Pairing Gaps in Neutron Stars

- General theory: Gorkov equations, BCS approximation
- In-medium interaction: Polarization effects
- Overview of nn pairing gaps in neutron matter
- Gaps in beta-stable matter: EOS, Medium effects, TBF
- Hyperon-nucleon pairing in neutron stars ?

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with J. Mur & A. Polls & A. Ramos ; Barcelona
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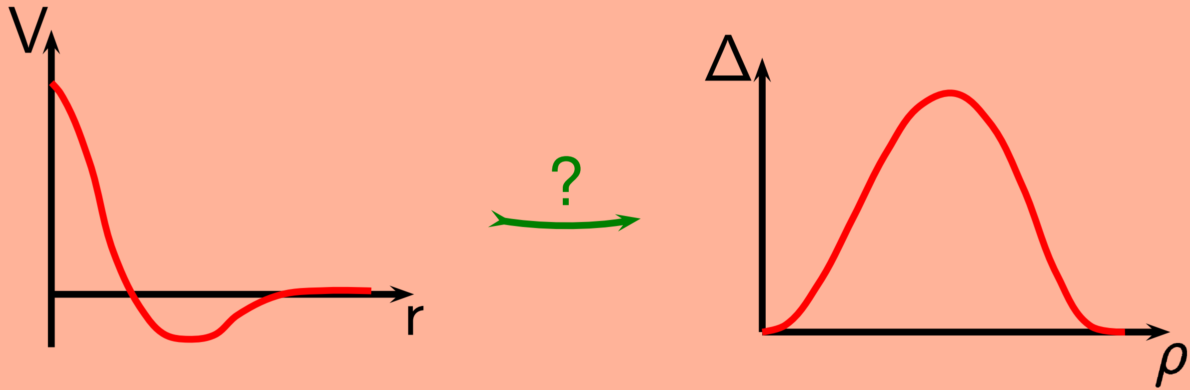
P. Schuck ; Orsay

Motivation:

- Theoretical goal:

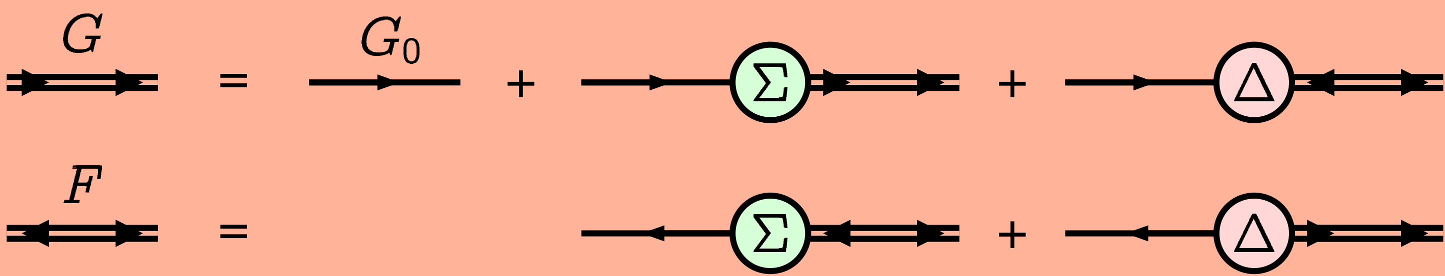
  Microscopic calculation of pairing gaps from fundamental interaction (bare potential):



- Experimental relevance:
  - Electron systems
  - T=1 nn,pp pairing:
    - Pairing force in finite (halo) nuclei
    - Superfluidity in neutron stars: glitches, cooling
  - T=0 np pairing:
    - Relevance for finite nuclei ($N \approx Z$) ?
    - Deuteron correlations, production
  - Pairing in trapped atomic gases
  - Color superconductivity

Superfluid Fermi Systems:

- General Framework: Gorkov Equations:



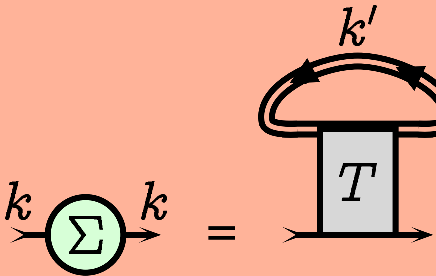
Generalization of Dyson equation:
Gap function Δ is analog of self-energy Σ

Formal solution:

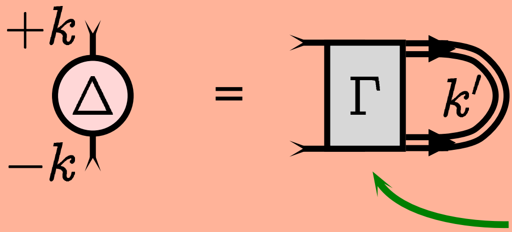
$$G(k) = \frac{k_0 + \epsilon(-k)}{D(k)} \quad D(k) = [k_0 - \epsilon(+k)][k_0 + \epsilon(-k)] - |\Delta(k)|^2$$

$$F^\dagger(k) = \frac{-\Delta^*(k)}{D(k)} \qquad \epsilon(k) = \frac{\boldsymbol{k}^2}{2m} + \Sigma(\boldsymbol{k}, k_0) - \mu$$

- Gap Equation (4-dim):



$$\Sigma(k) = i\int\frac{d^4k'}{(2\pi)^4}\langle k, k'|T|k, k'\rangle G(k')$$



$$\Delta(k) = i\int\frac{d^4k'}{(2\pi)^4}\langle k, -k|\Gamma|k', -k'\rangle F(k')$$

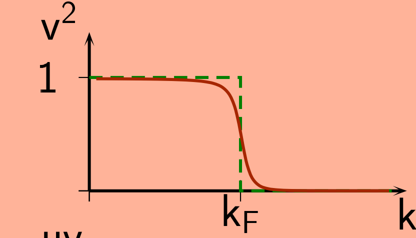
Irreducible interaction kernel

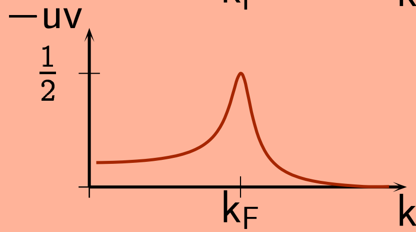
Γ: Irreducible interaction kernel built with V, G, F:
Self-consistent problem !
Approximation: use Γ of the normal system

- Simplest (BCS) approximation: $\Gamma = V$ (bare potential):

$$\Sigma(\boldsymbol{k}) = \sum_{k'} v_{k'}^2 \, \langle \boldsymbol{k}, \boldsymbol{k}' | V | \boldsymbol{k}, \boldsymbol{k}' \rangle_a$$

$$\Delta(\boldsymbol{k}) = \sum_{k'} (uv)_{k'} \underbrace{\langle +\boldsymbol{k}', -\boldsymbol{k}' | V | +\boldsymbol{k}, -\boldsymbol{k} \rangle_a}_{\langle \boldsymbol{k}' | V | \boldsymbol{k} \rangle}$$





Mean field approximation !

