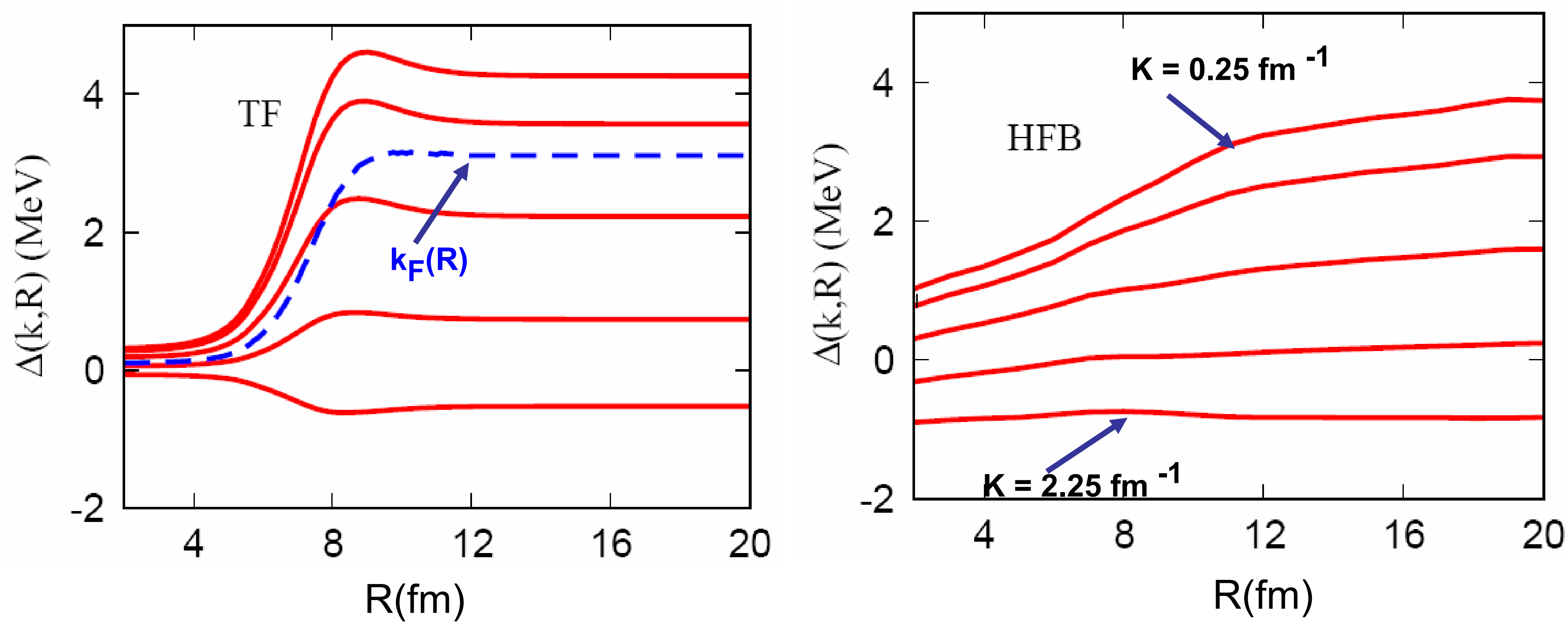
Calculated gaps for unbound states in a cell without nucleus

5-10% difference

Calculated gaps for unbound states in a cell with nucleus

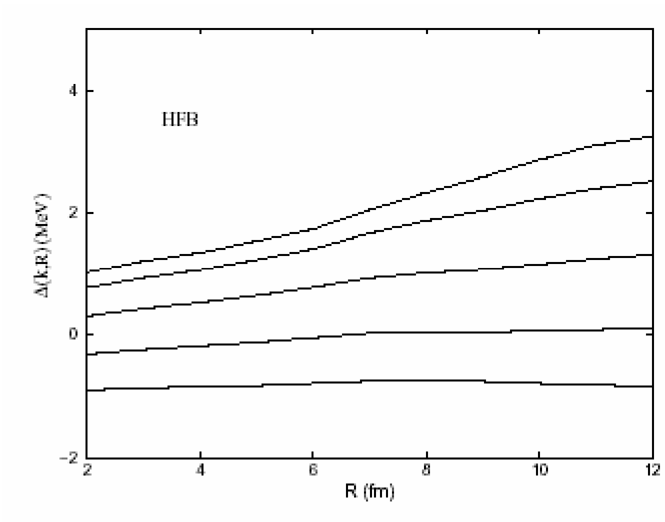
Spatial description of (non-local) pairing gap

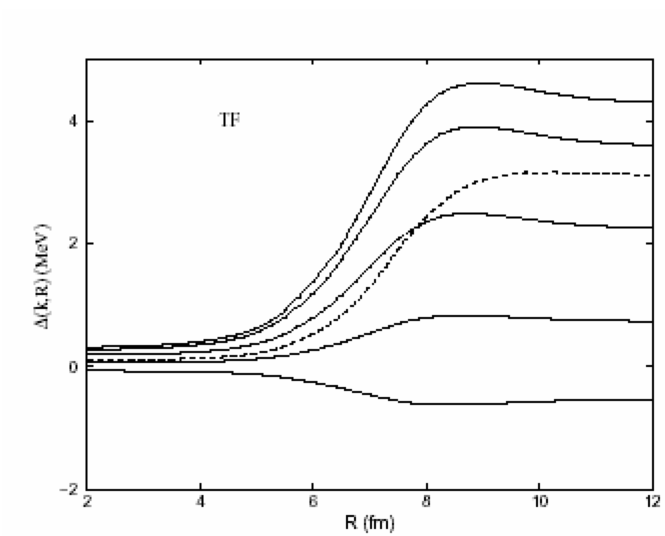
The range of the force is small compared to the coherence length, but not compared to the diffusivity of the nuclear potential



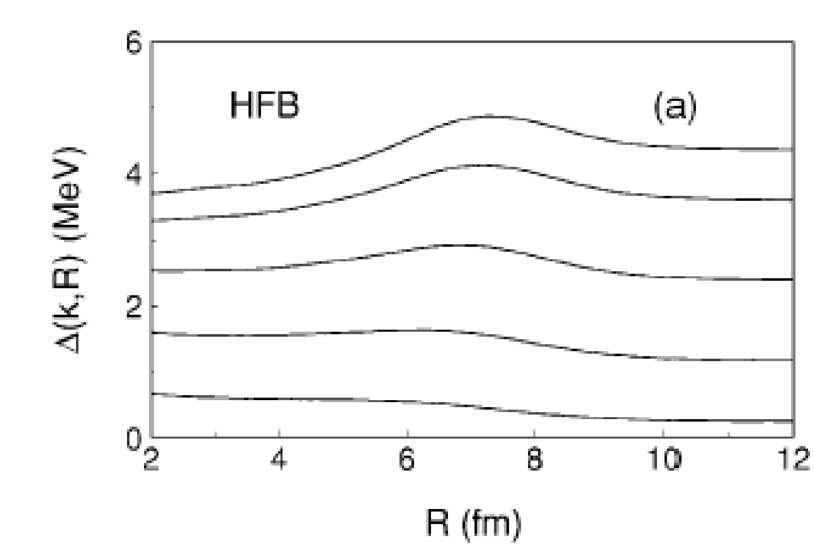
The local-density approximation overstimates the decrease of the pairing gap in the interior of the nucleus (proximity effects).

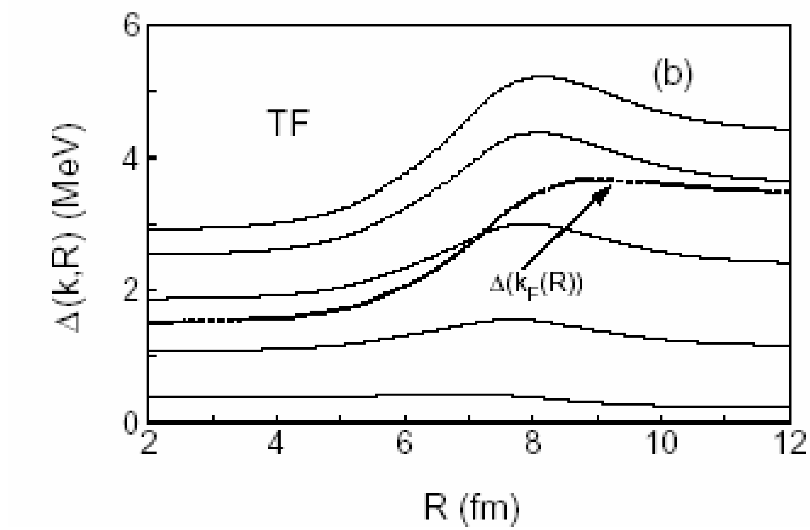
Argonne





Gogny





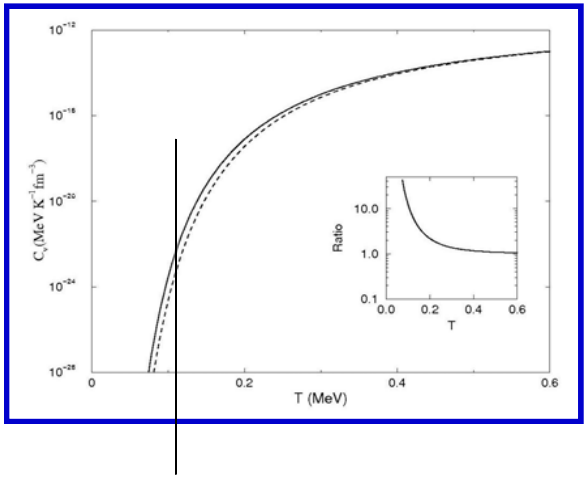
Observation of thermal emission → Constraints on internal structure

THERMAL EMISSION → THERMAL DIFFUSIVITY → SPECIFIC HEAT

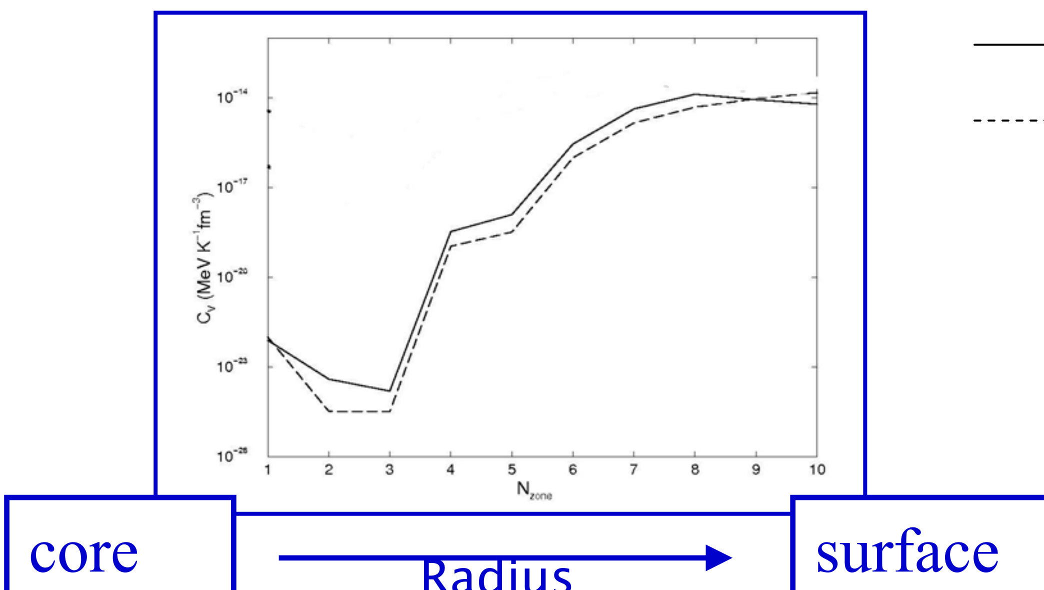
FOR A SUPERFLUID SYSTEM …

$$C_V^{(\mathrm{sf})} = C_V^{(\mathrm{norm})} \frac{\sqrt{2}}{\pi^{3/2}} \left(\frac{\Delta_F}{T}\right)^{5/2} e^{-\Delta_F/T}$$

Exponential dependence on Δ at the Fermi surface



At T=0.1 MeV the difference in the specific heat for calculation with or without nucleus is of an **order of magnitude**

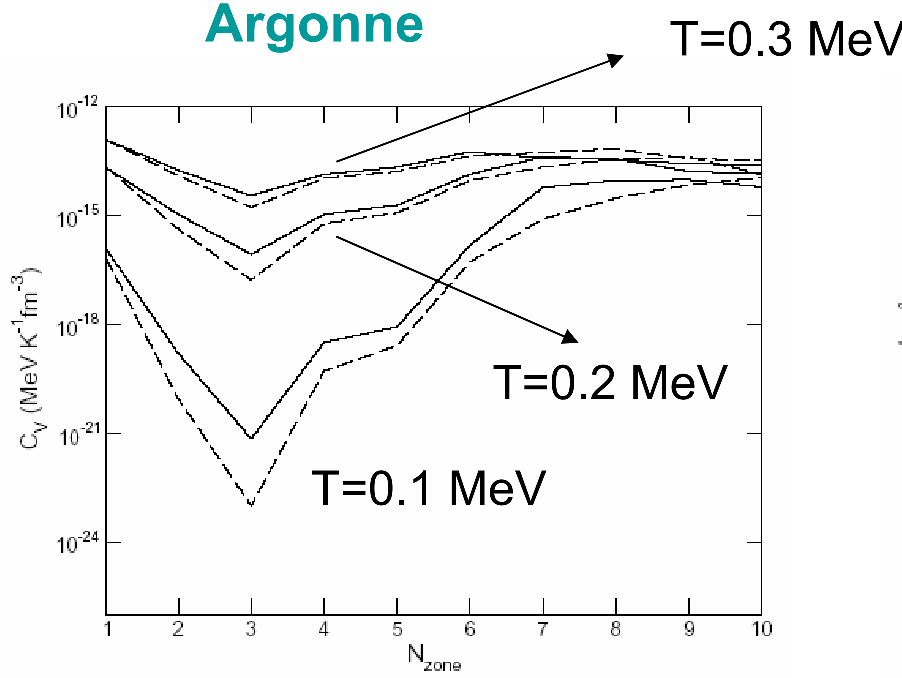


——— With nuclear cluster

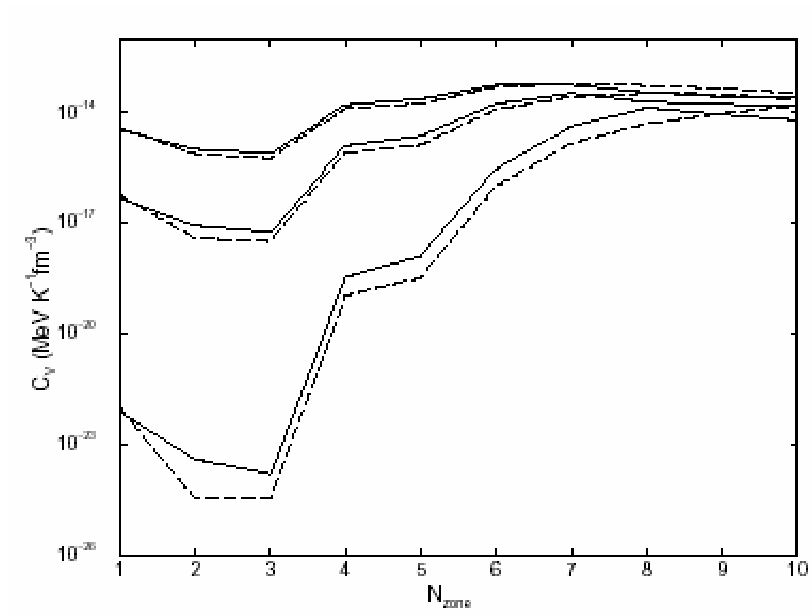
- - - - - - Without

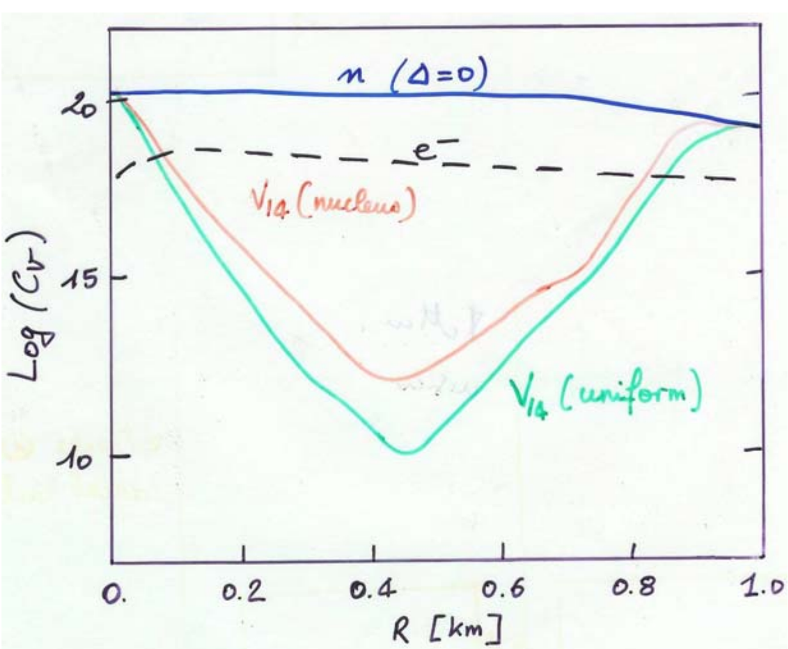
Role of nuclear cluster: (as a rule) it decreases the pairing gap and increases the specific heat

Argonne



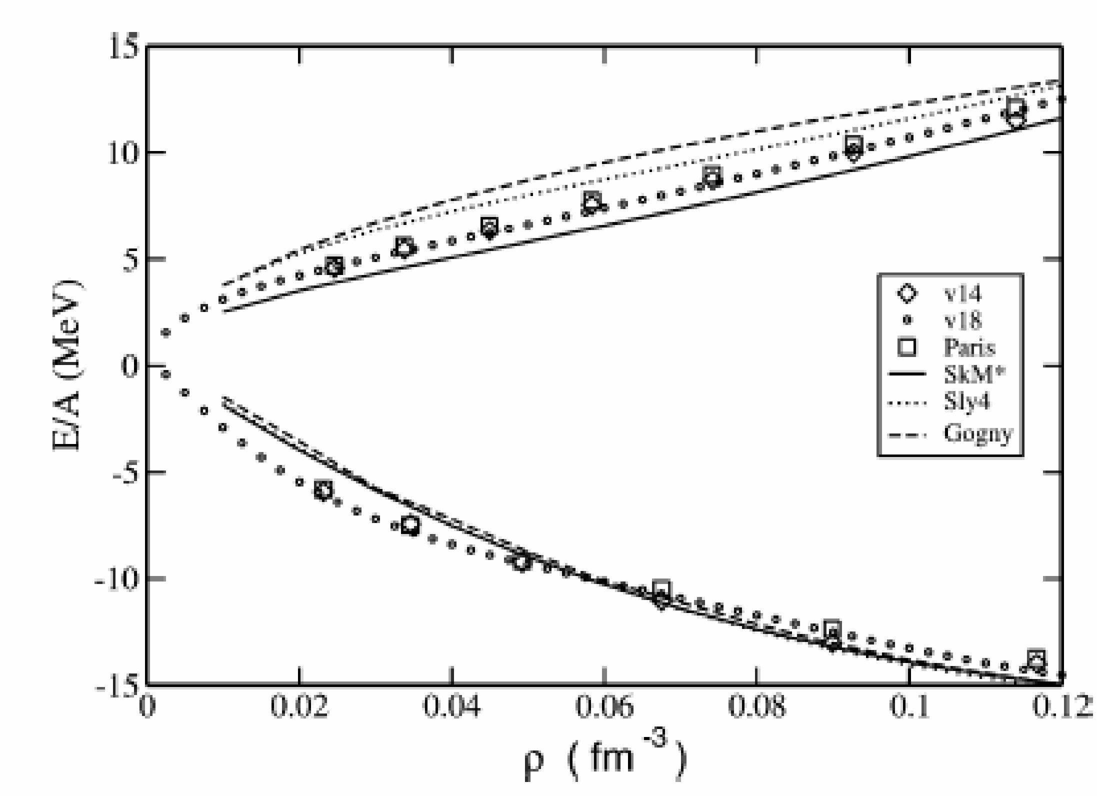
Gogny





The presence of nuclear clusters Influence the specific heat, but the main uncertainty is the absolute value of the gap

Calculations with self-consistent Hartree-Fock fields



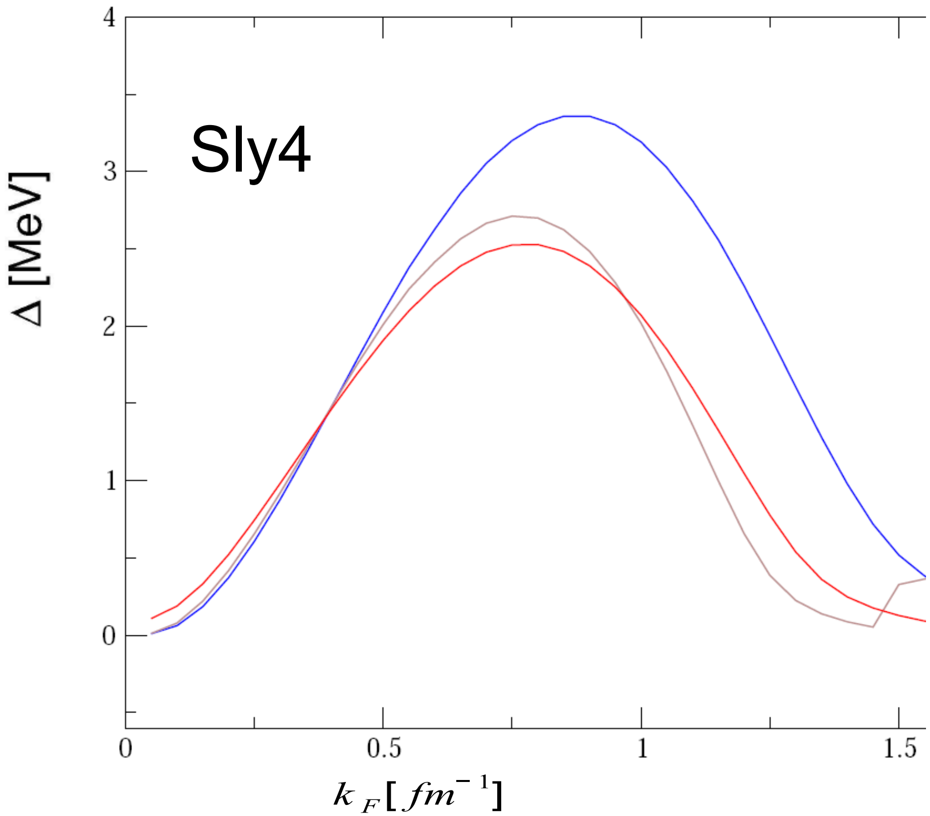
M. Baldo, C. Maieron, P. Schuck, X. Vinas , Nucl. Phys. 736(2004)241

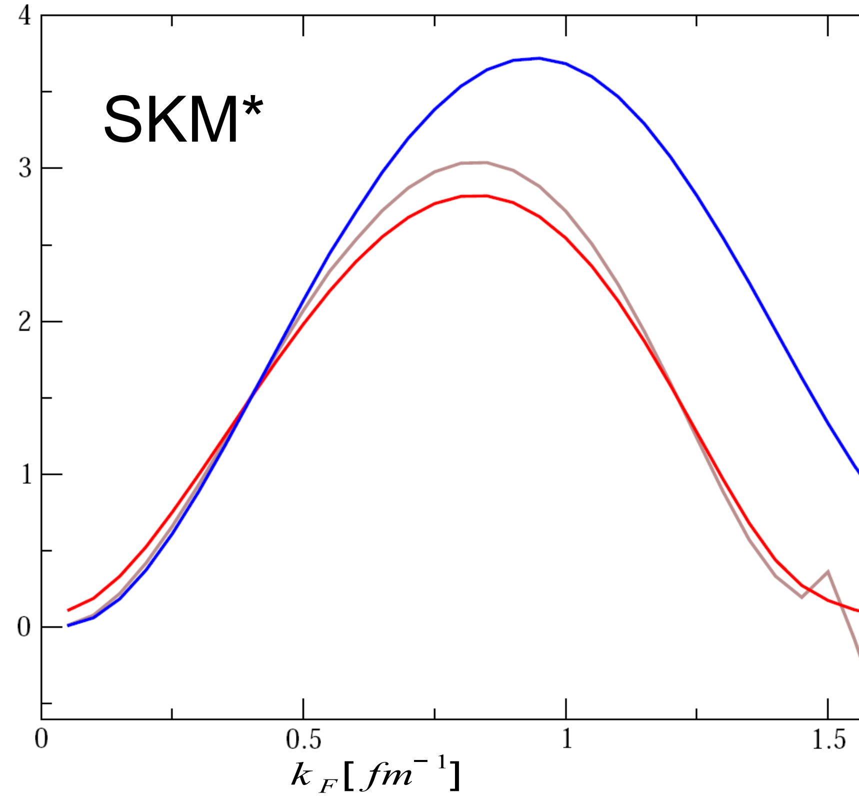
ZERO-RANGE INTERACTIONS

Self-consistent Hartree-Fock field
Density dependent pairing interaction with 60 MeV cutoff

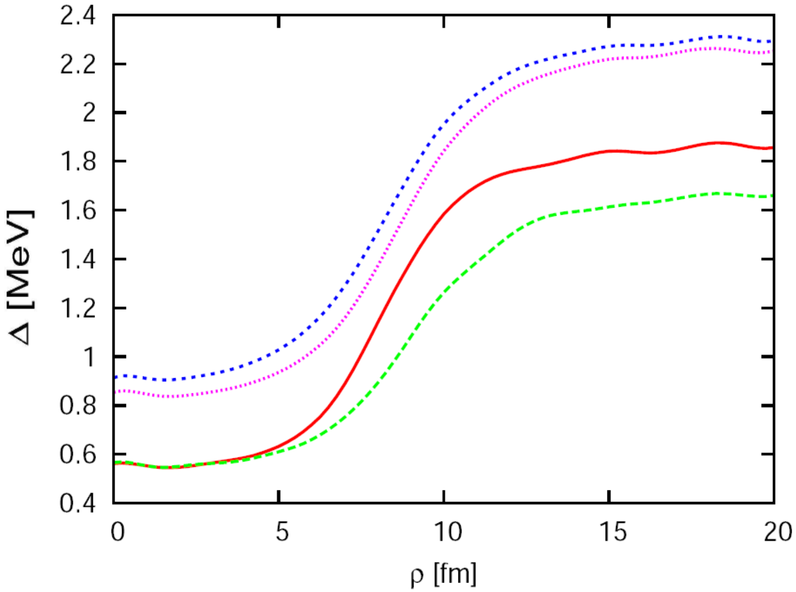
$$V_{eff}(\rho(R_{cm})) = -481\,(1 - 0.7(\rho/\rho_0)^{0.45})\ \delta(r_1 - r_2)\ MeV\ \ fm^3$$

