Time-Correlated Structure in Pulsar Timing Noise

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The Goal: To Identify Evidence of Non-rigid Body Rotation

- Timing Noise: Response of the Crust to Stochastic Torque
- All modes are excited (e.g. damped rotational modes, precession, vortex lattice modes)
- Previous studies using Fourier techniques have found no evidence of non-rigid body rotation (Boynton, Deeter, 1969; Boynton, 1981; Boynton, Deeter, et. al., 1984)

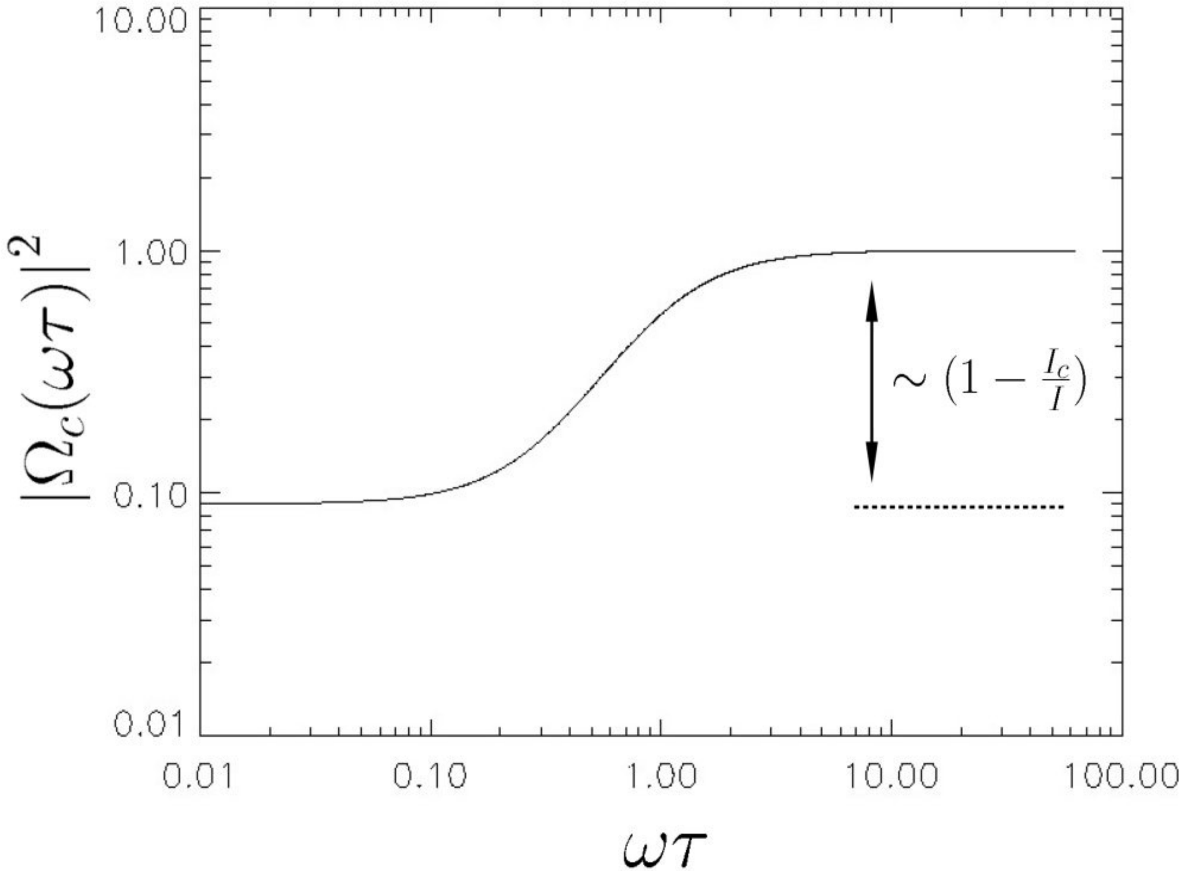
2-Component NS Model

$$I_c\dot{\Omega}_c = N(t) - \frac{I_r}{\tau}(\Omega_c - \Omega_s)$$

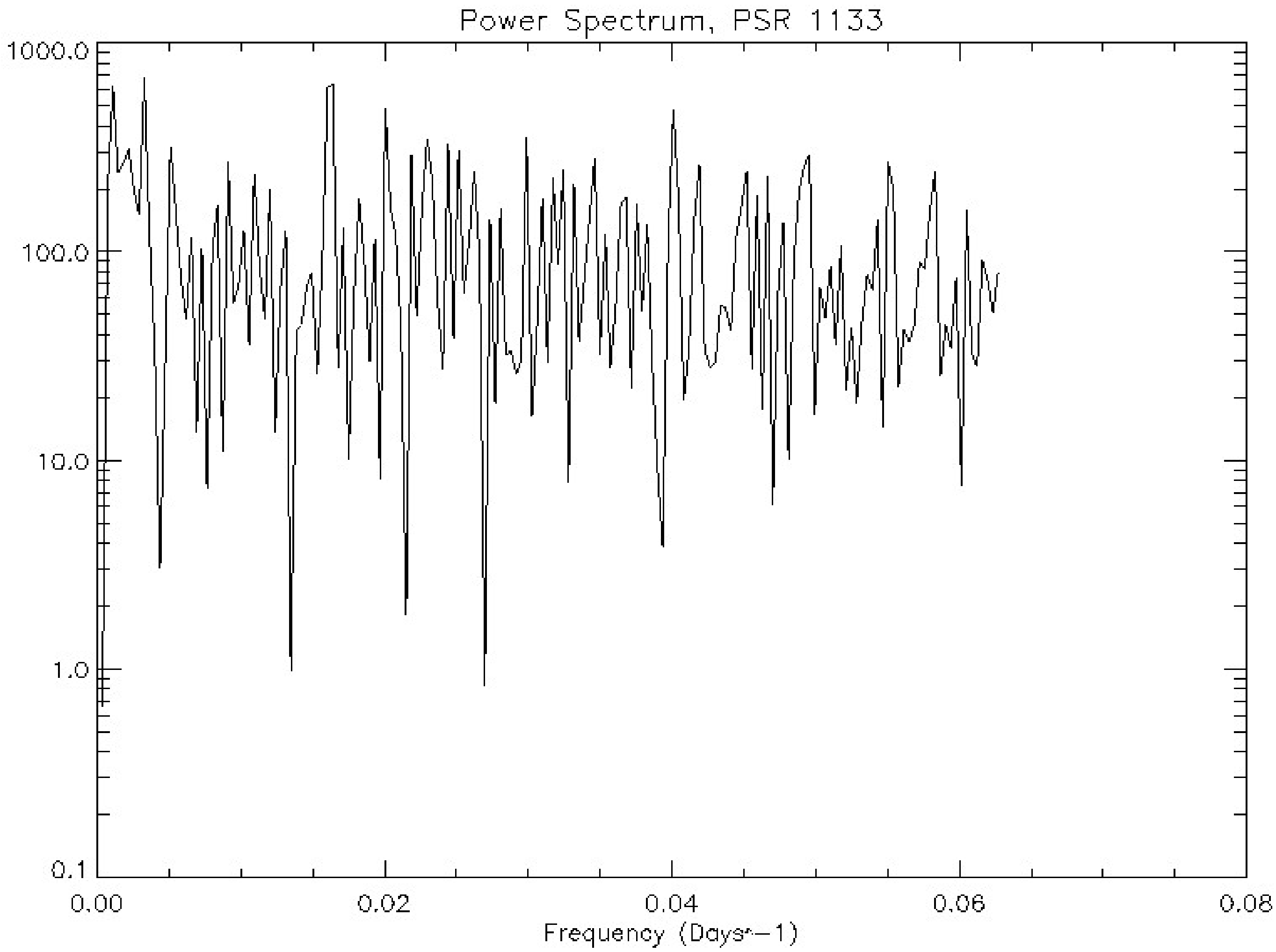
$$I_s\dot{\Omega}_s = \frac{I_r}{\tau}(\Omega_c - \Omega_s)$$

$$|\Omega_c(\omega)|^2 = [\frac{(\omega\tau)^4 + (\frac{I_c}{I})^2(\omega\tau)^2}{(\omega\tau)^4 + (\omega\tau)^2}]\frac{|N(\omega)|^2}{\omega^2\ I_c^2}$$

