Superfluidity in neutron stars

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Outline

1. Motivation
2. S-wave superfluidity in neutron matter
3. EFT for S-wave gaps
4. P-wave superfluidity
5. Summary and open problems

with P.T. Reuter

B. Friman, K. Hebeler GSI

C.J. Pethick NORDITA

G.E. Brown STONY BROOK UNIVERSITY

1. Motivation

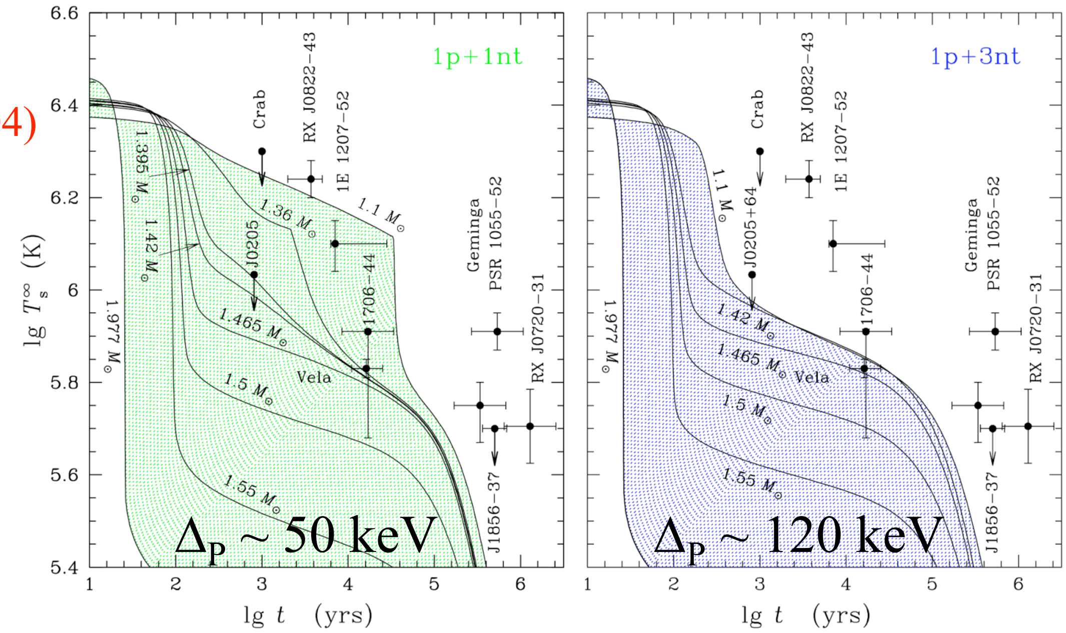
Superfluidity in strong-interaction systems: nuclei, halos to neutron stars

Impact on cooling: suppresses ν emission, but thermal quasiparticles can emit neutrino-antineutrino bremsstahlung

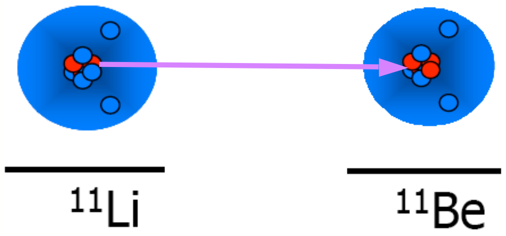
need consistent theory for superfluidity and electro-weak operators Leinson, Perez (2006)

superfluidity enhances cooling from P-wave superfluid just below T_c
Yakovlev, Pethick (2004); Blaschke et al. (2004) found too rapid cooling for gaps $\Delta_P > 30$ keV

in contrast to standard $\Delta_P \sim 0.1\text{-}1$ MeV
less severe dependence in Page et al. (2004)



similarly: beta decay of nn halo in ^{11}Li suppressed Sarazin et al. (2004)

