Equation of state in the inner crust: discussion of the shell effects

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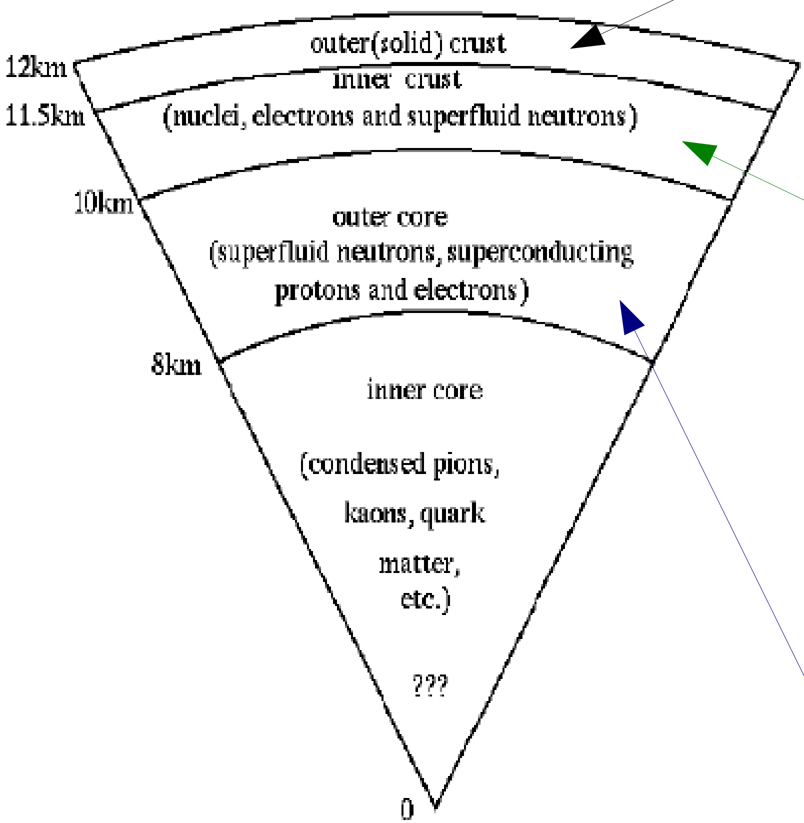
- "bare" calculations of the EoS
- Discussion of the shell effects (cluster / continuum)
- Equation of state in the inner crust with partial removal of continuum shell effects
- ...

Anatomy of a neutron star

Neutron star at equilibrium : statistical, beta-stability.



$\rho < 10^3$ g/cm^3 : ^{56}Fe bcc lattice + cloud of electrons

$\rho > 10^3$ g/cm^3 : electrons fully ionized

$\rho > 10^6$ g/cm^3 : electrons are relativistic

$\rho > 10^7$ g/cm^3 : ^{56}Fe → ^{84}Se → ...

$\rho > 10^9$ g/cm^3 : e capture:

n rich exotic nuclei ^{82}Ge → ...→ ^{118}Kr

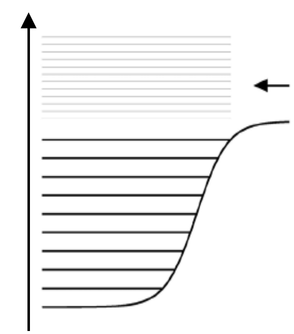
$\rho > 4.3\ 10^{11}$ g/cm^3 : neutron drip

→ nuclear clusters (lattice)

surrounded by a neutron gas

zone	density $\rho/\rho0$
11	0.0017
10	0.0024
...	...
1	0.5

microscopic description:



$\rho > 2.4\ 10^{14}$ g/cm^3 : nuclei dissolve

→ homogeneous nuclear matter

Still some open theoretical questions:

- Deformation of the nuclear cluster
- Is the neutron gas correctly described ?
  - shell effects,
  - band theory.
- How far the nuclear force is reliable ?
  - low densities,
  - large asymmetries
- Description of the pairing at low density
  - uniform matter: strong/weak,
  - non-uniform matter

Self-consistent Hartree-Fock-Bogoliubov mean-field based on a Skyrme interaction

Skyrme interaction : zero range “in medium” nuclear interaction
Fitted to reproduce the properties of nuclear matter
and tuned on nuclei (here SLy4).

Hartree-Fock-Bogoliubov : mean-field model with treat the pairing
correlations acting between all single particle
states.
well adapted for non-homogeneous systems.

More details in N. Sandulescu's talk

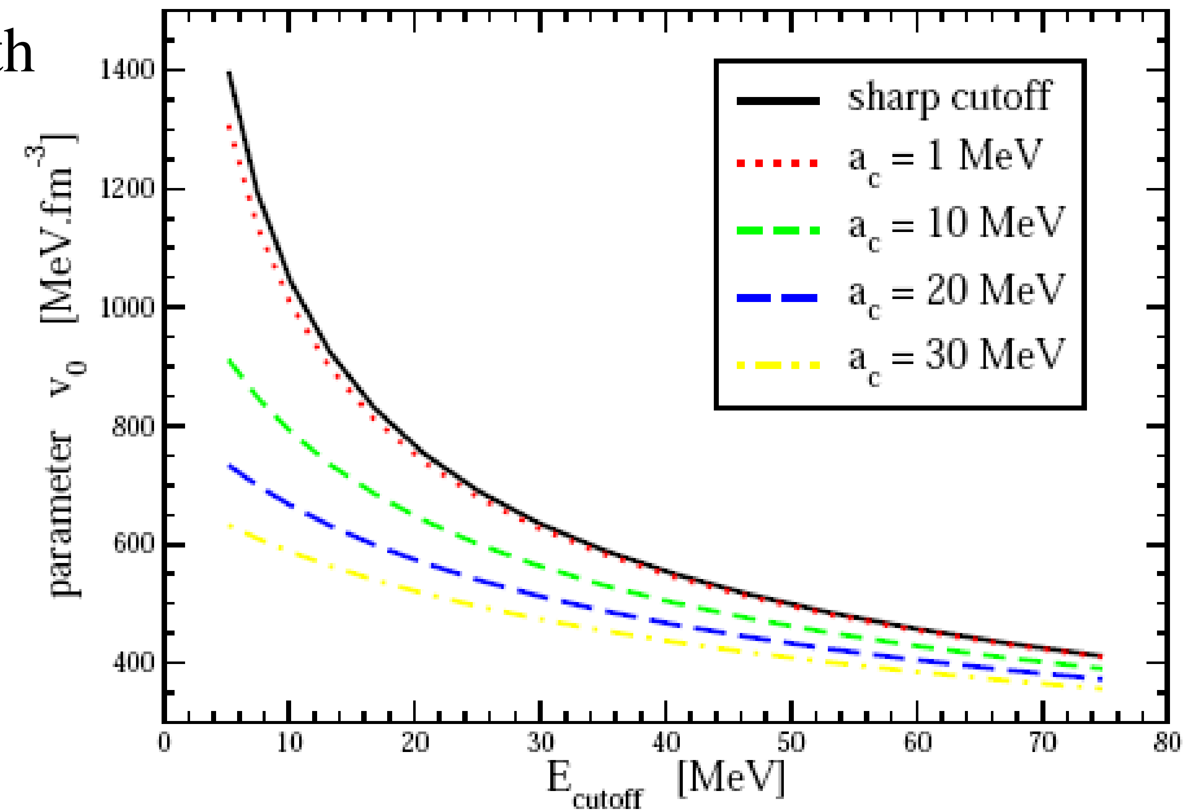
Difficulties : but only numerical, is to treat large boxes (R>30 fm)
-> smooth cut-off of the pairing field @ 20 MeV

Zero range pairing interaction

$$v(r_1 - r_2) = v_0\left[1 - \eta\left(\frac{\rho}{\rho_0}\right)^{\alpha}\right]\delta(r_1 - r_2)$$

For a sharp cut-off: $v_0 = \frac{\hbar^2}{m}\left(\frac{2\pi^2}{\pi/2a_{nn} - k_c}\right)$ Esbensen, Bertsch, Hencken, PRC 1997

