Superfluid Response & Neutrino Emission in the Inner Crust

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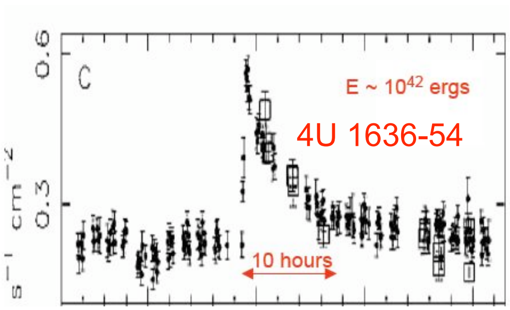
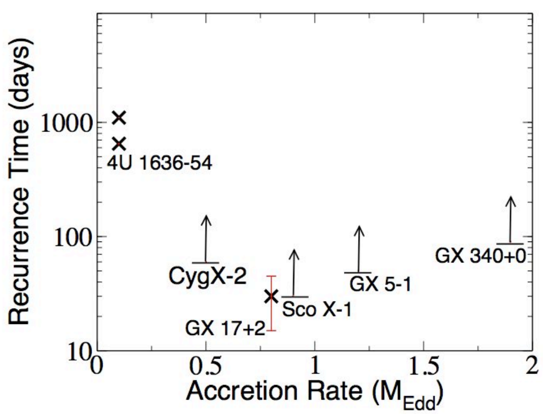
Superburst Recurrence Time



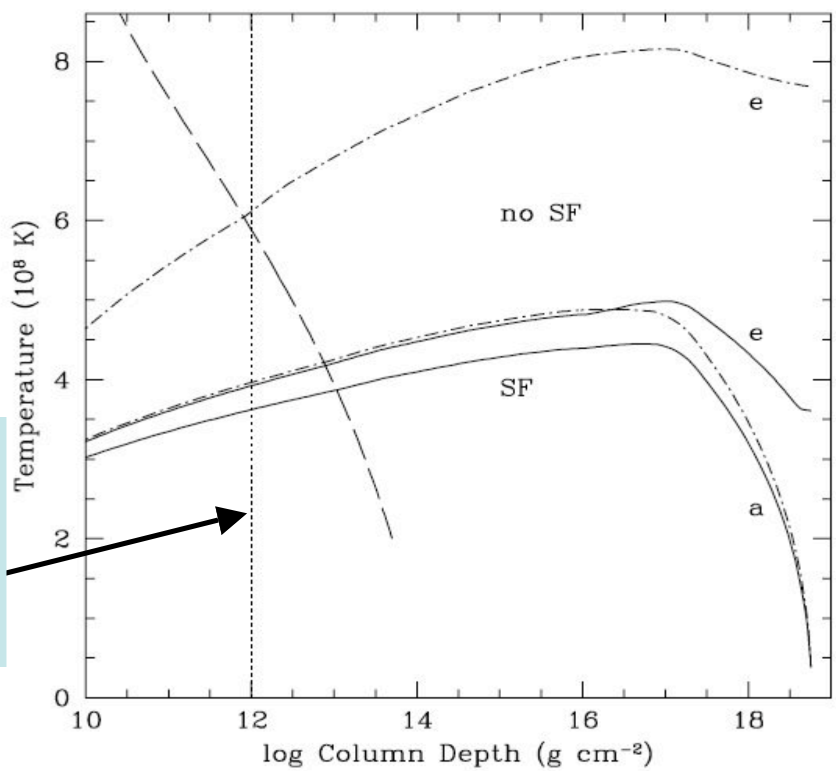
Neutron Star Thermometer:
Ignition (recurrence times) very sensitive to the *thermal profile of the neutron star crust*.

Superbursts are longer duration (hours) bursts with *recurrence times days-years*.
Likely to be ignition of carbon poor ashes produced during XRB activity.



Inferred ignition depth

Thermal Profile of the Crust



Woosley & Taam (1976), Cumming & Bildsten (2001)
Strohmayer & Brown (2002), Brown (2004)

Cumming et al. (2006)

Is the neutrino emission more efficient in the superfluid phase ?

Superfluid Rate: $$Q_{PBF} \approx 10^{21}\, T_9^7 \; erg/cm^3 s$$

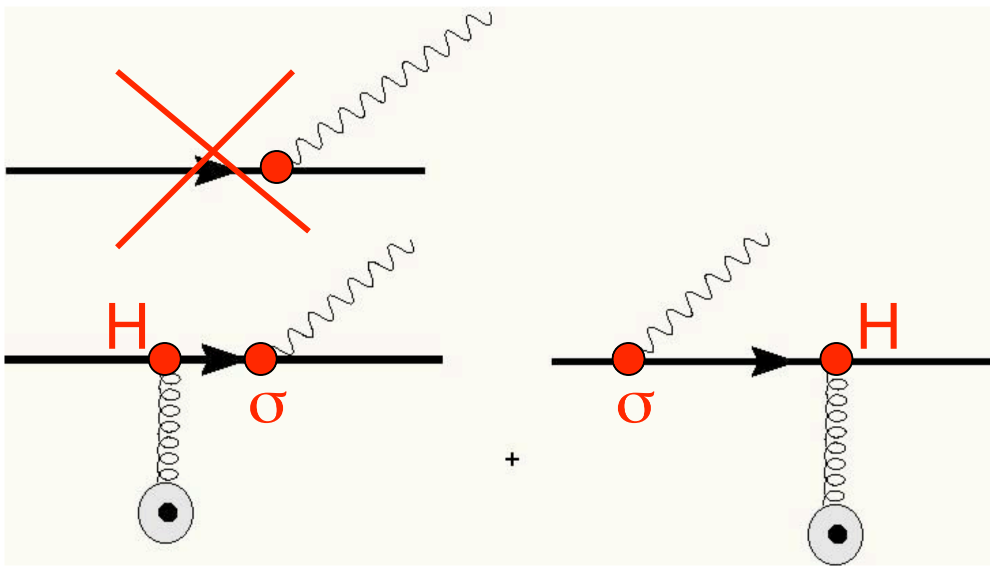
Normal Rate: $$Q_{Bremss} \approx 10^{19}\, T_9^8 \; erg/cm^3 s$$

Roughly a factor of 200 enhancement for $T \sim T_c \sim 5 \times 10^9$ K

Leinson & Perez (2006) recalculated the superfluid rate and find a large suppression.

