Nuclear Superfluidity and Thermal Properties of Neutron Stars

N. Sandulescu, Institute of Atomic Physics, Bucharest

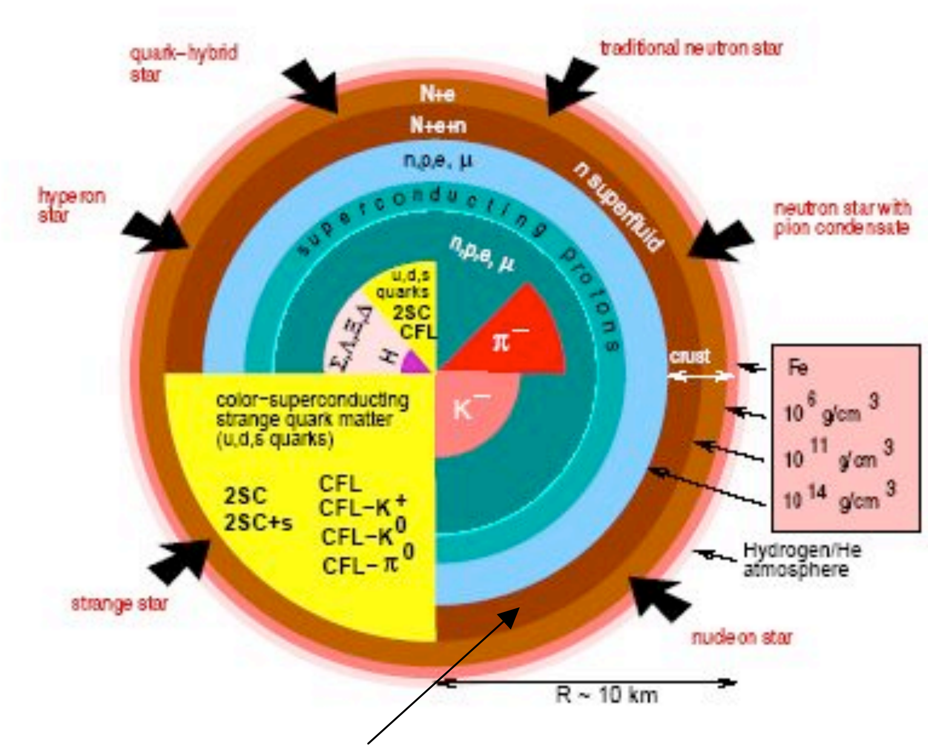
Outline

- Superfluid properties of inner crust matter in a self-consistent HFB approach
- Effects of superfluidity on specific heat and cooling time of the inner crust

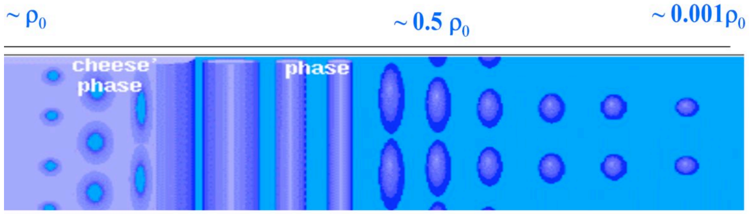
Collaboration:

E. Khan	*IPN-Orsay*
R. Liotta	*KTH-Stockholm*
J. Margueron	*IPN-Orsay*
C. Monrozeau	*IPN-Orsay*
Nguyen Van Giai	*IPN-Orsay*

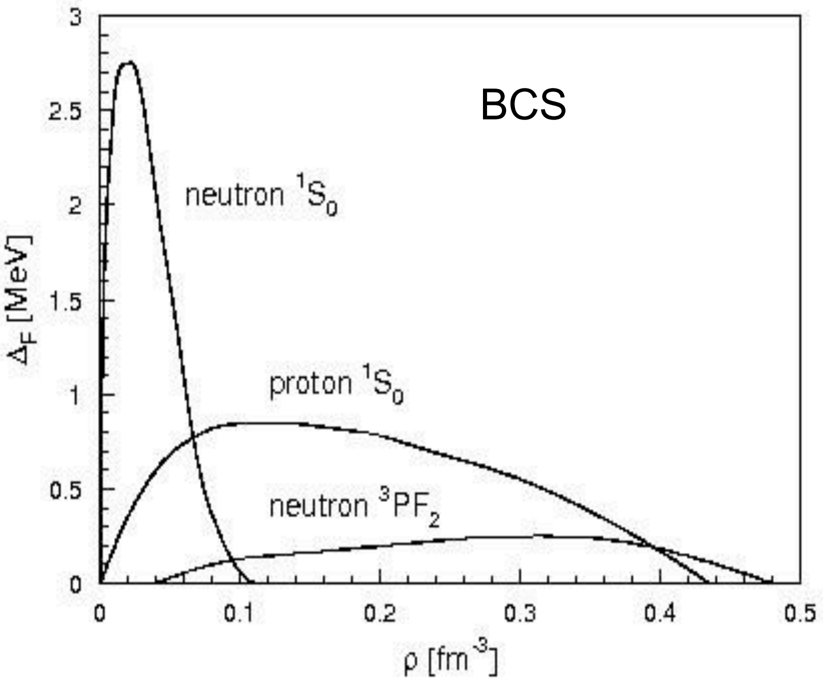
Nuclear Superfluidity in Neutron Stars



Inner Crust

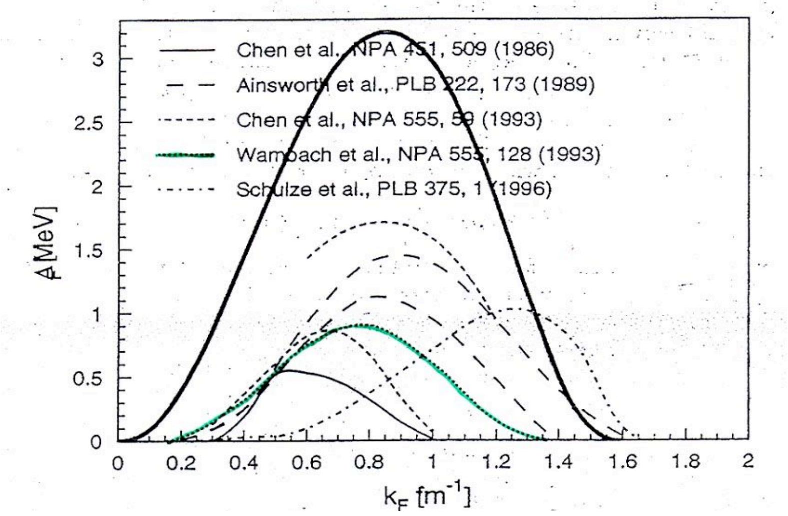


Pairing gap in neutron stars matter

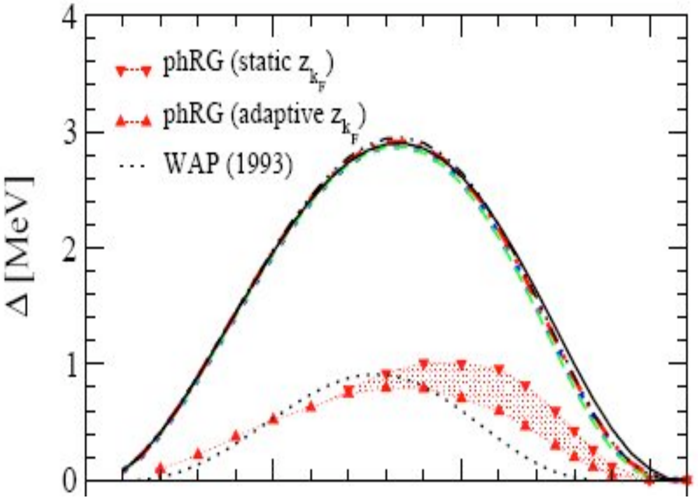


(U.Lombardo, H.-J. Schulze, LNP578, 2001)

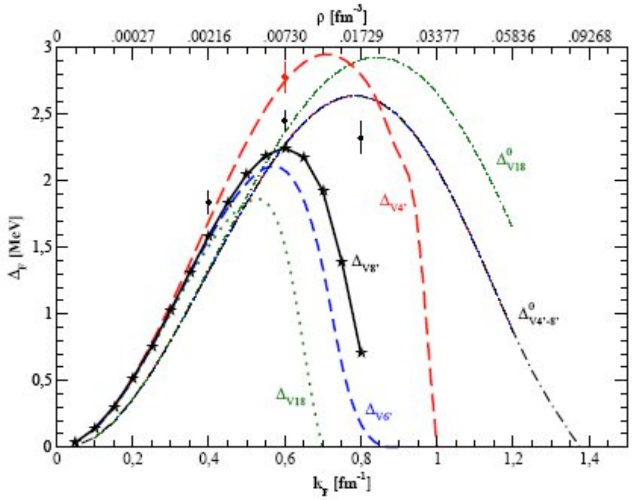
$^1S_0$ Pairing Gap in Neutron Matter



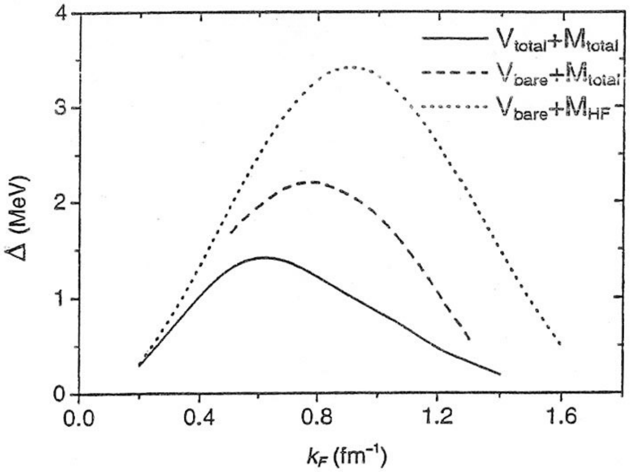
U.Lombardo, H.-J. Schulze, LNP578, 2001



A. Schwenk et al, NPA713 (2003) 191



A. Fabrocini et al, PRL95 (2005)

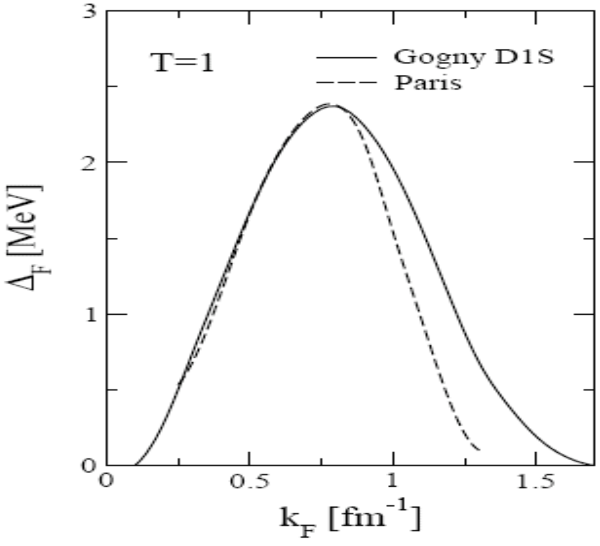


C.Shen et al , Phys.Rev.C67(2003)

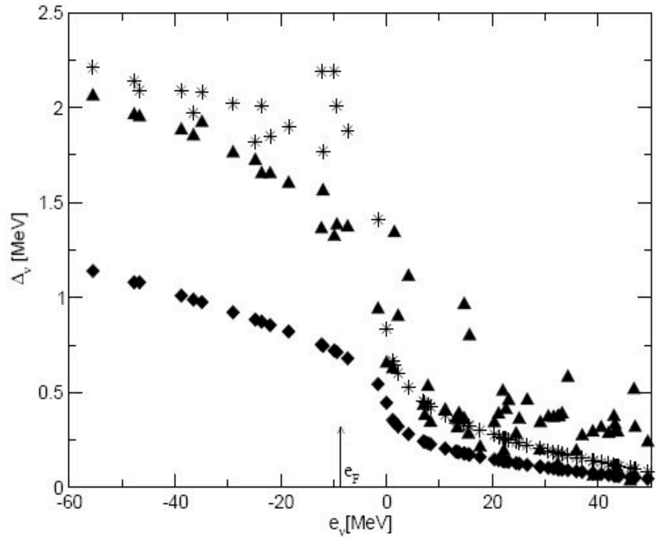
Pairing Gap in Nuclei

Nuclei: - *"effective" forces (e.g.. Gogny)*
- *"realistic" force + in-medium effects ?*

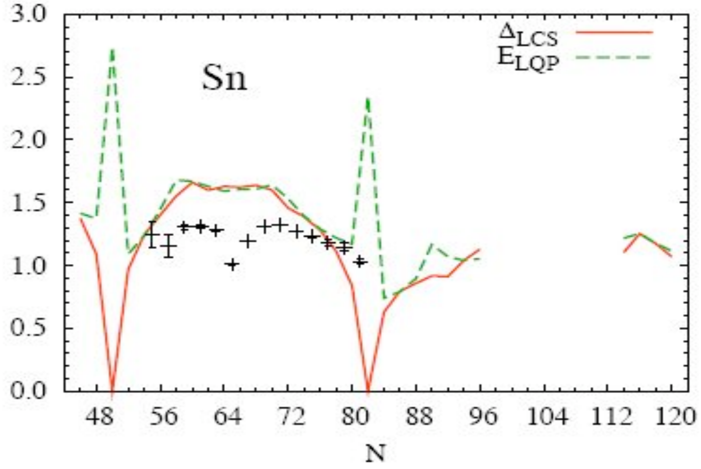
Gap in infinite matter



?



