Neutron Star Cooling
Pairing & Magnetic Fields

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- Basic Neutron Star Cooling
- Troubles: Surface Effects and Pairing
- Minimal Cooling
- The “Magnificent Seven”: Strong Toroidal Fields ?
- Conclusions

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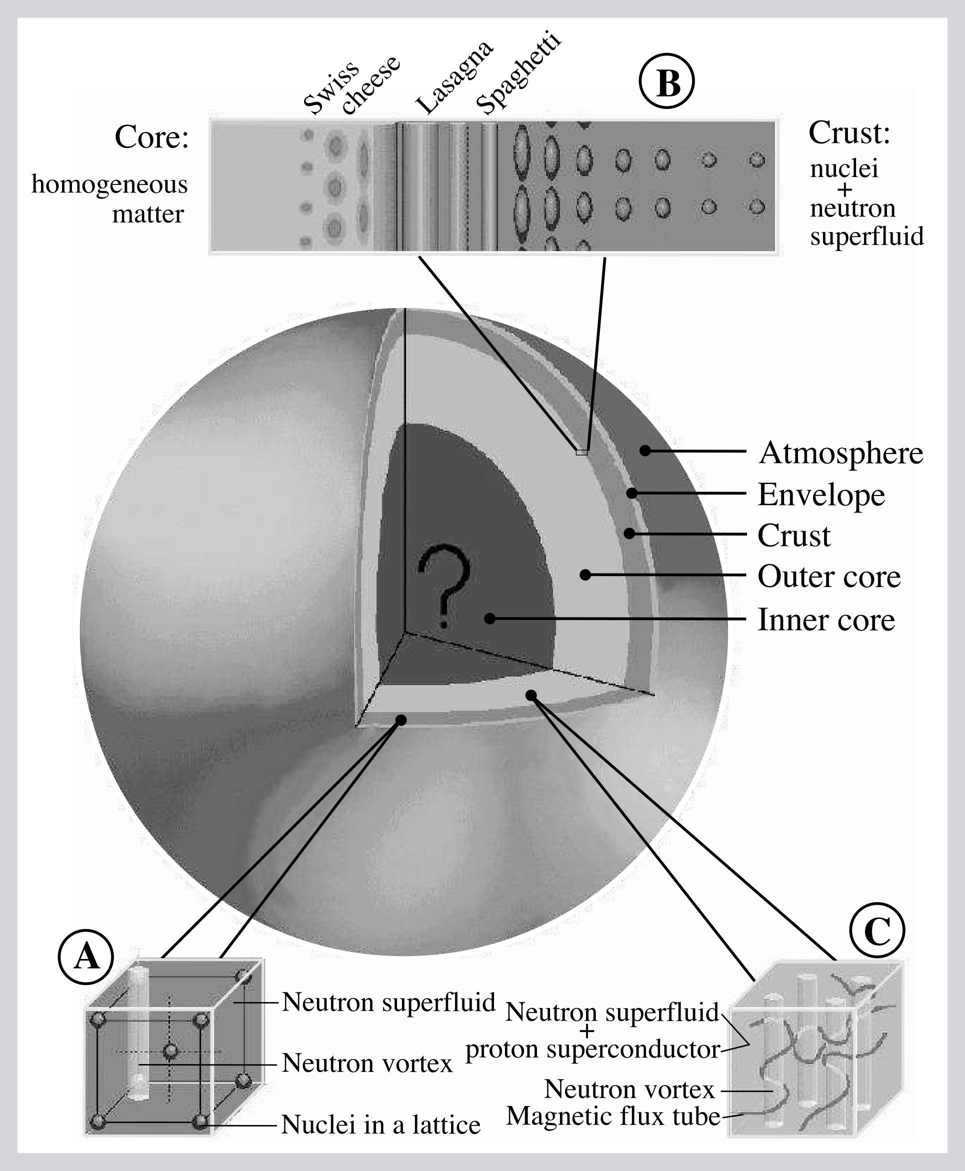
Overall View of a Neutron Star



Basic Equations

Schwarzschild metric:

$$ds^2 = -e^{2\phi} c^2 dt^2 + \frac{dr^2}{1 - 2Gm/c^2 r} + r^2 d\Omega^2$$

Proper time: $d\tau = e^{\phi}\, dt$

Proper length: $dl = \frac{dr}{\sqrt{1 - 2Gm/c^2 r}}$

Energy balance:

$$\frac{d(Le^{2\Phi})}{dr} = -\frac{4\pi r^2 e^{\Phi}}{\sqrt{1 - 2Gm/c^2 r}} \left(\frac{d\epsilon}{dt} + e^{\Phi}(q_\nu - q_h) \right) \quad \text{and} \quad L(r=0) = 0$$

$$\frac{d\epsilon}{dt} = \frac{d\epsilon}{dT}\frac{dT}{dt} = c_v \frac{dT}{dt}$$

Energy transport:

$$\frac{d(Te^{\Phi})}{dr} = -\frac{1}{\lambda} \frac{Le^{\Phi}}{4\pi r^2 \sqrt{1 - 2Gm/c^2 r}} \quad \text{and} \quad T(r = r_b) \equiv T_b = T_b(L_b)$$

$$L(r = r_b) \equiv L_b = L_\gamma \quad \text{and} \quad L_\gamma \equiv 4\pi R^2 \sigma_B T_e$$

Reduce it to One Simple Equation

Energy balance:

$$\boxed{\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H}$$

⇒ 3 essential ingredients are needed:

- C_v = total stellar specific heat
- L_γ = total surface photon luminosity
- L_ν = total stellar neutrino luminosity

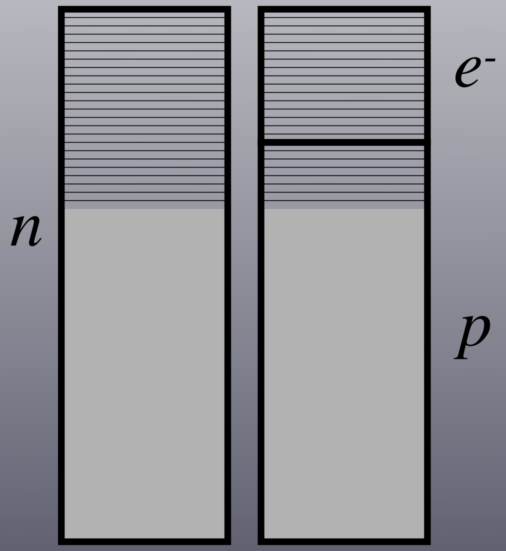
Neutrino Emission

Basic mechanism: β and inverse β decays:

$$n \longrightarrow p + e^- + \overline{\nu}_e \quad \text{and} \quad p + e^- \longrightarrow n + \nu_e$$

Energy conservation:

$$E_{Fn} = E_{Fp} + E_{Fe}$$



Momentum conservation:

“Triangle rule”: $p_{Fn} < p_{Fp} + p_{Fe}$

$$n_i = \frac{k_{F\,i}^3}{3\pi^2} \Rightarrow n_n^{1/3} \le n_p^{1/3} + n_e^{1/3} = 2n_p^{1/3}$$

$$\boxed{x_p \equiv \frac{n_p}{n_n + n_p} \ge \frac{1}{9} \approx 11\%}$$

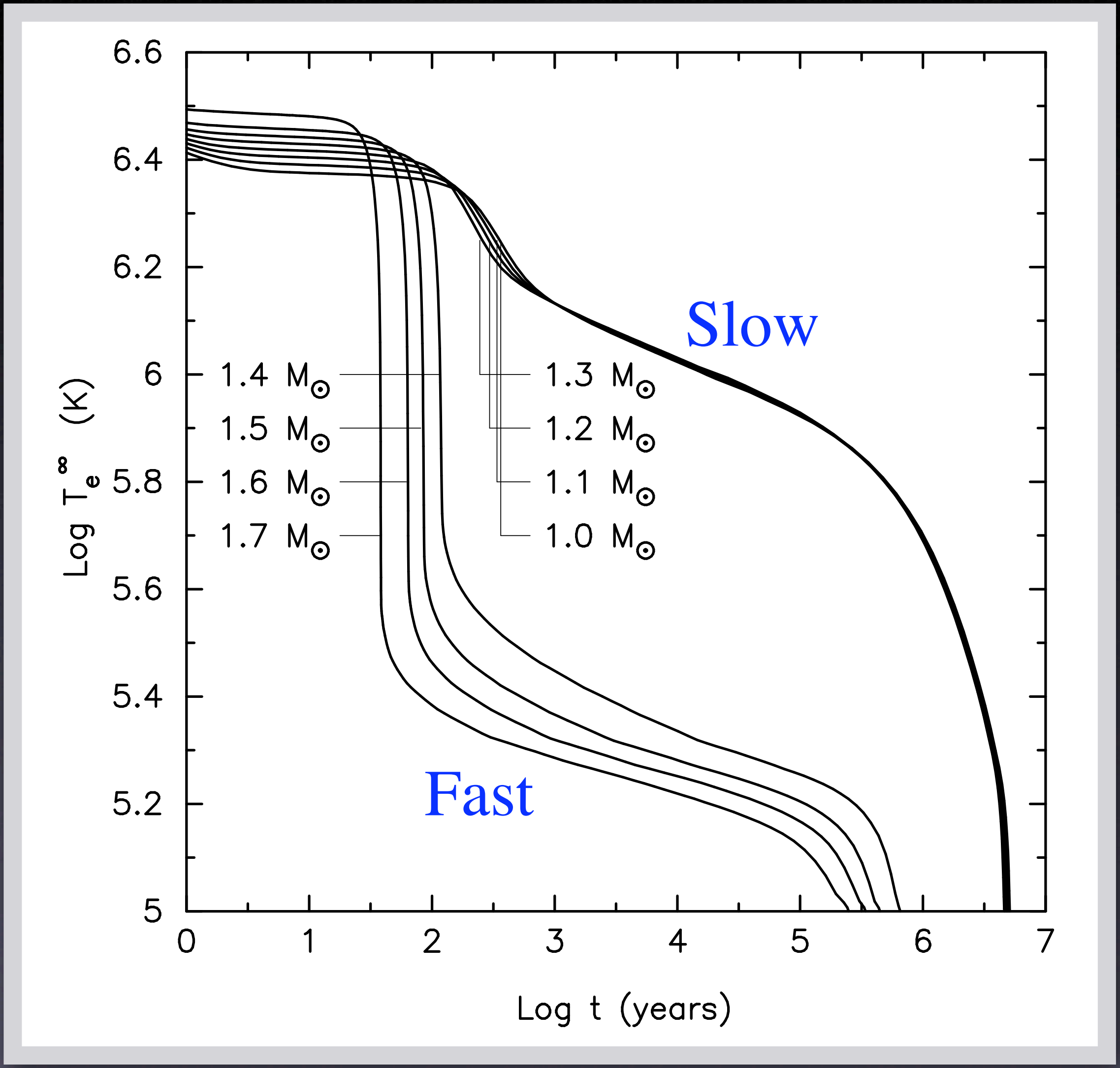
Direct URCA process in neutron stars, Lattimer, Pethick, Prakash & Haensel, 1991 PRL 66, 2701