- Partial wave expansion $\rightarrow {}^1S_0$ channel:

$$\Delta(k') = -\frac{1}{\pi} \int_0^\infty dk \, k^2 \frac{V_{^1S_0}(k, k')}{\sqrt{\left[k^2/2m + \Sigma(k) - \mu\right]^2 + \Delta(k)^2}} \Delta(k)$$

BCS at Low Density: Weak-Coupling Approximation

- Gap equation:

$$\Delta_k = -\sum_{k'} V_{kk'} \frac{1}{2E_{k'}} \Delta_{k'} \quad , \quad E_k = \sqrt{(e_k - e_F)^2 + \Delta_k^2}$$

- Transformation:

$$T_{kk'} = V_{kk'} - \sum_{k''} T_{kk''} \frac{\mathrm{sgn}(k'' - k_F)}{2E_{k''}} V_{k''k'}$$

$$\Delta_k = -2 \sum_{k'<k_F} T_{kk'} \frac{1}{2E_{k'}} \Delta_{k'} \quad : \quad \text{restricted momentum space}$$

- Low density ($k_F \to 0$, $\Delta/e_F \to 0$):

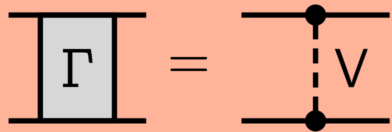
$$T_{kk'} \to T_{00} = \frac{4\pi a_{nn}}{m} \quad : \quad \text{free T} - \text{matrix}$$

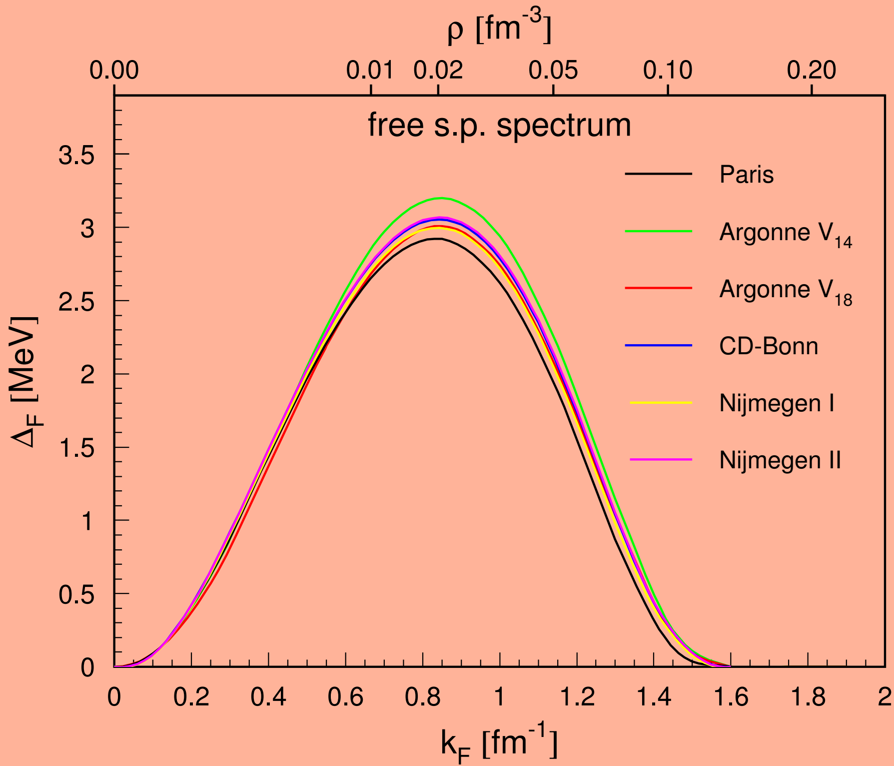
$$\frac{\Delta(k_F)}{e_F} \to \frac{8}{\mathrm{e}^2} \exp\left[\frac{\pi}{2k_F a_{nn}}\right]$$

No medium effects included !
Low-density approximation !

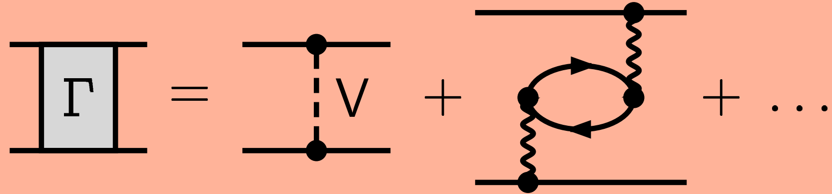
${}^1S_0\ nn$ Gap with and without Polarization Effects:

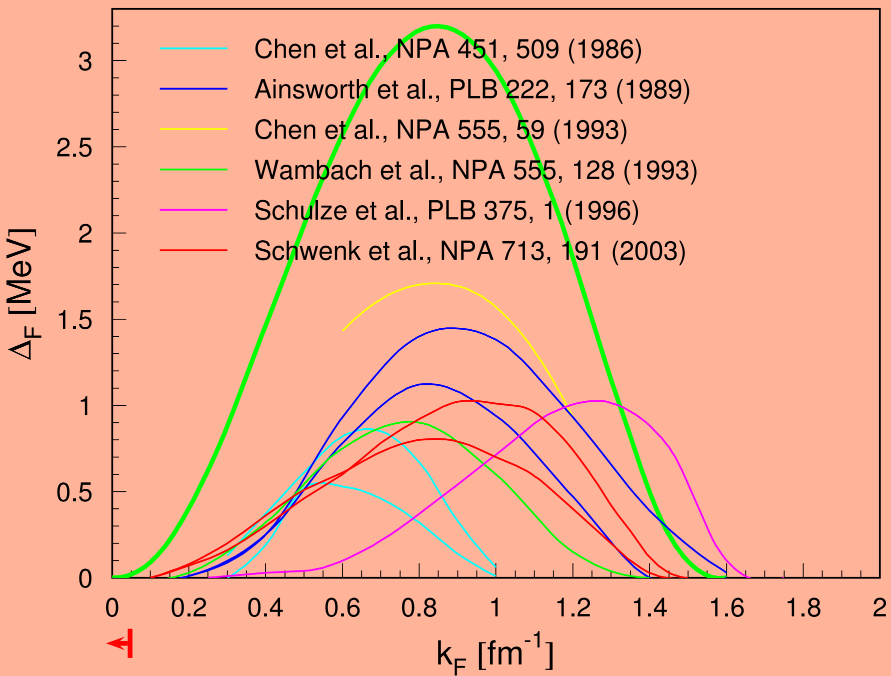
- Free potential:





● Including polarization:



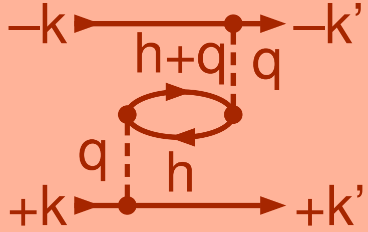


"low density"

First-Order Polarization Interaction:

- Interaction at low density (one hole line):

$$\langle \boldsymbol{k}' | W | \boldsymbol{k} \rangle =$$



+ ex.

$$\overset{\rho\to 0}{\approx} \Pi(q) \left[\underbrace{T(k/2, k'/2)} \right]^2$$

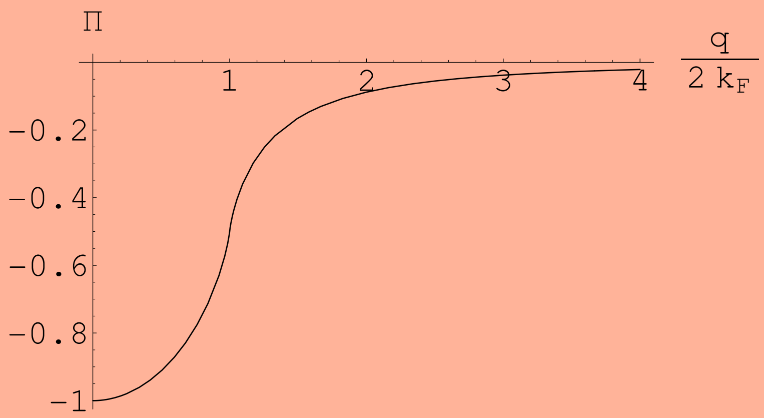
$$\frac{4\pi}{m} \left[\frac{1}{a_{nn}} - \frac{r_{nn}}{2} \frac{k^2 + k'^2}{8} \right]^{-1} \qquad a_{nn} = -18.8 \text{ fm}$$

$$r_{nn} = 2.8 \text{ fm}$$

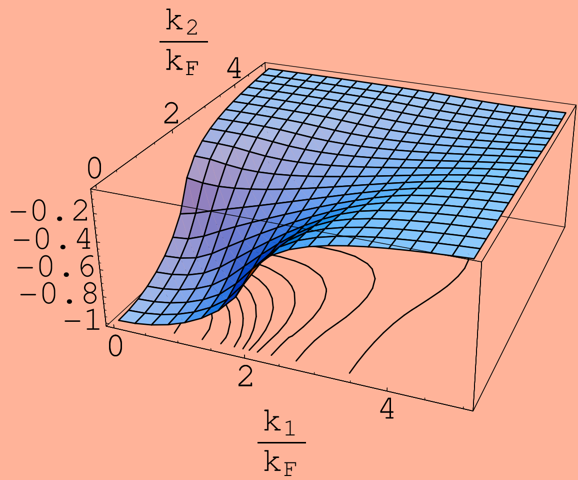
$$\hookrightarrow W_{^1S_0}(k, k') \approx -\frac{1}{2} \Pi(k, k') \, T^2(k/2, k'/2)$$ repulsive !

- The Lindhard function cuts off high momenta:

$\Pi(q)$:



$\Pi(k_1, k_2)$:

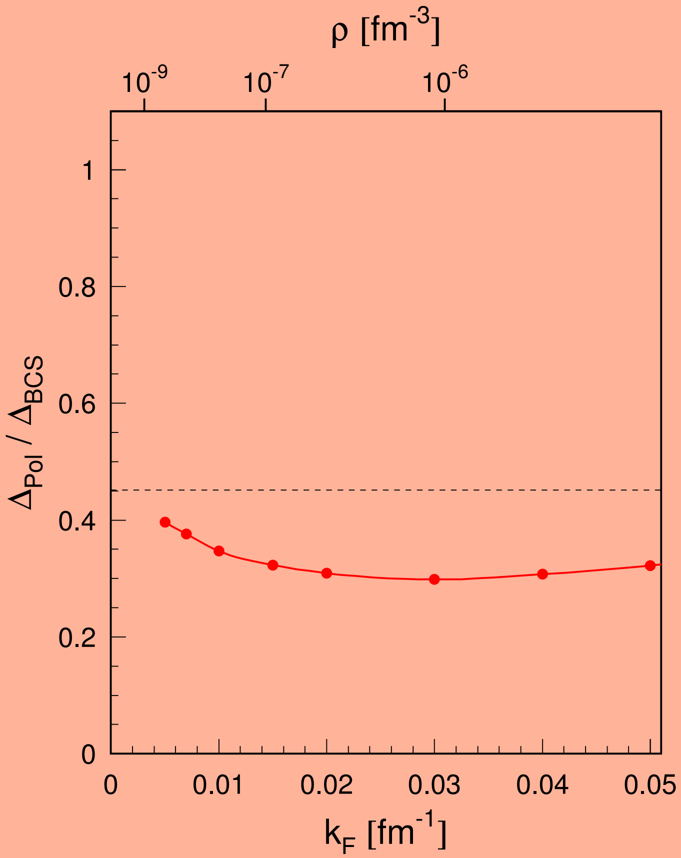


$$W_{^1S_0}(k,k') \overset{k_F\to 0}{\approx} -\frac{1}{2}\Pi(k,k')\,T_0^2\,, \quad T_0 \approx \frac{4\pi a_{nn}}{m}$$

determined by scattering length alone

Gap with Polarization Interaction:

- Numerical calculation: H.-J. S., A. Polls, A. Ramos; PRC 63, 044310



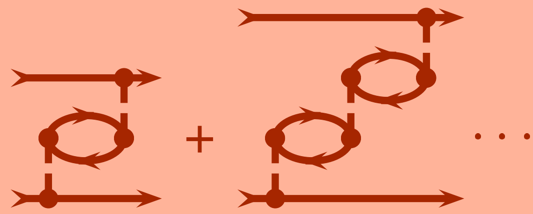
$$\leftarrow \frac{1}{(4e)^{1/3}}$$

$$\Delta_{\mathrm{BCS}}(k_F) \xrightarrow{k_F \to 0} \frac{k_F^2}{2m}\frac{8}{e^2}\exp\left[\frac{\pi}{2k_F a_{nn}}\right]$$

➥ Suppression of BCS gap by factor $\approx \mathbf{3}$ at low density !