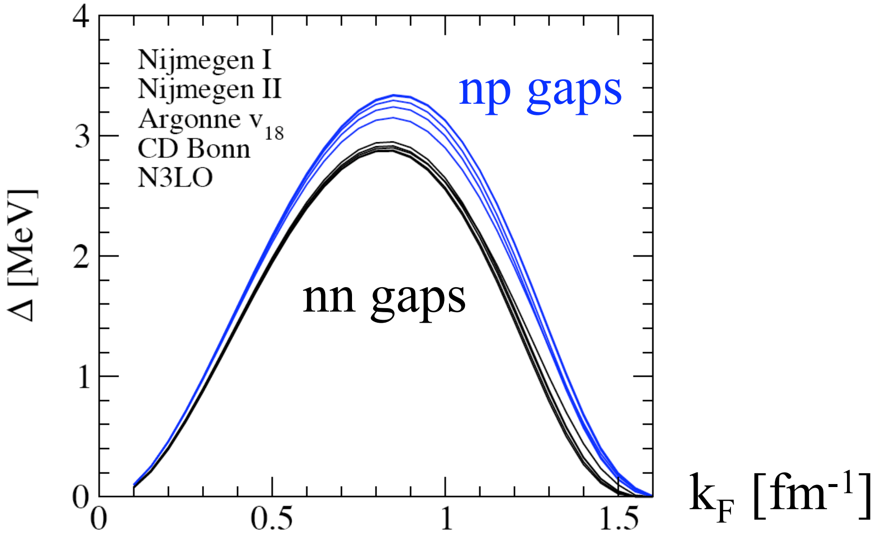


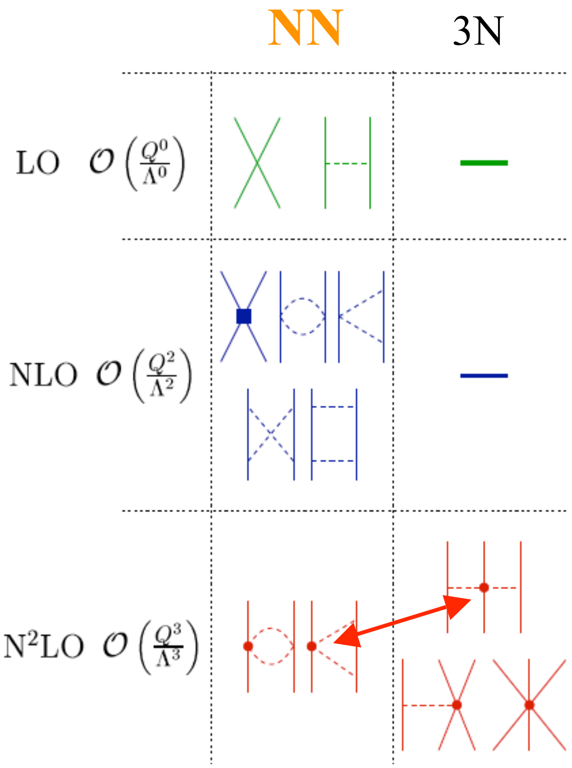
Rotational properties and vortices

Superfluidity in neutron matter

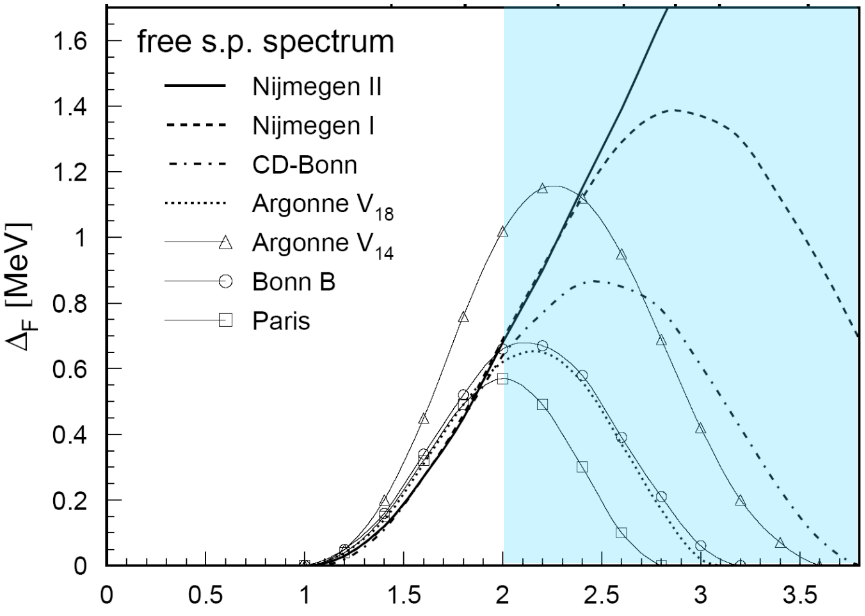
BCS gaps from nucleon-nucleon interactions (best via pions + contacts) and free dispersion, well constrained for momenta < 2 fm^{-1}

low-density:
$^1S_0$ pairing





higher-density:
$^3P_2$-$^3F_2$ pairing
cf. liquid ^{3}He



from Baldo et al.,
PR C58 (1998) 1921.

Superfluidity at extremely low densities from cold atoms

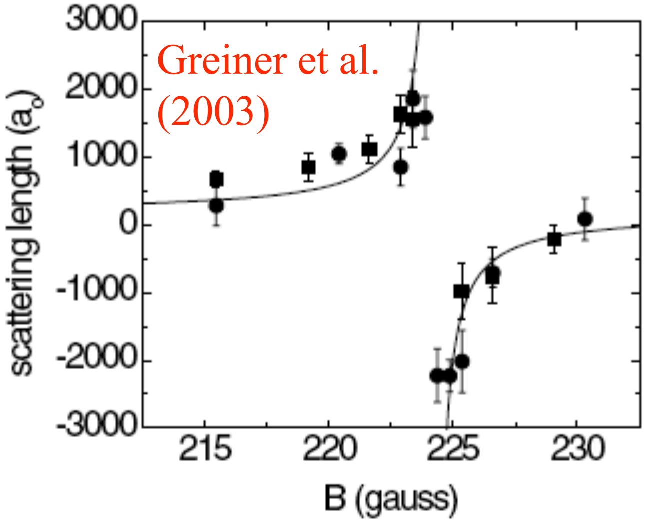
large neutron-neutron scattering length

$$a_{\mathrm{nn}} = -18.5 \pm 0.3\,\mathrm{fm}$$

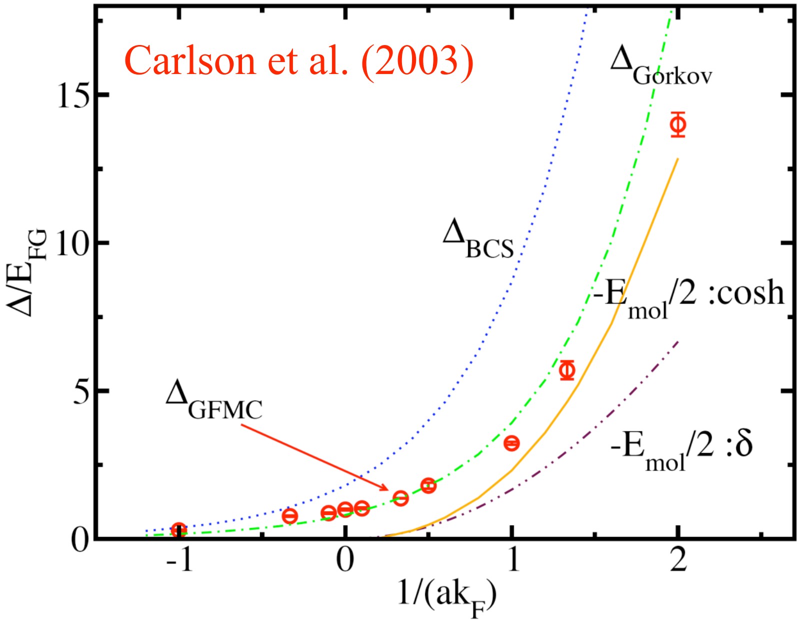
generate same properties by tuning scattering length of dilute systems to universal regime

$$0 \leftarrow 1/a_s \quad \ll \quad k_{\mathrm{F}} \quad \ll \quad 1/r_e\,, 1/R\,, \ldots \rightarrow \infty$$

strongly-interacting dilute


Greiner et al. (2003)

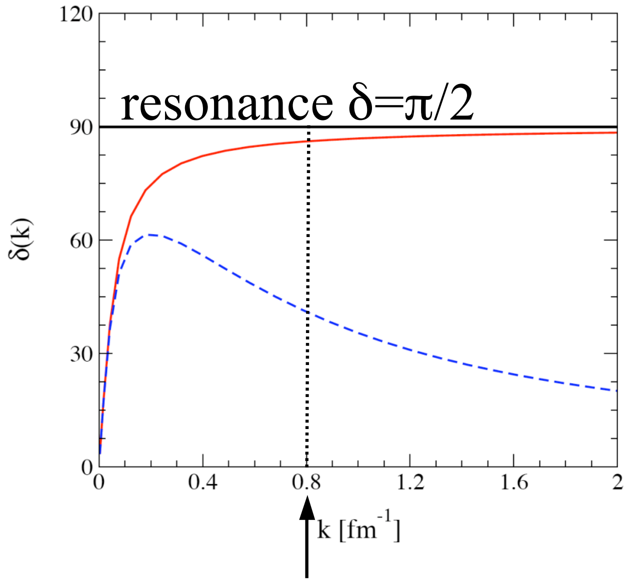
Fermi momentum sets scale, physics independent of interaction details, same for fermionic ^{6}Li or ^{40}K atoms or extremely low-density neutrons


Carlson et al. (2003)

insights to neutron superfluidity from cold atoms: $T_c \sim 0.2\text{-}0.3\ T_F$ Duke group (2007)