Frequency Response



Observations

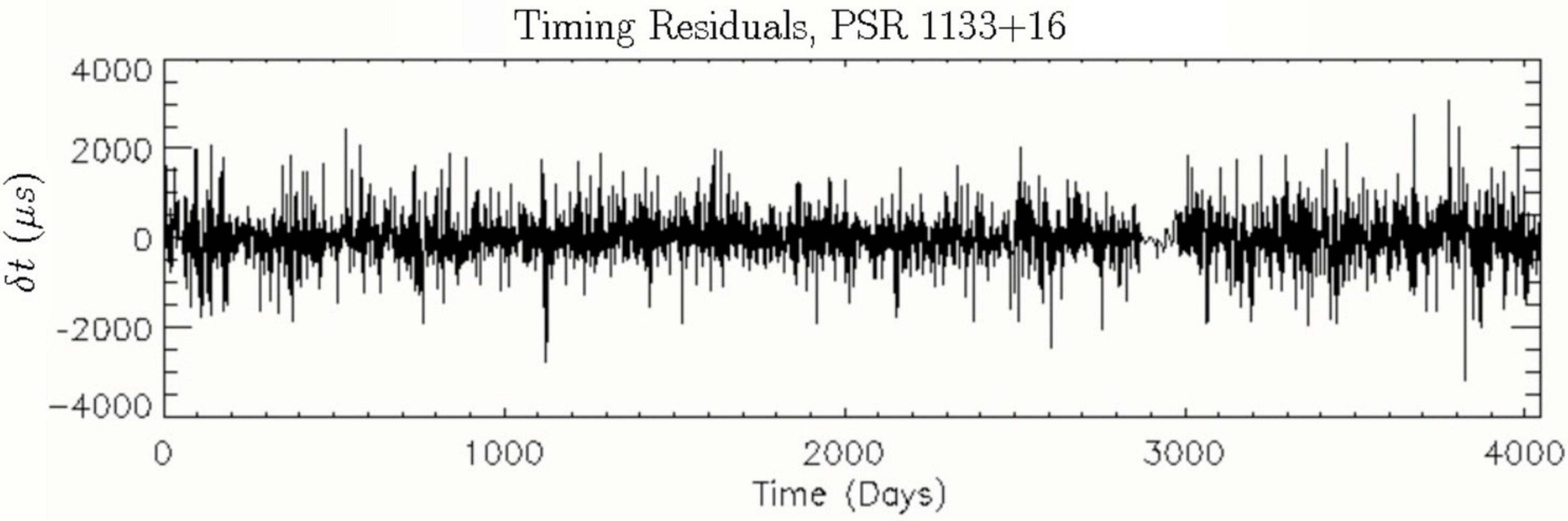


- We use methods in the time domain

PSR B1133+16

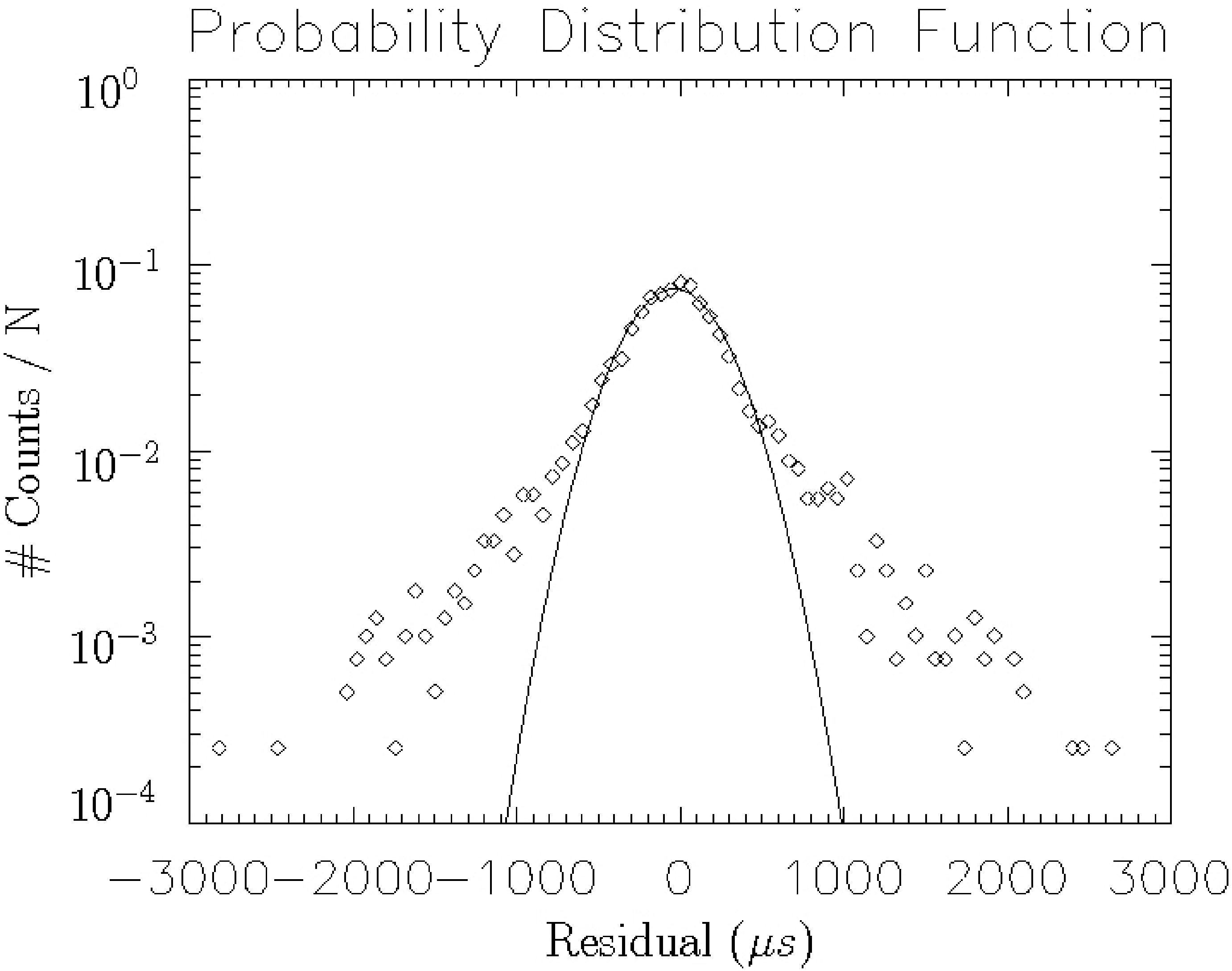
- Spin period: $P = 1.19\ s$
- Dipole field: $|B_d| = 5 \times 10^{12}\ G$
- Spin-down age: $\tau = 5 \times 10^{6}\ yrs$

The Data



Timing Residuals, PSR 1133+16
4000
2000
0
-2000
−4000
δt (μs)
0
1000
2000
3000
4000
Time (Days)

Distribution of Residuals



Auto-Correlation Function

- For evenly sampled data:

$$ACF(k) = \frac{1}{(n-k)\,\sigma^2} \sum_{i=1}^{n-k} (y_i - \bar{y})(y_{i+k} - \bar{y})$$

- For unevenly sampled data: we use the discrete correlation function, DCF (Edelson, Krolik, 1988)