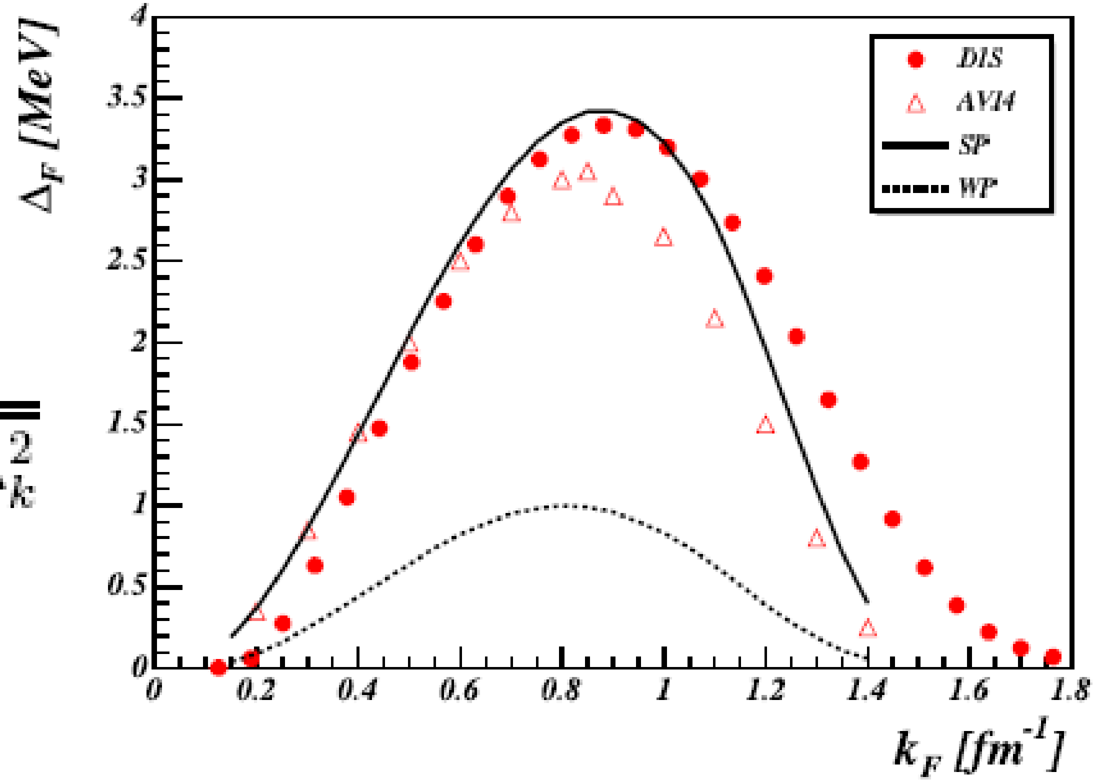
→ Could be extended to smooth cut-off with a Gaussian shape



Zero range pairing interaction

mimic the pairing gap in homogeneous nuclear matter

$$\Delta_p = -\sum_k v_{pk} \frac{\Delta_k}{2\sqrt{(\epsilon_k - \epsilon_F)^2 + \Delta_k^2}}$$

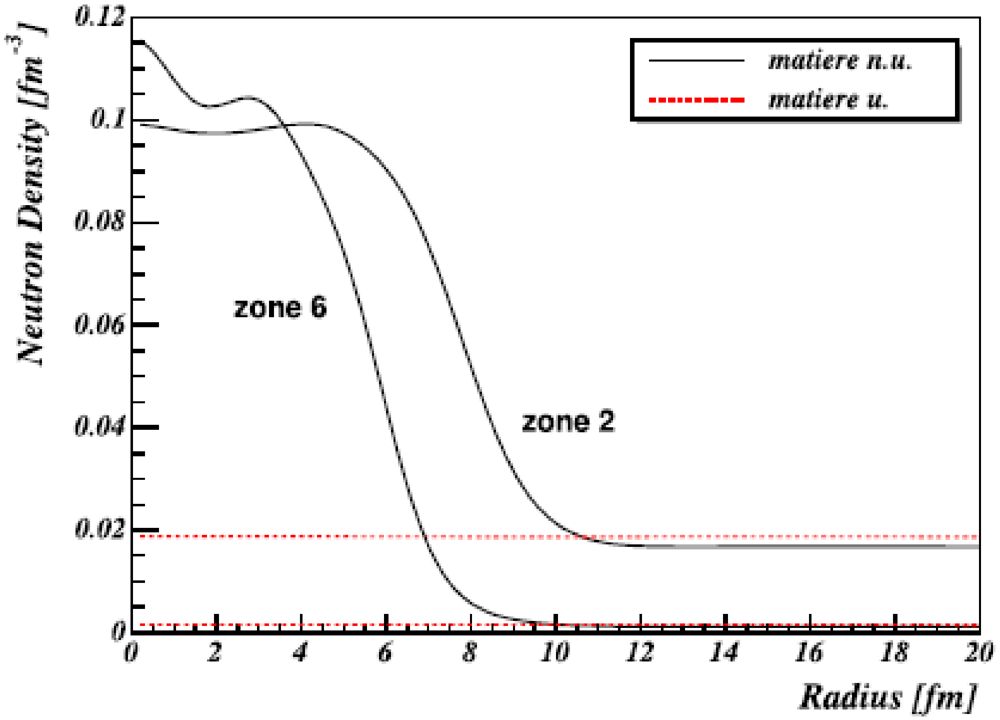


force	V_0 [MeV fm^3]	η	α
SP	-648	0.95	0.45
WP	-648	0.87	0.20

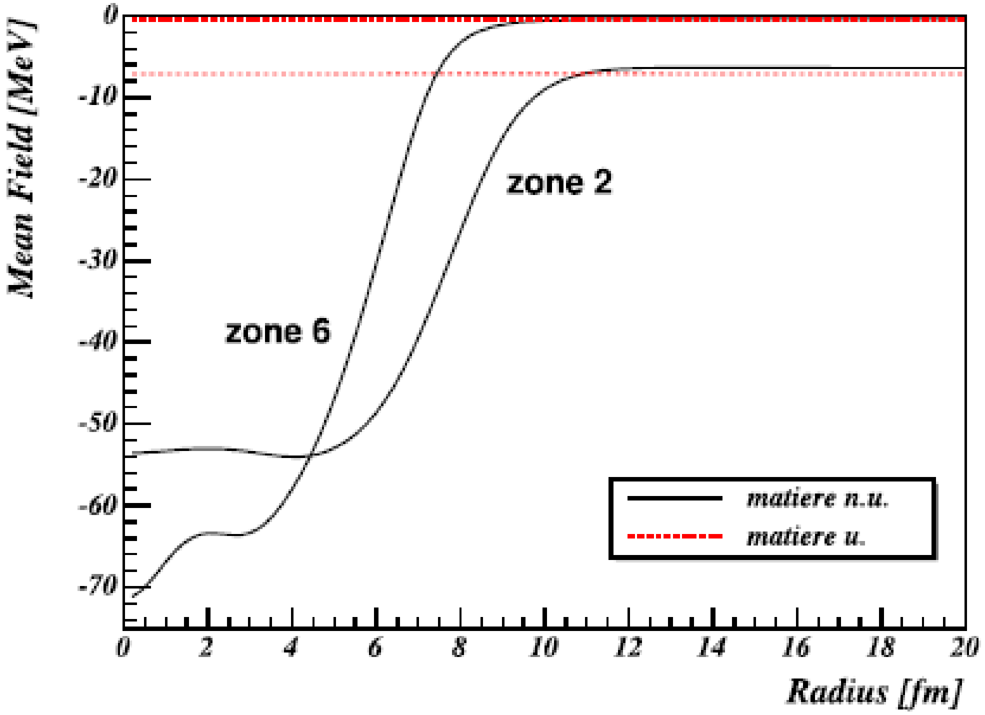
for a smooth cut-off of 20 MeV with a tail of 10 MeV

Self-consistent mean-field calculations with pairing (HFB) for Negele-Vautherin cells

