Thermal structure of magnetized neutron star envelopes

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- Conductive opacities, thermal structure, and cooling
- The effects of superstrong magnetic fields
- An application to magnetars

Basic estimates for thermal conductivities

In the "elementary theory" (with energy-independent effective frequency)

$$\varkappa = a\,\frac{n_e k^2 T}{m_e^* \nu}, \quad a = \left[\begin{array}{ll} 3/2 & (T \gg T_{\rm F}) \\ \pi^2/3 & (T \ll T_{\rm F}) \end{array}\right.$$

$$m_e^* = m_e \gamma_{\rm r}, \quad \gamma_{\rm r} = \sqrt{1+x_{\rm r}^2}, \quad x_{\rm r} = p_{\rm F}/m_e c = 0.01009\,(\rho Z/A)^{1/3}$$

$$T_{\rm F} = \frac{m_e c^2}{k}(\gamma_{\rm r}-1) \qquad \left(\frac{m_e c^2}{k} = 5.93\times 10^9\ {\rm K}\right)$$

Matthiessen rule: $\nu = \nu_{ei} + \nu_{ee}$

$$\nu_{ei}+\nu_{ee} \le \nu \le \nu_{ei}+\nu_{ee}+\delta\nu, \quad \delta\nu \ll \min(\nu_{ei}, \nu_{ee})$$

For non-degenerate electron gas:

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\,\frac{Z^2 e^4}{(kT)^{3/2}}\,n_i \Lambda_{ei}, \quad \Lambda_{ei} \sim \ln\frac{r_{\rm max}}{r_{\rm min}}$$

$$r_{\rm max}^{-2} = 4\pi(n_e + Z^2 n_i)e^2/kT, \quad r_{\rm min} = \max(\lambda_T,\, r_{\rm cl}), \quad \lambda_T = \sqrt{\frac{2\pi\hbar^2}{m_e kT}}, \quad r_{\rm cl} = \frac{Ze^2}{kT}$$

$$\nu_{ee} = \frac{8}{3}\sqrt{\frac{\pi}{m_e}}\,\frac{e^4}{(kT)^{3/2}}\,n_e \Lambda_{ee}$$

For strongly degenerate electron gas:

Electron-ion scattering

[Potekhin, Baiko, Haensel, Yakovlev (1999) *A&A*, **346**, 345]

$$\nu_{ei} = \frac{4\pi Z_i^2 e^4}{p_F^2 v_F} n_i \Lambda_{ei} \qquad v_F = \frac{p_F}{m_e^*} = c\,\frac{x_r}{\gamma_r} = c\,\beta$$

Electron-electron scattering

[Shternin & Yakovlev (2006) *PRD*, **74**, 043004]

$$kT_p = \hbar\omega_p = \hbar\sqrt{4\pi e^2 n_e / m_e^*} \qquad y = \sqrt{3}\,T_p/T = (571.6/T_6)\sqrt{\beta}\,x_r$$

$$\nu_{ee} = \frac{m_e c^2}{\hbar}\frac{6\alpha_f^{3/2}}{\pi^{5/2}} x_r y\sqrt{\beta}\, I(\beta,y) = 1.66\times10^{17} x_r y \sqrt{\beta}\, I(\beta,y)\ \mathrm{s}^{-1}$$

$$I(\beta,y) = \frac{1}{\beta}\left(\frac{10}{63} - \frac{8/315}{1+0.0435y}\right)\ln\left(1+\frac{128.56}{37.1y+10.83y^2+y^3}\right) + \beta^3\left(\frac{2.404}{B}+\frac{C-2.404/B}{1+0.1\beta y}\right)\ln\left[1+\frac{B}{A\beta y+(\beta y)^2}\right] + \frac{\beta}{1+D}\left(C+\frac{18.52\beta^2 D}{B}\right)\ln\left[1+\frac{B}{Ay+10.83(\beta y)^2+(\beta y)^{8/3}}\right]$$

$$A = 12.2 + 25.2\,\beta^3 \qquad C = 8/105 + 0.05714\,\beta^4$$

$$B = A\exp[(0.123636 + 0.016234\,\beta^2)/C] \qquad D = 0.1558\,y^{1-0.75\,\beta}$$

Partially degenerate electron gas

Electron-ion scattering in arbitrary magnetic field

[e.g., Potekhin (1999) *A&A*, **351**, 787]

$$\vec{j}_e = \sigma\cdot\vec{E}^* - \alpha\cdot\nabla T, \quad \vec{j}_T = \tilde{\alpha}\cdot\vec{E}^* - \tilde{\kappa}\cdot\nabla T, \quad \vec{E}^* = \vec{E} + \nabla\mu/e$$

$$\tilde{\alpha}_{ij}(\boldsymbol{B}) = k^2 T\alpha_{ji}(-\boldsymbol{B}) = k^2 T\alpha_{ji}(\boldsymbol{B}) \qquad \boxed{\varkappa = \tilde{\kappa} + k^2 T\alpha\cdot\sigma^{-1}\cdot\alpha}$$

$$\begin{bmatrix}\sigma_{ij}\\ \alpha_{ij}\\ \tilde{\kappa}_{ij}\end{bmatrix} = \int \begin{bmatrix} e^2 \\ e(\mu-\epsilon)/T \\ (\mu-\epsilon)^2/T\end{bmatrix} \frac{\mathcal{N}_B(\epsilon)}{m_e^*(\epsilon)}\tau_{ij}(\epsilon)\left(-\frac{\partial f^{(0)}}{\partial\epsilon}\right)d\epsilon \qquad \mathcal{N}_B(\epsilon) = \frac{m_e\omega_c}{2(\pi\hbar)^2}\sum_{n=0}^{n_{\max}} g_n p_n(\epsilon)$$