Discrete Correlation Function

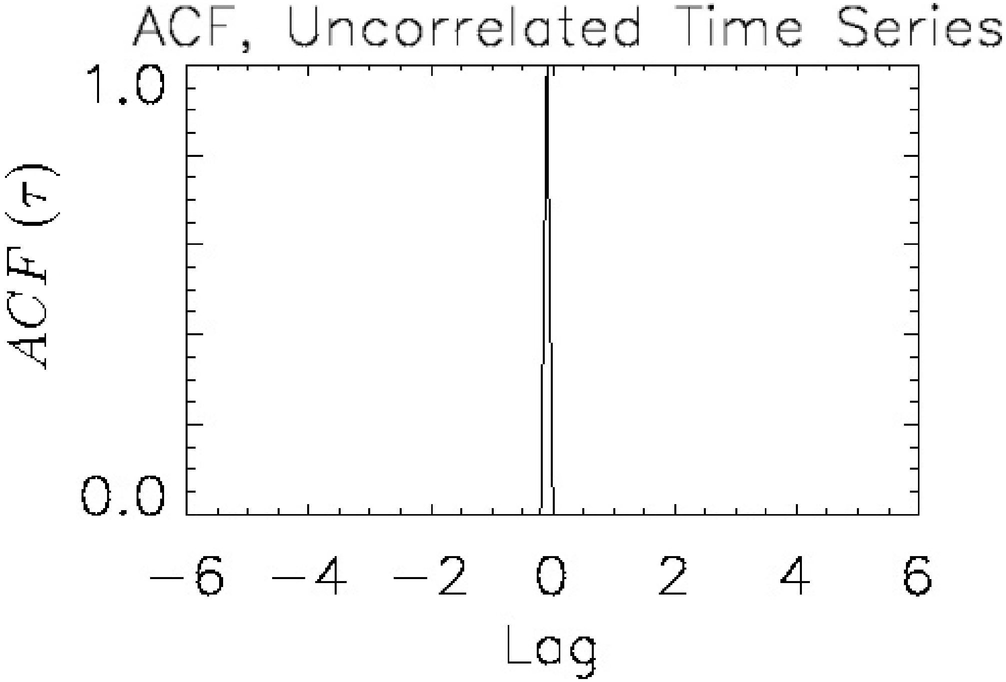
$$UDCF_{ij} = \frac{(y_i - \bar{y})(y_j - \bar{y})}{\sigma_y^2}$$

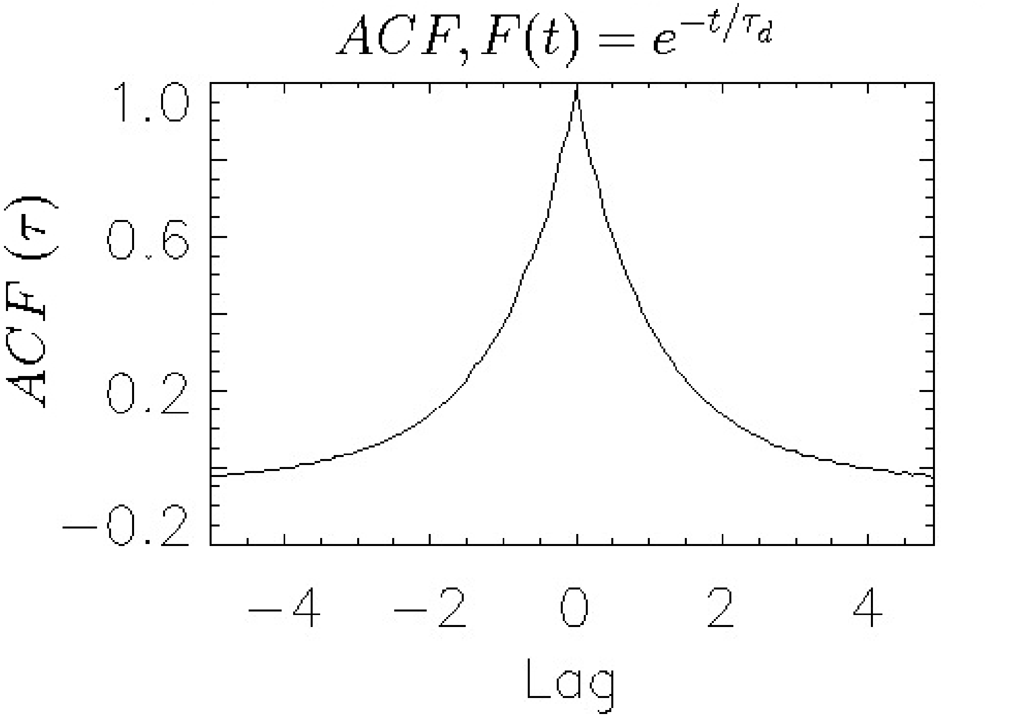
$$DCF(\tau) = \frac{1}{M} \sum_{i=1}^{M} UDCF$$