Densities



Mean-Field Potentials

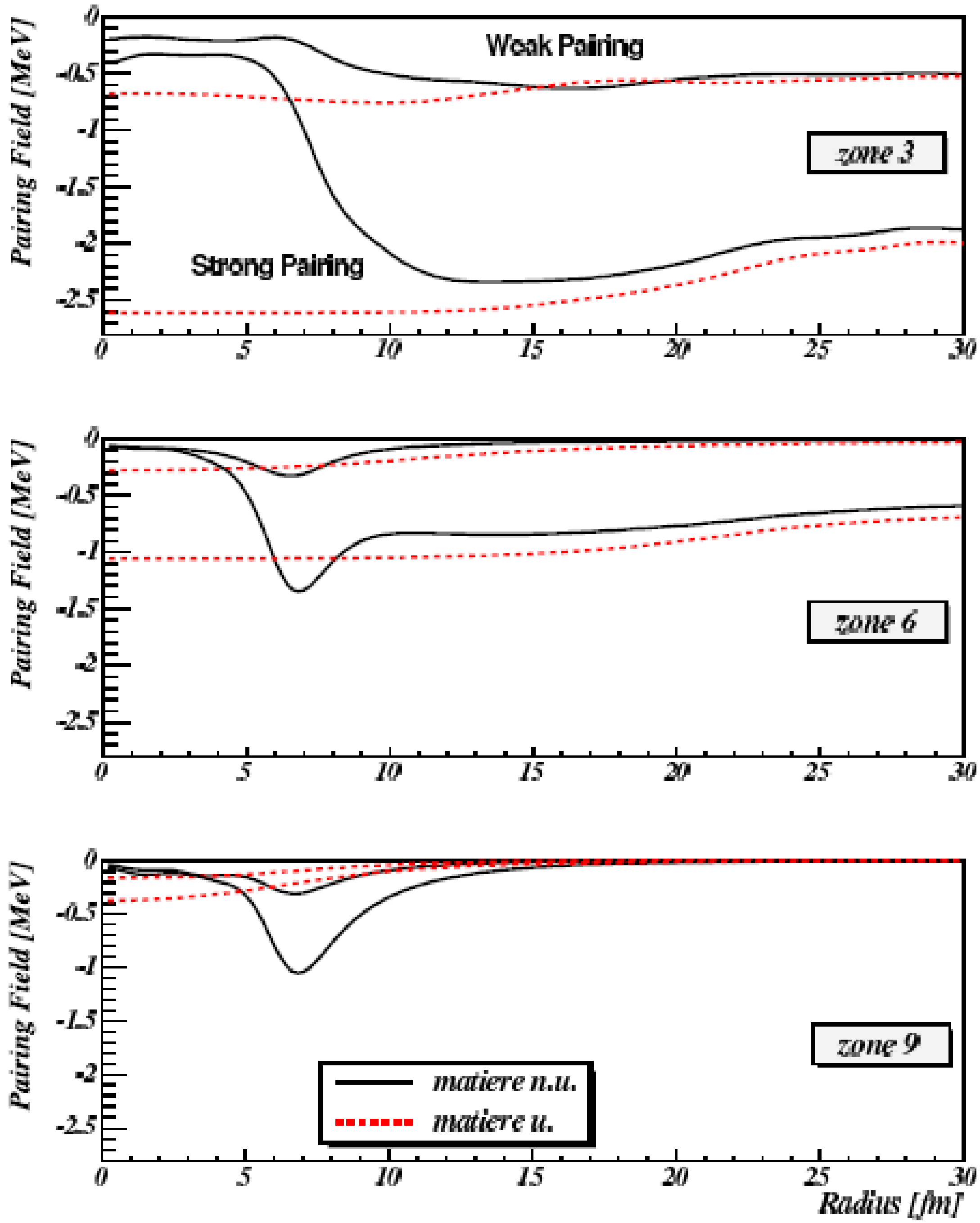


From Ch. Monrozeau, PhD thesis 2007

cf. talk of N. Sandulescu

Weak and Strong pairing fields

cf. talk of
N. Sandulescu



$$\xi \approx \frac{\hbar^2 k_F}{m\Delta}$$

Binding energy in W-S cell & EoS

$$\frac{E}{V}(\rho, N, Z) = \frac{Zm_p + Nm_n}{V} + \frac{A}{V} B(\rho, R_{box}) + \frac{E(lattice)}{V} + \frac{E(electrons)}{V}$$

$$\frac{E(lattice)}{V} = -0.89593 \frac{Z^2 e^2}{r_c}$$ bbc lattice

$$\frac{E(electrons)}{V} = \int dr_1 dr_2 \rho_e(r_1) \frac{e^2}{r_{12}} [\rho_e(r_2) - \rho_p(r_2)]$$

Equation of state

For a given density (ρ):

- minimisation of F/V
- β-stability: $\mu_p + \mu_e = \mu_n$

B, μ_p, μ_n

given by HF or HFB and

$$\mu_e = \frac{dE/V}{dN_e} = \hbar k_{Fe} + \frac{dE(electrons)/V}{dN_e}$$

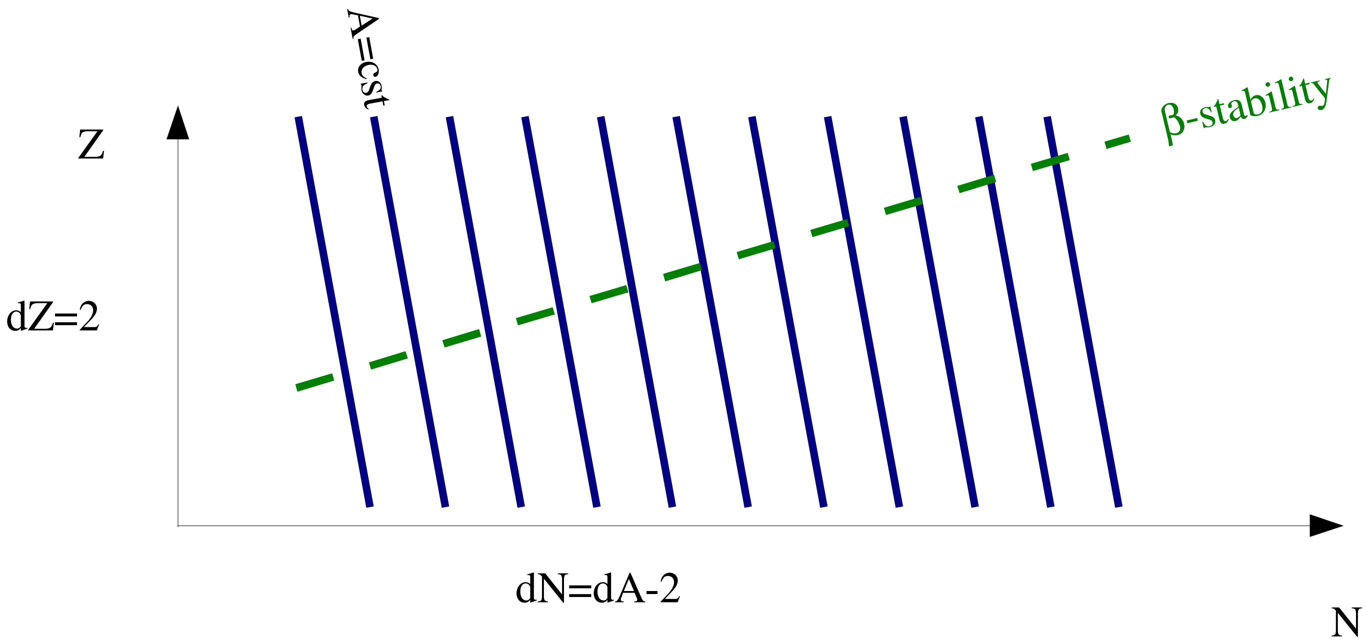
Mapping of (Z,N) cells @ constant density

For a given ρ :

$$\rho = \frac{N+Z}{\frac{4}{3}\pi R^3}$$

- For all the cells, the step is fixed dR=0.2 fm → dV
  ρ is fixed, then A change by step dA ~ ρdV=ρ4πR^{2}dR