$$p_n(\epsilon) = [(\epsilon/c)^2 - (m_e c)^2 - 2m_e\hbar\omega_c n]^{1/2}$$

$$\tau_{zz} = \tau_\parallel, \quad \tau_{xx} = \frac{\tau_\perp}{1+(\omega_g\tau_\perp)^2}, \quad \tau_{yx} = \frac{\omega_g\tau_\perp^2}{1+(\omega_g\tau_\perp)^2} \qquad n_e = \int \mathcal{N}_B(\epsilon)\left(-\frac{\partial f^{(0)}}{\partial\epsilon}\right)d\epsilon$$

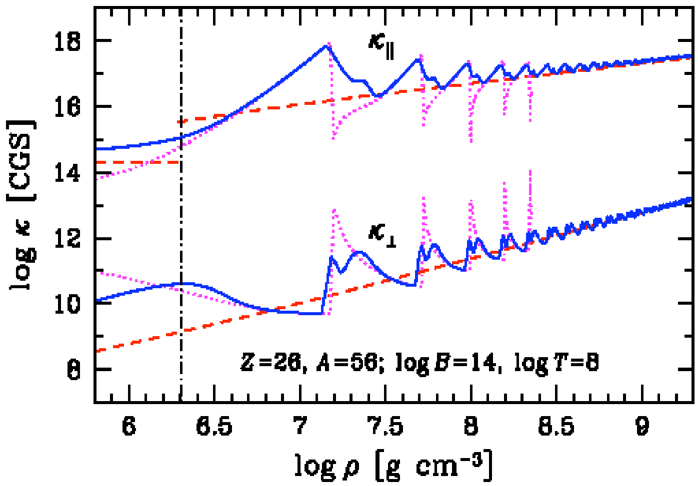
Particular case: no magnetic field

$$\boxed{\varkappa = k^2 T(\sigma_2 - \sigma_1^2/\sigma_0)} \qquad \sigma_n = \int \frac{\chi^n}{\nu_{ei}(\epsilon)}\frac{\mathcal{N}_0(\epsilon)}{m_e^*(\epsilon)}\frac{e^\chi}{(e^\chi+1)^2}d\chi \qquad \chi = \frac{\epsilon-\mu}{kT}$$

$$\mathcal{N}_0(\epsilon) = p^3/(3\pi^2\hbar^3) \qquad m_e^*(\epsilon) = \sqrt{m_e^2 + (p/c)^2}$$

Thermal conductivities in a strongly magnetized envelope

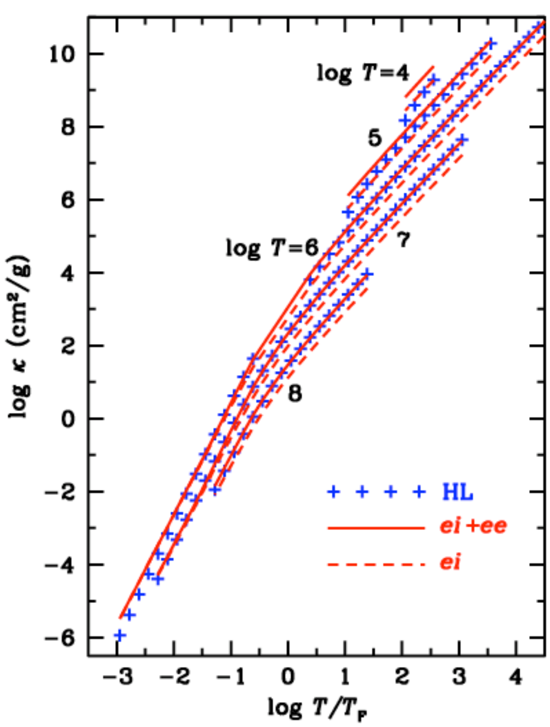
http://www.ioffe.ru/astro/conduct/

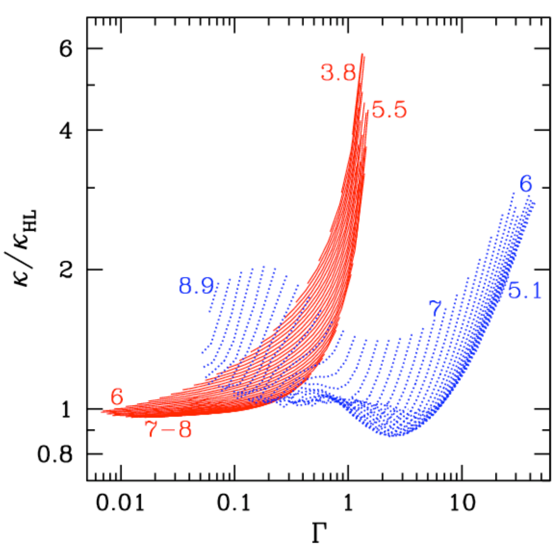


Solid – exact, dots – without T-integration, dashes – magnetically non-quantized

[Ventura & Potekhin (2001), in *The Neutron Star – Black Hole Connection,* ed. Kouveliotou *et al*. (Dordrecht: Kluwer) 393]

UPDATED ! – Cassisi, Potekhin, Pietrinferni, Catelan, & Salaris (2007) *ApJ* **661**, 1094