G. Gori et al, Phys.Rev. C72 (2005) 011302

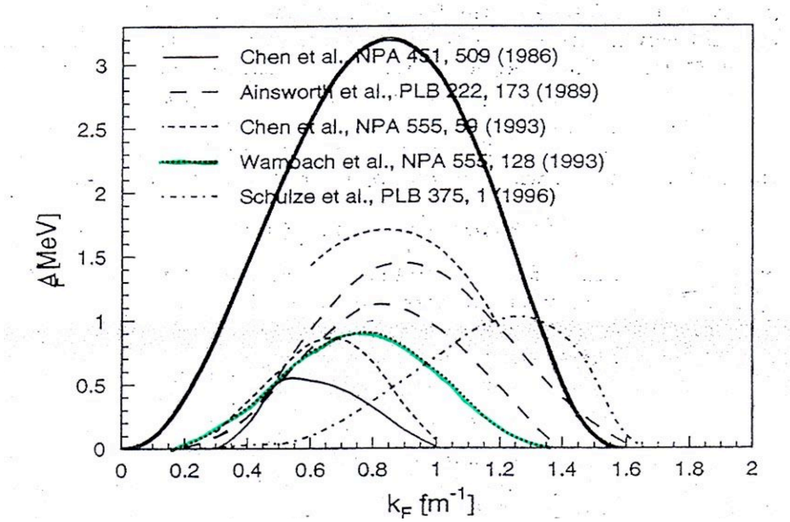


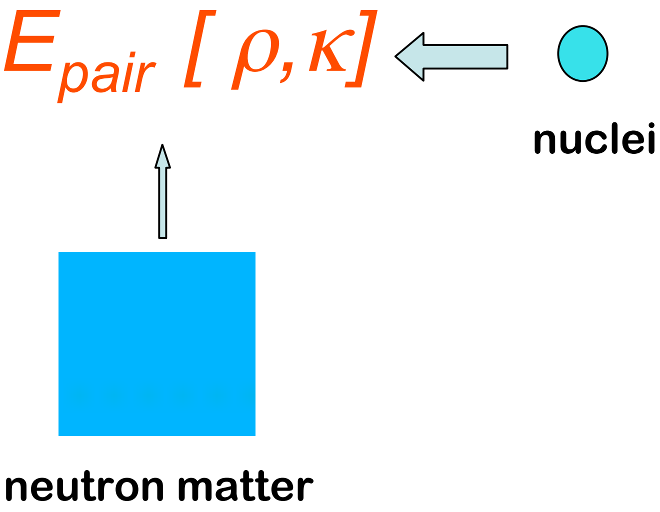
T. Duguet et al, Les Houches School, may 2007

Pairing in the Inner Crust ?

$$E_{nuc} = E_{Skyrme} + E_{pair}[\rho,\kappa]$$

Pairing in uniform neutron matter ?





$E_{pair}[\rho,\kappa]$

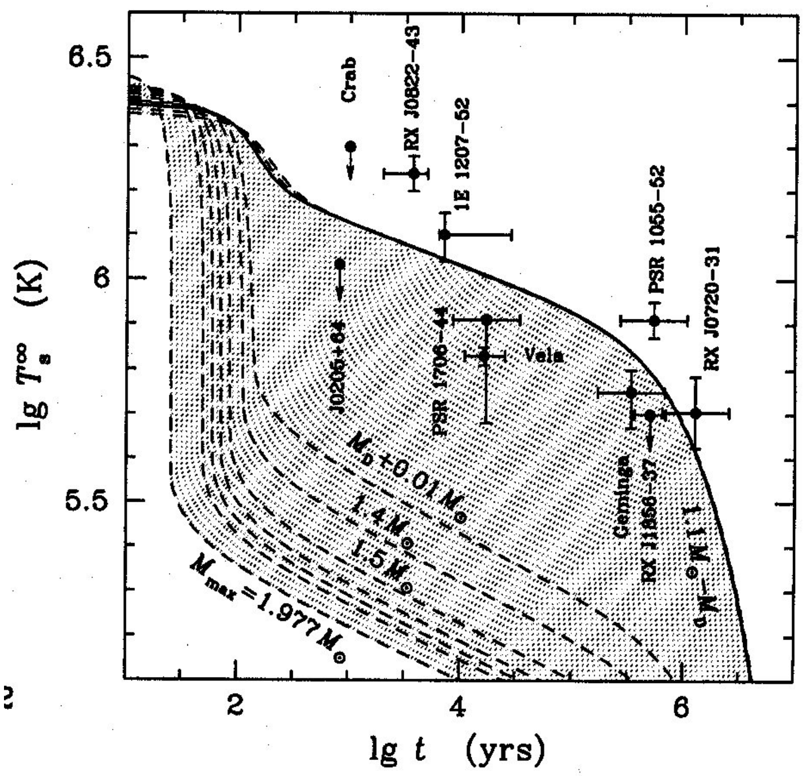
nuclei

neutron matter

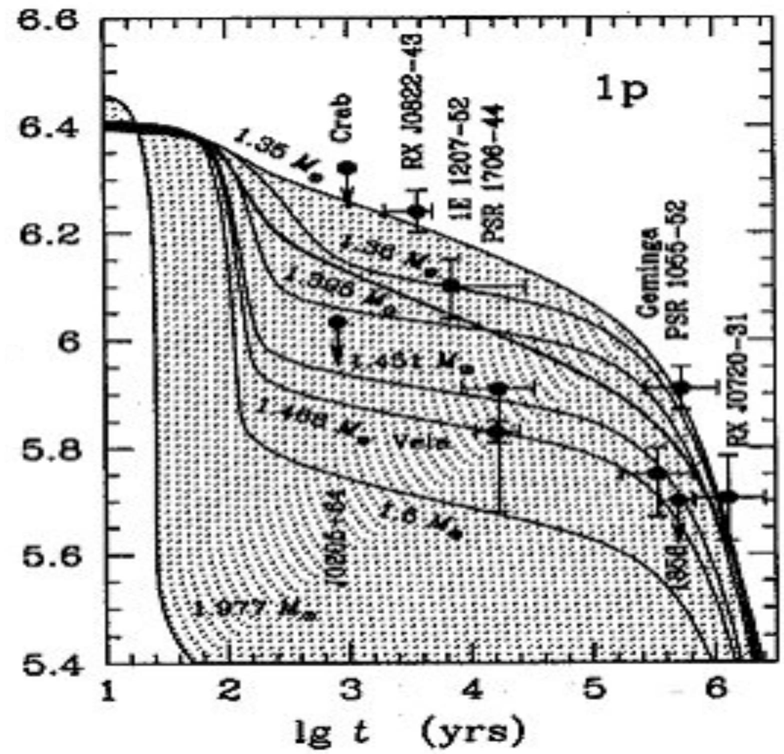
Observational consequences related to the uncertainty in the $^1S_0$ pairing ?

Nuclear Superfluidity and Neutron Stars Cooling

No superfluidity



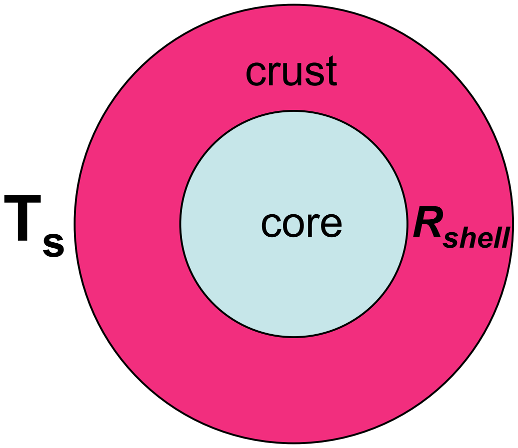
Superfluidity

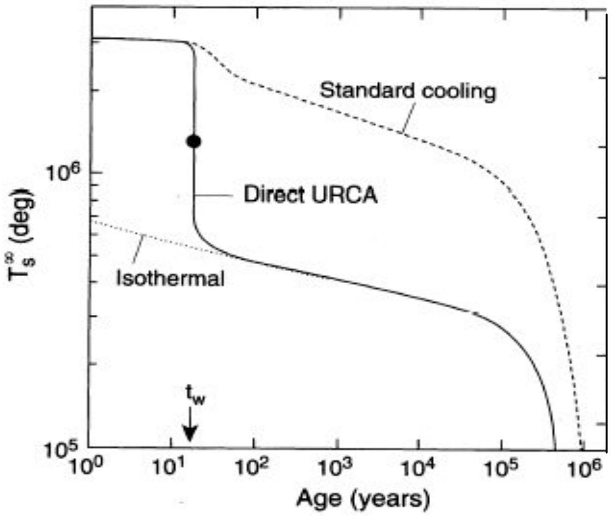


(D.G. Yakovlev et al, ApJ 2004)

- **URCA processes:** ***direct*** **:** **n > p +e +ν; p+e > n+ ν**
  modified **:** **n+n >n+ p +e + ν;n+p+e >n+ n+ ν**

Cooling Time of Neutron-Stars Crust





J.M. Lattimer et al, ApJ 1994

- *G. E. Brown et al, PRD37, 1998*

$$t_w \approx R_{shell}^2 \frac{C_V}{k}$$

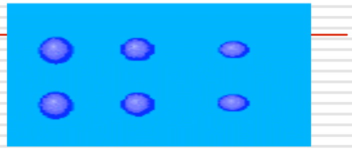
- *J.M. Lattimer et al, ApJ425, 1994*

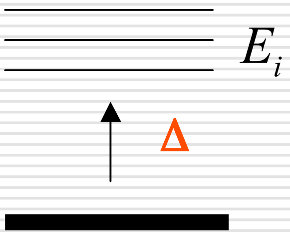
$$t_w \propto R_{shell}^n \qquad 1.7 \le n \le 1.8$$

t_w is strongly affected by the inner crust superfluidity !

Superfluidity and Specific Heat of Crust Matter

$$C^{(t)}_V = \mathbf{C_V\,(n)} + C_V\,(e) + C_V(\text{lattice})$$





- *normal phase :* $C_V\,(n) > C_V\,(e)$
- *suprafluid phase :* $C_V(n) \Leftrightarrow C_V(n;\Delta=0)\,e^{-\Delta/kT}$
- issues: ***- effect of nuclear clusters on cooling time ?***
  - P.M.Pizzochero et al, ApJ569, 2002

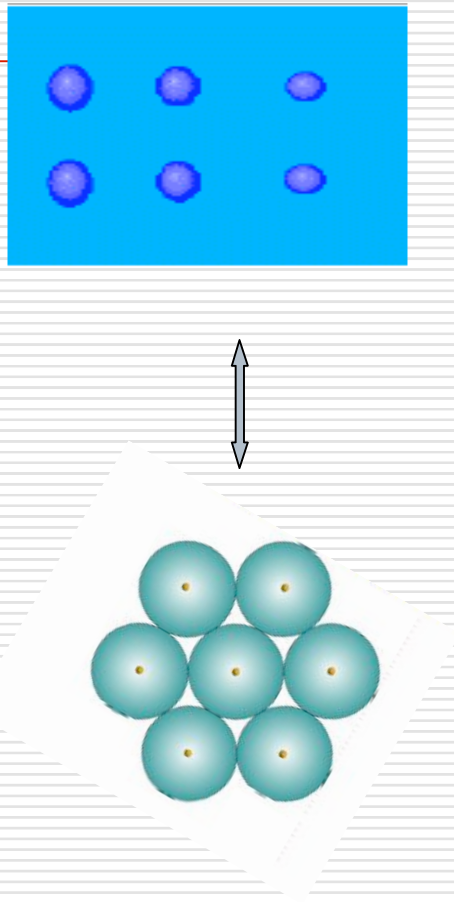
  - cooling time versus pairing intensity ?

  - effects of te collective excitations ?

Inner crust: microscopic treatment

$$E = E_{ph} + E_{pairing} + E_{electrons}$$

δF=0, β–equilibrium



Self-consistent mean field calculations (HFB)

I) Inner crust structure: N/Z, R_{ws}

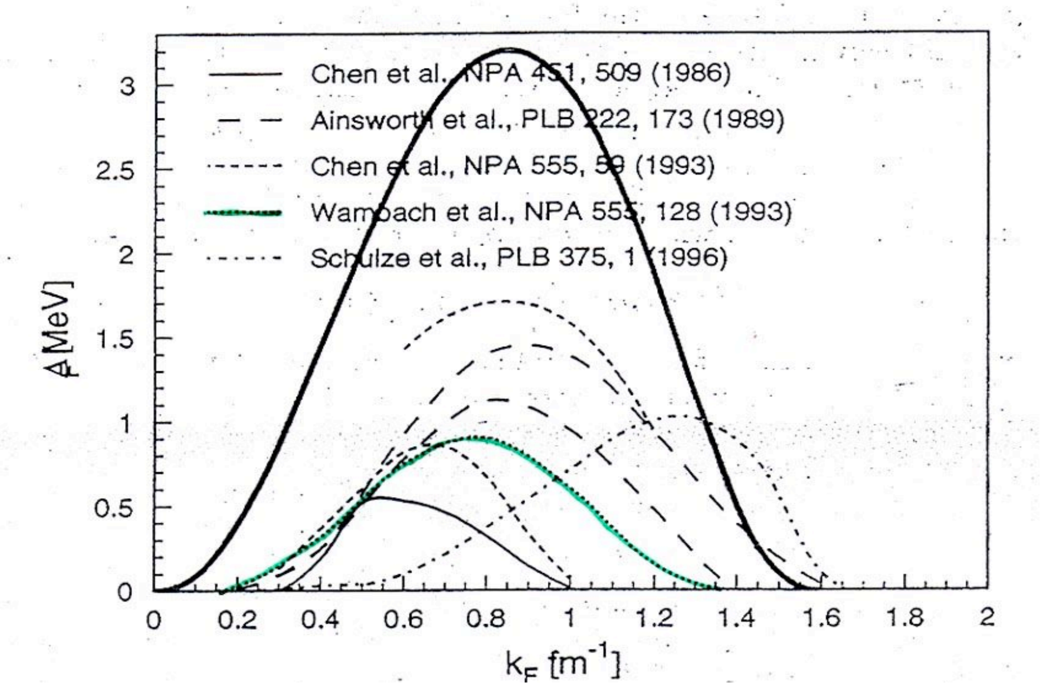
*II) Pairing properties : **$\Delta(r,T,\omega)$, E_i** ,…*

*III) Collective excitations: **QRPA***

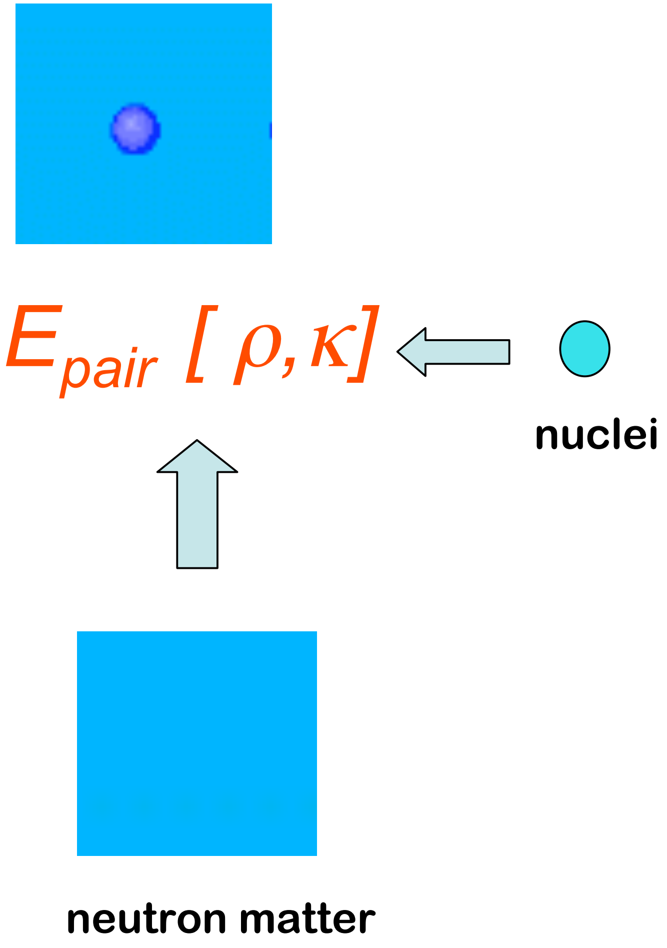
Pairing Correlations

$$E_{nuc} = E_{Skyrme} + E_{pair}[\rho,\kappa]$$

Pairing in uniform neutron matter ?