Fast vs Slow Neutrino Emission

Name	Process	Emissivity (erg $\mathrm{cm}^{-3}\ \mathrm{s}^{-1}$)	
Modified Urca cycle (neutron branch)	$n + n \to n + p + e^- + \bar{\nu}_e$ $n + p + e^- \to n + n + \nu_e$	$\sim 2\times10^{21}\ R\, T_9^8$	Slow
Modified Urca cycle (proton branch)	$p + n \to p + p + e^- + \bar{\nu}_e$ $p + p + e^- \to p + n + \nu_e$	$\sim 10^{21}\ R\, T_9^8$	Slow
Bremsstrahlung	$n + n \to n + n + \nu + \bar{\nu}$ $n + p \to n + p + \nu + \bar{\nu}$ $p + p \to p + p + \nu + \bar{\nu}$	$\sim 10^{19}\ R\, T_9^8$	Slow
Cooper pair formations	$n + n \to [nn] + \nu + \bar{\nu}$ $p + p \to [pp] + \nu + \bar{\nu}$	$\sim 5\times10^{21}\ R\, T_9^7$ $\sim 5\times10^{19}\ R\, T_9^7$	Medium
Direct Urca cycle	$n \to p + e^- + \bar{\nu}_e$ $p + e^- \to n + \nu_e$	$\sim 10^{27}\ R\, T_9^6$	Fast
π^- condensate	$n + < \pi^- > \to n + e^- + \bar{\nu}_e$	$\sim 10^{26}\ R\, T_9^6$	Fast
K^- condensate	$n + < K^- > \to n + e^- + \bar{\nu}_e$	$\sim 10^{25}\ R\, T_9^6$	Fast

The Cooling of Compact Stars, Page, Geppert, Weber, Nucl. Phys. A 777, p. 497-530 (2006). [Special issue on Nuclear Astrophysics]

Direct vs Modified Urca Processes

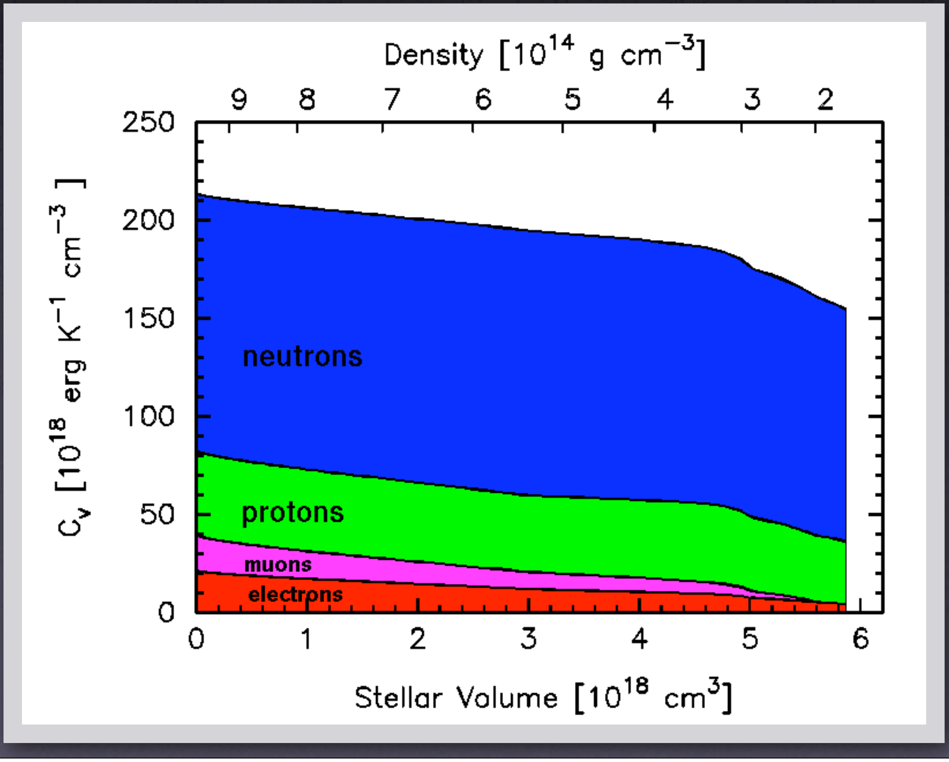


The cooling of neutron stars by the direct URCA process, Page & Applegate, 1992 ApJ 394 L17

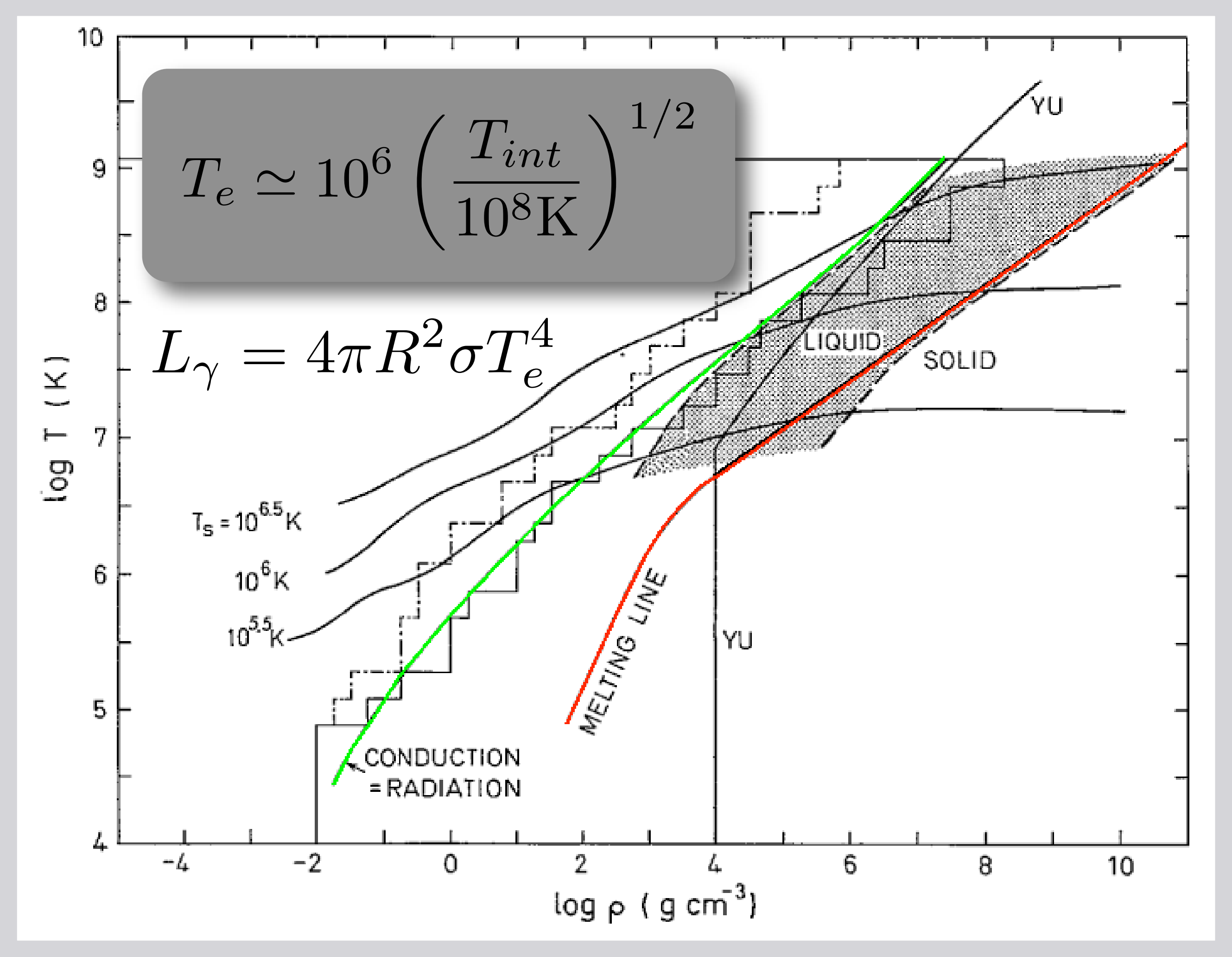
Specific Heat

Sum over all degenerate Fermion: $C_v = \sum_i C_{v\ i}$

$$C_{v\ i} = N_i(0) \frac{\pi^2}{3} k_B^2 T \quad \text{with} \quad N_i(0) = \frac{m_i^* p_{Fi}}{\pi^2 \hbar^3}$$



Envelope and Photon Emission



$$T_e \simeq 10^6 \left(\frac{T_{int}}{10^8 \mathrm{K}} \right)^{1/2}$$

$$L_\gamma = 4\pi R^2 \sigma T_e^4$$

Structure of neutron star envelopes, Gudmundsson, Pethick, Epstein, 1983 ApJ 272, 286

Some Simple Analytical Solutions

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu$$

$$C_v = CT \quad L_\nu = NT^8 \quad L_\gamma = ST^{2+4\alpha}$$

$$L_\gamma = 4\pi R^2 \sigma T_e^4 \ \text{ with } \ T_e \propto T^{0.5+\alpha}$$

- Neutrino Cooling Era: $L_\nu >> L_\gamma$

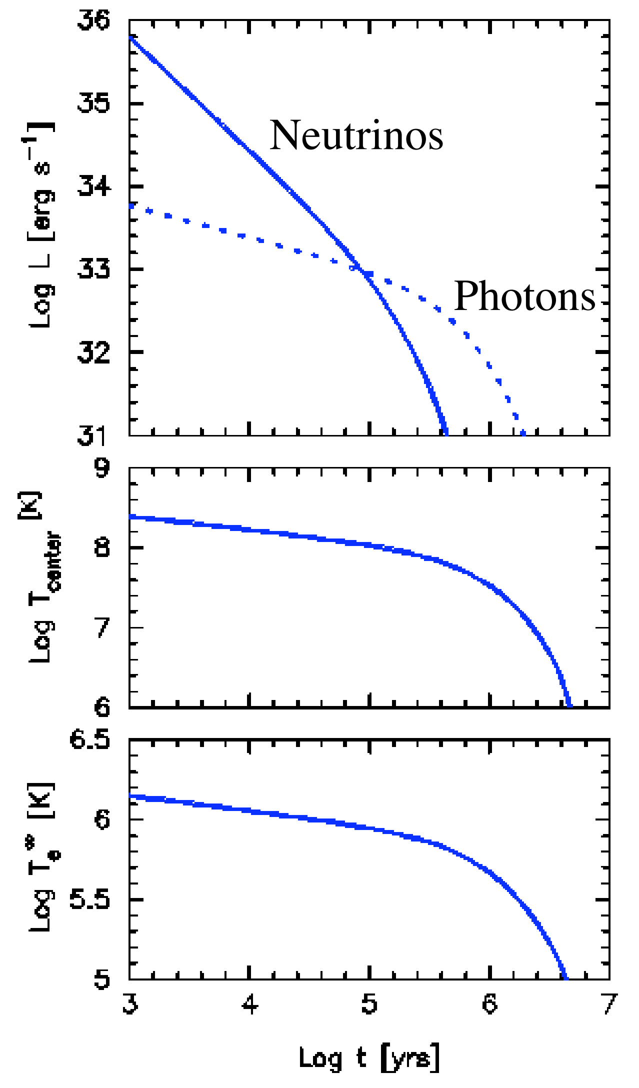
$$\frac{dT}{dt} = -\frac{N}{C} T^7 \Rightarrow t - t_0 = A\left[\frac{1}{T^6} - \frac{1}{T_0^6}\right]$$

$$\boxed{T \propto t^{-1/6}}$$

- Photon Cooling Era: $L_\gamma >> L_\nu$

$$\frac{dT}{dt} = -\frac{N}{S} T^{1+\alpha} \Rightarrow t - t_0 = A\left[\frac{1}{T^\alpha} - \frac{1}{T_0^\alpha}\right]$$