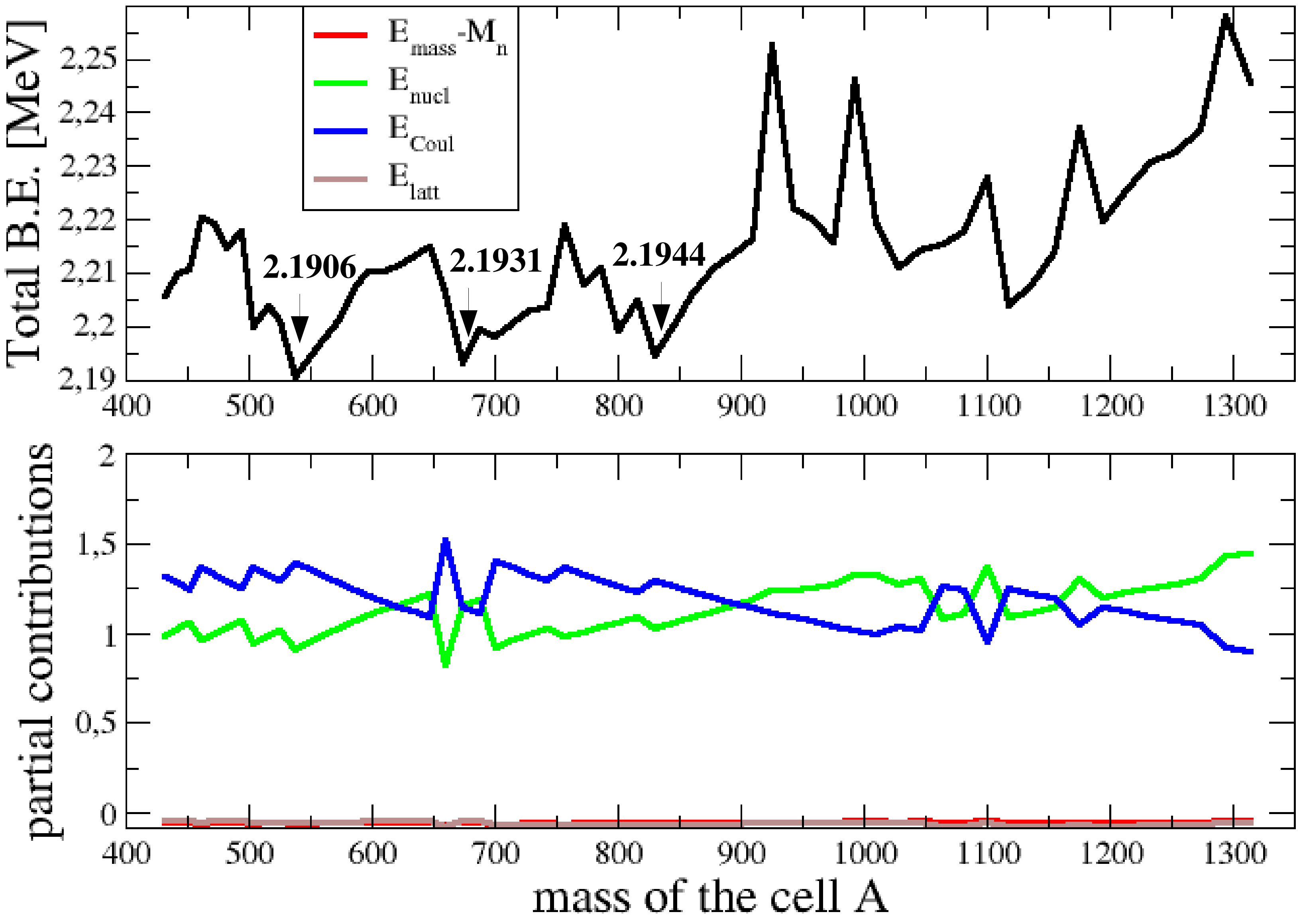
- Define a windows (N,Z) where to scan:
  20 < Z < 60 with dZ=2 → dN=dA-dZ



ρ/ρ0	zone #
0.126	z03
0.055	z04
0.036	z05
0.023	z06
0.009	z07
0.005	z08
0.003	z09
0.002	z10
0.001	z11

HF: Binding energy (zone 05)