Suppression Factor $\sim V_F^4 \sim 10^{-6}$

Bremsstrahlung 101:



Kinematically forbidden

Need acceleration for radiation

$$\frac{1}{\frac{p\cdot q}{m}+\frac{q^2}{2m}+\omega}+\frac{1}{\frac{p\cdot q}{m}-\frac{q^2}{2m}-\omega}\approx\frac{1}{\omega}\frac{p}{m}\frac{q}{\omega}$$

Radiation without acceleration set q=0:

$$\frac{H\,\sigma}{\omega}-\frac{\sigma\,H}{\omega}\approx\frac{1}{\omega}[H,\sigma]$$

Neutrino Bremsstrahlung:

$$L = \frac{G_F}{2\sqrt{2}}\; l_\nu(x)\; j^\mu(x)$$

$$j^\mu(x) = \bar{\psi}(x)\,\gamma^\mu(c_V - c_A\gamma_5)\,\psi(x)$$

$$\xrightarrow{NR} c_V\;\psi^+\psi\,\delta^{\mu 0} - c_A\;\;\psi^+\,\sigma^i\,\psi\,\delta^{\mu i}$$

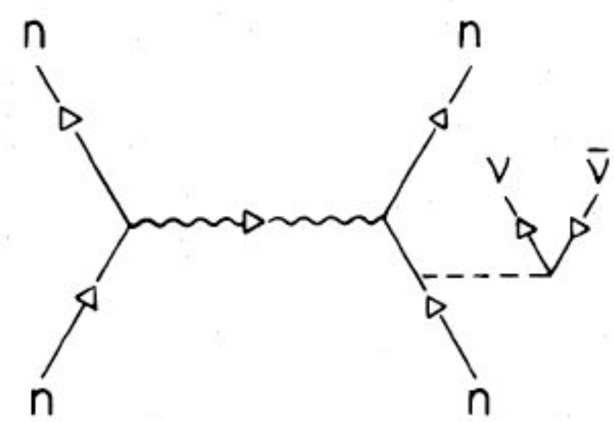
Neutrinos couple to density and spin:

$[H_{nuclear}, \rho]=0$, but $[H_{nuclear}, \sigma]\neq 0$

Pion exchange does not conserve spin:

$H_{nuclear} \sim V_{OPE}$

$$V_{\mathrm{OPE}} = \left(\frac{f}{m_\pi}\right)^2 \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\left(\frac{-1}{k^2+m_\pi{}^2}\right)\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)})$$



Friman & Maxwell (1979)

Vector response is suppressed in non-relativistic systems

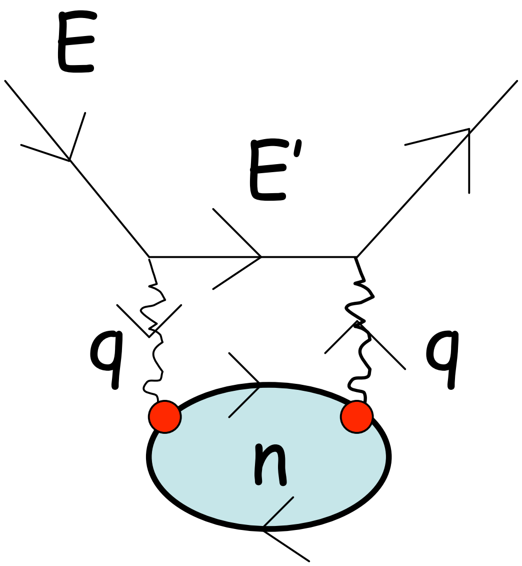
Vector Current

$$\frac{|M_\rho|^2}{|M_\sigma|^2} \approx \frac{p^4}{m^4}\,\frac{q^2}{\omega^2} \approx V_F^4\,\frac{q^2}{\omega^2}$$

Axial-vector Current

Neutron stars cool because:
(I) Weak interactions involve axial currents
(II) Nuclear interactions have a tensor component.

Neutrino interactions in a dense medium



$$\frac{d^2\sigma}{V\, d\cos\theta\, dE'} \approx G_F^2 \frac{E}{E'} \,\mathrm{Im}\left[L_{\mu\nu}(k,k+q)\,\Pi^{\mu\nu}(q)\right]$$

$$L_{\mu\nu} = \mathrm{Tr}[\, l_\mu(k)\; l_\nu(k+q)\,]$$

$$\Pi^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}[\, j^\mu(p)\;\; j^\nu(p+q)\,]$$

$$\frac{d^2\sigma}{V\, d\cos\theta\, dE'} \approx \frac{G_F^2 n}{8\pi^2}\; E'^2\left[c_V^2(1+\cos\theta)S_\rho(\omega,|\vec{q}\,|) + c_A^2(3-\cos\theta)S_\sigma(\omega,|\vec{q}\,|)\right]$$

$$S_\rho(\omega,|\vec{q}\,|) = \frac{1}{n}\int_{-\infty}^{\infty} dt\; e^{i\omega t}\,\langle \rho(\vec{q},t)\,\rho(-\vec{q},0)\rangle$$

Density Fluctuations

$$S_\sigma(\omega,|\vec{q}\,|)\,\delta_{ij} = \frac{1}{n}\int_{-\infty}^{\infty} dt\; e^{i\omega t}\,\langle \sigma_i(\vec{q},t)\,\sigma_j(-\vec{q},0)\rangle$$

Spin-Density Fluctuations

Sum Rules:

Conservation laws impose constraints on the response functions

F-sum rule:

$$\int \frac{d\omega}{2\pi}\, \omega\, S_O(q,\omega) = \langle \Phi | [H, O(q)], O(q)] | \Phi \rangle$$

$$\langle \Phi | [H, \rho(q)], \rho(q)] | \Phi \rangle = \frac{q^2}{2m}$$

$$\langle \Phi | [H, \sigma(q)], \sigma(q)] | \Phi \rangle = W_{Tensor} + O(q)$$

Depends on the nature of the ground state.