$$\boxed{T \propto t^{-1/\alpha}}$$



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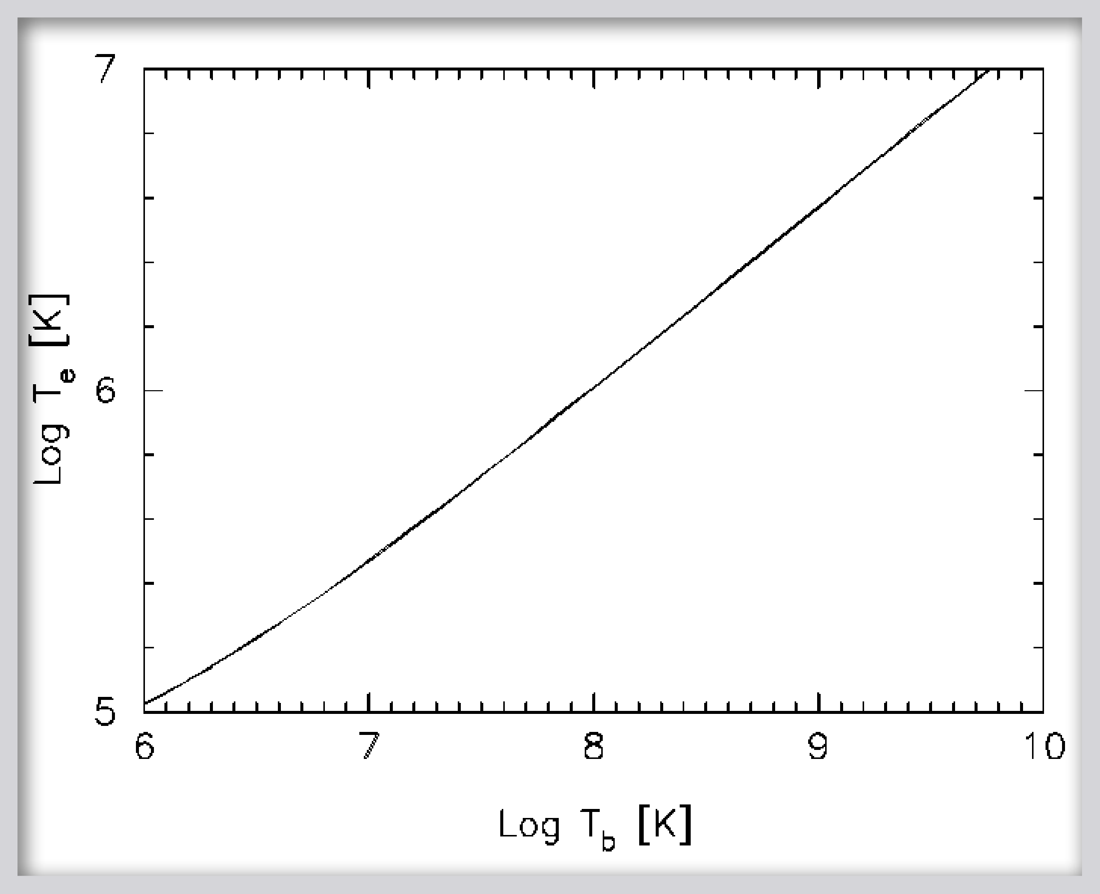
Troubles (1): Envelope Chemical Composition

The "T_e - T_b" relationship for heavy element envelopes

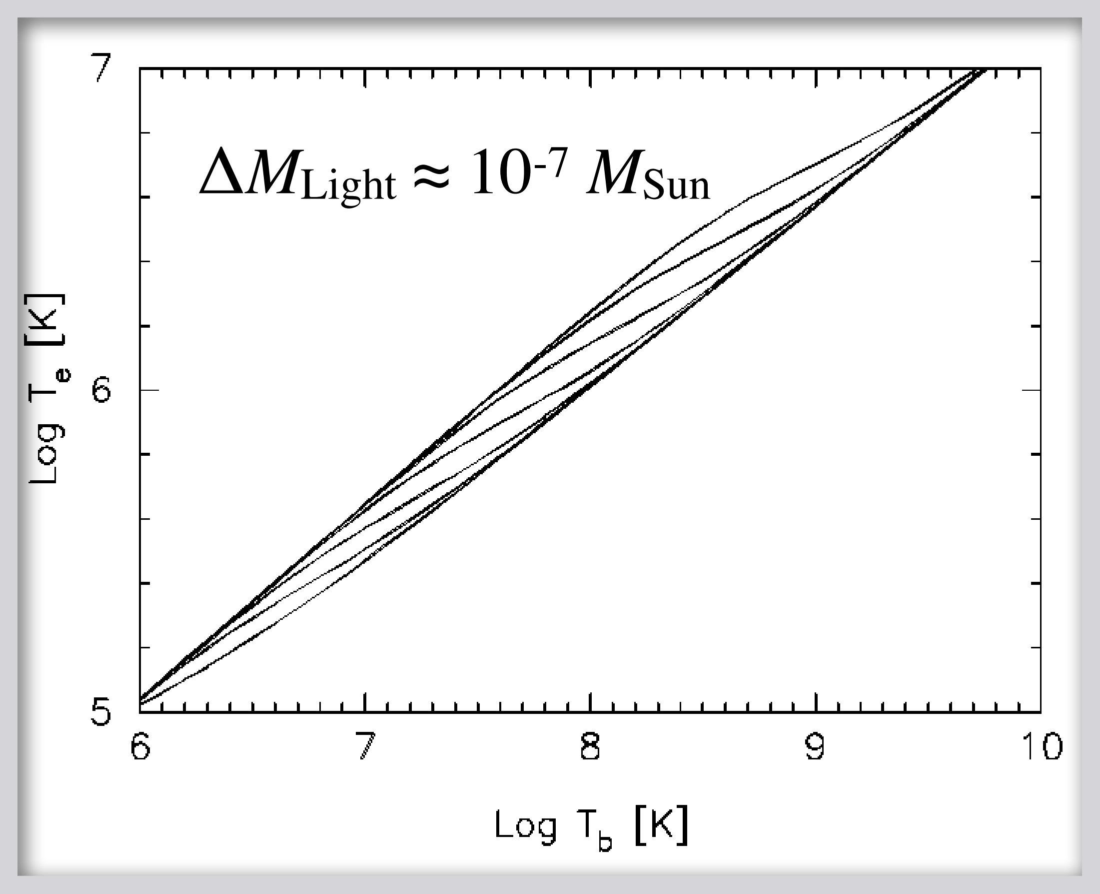


Troubles (1): Envelope Chemical Composition

The "T_e - T_b" relationship for heavy element envelopes

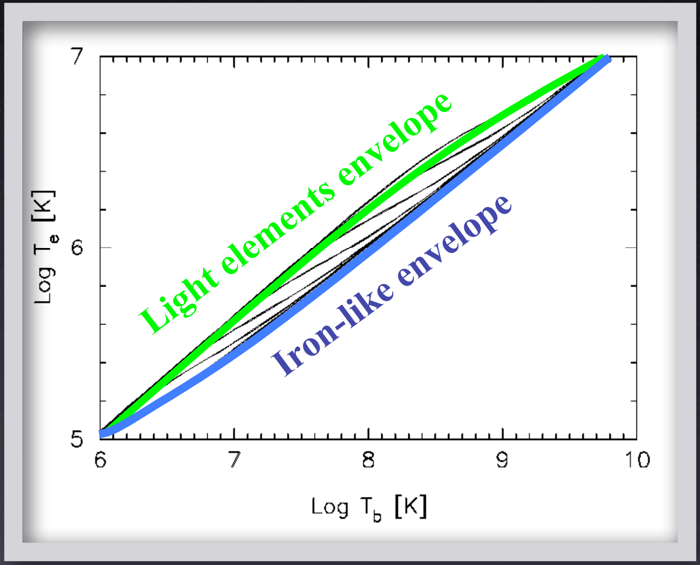
... and for light element envelopes

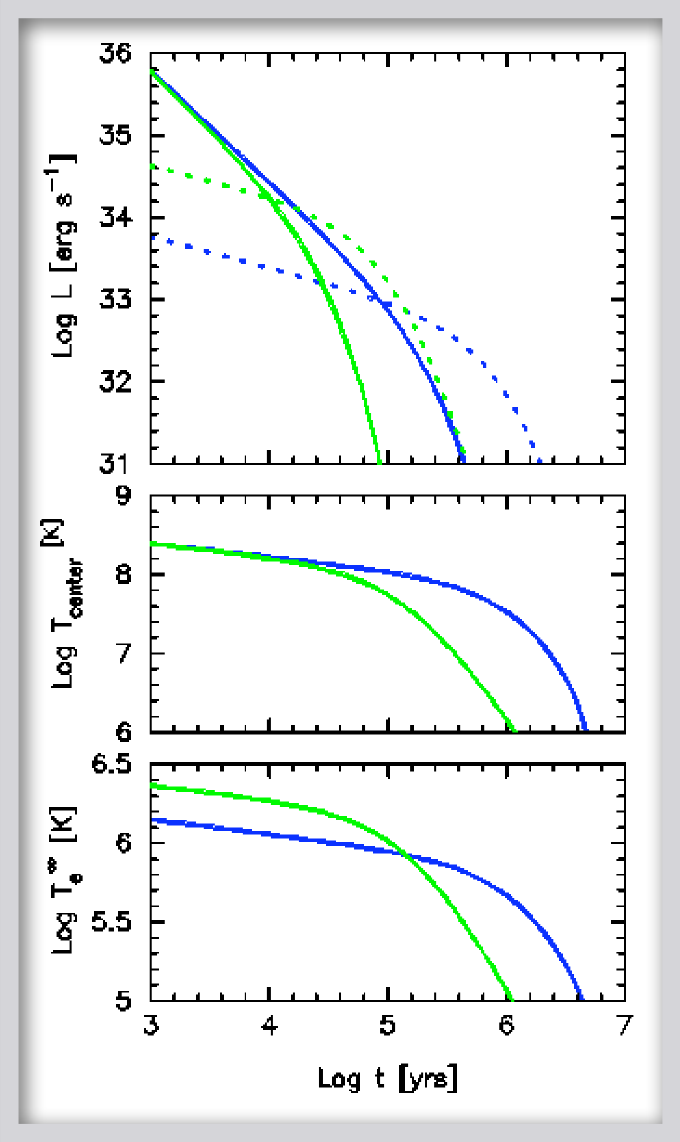
Thermal conductivity in the liquid phase $\lambda \propto \frac{1}{Z}$



Cooling Neutron Stars with Accreted Envelopes, Chabrier, Potekhin, Yakovlev, 1997 ApJ 477, L99