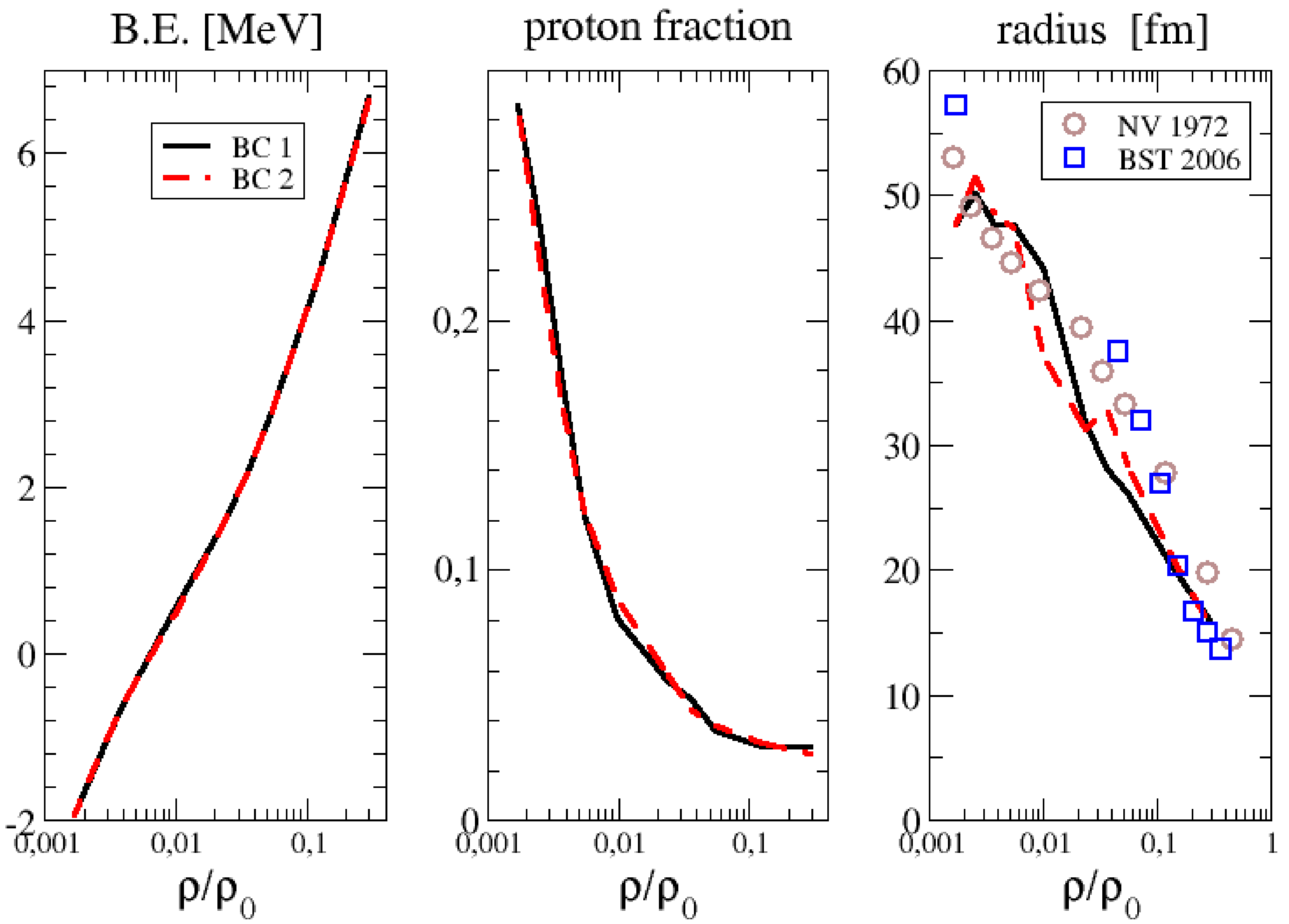
preliminary results



HF: B.E., proton fraction and radius

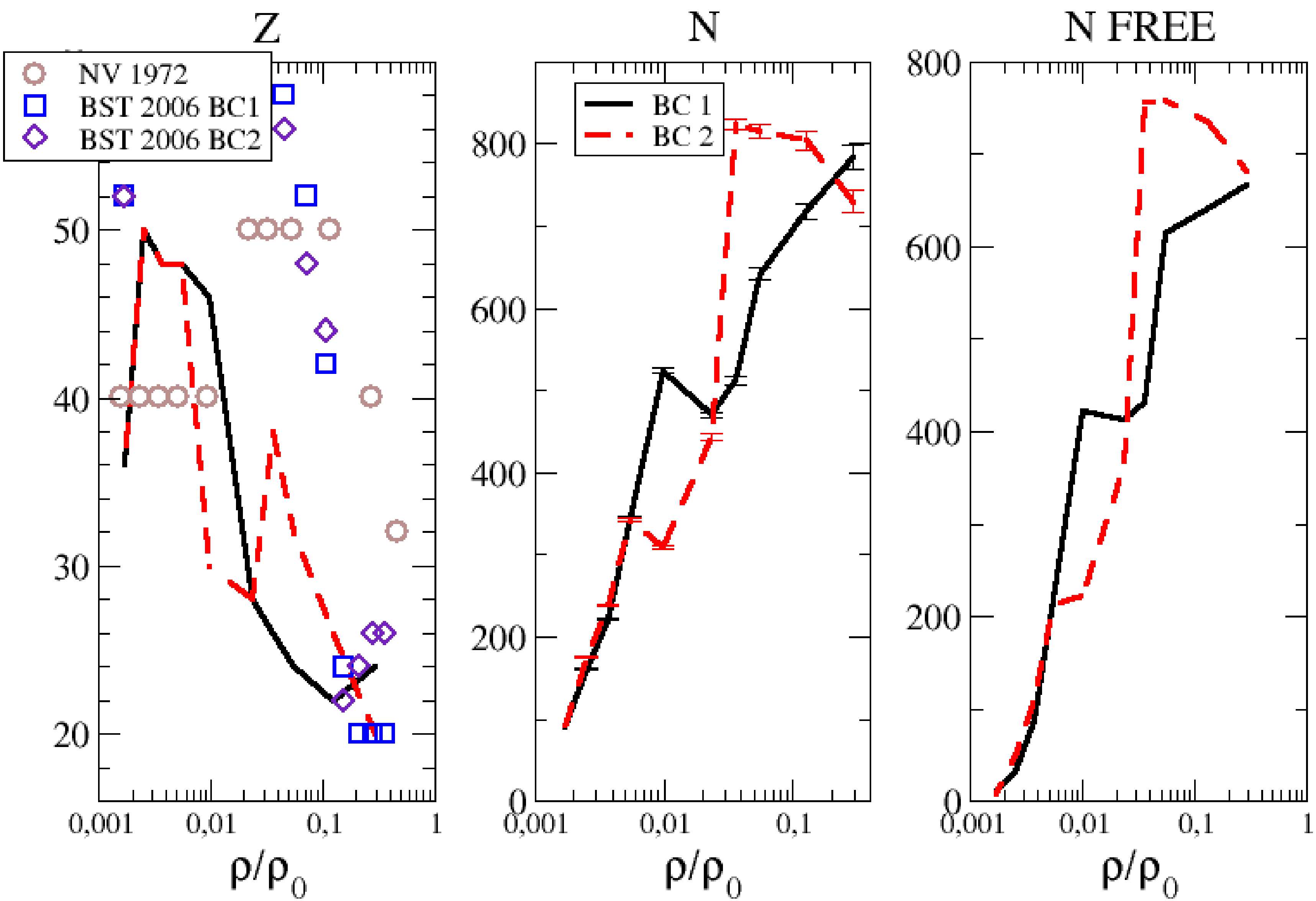
preliminary results

17914 cells



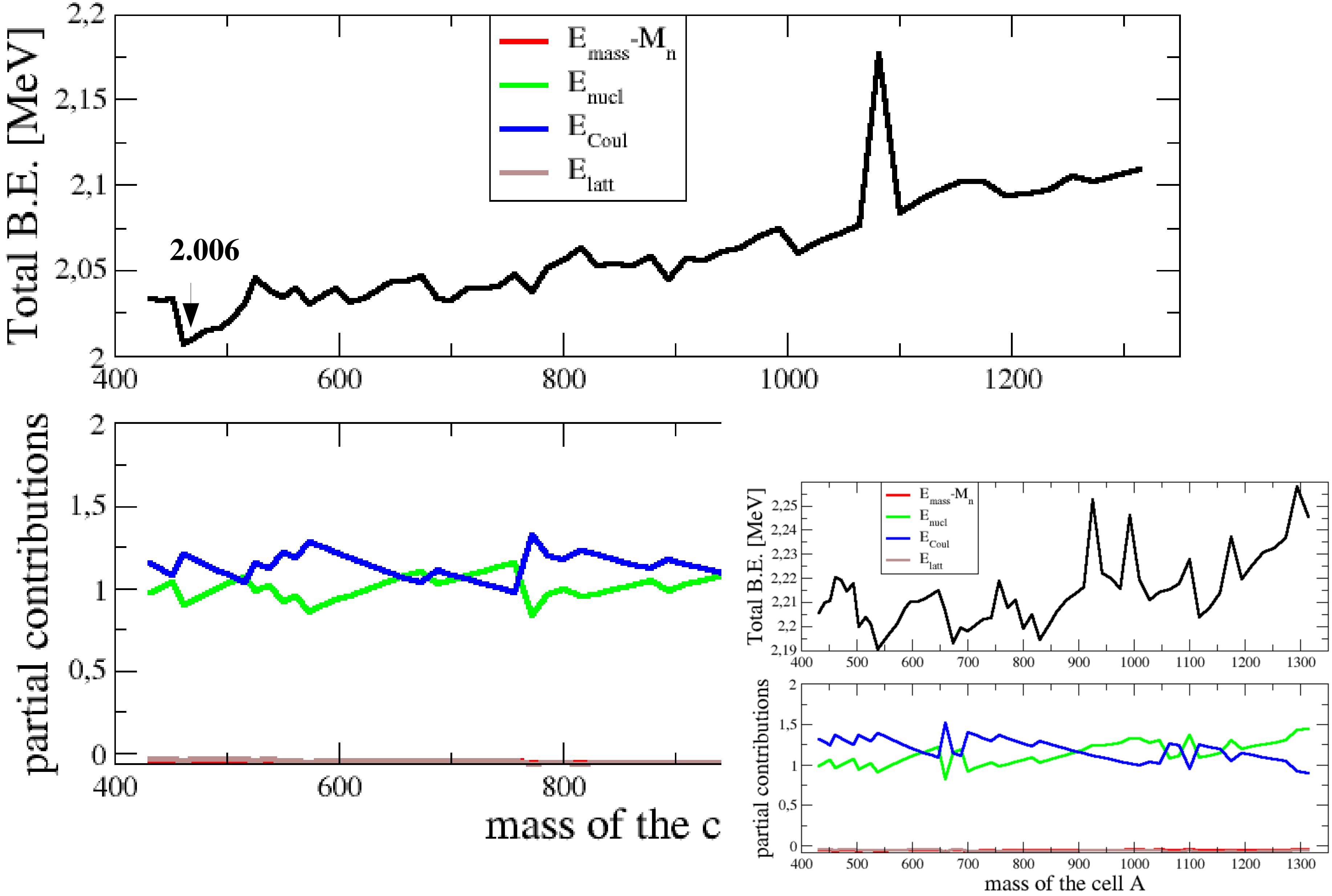
HF: (Z,N)