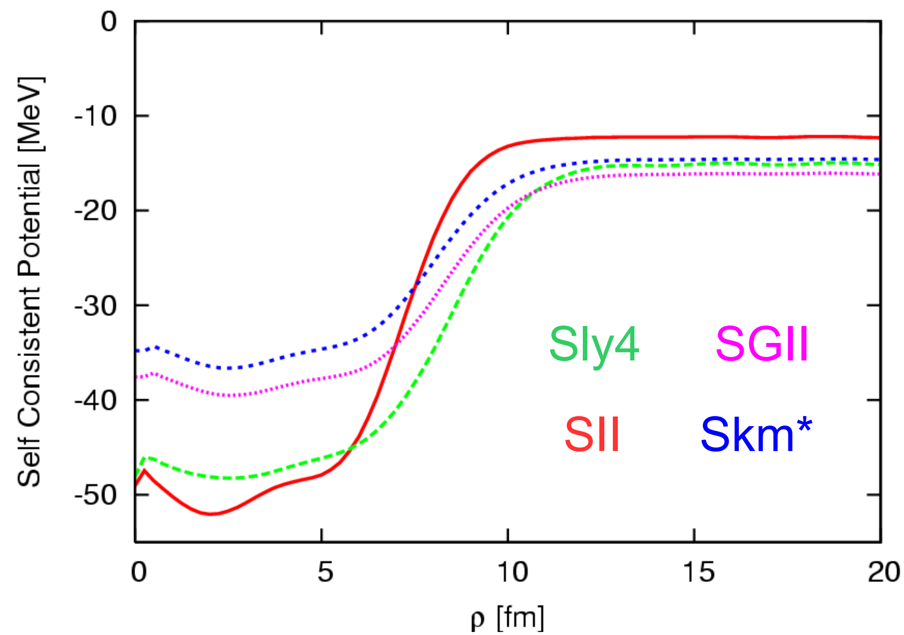
E. Garrido et al. Phys. Rev. C60(1999)64312

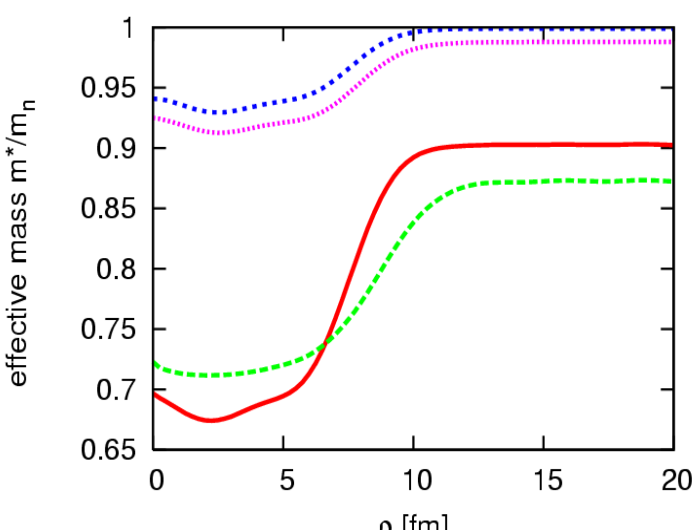




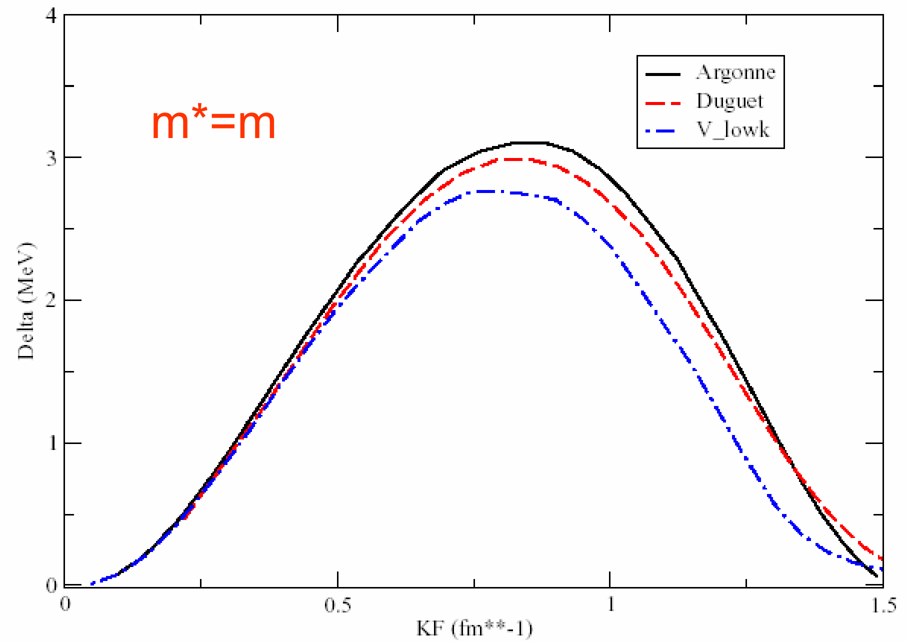
Spatial description of pairing gap calculated with different HF fields.

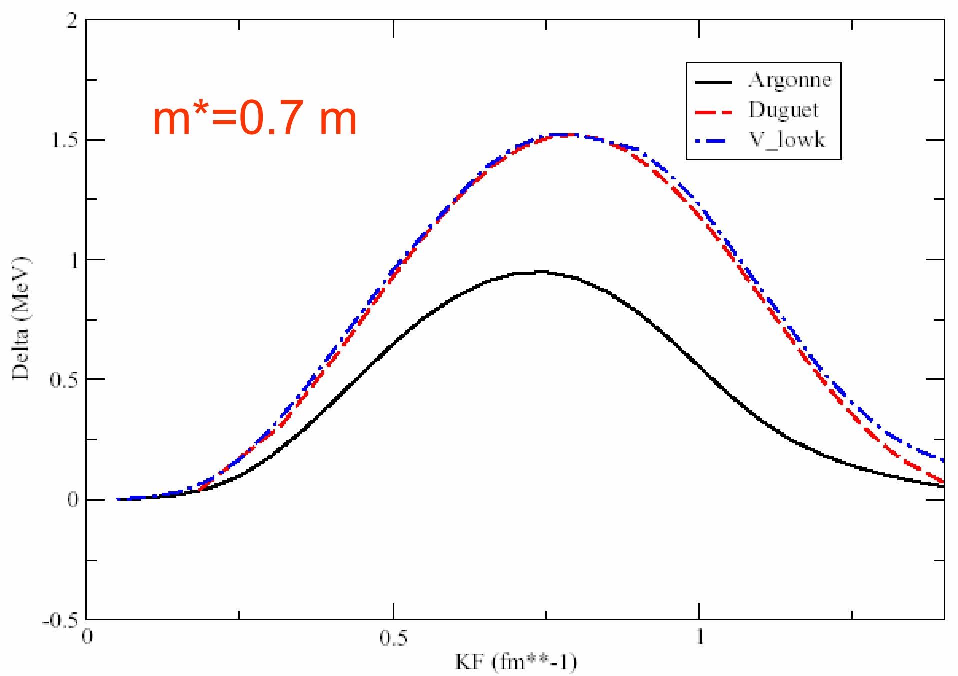






The difference in effective mass leads to important consequences in vortex pinning
(Talk by Barranco)


m*=m
Argonne
Duguet
V_lowk
Delta (MeV)
KF (fm**-1)
0
1
2
3
4
0
0.5
1
1.5


m*=0.7 m
Argonne
Duguet
V_lowk
Delta (MeV)
KF (fm**-1)
-0.5
0
0.5
1
1.5
2
0
0.5
1

Beyond the mean-field approximation

Main uncertainty: many-body effects

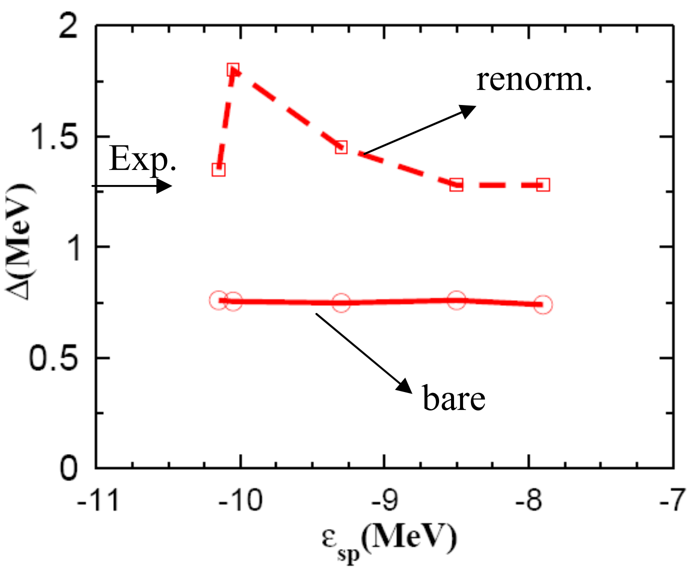
A reasonable first approximation: just reduce the gap in accordance with neutron matter results (Baldo, Sandulescu…).

However, one would like to consider in detail the interface between the cluster and the neutron sea. This is essential for vortex pinning!

First attempt: neglect self-energy effects (low density), only include induced interaction from the exchange of medium fluctuations.

Calculations are performed in a parallel way in atomic nuclei ($E_f<0$) and In the inner crust ($E_f>0$)

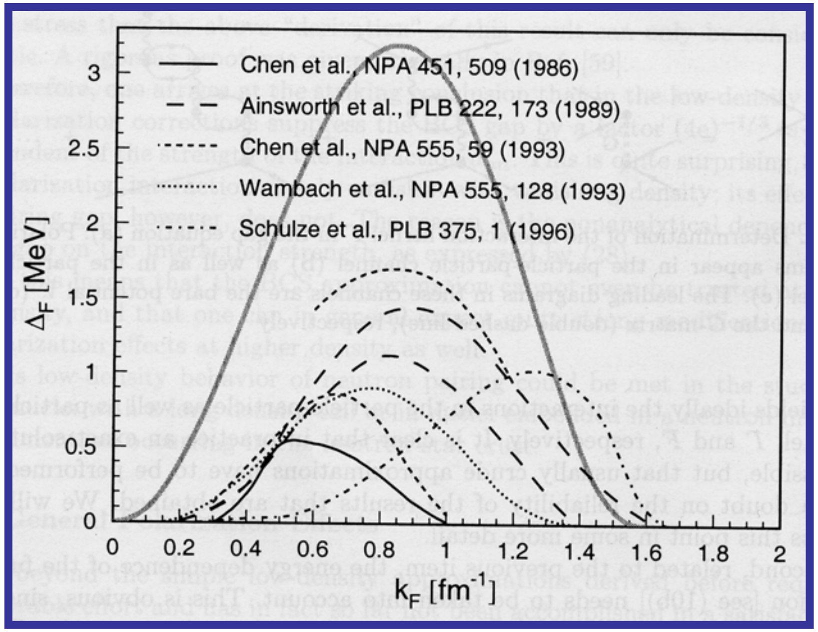
PAIRING GAP IN FINITE NUCLEI



Medium effects **increase** the gap in ^{120}Sn

F. Barranco et al., Eur. J. Phys. A21(2004) 57

PAIRING GAP IN NEUTRON MATTER



Medium effects **decrease** the gap

Induced interaction with effective Skyrme forces

Density channel
(S = 0)

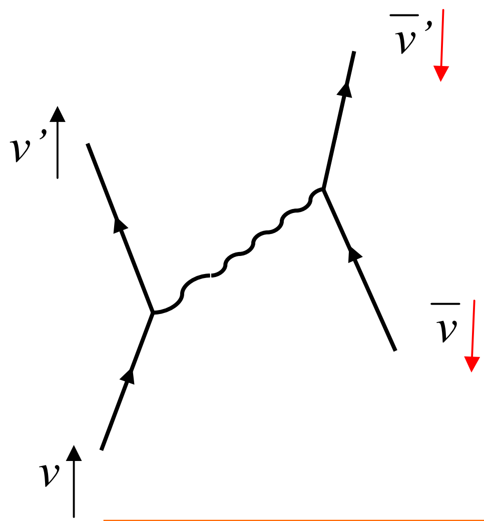
Skyrme interaction

Spin channel
(S = 1)

$$V(\vec{r},\vec{r'}) = [F_0(r) + F'_0(r)\vec{\tau}_1 \cdot \vec{\tau}_2]\,\delta(\vec{r}-\vec{r'}) + [G_0(r) + G'_0(r)\,\vec{\tau}_1 \cdot \vec{\tau}_2]\,\delta(\vec{r}-\vec{r'})\,\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

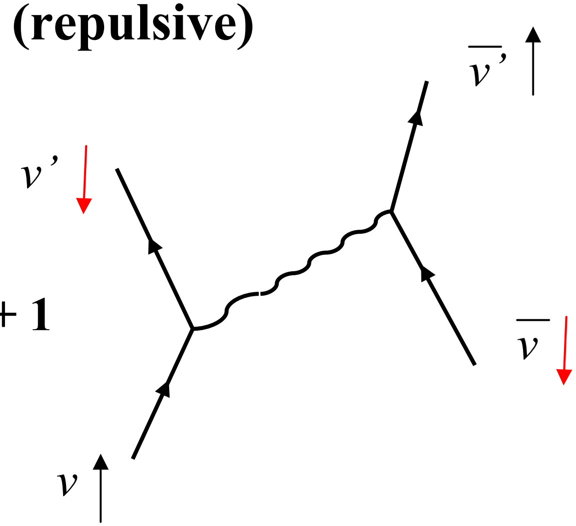
S=0 (attractive)

$S_z = 0$



S=1 (repulsive)

$S_z = +1$



$$2\sum_{J^\pi M i}\frac{(f^2)^{\nu'm'}_{\nu m;J^\pi Mi} - (g^2)^{\nu'm'}_{\nu m;J^\pi Mi}}{E_0-E_{int}}$$

$$f^{\nu'm'}_{\nu m;J^\pi Mi} = i^{l-l'}\langle j'm'|(i)^J Y_{JM}|jm\rangle \times \int dr\, \varphi_{\nu'}\,[\,(F_0+F_0')\,\delta\rho^i_{J^\pi n} + (F_0-F_0')\,\delta\rho^i_{J^\pi p}]\,\varphi_\nu$$

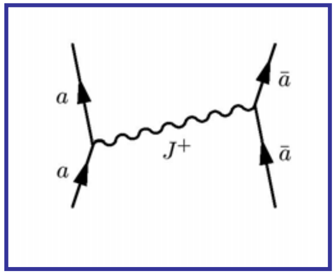
$$g^{\nu'm'}_{\nu m J^\pi Mi} = \sum_{L=J-1}^{J+1} i^{l-l'}\langle j'm'|(i)^L[Y_L\times\sigma]_{JM}|jm\rangle \times \int dr\,\varphi_{\nu'}\,[\,(G_0+G_0')\delta\rho^i_{J^\pi Ln} + (G_0-G_0')\,\delta\rho^i_{J^\pi Lp}]\,\varphi_\nu$$

FINITE NUCLEI (^{120}Sn):

The induced interaction arising from the coupling to surface and spin modes is attractive and leads to a pairing gap of about 0.7 MeV (50 % of the experimental value).Excluding the coupling to spin modes, the gap increases to about 1.1 MeV.

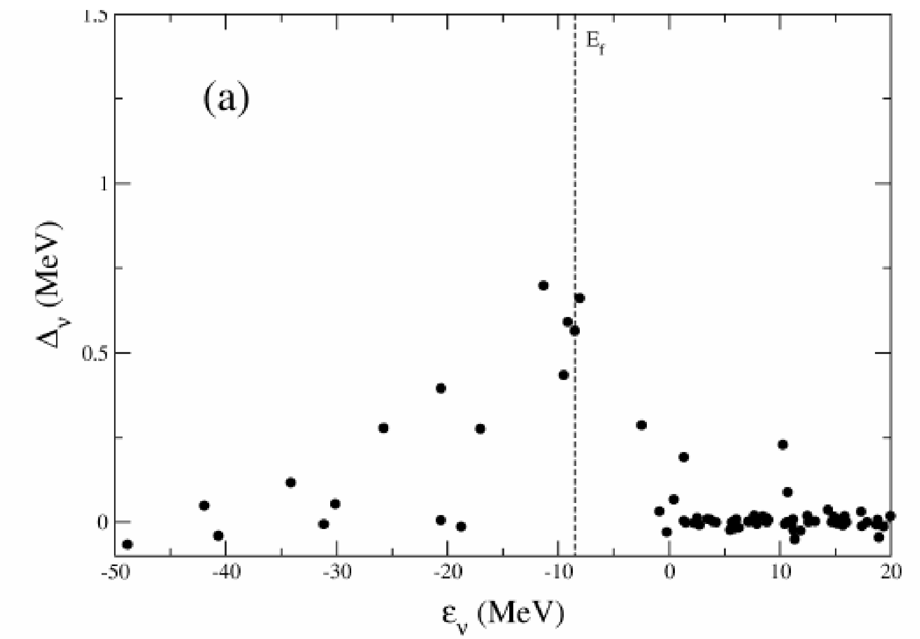
One must then add the bare interaction.

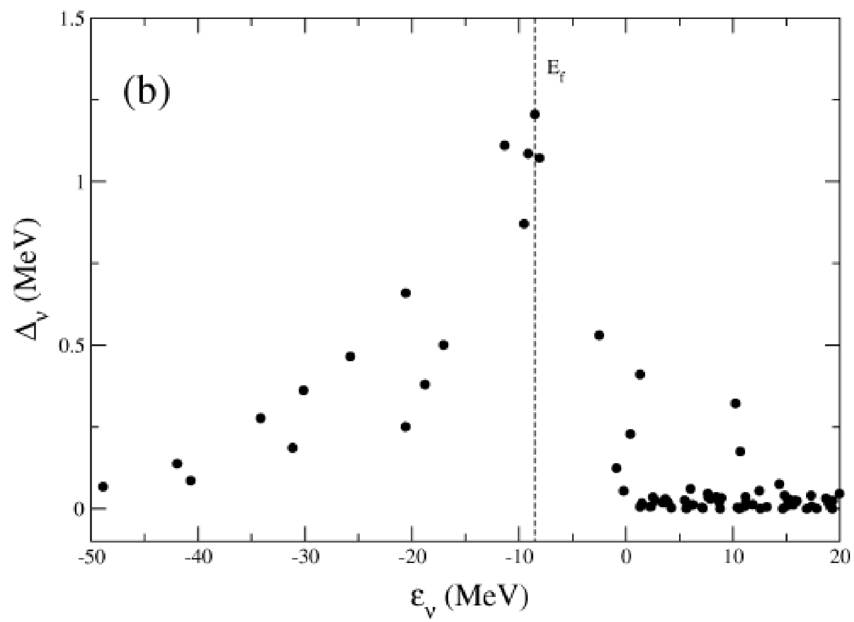
G. Gori et al.,
Phys. Rev. C72(2005)11302



Surface + Spin modes

Surface modes only



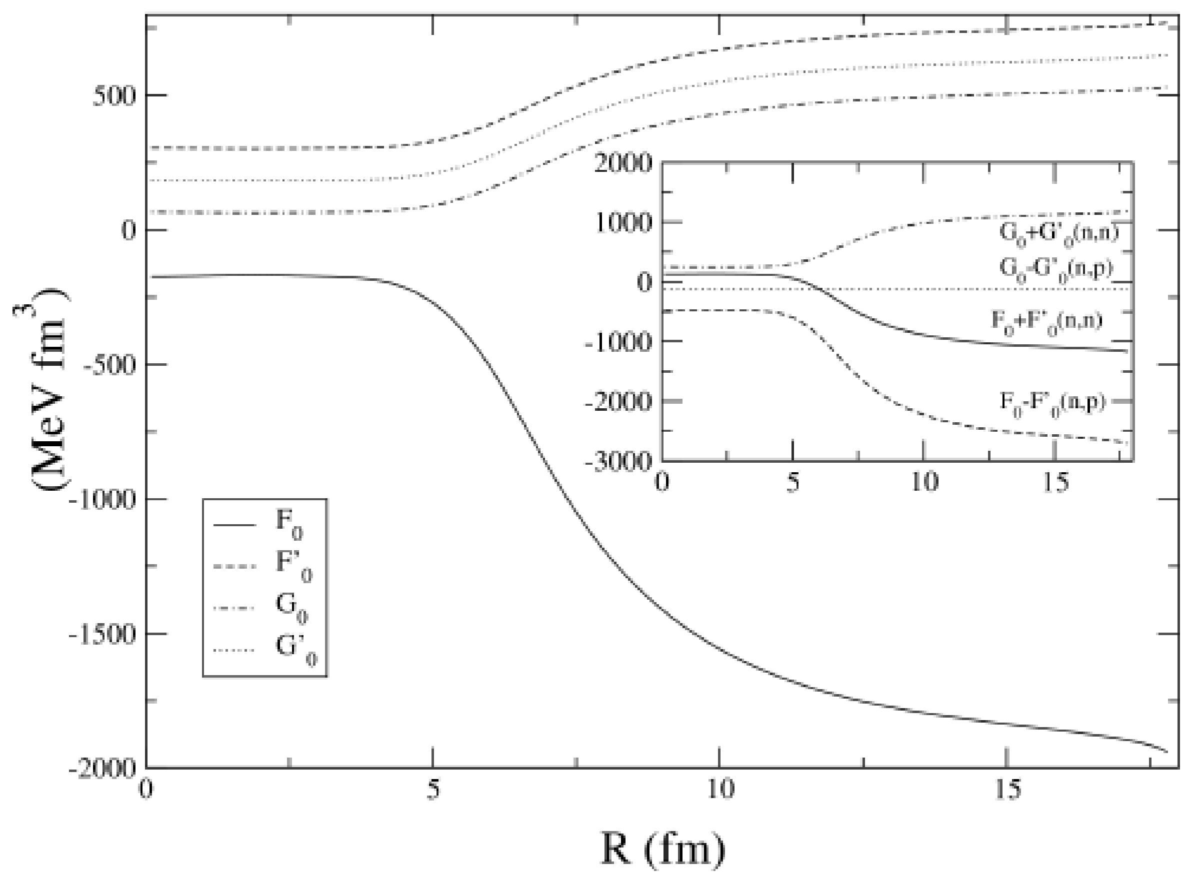


Why such a difference with neutron matter?

The proton-neutron interaction in the particle vibration coupling plays an essential role. If we cancel it, a net repulsive effect is obtained for the induced interaction.

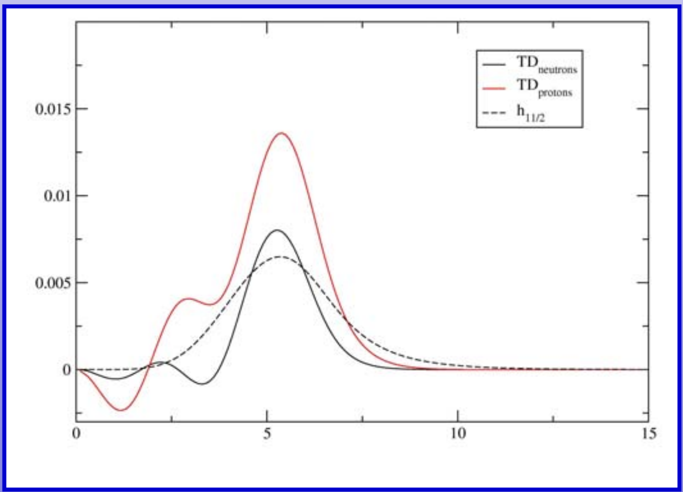
Strong difference between induced interaction in neutron and nuclear matter

Landau parameters of SkM* force in ^{120}Sn



Why such a difference with neutron matter?