In neutron matter W_{tensor} ~ 50 MeV at nuclear density.

Akmal & Pandharipande (2003)

Fermion Superfluids

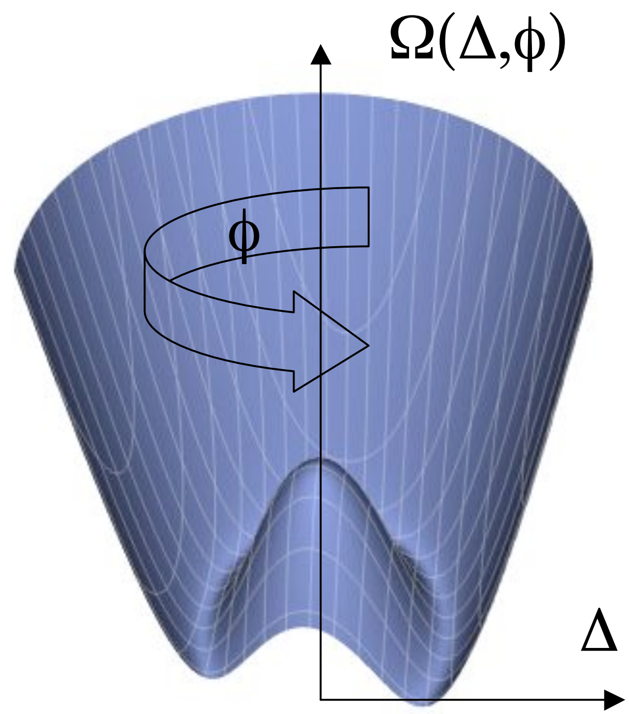
Arbitrarily weak interaction destabilizes the Fermi Gas
(Bardeen, Cooper and Schreiffer (1957))

$$H = \sum_{k,s=\uparrow,\downarrow} \left(\frac{k^2}{2m} - \mu_s\right) a_s^\dagger a_s + g \sum_{k,p,q} a_{k+q\uparrow}^\dagger a_{p-q\downarrow}^\dagger a_{k\uparrow} a_{p\downarrow}$$

$\Delta = g \langle a_{-k} a_k \rangle \quad \Delta^* = g \langle a^\dagger_{-k} a^\dagger_k \rangle$

$\Delta \rightarrow |\Delta| e^{i\phi}$

$$E(p) = \sqrt{(\frac{p^2}{2m} - \mu)^2 + \Delta^2}$$



Condensation & Greens Functions

Partition Function: Z = Tr[Exp[-β(H−μN))

$$\mathcal{Z} = \int_{\psi(\beta)=-\psi(0)} D(\bar{\psi},\psi)\exp\Big\{-\overbrace{\int_0^\beta d\tau d^d r}^{x\equiv(\tau,\mathbf{r})\quad \int dx}\Big[\sum_\sigma \bar{\psi}_\sigma \overbrace{\left(\partial_\tau + \frac{\hat{\mathrm{p}}^2}{2m} - \mu\right)}^{[\hat{G}_0^{(\mathrm{p})}]^{-1}}\psi_\sigma - g\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow\Big]\Big\}$$

Hubbard-Strantanovich Decoupling:

$$e^{g\int dx\,\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow} = \int D(\bar{\Delta},\Delta)\exp\Big\{-\int dx\,\overbrace{\Big[\frac{1}{g}|\Delta(\mathbf{r},\tau)|^2 + (\bar{\Delta}\psi_\downarrow\psi_\uparrow + \Delta\bar{\psi}_\uparrow\bar{\psi}_\downarrow)\Big]}^{\frac{1}{g}(\bar{\Delta}+g\bar{\psi}_\uparrow\bar{\psi}_\downarrow)(\Delta+g\psi_\downarrow\psi_\uparrow)-g\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow}\Big\}$$

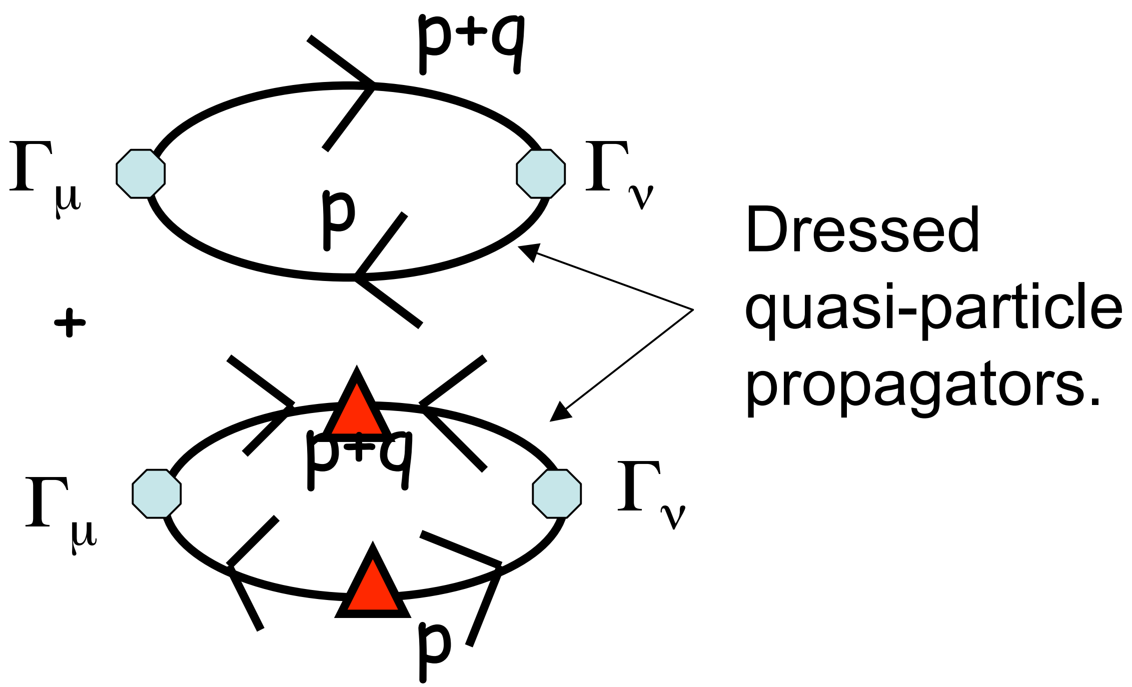
$$\mathcal{Z} = \int D(\bar{\psi},\psi)\int D(\bar{\Delta},\Delta)e^{-\int dx\frac{|\Delta|^2}{g}}\exp\Big[-\int dx\,\overbrace{(\bar{\psi}_\uparrow\quad\psi_\downarrow)}^{\text{Nambu spinor}}\overbrace{\begin{pmatrix}[\hat{G}_0^{(\mathrm{p})}]^{-1} & \Delta\\ \bar{\Delta} & [\hat{G}_0^{(\mathrm{h})}]^{-1}\end{pmatrix}}^{\text{Gor'kov Ham. }\hat{\mathcal{G}}^{-1}}\begin{pmatrix}\psi_\uparrow\\ \bar{\psi}_\downarrow\end{pmatrix}\Big]$$