U.Lombardo, H.-J. Schulze, LNP578, 2001



$E_{pair}[\rho,\kappa]$

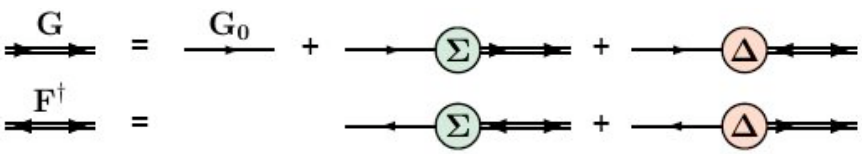
nuclei

neutron matter

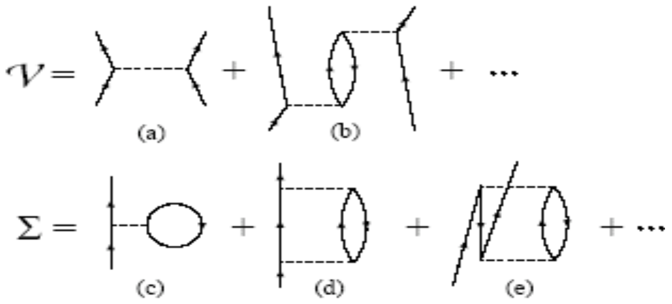
Pairing in Nuclear Matter: beyond BCS

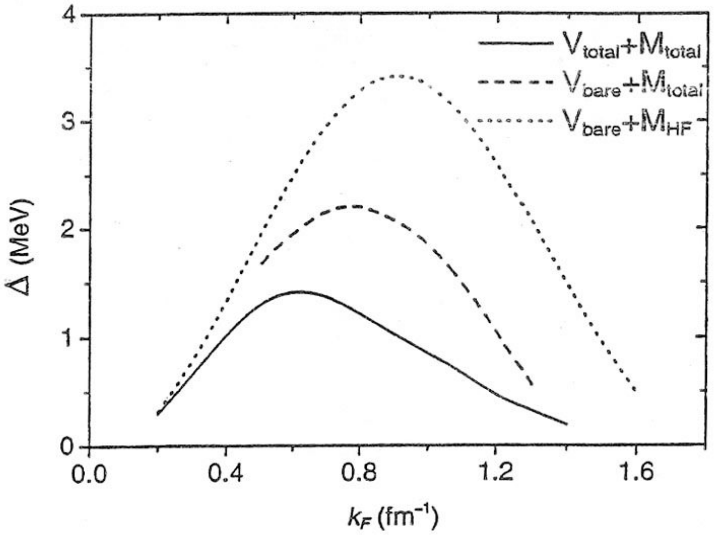
Gorkov equations

$$G = -i < 0|T(aa^{+})|0>; \qquad F = -i < 0|T(a^{+}a^{+})|0>$$



$$\Delta_k(\omega) = \sum_k \int d\omega' V_{kk'}(\omega,\omega') \frac{\Delta_{k'}(\omega')}{[\omega' - \varepsilon_{k'}(\omega')][\omega' + \varepsilon_{k'}(-\omega')]}$$





C.Shen, U.Lombardo,P.Schuck,W.Zuo, N.S, Phys.Rev.C67(2003)

Efective Pairing Interactions

V_{bare} $\xLeftrightarrow{k_F < 0.9}$ Gogny force

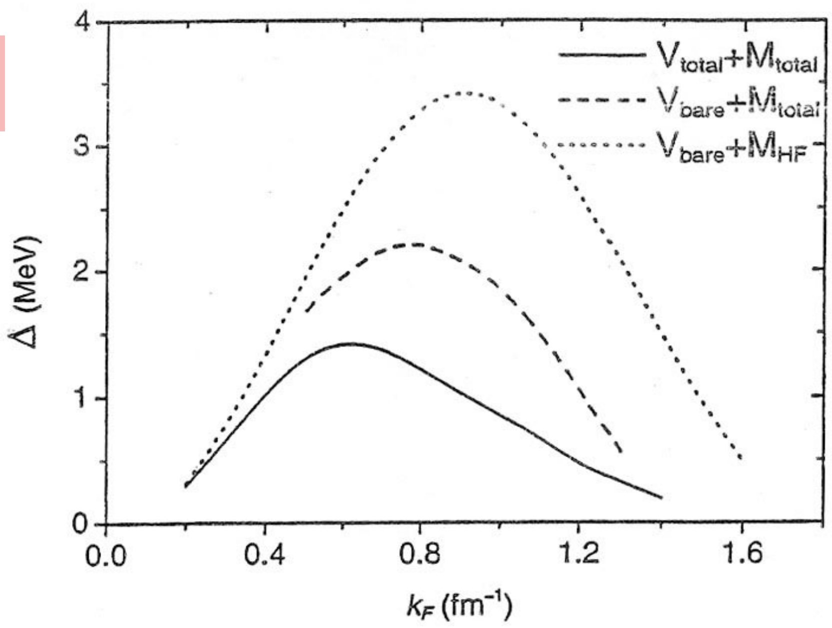
$\Updownarrow$

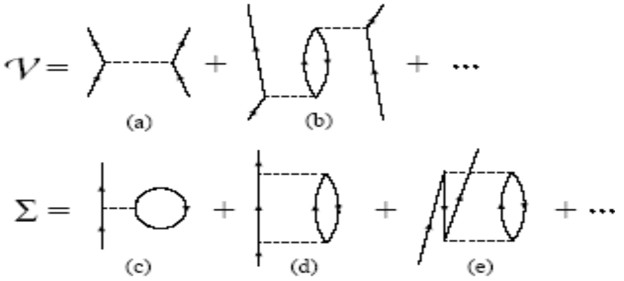
$V_{pair} = V_0[1-\eta(\rho/\rho_0)^\alpha]\delta(r-r')$

$\alpha = 0.45;\ \eta = 0.7$

I) $V_0 = -430$ $\Longleftrightarrow$ $\Delta_{max} = 3$ MeV

II) $V_0 = -330$ $\Longleftrightarrow$ $\Delta_{max} = 1$ MeV



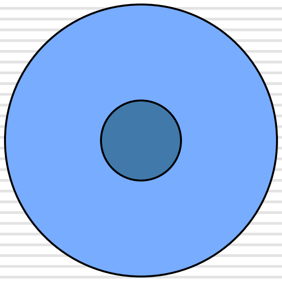


C.Shen et al Phys.Rev.C67(2003)

Finite-Temperature HFB Approach

$$E_{nuc} = E_{Skyrme} + E_{pair}\,[\,\rho,\kappa\,]$$

$$\begin{pmatrix} h_T(r)-\lambda & \Delta_T(r) \\ \Delta_T(r) & -h_T(r)+\lambda \end{pmatrix}\begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix}$$



$$\rho_T(r) = \frac{1}{4\pi}\sum_i (2j_i+1)[V_i^*(r)V_i(r)(1-f_i)+U_i^*(r)U_i(r)f_i]$$

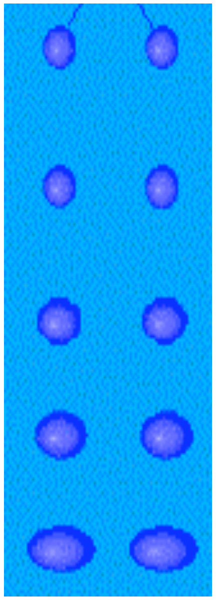
$$\Delta_T(\mathbf{r}) = V_{pair}\,\kappa_T(\mathbf{r})$$

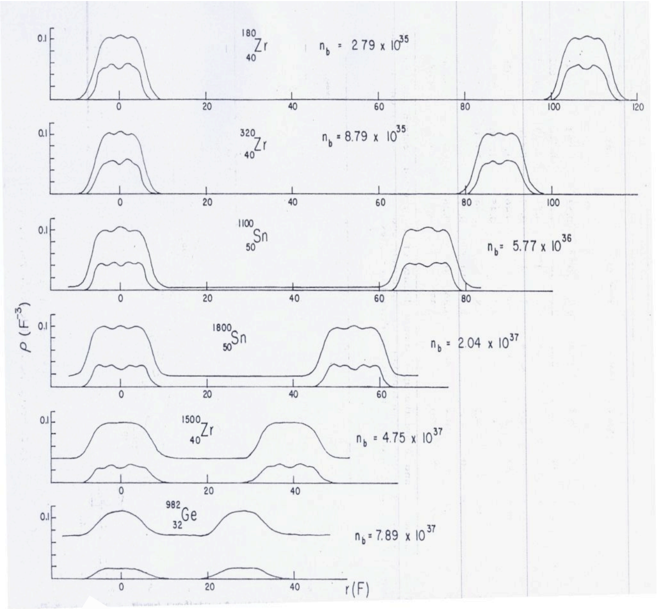
$$\kappa_T(r) = \frac{1}{4\pi}\sum(2j_i+1)U_i^*(r)V_i(r)(1-2f_i)$$

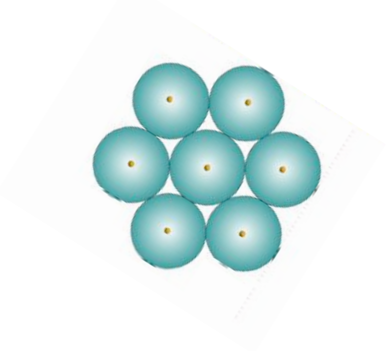
where : $f_k = \dfrac{1}{1+\exp(E_k/k_BT)}$

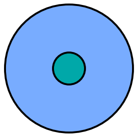
Inner Crust Structure

J.W. Negele, D. Vautherin, NPA207 (1973) 298



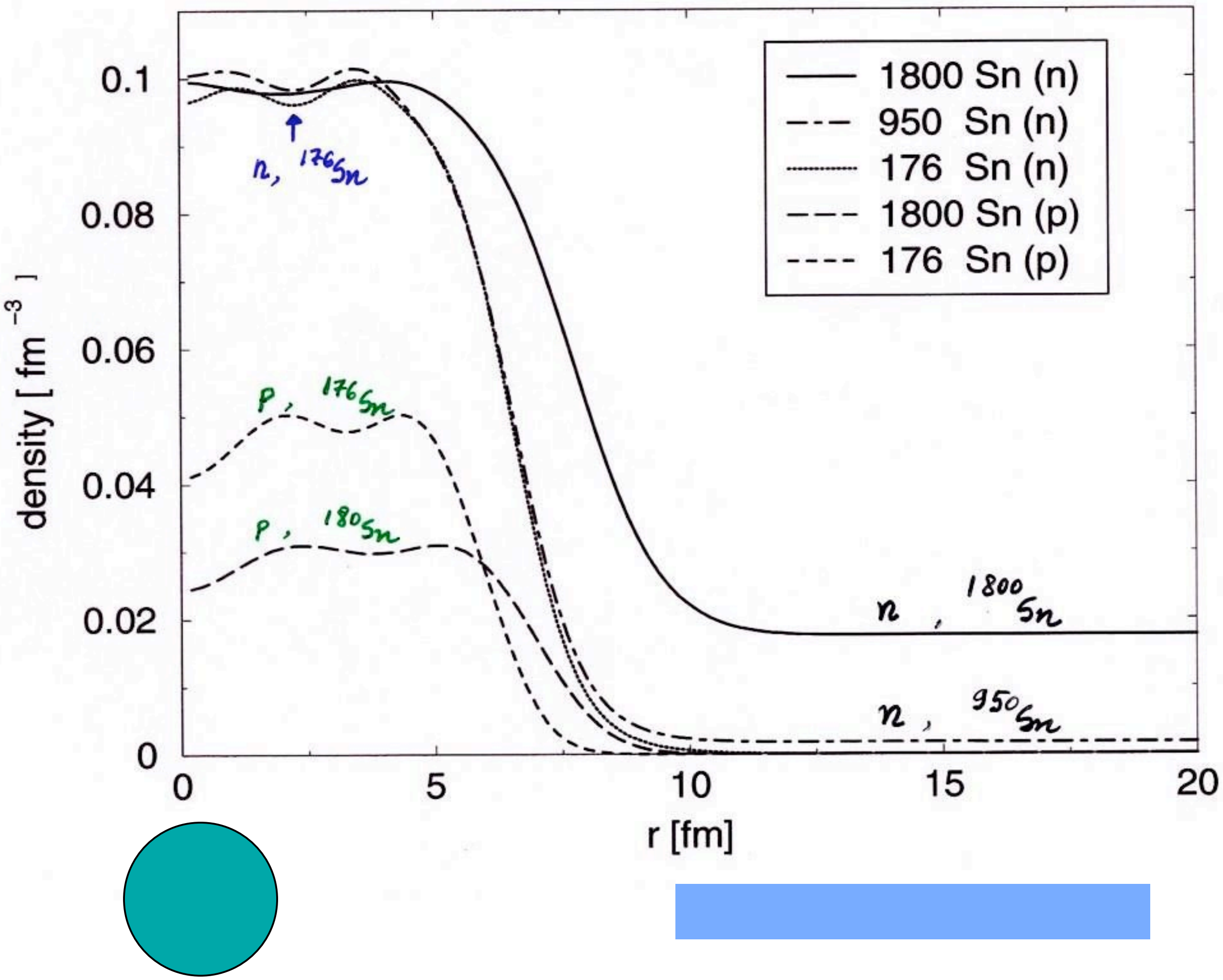


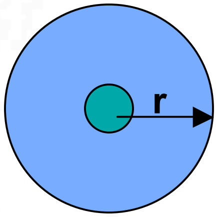




		Z	N	
$\rho_0/100$	⇔	40	460	$^{500}Zr_{40}$
$\rho_0/42.9$	⇔	50	900	$^{950}Sn_{50}$
$\rho_0/7.84$	⇔	50	1750	$^{1800}Sn_{50}$
$\rho_0/3.37$	⇔	40	1460	$^{1500}Zr_{40}$
$\rho_0/2.03$	⇔	32	950	$^{982}Ge_{32}$