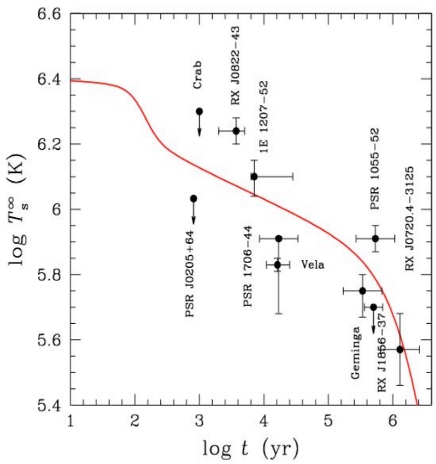




Conductive opacities of helium as functions of degeneracy (left) and Coulomb coupling parameter (right): comparison to Hubbard & Lampe tables [Cassisi, Potekhin, Pietrinferni, Catelan, & Salaris (2007) *ApJ* **661**, 1094]

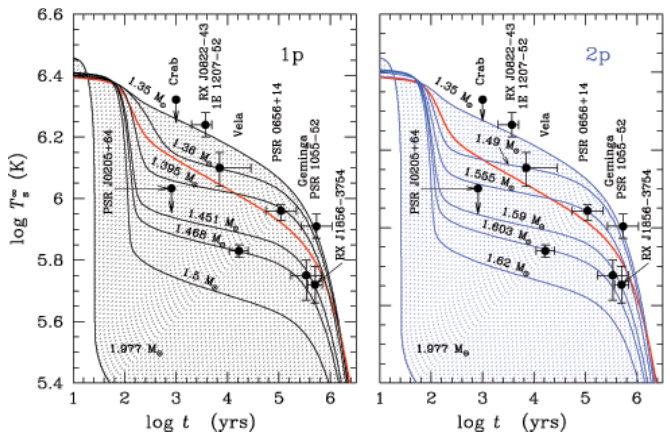
Thermal evolution

"Basic cooling curve"
of a neutron star
(no superfluidity, no exotica)



Neutron star cooling
[Yakovlev *et al.* (2005) *Nucl. Phys. A* **752**, 590c]

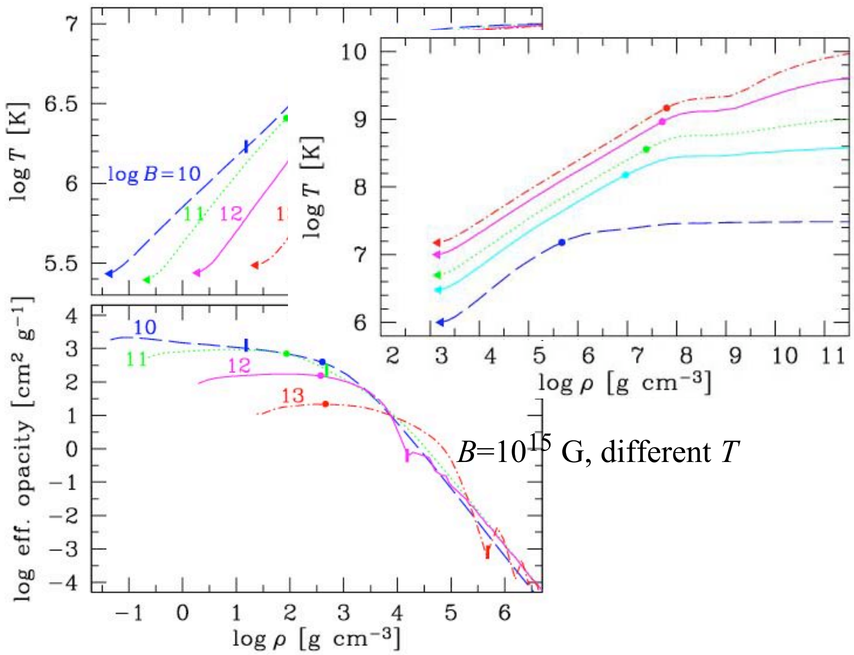
Cooling of neutron stars
with proton superfluidity in the cores



Cooling of neutron stars
with nucleon and exotic cores

[based on Yakovlev *et al.* (2005)
Nucl. Phys. A **752**, 590c]