Mean-Field Approximation: Ignore the fluctuations in the gap

Response to External Perturbations

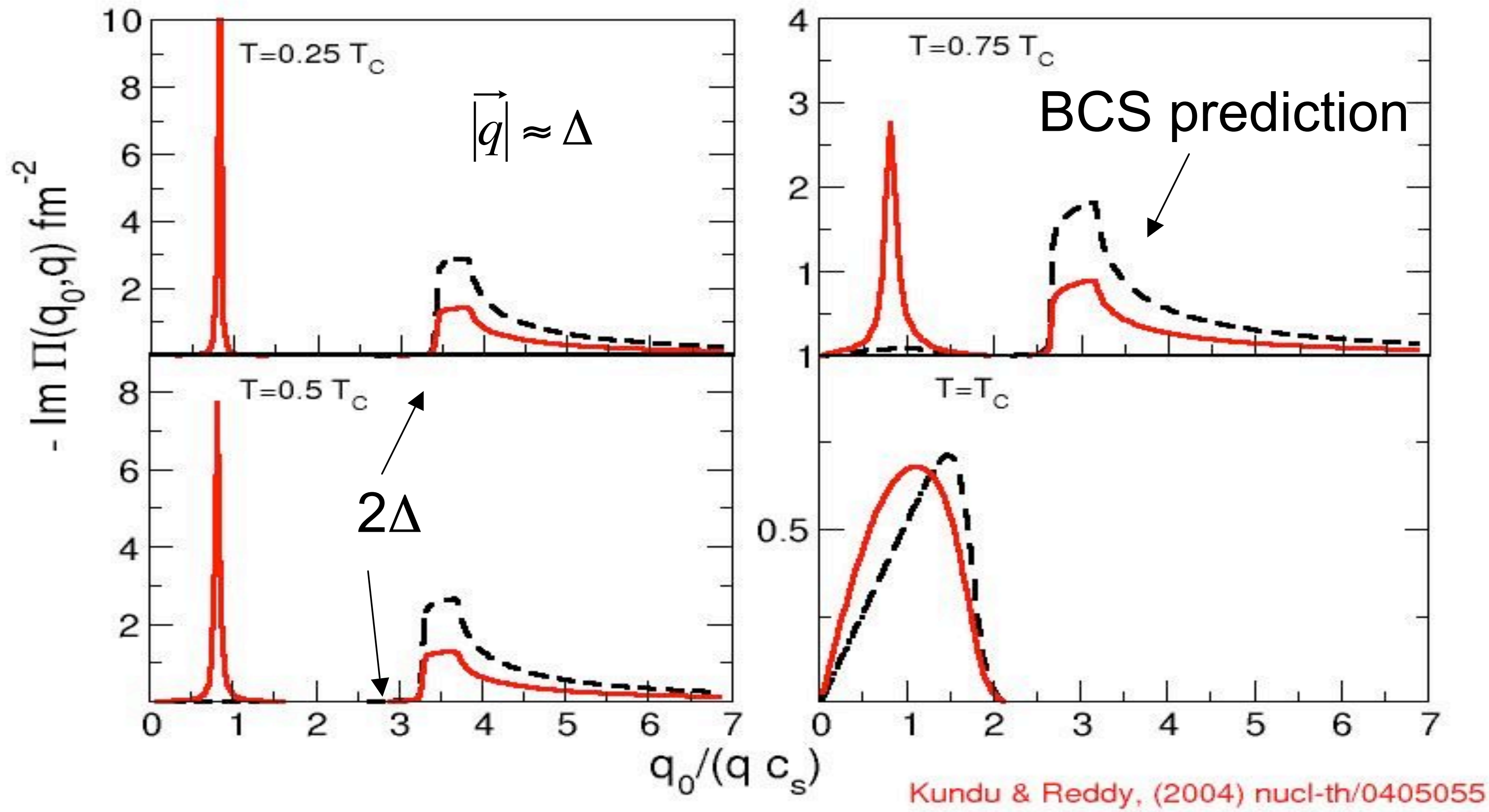
$$\Pi_{\mu\nu}(\vec{q},q_0) = \int d^4p\ Tr\ [\ G(p)\ \Gamma_\mu G(p+q)\Gamma_\nu\ ]$$

=



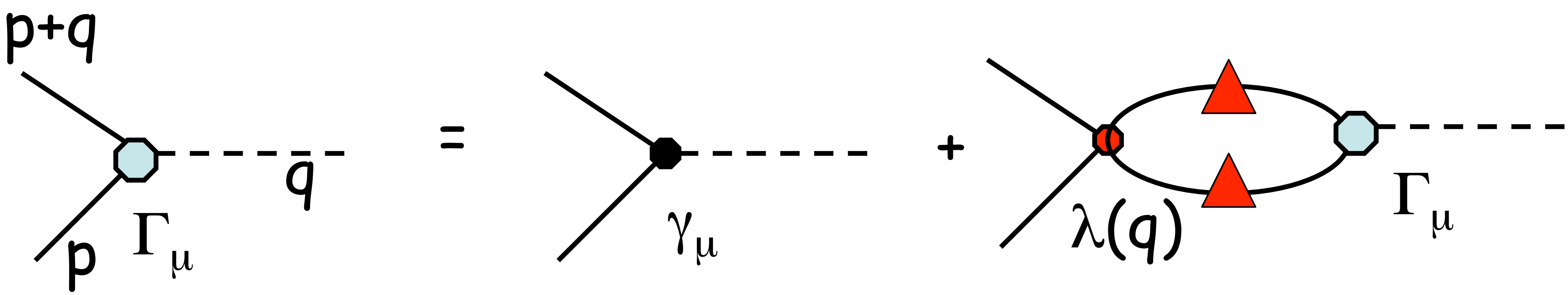
Gap modifies excitation spectrum Pairing introduces coherence effects

Spectrum of density fluctuations in Superfluids



BCS rate does not vanish at q=0 !