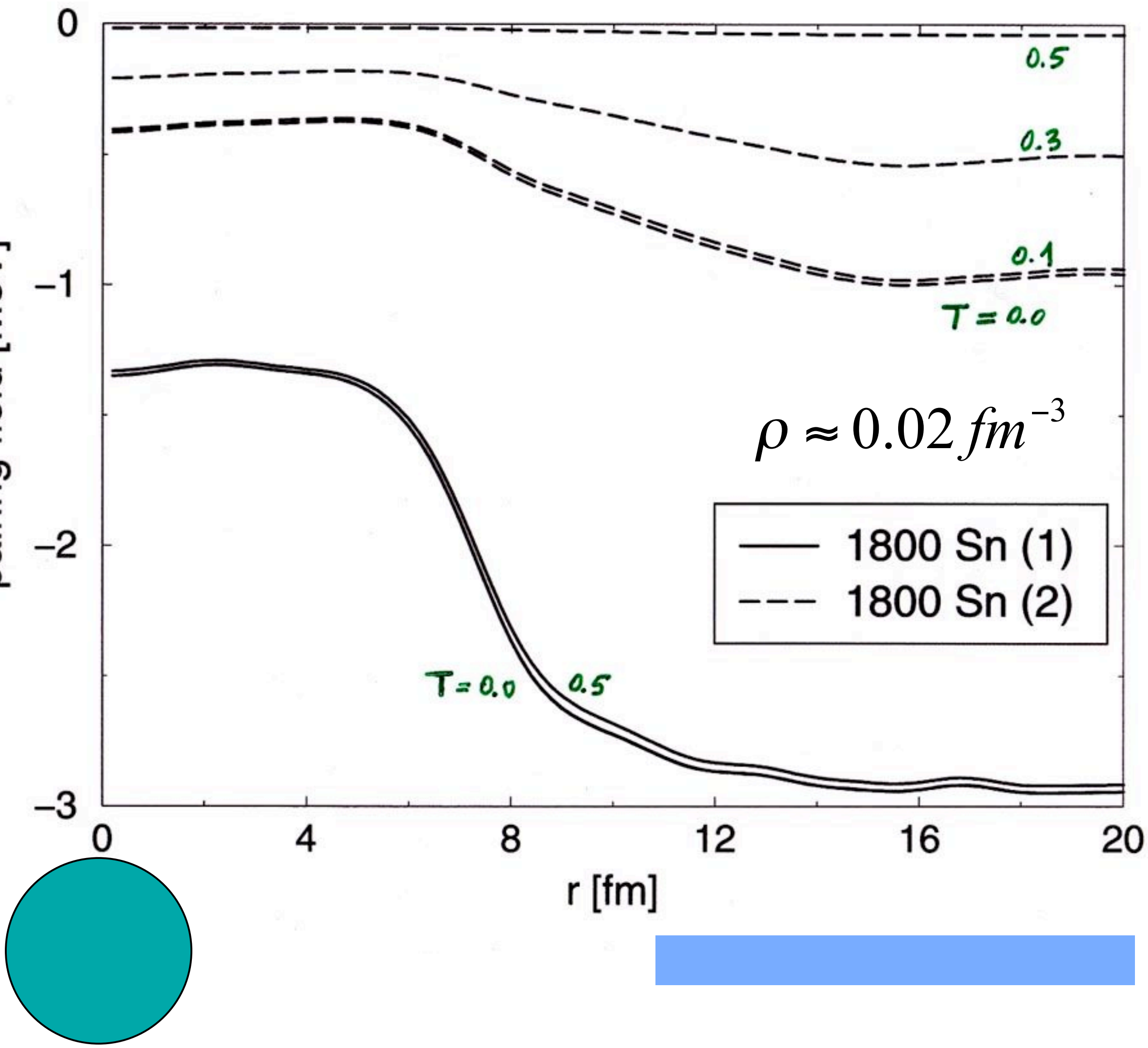
Density in the Wigner-Seitz Cells



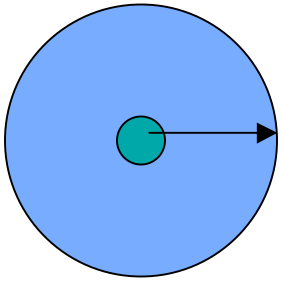


N.S. ,Nguyen Van Giai,R.J.Liotta, Phys.Rev.C69(2004)045802

Pairing Field in the Wigner-Seitz Cells

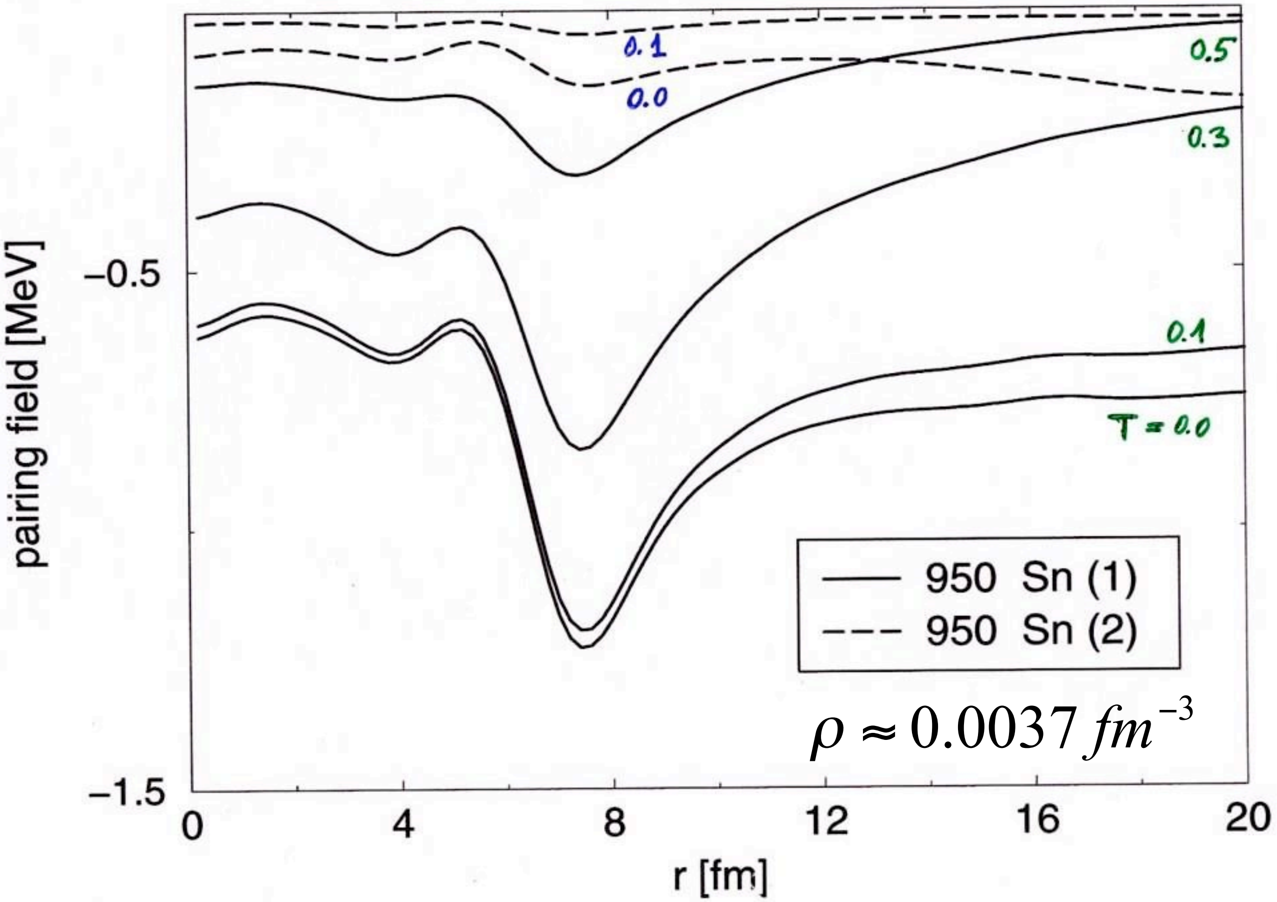


$\rho \approx 0.02\, fm^{-3}$



N.S, Phys.Rev.**C70** (2004) 025801

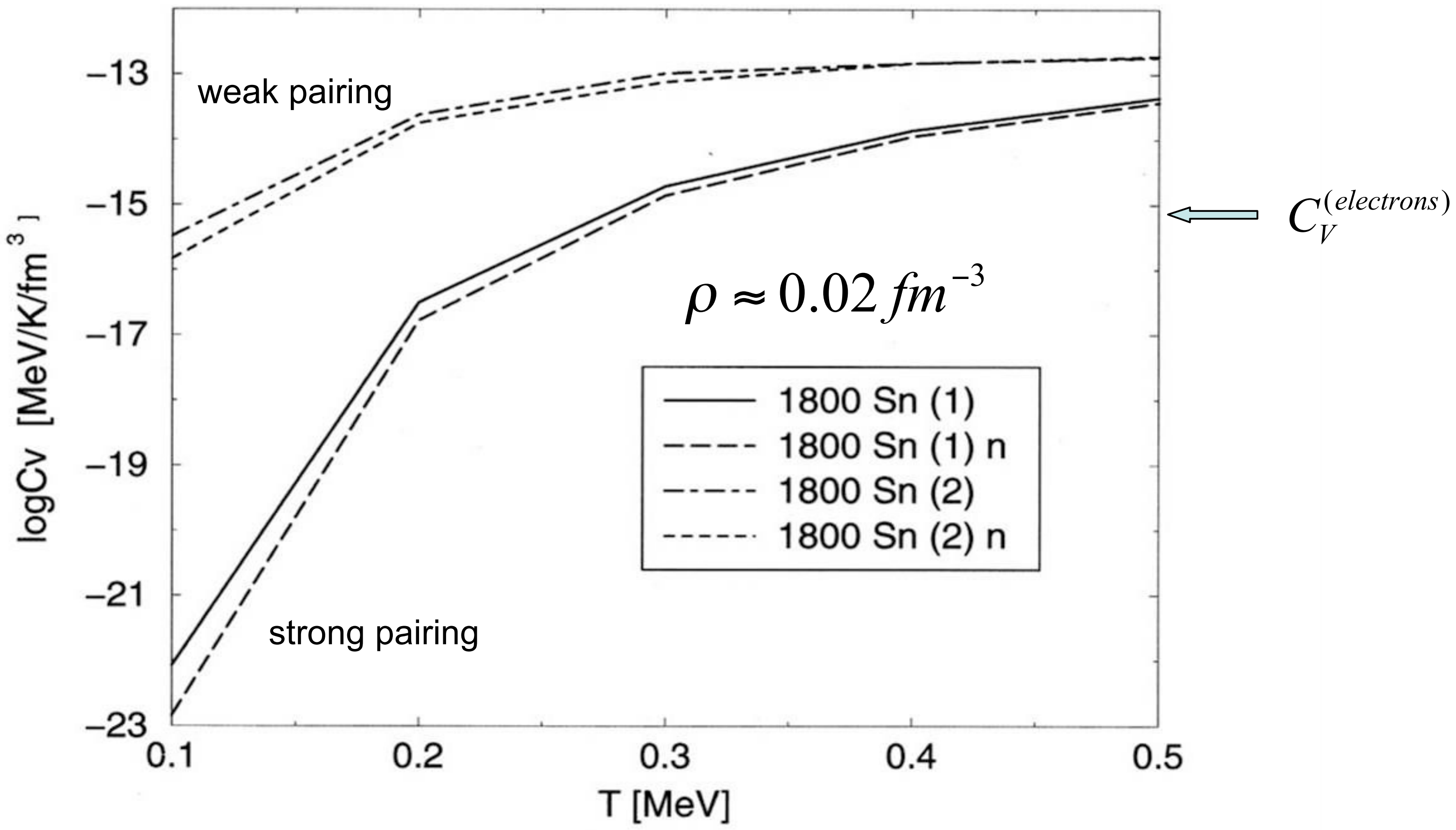
Pairing Field in the Wigner-Seitz Cells



$\rho \approx 0.0037\, fm^{-3}$

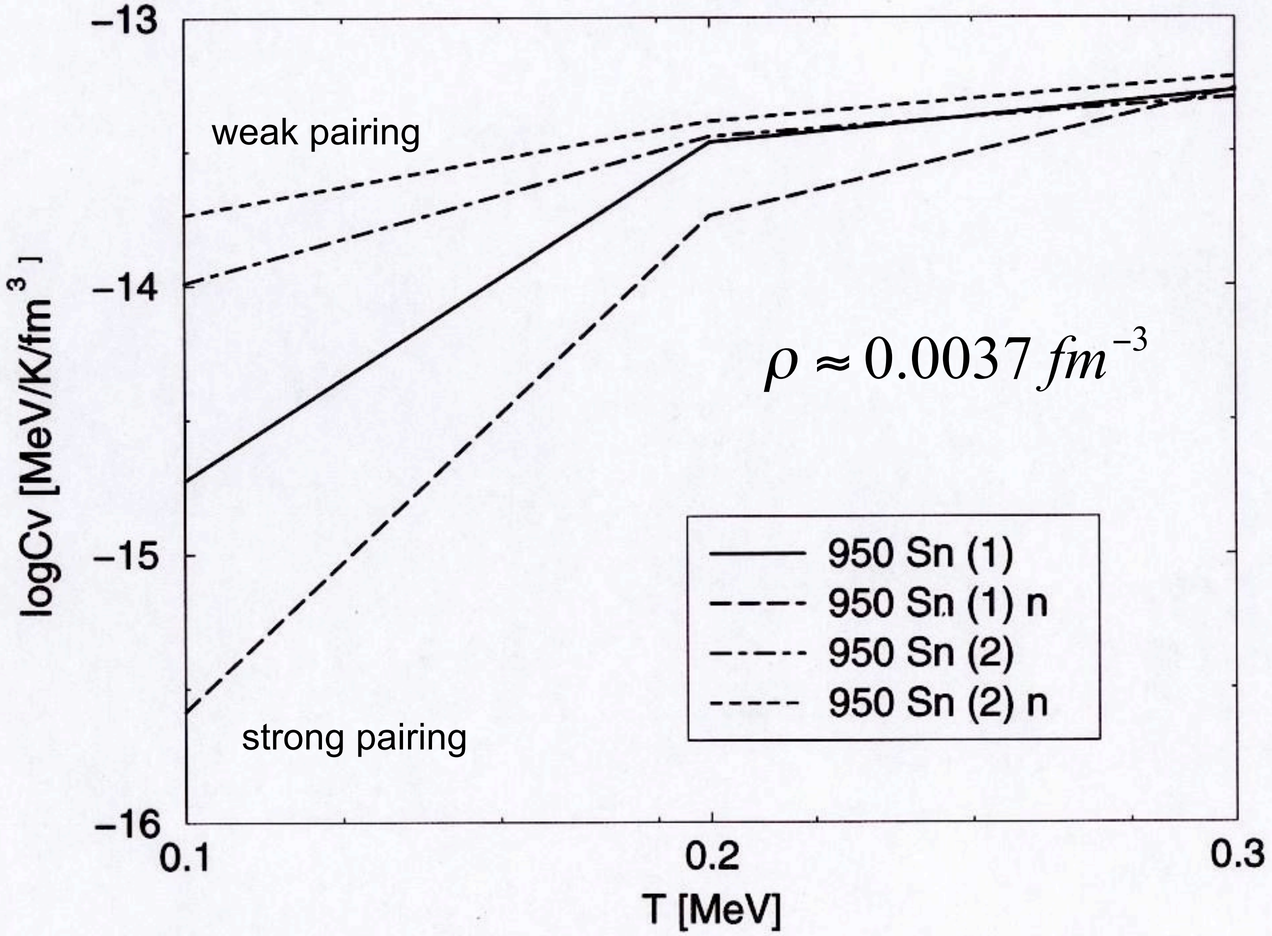
N.S, Phys.Rev.**C70** (2004) 025801

Specific Heat in the FT-HFB Approach



N.S, Phys.Rev.**C70** (2004) 025801

Specific Heat in the FT-HFB Approach



weak pairing

strong pairing

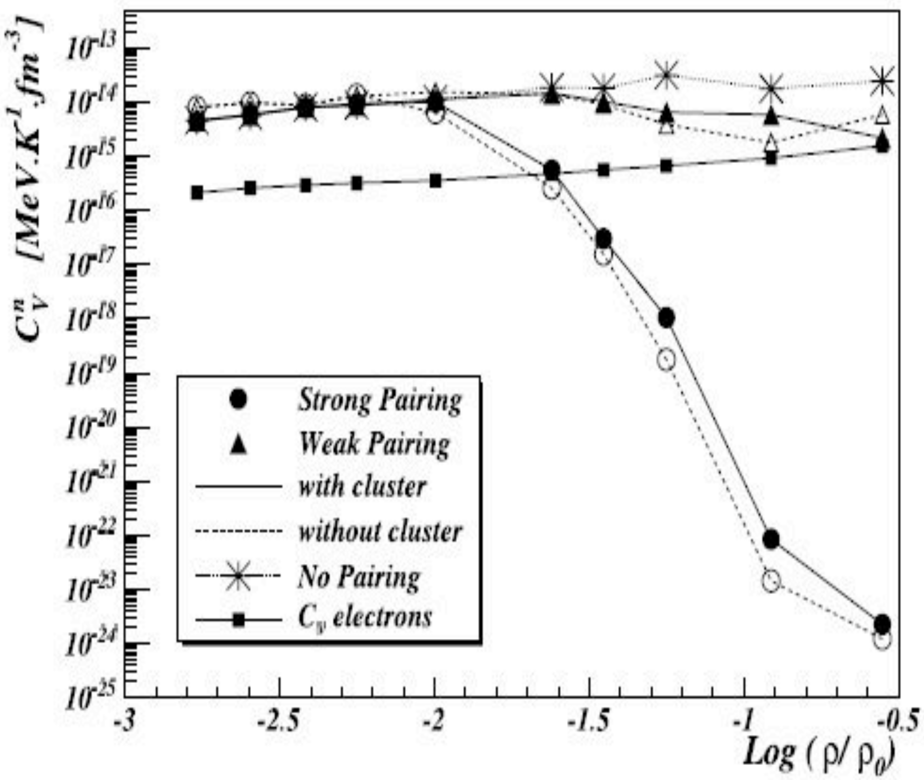
$\rho \approx 0.0037\, fm^{-3}$

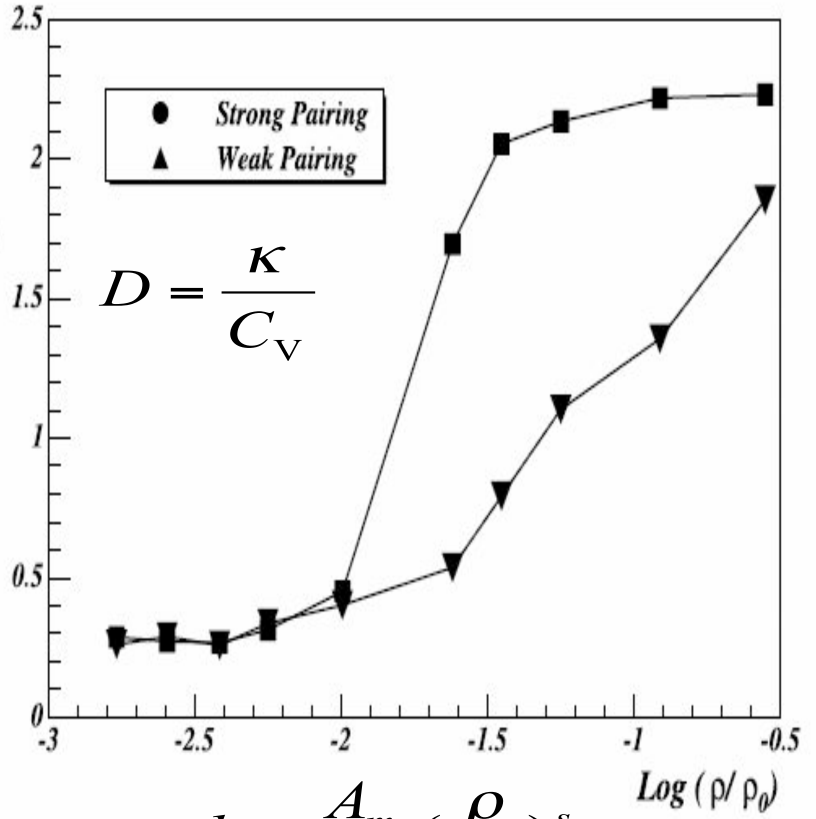
$\Leftarrow C_V^{(electrons)}$

N.S, Phys.Rev.**C70** (2004) 025801

Specific Heat and Diffusivity Across the Inner Crust

C. Monrozeau, J. Margueron, N. S, Phys Rev. C, in press