preliminary results



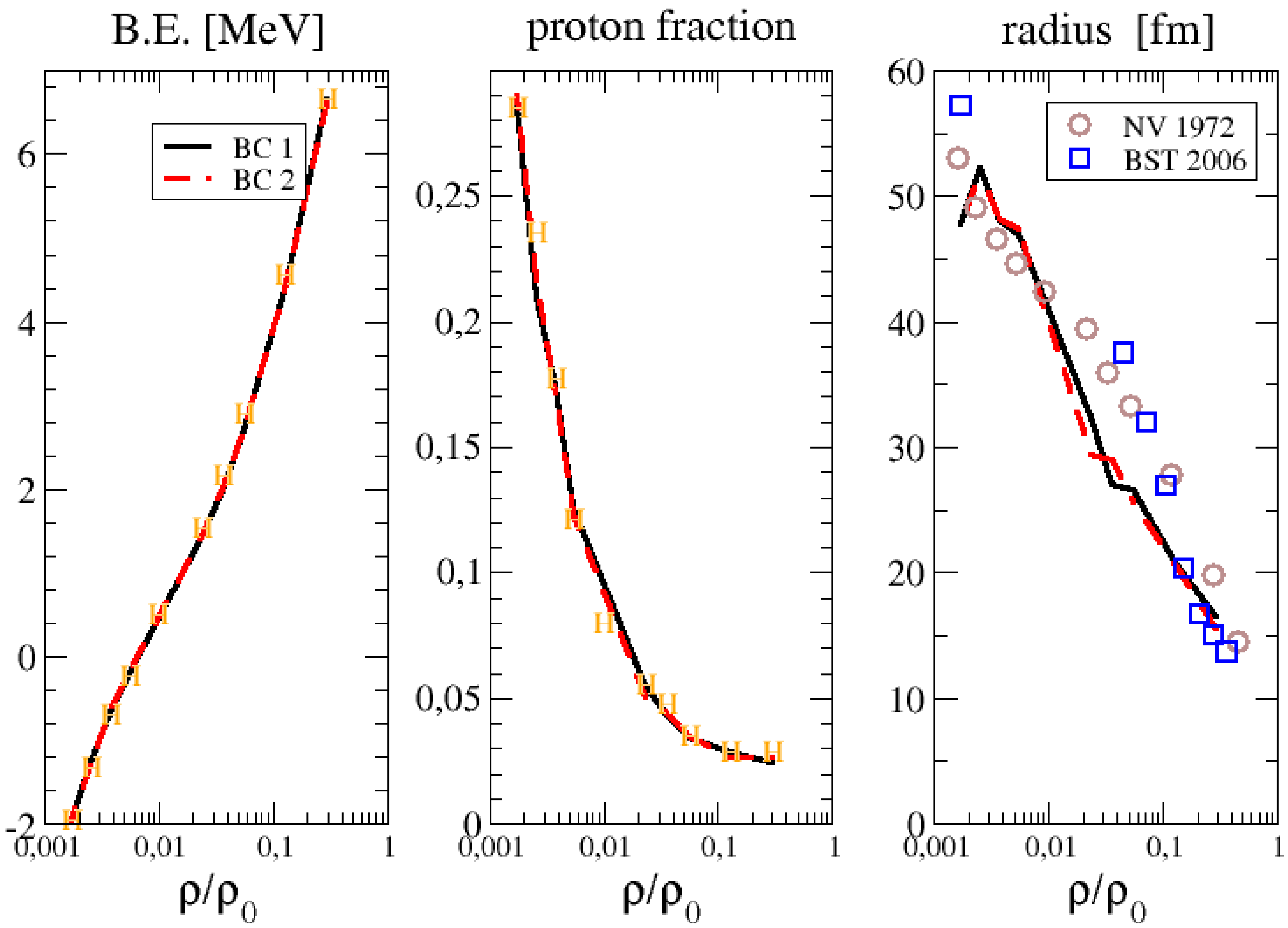
HFB: Binding energy (zone 05)

preliminary results



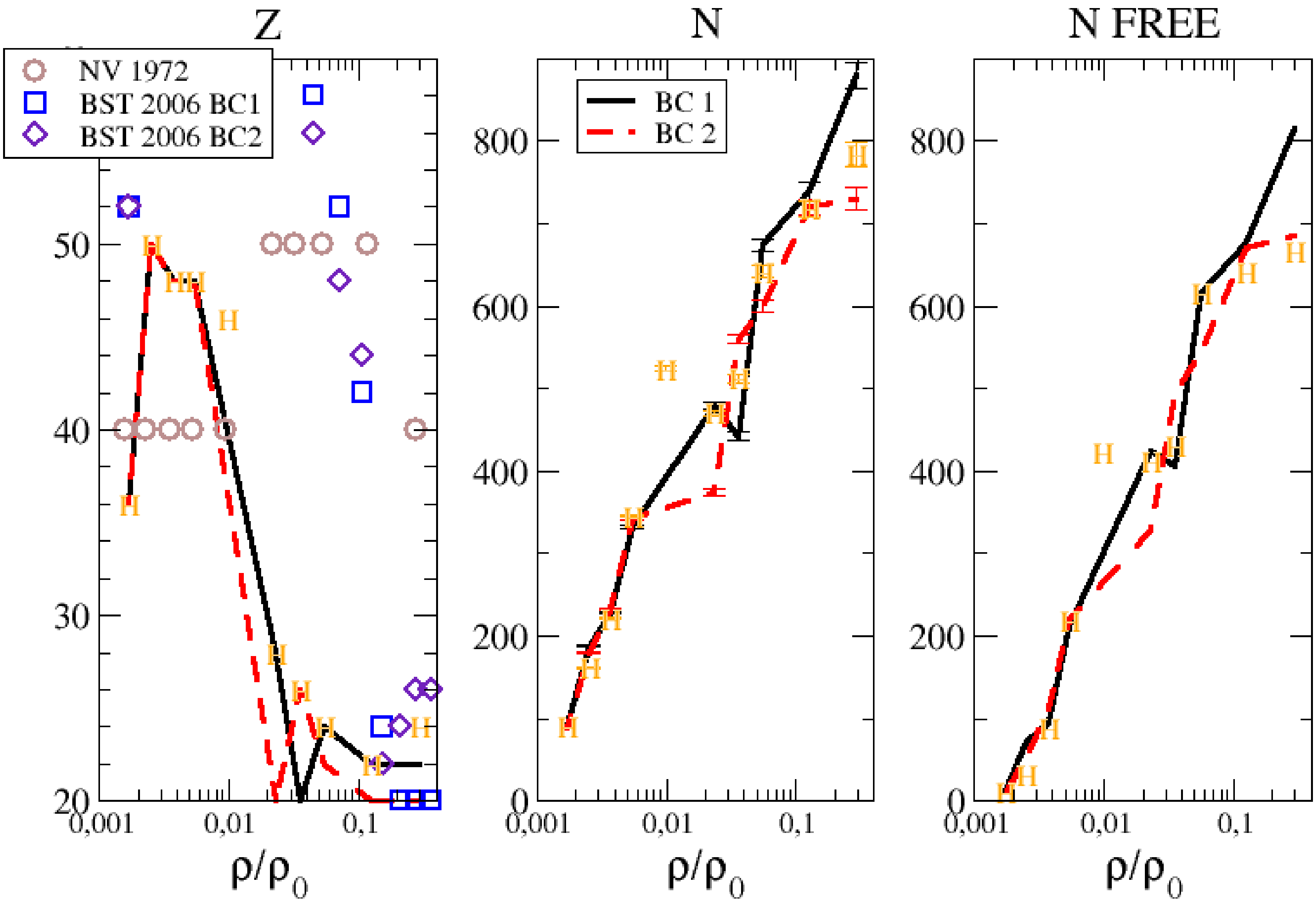
HFB: B.E., proton fraction, radius

preliminary results



HFB: (Z,N)

preliminary results



How solid are the results ?

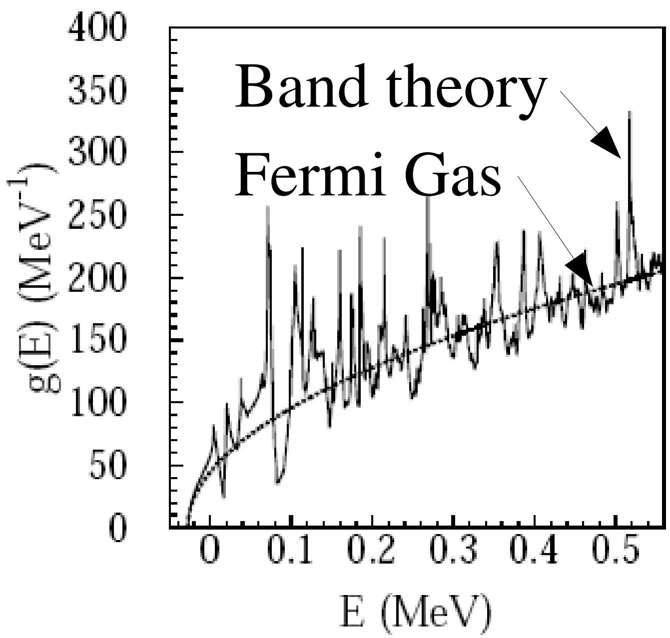
Are there spurious fluctuations spoiling the research of the minimum energy cell ?

What about unbound neutrons ?