GFMC results consistent with reduction from BCS gaps

Neutron matter in stars is less dilute



a_{nn} only

a_{nn} + effective range

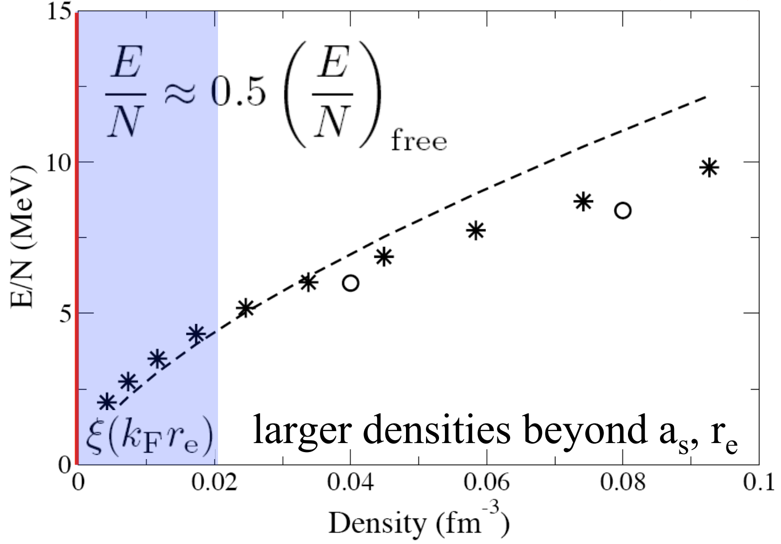
$r_e = 2.7$fm

effective range appreciable, weakens interactions at higher momenta

fewer particles interact strongly

low-density k_F at maximum S-wave gap

Carlson et al. (2003)



All microscopic calculations for neutron matter equation of state similar

even above universal regime

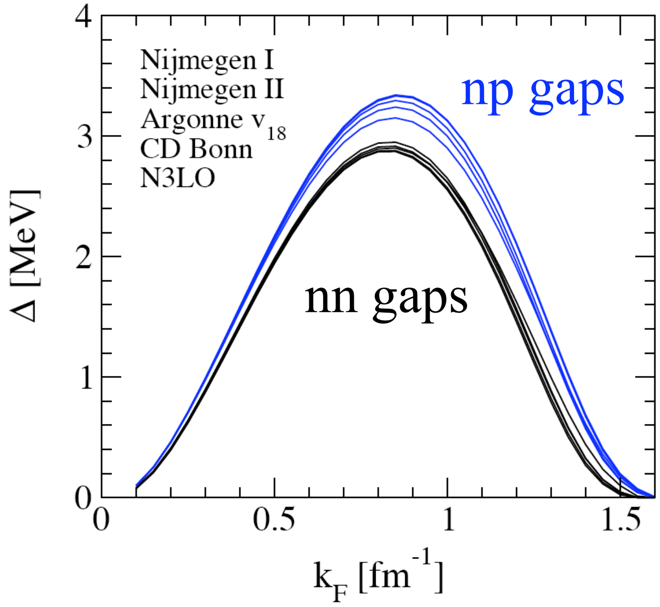
large a_{nn} + effective range: $\frac{E}{N} = \xi(k_F r_e) \frac{3k_F^2}{10m}$

theoretically simpler, can solve in di-fermion EFT for large a_s + large r_e

AS, Pethick (2005)

2. S-wave superfluidity in neutron matter

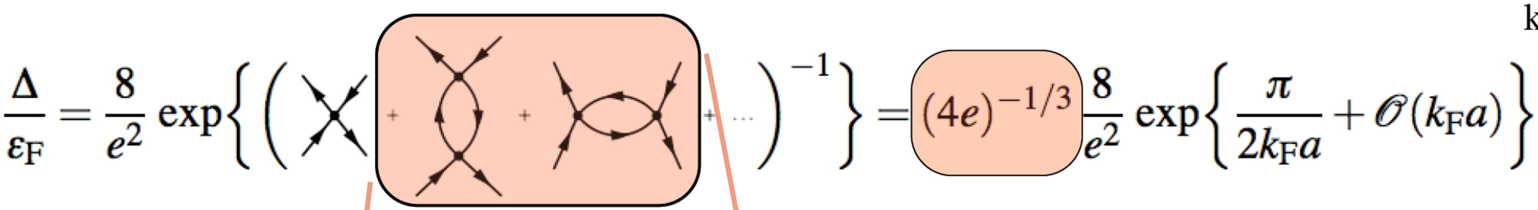
BCS gaps well constrained by NN scattering,
charge dependences resolved Hebeler et al. (2007)



Induced interactions beyond BCS:

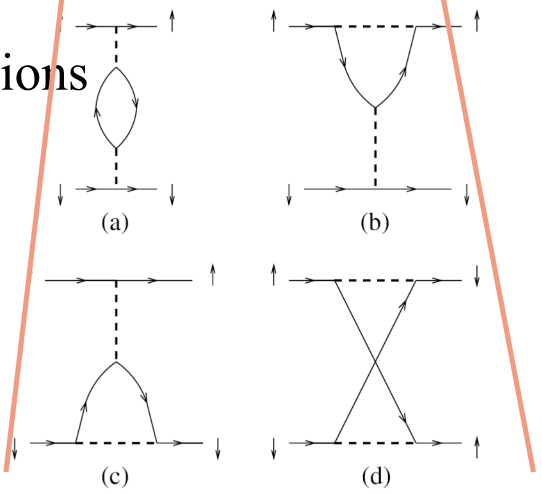
spin fluctuations repulsive, suppress $^1S_0$ gap even for perturbative $k_F a_s$

Gorkov et al. (1961); Heiselberg et al. (2000)



$$\frac{\Delta}{\varepsilon_F} = \frac{8}{e^2}\exp\left\{\left(\quad + \quad + \quad + \cdots \right)^{-1}\right\} = (4e)^{-1/3}\frac{8}{e^2}\exp\left\{\frac{\pi}{2k_F a} + \mathcal{O}(k_F a)\right\}$$

screening and
vertex corrections



dominated by low-lying particle-hole excitations, long-range physics, <u>assumes large separation of clusters</u>

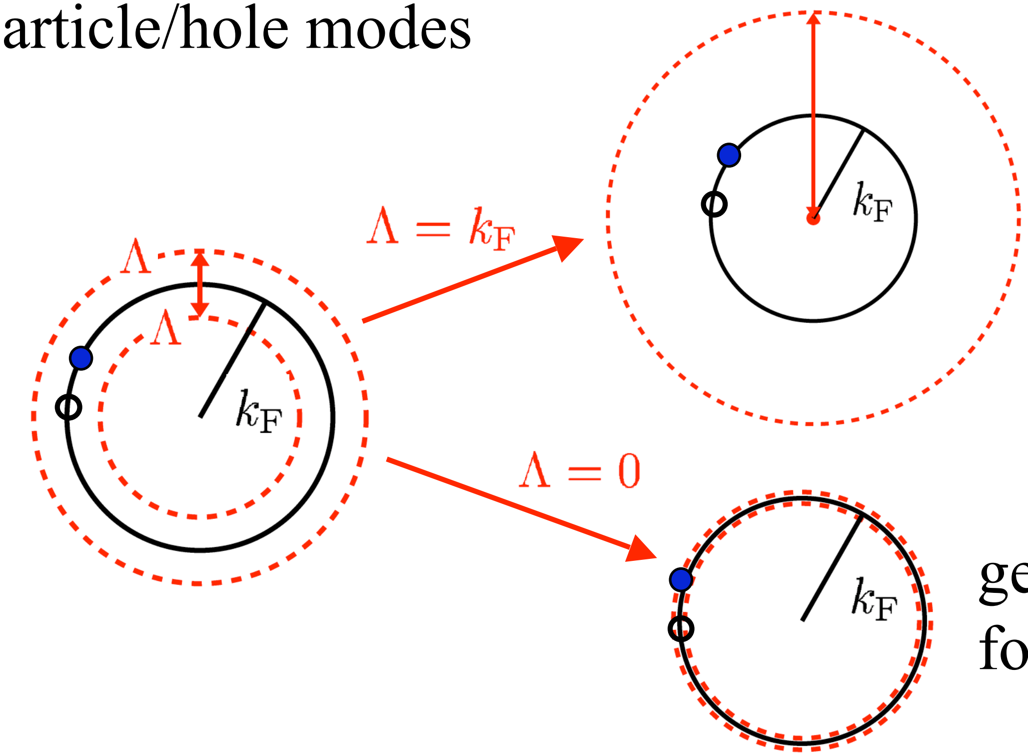
use renormalization group to include higher-order particle-hole contributions