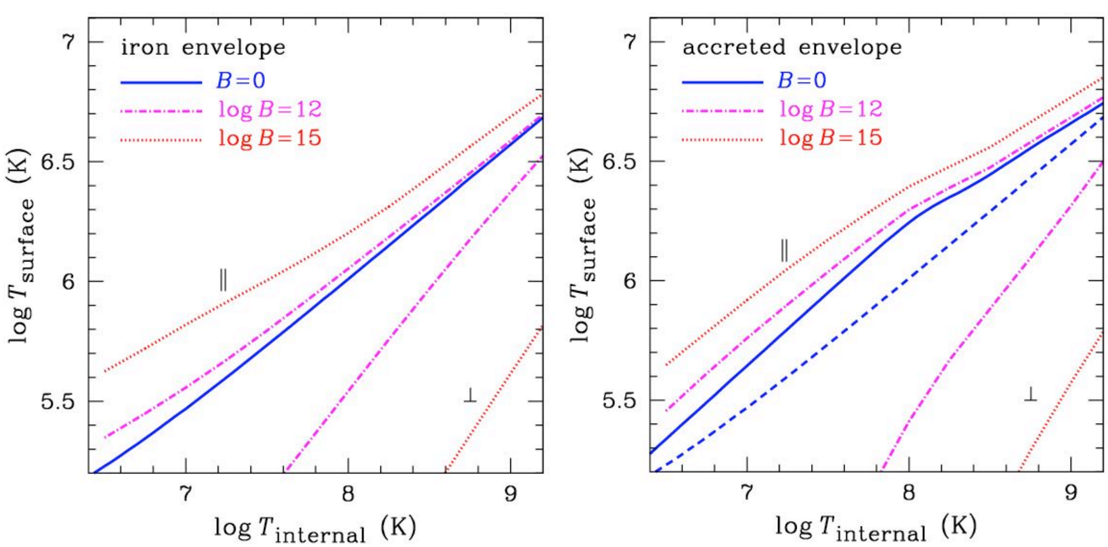
Thermal structure with a magnetic field



B=10^{15} G, different T

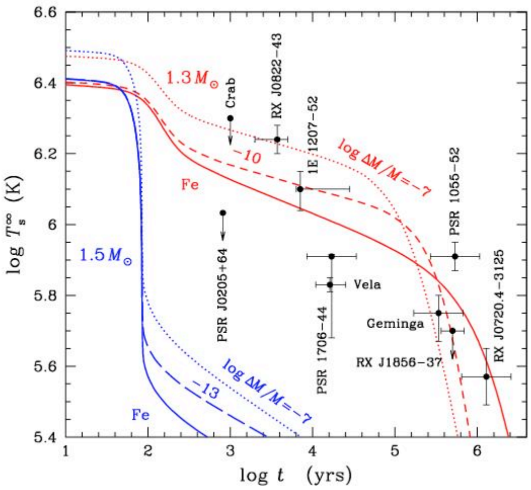
T_b=10^7 K, different B

Temperature drops in magnetized envelopes of neutron stars

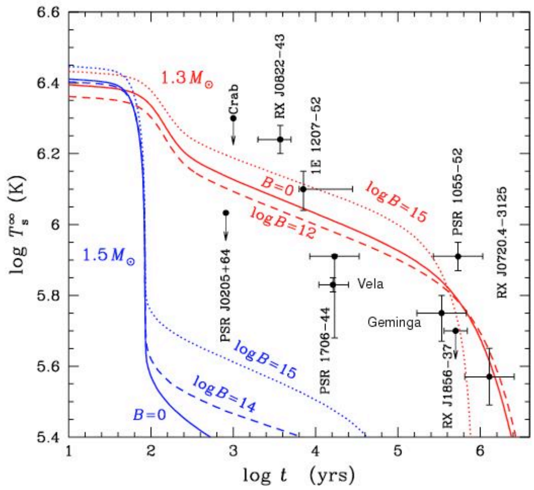


[based on Potekhin, Yakovlev, Chabrier, & Gnedin (2003) *ApJ* **594**, 404]

Cooling of neutron stars
with ***accreted envelopes***

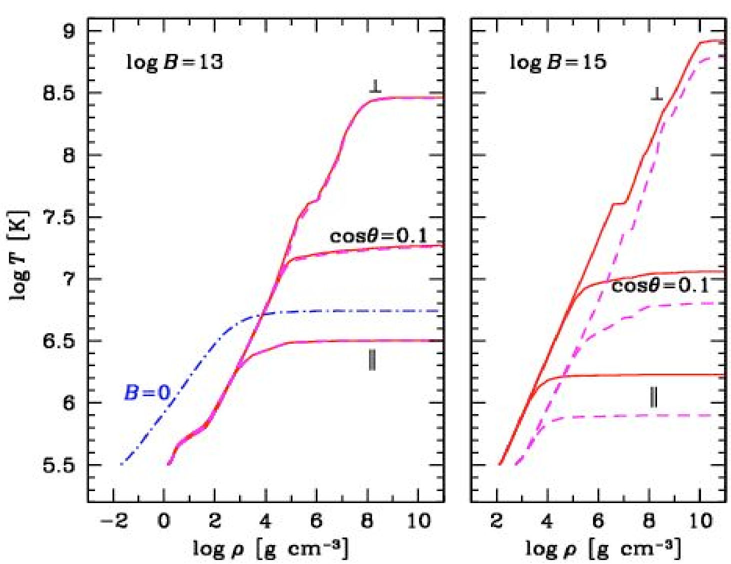


Cooling of neutron stars
with ***magnetized envelopes***



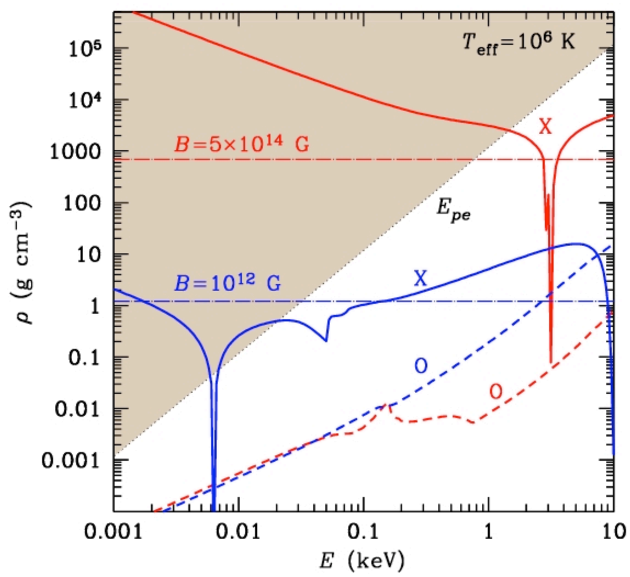
[Chabrier, Saumon, & Potekhin (2006) *J.Phys.A: Math. Gen.* **39**, 4411;
used data from Yakovlev *et al.* (2005) *Nucl. Phys. A* **752**, 590c]