- Range of validity :

$$\langle \boldsymbol{k}' | W_\infty | \boldsymbol{k} \rangle \sim \frac{T_q}{1 - \Pi(q) T_q}$$

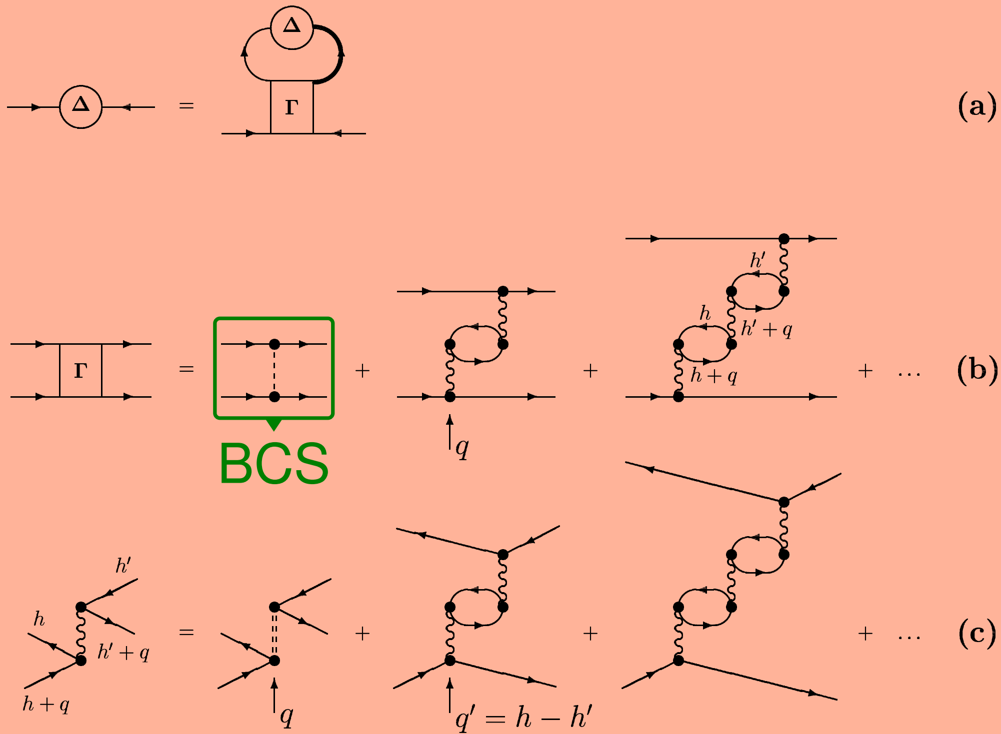


Expansion parameter: $\alpha \sim \frac{m k_F}{\pi^2} \frac{4\pi a_{nn}}{m} \sim k_F a_{nn}$

➥ First-order polarization good at $k_F \ll 1/|a_{nn}| \approx 0.05\ \mathrm{fm}^{-1}$
Corresponds to very low density $\rho \lesssim 10^{-4} \rho_0$
No theoretical control at higher density !

Beyond First Order: Babu-Brown Approach:

H.-J. S., J. Cugnon, A. Lejeune, M. Baldo, U. Lombardo; PLB 375, 1 (1996)



Too difficult to solve exactly:

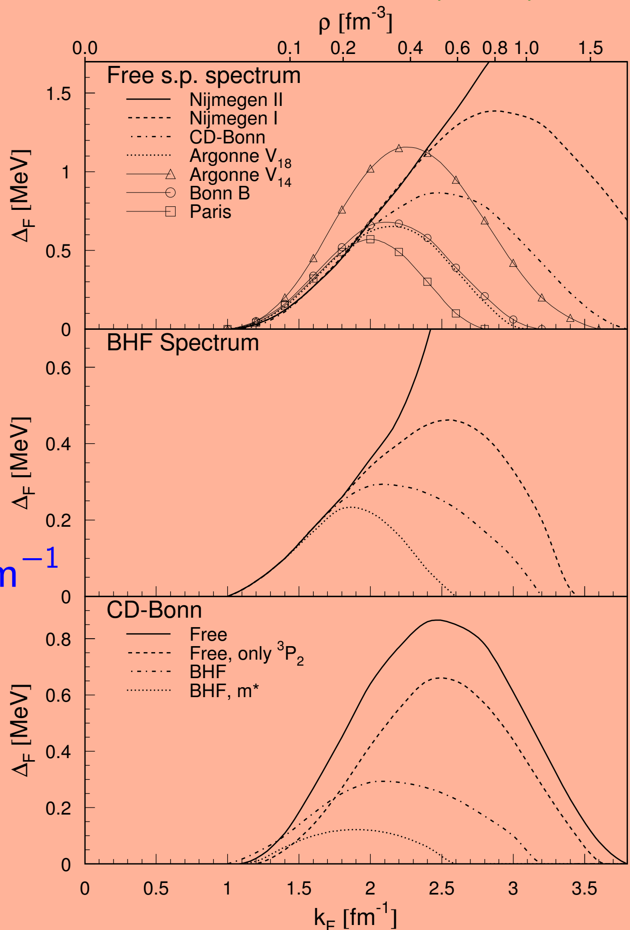
Strong approximations necessary

↪ Large uncertainty of results

$^3PF_2$ nn Gap in Neutron Matter:

M. Baldo, Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, H.-J. S.; PRC 58, 1921 (1998)

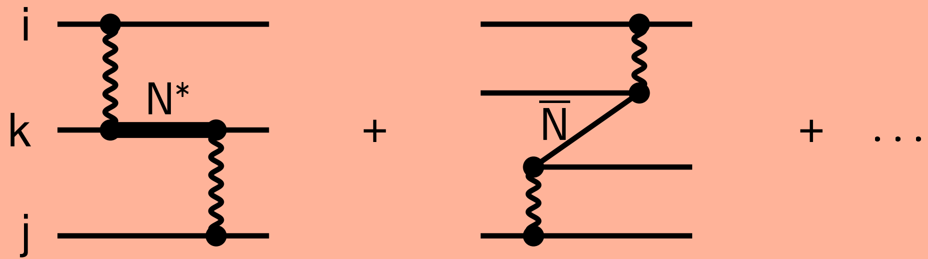
- BCS results with bare nn potentials:



- Unconstrained by phase shifts at $k_F \gtrsim 2\,\mathrm{fm}^{-1}$
- Self-energy effects are large
- $P-F$ coupling is important
- Polarization effects are unknown
  (Schwenk & Friman, PRL 92: $\Delta_{^3P_2} < 10^{-2}$ MeV)
- TBF are important ...