AS, Friman, Brown (2003)

RG approach to interacting Fermi systems

follows Shankar, RMP 66 (1994) 129.

cutoff Λ around Fermi surface defines effective theory for low-lying particle/hole modes



start from full space + NN int.

integrate out mom. shells successively

generate induced interactions for low-lying modes

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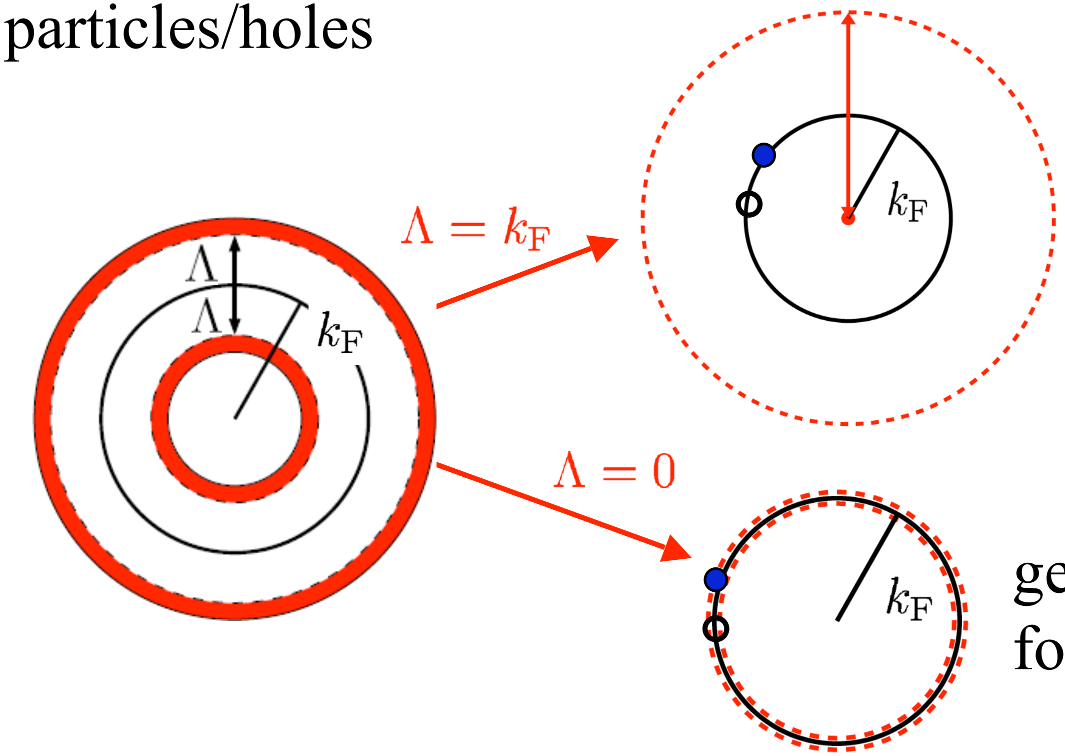


start from full space + NN int.

integrate out mom. shells successively

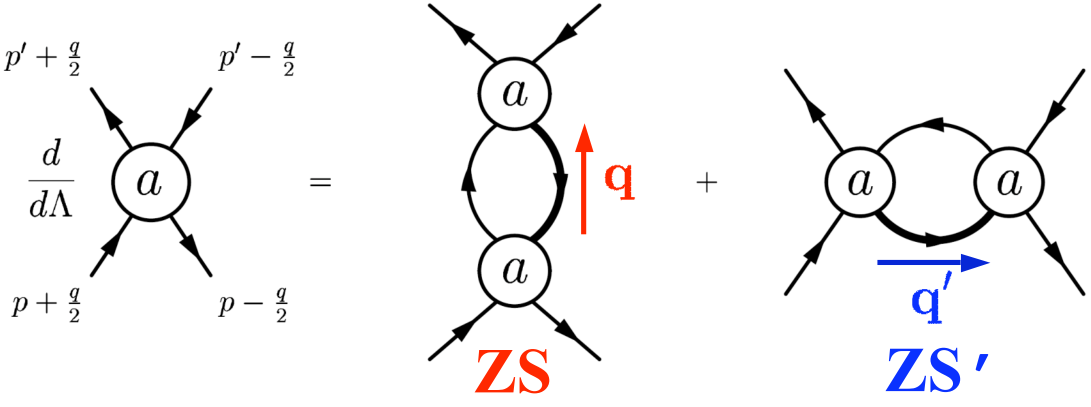
generate induced interactions for low-lying modes

Change of 4-pt vertex

Intermediate states:

thin from mom. shells,

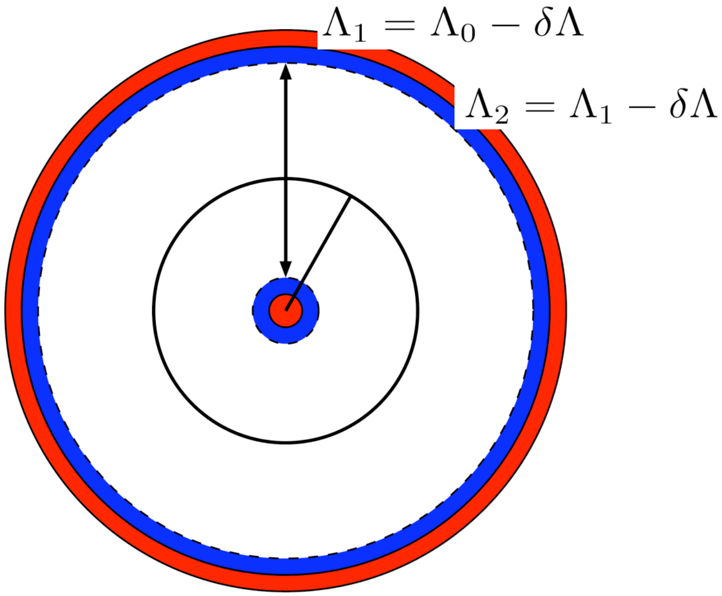
thick from fast p/h > Λ

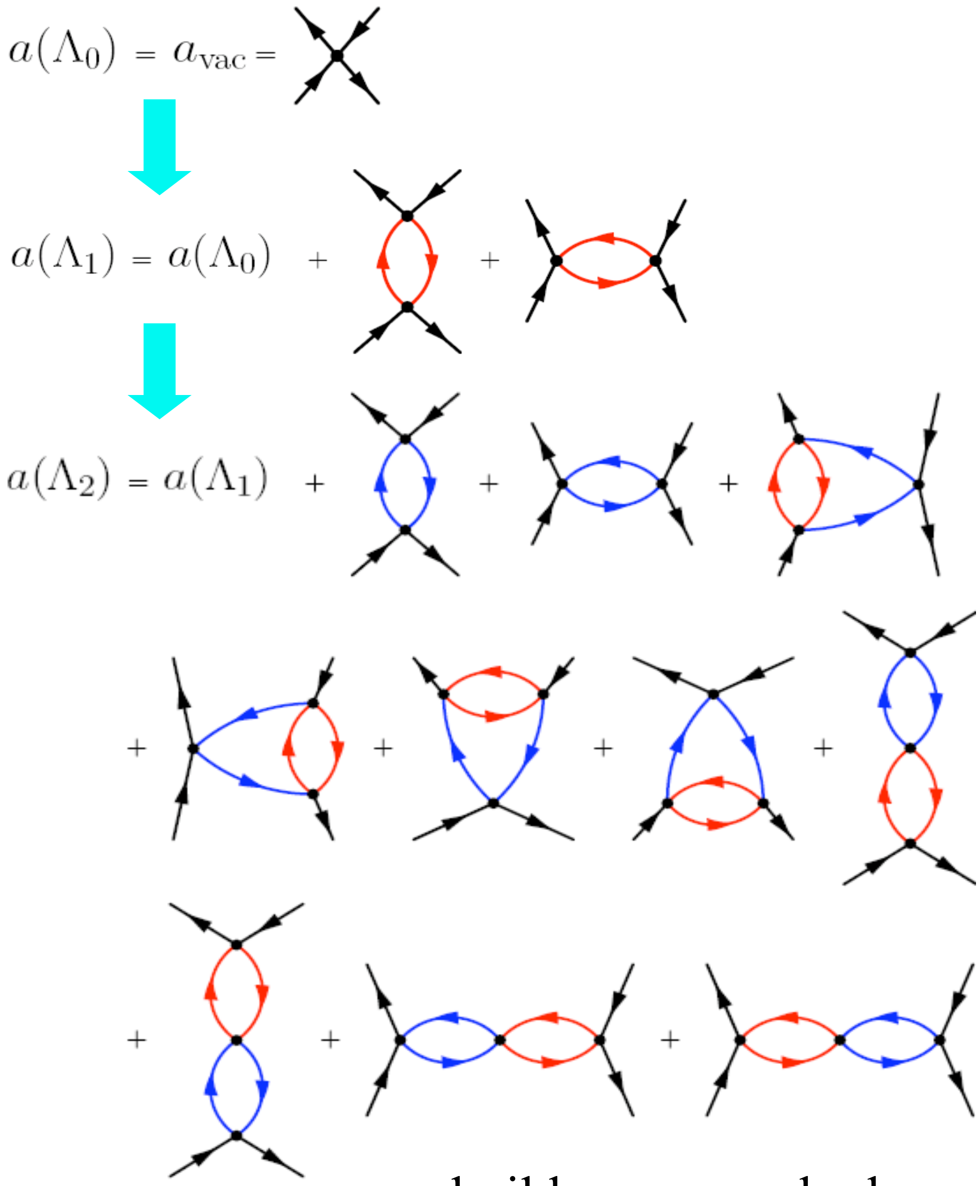


Efficacy of the RG method

Start from free-space
NN interaction

After two shells:





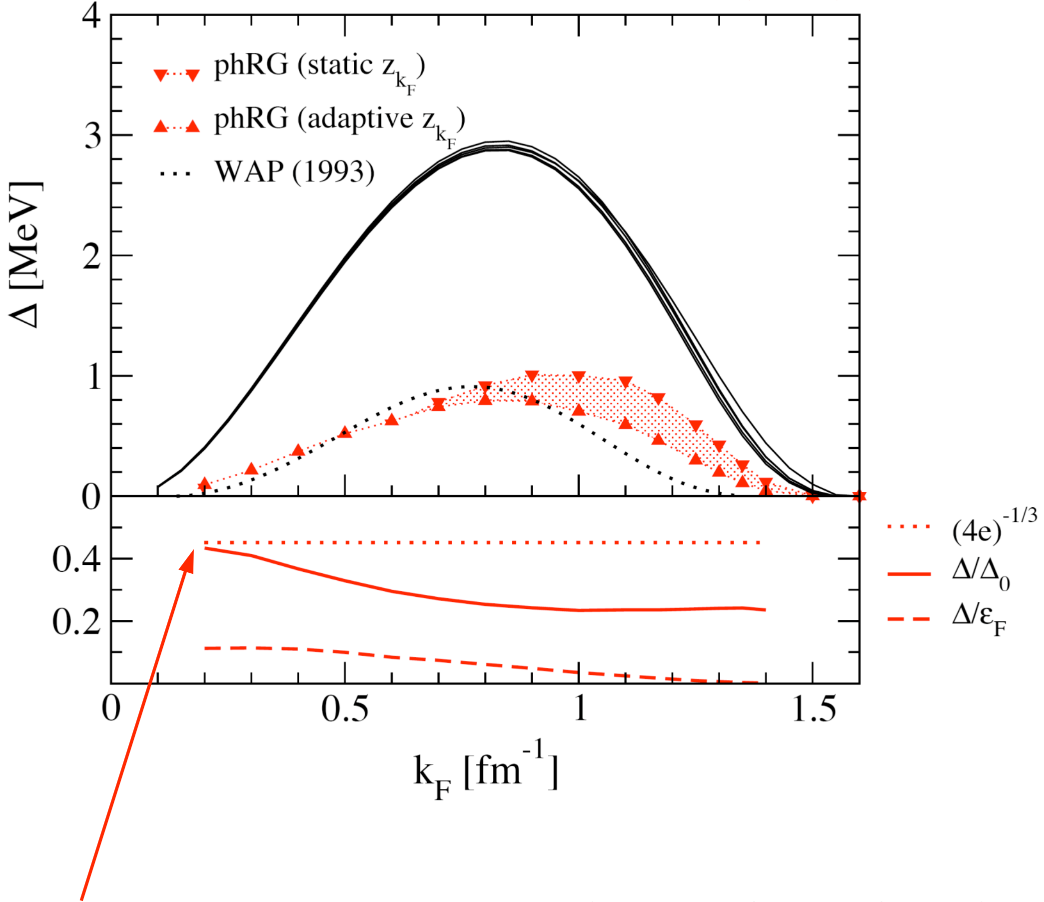
$+ \, \mathcal{O}(a_{\text{vac}}^4)$

builds up many-body
correlations (~ parquet)