Troubles (1): Envelope Chemical Composition





Troubles (2): Nucleon Pairing

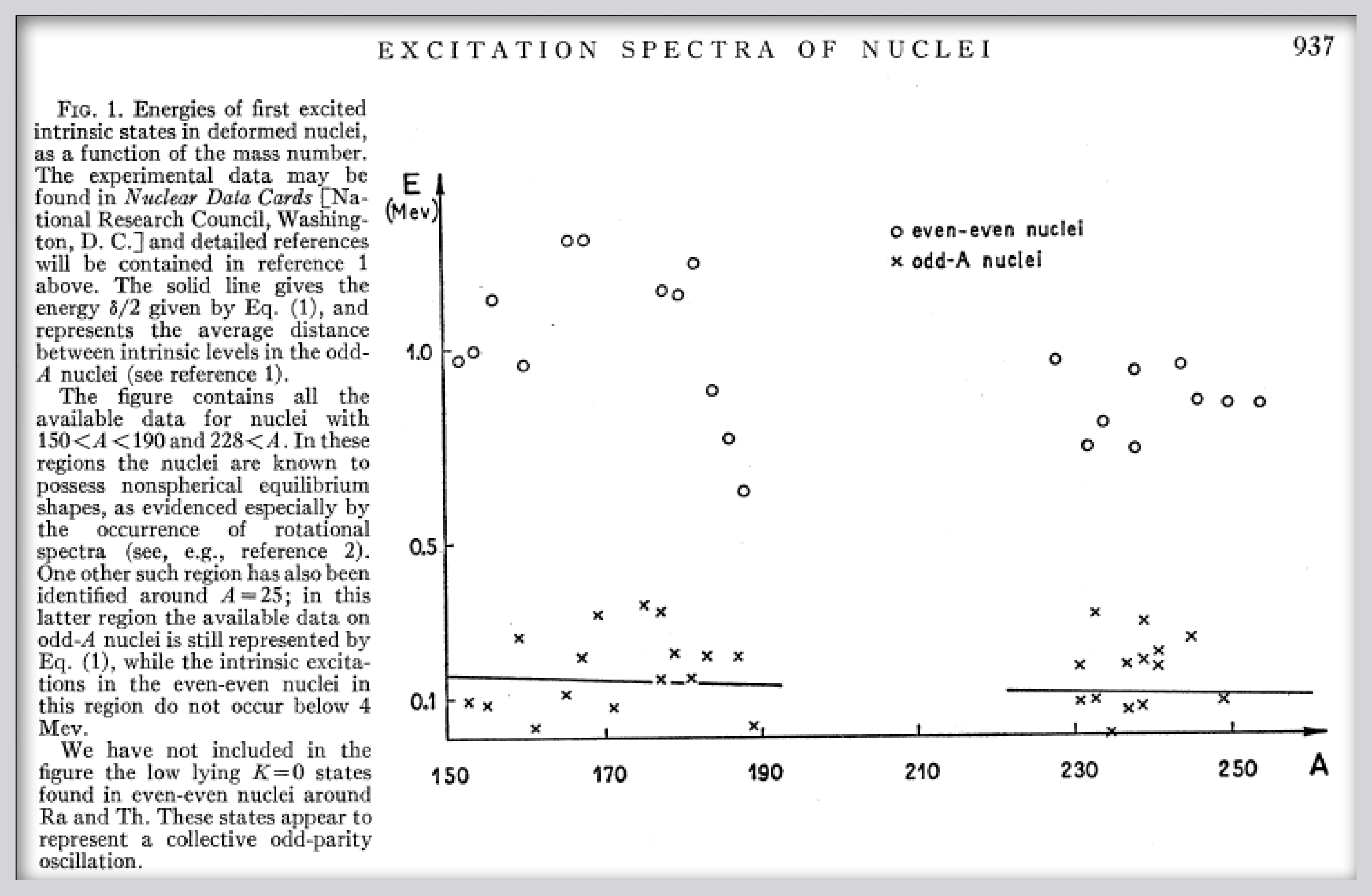


Fig. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.

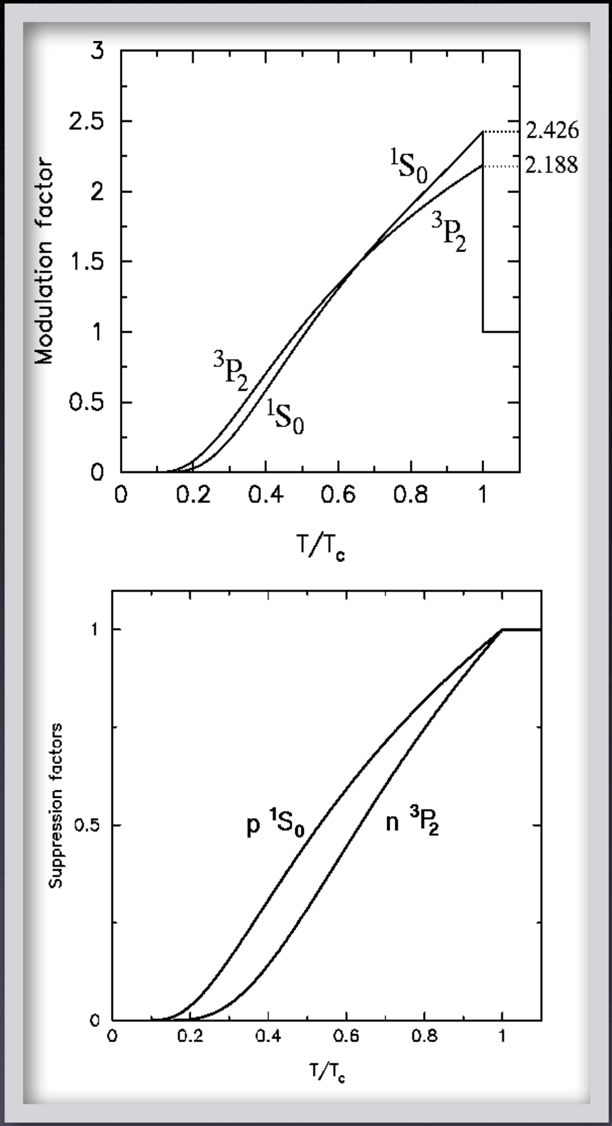
Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State, Bohr, Mottelson, Pines, 1958 PhRv 110, 936

Suppression of C_v and Q_v by Pairing

The presence of a pairing gap in the single particple excitation spectrum results in a Boltzmann-like [exp(-$\Delta/k_\mathrm{B}T$)] suppression of C_v and Q_ν:

$$C_v \rightarrow C_v^{\mathrm{Paired}} = R_c C_v^{\mathrm{Normal}}$$

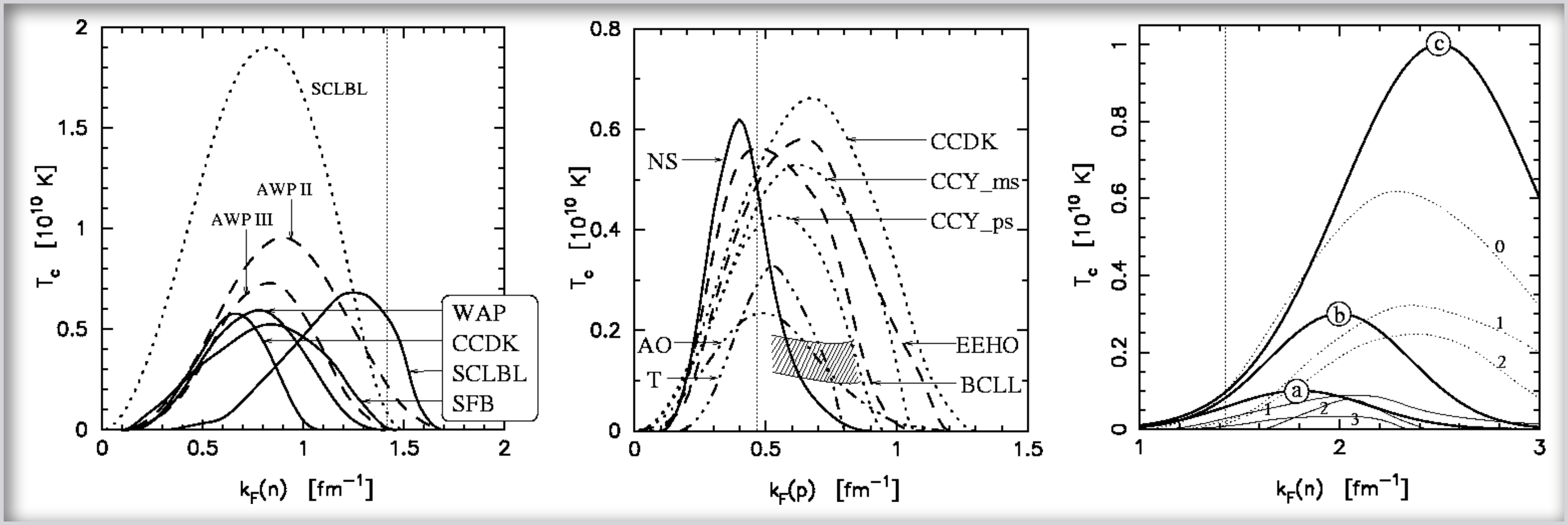
$$Q_\nu \rightarrow Q_\nu^{\mathrm{Paired}} = R_\nu Q_\nu^{\mathrm{Normal}}$$



See, e.g., *Neutrino emission from neutron stars*, Yakovlev, Kaminker, Gnedin & Haensel, Phys. Rep. 354, 1 (2001)

Trouble (2): Pairing T_c

Neutron ${}^1S_0$ Proton ${}^1S_0$ Neutron ${}^3P_2$



Enormous uncertainties on the actual values of T_c for pairing in the core (proton ${}^1S_0$ and neutron ${}^3P_2$)

Effect of Pairing on the Cooling

Slow cooling

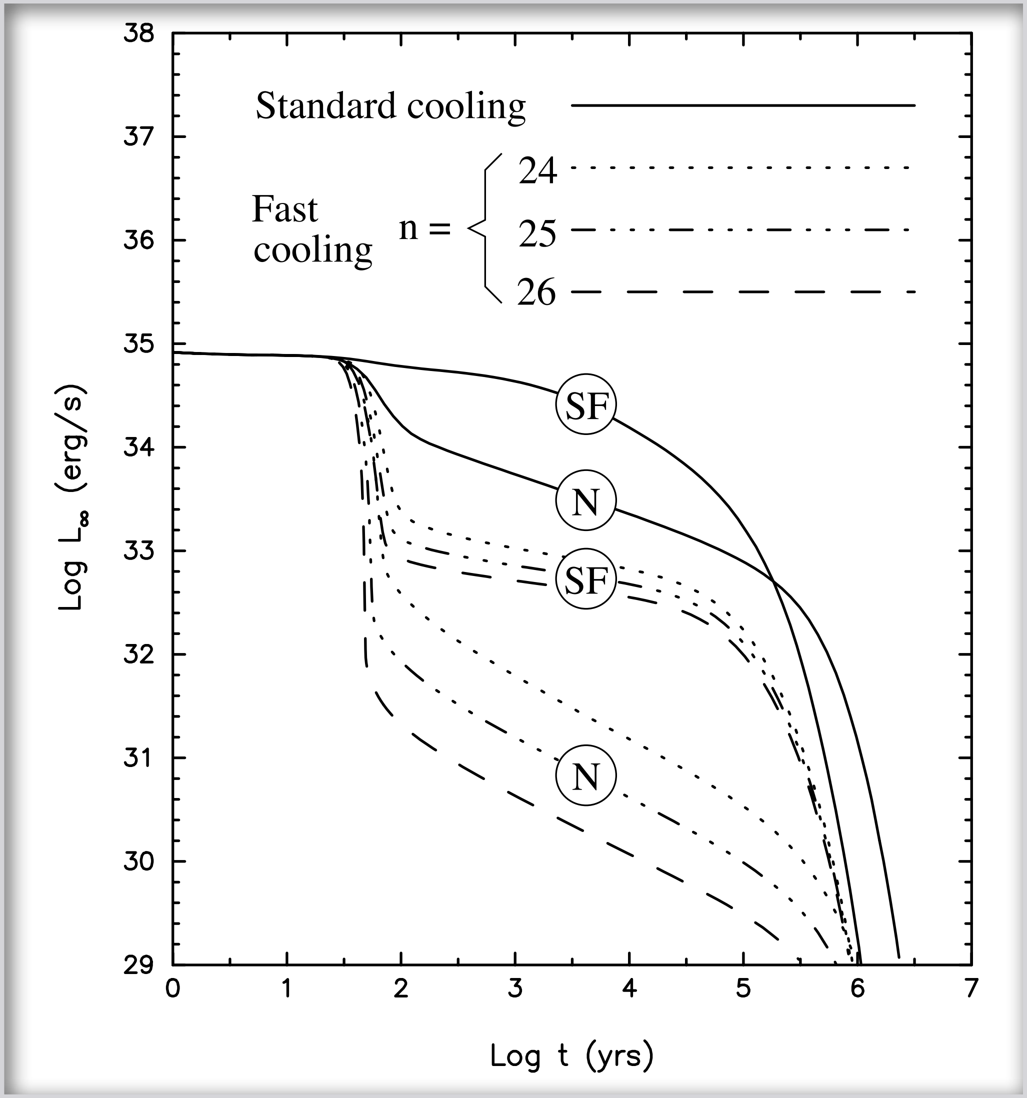
$q_\nu \sim 10^{21}\ T_9^8\ \mathrm{erg\ cm^{-3}\ s^{-1}}$