Three-Nucleon Forces in Brueckner Theory:

- Small correction ($\approx$ 1 MeV at ρ_0) for correct saturation:



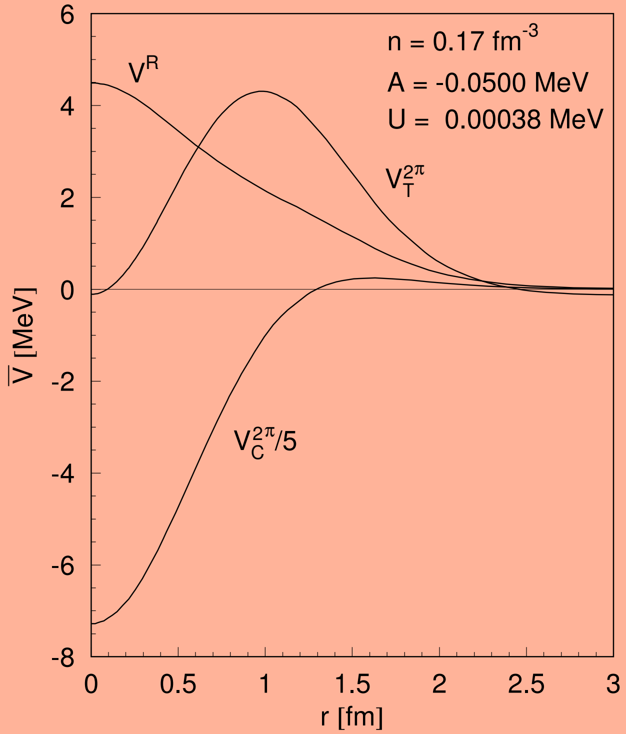
- Urbana Model: Two pion exchange + phenom. repulsion:

$$V_{ijk} = \sum_{\text{cyc.}} \Big[\quad A \{X_{ij}, X_{jk}\} \{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + \frac{A}{4} [X_{ij}, X_{jk}] [\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] + U \, T_{ij}^2 T_{jk}^2 \Big]$$

Fix parameters A, U for correct saturation point

- Effective two-body force after averaging:

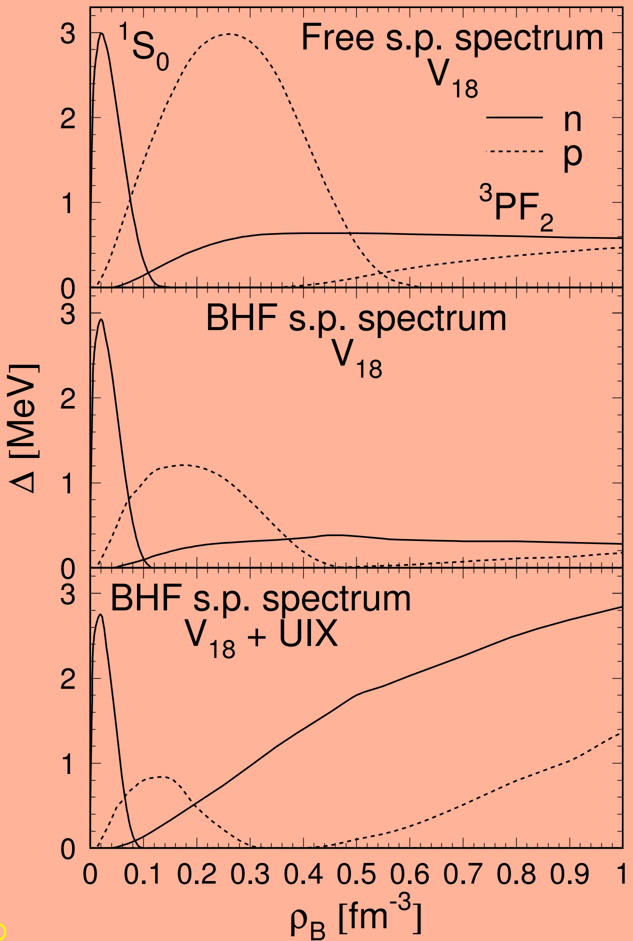
$$\overline{V}_{ij}(r) = \rho \int d^3 r_k \sum_{\sigma_k,\tau_k} g(r_{ik})\, g(r_{jk})\, V_{ijk}$$
$$= \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \left[\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j V_C^{2\pi}(r) + S_{ij}(\hat{\boldsymbol{r}}) V_T^{2\pi}(r)\right] + V^R(r)$$



→ $V_C^{2\pi}(r)$ is repulsive in $^1S_0$, but attractive in $^3PF_2$ wave !

Gaps in Neutron Star Matter:

X.-R. Zhou, H.-J. S., E.-G. Zhao, Feng Pan, J.P. Draayer; PRC 70, 048802 (2004)



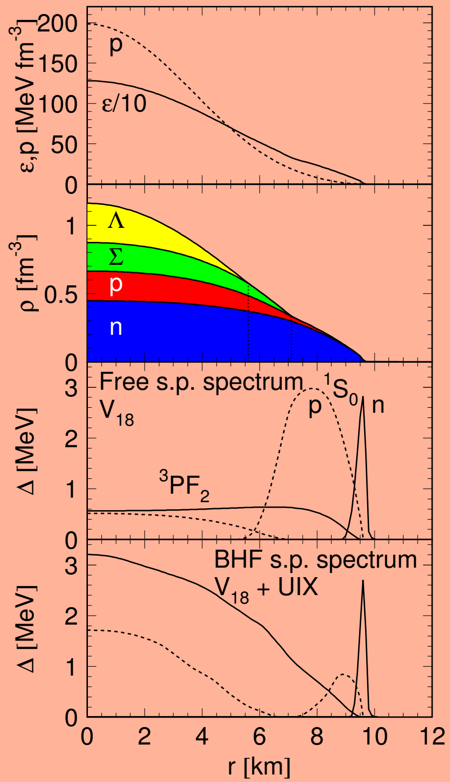
EOS: BHF (V18 + UIX + NSC89)

- Self-energy effects suppress gaps
- TBF suppress pp $^1S_0$ but strongly enhance $^3PF_2$ gaps !

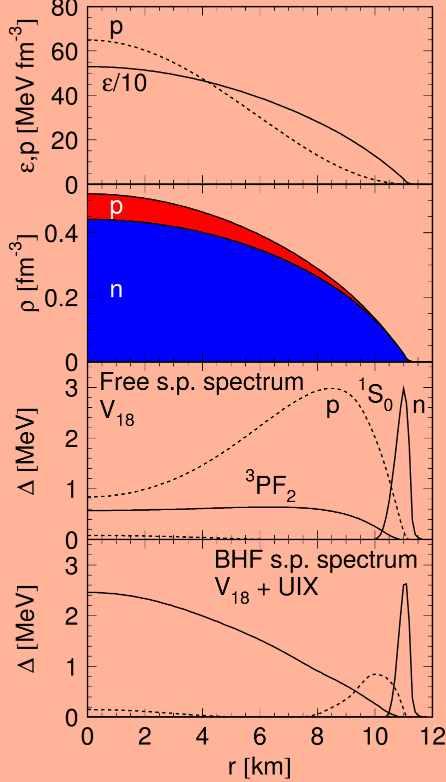
Neutron Star Profile: Particle Densities & Gaps:

EOS: BHF (V18 + UIX + NSC89) , M = 1.2 $M_\odot$

with hyperons:



without hyperons:



Polarization effects (including pn interaction) ?