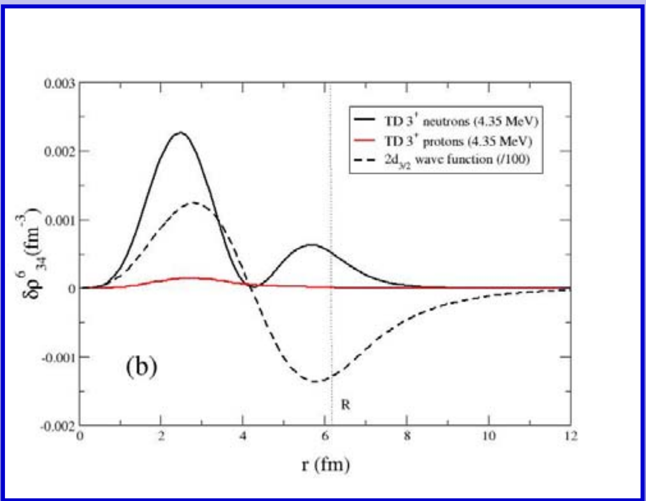
Crucial: the surface nature of density modes. This assures an important overlap between the transition density and the single-particle wave-function at the Fermi energy.



$$\langle j'm'JM|V_{res}|jm\rangle = \quad (-)^{M+J}\langle j'm'|(i)^J Y_{J-M}|jm\rangle \int \mathrm{d}r \varphi_{j'}(r)\varphi_j(r)\zeta_{JL}(r),$$

$$\zeta_{JL}(r) = [F_0(r) + F_0'(r)]\rho^{(1)\lambda}_{JL_n}(r) + [F_0(r) - F_0'(r)]\rho^{(1)\lambda}_{JL_p}(r).$$

Volume nature of Spin-modes



$$\langle j'm'JM|V_{res}|jm\rangle = \quad (-)^{J+M+1}\sum_{L=J-1}^{J+1}\langle j'm'|(i)^L[Y_L \times \sigma]_{J-M}|jm\rangle \int \mathrm{d}r \varphi_{j'}(r)\varphi_j(r)\xi_{JL}(r),$$

$$\xi_{JL}(r) = [G_0(r) + G_0'(r)]\rho^{(1)\lambda}_{JL_n}(r) + [G_0(r) - G_0'(r)]\rho^{(1)\lambda}_{JL_p}(r).$$

Induced interaction and proximity effects in neutron stars

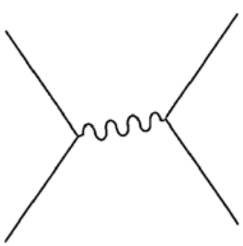
Wigner-Seitz cell

HFBCS — Skyrme interaction

↓

QRPA — residual interaction derived from mean field potential

↓



v_{ind}

Pairing calculation (in Wigner-Seitz cell)

- single-particle levels from Skyrme interaction
- pairing interaction matrix elements:

$$v = v_{induced} + v_{bare}$$

Pairing gap Δ

Induced interaction in the inner crust: computational difficulties

- high level density
- large number of particle-hole configurations (up to 5000)
- convergence needs calculation of phonons up to high multipolarity (J ~ 30h) (natural and unnatural parities)
- need for parallel version of the codes

Limitations of the calculation

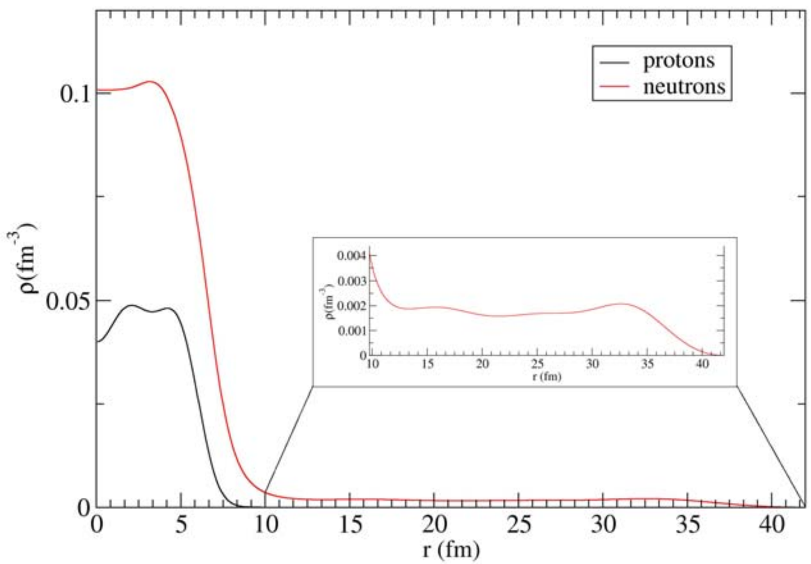
- RPA calculation
- No self-energy effects

Example of calculation: mean field (HF) – cell ^{588}Sn

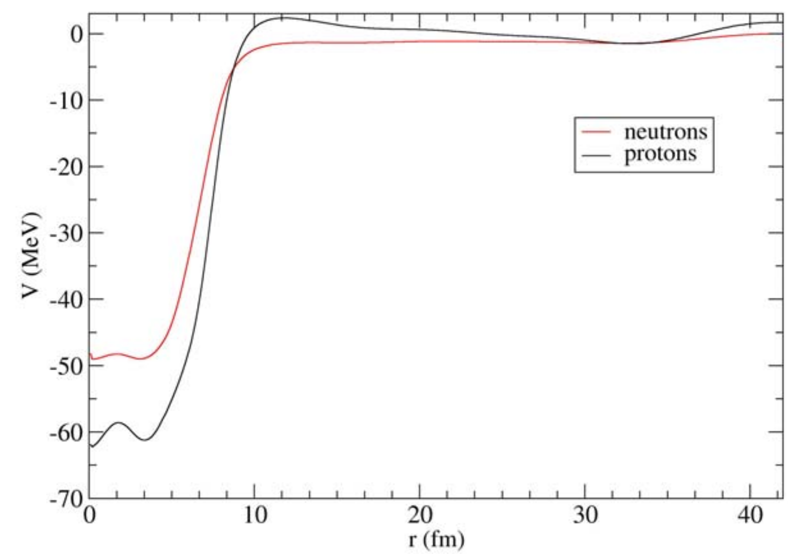
R = 42 fm $\rho = 0.020\,fm^{-3} \approx 0.13\rho_0$ $E_F^n = 1.7\,MeV$

SkM* interaction $k_F = 0.33\,fm^{-1}$

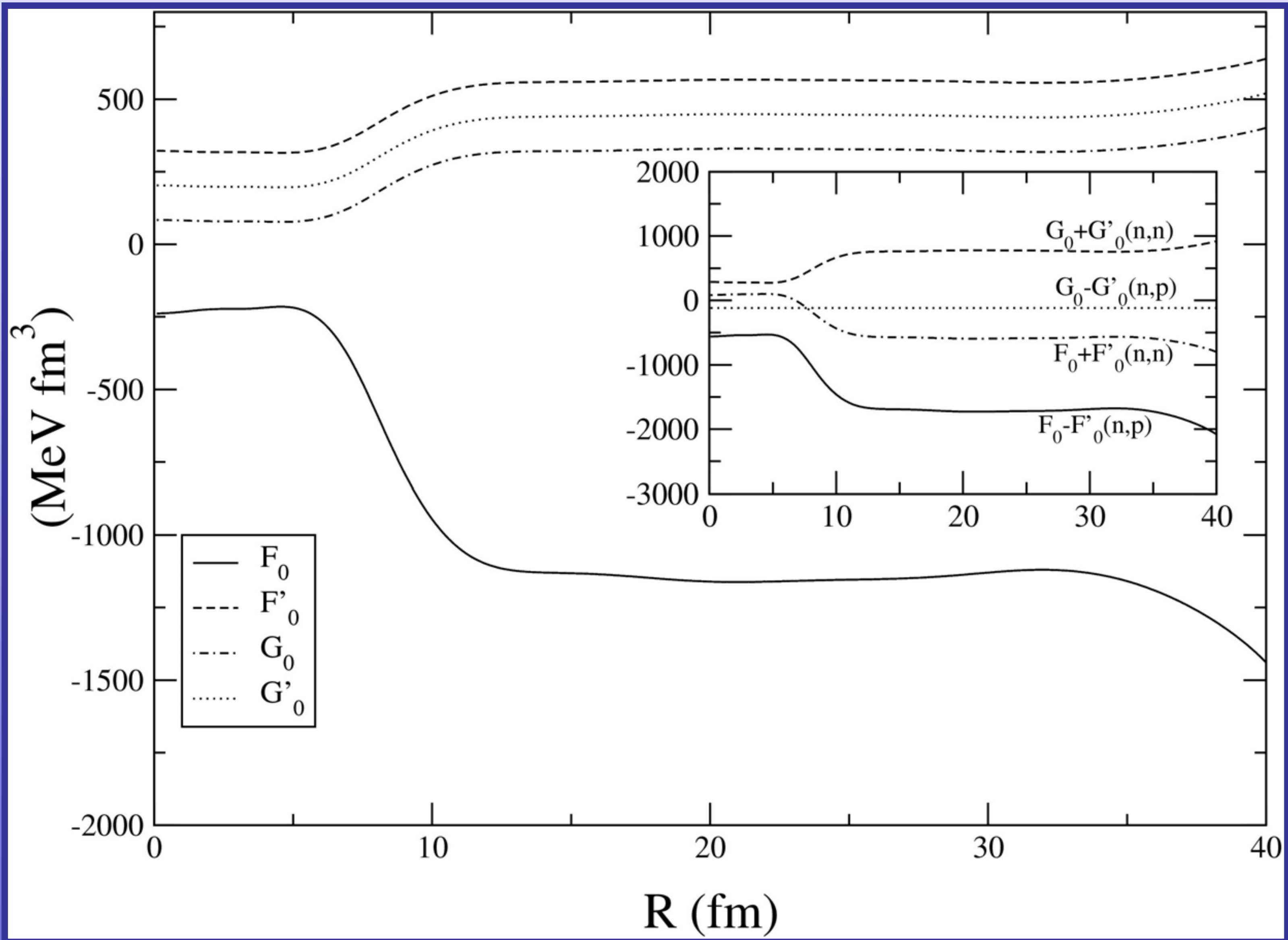
proton and neutron densities



proton and neutron potentials

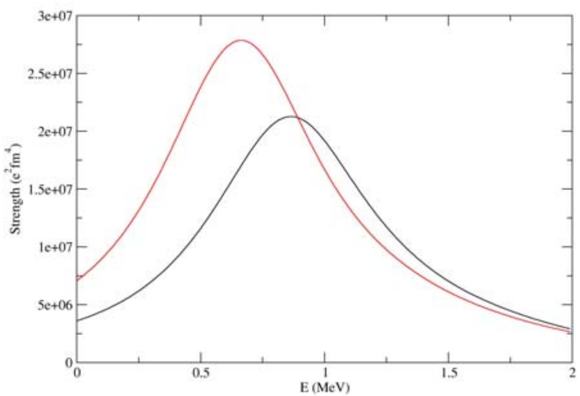


Landau–Migdal parameters

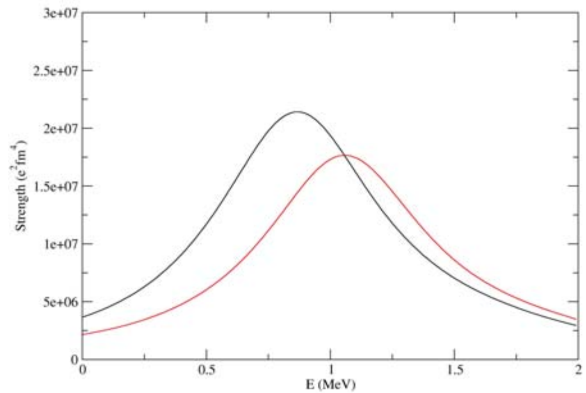


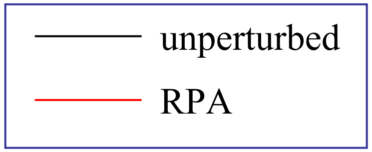
Example of calculation: response to external fields (RPA) – cell ^{588}Sn

$F(r)=r^2\ Y_2$

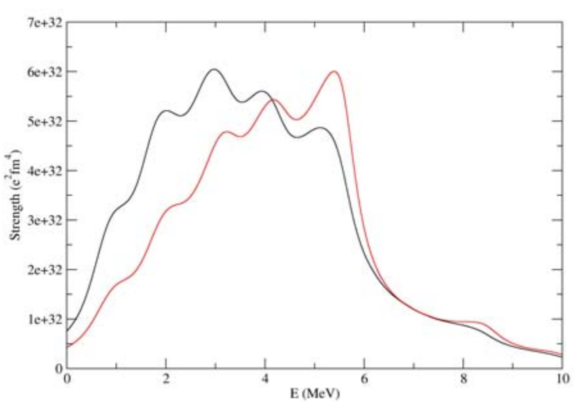


$F(r)=r^2\ [Y_2 x \sigma]$

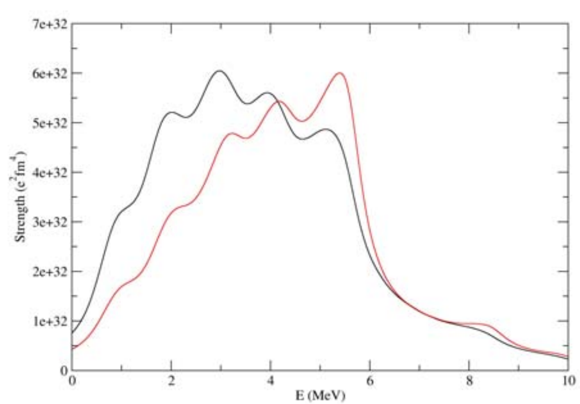




$F(r)=r^{10}\ Y_{10}$

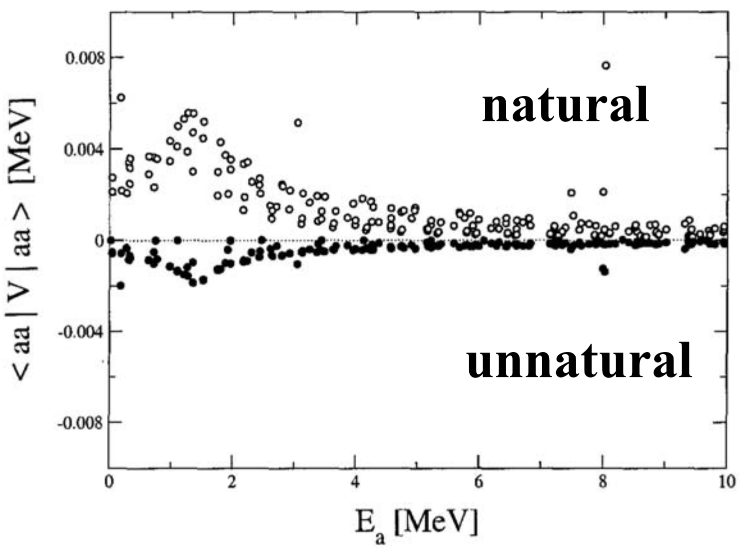


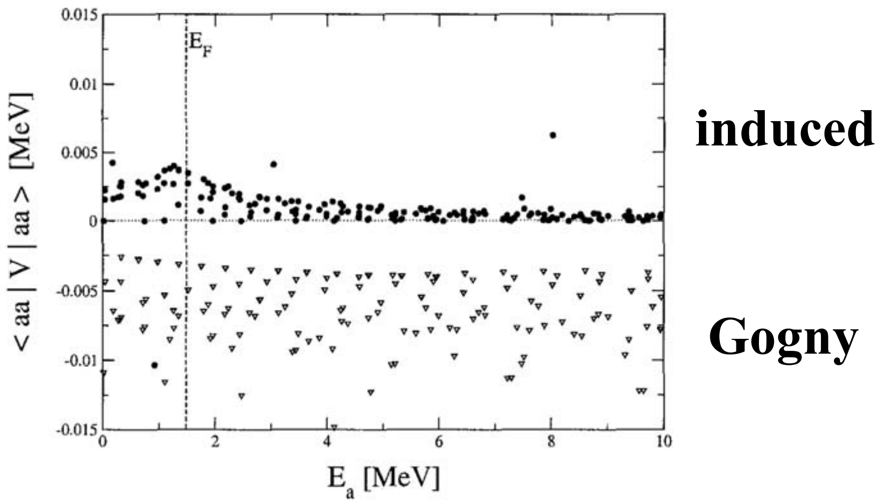
$F(r)=r^{10}\ [Y_{10} x \sigma]$

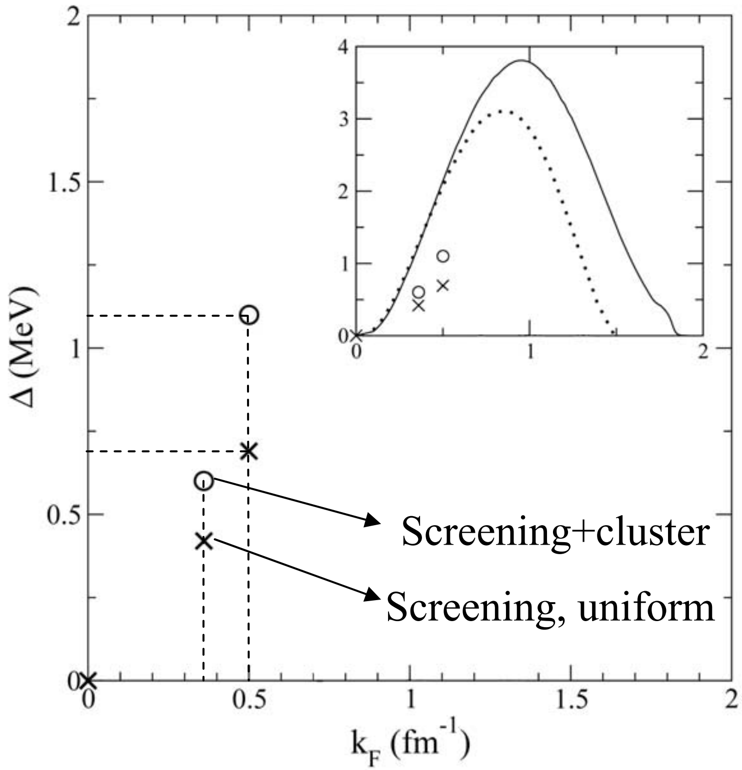


Preliminary results

^{588}Sn cell , Skm*
R = 42 fm , k_F = 0.33 fm^{-1}







The presence of the cluster increases the gap by about 50%

Simple calculation in uniform matter:

$$V_{ph} = (F_0 + G_0 \vec{\sigma}_1 \cdot \sigma_2)\, \delta(\vec{r}_1 - \vec{r}_2)$$

$$V_{tot}(k_1, k_2) = V_{bare}(k_1, k_2) + \frac{1}{2k_1k_2} \int_{|k_1-k_2|}^{k_1+k_2} dk k \left(-\frac{1}{2}\, \frac{F_0^2 R_k^0}{1 + F_0 R_k^0} + \frac{3}{2}\, \frac{G_0^2 R_k^0}{1 + G_0 R_k^0} \right)$$

$$\Delta_k = -\frac{1}{4\pi^2} \int_0^\infty dk' k'^2 [V_{bare}(k,k') + V_{ind}(k,k')] \frac{\Delta_{k'}}{\sqrt{(e_{k'} - e_F)^2 + \Delta_{k'}^2}}$$

H.J. Schultze et al., Phys. Lett. B375(1996)1

An open problem: proper treatment of non-local interactions

N. Van Giai et al., Ann. Phys. 214(1992) 293

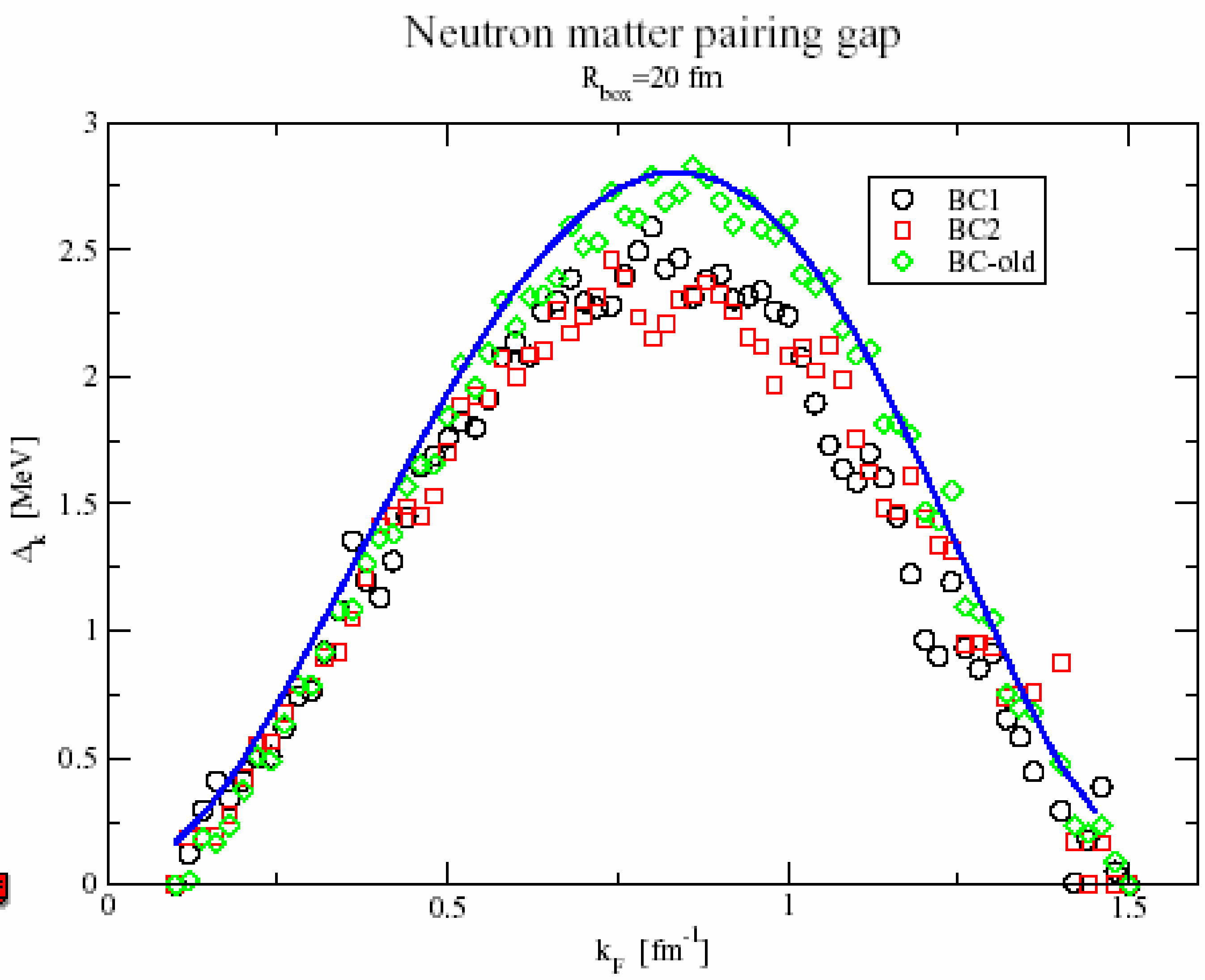
CONCLUSIONS

At the mean field level and within the WIgner-Seitz approximation, the presence of nuclear clusters influence the spatial dependence of the pairing gap, and its absolute value by 5-10%, and the specific heat by up to 1-2 orders of mangntude > (cooling time).

The results are sensitive to the pairing force and to the effective mass of the Hartree-Fock mean field, but the main features are the same.

The main uncertainty in the calculation are the medium polarization effects. They act in a distinct different manner in finite nuclei and in uniform matter. A detailed study of the interface between the clsuter and neutron sea is difficult but is required for vortex pinning.

A preliminary calculation shows that the induced interaction (exchange of medium fluctuations) increase the gap as compared to uniform matter.

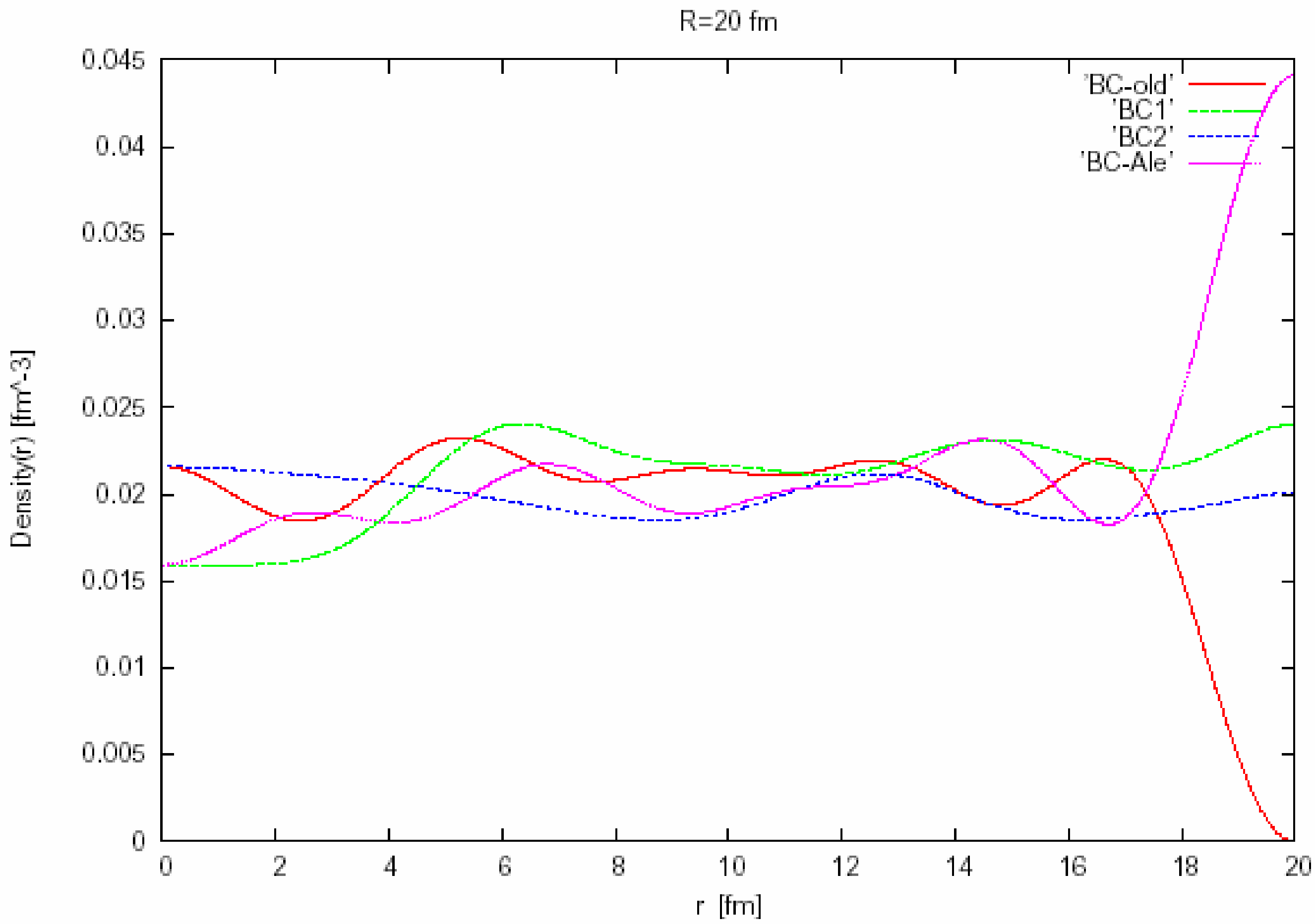


Neutron matter pairing gap
R_box=20 fm
BC1
BC2
BC-old
Δ_k [MeV]
k_F [fm^-1]
0
0.5
1
1.5
2
2.5
3

k_F, fm^{-1}	Z	k_F^{as}, fm^{-1}		$\Delta(0)$, MeV		Δ_{as}, MeV		Δ_F, MeV		Δ_{inf}, MeV		Δ^0_{inf}, MeV
		BC1	BC2	BC1	BC2	BC1	BC2	BC1	BC2	BC1	BC2	
0.2	52	0.1156	0.1095	0.088	0.132	0.042	0.046	0.043	0.058	0.126	0.106	0.40
0.6	58	0.5786	0.5783	1.464	1.471	1.947	1.899	1.919	1.893	2.321	2.320	2.42
	56	0.5783	0.5786	1.456	1.428	1.899	1.912	1.893	1.891	2.319	2.321	
0.7	52	0.6758	0.6753	1.665	1.650	2.358	2.288	2.300	2.247	2.680	2.678	2.76
	48	0.6763	0.6763	1.679	1.648	2.312	2.368	2.290	2.325	2.682	2.682	
0.8	42	0.7732	0.7724	1.767	1.726	2.614	2.546	2.555	2.445	2.883	2.882	2.93
	44	0.7729	0.7727	1.747	1.834	2.580	2.679	2.525	2.560	2.883	2.883	
0.9	24	0.8694	0.8693	1.862	1.664	2.777	2.625	2.636	2.506	2.919	2.919	2.92
	22	0.8725	0.8664	1.936	1.654	2.677	2.680	2.617	2.544	2.918	2.919	
1.0	20	0.9499	0.9613	1.249	1.966	2.199	2.635	2.023	2.517	2.800	2.773	2.68
	24	0.9612	0.9574	1.894	1.504	2.705	2.507	2.519	2.288	2.774	2.782	
1.1	20	1.0315	1.0531	0.996	1.889	1.477	2.411	1.318	2.317	2.550	2.458	2.26
	26	1.0434	1.0649	1.927	1.296	2.469	2.242	2.280	2.020	2.500	2.408	
1.2	20	1.1243	1.1321	1.556	0.992	1.340	2.017	1.210	1.558	2.113	2.066	1.66
	26	1.1278	1.1160	0.760	0.991	1.549	0.963	1.249	0.862	2.092	2.163	

It is worth to mention that the difference between the asymptotic Δ_{as} value and the infinite neutron matter prediction Δ_{inf} is a measure of validity of the LDA for the gap calculation outside the central nuclear cluster. One can see that, as a rule, the LDA works within 10% accuracy, but sometimes the difference is greater which is an evidence of the so-called proximity effect. The Fermi average value Δ_F is usually very close to Δ_{as} value. It is explained with the

M. Baldo, E.E. Saperstein, S.V. Tolokonnikov, nucl-th/0609031



R=20 fm
'BC-old'
'BC1'
'BC2'
'BC-Ale'
Density(r) [fm^-3]
r [fm]
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
2
4
6
8
10
12
14
16
18
20

A few basic questions about pairing correlations

1. **Does superfluidity affect the results found by Negele and Vautherin?**

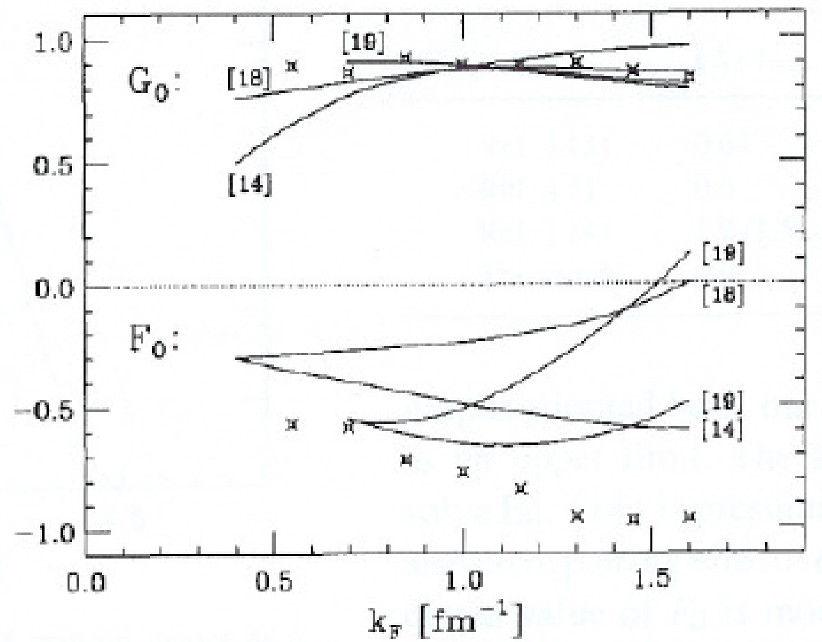
2. **What is the spatial dependence of the pairing gap?**

   How important are the nuclear clusters?