Superstrong fields: Energy transport below the plasma frequency may affect the temperature profile and T_s



Temperature profiles in the accreted envelope of a neutron star with “ordinary” (left panel) and superstrong (right) magnetic field, for the local effective temperature $10^{5.5}$ K, with (solid lines) and without (dashed lines) plasma-frequency cut-off

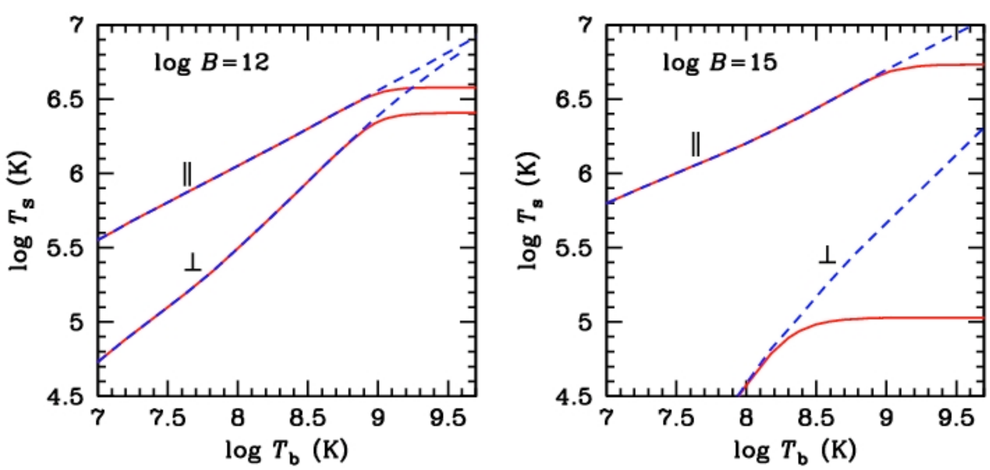
[Potekhin, Yakovlev, Chabrier, & Gnedin (2003) *ApJ* **594**, 404]



Photon-decoupling densities for X- and O-modes for a partially ionized H amosphere, for magnetic field strengths typical of pulsars (blue lines) and magnetars (red lines).

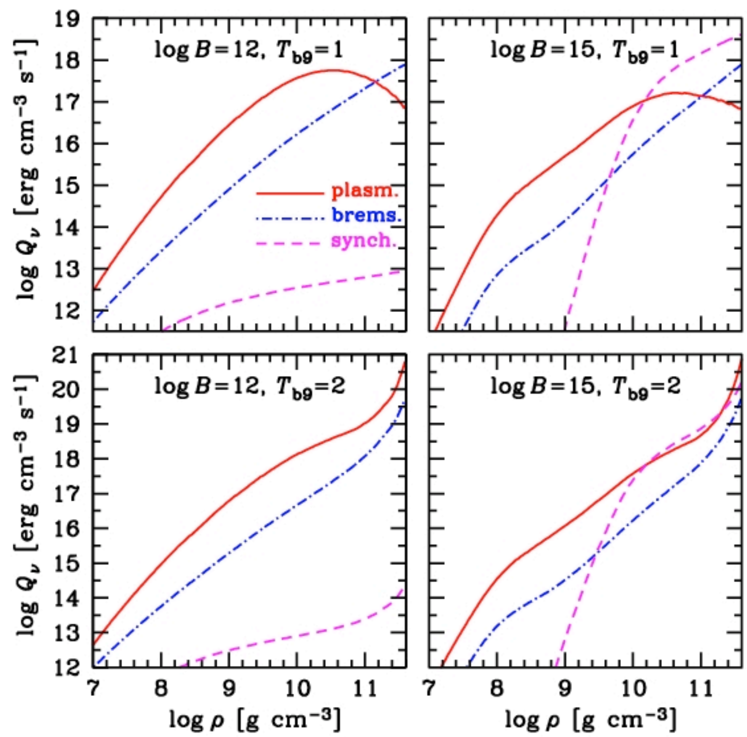
Dot-dashed lines correspond to the radiative surface, the shadowed region corresponds to $E < E_{pl}$.