and

Fast cooling

$q_\nu \sim 10^{n}\ T_9^6\ \mathrm{erg\ cm^{-3}\ s^{-1}}$

controlled by pairing



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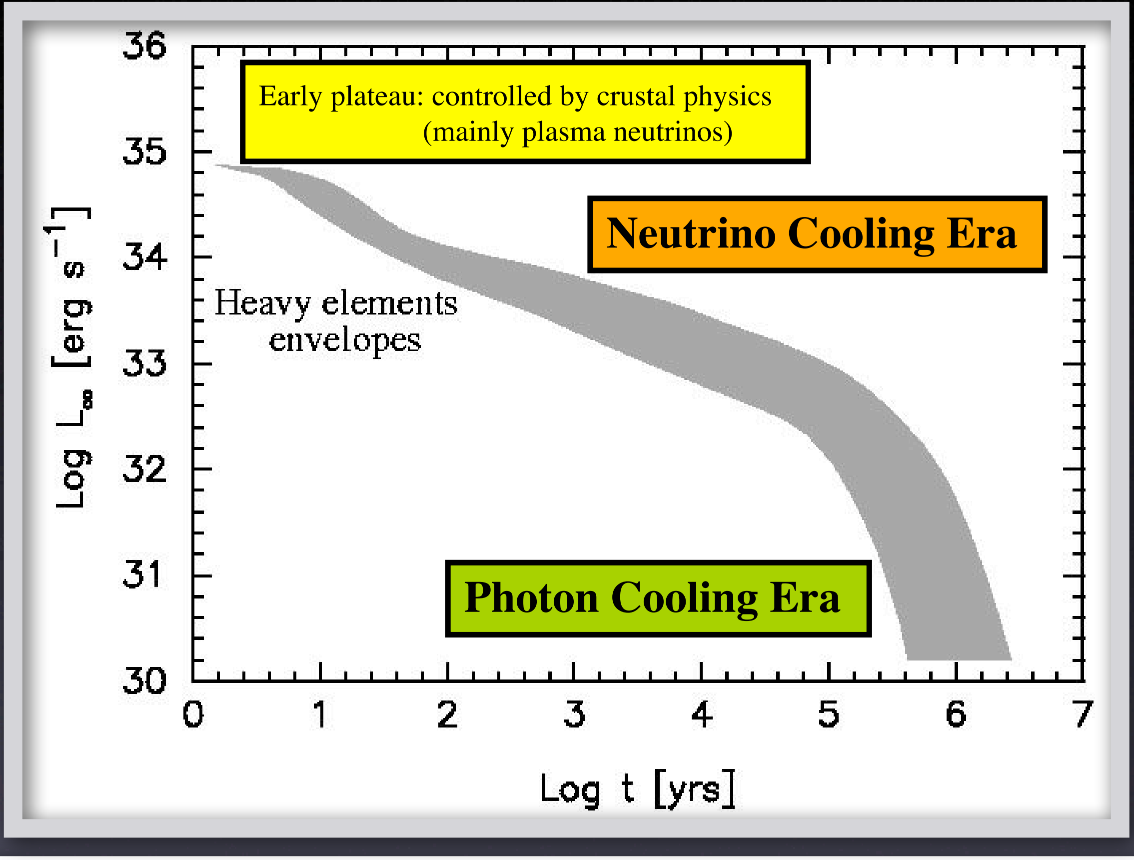
Minimal Cooling



Early plateau: controlled by crustal physics (mainly plasma neutrinos)

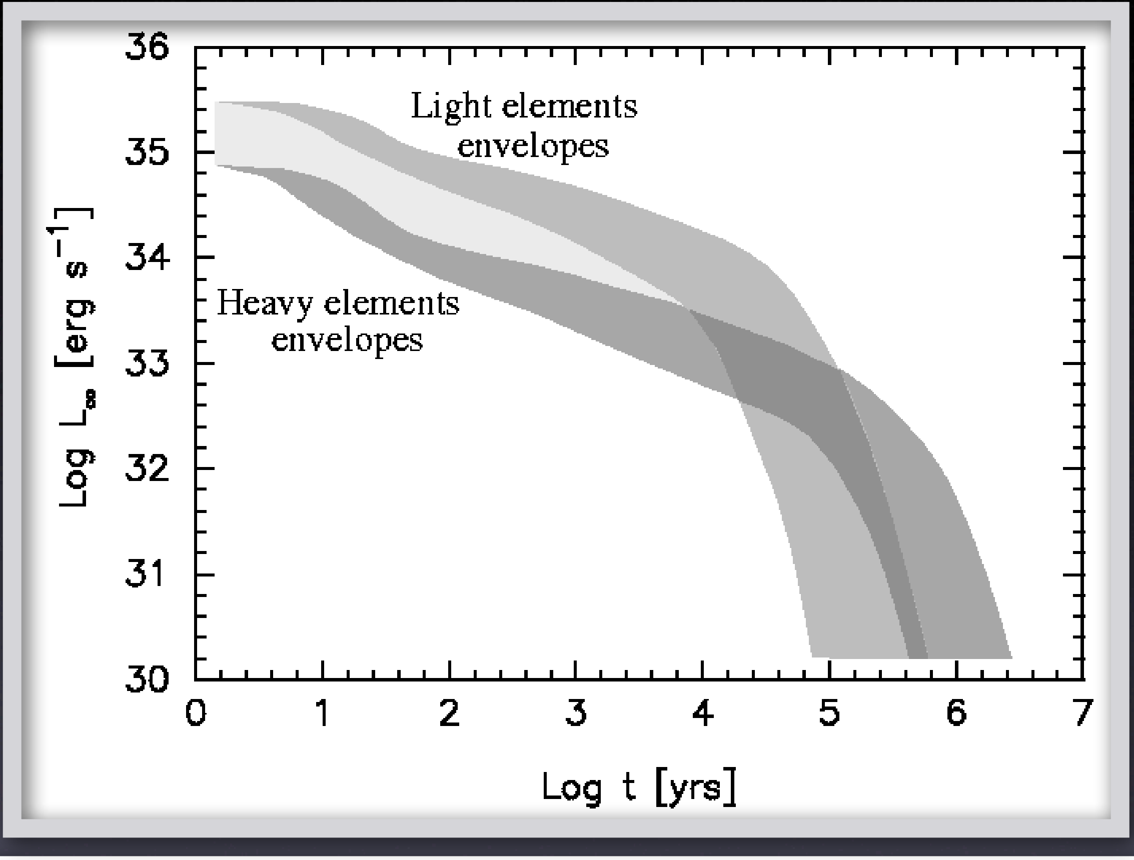
Neutrino Cooling Era

Heavy elements envelopes

Photon Cooling Era

Minimal Cooling: exclude anything beyond just nucleons and leptons (i.e., no meson condensates, no hyperons, no deconfined quarks, ... no nothing) but include all uncertainties on “standard” physics.

Minimal Cooling



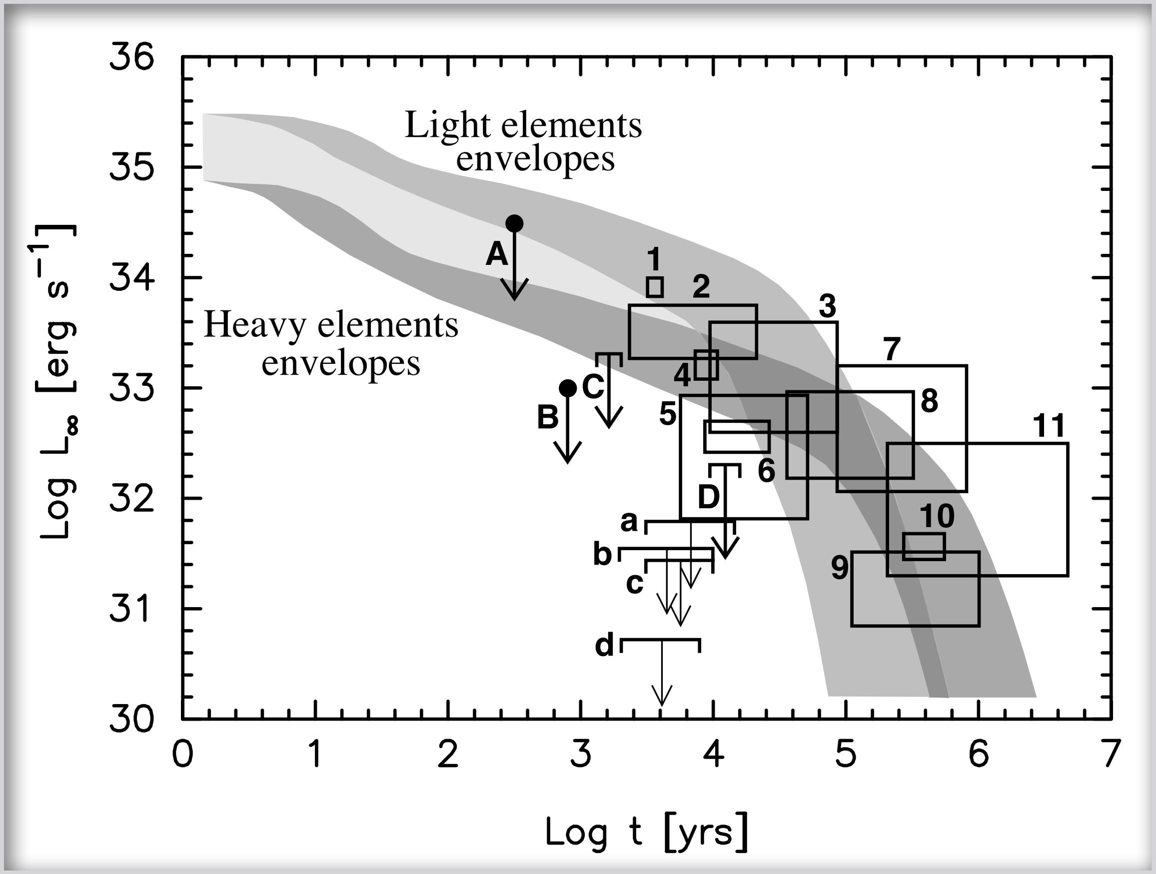
Minimal Cooling: exclude anything beyond just nucleons and leptons (i.e., no meson condensates, no hyperons, no deconfined quarks, ... no nothing) but include all uncertainties on "standard" physics.