What about pairing involving hyperons ? ...

Dispersive Self-Energy Effects:

M. Baldo, U. Lombardo, H.-J. S., W. Zuo; PRC 66, 054304 (2002)

- Four-dimensional gap equation:

$$\Delta(k',\omega') = -\int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi i} \Gamma(k',\omega';k,\omega) \frac{\Delta(k,\omega)}{D(k,\omega)}$$

$$D(k,\omega) = \left[\underbrace{M(k,+\omega)}_{\frac{k^2}{2m}+\Sigma(k,\mu+\omega)-\mu} - \omega - i0\right]\left[M(k,-\omega) + \omega - i0\right] + \Delta(k,\omega)^2$$

- For static potential $\Gamma(k,\omega;k',\omega') = V(k,k')$:

$$\hookrightarrow \Delta(\boldsymbol{k}') = -\sum_{\boldsymbol{k}} V(\boldsymbol{k}',\boldsymbol{k}) \left(\frac{1}{\pi} \int_0^\infty d\omega \, \mathrm{Im} \frac{1}{D(\boldsymbol{k},\omega)} \right) \Delta(\boldsymbol{k})$$

gap is also energy independent

- Neglect imaginary part of Σ: $\mathrm{Im}\, M = 0$:

$$\hookrightarrow \Delta(k') = -\sum_k \frac{V(k',k)\, Z(k)}{2\sqrt{M_s(k)^2 + \Delta(k)^2}}\, \Delta(k)$$

with

$$M_s(k) \equiv \frac{M(k,+e_k) + M(k,-e_k)}{2} \quad , \quad e_k = M(k, e_k)$$

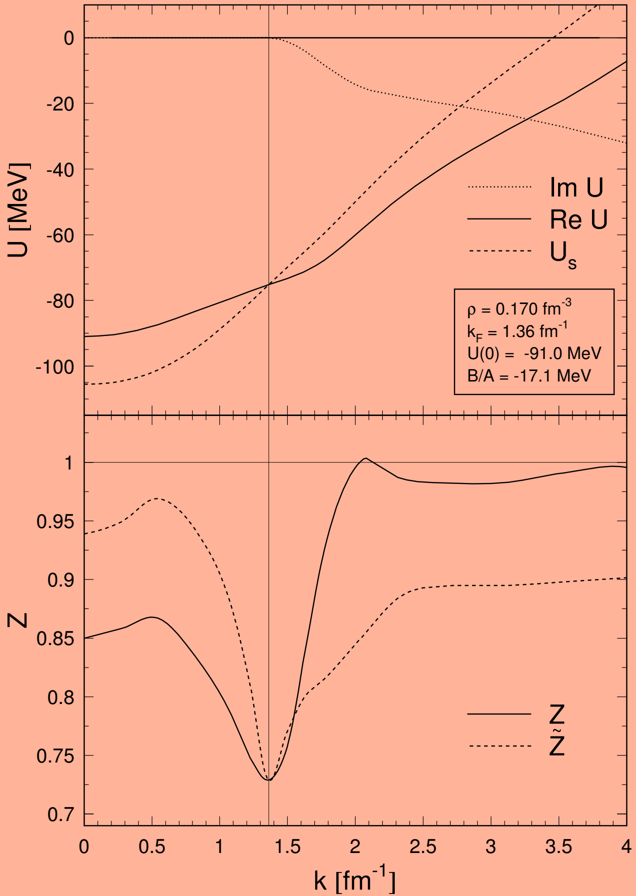
$$Z(k) = \left[1 - \frac{\partial \Sigma}{\partial \omega}\bigg|_{\omega = e_k} \right]^{-1}$$

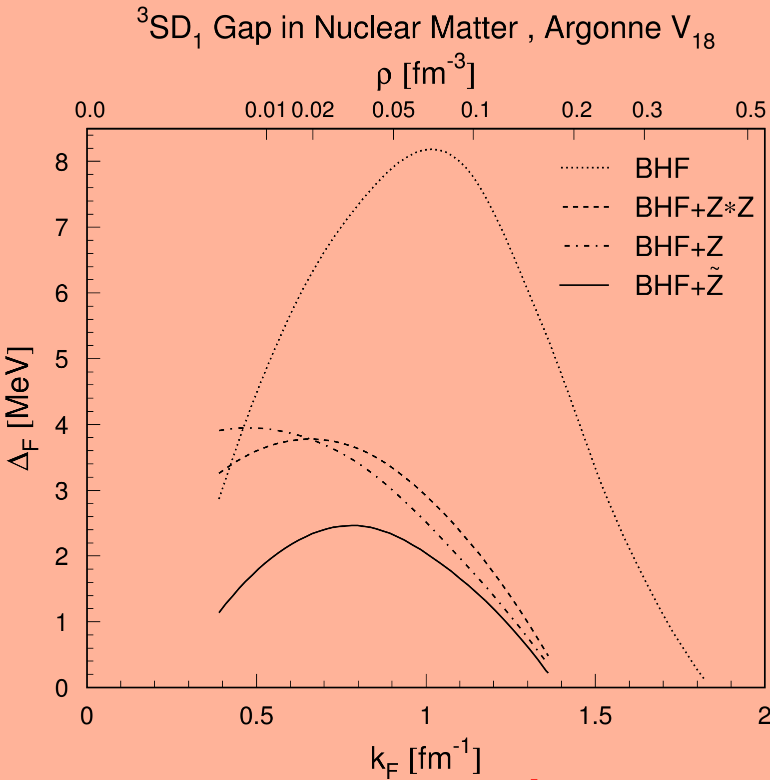
i.e., gap equation involves spectroscopic factor $Z(k)$ and symmetrized s.p. energy $M_s(k)$

- For the exact result ($\mathrm{Im}\, M \neq 0$) replace

$$Z(k) \to \widetilde{Z}(k) \equiv \sqrt{[\mathrm{Re} M_s(k)]^2 + \Delta(k)^2}\, \frac{2}{\pi} \int_0^\infty d\omega\, \mathrm{Im} \frac{1}{D(k,\omega)}$$

• Numerical Results ($^3SD_1$ np pairing in symmetric matter):





↪ Strong suppression due to Z-factor ≈ 0.7 . . . 0.8 !