$$D = \frac{\kappa}{C_{\mathrm{V}}}$$

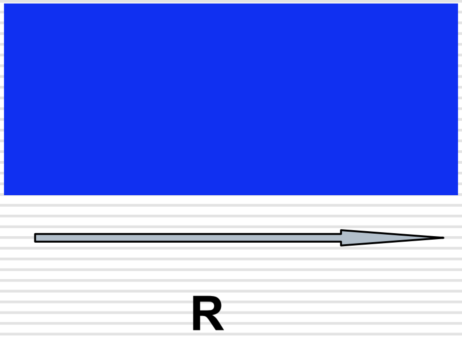
$$k \cong \frac{A_m}{T^m} \left(\frac{\rho}{\rho_0}\right)^s$$

(Lattimer et al, ApJ 425, 1994)

Thermal Diffusivity and Cooling Time

Constant thermal diffusivity

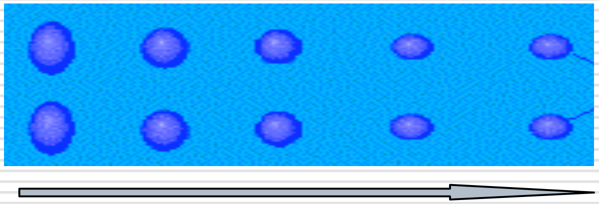
$$t_{diff} = \gamma \frac{R^2}{D} \; ; \; D = \frac{\kappa}{C_V}$$



Non-constant thermal diffusivity

$$t_{diff} = \gamma \int_{\rho_c}^{\rho_{shell}} \frac{1}{D[\rho, T(\rho)]} dR^2[\rho]$$

$$t_{diff} = \gamma \sum \frac{x_i^2}{k_i} C_V(i)$$

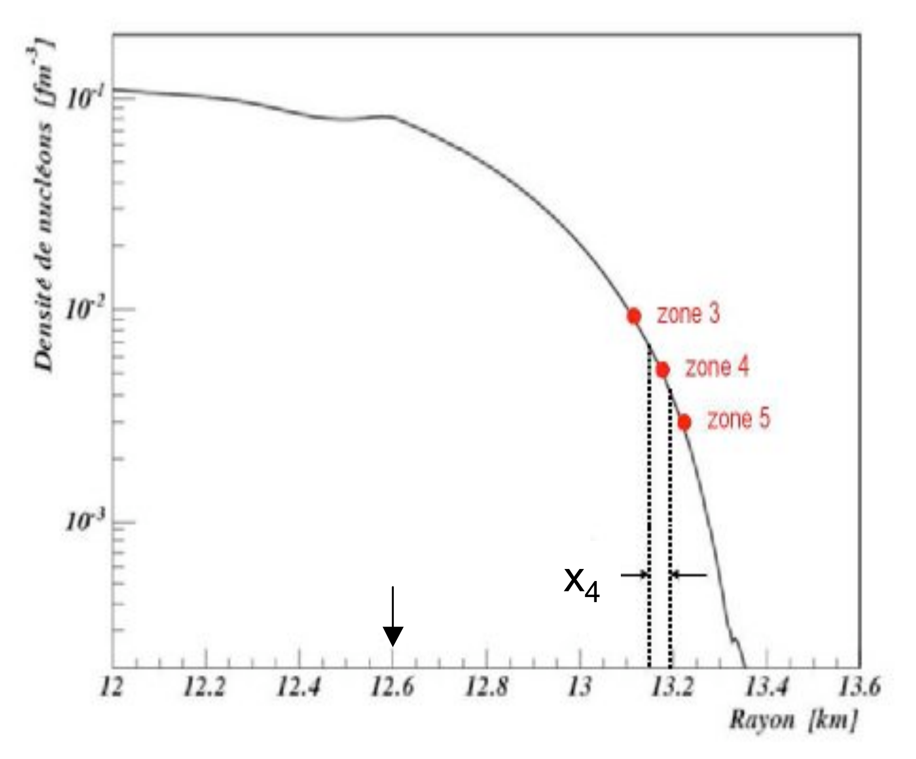


$$R = R[\rho]$$

given by TOV equation

Tolman – Oppenheimer – Volkov equation

$$\frac{dP}{dr} = -\frac{G[M(r) + 4\pi r^3 P/c^2](P/c^2 + \rho)}{r^2 - 2GM(r)r/c^2}$$

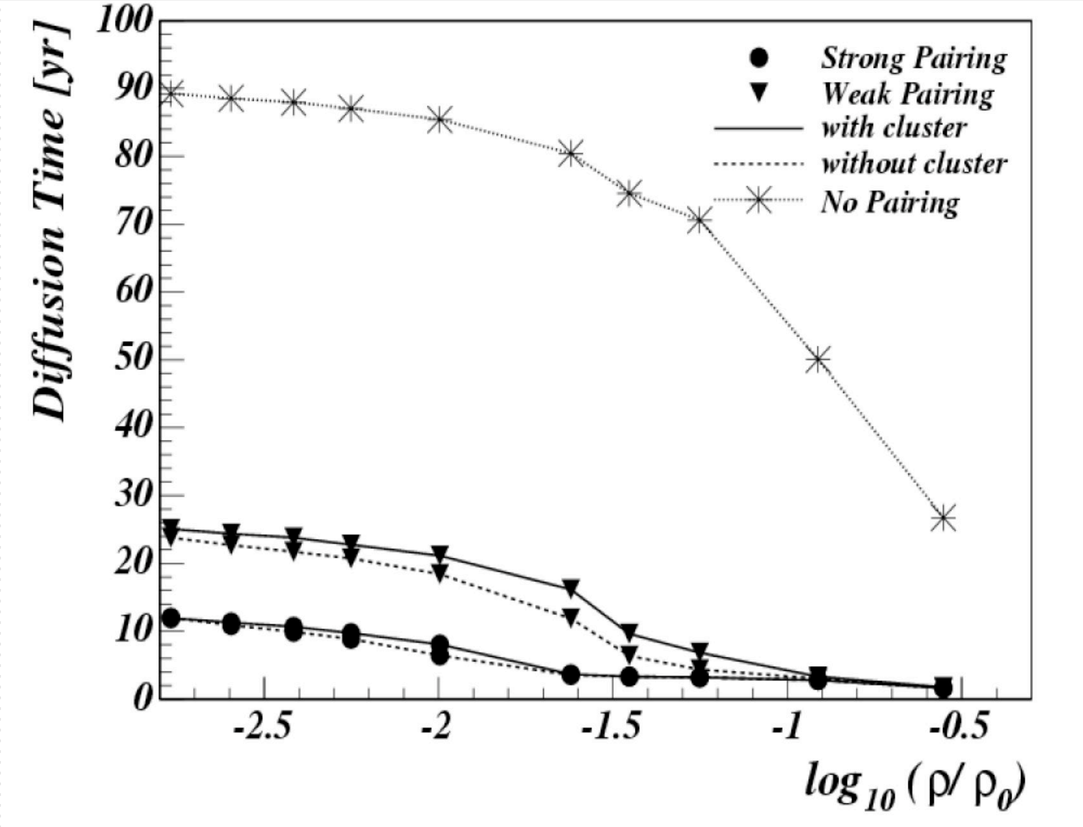


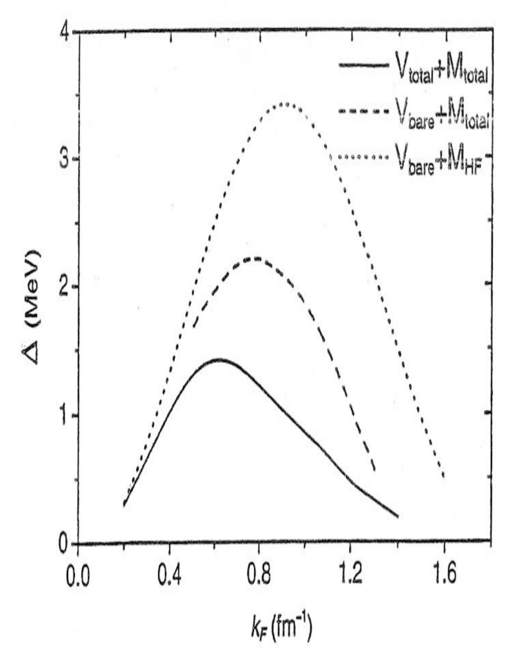
EOS

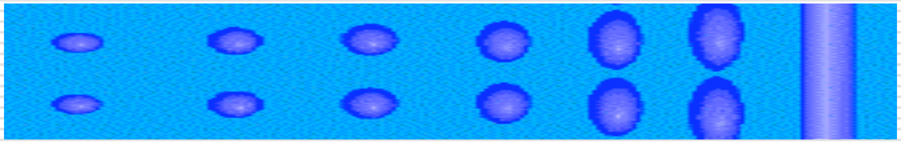
- *outer crust* : Baym-Pethick-Sutherland
- *inner crust* : Negele - Vautherin
- *core* : Glendenning-Moszkowski (GM1)