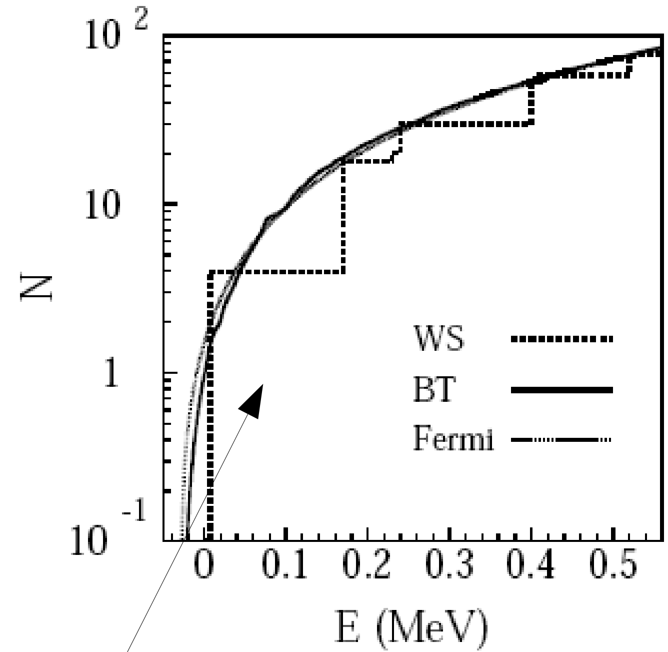
Unbound neutrons :

the problem of spurious shell effects

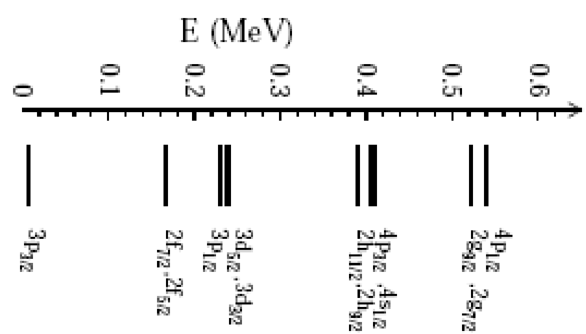
Density of states (unbound neutrons)



Integrated density of states (unbound neutrons)



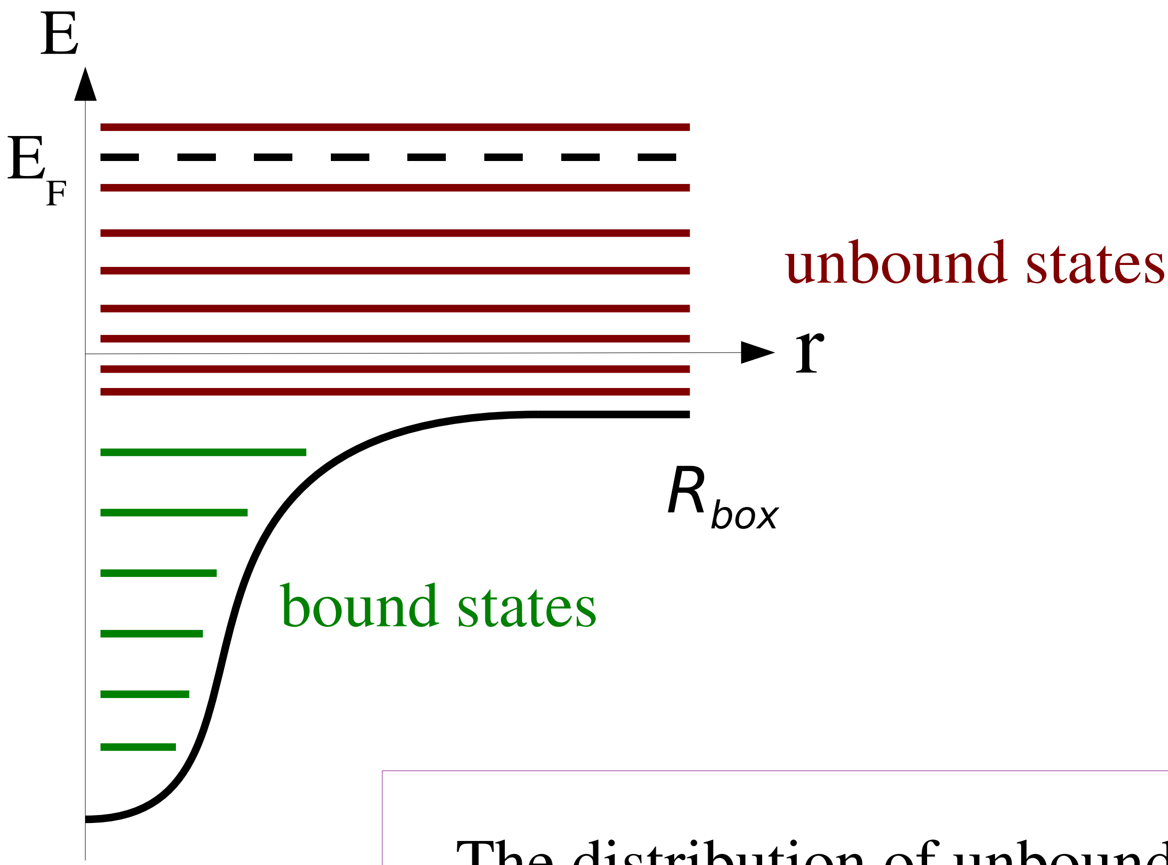
WS approximation:



Shell effects : $\epsilon \sim 100\, keV$

N. Chamel et al., Phys. Rev. C 75, 055806 (2007)

Modelization of the continuum with finite size boxes eigen-states



average level spacing in the continuum :

$$\Delta\epsilon \sim \frac{\hbar^2}{2mR_{box}^2}$$

The distribution of unbound states can vary depending :

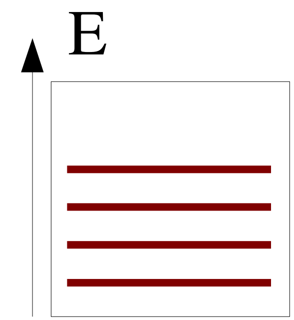
- on the box radius : $\Delta\epsilon \downarrow if\ R_{box} \uparrow$
- on the boundary conditions.

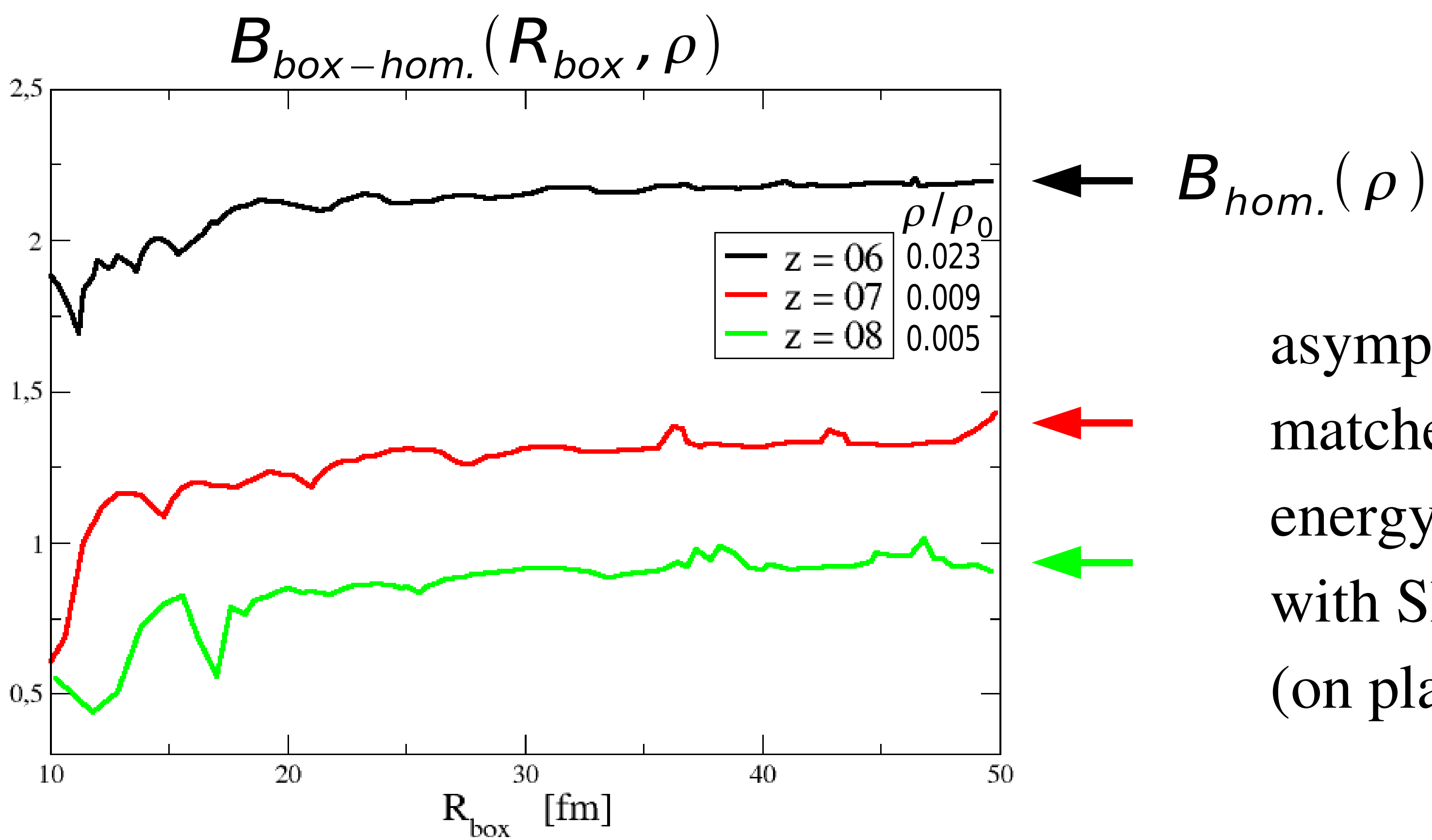
Estimation of shell effects for unbound neutrons