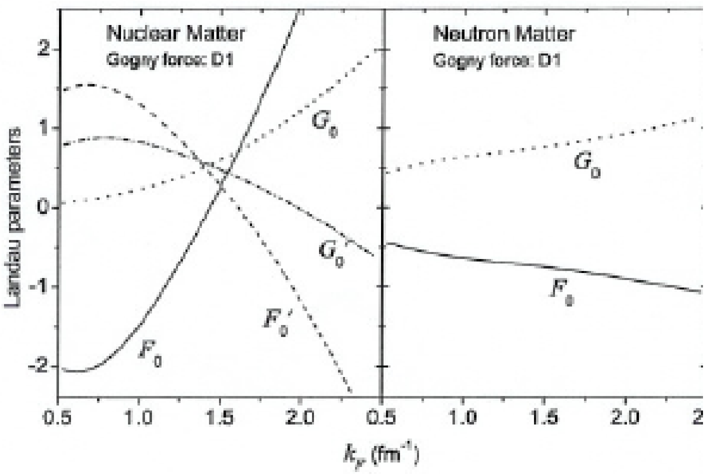
3. **How much are the gaps affected by many-body processes ?**

4. **Can we prove experimentally that the crust is really superfluid?**

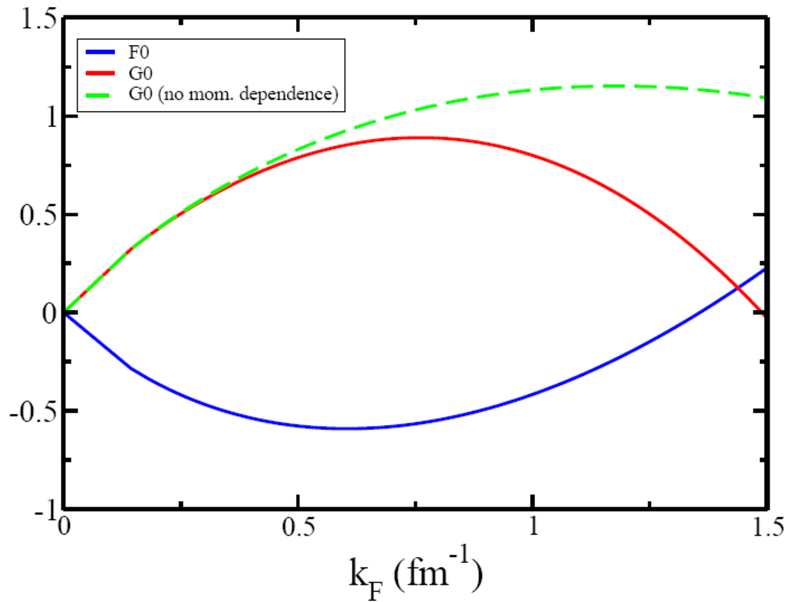
G-matrix



Gogny force

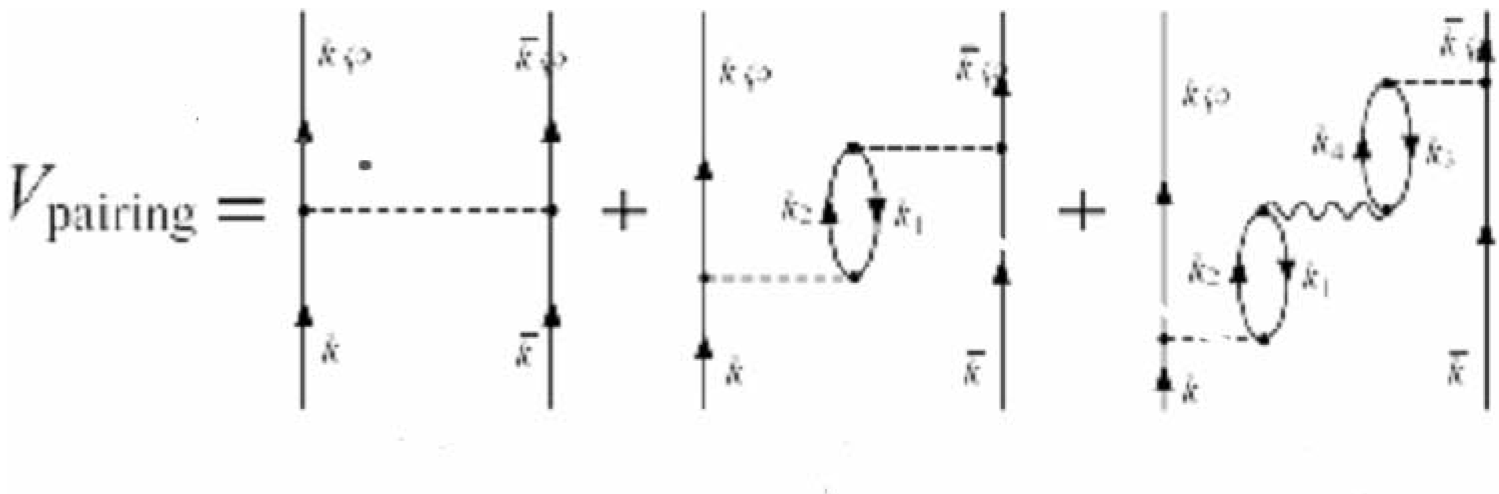


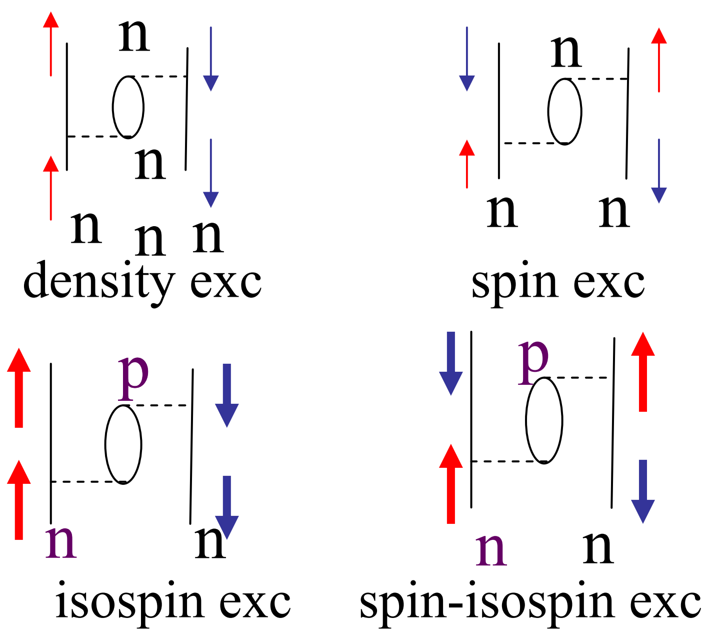
SkM* force

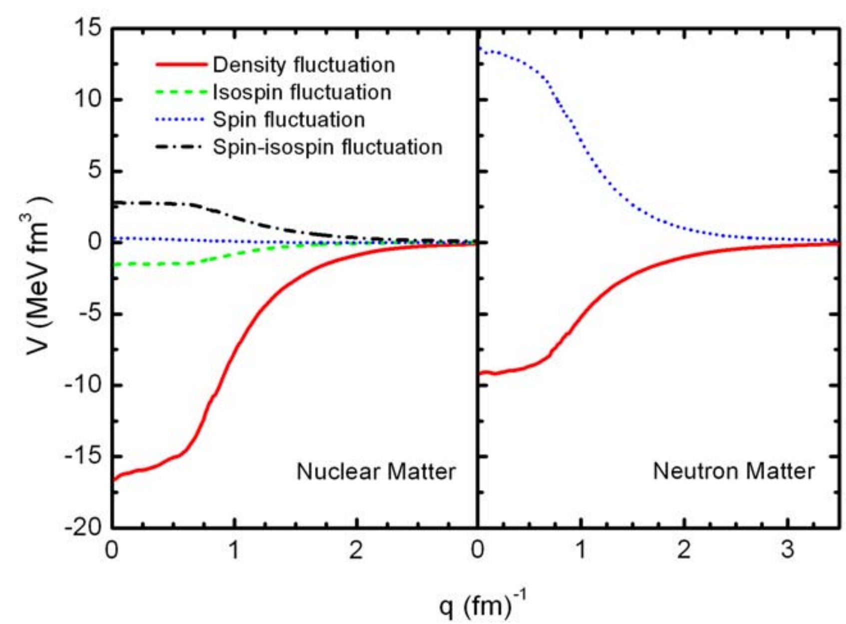


Density dependence of Landau parameters (at k=0)

Pairing interaction in neutron and nuclear matter and exchange of p.h. excitations

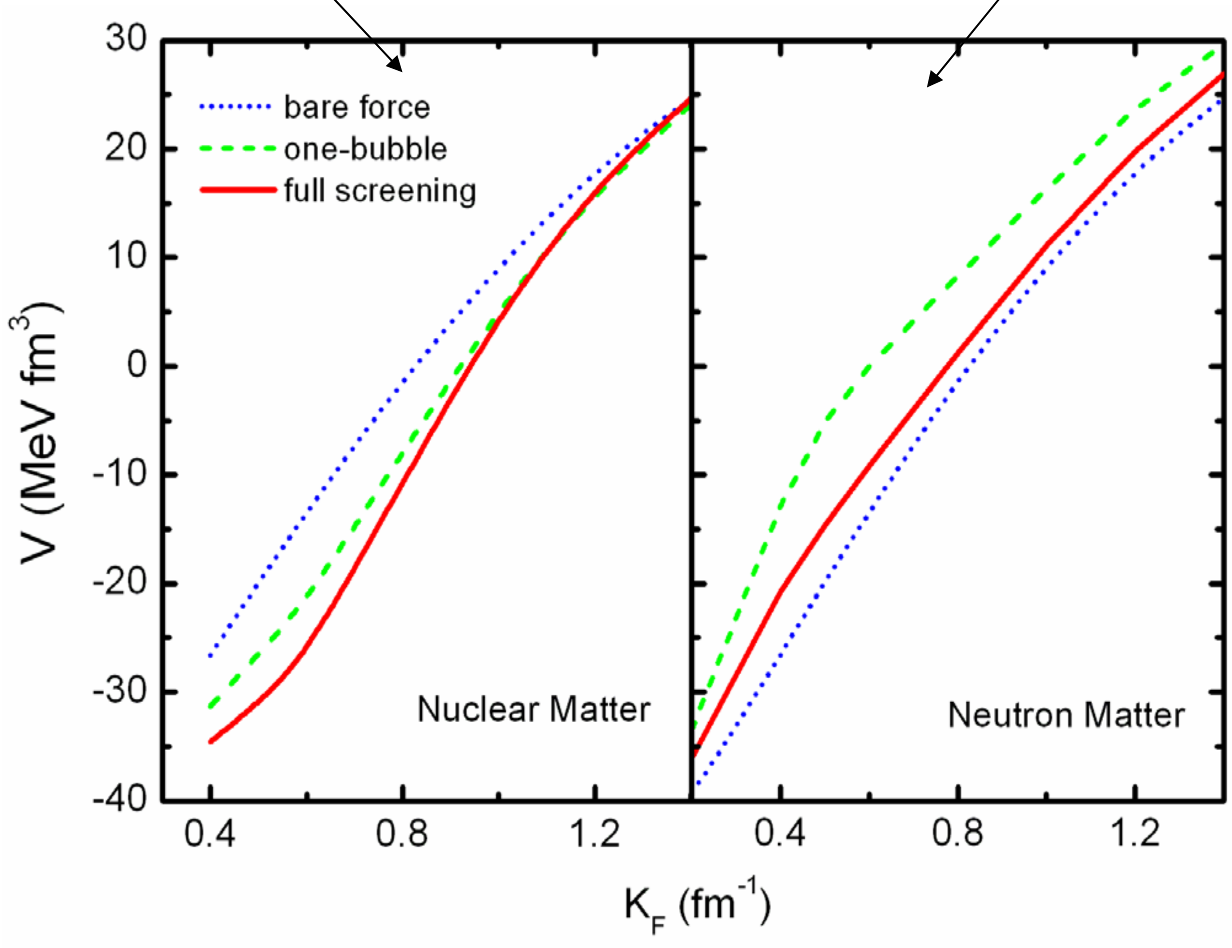






antiscreening
$V_{ind} > V_{dir}$

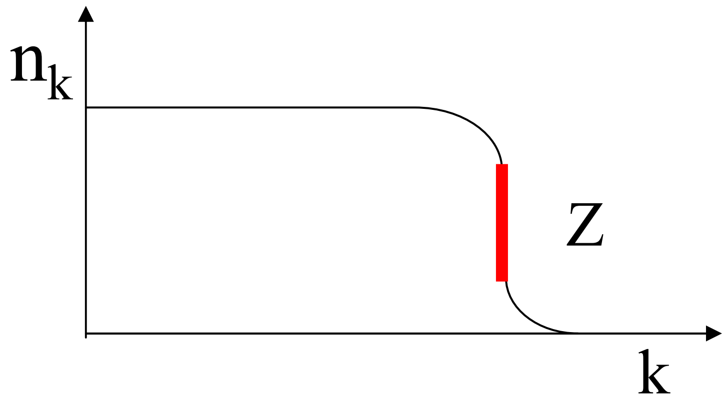
screening
$V_{ind} < V_{dir}$



L.G. Cao, U. Lombardo, P. Schuck, PRC 74(2006)64301

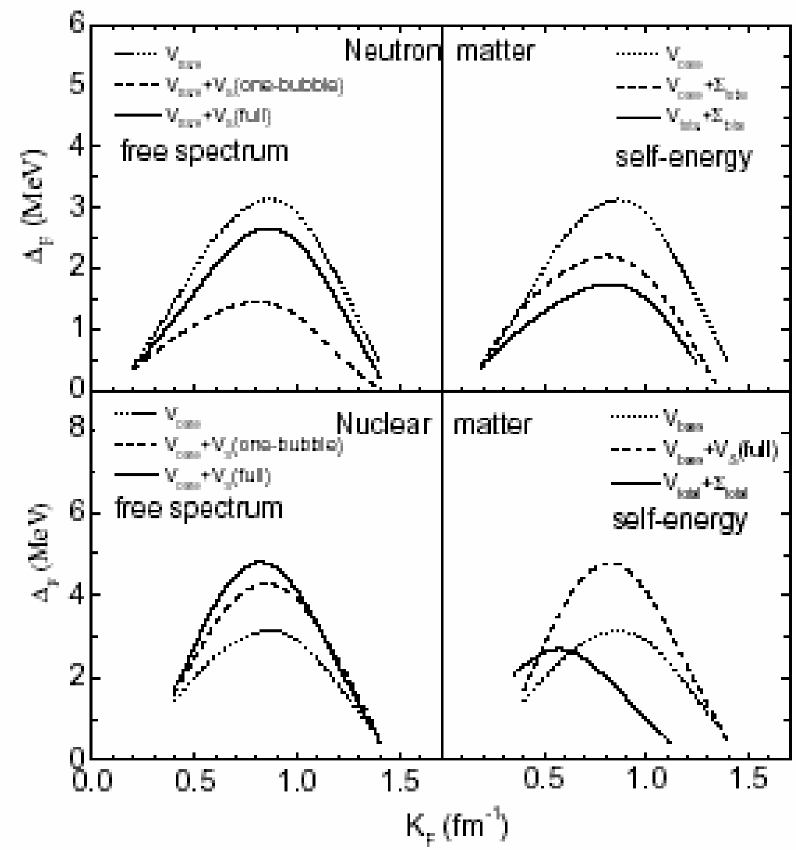
Z=1 free Fermi gas

Z<1 correlated Fermi system



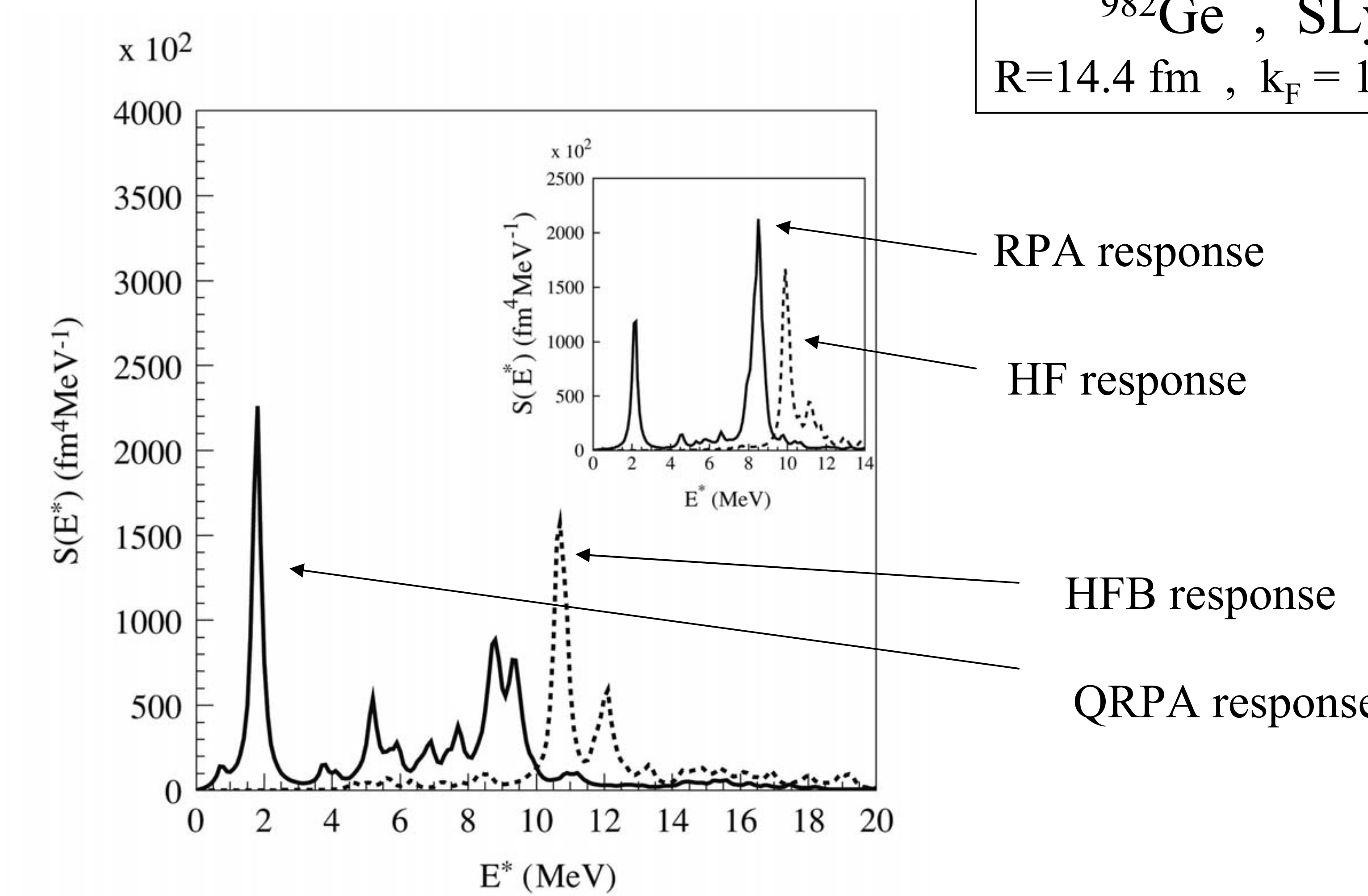
Generalized gap equation

$$\Delta_p = -\frac{1}{2}\int d^3p' \frac{Z_p V_{pp'} Z_{p'}}{\sqrt{(\varepsilon_{p'} - \varepsilon_F)^2 + \Delta_{p'}^2}} \Delta_{p'}$$



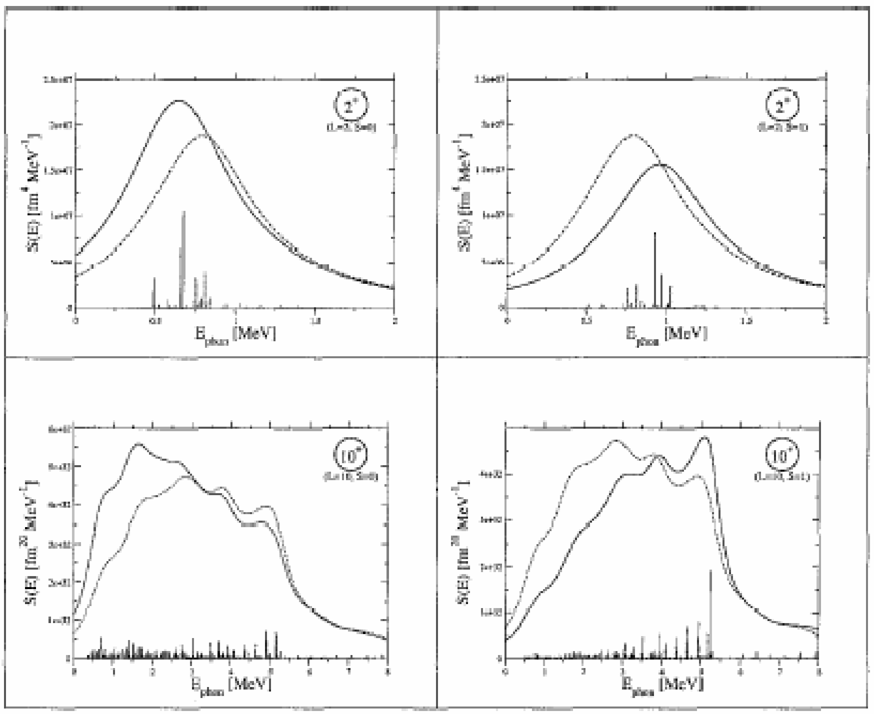
Example of QRPA versus RPA response: cell ^{982}Ge

^{982}Ge , SLy4
R=14.4 fm , k_F = 1.32 fm^{-1}



(E.Khan et al., PRC 71 (2005) 042801)

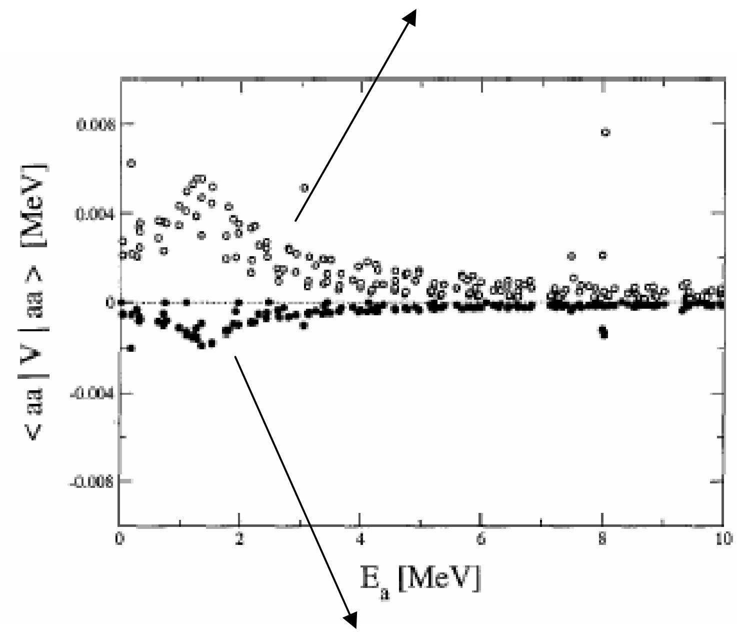
Going beyond mean field within the Wigner-Seitz cell: including the effects of polarization (exchange of vibrations) and of finite nuclei at the same time