Results provided by Isaac Vidana

Cooling Time of The Inner Crust

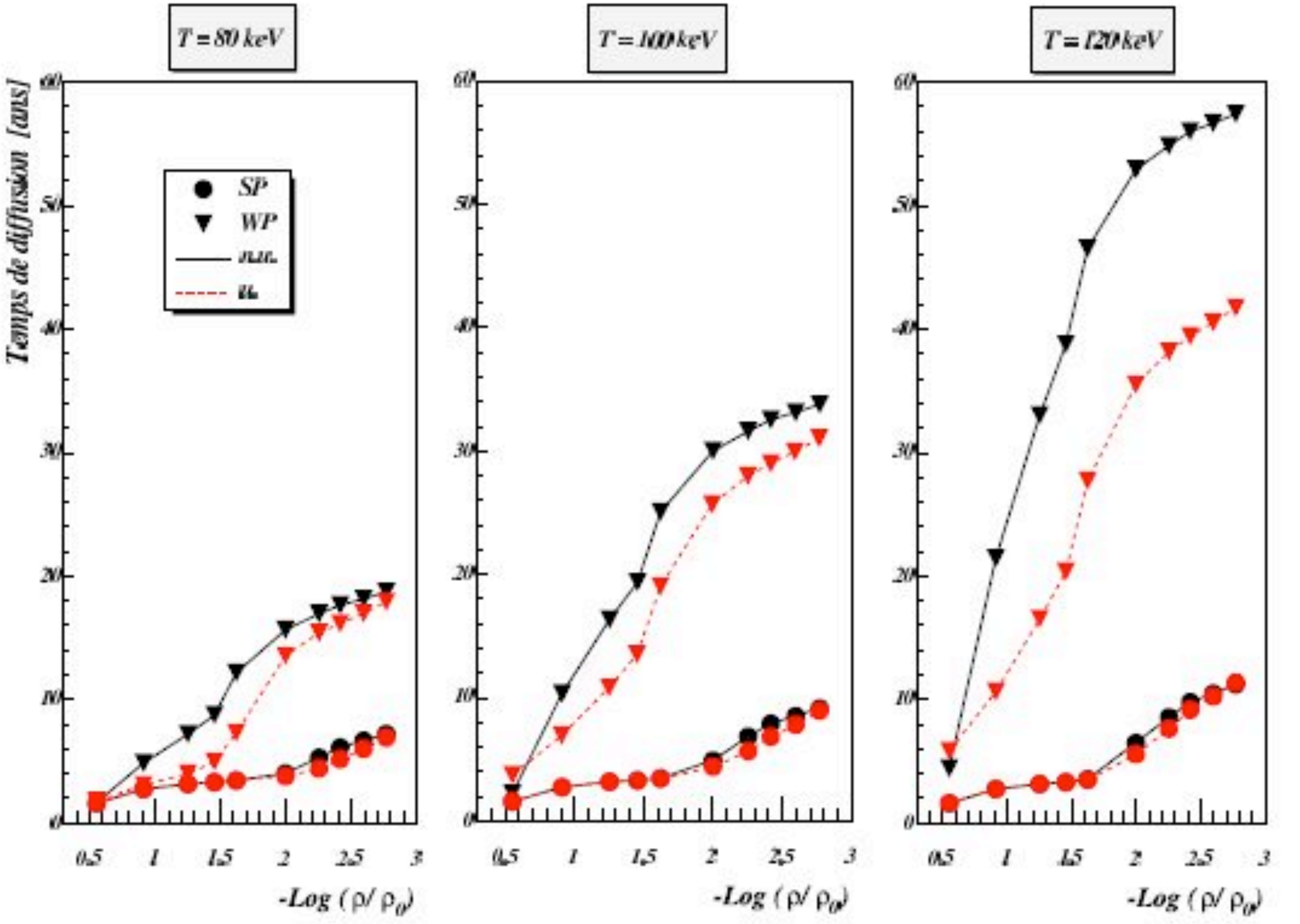






C. Monrozeau, J. Margueron, N. S, Phys Rev. C, in press

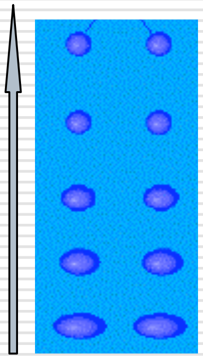
Cooling time for various crust temperatures



C. Monrozeau, PhD thesis (Orsay, 2007)

Open Questions

- ***What is the influence of the* detailed *structure of the inner crust upon the cooling time ?***
- ***What is the effect of the temperature gradient on cooling time ?***



$$t_{diff} = \lambda \int_{\rho_c}^{\rho_{shell}} \frac{1}{D[\rho, T(\rho)]} dR^2[\rho]$$

- ***Contribution of colective modes on specific heat and cooling time ?***

Collective Modes in the Crust of Neutron Stars

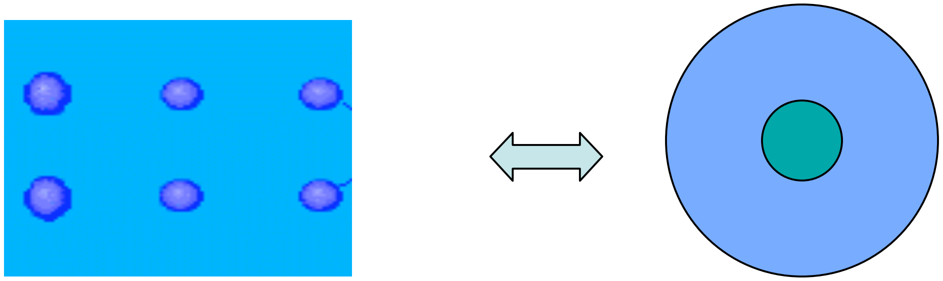
Non-uniform condensate:

coherence length : $\zeta \sim \hbar v_F / \pi \Delta_F$

distance between clusters: **L**

(a) **L >> ζ** : ~ the case of uniform condensate

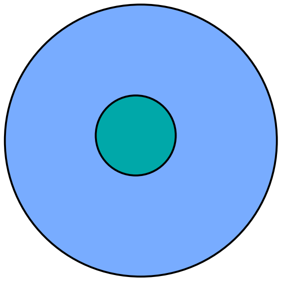
(b) **L < ζ** : need of microscopic calculations !



QRPA response

$$\rho' = \mathbf{GF} \quad \Longleftrightarrow \quad G = (1 - G_0 V)^{-1} G_0 = G_0 + G_0 V G.$$

$$G_0^{\alpha\beta}(r\sigma, r'\sigma';\omega) = \sum_{ij} \frac{\mathcal{U}_{ij}^{\alpha 1}(r\sigma)\bar{\mathcal{U}}_{ij}^{*\beta 1}(r'\sigma')}{\hbar\omega - (E_i + E_j) + i\eta} - \frac{\mathcal{U}_{ij}^{\alpha 2}(r\sigma)\bar{\mathcal{U}}_{ij}^{*\beta 2}(r'\sigma')}{\hbar\omega + (E_i + E_j) + i\eta},$$



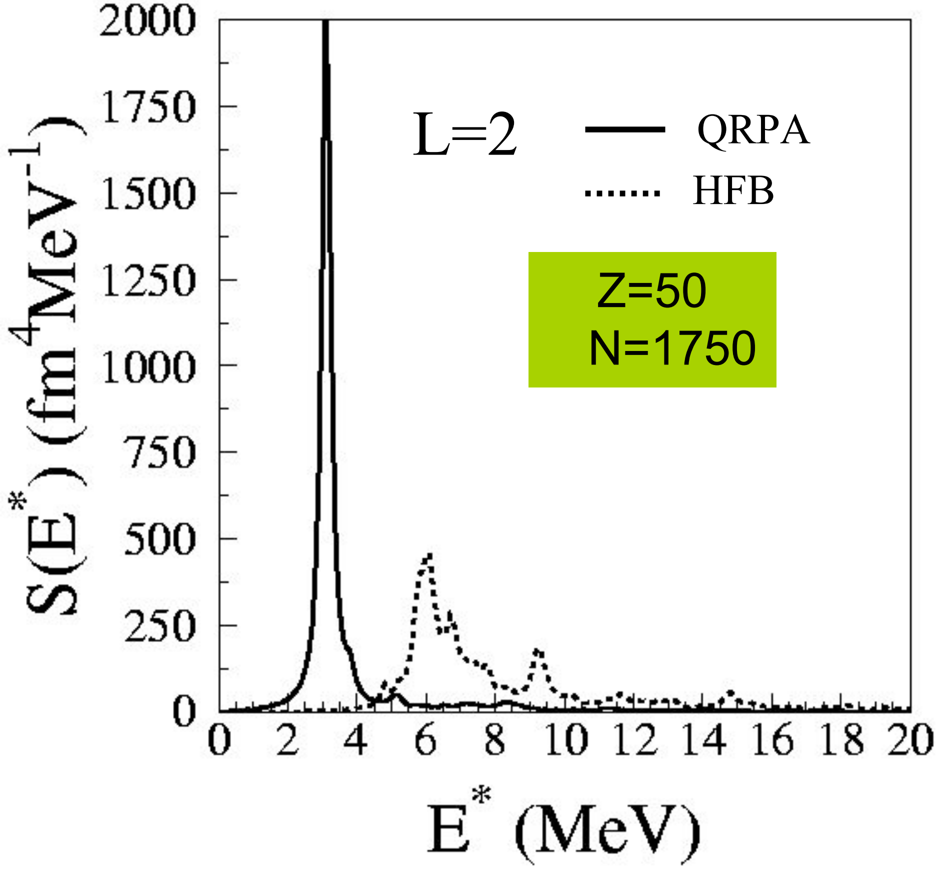
Rezidual interaction:

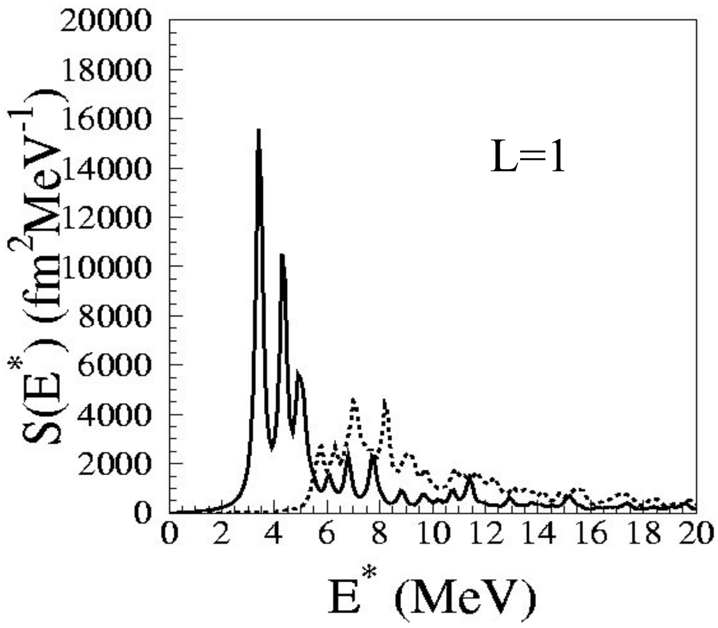
$$\boxed{\mathbf{V}^{\alpha\beta}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \frac{\partial^2 \mathcal{E}}{\partial\rho_\beta(\mathbf{r}'\sigma')\partial\rho_{\bar{\alpha}}(\mathbf{r}\sigma)}} \quad \Longleftrightarrow \quad V = \begin{pmatrix} V^{ph,ph} & V^{ph,pp} & V^{ph,hh} \\ V^{pp,ph} & V^{pp,pp} & V^{pp,hh} \\ V^{hh,ph} & V^{hh,pp} & V^{hh,hh} \end{pmatrix}$$