S-wave superfluidity including induced interactions

induced interactions dominated by spin fluctuations suppress S-wave gap to $\Delta \approx 0.8$ MeV

magnitude/sign as expected


AS, Friman, Brown (2003)

band/uncertainty at larger density due to approximate self-energy treatment

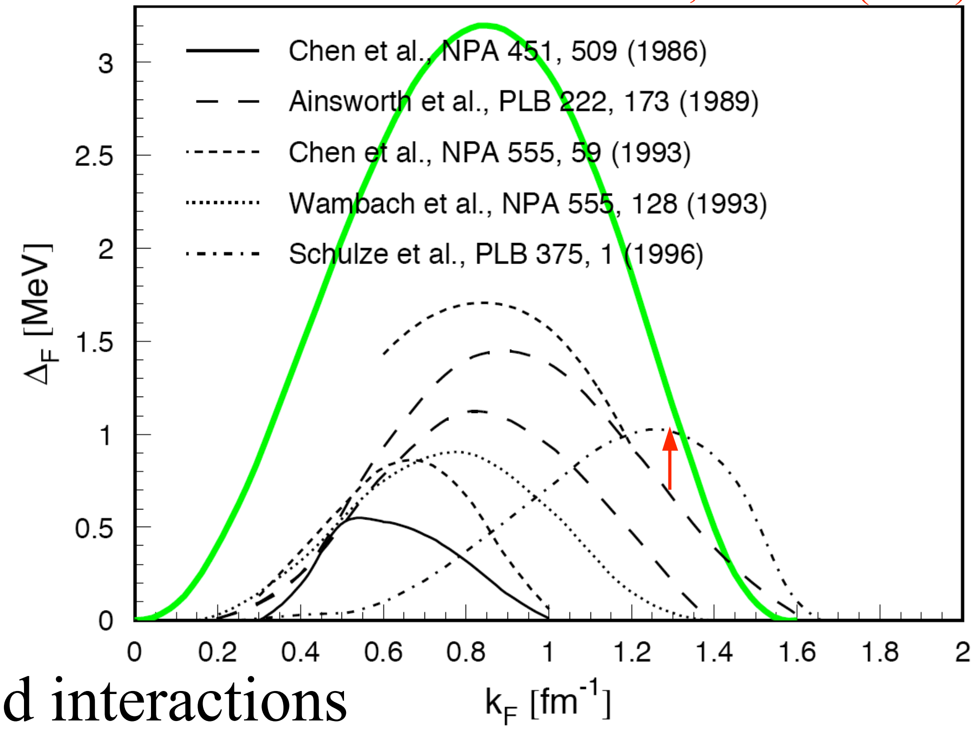
$m^*/m \approx 1$ below maximum

similar to Wambach et al. (1993)

nonperturbative RG reproduces Gorkov et al. $(4e)^{-1/3}$ suppression at low density

Discussion of previous gaps including induced interactions

from Lombardo, Schulze (2000)



Chen et al. (1986)

qualitative result, weak-coupling
in terms of q=0 Landau parameters

Ainsworth et al. (1989) superceded by
Wambach et al. (1993) (same authors, technique)

pseudo-potential +
Bethe-Salpeter equations for
induced interactions for finite q

Chen et al. (1993)

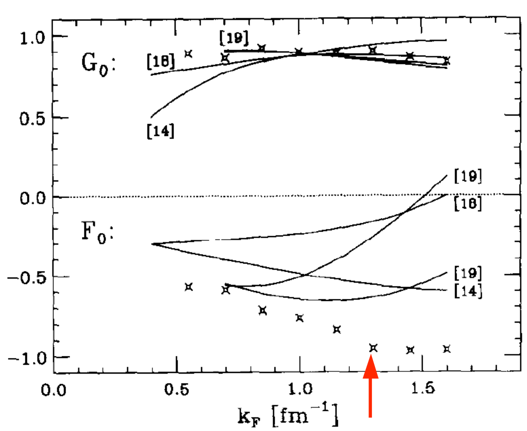
low order CBF, perturbative induced interactions

all above make approximations that disagree with BCS benchmark

Wambach et al. (1993) seems most reliable

Schulze et al. (1996)

based on q=0 induced interactions extrapolated to
finite q with averaging prescription,
strange results with $F_0 \approx$ -1 very close to instability

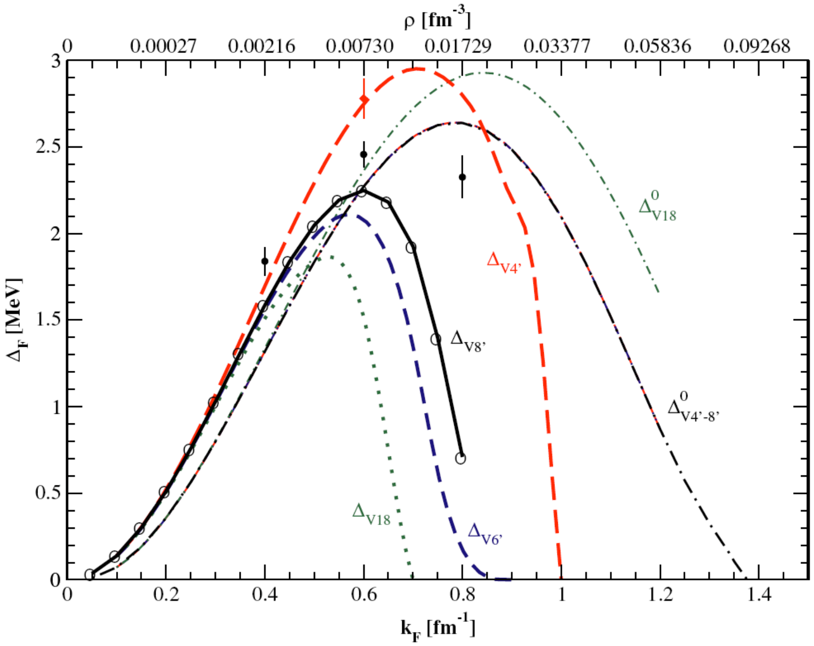


MC results for S-wave gaps

Fabrocini et al. (2005)

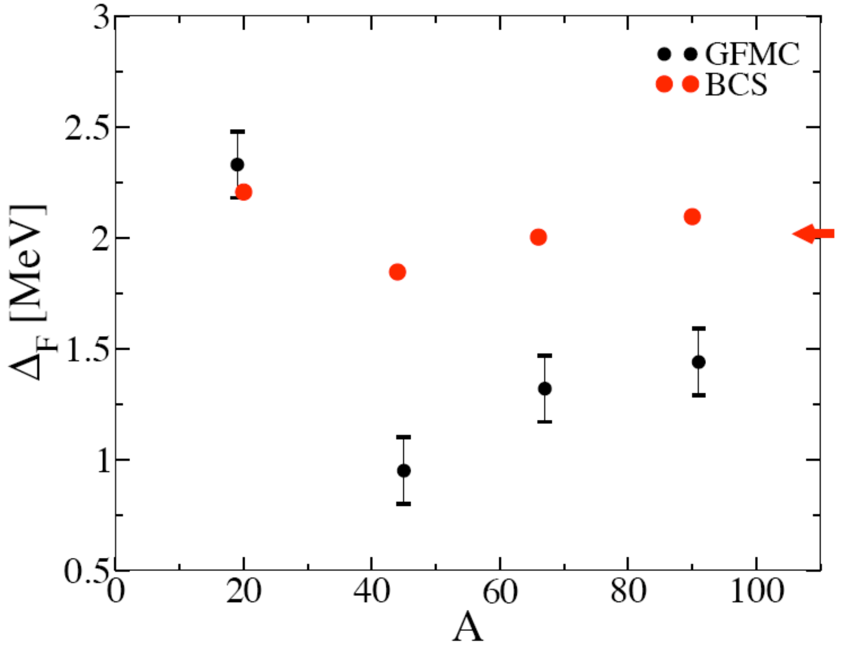
AFDMC for N=12-18 in box,
gaps from odd-even energies