Minimal Cooling vs Data

1. RX J0822-4247 (in SNR Puppis A)
2. 1E 1207.4-5209 (in SNR PKS 1209-52)
3. PSR 0538+2817
4. RX J0002+6246 (in SNR CTB 1)
5. PSR 1706-44
6. PSR 0833-45 (in SNR ``Vela'')
7. PSR 1055-52
8. PSR 0656+14
9. PSR 0633+1748 (``Geminga'')
10. RX J1856.5-3754
11. RX J0720.4--3125

A. CXO J232327.8+584842 (in SNR Cas A)
B. PSR J0205+6449 (in SNR 3C58)
C. PSR J1124--5916 (in SNR G292.0+1.8)
D. RX J0007.0+7302 (in SNR CTA 1)

a. ? (in SNR G315.4--2.3)
b. ? (in SNR G093.3+6.9)
c. ? (in SNR G084.2--0.8)
d. ? (in SNR G127.1+0.5)



Minimal Cooling of Neutron Stars: A New Paradigm, Page, Lattimer, Prakash & Steiner, 2004 ApJS 155, 623

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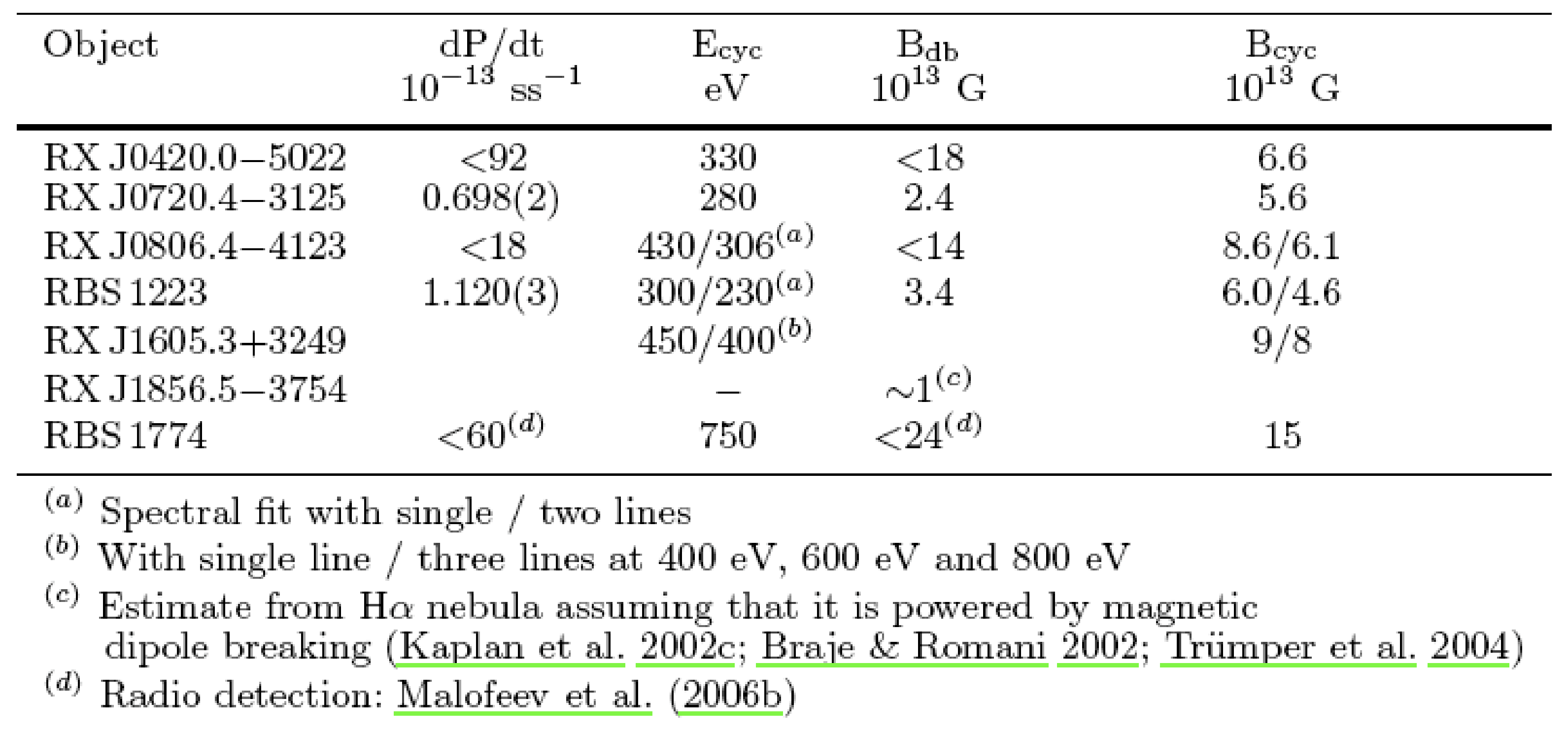
The “Magnificent Seven”: Strong B



Object	dP/dt 10^{-13} ss^{-1}	E_{cyc} eV	B_{db} 10^{13} G	B_{cyc} 10^{13} G
RX J0420.0−5022	<92	330	<18	6.6
RX J0720.4−3125	0.698(2)	280	2.4	5.6
RX J0806.4−4123	<18	430/306(a)	<14	8.6/6.1
RBS 1223	1.120(3)	300/230(a)	3.4	6.0/4.6
RX J1605.3+3249		450/400(b)		9/8
RX J1856.5−3754		–	~ 1(c)	
RBS 1774	<60(d)	750	<24(d)	15

(a) Spectral fit with single / two lines

(b) With single line / three lines at 400 eV, 600 eV and 800 eV

(c) Estimate from Hα nebula assuming that it is powered by magnetic dipole breaking (Kaplan et al. 2002c; Braje & Romani 2002; Trümper et al. 2004)

(d) Radio detection: Malofeev et al. (2006b)

The Magnificent Seven: Magnetic fields and surface temperature distributions,
F Haberl, 2006 astro.ph/069066

The Composite Spectrum of RX J1856

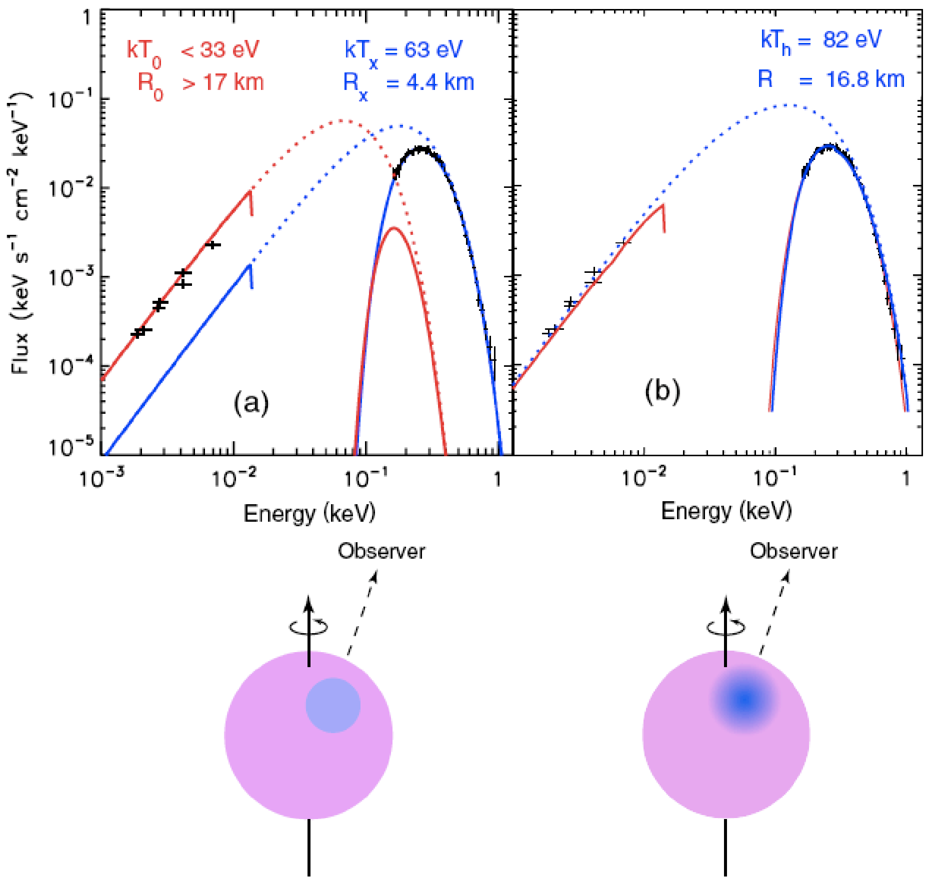


Figure 1. Blackbody fits to the optical and X-ray spectra of RX J1856.5-3754 for a two-component model (a) and a model with a continuous temperature distribution (b).

The puzzles of RX J1856.5-3754: Neutron Star or Quark Star?, Truemper, Burwitz, Haberl & Zavlin, 2004 NuPhS 132, 560

Heat Transport with Strong B

$$\vec{F} = -\kappa \cdot \vec{\nabla} T$$

$$\kappa = \begin{pmatrix} \kappa_\perp & \kappa_\wedge & 0 \\ -\kappa_\wedge & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