E.Khan, N. Sandulescu .,M.Grasso, Nguyen Van Giai, Phys. Rev. C66 (2002)024309

Supergiant resonances in the crust of neutron stars

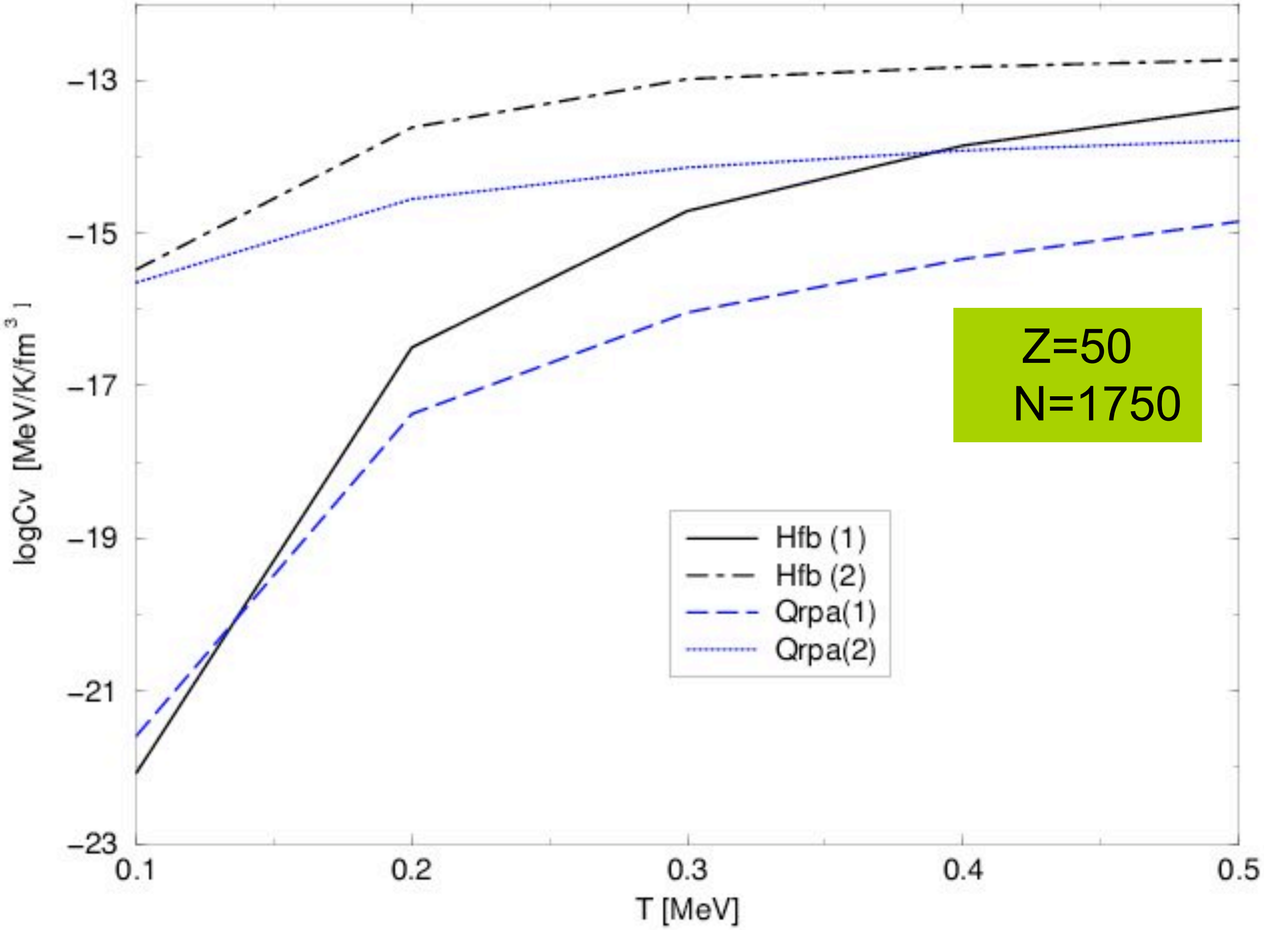
QRPA response: $\rho' = GF$ $G = (1 - G_0 V)^{-1} G_0 = G_0 + G_0 V G.$





E.Khan, N.Sandulescu, Nguyen Van Giai, Phys.Rev.C71 (2005) 042801R

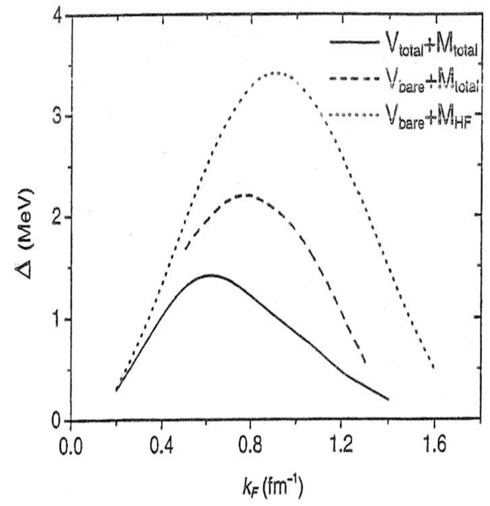
Specific heat of collective modes



Z=50
N=1750

(N. Sandulescu , nul-th/061201)

Summary and (No) Conclusions



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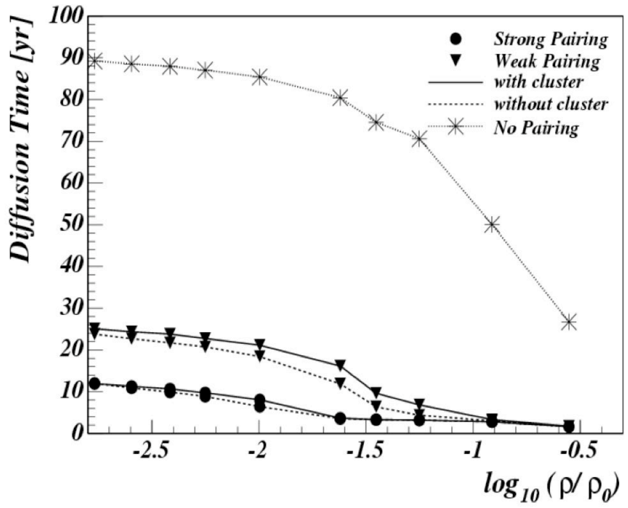
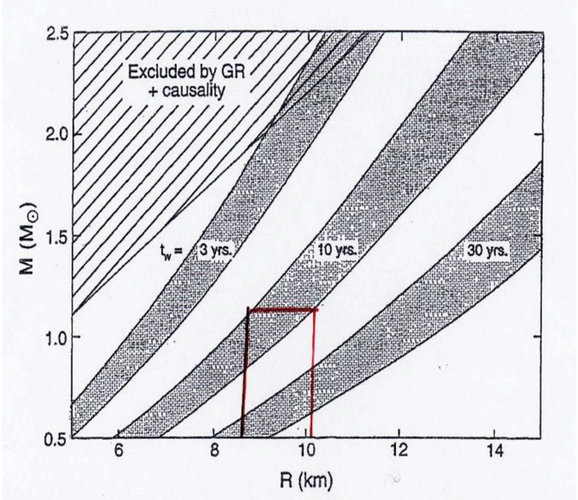
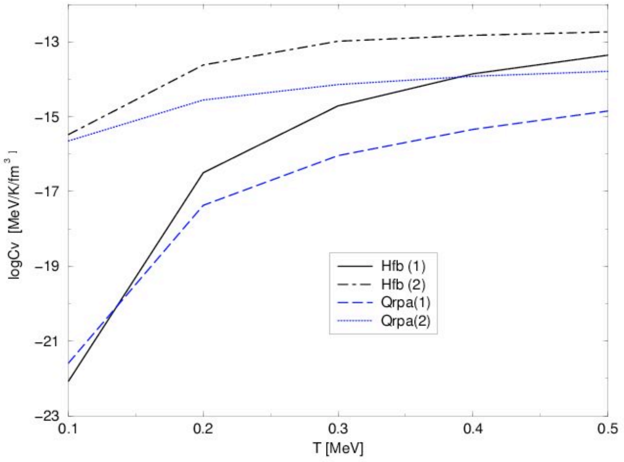


?

Consequences ?



Lattimer et al, ApJ425(1994)802



Mass-Radius Constraints from Cooling Time

Lattimer et al, ApJ425(1994)802

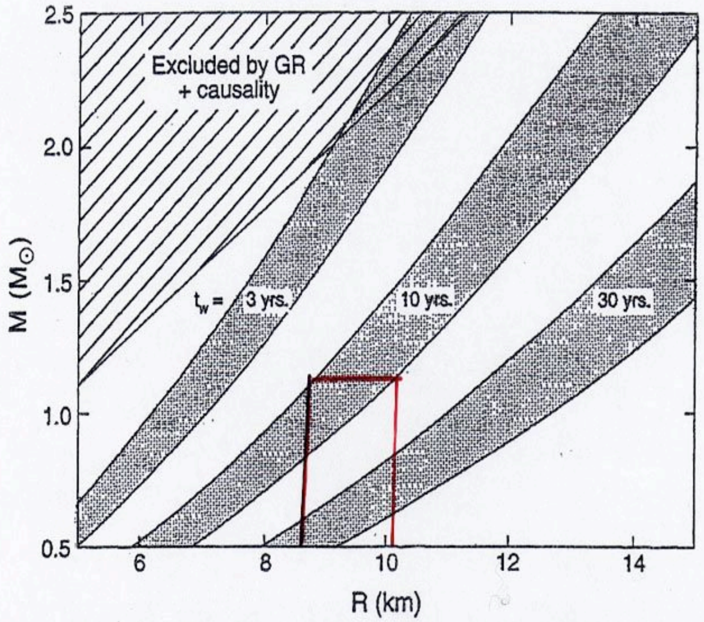


FIG. 12.—Shaded areas are the allowed regions of mass and radius for a neutron star observed to have the indicated values of the rapid cooling time t_w. It is assumed that $\rho_{core} = 0.5\rho_0$ and that the neutron star crust is superfluid. The mass-radius region excluded by general relativity and causality is indicated by the

$1.15\ M_0 < M < 1.5\ M_0$

t_w= 10 years

No Superfluidity:

6.8 km < R < 8.5 km

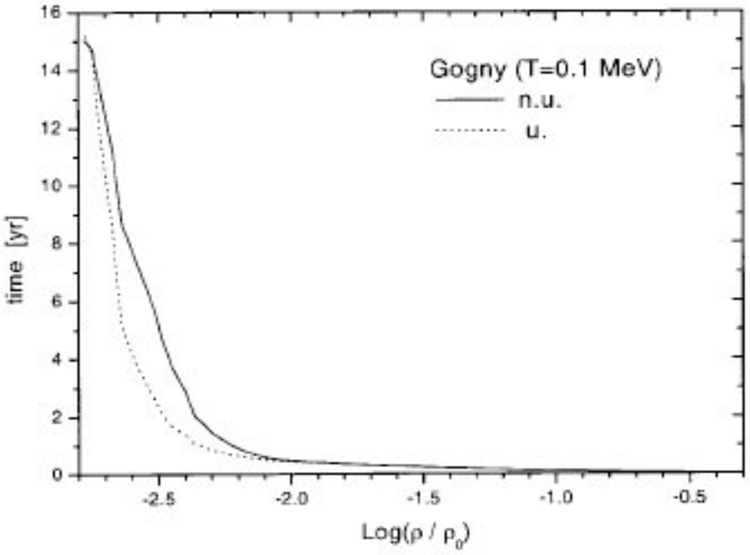
Superfluidity:

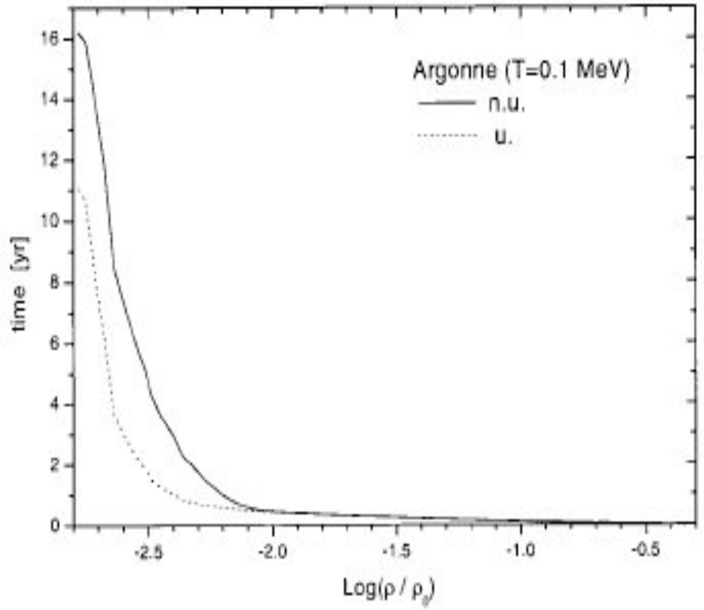
9 km < R < 11.5 km

Interaction	$\mathcal{T}(n.u.)$ [ans]	$\mathcal{T}(u.)$ [ans]
SP	9.1	8.9
WP	33.8	31.1
SP'	11.9	11.9
WP'	25.0	23.7
Argonne [Piz02]	16	11
Gogny [Piz02]	15	15

TAB. 7.1 – *Temps de diffusion obtenus avec différentes forces effectives d'appariement, à T=100 keV, dans la matière non-uniforme et uniforme. Les résultats de l'étude [Piz02] sont également donnés.*

Cooling time in previous calculations





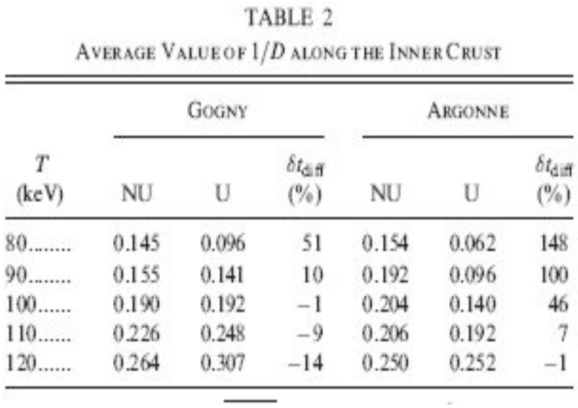
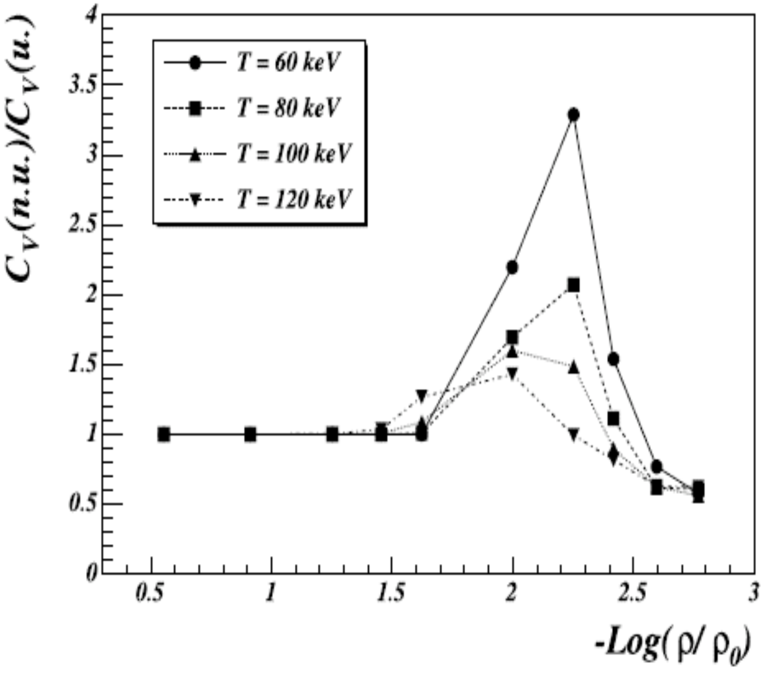