Collective (Goldstone) modes



Generalized Ward Identity:

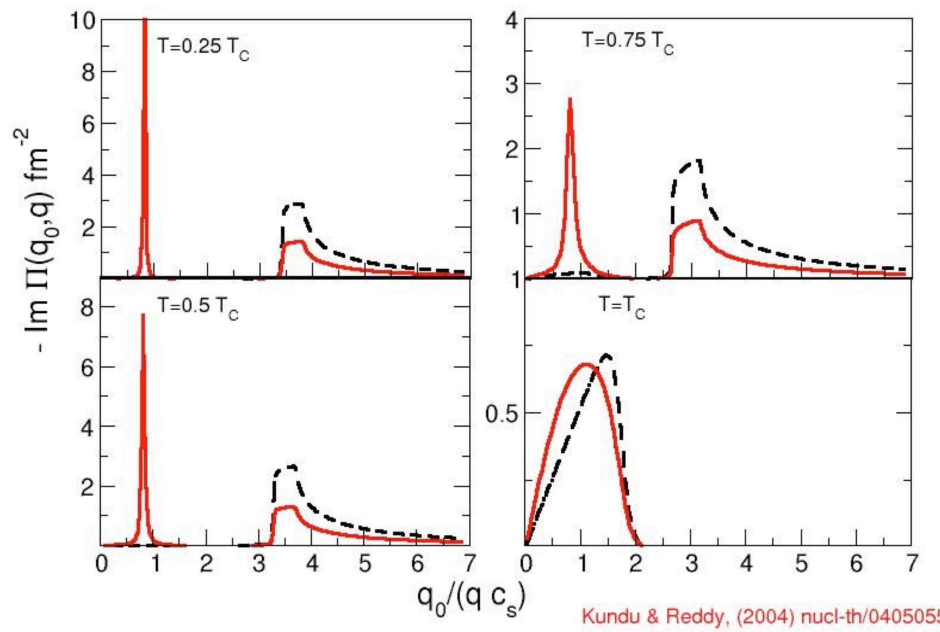
$$\sum_\mu q_\mu \Gamma_\mu = \tau_3^{NG} \tilde{G}^{-1}(p) - \tilde{G}^{-1}(p+q)\tau_3^{NG}$$

if $\lim_{q\to 0} \lambda(q)$ is finite

$\Gamma(q) = (\vec{q}, q_o)$ is singular at

$$q_o = c_s q$$

Goldstone mode



Bogoliubov, Nuovo Cimento, **7**, 6 (1958)
Anderson, Phys. Rev. **112**, 1900 (1958)
Nambu, Phys. Rev. **117**, 648 (1960)

Current Conservation

At q=0 we can rewrite vertex in terms of Greens functions

$$\omega\,\Gamma_0 = G^{-1}(p_0+\omega,\,p)\,\tau_3 - \tau_3\,G^{-1}(p_0,\,p)$$

Response Function vanishes when we use this vertex function as required by current conservation.

In the low temperature limit the vertex function is:

$$\Gamma_0 \cong \tau_3 + i\tau_2\Delta\,\frac{\omega}{\omega^2 - c_s^2 q^2}$$

Nambu (1960)

This suppresses the rate by a factor of c_s^4 or v_F^4.

Leinson & Perez (2006)

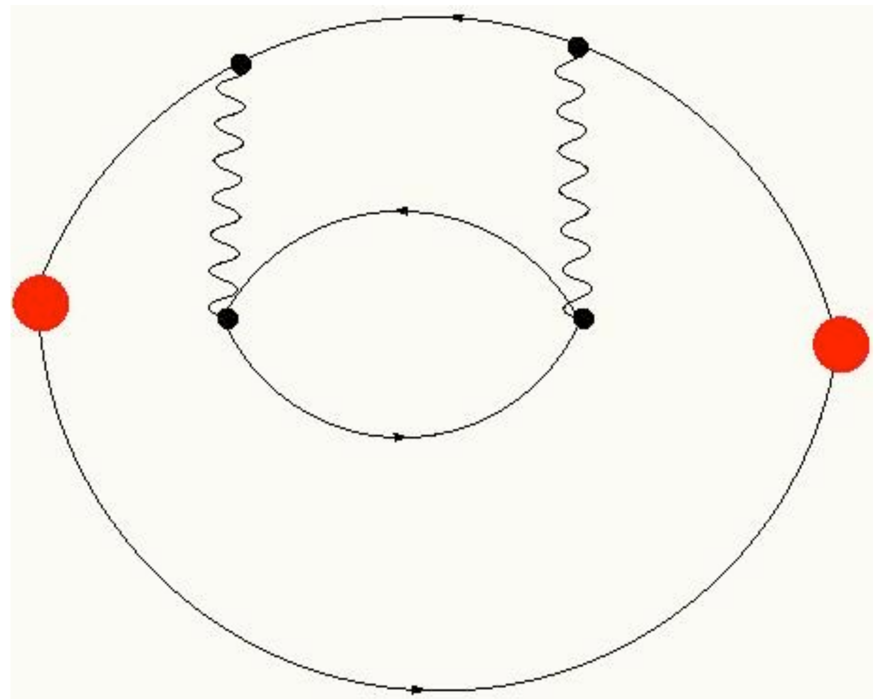
How will tensor forces change this result ?

Work in progress:

Reddy & Steiner

Need to calculate the "bremsstrahlung" diagram in the superfluid.