$$\tau - \frac{\Delta\tau}{2} \leq t_j - t_i \leq \tau + \frac{\Delta\tau}{2}$$

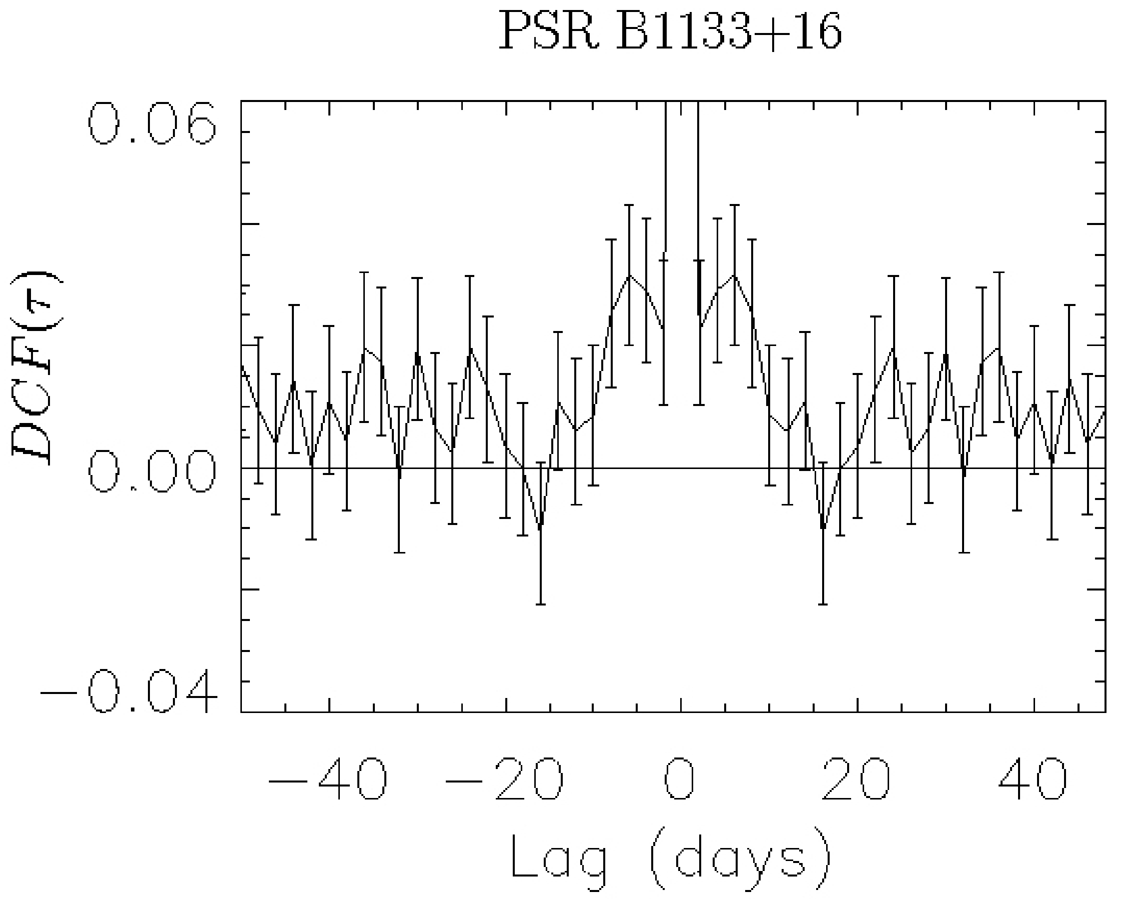
- Benefits
  - No binning required
  - Uses all data points
  - No penalty for gaps in the data

Auto-Correlation Function





Discrete Correlation Function



$$S_{corr} = DCF[1] \cdot DCF[2] \cdot DCF[3] \cdot DCF[4]$$
$$S_{corr} \simeq 26\ \sigma$$

Significance

- From Monte Carlo simulations, the probability of the null hypothesis is less than 10^-4
- Further confirmation: Lagged Dispersion Statistic