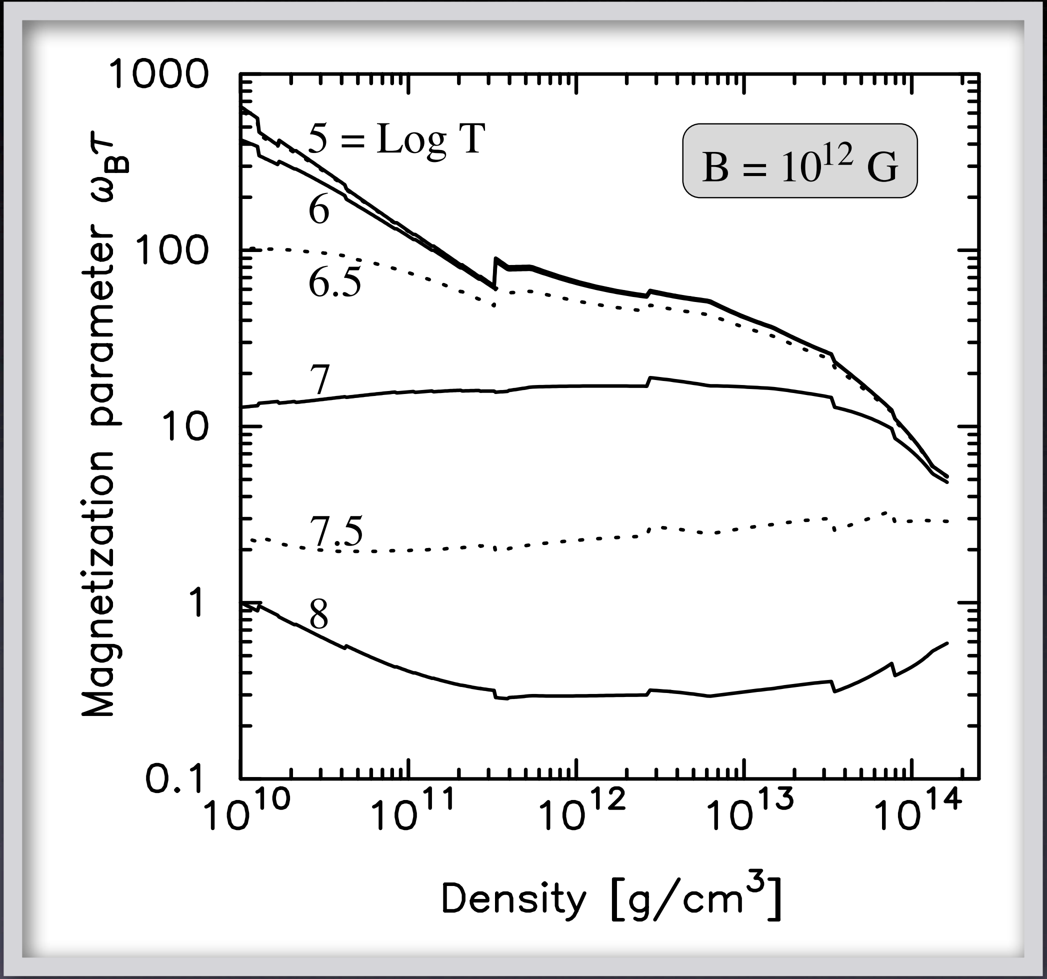
$$\kappa_\parallel = \kappa_0$$

$$\kappa_\perp = \frac{\kappa_0}{1+(\omega_B \tau)^2}$$

$$\kappa_\wedge = \frac{\kappa_0 \; \omega_B \tau}{1+(\omega_B \tau)^2}$$

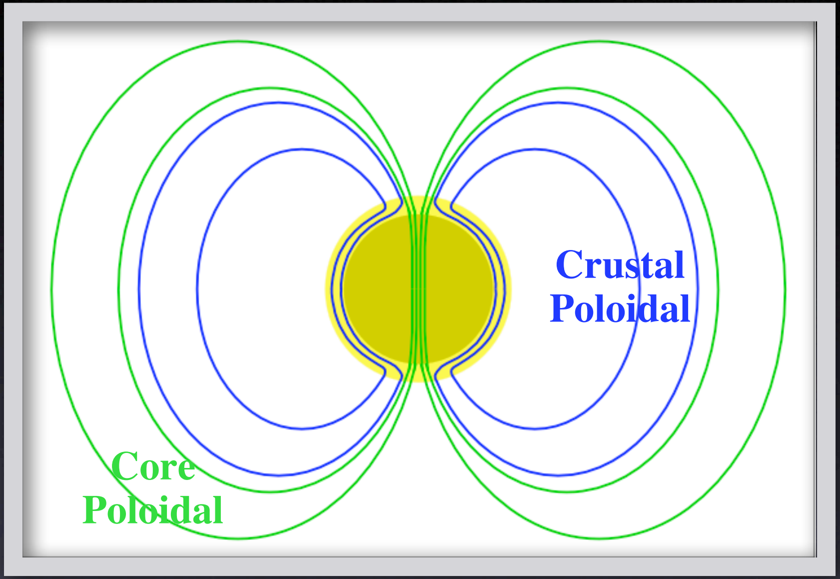
$\omega_B = \frac{eB}{m_e^* c}$ = electron cyclotron frequency

τ = electron relaxation time

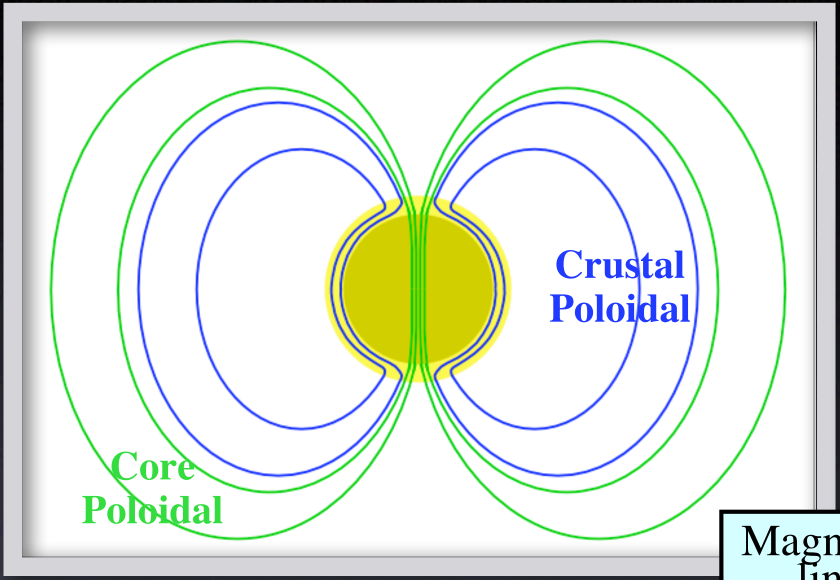


Temperature distribution in magnetized neutron star crusts, Geppert, Küker & Page, 2004 A&A 426, 267

Crust + Core Poloidal Field



Crust + Core Poloidal Field



Magnetic field lines are isothermal !

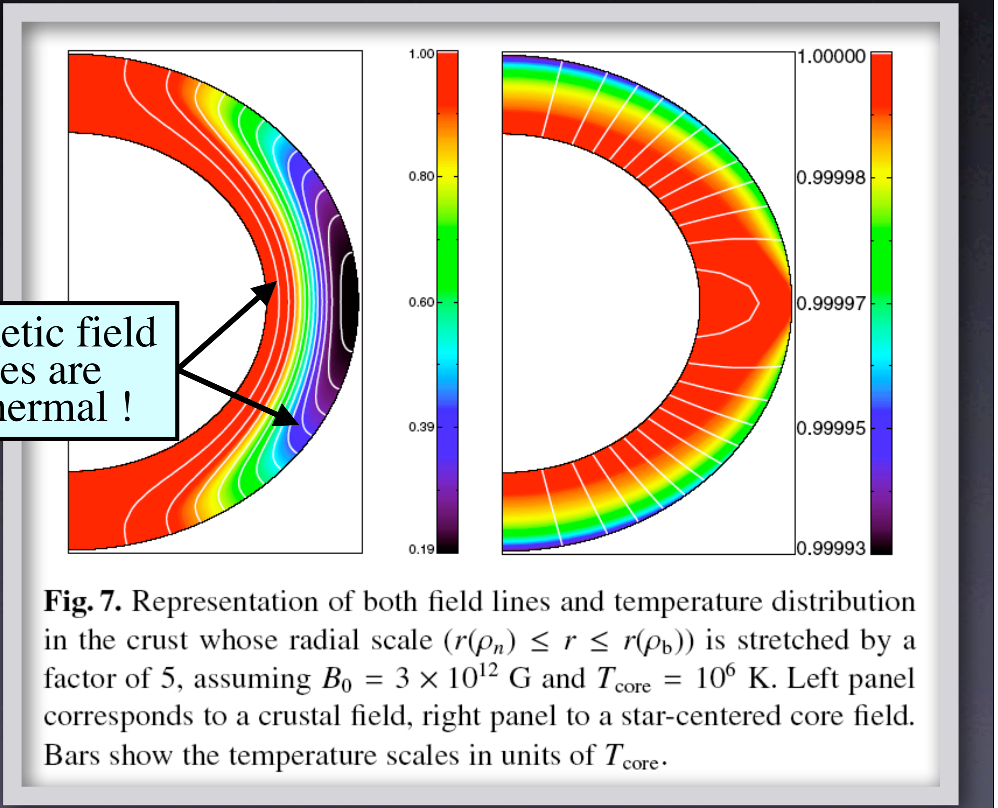
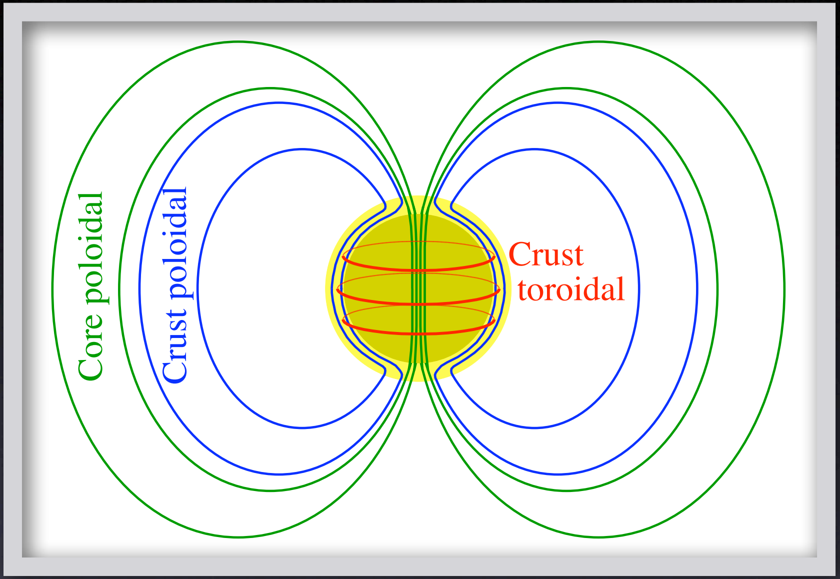


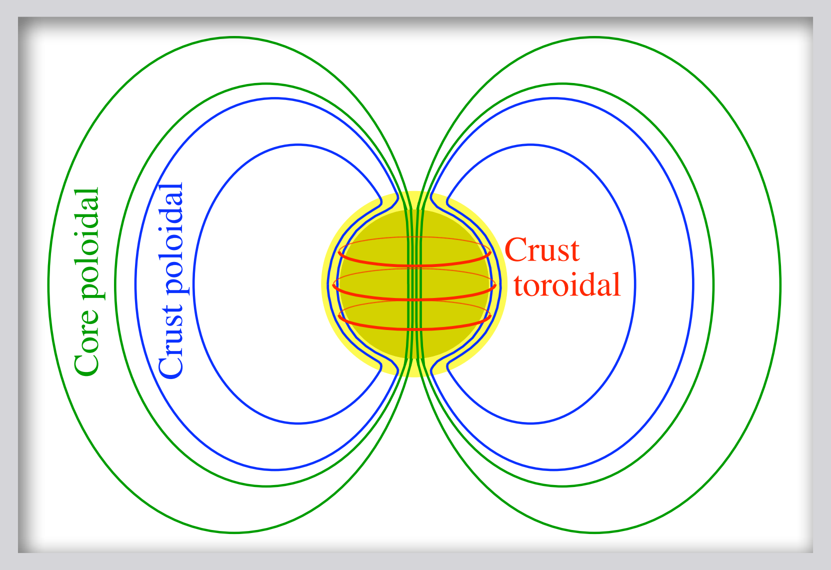
Fig. 7. Representation of both field lines and temperature distribution in the crust whose radial scale ($r(\rho_n) \leq r \leq r(\rho_b)$) is stretched by a factor of 5, assuming $B_0 = 3 \times 10^{12}$ G and $T_{\rm core} = 10^6$ K. Left panel corresponds to a crustal field, right panel to a star-centered core field. Bars show the temperature scales in units of $T_{\rm core}$.