When T~ Δ expect a rate similar to that in the normal phase.



Conclusions

Neutrino emission due to Cooper-pair recombination is highly suppressed. As calculated in Leinson and Perez (2006).

Suppression is due to current conservation. Need to accelerate particles in order to radiate.

Need to calculate how tensor forces will influence the spin response in the presence of superfluidity. Expect a rate similar to Friman & Maxwell for $T \sim \Delta$.

Need to explore neutrino reactions involving nuclei in the crust.

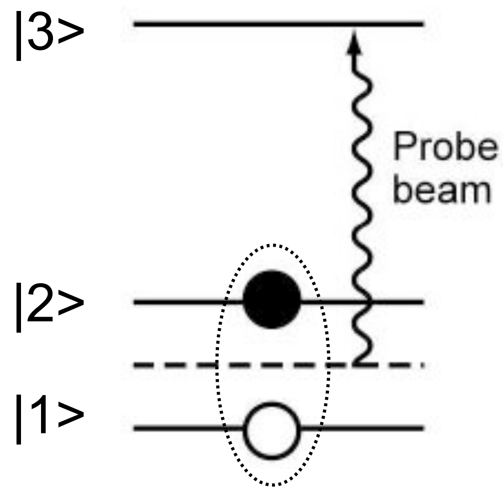
Bob's question:

Neutrino rates in matter can differ by factors of a few but if larger factors are proposed - be skeptical - ask lots of questions.

Forget about PBF in the vector channel.

Response Functions From Cold-Atom Expt.

Laser probe can be tuned to produce specific transitions



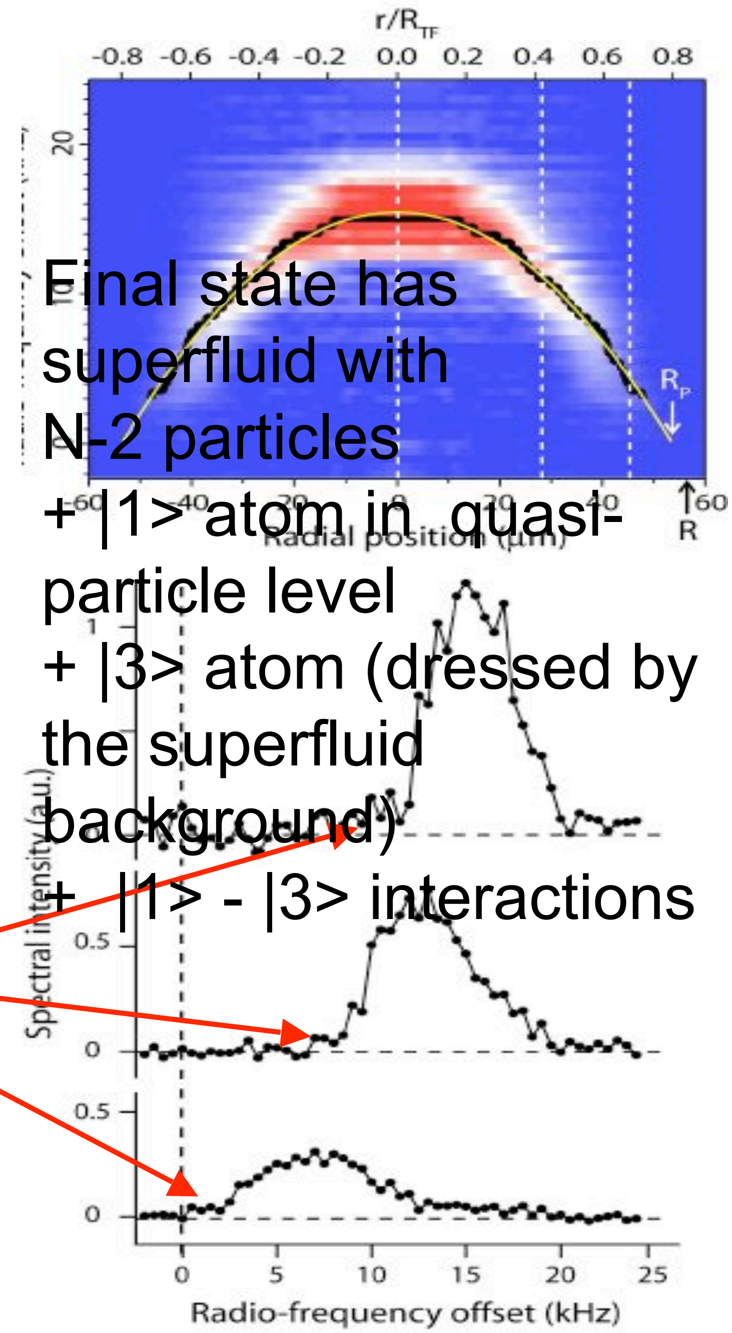
Example: |2>→|3> Transition

Threshold has information about the gap:

$\omega_{th} \sim 0.3\ \varepsilon_F$
Implies
$\Delta \sim 0.5\ \varepsilon_F$

$$S(\omega) \propto \int d^3k\ n(k)\ \delta(\omega - \tilde{\omega}_k)$$

$$\tilde{\omega}_k = E_{QP}(k) + E_3(-k) + V_{13} - \mu$$



Final state has superfluid with N-2 particles
+ |1> atom in quasi-particle level
+ |3> atom (dressed by the superfluid background)
+ |1> - |3> interactions

Pairing in Fermi Systems

- Electronic Superconductors :(Δ~10^{-3} eV)/(E_F~10 eV) ~ 10^{-4}
- Nuclei and Nuclear Matter : (Δ~1 MeV)/(E_F~10 MeV) ~ 10^{-1}
- Dense Quark Matter: (Δ~100 MeV)/(E_F~400 MeV) ~ 1/4

Cold atom experiments (^{6}Li and ^{40}K atoms) can tune the interaction through Feshback resonances. Explore BCS, BEC and the cross-over region !