RPA response

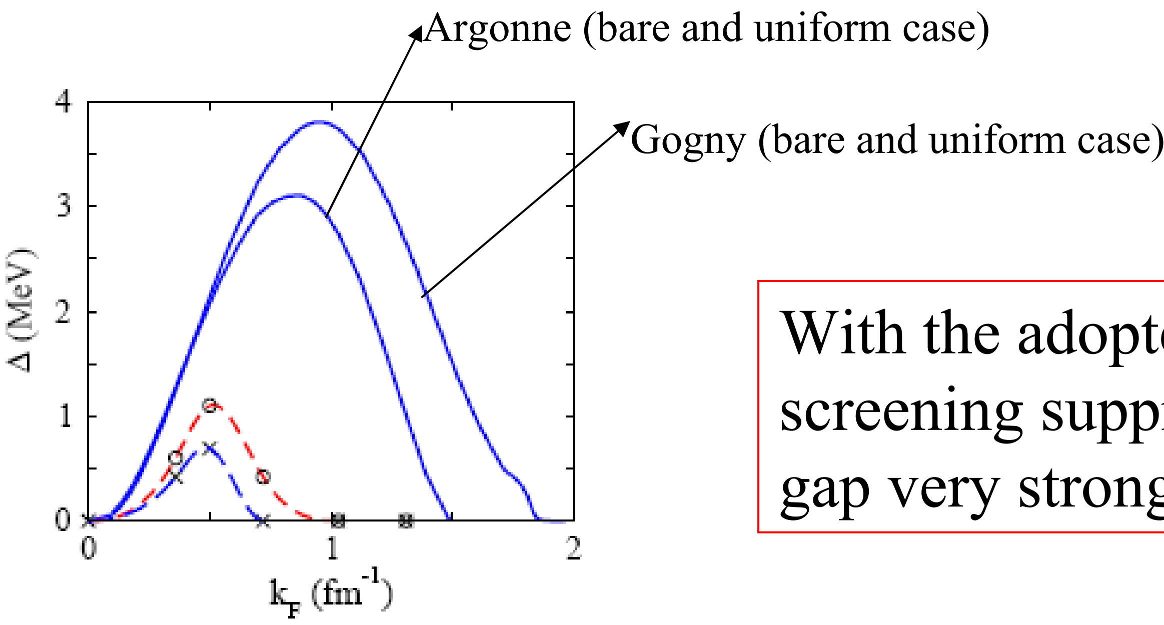
Spin modes

Induced pairing interaction

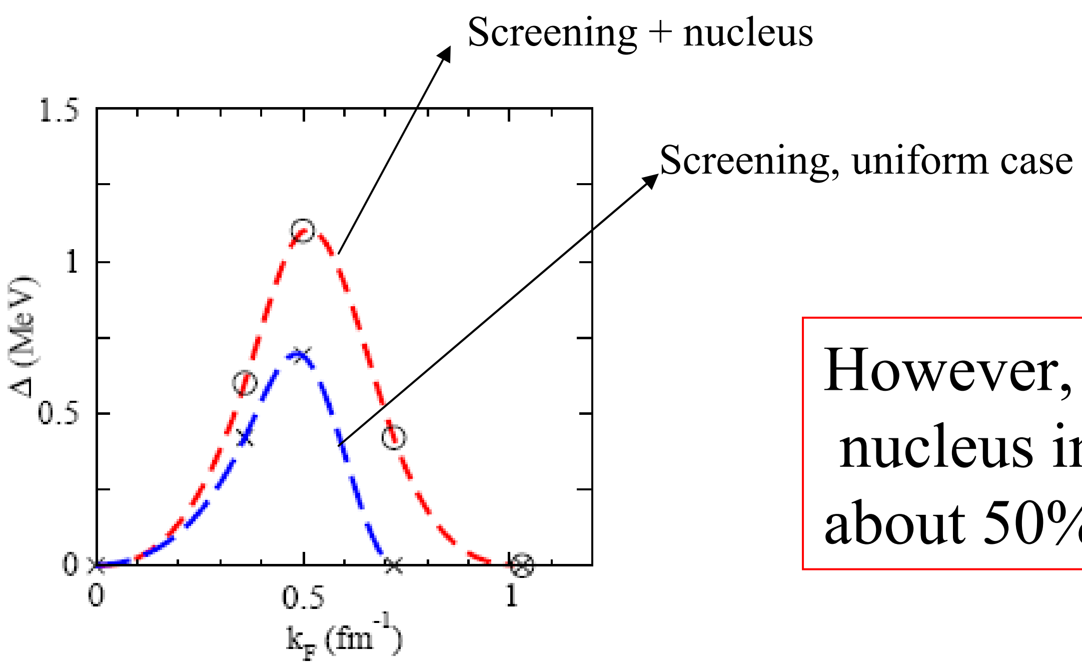


Density modes

G. Gori, F. Ramponi, F. Barranco,R.A. Broglia,
G. Colo, D. Sarchi,E. Vigezzi, NPA731(2004)401



With the adopted interaction, screening suppresses the pairing gap very strongly for $k_F > 0.7\ fm^{-1}$



However, the presence of the nucleus increases the gap by about 50%

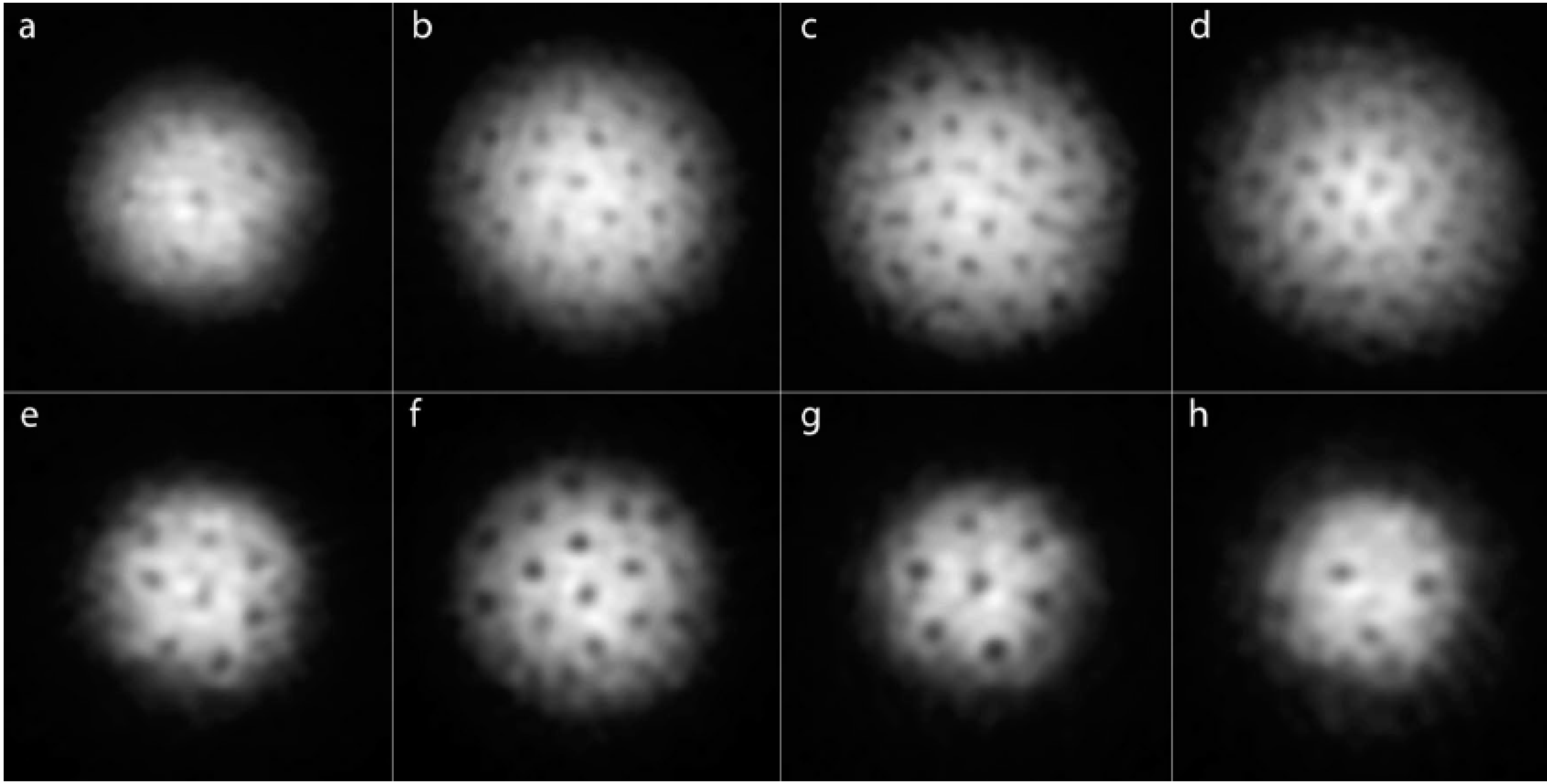
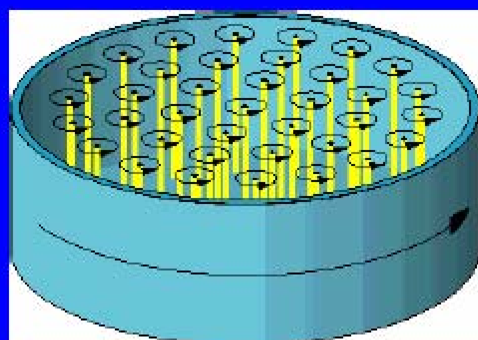


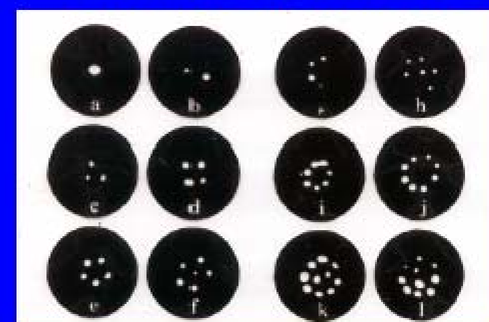
Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu m \times 880\ \mu m$.

Zwierlein et al. Nature 435(2005)1047

The neutron superfluid's rotation



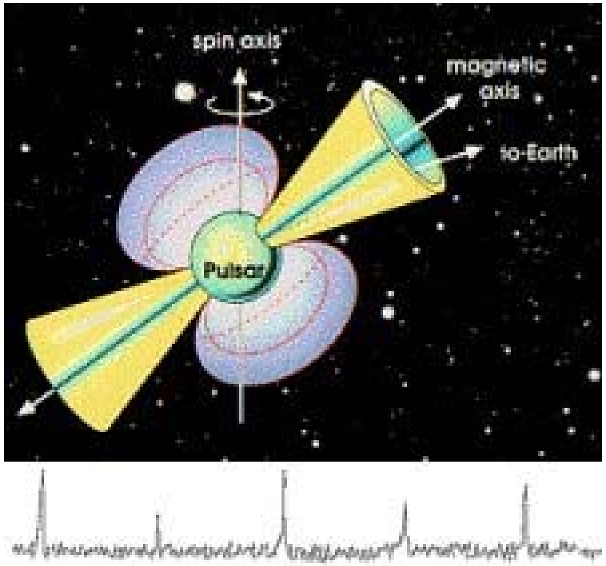
Rotating superfluid He



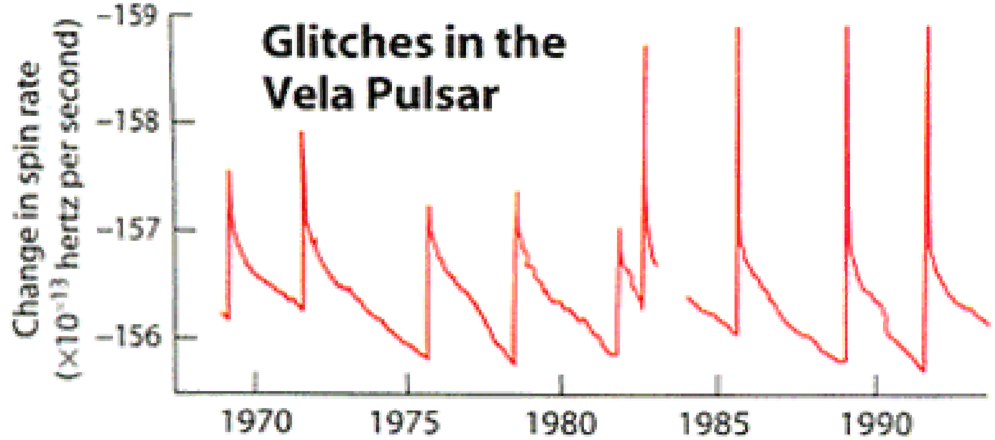
Distribution of vortices determines the fluid's angular momentum.

A typical neutron star contains $\sim 10^{17}$ neutron vortices.

Glitches



As a rule, rotational period of a neutron star slowly increases because the system loses energy emitting electromagnetic radiation.



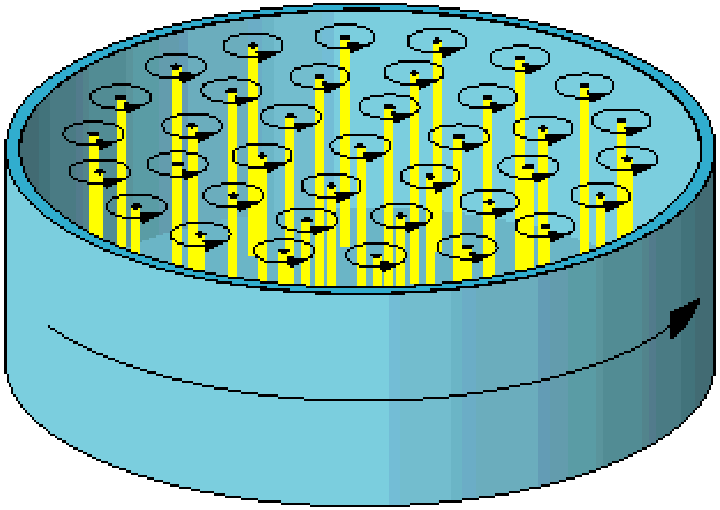
One of the proposed explanations → Superfluid nature of nucleons in the inner crust

P.W. Anderson and N.Itoh, Nature 256(1975)25

A superfluid in a rotating container develops an array of microscopic linear vortices

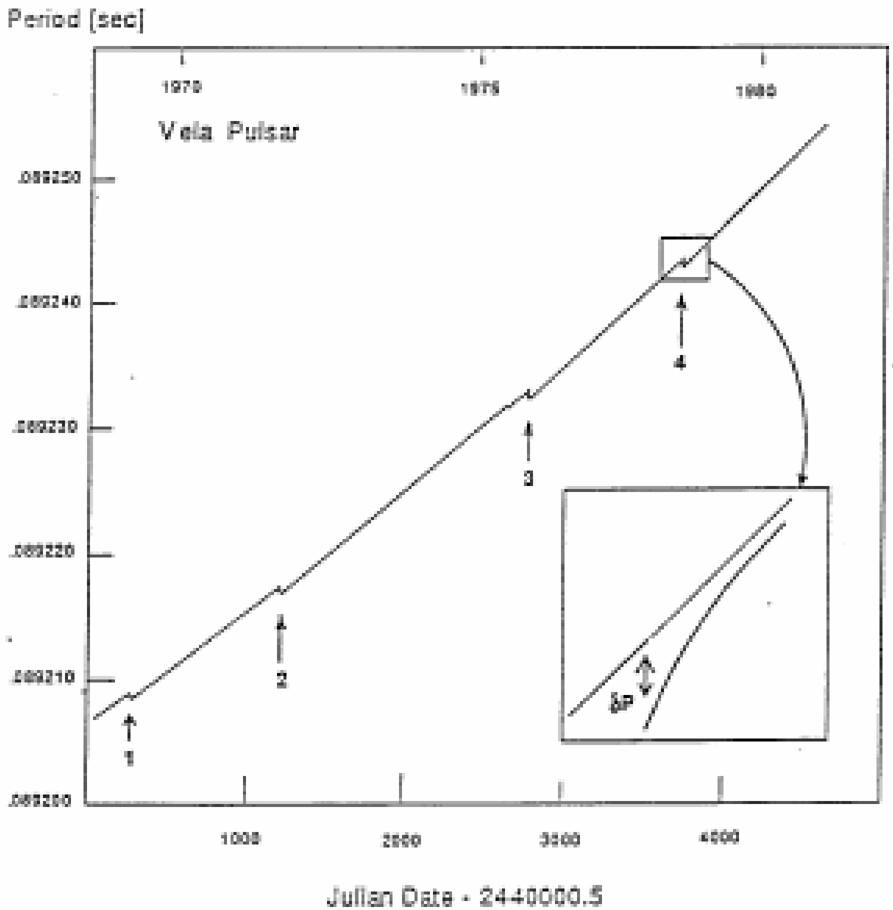
Quantized circulation of superfluid velocity about vortex:

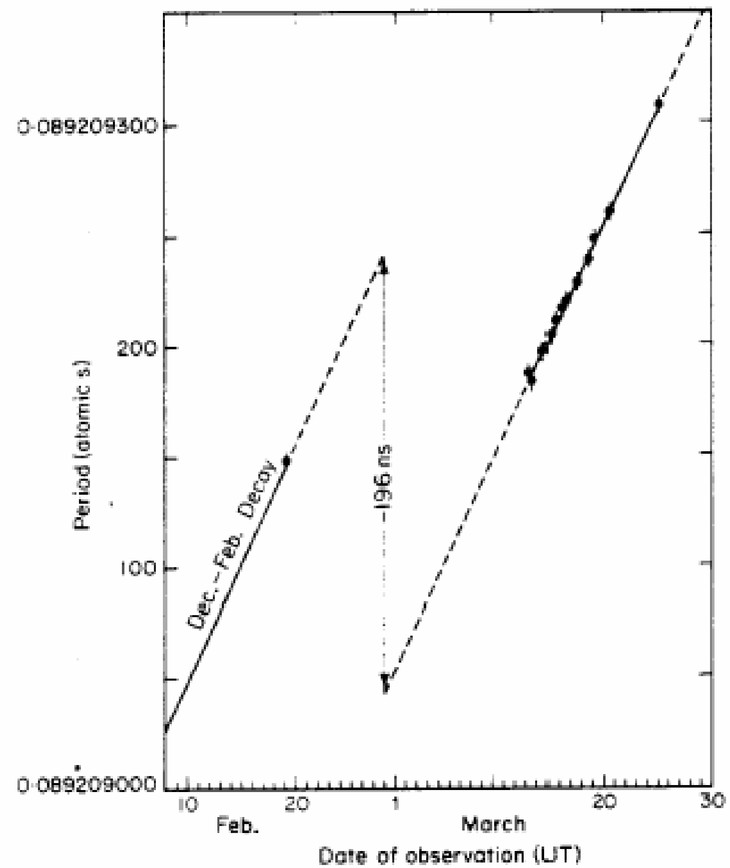
$$\oint_{\mathcal{C}} \mathbf{v_s} \cdot d\ell = \frac{2\pi\hbar}{2m_n}$$

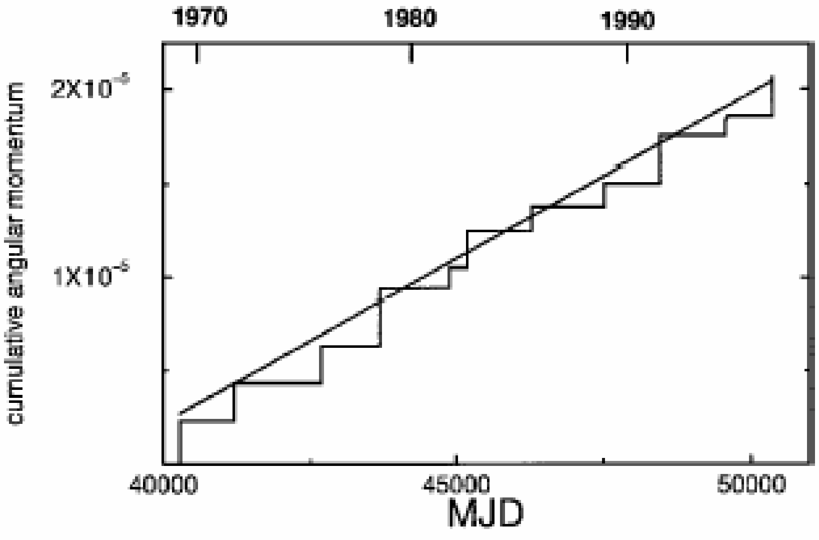


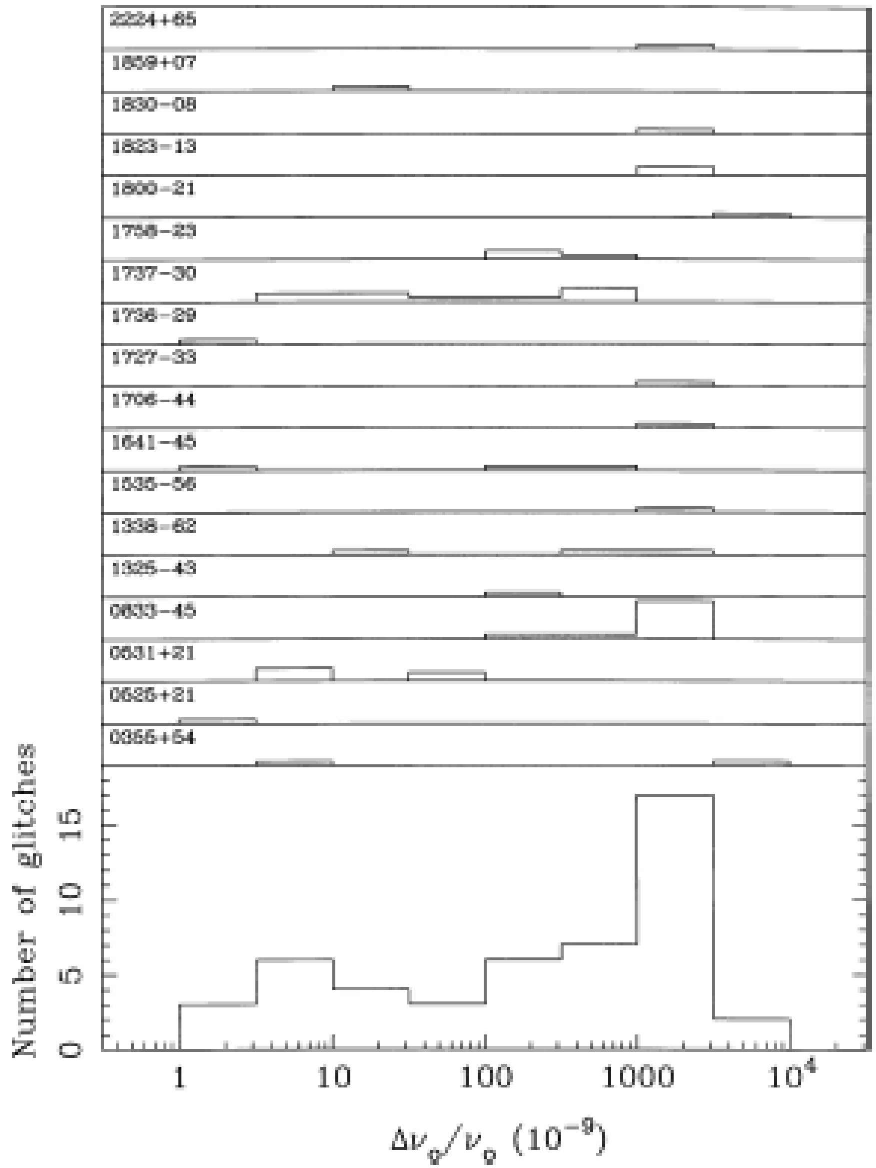
Vortices may pin to **container impurities**, what may modify their dynamics. Sudden unpinning at critical period difference, due to Magnus force, would cause the glitch.

P.W. Anderson and N.Itoh, Nature 256(1975)25


Period [sec]
1970
1975
1980
Vela Pulsar
.089250
.089240
.089230
.089220
.089210
.089200
1
2
3
4
δP
1000
2000
3000
4000
Julian Date - 2440000.5


0·089209300
200
100
0·089209000
Period (atomic s)
Dec.-Feb. Decay
196 ns
10
20
1
20
30
Feb.
March
Date of observation (UT)





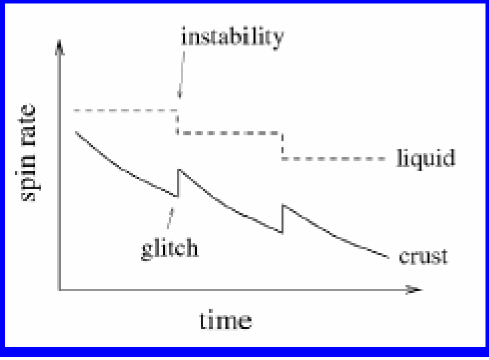
A.G. Lyne, S.L.Shemar, F. Graham Smith, Mon. Not. Roy. Soc. 315(2000)534

Mechanism of glitches

Pulse structure not notably affected by glitch => phenomenon internal in the neutron star. Long time scales for response (relaxation ~ months) => well-oiled machinery – superfluidity! [Metastable superfluid flow (Packard 1972).]

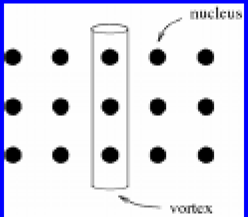
Pulses connected via magnetic field - to the crust.

Neutron liquids in star act as a reservoir of angular momentum L.
Crust neutron superfluid carries ~ 3% of total L.
Sudden transfer of L_{sf} to crustal solid speeds it up => glitch



G. Baym, Denver 2007

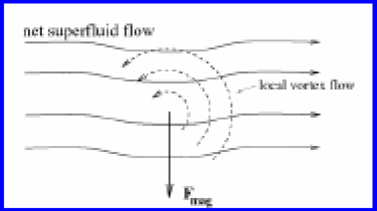
Vortex model of glitches



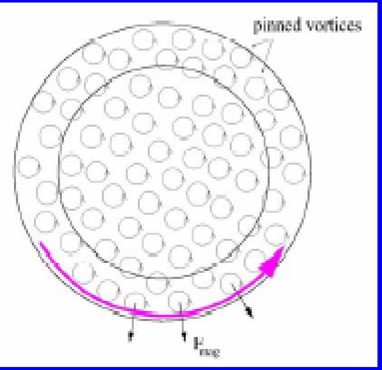
Pin vortices to (or between) nuclei in inner crust (*Anderson & Itoh 1975*). E~ 3Mev/nucleus.

$n_{vortices}$ fixed => $\Omega_{superfluid}$ fixed; Ω_{normal} decreases as star radiates.

As $\Omega_{sf} - \Omega_{n}$ grows, Magnus force $=\rho_s \times (\mathbf{v}_{vortex} - \mathbf{v}_{superfl})$ drives unpinning (glitch) and outward relaxation.