$T_s - T_b$



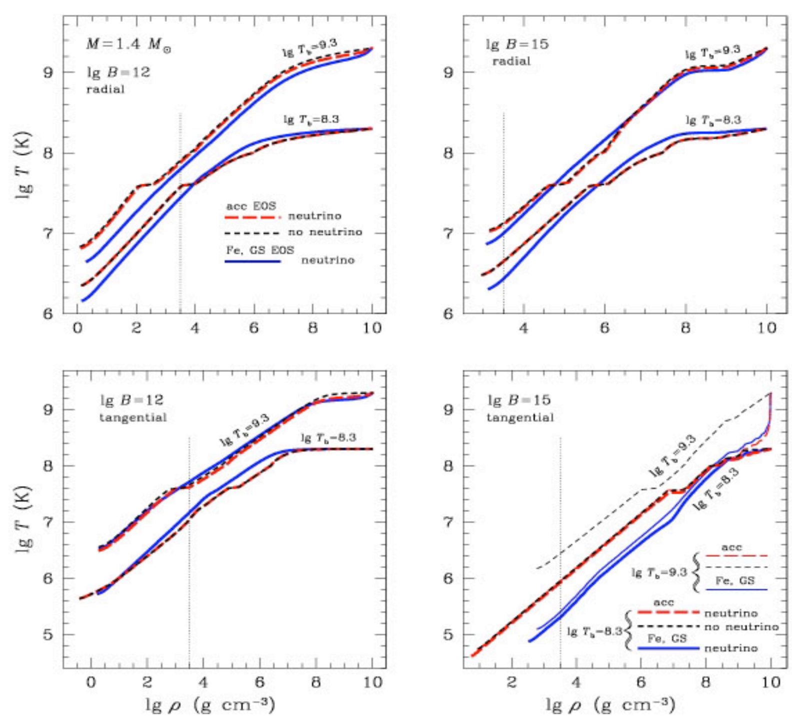
Effective temperature of the surface as a function of the internal temperature with account of the neutrino emission

Neutrino emission rate in the outer crust



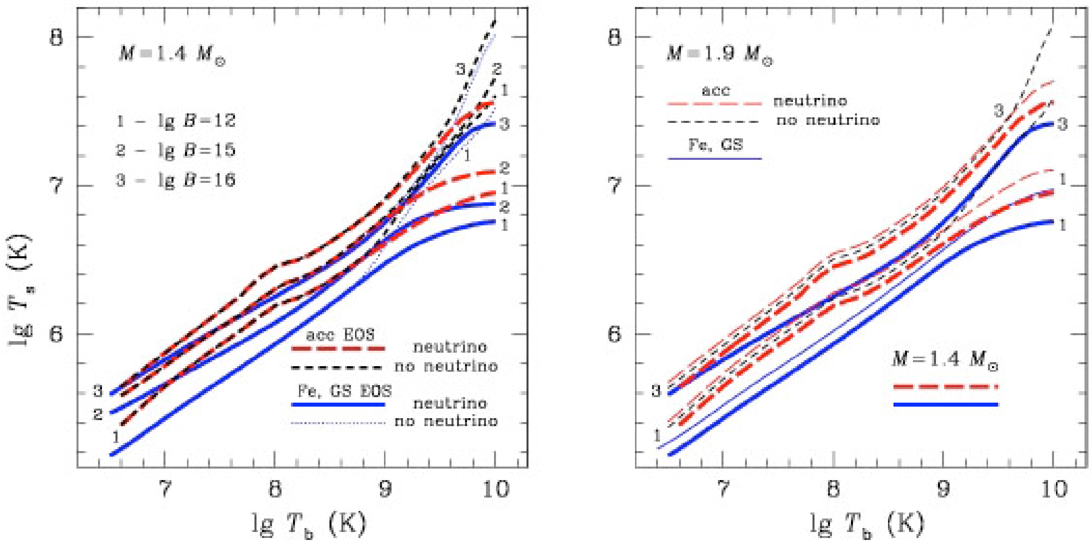
Temperature profiles in magnetized envelopes of neutron stars

The effects of neutrino emission, chemical composition, and magnetic fields

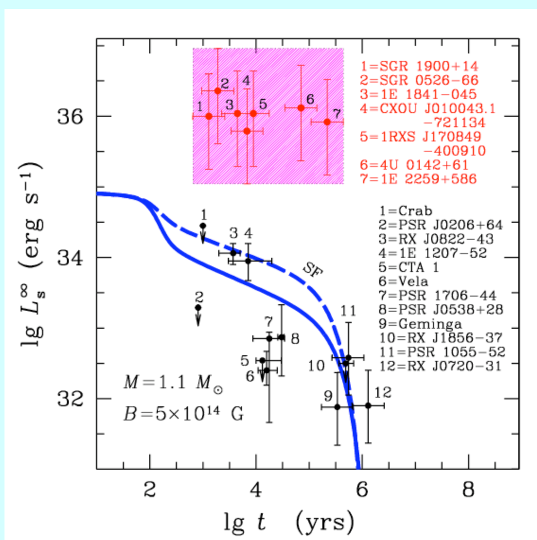


Temperature drops in magnetized envelopes of neutron stars

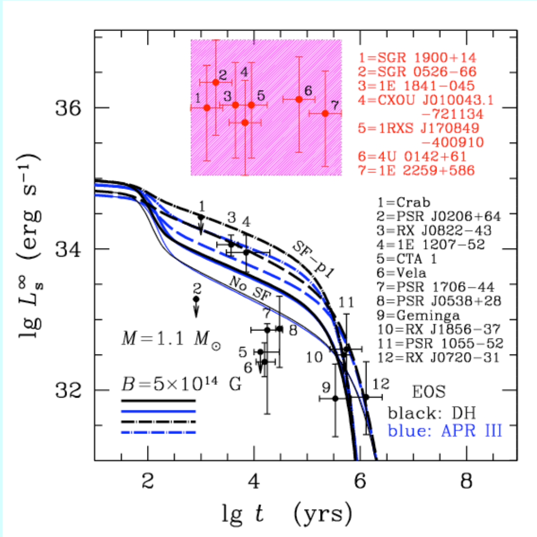
The effects of neutrino emission, chemical composition, and magnetic fields