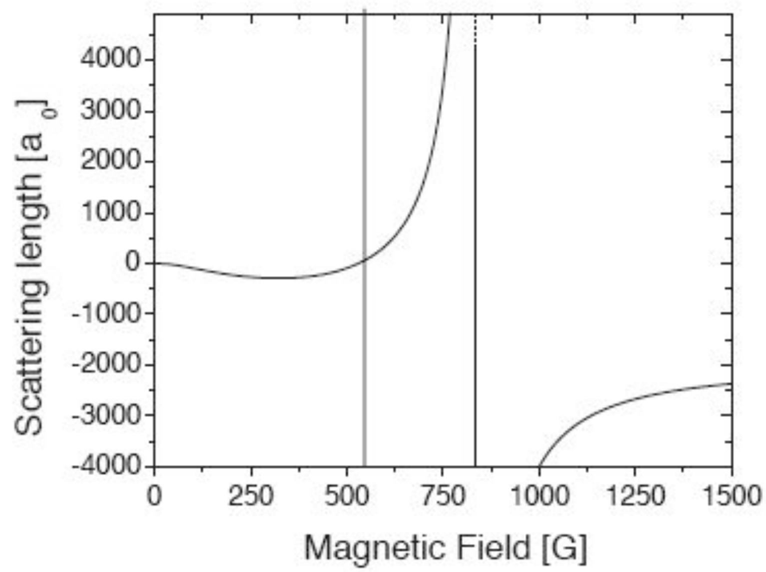
Several Groups: Hulet et al. (Rice); Ketterle et al. (MIT); Thomas et al. (Duke); D. Jin (Boulder).

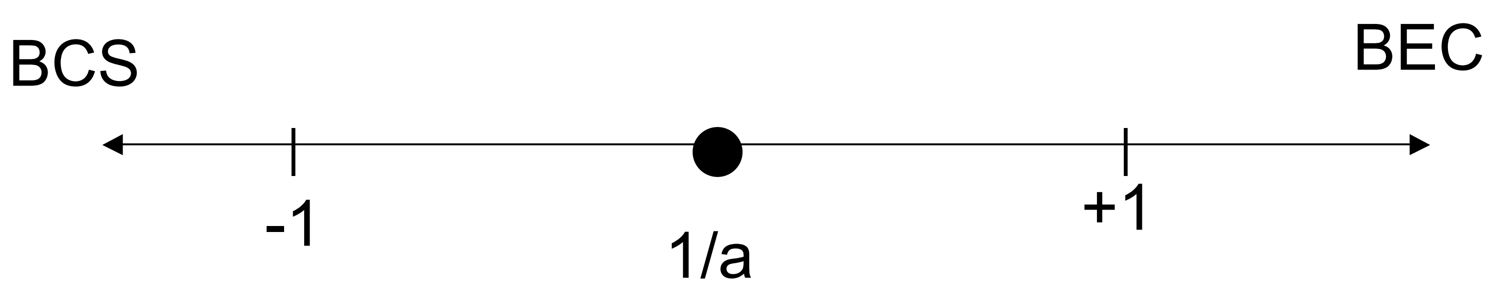
Universal System: Unitary Fermi Gas

Strongly-Coupled Fermions with short-range interactions

$$\mathcal{H} = \sum_{k=1}^{A} \left(-\frac{\hbar^2}{2m_k}\nabla_k^2\right) + \sum_{i<j} v(r_{ij})$$

	Cold Fermi A
scattering Length (a)	tunable
Effective range (r_o)	0





Universal Constants at a=∞

$k_F=(3\pi^2\rho)^{1/3}$ is the only scale in the problem.

$$\mu = \xi\,\varepsilon_F = \xi\,\frac{k_F^2}{2m}$$

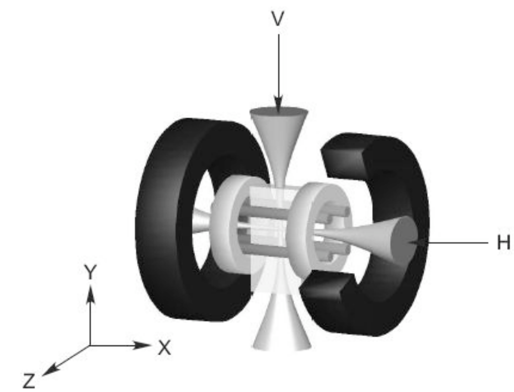
$$P = \xi\,P_{FG}$$

$$\Delta = \eta\,\varepsilon_F$$

Experiment can measure ξ and η

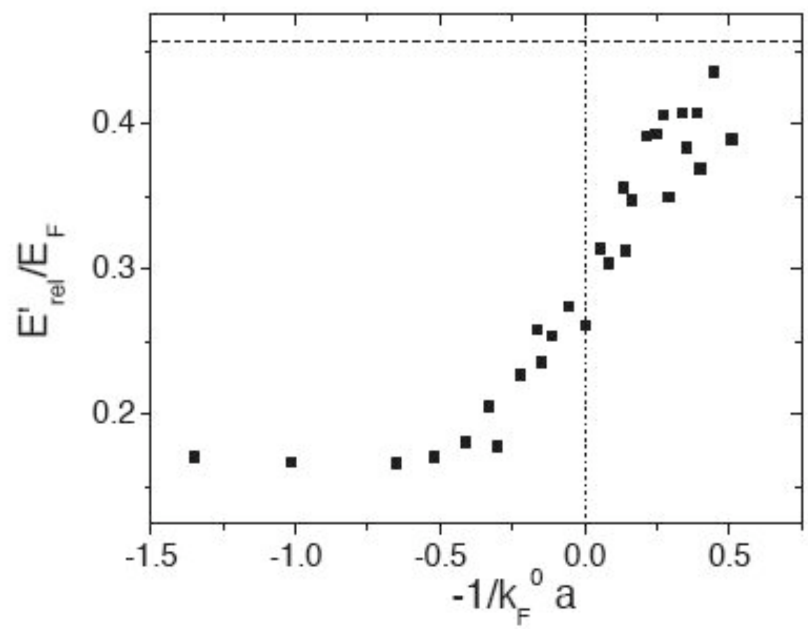
Measuring ξ from Energy Release

Magnetic trap creates a harmonic oscillator potential to trap atoms: 10^6-10^7 atoms in ~ 100 μm^3