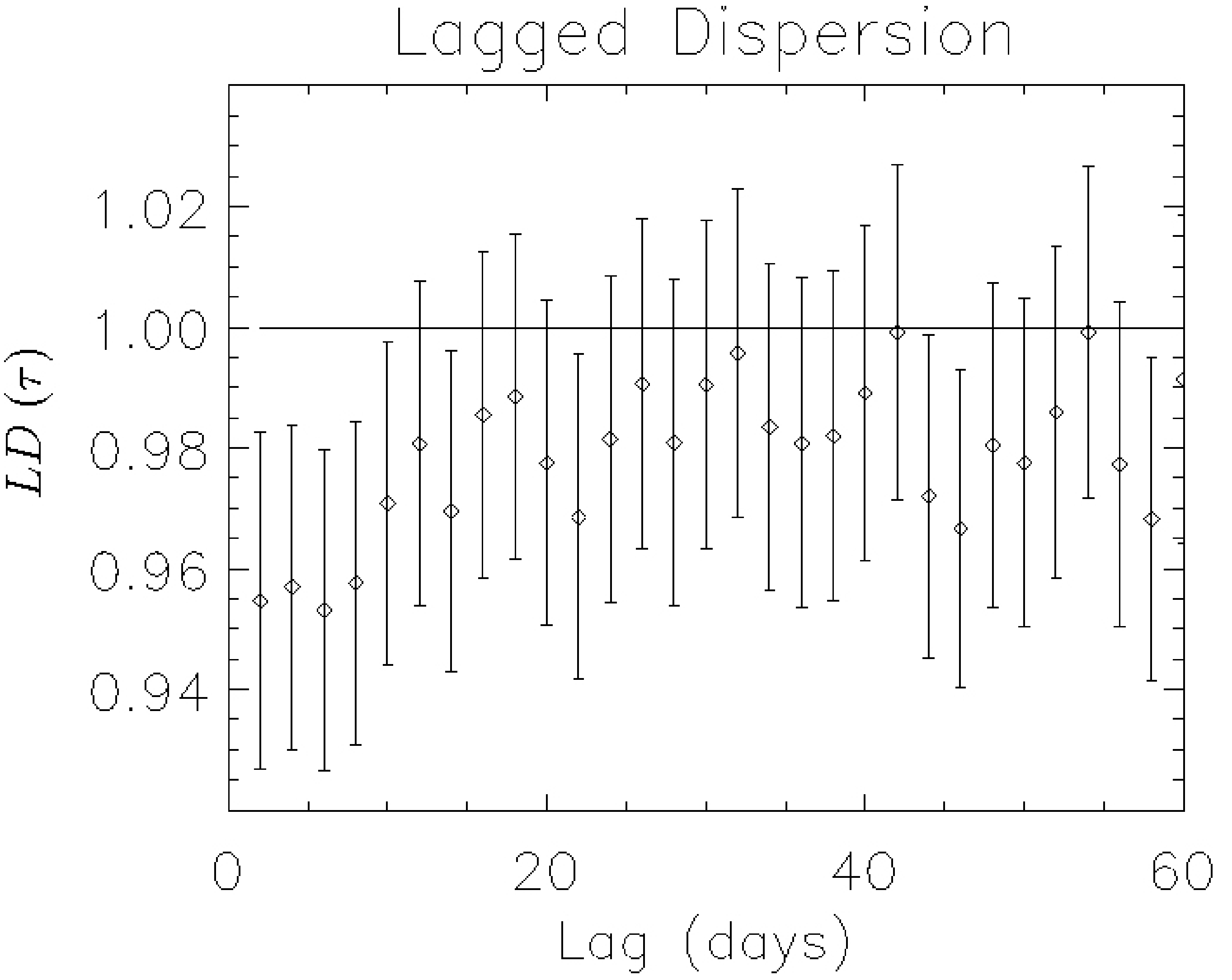
Lagged Dispersion Statistic

$$\Delta y_{ij} = y_j - y_i$$

$$\tau - \frac{\Delta\tau}{2} \leq t_j - t_i \leq \tau + \frac{\Delta\tau}{2}$$

$$LD(\tau) = \frac{1}{M}\Sigma(\Delta y_{ij} - \bar{\Delta y})^2$$

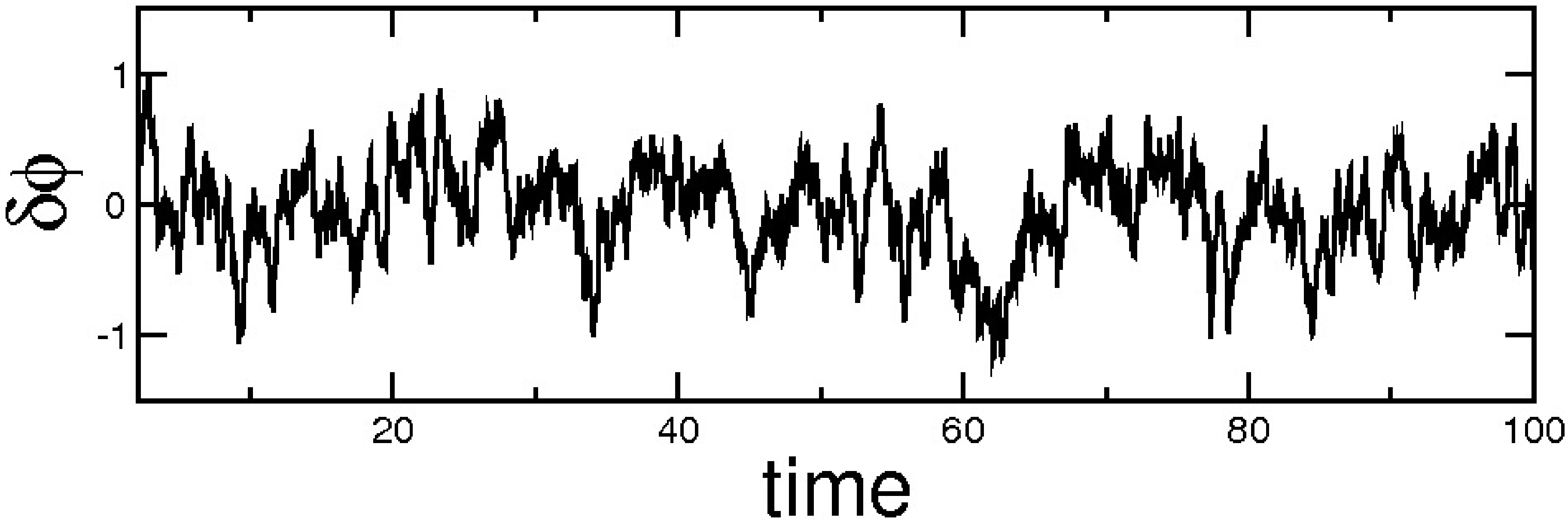
Lagged Dispersion

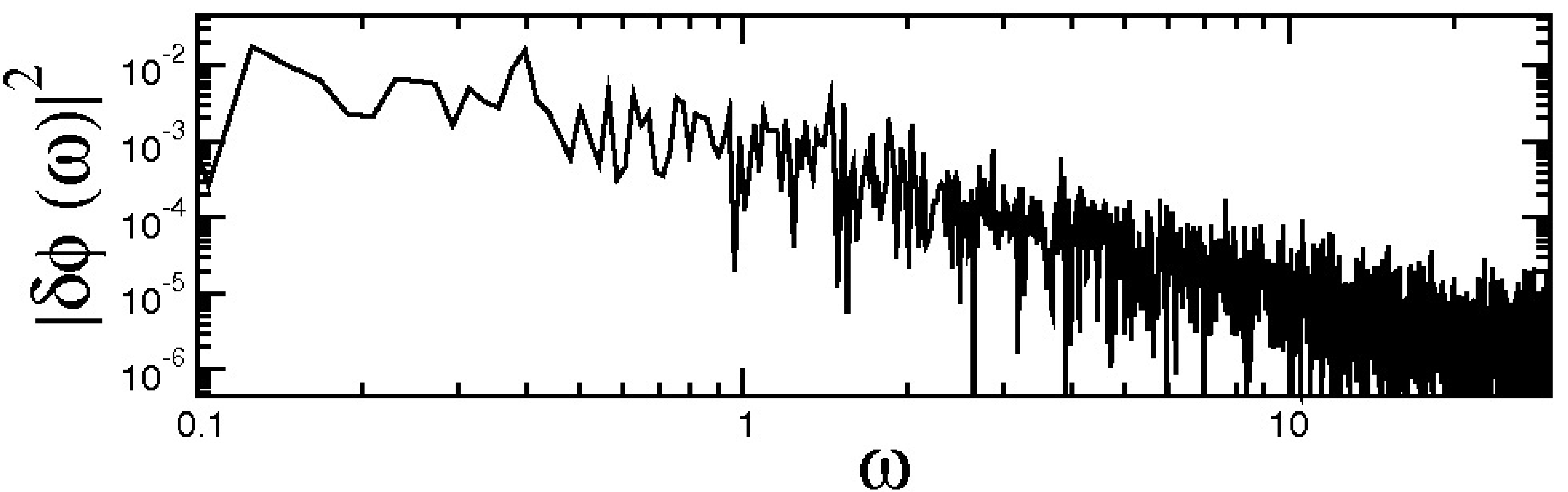