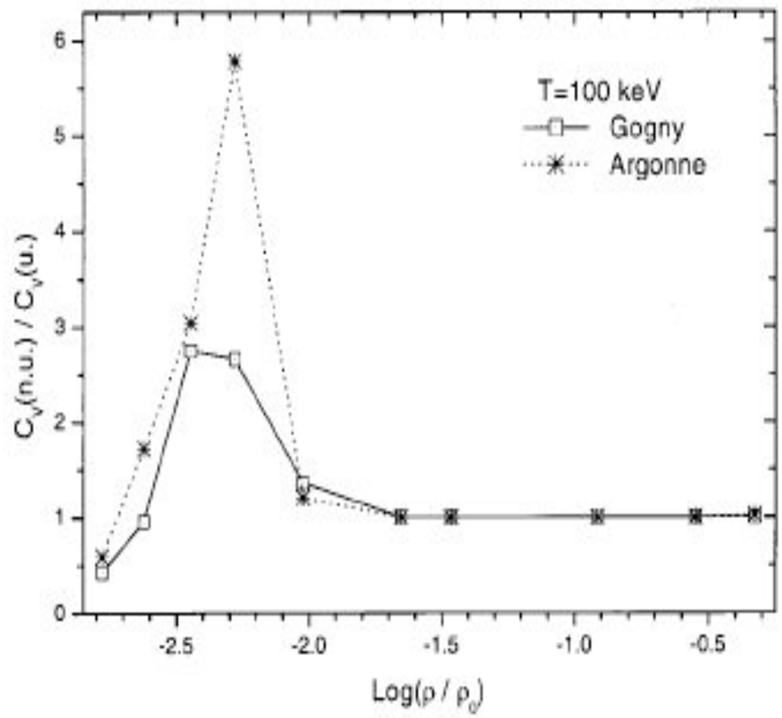
TABLE 2

AVERAGE VALUE OF $1/D$ ALONG THE INNER CRUST

T (keV)	GOGNY NU	GOGNY U	GOGNY $\delta t_{\rm diff}$ (%)	ARGONNE NU	ARGONNE U	ARGONNE $\delta t_{\rm diff}$ (%)
80........	0.145	0.096	51	0.154	0.062	148
90........	0.155	0.141	10	0.192	0.096	100
100......	0.190	0.192	−1	0.204	0.140	46
110......	0.226	0.248	−9	0.206	0.192	7
120......	0.264	0.307	−14	0.250	0.252	−1

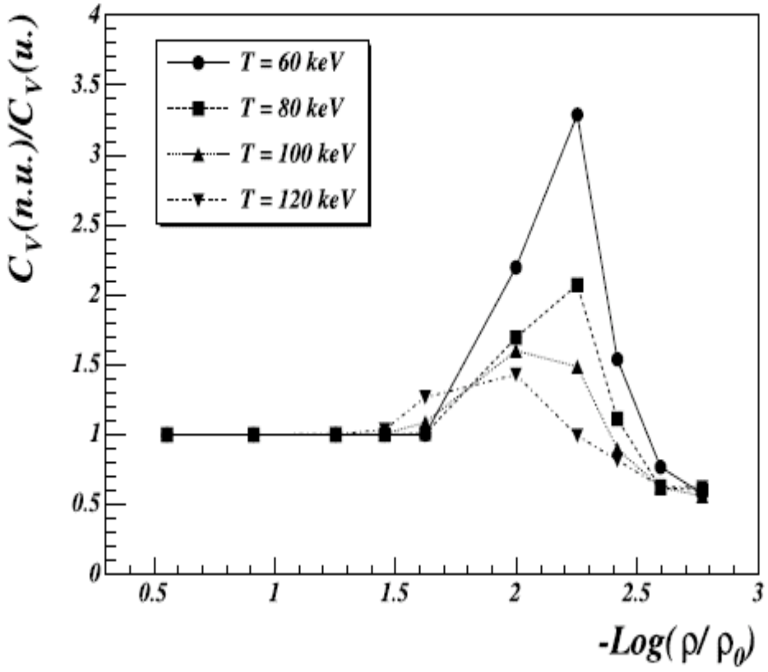
Total specific heat : nuclear cluster contribution

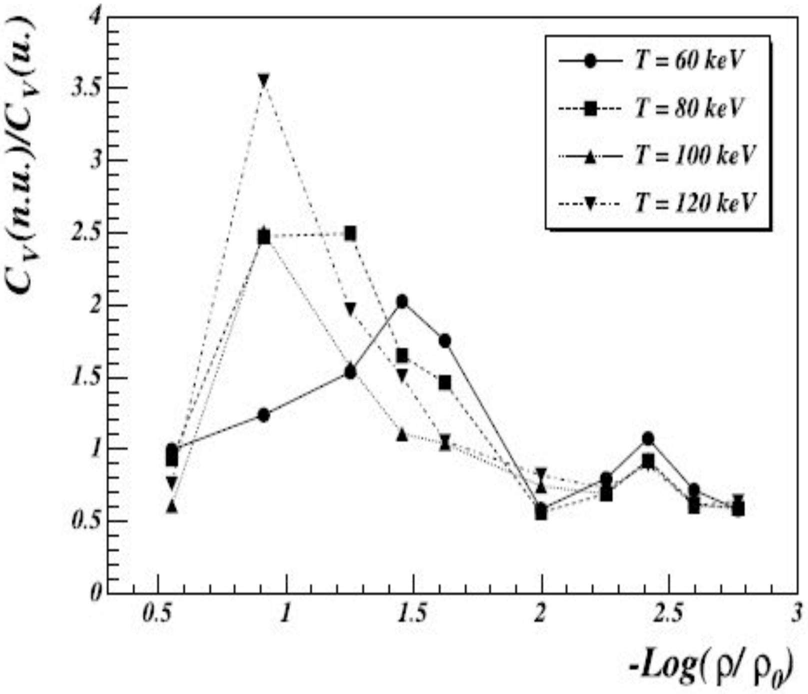




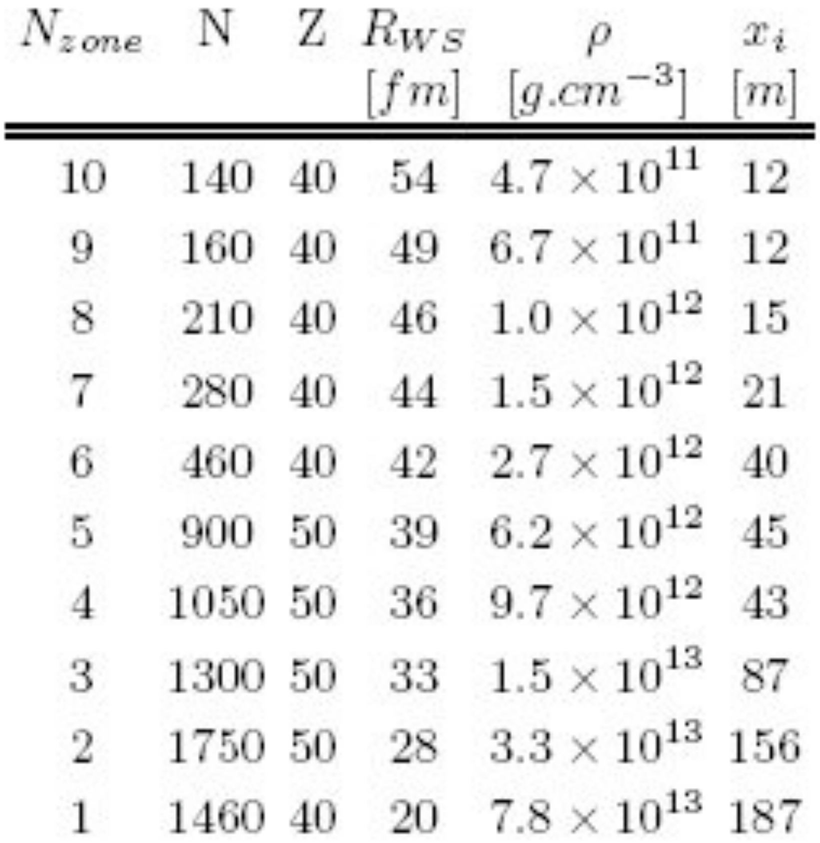
- P.M.Pizzochero et al, ApJ569, 2002

Total Specific Heat : nuclear cluster contribution





Properties of the Wigner–Seitz Cells



N_{zone}	N	Z	R_{WS} $[fm]$	ρ $[g.cm^{-3}]$	x_i $[m]$
10	140	40	54	4.7×10^{11}	12
9	160	40	49	6.7×10^{11}	12
8	210	40	46	1.0×10^{12}	15
7	280	40	44	1.5×10^{12}	21
6	460	40	42	2.7×10^{12}	40
5	900	50	39	6.2×10^{12}	45
4	1050	50	36	9.7×10^{12}	43
3	1300	50	33	1.5×10^{13}	87
2	1750	50	28	3.3×10^{13}	156
1	1460	40	20	7.8×10^{13}	187

Specific Heat of Collective Modes

- *specific heat:* $$C_V = \frac{1}{T}\frac{\partial S_{coll}}{\partial T}$$

- *entropy:* $S_{coll} = \Sigma_\nu [(1+g_\nu)\ln(1+g_\nu)-g_n \ln g_n]$

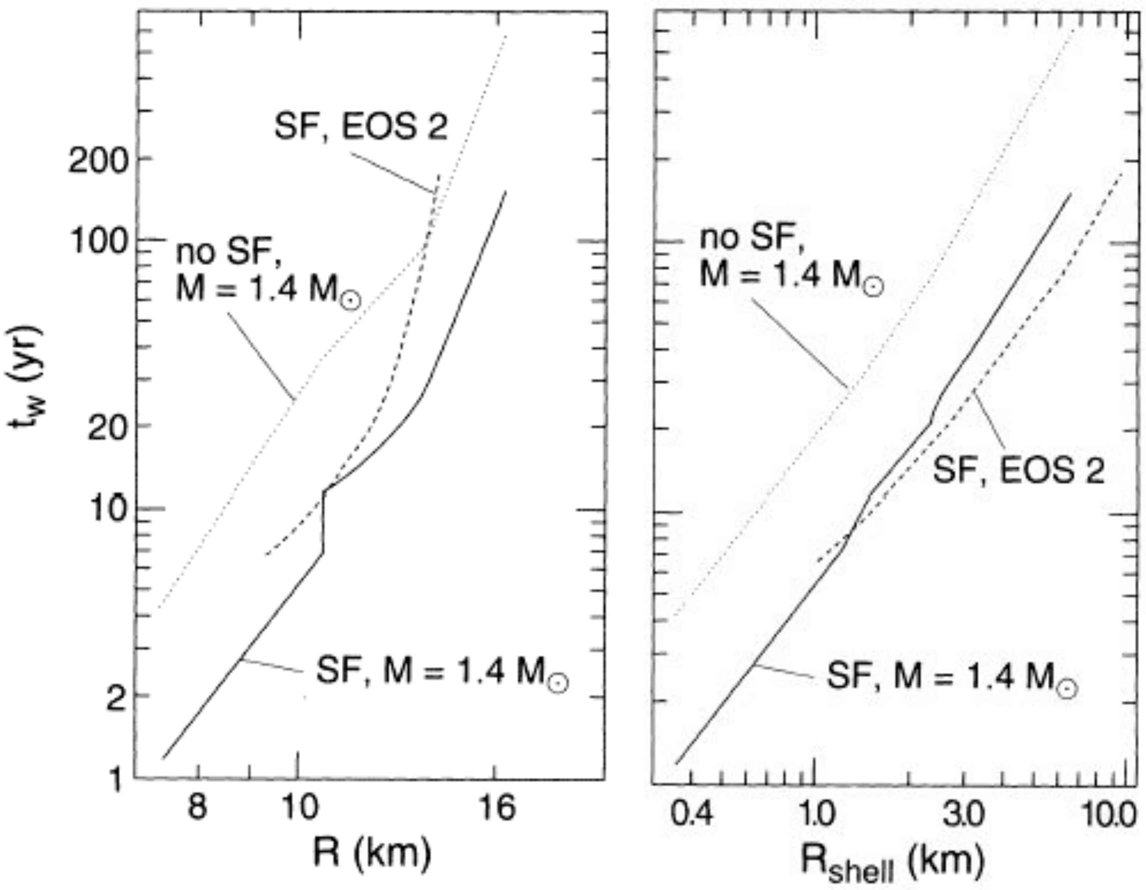
  $- \Sigma_{ij} [(1+g_{ij})\ln(1+g_{ij})-g_{ij}\ln g_{ij}]$

$g_\nu=[\exp(\Omega_\nu/kT)-1]^{-1}$ (Ω_ν –QRPA spectrum)

$g_{ij}=[\exp((E_i+E_j)/kT)-1]^{-1}$ (E_i - HFB spectrum)

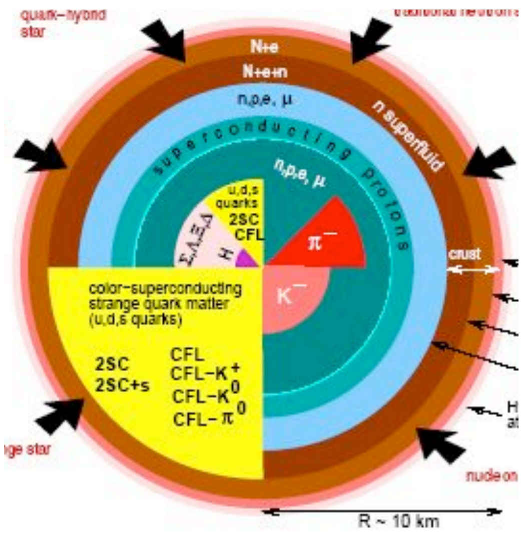
Cooling time versus stars size

Lattimer et al, ApJ425(1994)802



$$t_w \approx \frac{R_{shell}^2}{D}$$

$$D = \frac{\kappa}{C_V}$$



Superfluidity is changing t_w by a factor of 3 !