Collective outward motion of many (~ 10^{14}) vortices produces large glitch

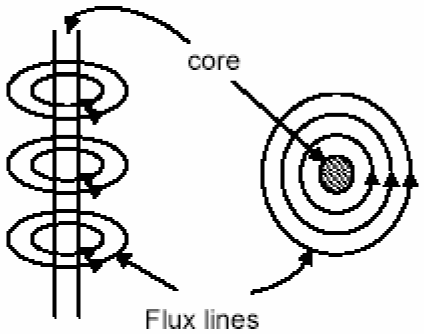


Vortex theory for glitches

- **Quantized vortex lines**

superfluid $\rightarrow$ irrotational flow $\rightarrow$ $\nabla\times\vec{v}_s = 0$

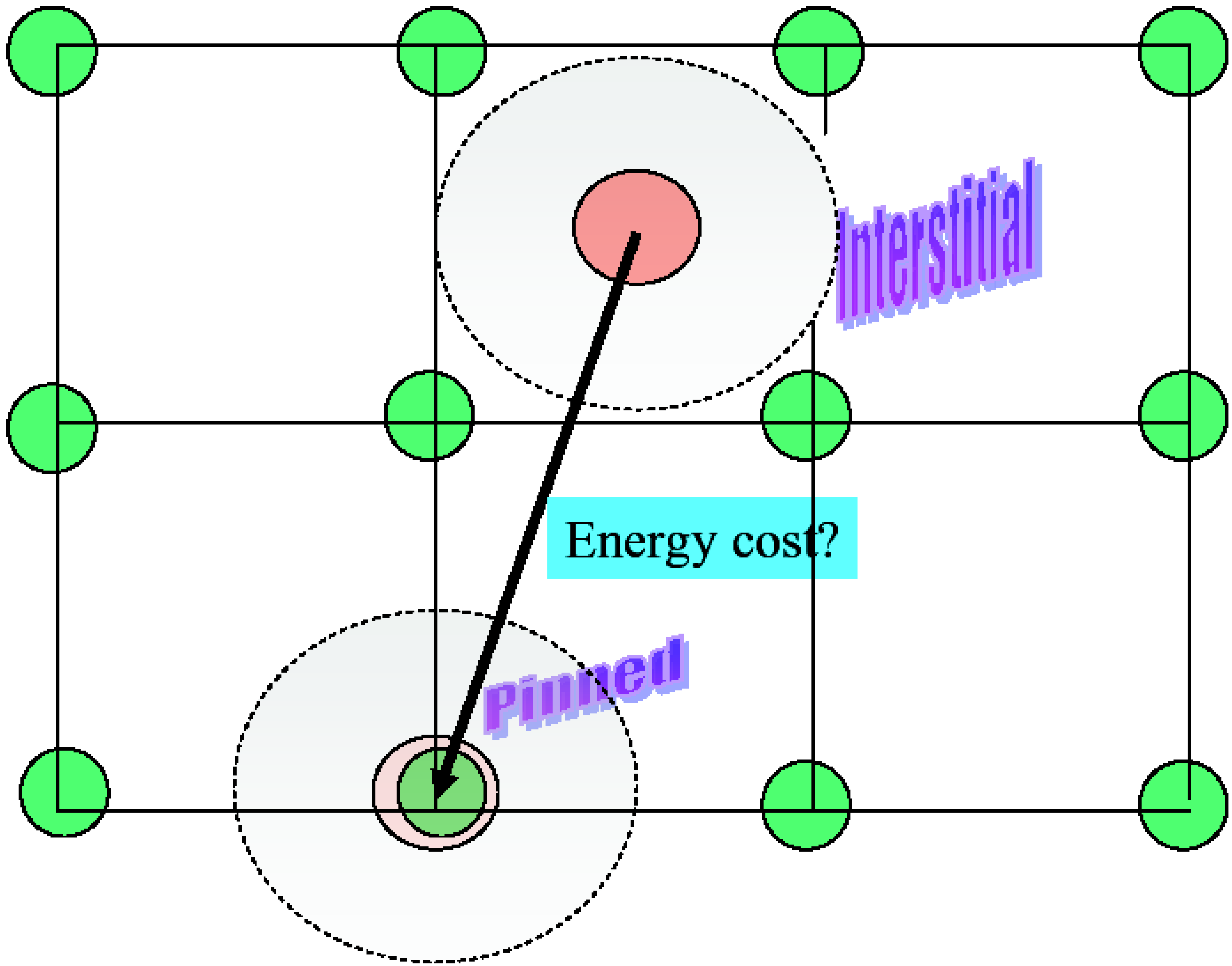
classical vortex $\rightarrow$ $\vec{v}_s = \frac{C}{r}\hat{e}_\theta$ $\rightarrow$ $\nabla\times\vec{v}_s = 2\pi C\,\delta^{(2)}(\vec{r})$



quantized vortex lines $\rightarrow$ $C = k\,\frac{\hbar}{2m_N}$ $(k = 1, 2, \cdots)$

quantized vorticity $\rightarrow$ $\oint \vec{v}_s \cdot d\vec{l} = k\,\frac{h}{2m_N}$ $(k = 1, 2, \cdots)$

A basic issue for a model of glitches based on vortex unpinning
To determine the favoured vortex configuration



A simple argument:

For sufficiently large densitites, pairing is smaller within the nuclear volume than outside;

Vortex destroys pairing within its core;

Then it is energetically convenient for the vortex to be placed on top of the nucleus, rather than far from it: in this way, one saves pairing energy.

$$\Delta E_{pin} \equiv \left[\varepsilon_{cond}(n_N) - \varepsilon_{cond}(n_G)\right] \cdot V_N$$

But we need a realistic estimate of the vortex-nucleus interaction

Microscopic quantum calculation of the vortex-nucleus system

The characteristic ansatz for the study of a vortex is
(c.f.Bohr&Mottelson,PR125(1962)495)

$$\Delta(\rho, z, \phi) = \Delta(\rho, z)\, e^{i\nu\phi}$$

where $\Delta(\rho, z)$ is a real function and ν=0,1,2,... is the vortex index. When ν=0 the no vortex standard HFB situation is recovered .

The parity of this function is given by the change in sign when

$$\phi \rightarrow \phi + \pi$$

$$z \rightarrow -z$$

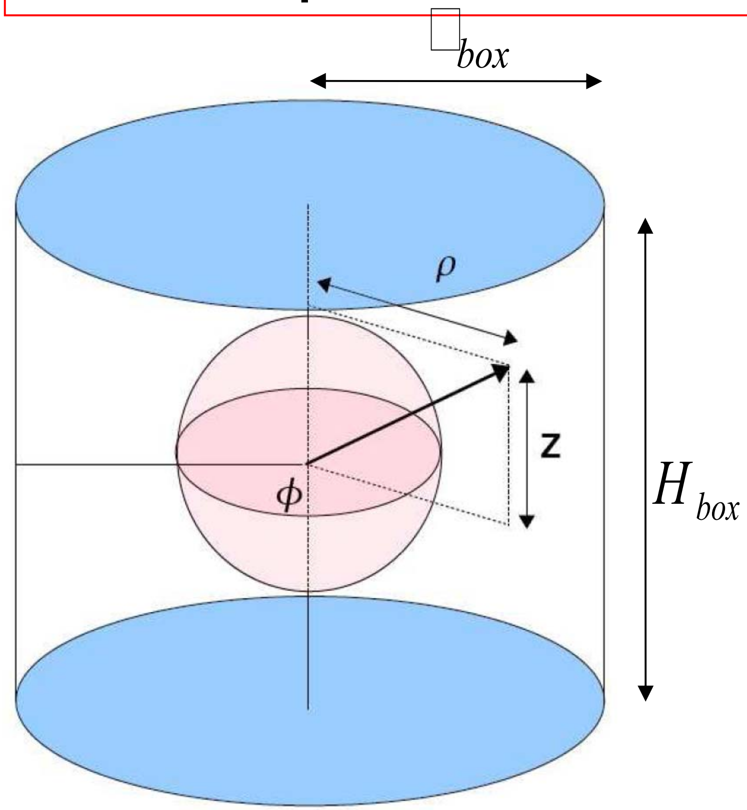
For a system with mirror symmetry with respect to x-y plane,

$$\Delta(\rho, z) = \Delta(\rho, -z)\,,$$

we have

$$\pi = (-1)^{\nu}$$

We solve the HFB (De Gennes) equations expanding on a single-particle basis in cylindrical coordinates



$$\begin{pmatrix} \varepsilon_i - \lambda & \Delta \\ \Delta & -(\varepsilon_i - \lambda) \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix} = E_i \begin{pmatrix} U_i \\ V_i \end{pmatrix}$$

- **HF: Skyrme interaction**
- **Pairing: density-dependent reproducing the gap of N-N bare interaction**
- **Protons are constrained to have a spherical geometry**
- **No spin-orbit interaction**

P. Avogadro, F. Barranco, R.A. Broglia, E. Vigezzi, Phys, Rev. C75 (2007)012085

$$u_\alpha(\rho,\phi,z) \square \sum_{nk} J_{nm}(\rho)\ \sin(kz)\ e^{im\phi} \qquad u_{nk;m;\alpha}$$

$$v_\alpha(\rho,\phi,z) \square \sum_{nk} J_{nm-\nu}(\rho)\sin(kz)\, e^{i(m-\nu)\phi}\ v_{nk;m;\alpha}$$

$$Vel_{vortex}(\rho,z) = -\frac{i\hbar}{m\rho n(\rho,z)}\sum_\alpha v_\alpha^*(\rho,z,\phi)\frac{\partial v_\alpha(\rho,z,\phi)}{\partial\phi}$$

Using a zero-range pairing interaction,

only local quantities are needed

➔

$$\eta(\rho,z) = \sum_\alpha v_\alpha(\rho,\phi,z)v_\alpha^*(\rho,\phi,z)$$

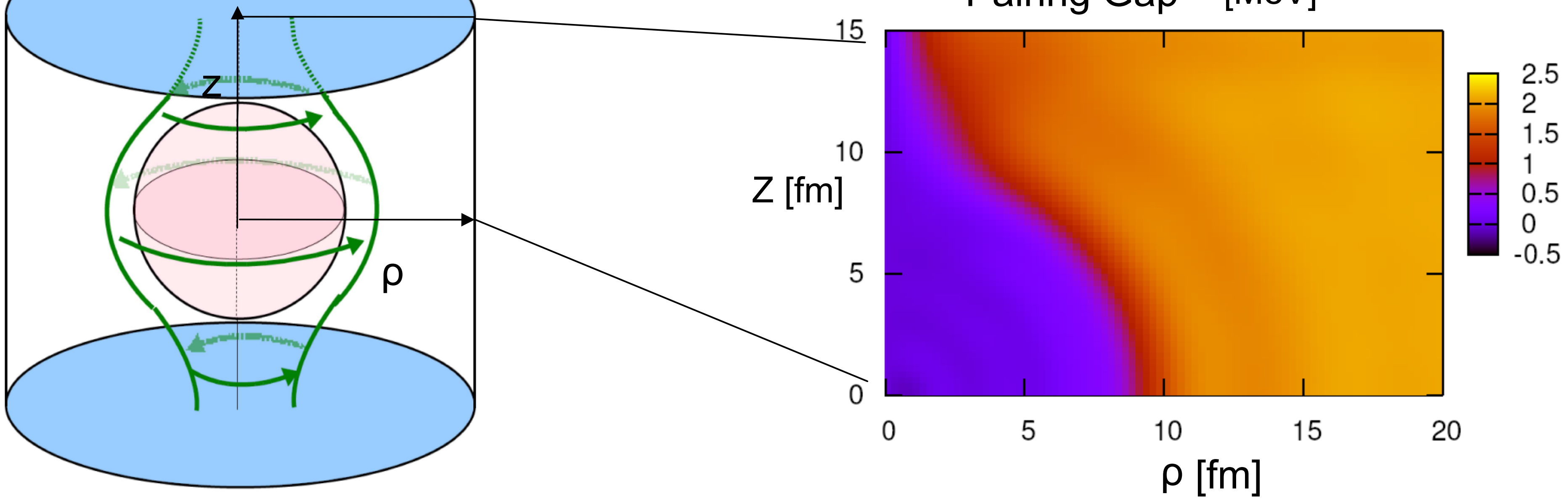
$$V(\rho,z) = Skyrme\ Density\ Functional$$

$$\kappa(\rho,\phi,z) = \sum_\alpha u_\alpha(\rho,\phi,z)v_\alpha^*(\rho,\phi,z)$$

$$\Delta(\rho,\phi,z) = \Delta(\rho,z)e^{i\nu\phi} = \frac{g}{2}\sum_\alpha u_\alpha(\rho,\phi,z)v_\alpha^*(\rho,\phi,z)$$

Vortex pinned on a nucleus

SII □= 5.8 MeV



Pairing Gap $$\Delta(\rho,z,\phi) = 481\left(1-0.7\left(\frac{n(\rho,z)}{0.08}\right)^{0.45}\right)\tilde{n}(\rho,z,\phi)$$

Abnormal density $$\tilde{n}(\rho,z,\phi) = \sum_{qm} u^{qm}(\rho,z,\phi)(v^{qm}(\rho,z,\phi))^*$$

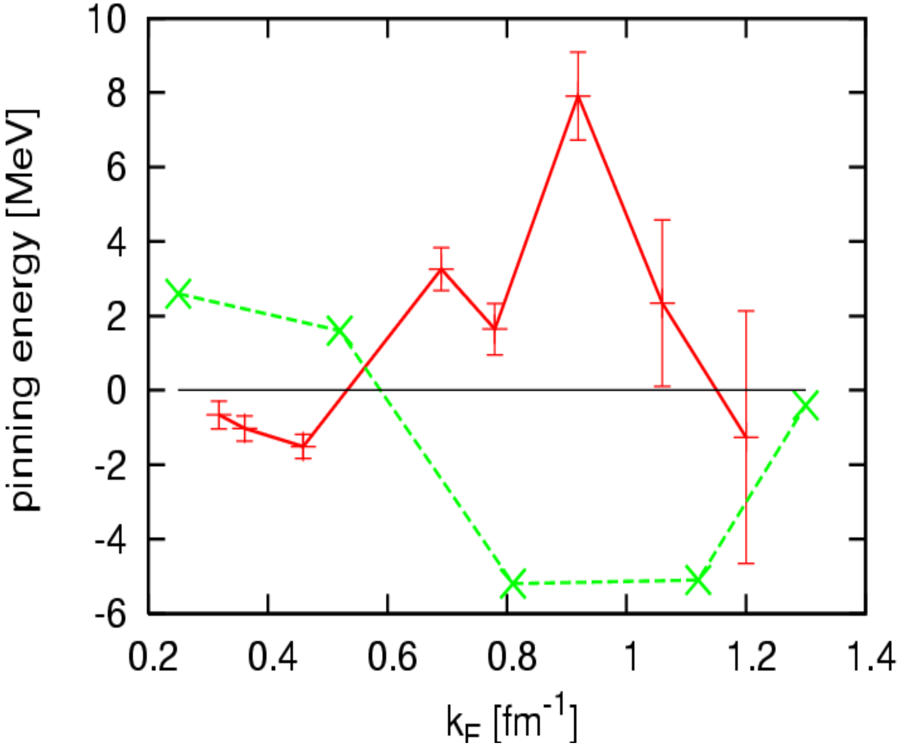
m is the angular momentum q

is the quasi-particle index,

Neutron density $$n(\rho,z) = 2\sum_{qm}|v^{qm}(\rho,z,\phi)|^2$$

Velocity field $$Vel = -\frac{i\hbar}{m\rho n(\rho,z)}\sum_{qm} v^*_{qm}(\rho,z,\phi)\frac{\partial v_{qm}(\rho,z,\phi)}{\partial\phi}$$

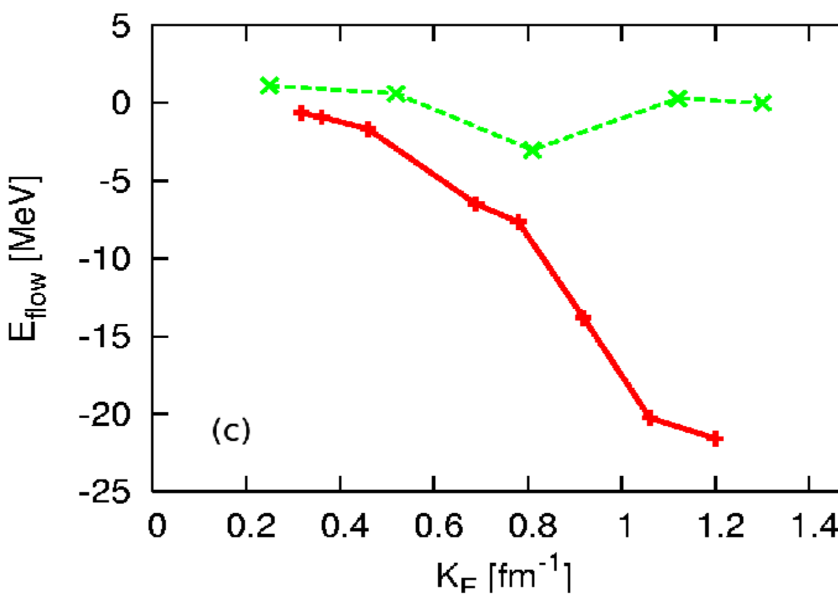
Pinning Energy: results

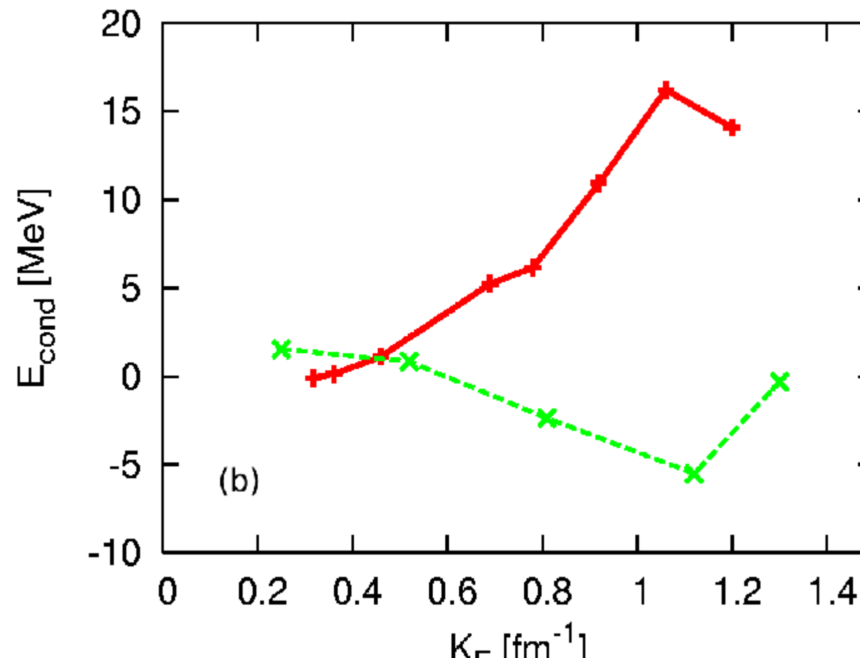


Our results : RED

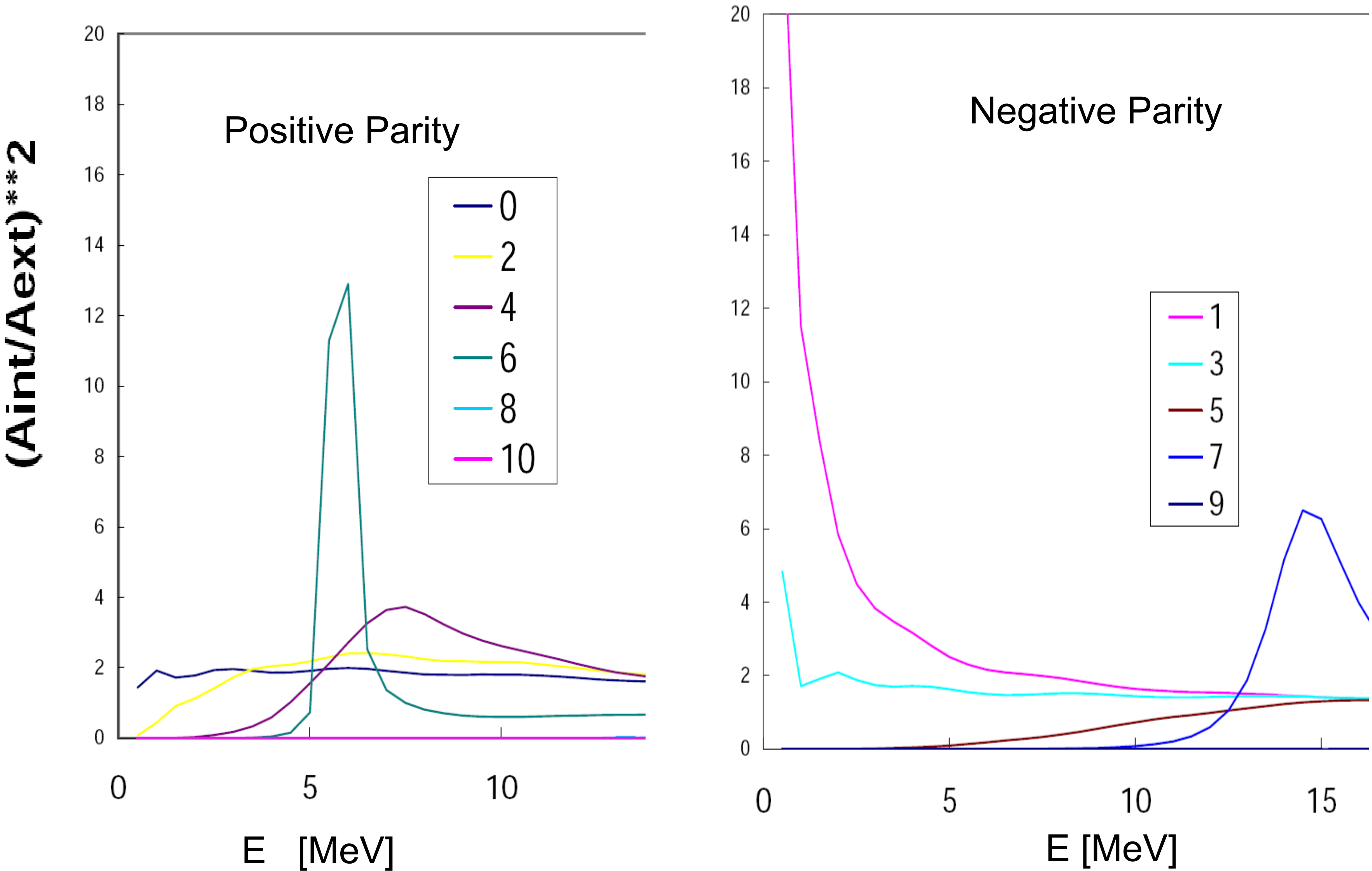
Pizzochero & Donati: GREEN

P.M. Pizzochero and P. Donati, Nucl. Phys. A742,363(2004) Semiclassical model with spherical nuclei.

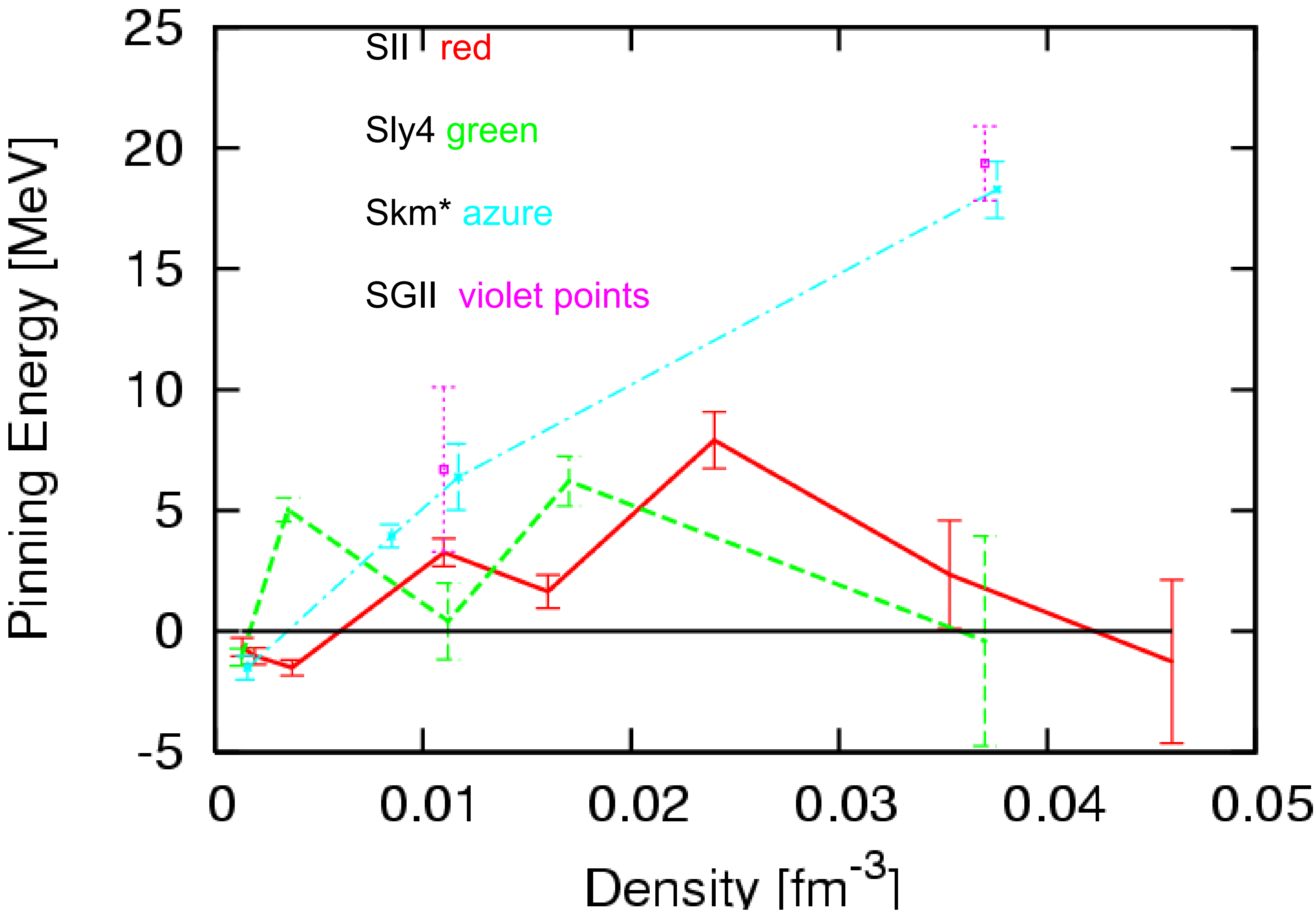




Detailed nuclear structure effects play an essential role!



Pinning Energies



Conclusions

Significant advances in the study of the inner crust have been made in the last years, using techniques typically used in the fieds of nuclear structure and in nuclear matter.
They concern its isotopic composition, its thermal properties, proximity and medium polarization effects, and the structure of vortices.

Does superfluidity affect the results found by Negele and Vautherin?

What is the spatial dependence of the pairing gap?
How important are the nuclear clusters?

How much are the gaps affected by many-body processes ?

Can we prove experimentally that the crust is really superfluid?