Conclusions

- We find correlation timescale of 10 days in PSR B1133+16
- Interpretation: Deviations from rigid body rotation
- DCF and LD (Time Domain) are better suited for analyzing noisy data than Fourier techniques

Error Sensitivity Simulation





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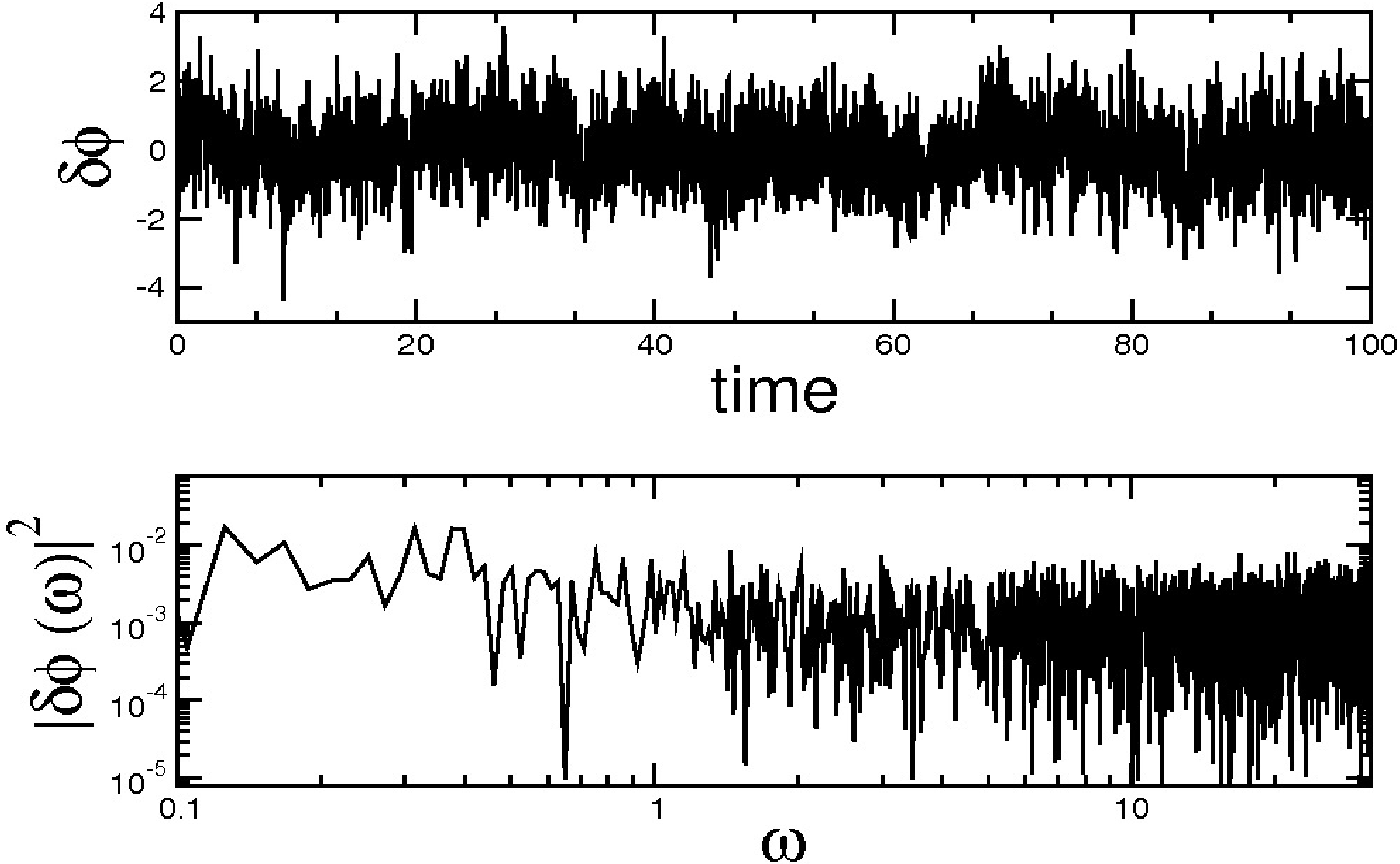