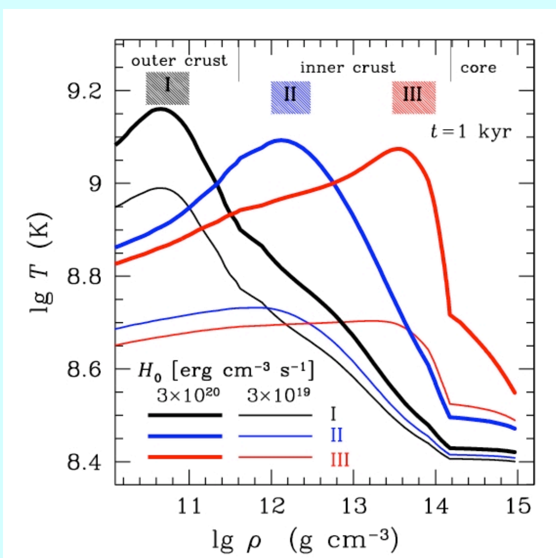


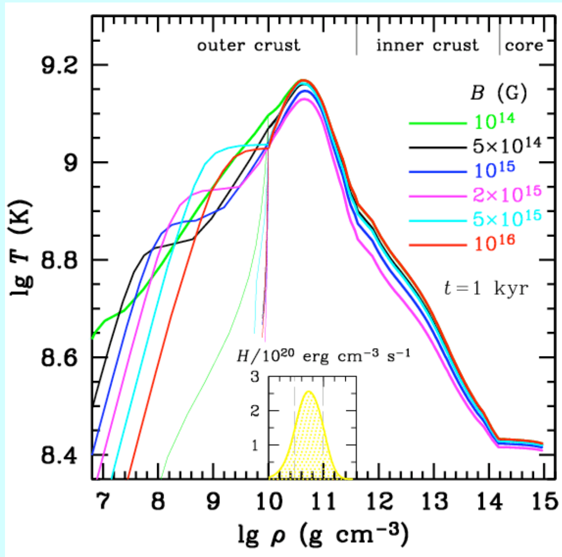
Application: Restrictions to models of crustal heating of magnetars

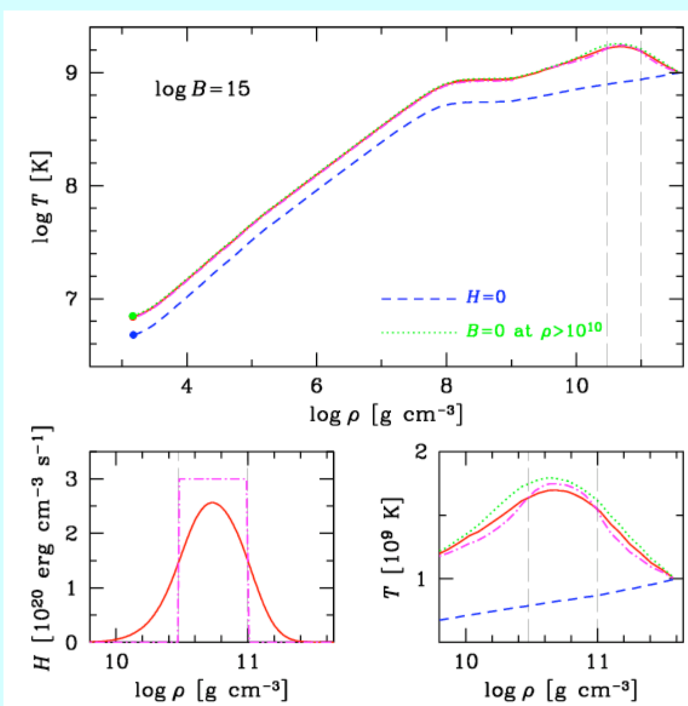


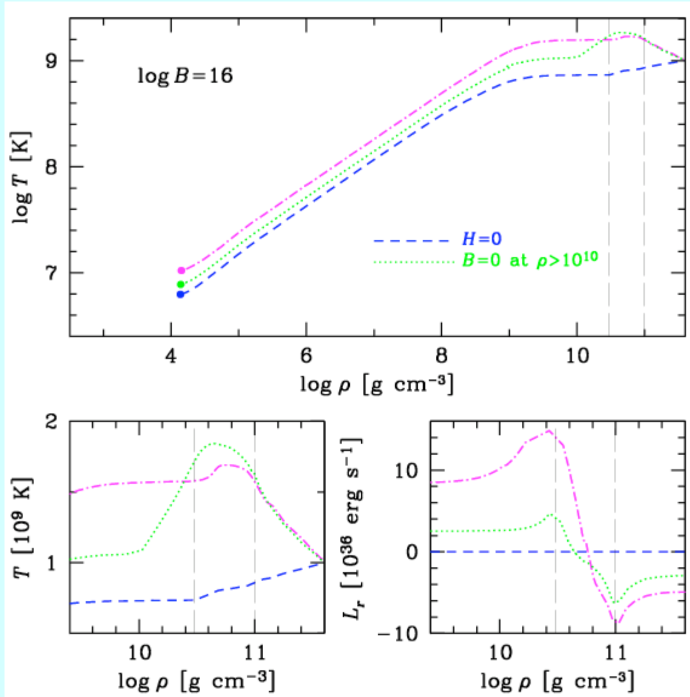
Application: Restrictions to models of crustal heating of magnetars