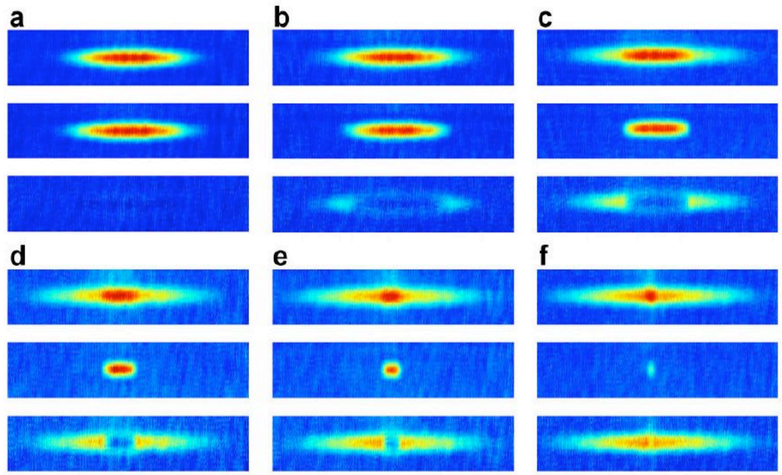
ξ	Expt
0.51 (4)	Kinast, et al., Science (2005)
0.32 (+.13,-.1)	Bartenstein, et al., PRL (2004)
0.36(15)	Bourdel, et al., PRL (2004)
0.46(5)	Partridge, et al., PRL (2004)
0.45(5)	Stewart, et al., PRL (2006)
0.41(15)	Tarruell, et al., cond-mat/0701181

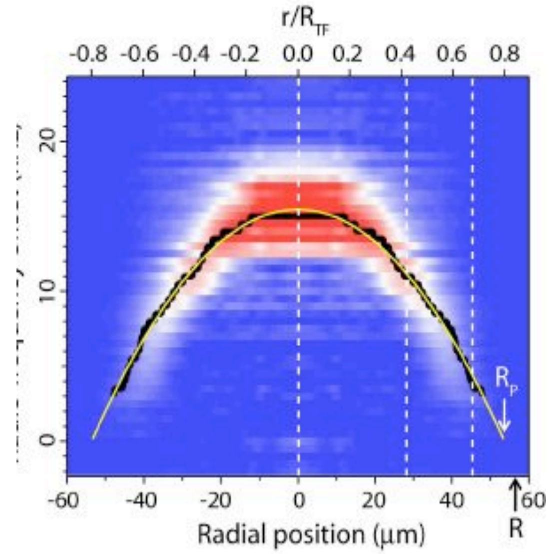
Ioffe-Prichard Trap

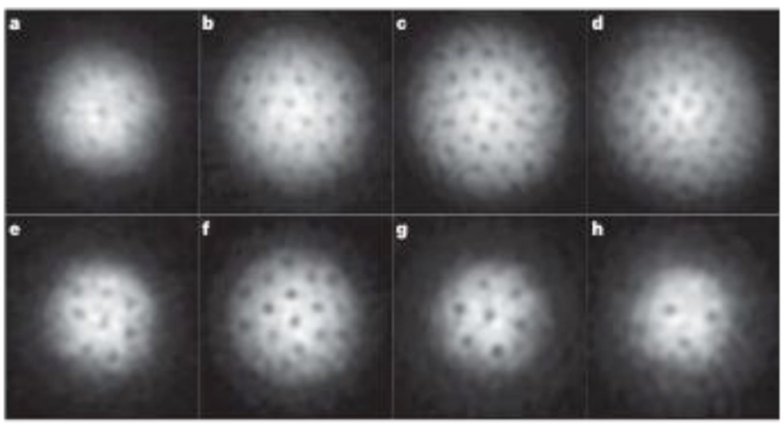


Rich Set of Experimental Results

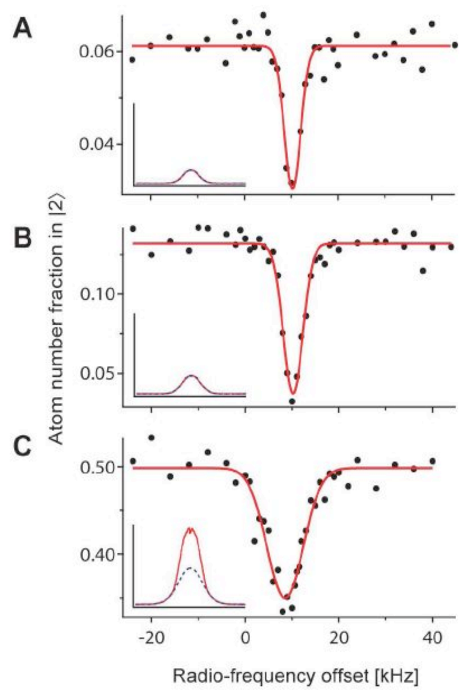
Radial
Density
and
polarization

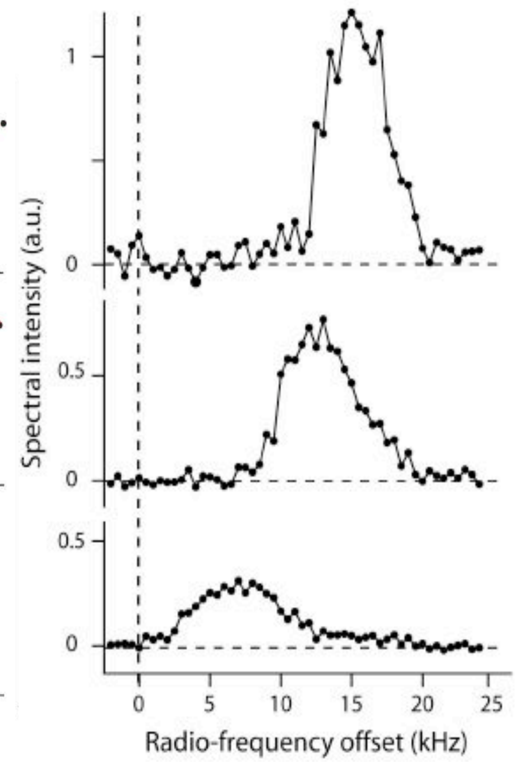






Vortices





RF response