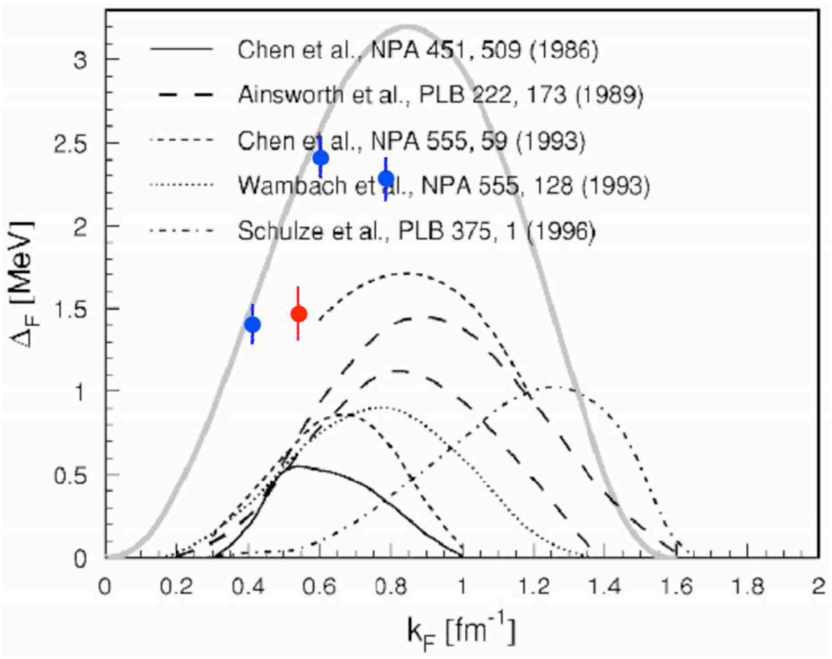
no effect of induced interactions at low densities, curves are lowest order CBF without screening/vertex corr.



Carlson, Gezerlis et al. (DNP 2007) see talk by Joe Carlson

GFMC for larger N<100





3. EFT for S-wave gaps

di-fermion effective field theory for large a_s and large r_e

Kaplan (1997), Bedaque, van Kolck (1998), Beane, Savage (2001), following Weinberg (1963)

both a_s and r_e are low-momentum scales, need to be iterated to all orders

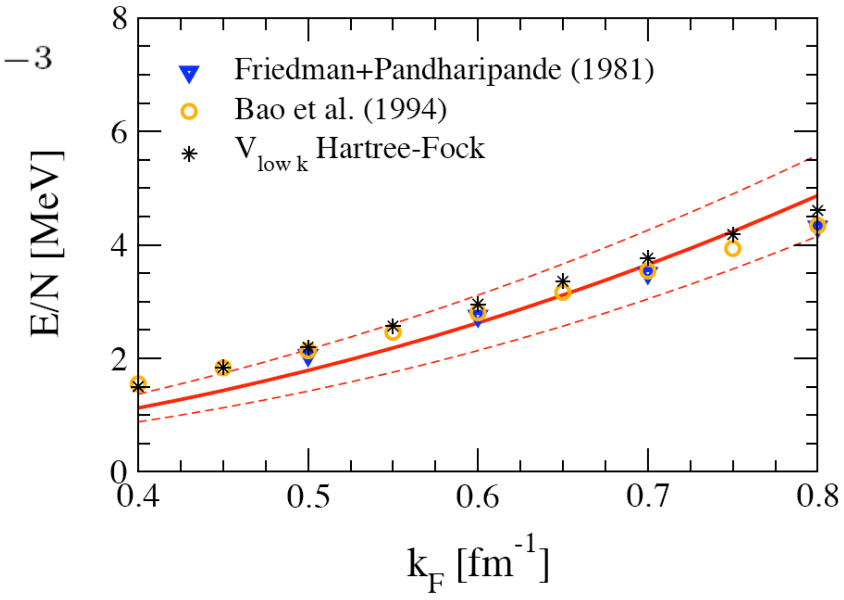
$$\mathcal{L} \;=\; \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2} \right) \psi - d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4} - \Delta \right) d \;-\; g \left(d^\dagger \psi\psi + d\, \psi^\dagger \psi^\dagger \right)$$

Δ, g: low-energy constants, matched to a_s and r_e, reliable for $k < 0.8\ \mathrm{fm}^{-1}$

for large r_e: leading order requires summing ladders, leads to average coupling $\sim (1+C\, k_F r_e)^{-1}$, with particle-hole, hole-hole loops subleading

AS, Pethick (2005)

neutron matter for $k_{\mathrm{F}} r_{\mathrm{e}} \lesssim 2$ or $\rho < 0.02\,\mathrm{fm}^{-3}$



errors due to particle-hole, hole-hole loops and weak pairing

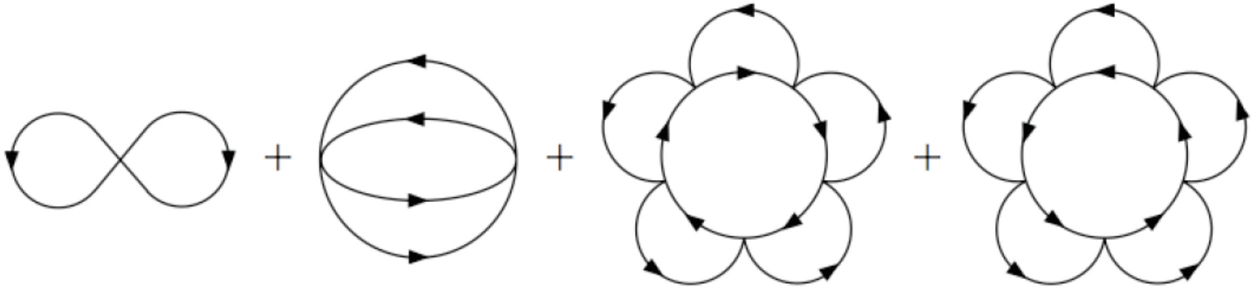
all microscopic results for E/N within errors of leading-order di-fermion EFT

di-fermion EFT gaps at lower densities

Reuter, AS, preliminary.