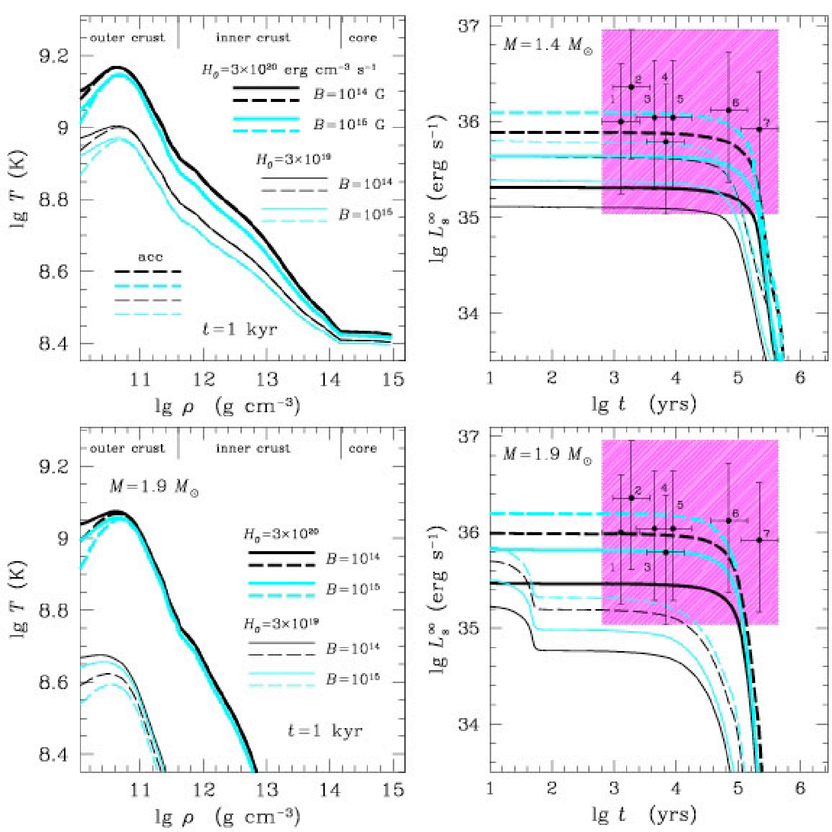



outer crust
inner crust
core
B (G)
10^14
5×10^14
10^15
2×10^15
5×10^15
10^16
t = 1 kyr
H/10^20 erg cm^-3 s^-1
lg T (K)
lg ρ (g cm^-3)

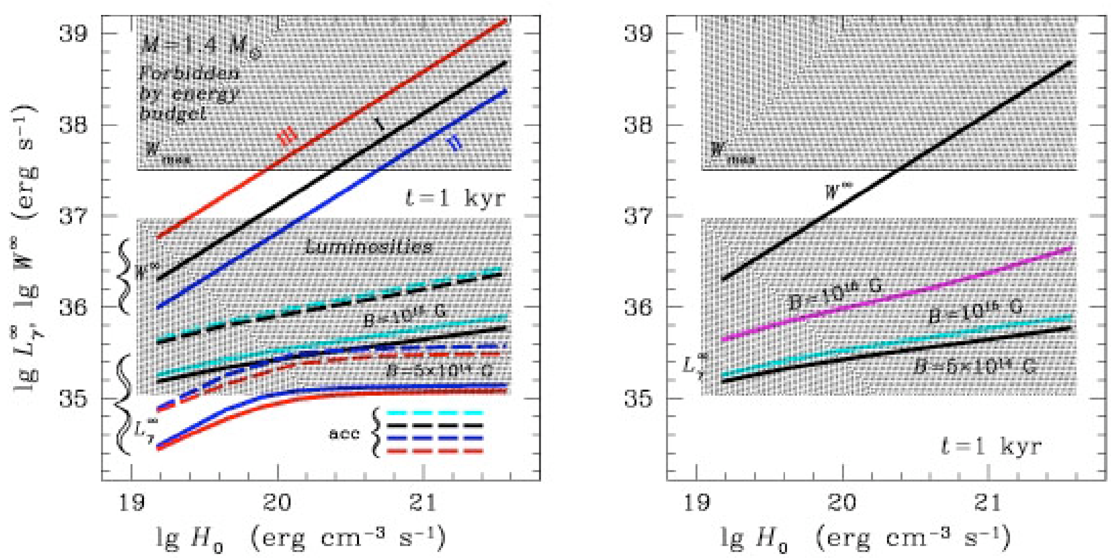

log B = 15
H = 0
B = 0 at ρ > 10^10
log T [K]
log ρ [g cm^-3]
H [10^20 erg cm^-3 s^-1]
T [10^9 K]



log B = 16
log T [K]
H=0
B=0 at ρ>10^10
log ρ [g cm^-3]
T [10^9 K]
L_r [10^36 erg s^-1]



outer crust
inner crust
core
H_0=3×10^20 erg cm^-3 s^-1
B=10^14 G
B=10^15 G
H_0=3×10^19
acc
t=1 kyr
lg T (K)
lg ρ (g cm^-3)
M=1.4 M_⊙
M=1.9 M_⊙
lg L_s^∞ (erg s^-1)
lg t (yrs)



Conclusions

- ***Magnetic fields*** make the temperature distribution highly anisotropic and can be important for evaluation of the effective temperature from observations.
- A ***superstrong*** magnetic field
  - on the average, makes the envelope more heat-transparent,
  - accelerates cooling at late epochs,
  - leads to theoretical uncertainties, which require further study.
- Reconciliation of crustal heating models with effective temperatures inferred form observations of some magnetars sensitively depends on the effects of superstrong magnetic fields and chemical composition of the outer envelopes.