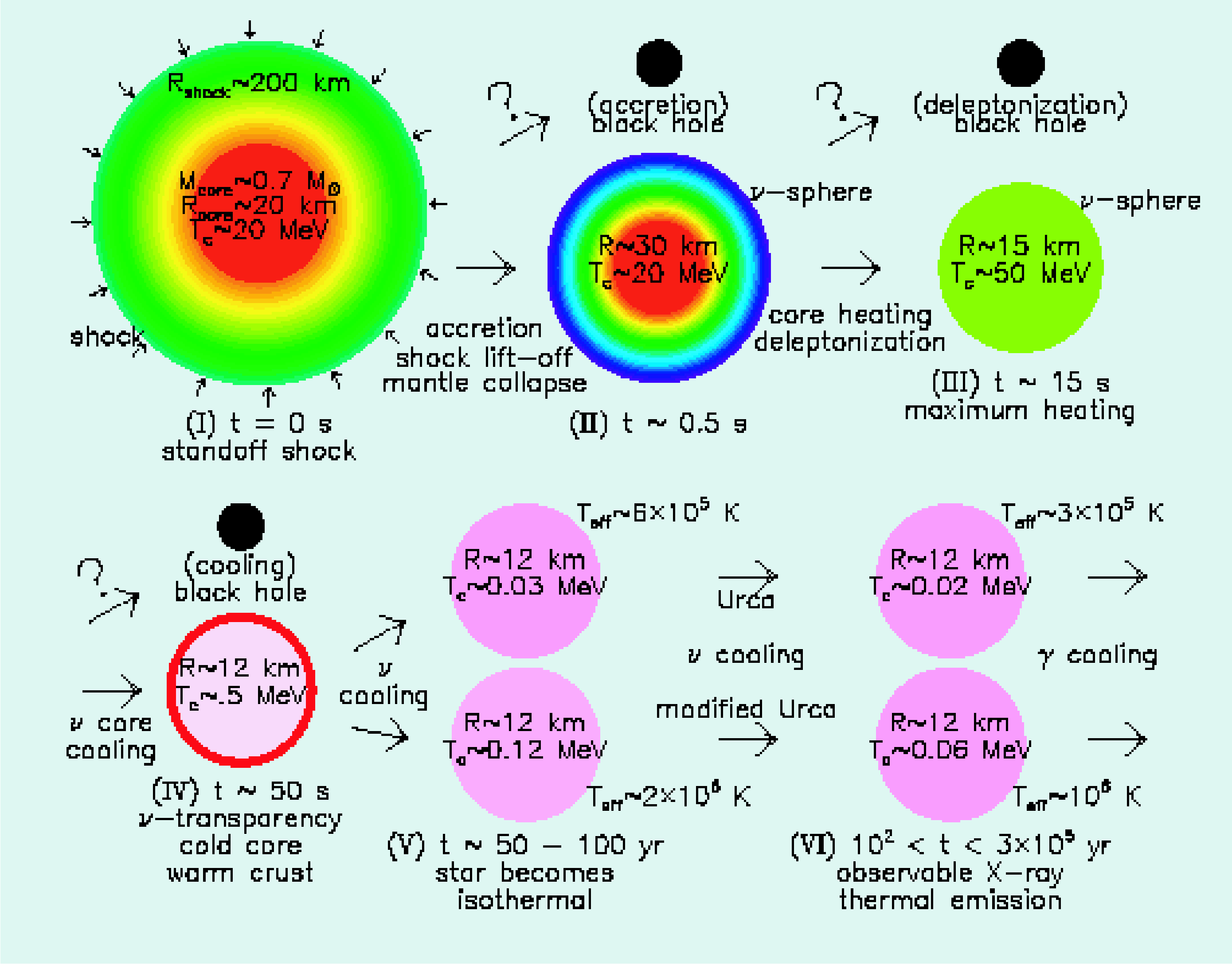
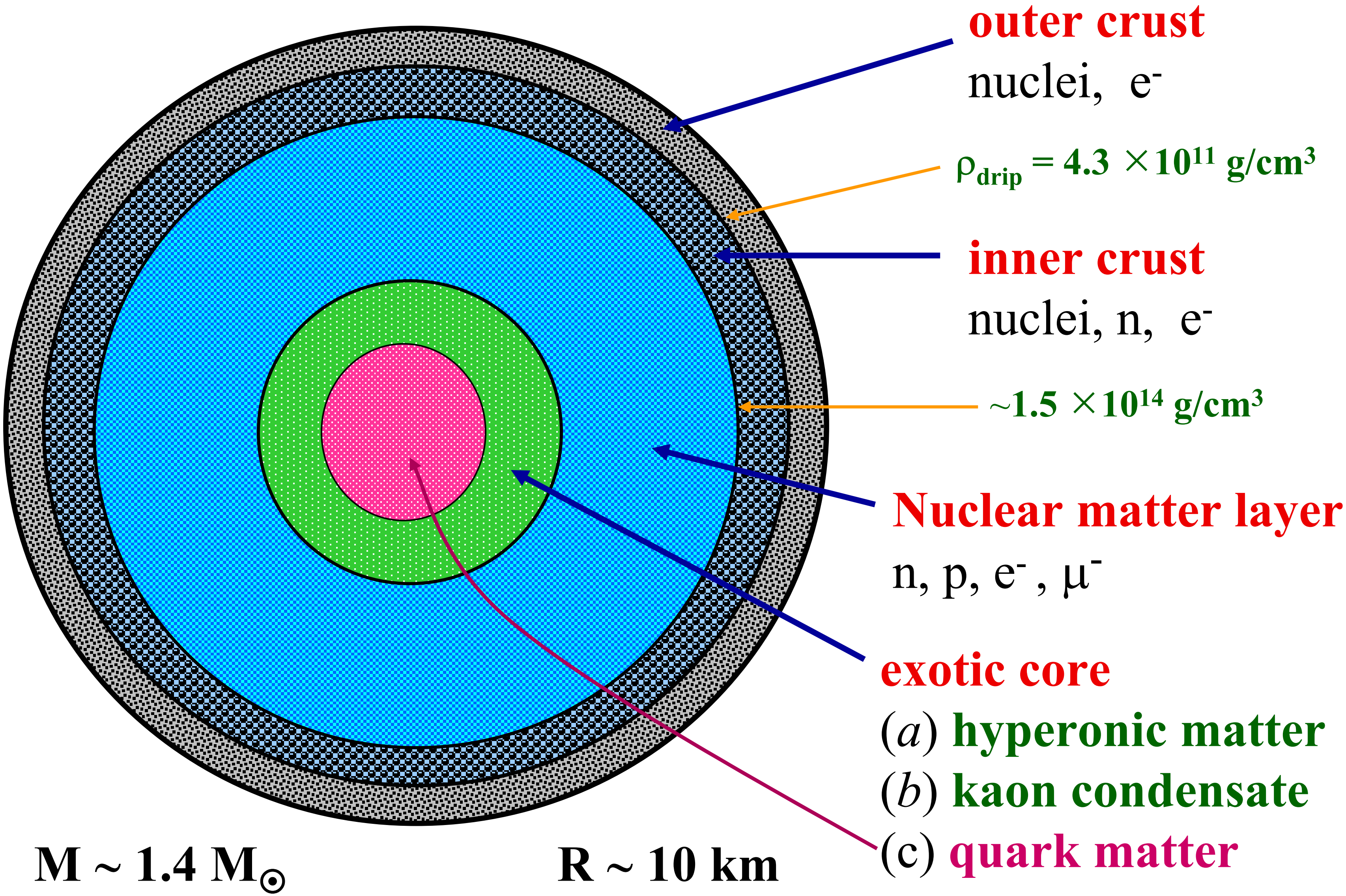


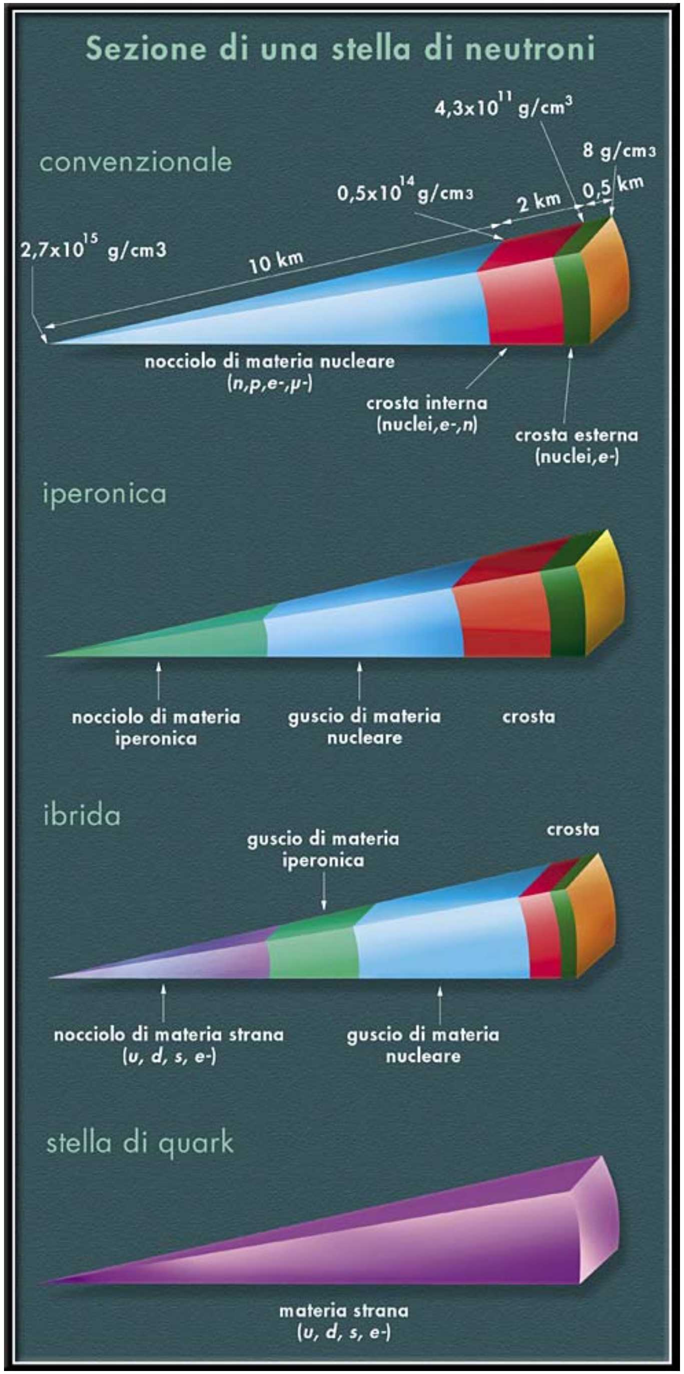
Fig. 1. The main stages of evolution of a neutron star. Roman numerals indicate various stages described in the text. The radius R and central temperatures T_c for the neutron star are indicated as it evolves in time t.

J.M. Lattimer , M. Prakash, Science 304(2004)

Schematic cross section of a Neutron Star



“Neutron Stars”



“traditional” Neutron Stars

Hyperon Stars

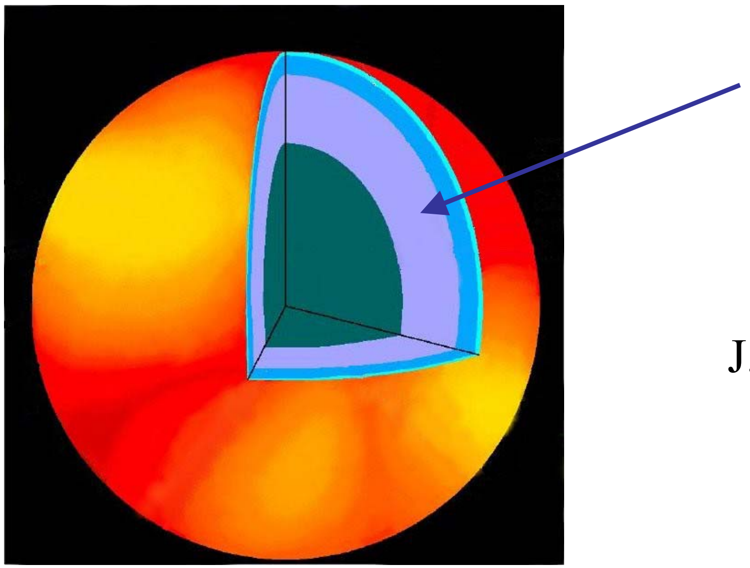
} **Hadronic Stars**

Hybrid Stars

Strange Stars

} **Quark Stars**

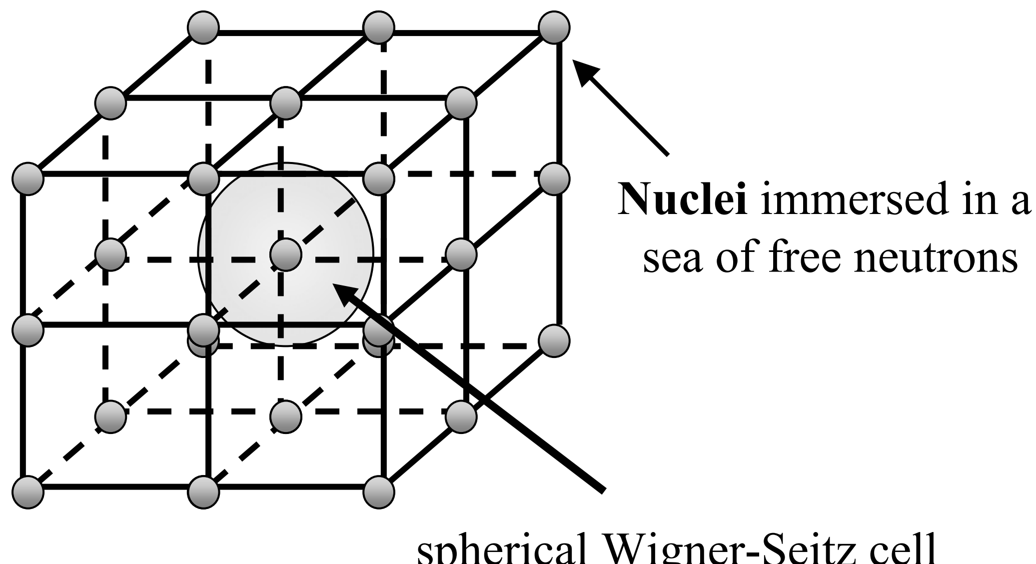
Microscopic quantal calculations of the isotopic composition of the inner crust



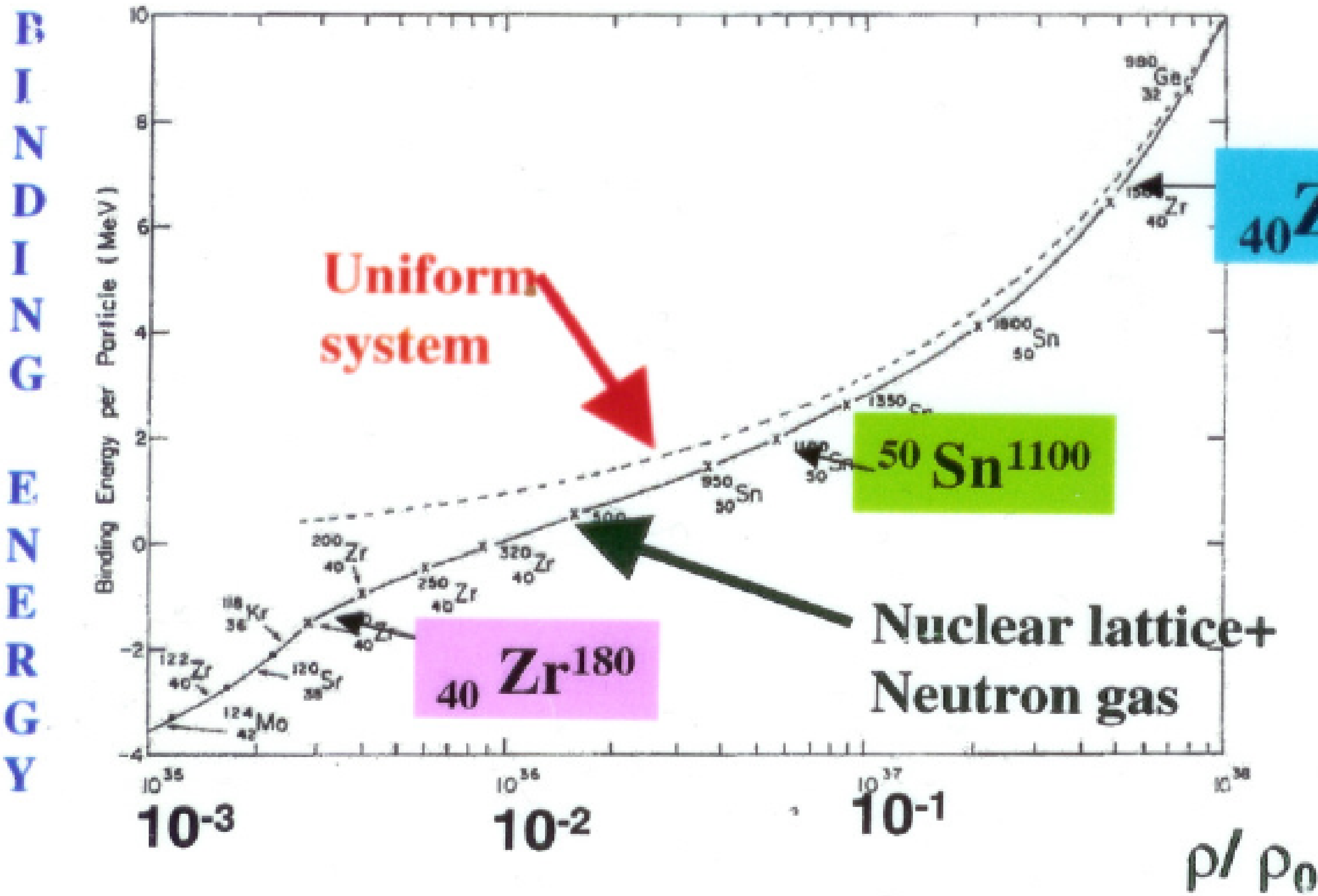
$0.001\ \rho_0 < \rho < 0.5\ \rho_0$

$\rho_0 = 2.8 \cdot 10^{+14}$ g cm^{-3}

Microscopic calculations (HF with Skyrme)
J.W. Negele and D. Vautherin, NPA207 (1972) 298



The inner crust: coexistence of finite nuclei with a sea of free neutrons

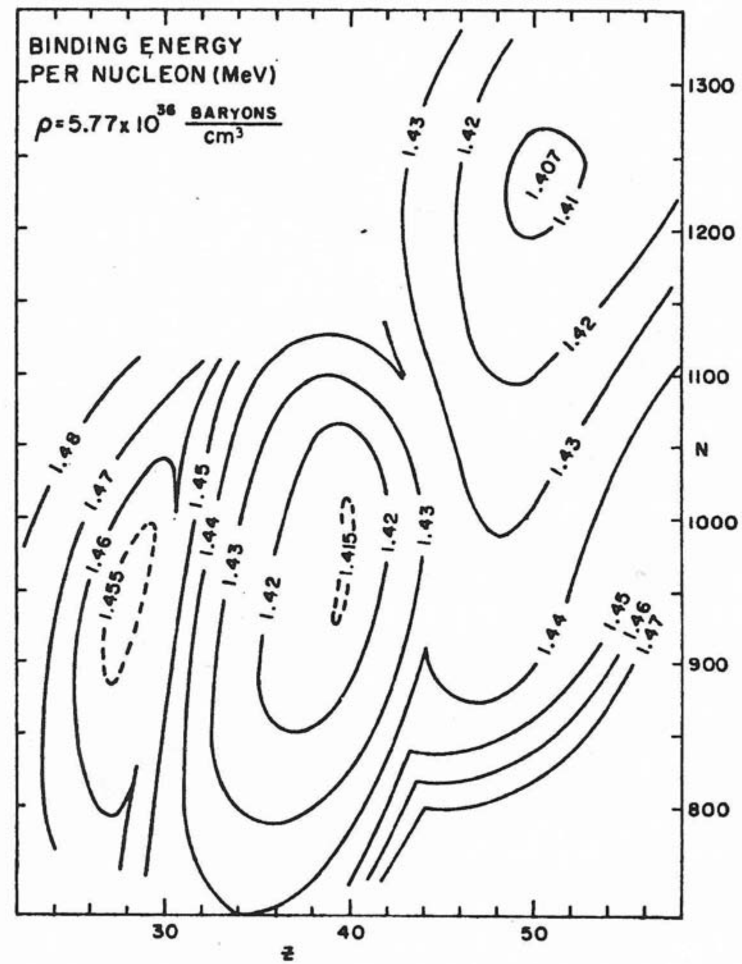


J. Negele, D. Vautherin
Nucl. Phys. A207 (1974) 298

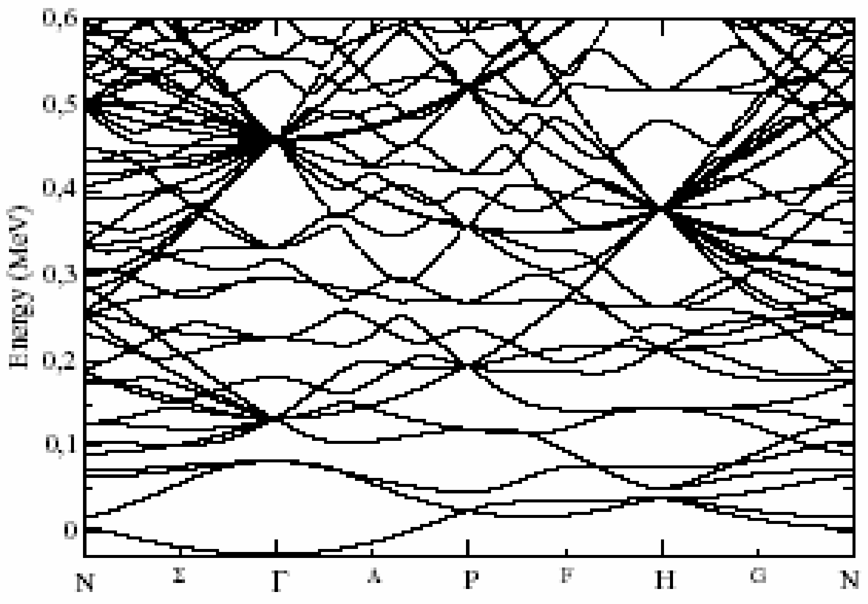
Looking for the energy minimum at a fixed (average) baryon density

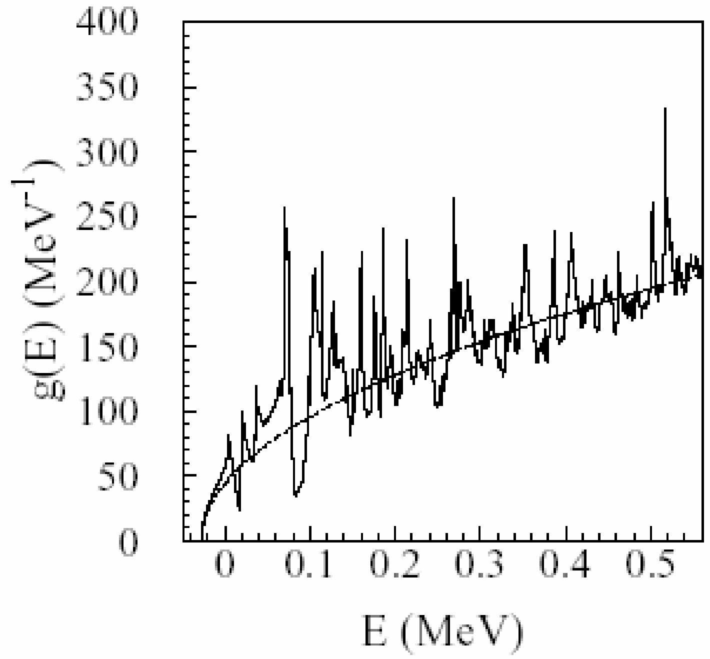
Density = 1/30 saturation density

No pairing

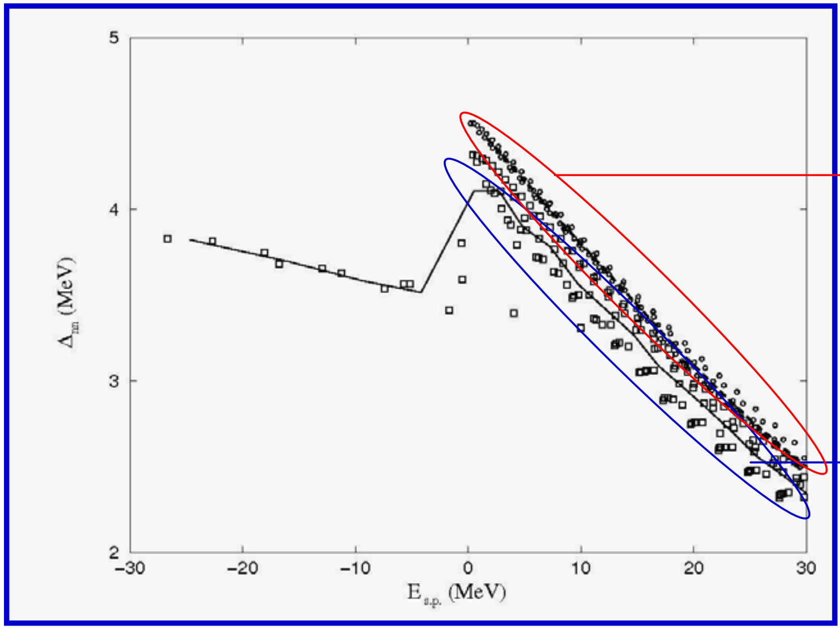


First calculation of band structure beyond the Wigner-Seitz approximation





N. Chamel, S. Naimi, E. Khan, J. Margueron, nucl-th/07_01851

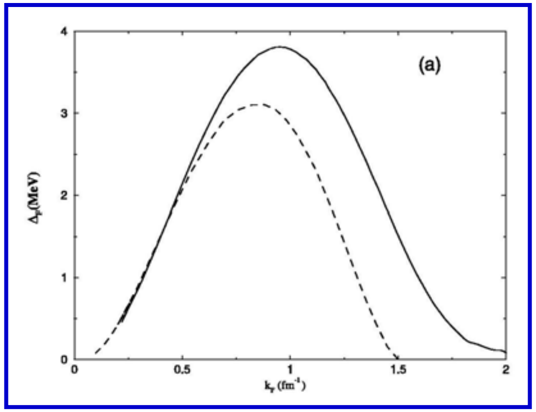


Calculated gaps for unbound states in a cell without nucleus

5–10% decrease around the Fermi energy

Calculated gaps for unbound states in a cell with nucleus

A comment on the need of an HFB treatment of the lattice …

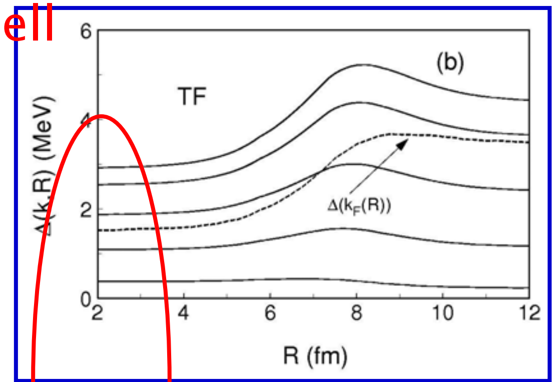


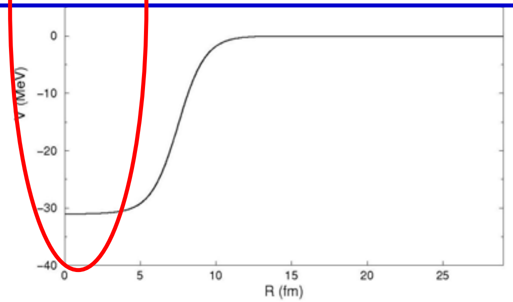
——— Gogny

- - - - - Argonne

Pairing gap sensitive to the nucelon-nucleon interaction!!!