Proton $^1S_0$ Pairing in Neutron Stars:

M. Baldo, H.-J. S.; PRC 75, 025802 (2007)

- Strong in-medium effects on protons due to large neutron background
- Consider complete set of medium effects: m*, Z, TBF, Polarization:

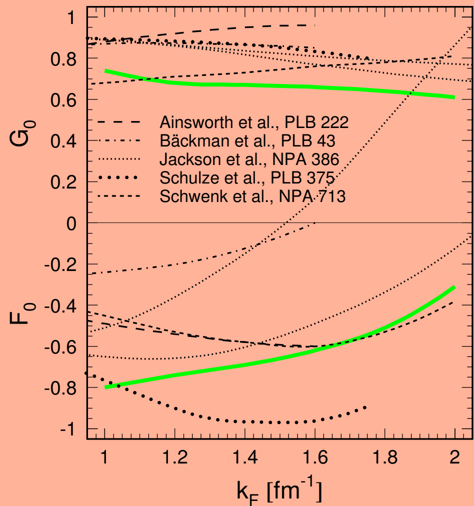
$$\Delta(k') = -\sum_k \frac{Z(k)[V + V_{\text{TBF}} + V_{\text{Pol}}](k',k)}{2\sqrt{M_s(k)^2 + \Delta(k)^2}} \Delta(k)$$

- Weak-coupling approximation:

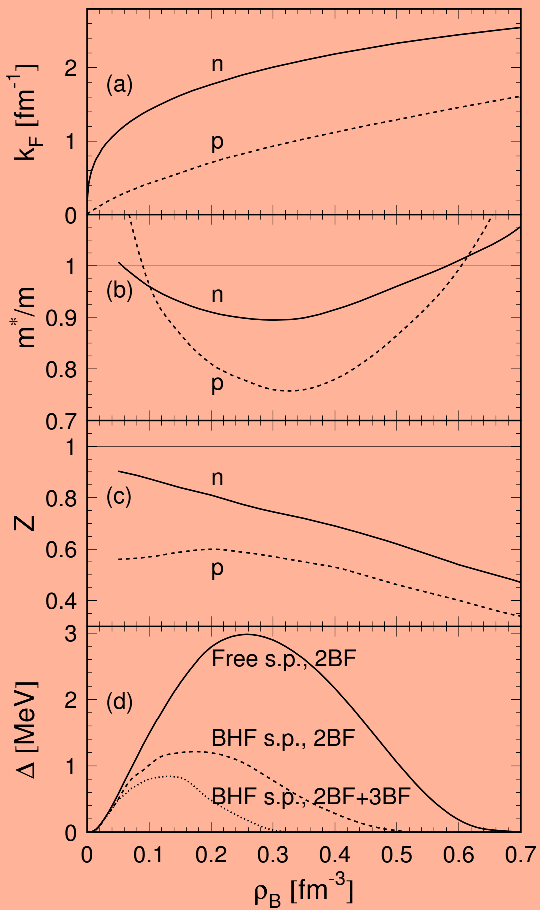
$$\Delta = c\,\mu\,e^{1/\lambda}\,, \quad \lambda = k_F m^* Z^2 V_{\text{eff}}$$

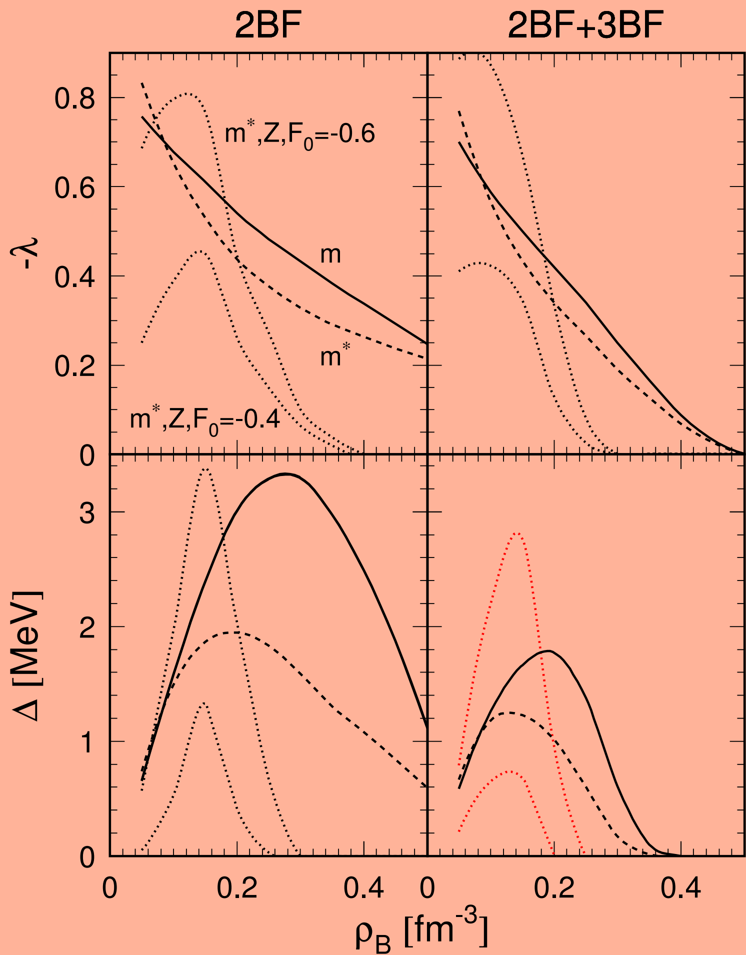
- Approximation for Landau parameters:

$$G_0 = 0.7\,;\ F_0 = -0.4, -0.6$$



• Results:

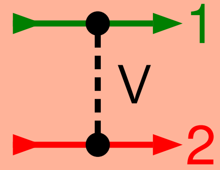




↪ Reduction by m^*, Z, TBF; Enhancement by polarization !

Pairing in Asymmetric Matter:

- Principal equations:



$$\Delta_{k'} = -\sum_k V_{kk'} \frac{\Delta_k}{2E_k}\left[1 - f(E_k^+) - f(E_k^-)\right]$$

$$\rho_1 + \rho_2 = \sum_k \left[1 - \frac{\epsilon_k}{E_k}\left[1 - f(E_k^+) - f(E_k^-)\right]\right]$$

$$\rho_1 - \rho_2 = \sum_k \left[f(E_k^-) - f(E_k^+)\right]$$

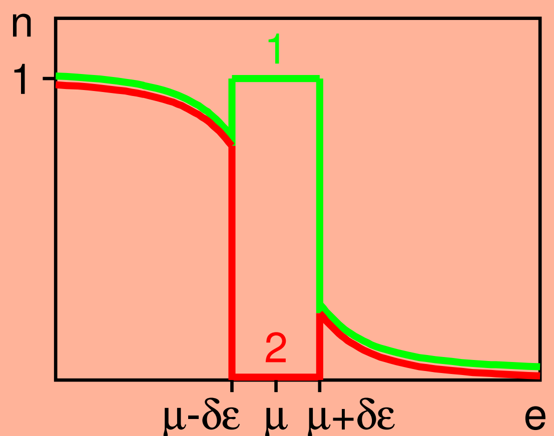
$$\mu = (\mu_1 + \mu_2)/2$$

$$\delta\mu = (\mu_1 - \mu_2)/2$$

$$E_k^{\pm} = E_k \pm \delta\mu$$

- At zero temperature: $f(E_k^+) = 0$, $f(E_k^-) = \theta(\delta\mu - E_k)$:

Unpaired particles concentrated in region around μ, Pauli-blocking the gap equation

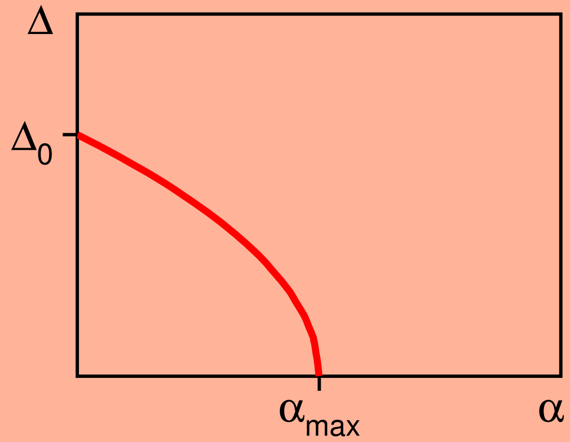


→ Strong suppression of the gap with asymmetry

- Solution in weak-coupling approximation $\Delta \ll \mu$:

$$\frac{\Delta_\alpha}{\Delta_0} = \sqrt{1 - \frac{\alpha}{\alpha_{\mathrm{max}}}}\,, \quad \alpha = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\alpha_{\mathrm{max}} = \frac{3\Delta_0}{4\mu} = \frac{6}{e^2}\exp\left[\frac{\pi}{2k_F a}\right]$$



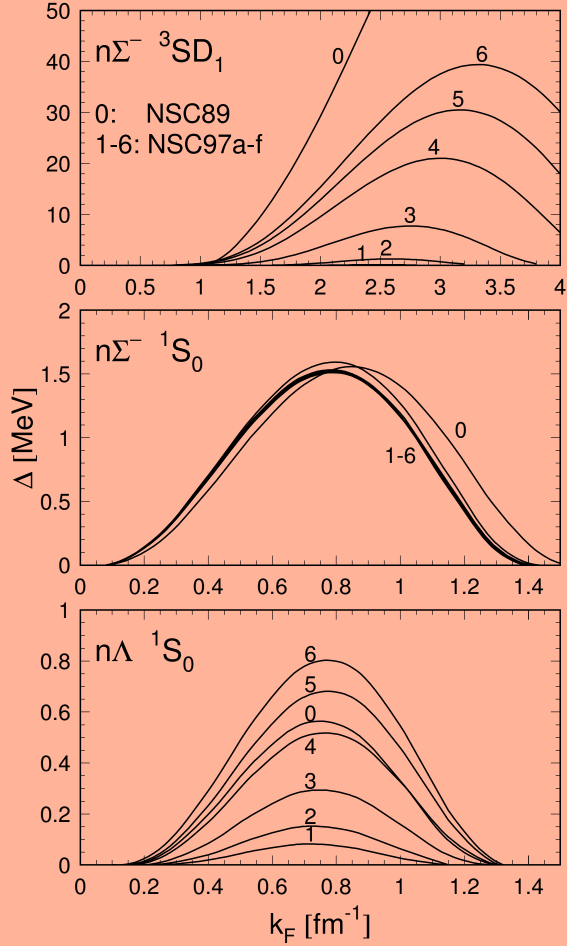
↪ Very small maximal asymmetry allowing pairing !
if $\Delta_0 \ll \mu$ . . .

Hyperon-Nucleon Pairing in Neutron Stars:

Xian-Rong Zhou, H.-J. S., Feng Pan, J.P. Draayer; PRL 95, 051101 (2005)

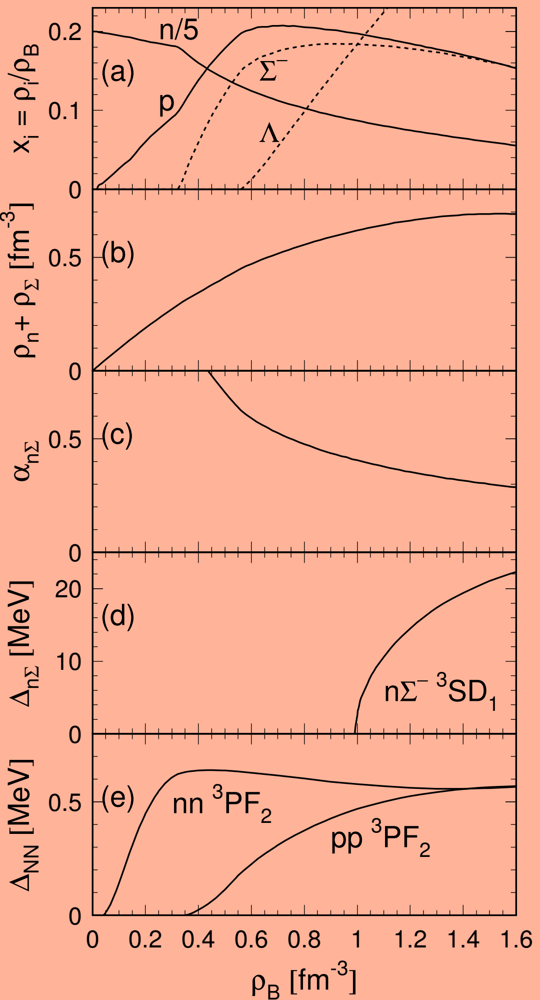
- NY Gaps in symmetric hyperon-nucleon matter:

  YY pairing unknown due to unknown potentials



↪ Nijmegen potentials predict very large $n\Sigma^- \, {}^3SD_1$ gaps ! (no hard core, very attractive)

● $n\Sigma^- \, {}^3SD_1$ pairing in neutron star matter:



→ Suppression of $nn\ {}^3PF_2$ pairing !

Suppression of direct Urca Σ^- cooling !

However, at high density many uncertainties:

- EOS, composition of matter ?
- NY potentials ?
- Medium effects on pairing ?
- Separation of paired/unpaired phases ?

→ Currently, YN pairing cannot be excluded

Summary

- Pairing in beta-stable matter:
  - Medium effects suppress $^1S_0$ nn and pp gaps
  - $^3PF_2$ gaps uncertain
  - Hyperon pairing ?

- Future Applications:
  - Gaps in neutron stars: cooling, glitches
  - Microscopic pairing forces in finite nuclei (crust)