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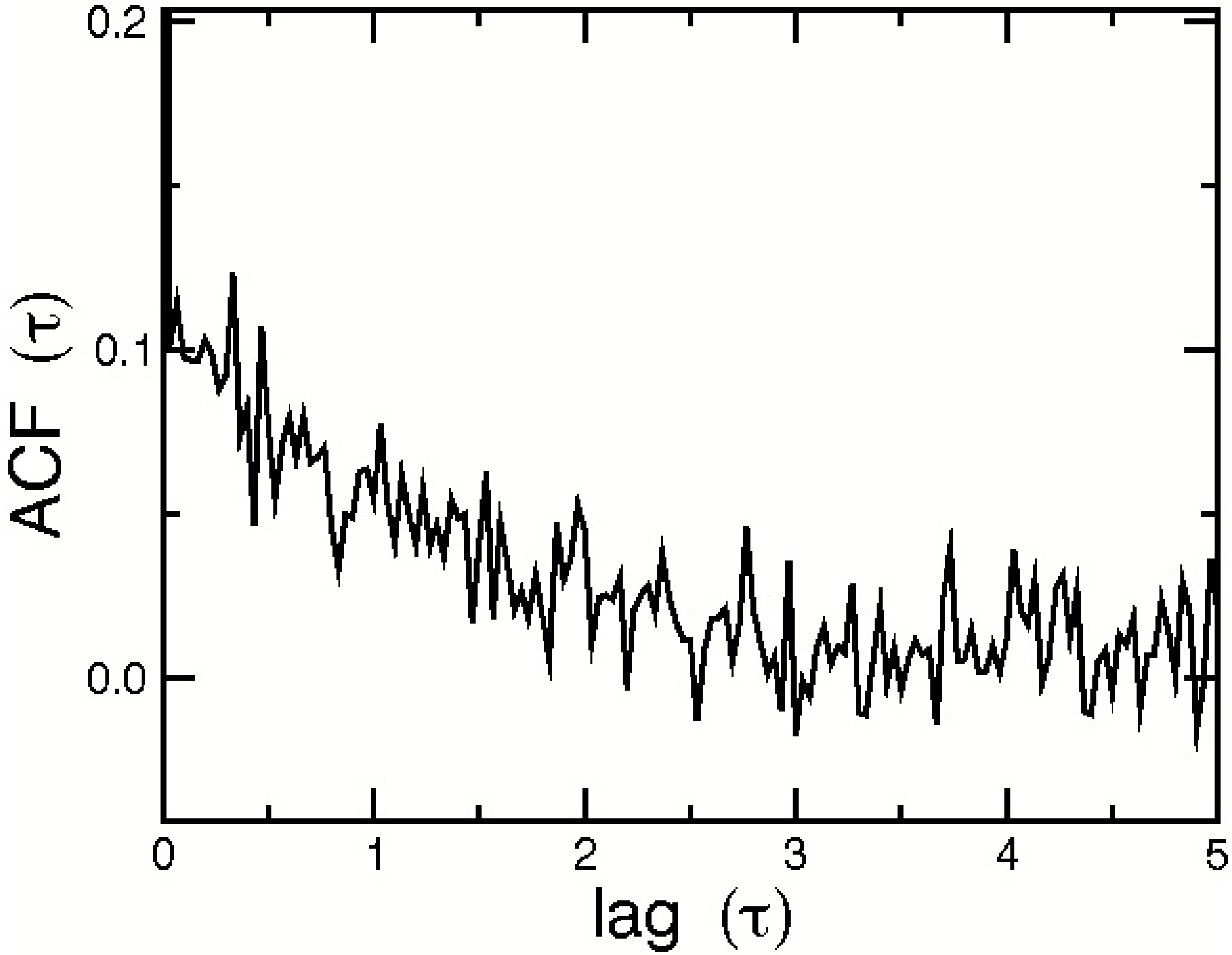

Future Work

$$I_c\dot{\Omega}_c = N(t) - \frac{I_r}{\tau}(\Omega_c - \Omega_s)$$

$$I_s\dot{\Omega}_s = \frac{I_r}{\tau}(\Omega_c - \Omega_s)$$

- Constrain values of moment of inertia ratio using our results