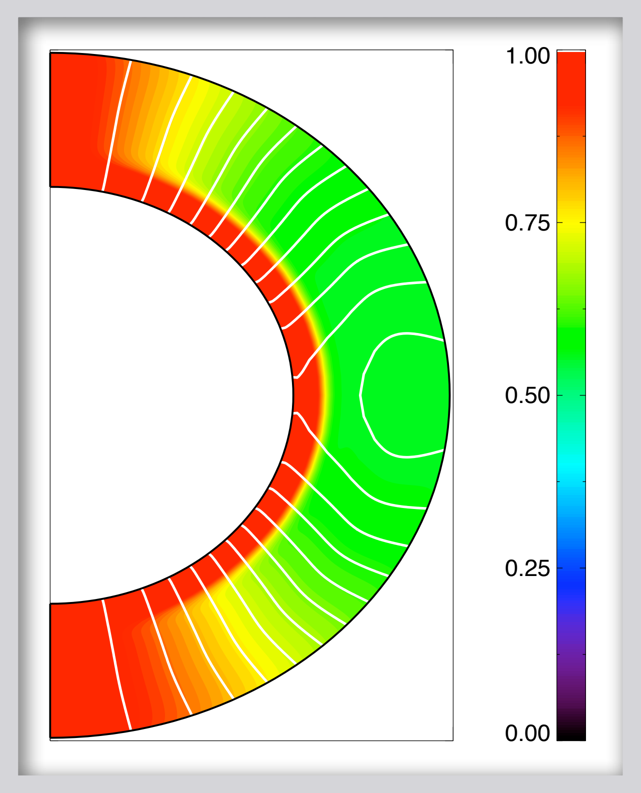
Temperature distribution in magnetized neutron star crusts, Geppert, Küker & Page, 2004 A&A 426, 267

Add a Toroidal Component



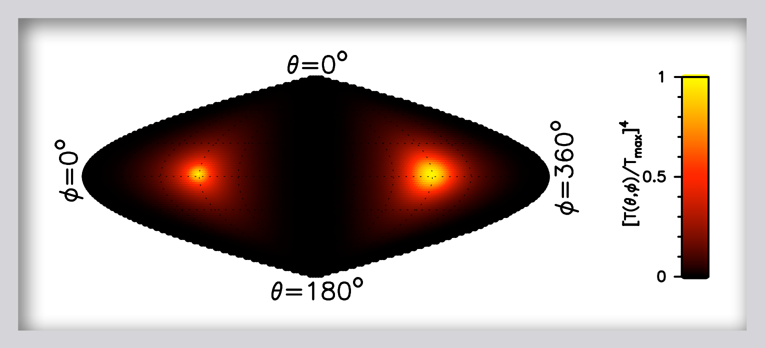
Add a Toroidal Component





Temperature distribution in magnetized neutron star crusts. II. The effect of a strong toroidal component, Geppert, Küker & Page, 2006 A&A 457, 937

Composite BB Fit for RX J1856



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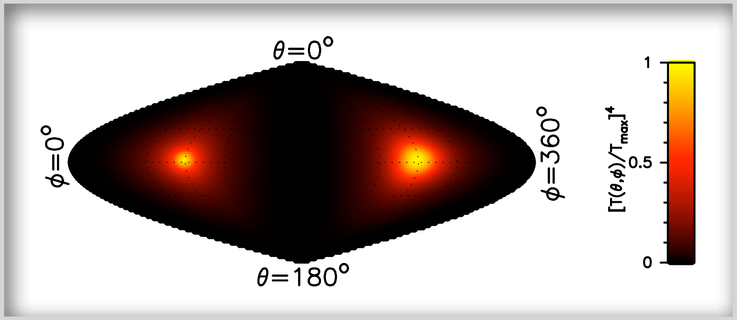


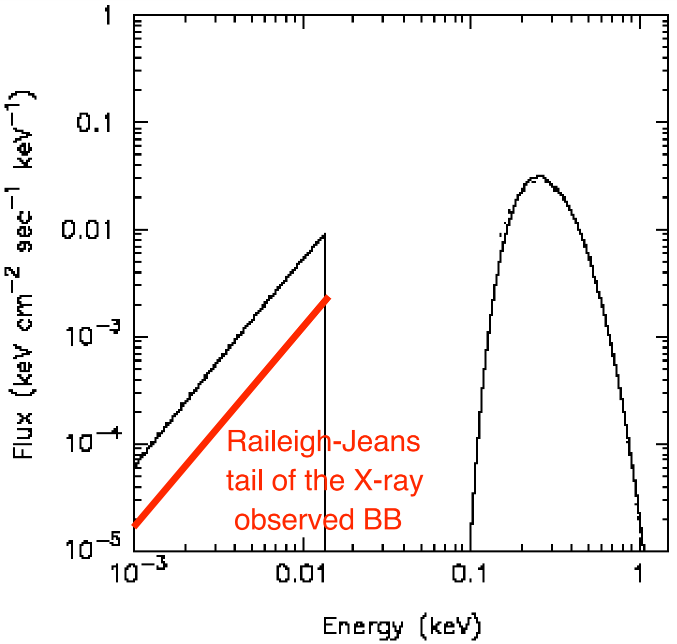
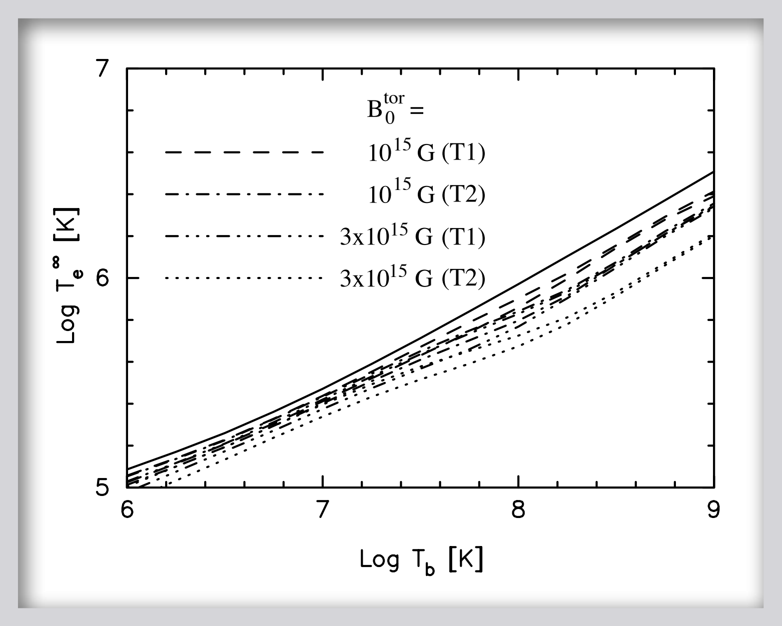


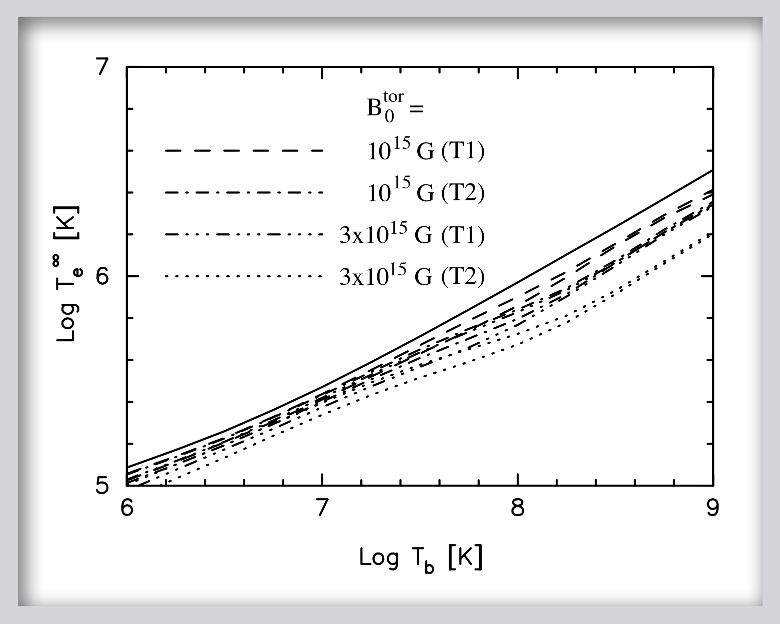
Fig. 10. Fit of the spectrum of RX J1856.5-3754. Dotted lines show the two blackbodies fit to the data from Trümper *et al.* (2004). The continuous line show our results: the star has a radius R = 14.4 km and R_∞ = 17.06 km for a 1.4 $M_\odot$, at a distance of 122 pcs ($N_H = 1.6 \times 10^{20}$ cm^{-2} for interstellar absorption) and the observer is assumed to be aligned with the rotation axis. The magnetic field structure corresponds to model c of Figure 6 adjusted to the 14.4 km radius with $T_b = 6.8 \times 10^7$ K, resulting in $T_{\rm eff}^\infty = 4.62 \times 10^5$ K and $T_{\rm max}^\infty = 8.54 \times 10^5$ K

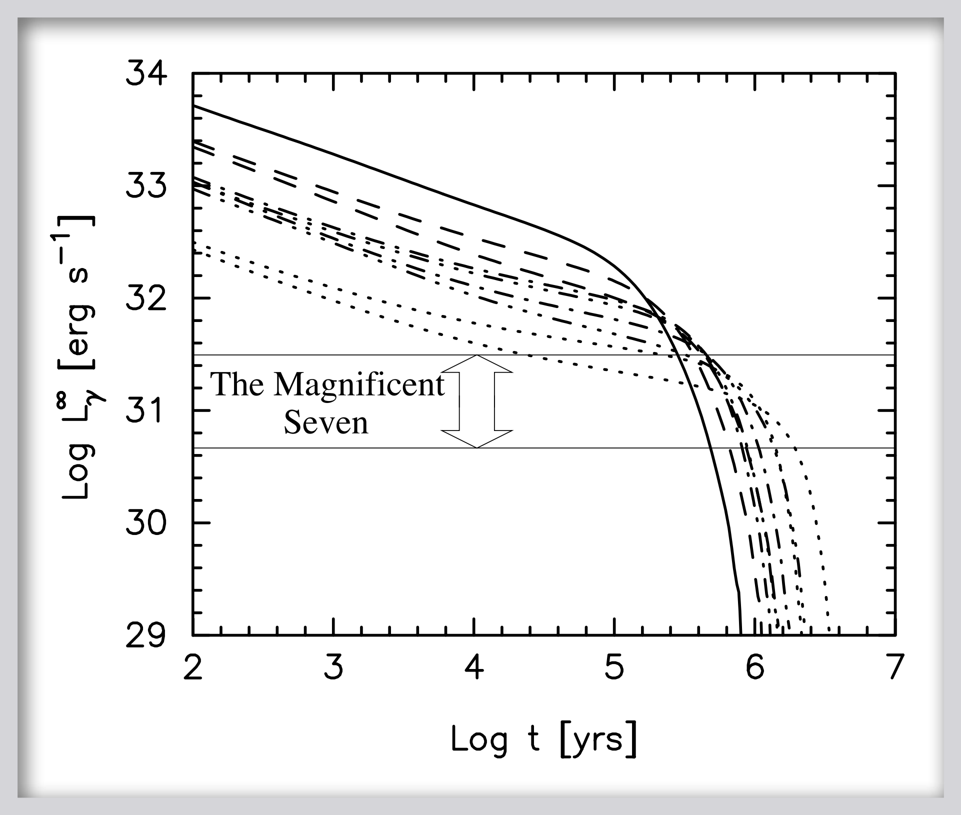
Temperature distribution in magnetized neutron star crusts. II. The effect of a strong toroidal component, Geppert, Küker & Page, 2006 A&A 457, 937

Long Live the Magnificent Seven !



Long Live the Magnificent Seven !





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Conclusions

- Many options for fast cooling, complicated by possible pairing of nucleons (or/and hyperons, quarks).
- Minimal Cooling: little evidence for fast cooling, but nevertheless we have some conspicuous cases.
- Still large uncertainties on observed luminosities (and ages).
- The "Magnificent Seven": are they permeated by strong toroidal fields ? Is this telling us something ?


"That's all Folks!"