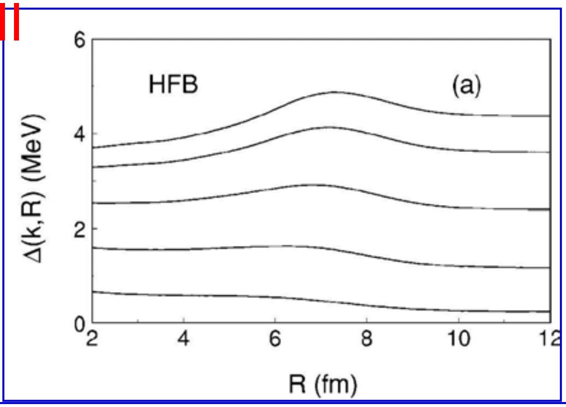
LDA description of pairing properties of WS cell





Higher density: smaller pairing field

HFB description of pairing properties of WS cell



Smoother pairing gap because Cooper-pair wave-function is delocalized

We need full HFB

- to consider quantal fluctuations;
- to get a reliable description of pairing field!

A few questions about pairing correlations in the inner crust

1. **Does superfluidity affect the results found by Negele and Vautherin?**

2. **What is the spatial dependence of the pairing gap?**

   How important are the nuclear clusters?

3. **How much are the gaps affected by many-body processes ?**

4. **Can we prove experimentally that the crust is really superfluid?**

New calculation of the optimal properties of the WIgner-Seitz cell including pairing

The ‘global’ functional: matching Fayans functional (for finite nuclei) with BBG calculation for neutron matter

$$F_m(r) = \left(1 + \exp\left((r - R_m)/d_m\right)\right)^{-1}.$$

$$\mathcal{E}\left(\rho_\tau(\mathbf{r}), \nu_\tau(\mathbf{r})\right) = \mathcal{E}^{\text{ph}}\left(\rho_\tau(\mathbf{r}), \nu_\tau(\mathbf{r})\right) F_m(r) + \mathcal{E}^{\text{mi}}\left(\rho_\tau(\mathbf{r}), \nu_\tau(\mathbf{r})\right)\left(1 - F_m(r)\right),$$

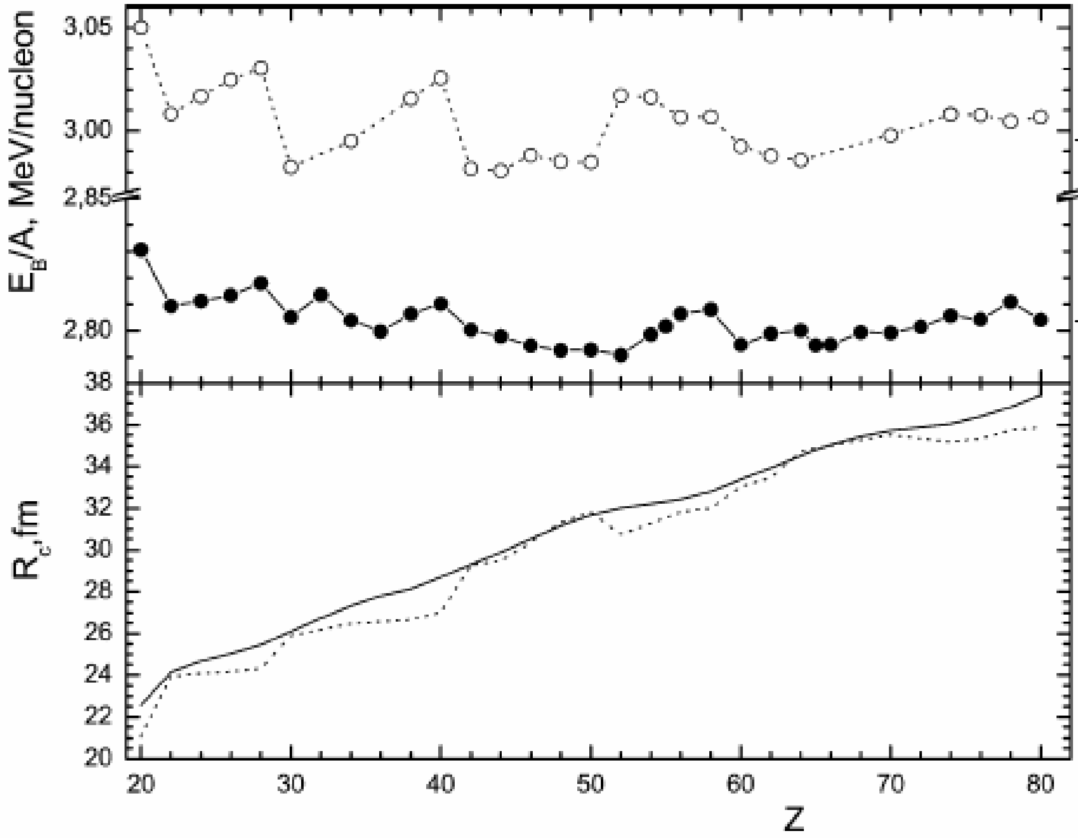
Phenomenological functional with gradient terms: ‘knows how to deal with the surface’

Microscopic, ‘exact’ description of neutron matter

$$\rho_p(R_m) = 0.1\,\rho_p(0)$$

Matching condition

Simplified pairing description: constant G(ρ) which reproduces the BCS gap obtained in neutron matter with the bare N-N force



Without pairing

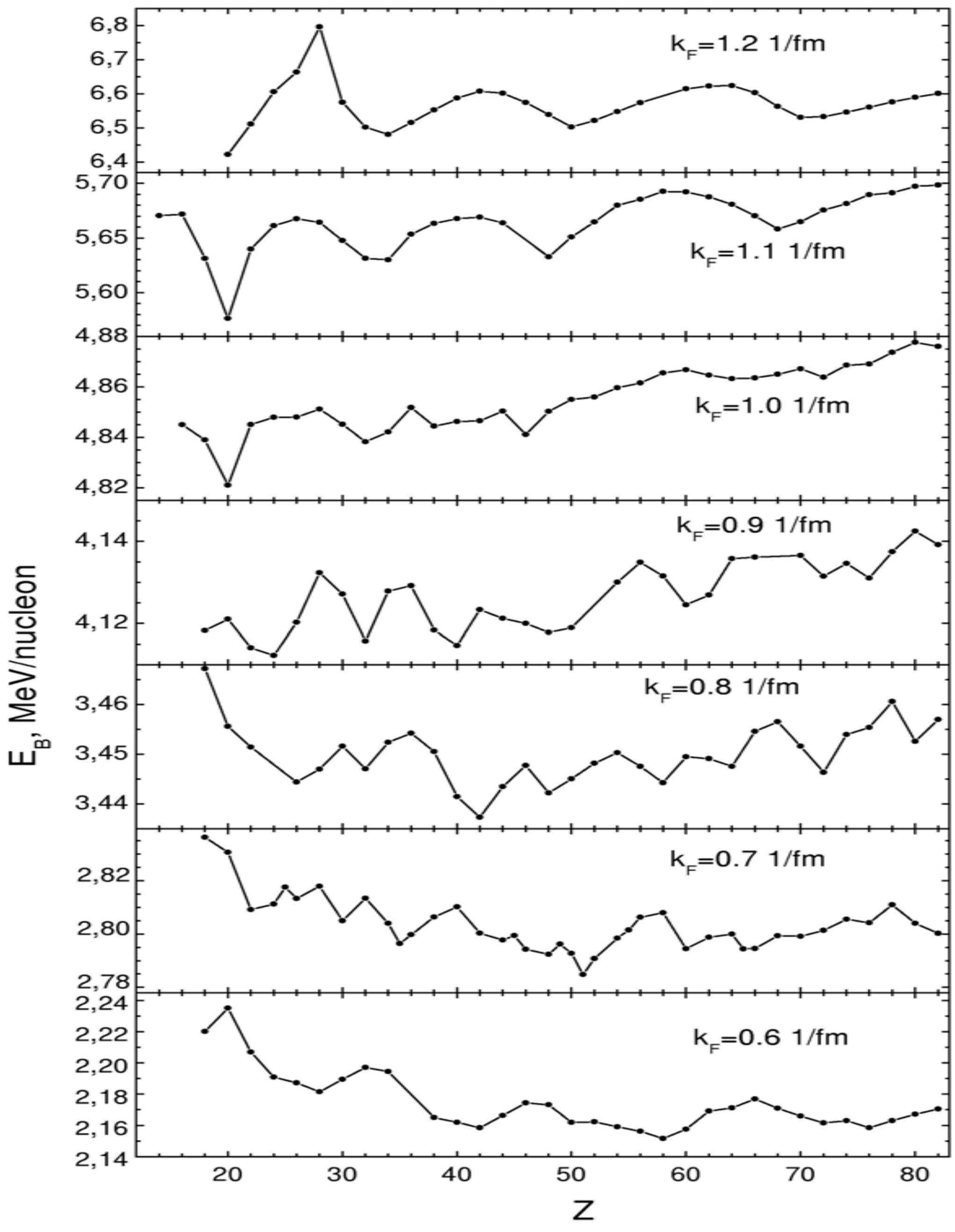
With pairing: smoothing of shell effects

M. Baldo, U. Lombardo, E.E: Saperstein, S.V. Tolokonnikov, Nucl. Phys. A750(2005)409

In search of the energy minimum as a function of the Z value inside the WS cell

NPA 750 (2005) 409

M.B. , U.Lombardo, E.E. Saperstein and S.V. Tolokonnikov.



Comparing with Negele & Vautherin [5]

k_F, fm^{-1}	Z		A	R_c, fm	x		
	Our study	[5]			Our study	[5]	[23]
0.6	58	50	1612.10	37.505	0.036	0.037	0.0004
0.7	51	50	1573.70	31.890	0.032	0.037	0.0010
0.8	42	50	1409.10	26.895	0.030	0.028	0.0019
0.9	24	50	857.02	20.255	0.028	0.028	0.0034
1.0	20	40	658.07	16.693	0.030	0.027	0.0057
1.1	20	40	634.62	14.993	0.032	0.027	0.0086
1.2	20	40	626.47	13.684	0.032	0.027	0.0125

[23] Uniform nuclear matter (M.B.,Maieron,Schuck,Vinas NPA 736, 241 (2004))

A few basic questions about pairing correlations

1. **Does superfluidity affect the results found by Negele and Vautherin?**

2. **What is the spatial dependence of the pairing gap?**
   How important are the nuclear clusters?

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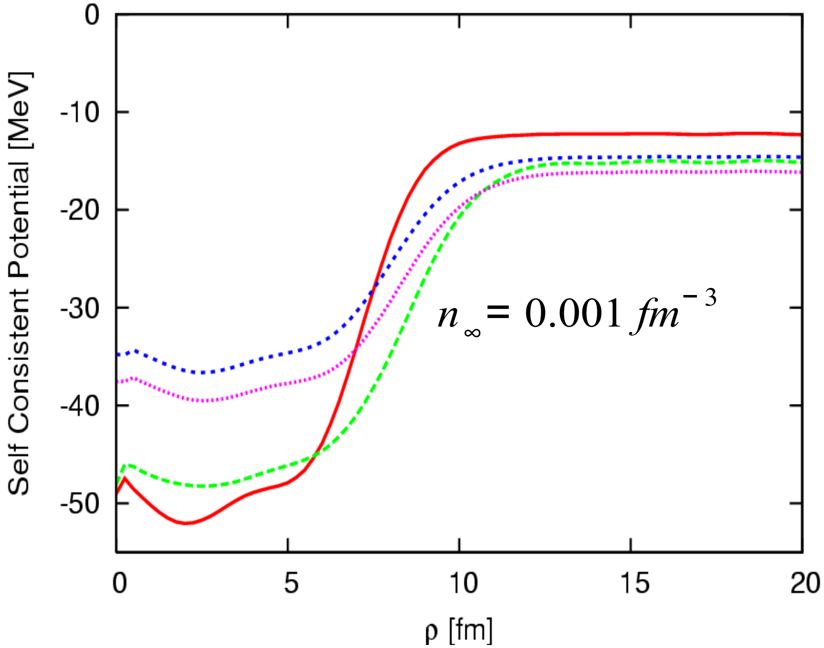
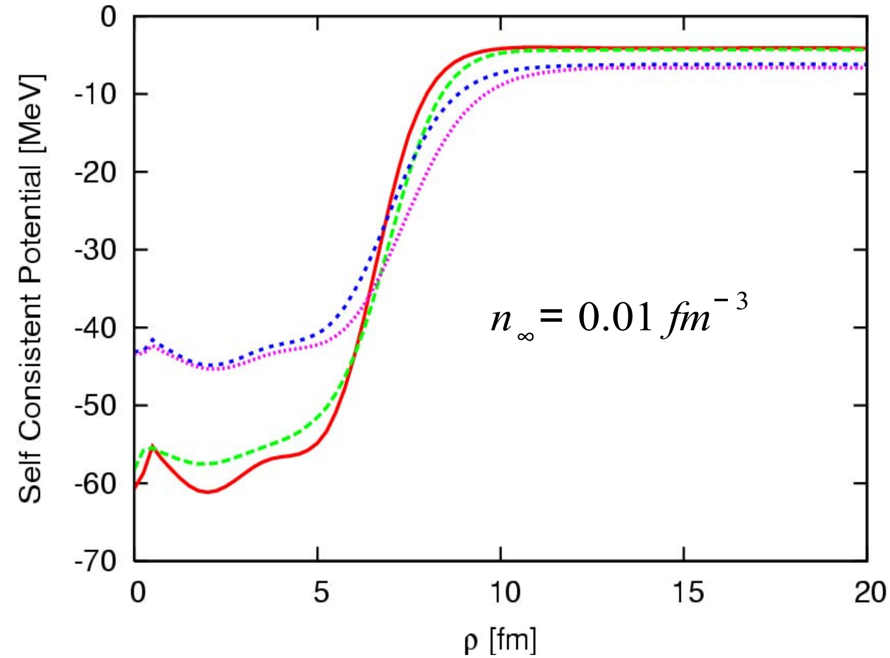
4. **Can we prove experimentally that the crust is really superfluid?**

Self consistent potentials



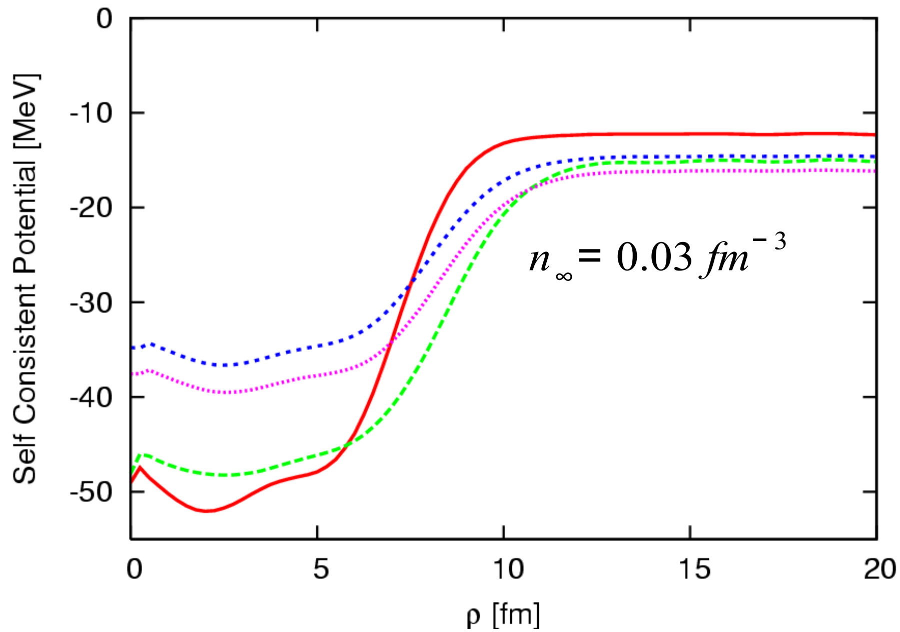


Red= SII

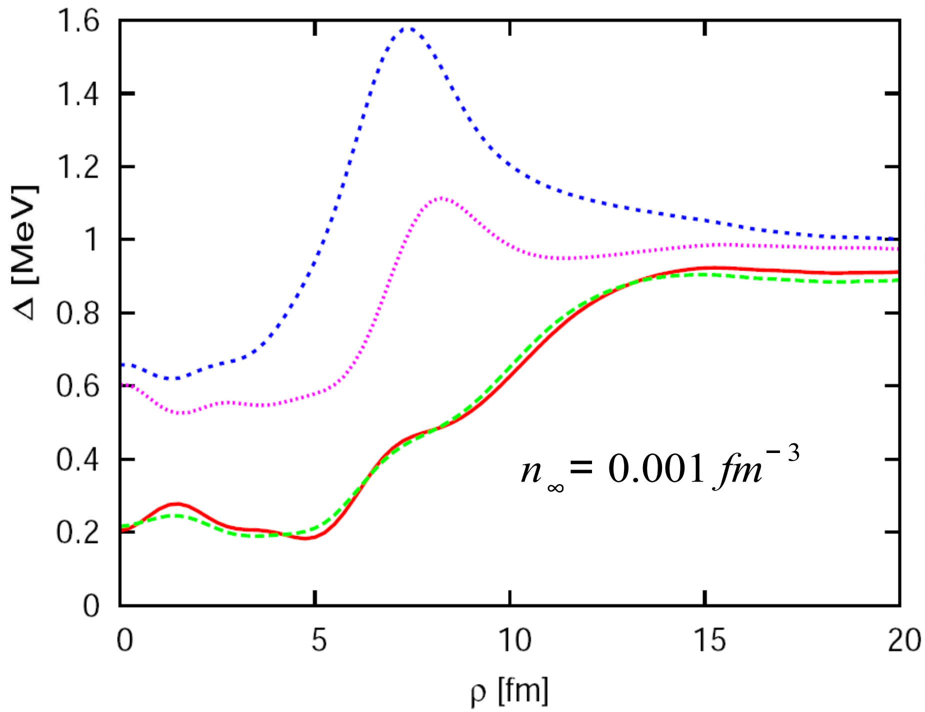
Green= Sly4

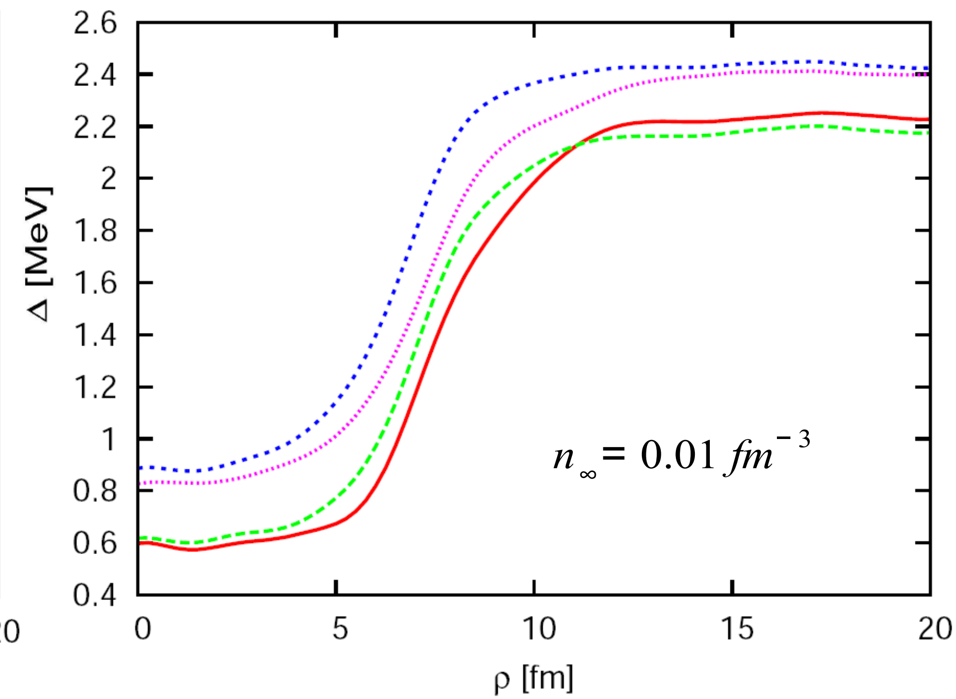
Blue= Skm*

Violet= SGII



The SII and Sly4 self consistent potentials are deeper than the Skm* and SGII ones.



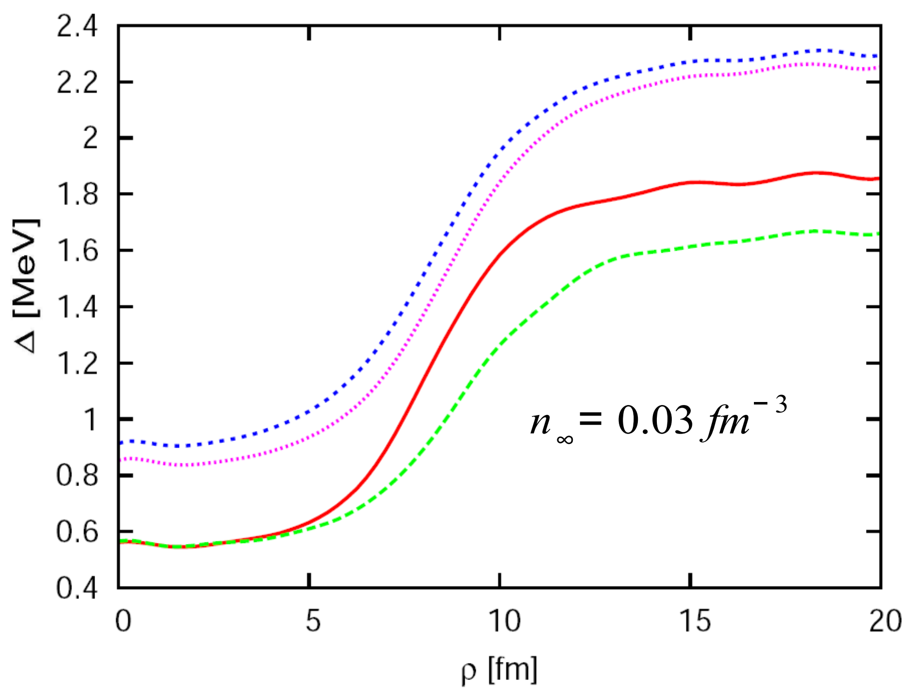


SII red

Sly4 green

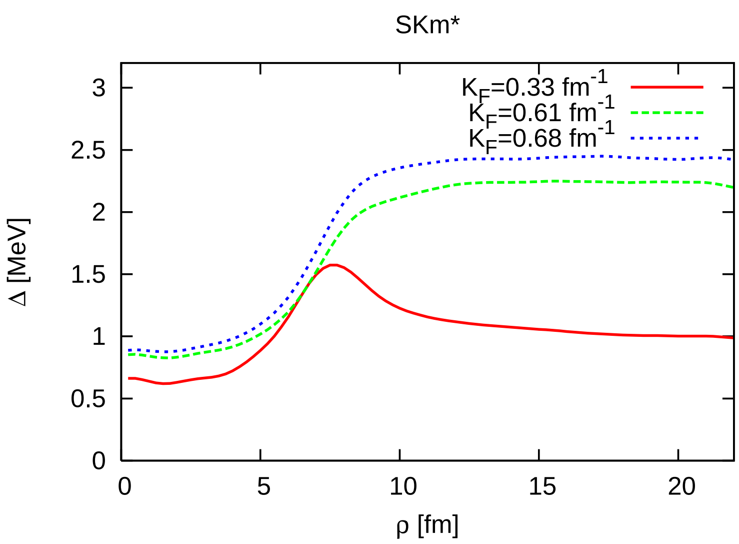
Skm* blue

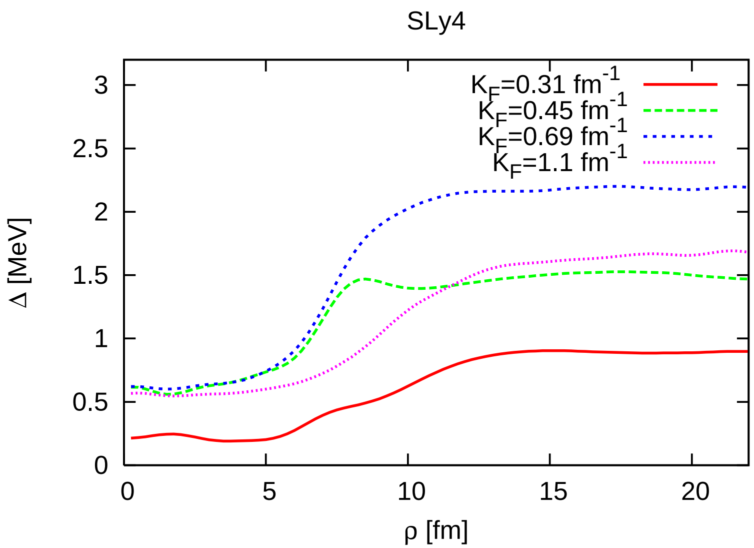
SGII violet

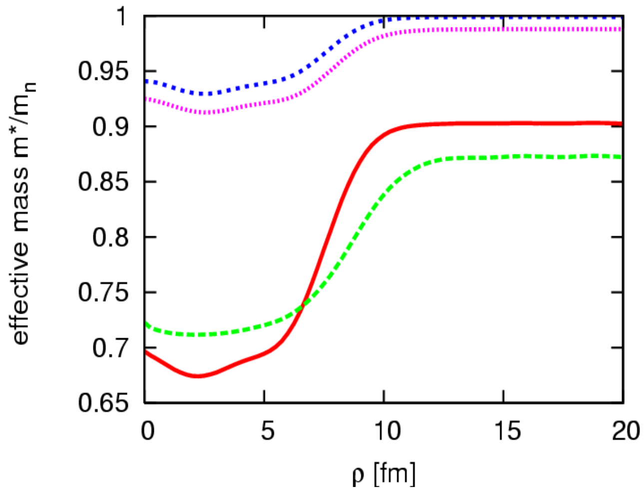


The pairing gap calculated with the different Skyrme forces for the cells with nuclei

Dependence on the HF mean field







Cooling time : effect of superfluidity and of inhomogeneity