Effective action in Gorkov basis to subleading order, based on ladders and leading particle-hole loop

$\Gamma[\mathcal{G}] =$

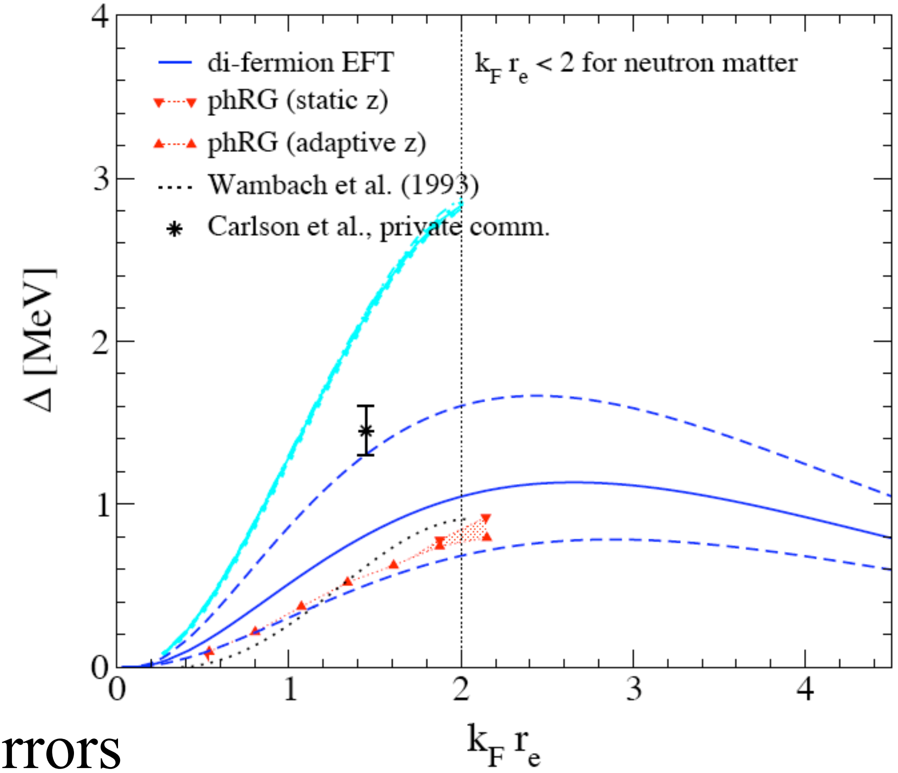


expansion in effective coupling

$$\frac{1}{g} \equiv \frac{1}{a_s k_F} - \frac{r_e k_F}{2} - \frac{4}{\pi}$$

with S-wave gap to subleading order

$$\boxed{\Delta = 8\,\epsilon_F \exp\left(\frac{\pi}{2\,g} + \ln(0.45) + c\,g + \cdots\right)}$$



first error estimate possible,
existing reliable S-wave gaps within errors

4. P-wave superfluidity

similar to phases in liquid ^{3}He:

for neutrons, tensor and spin-orbit interactions crucial

would condense $^3P_{\text{wrong } J=0}$ pairs without spin-orbit,
pion exchange only as in Khodel et al. (2006) unrealistic

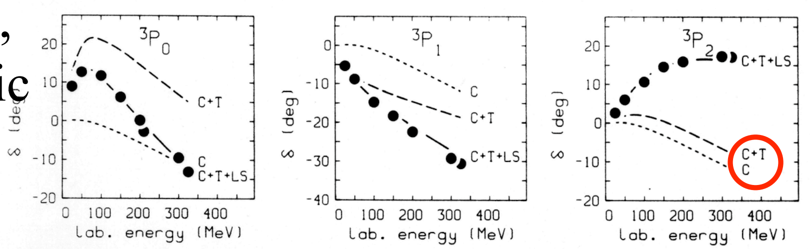


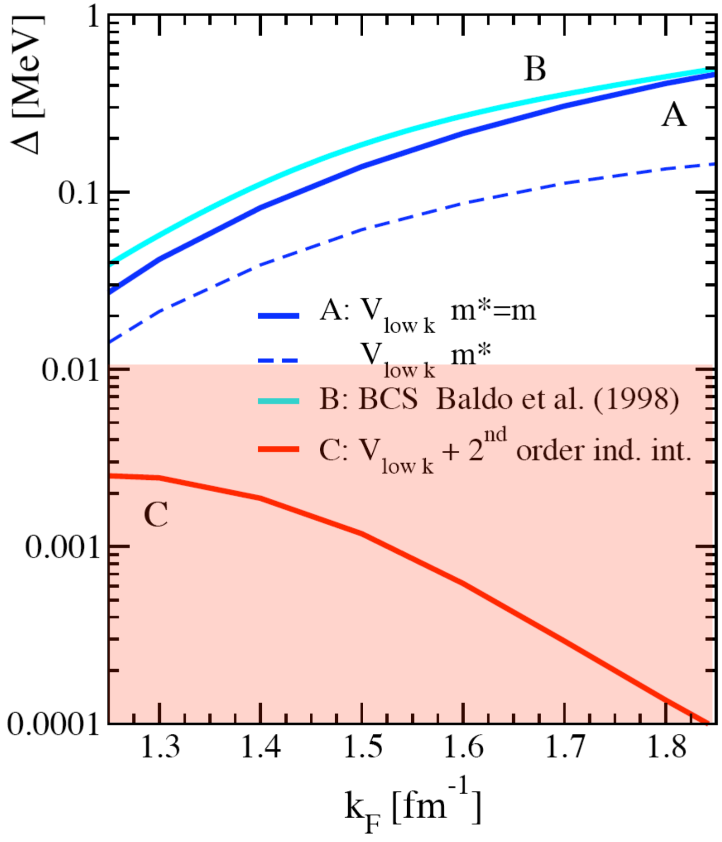
Fig. 3.3. ***NN* phase shifts in triplet *P* waves.** Shown are predictions using a central

without tensor/spin-orbit: spin fluctuations
attractive in S=1, would increase P-wave gap
Pethick, Ravenhall (1991); Jackson et al. (1982)

first perturbative results including
spin, spin-orbit and tensor induced
interactions AS, Friman (2004)
<50% corrections to pairing interactions

P-wave gaps < 10 keV possible
due to repulsive induced spin-orbit
interactions

implies that core neutrons may be
superfluid only at late times



5. Summary and open problems

S-wave superfluidity in neutron matter well constrained by NN scattering

induced interactions are essential for gaps

tractable di-fermion EFT results for S-wave gaps at lower densities,
reliable existing results within errors at subleading order

will learn more from intersections with cold atoms

spin-orbit interactions crucial for P-wave pairing, gaps may be small,
core neutrons only superfluid at late times? possible solution to precession problems

impact of clustering/pasta on S-wave gaps

consistent neutrino emission from S-wave superfluid in di-fermion EFT

improved treatment of spin-orbit, tensor ind. interactions for P-wave gaps

induced interactions in asymmtric matter, proton superconductivity
expect proton gaps < S-wave neutron gaps due to neutron polarization Wambach et al. (1991)

impact of 3N interactions on neutron P-wave gaps, proton gaps