Simulate homogeneous pure neutron matter in a cell:

Z=0, only N → no cluster

Set of cells with radius [10:50] fm @ fixed density ρ

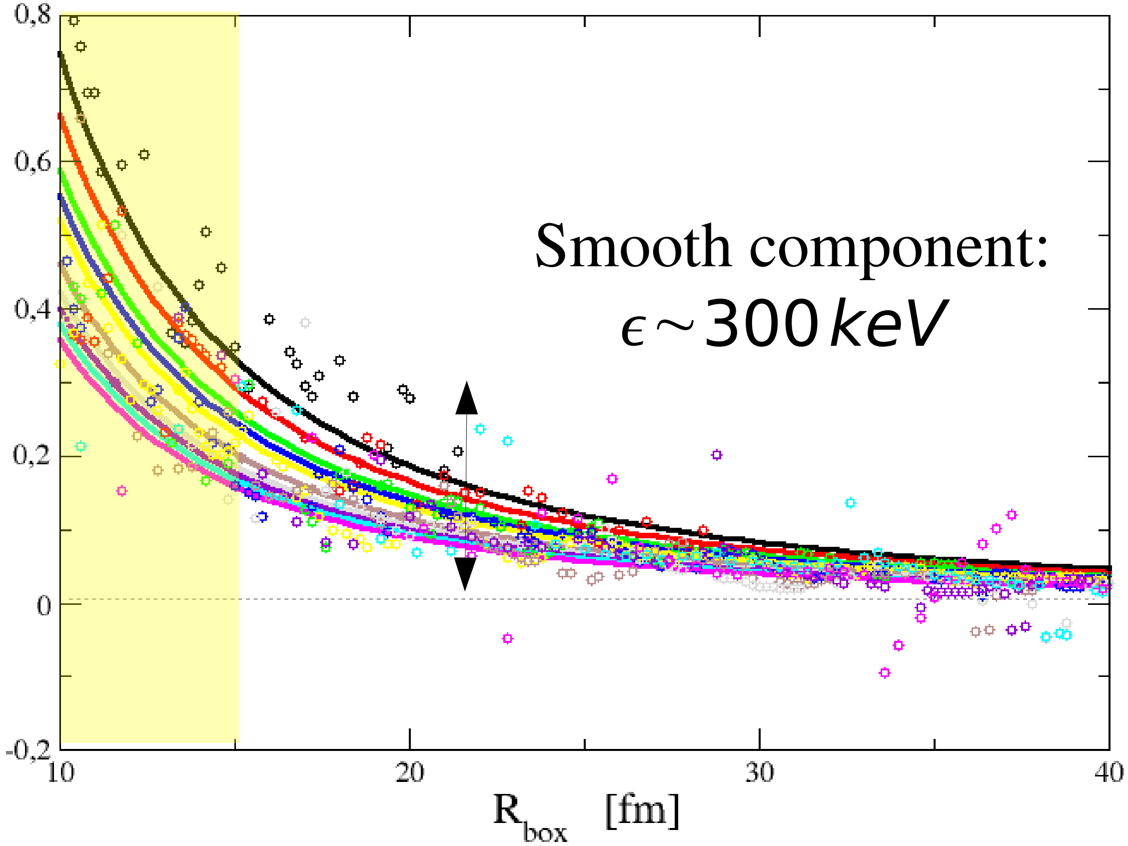




asymptotically matches homogeneous energy calculated with Skyrme interaction (on plane wave basis).

Smooth and residual shell effects

$$B_{hom.}(\rho) - B_{box-hom.}(R_{box}, \rho)$$

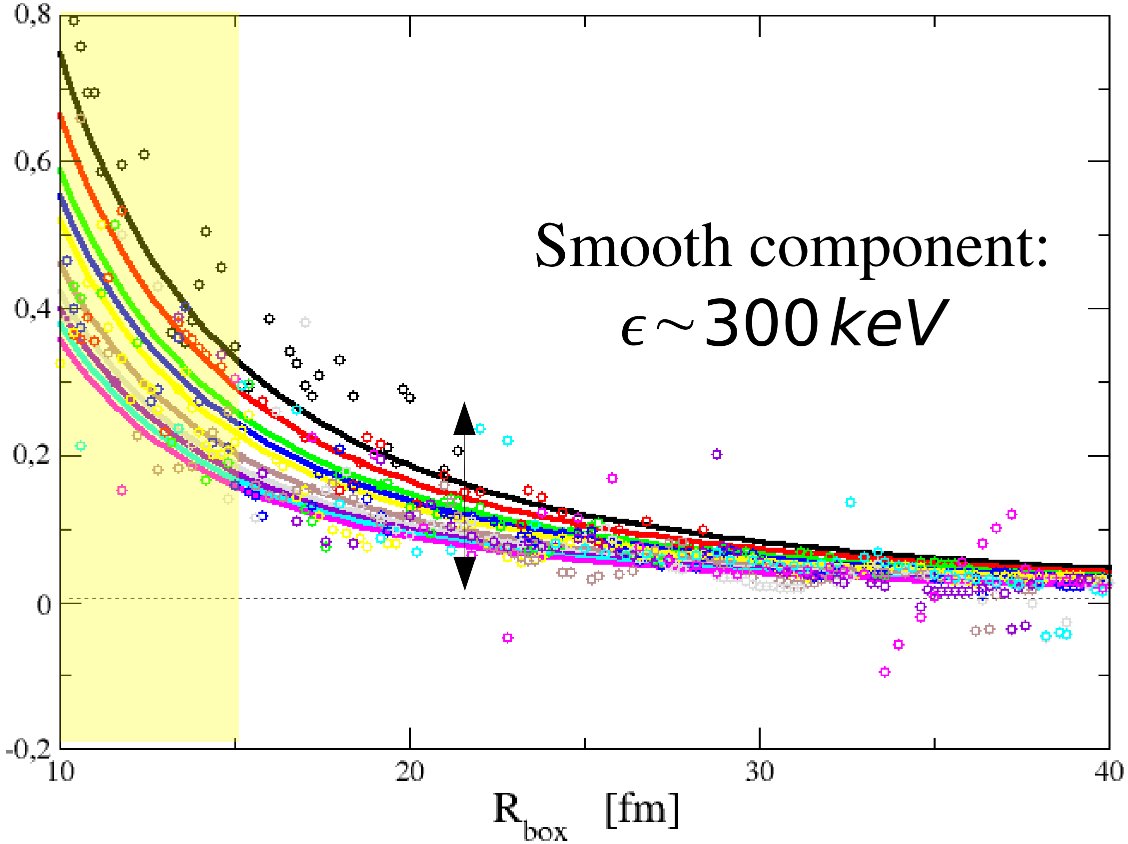


The finite size effect is fitted with

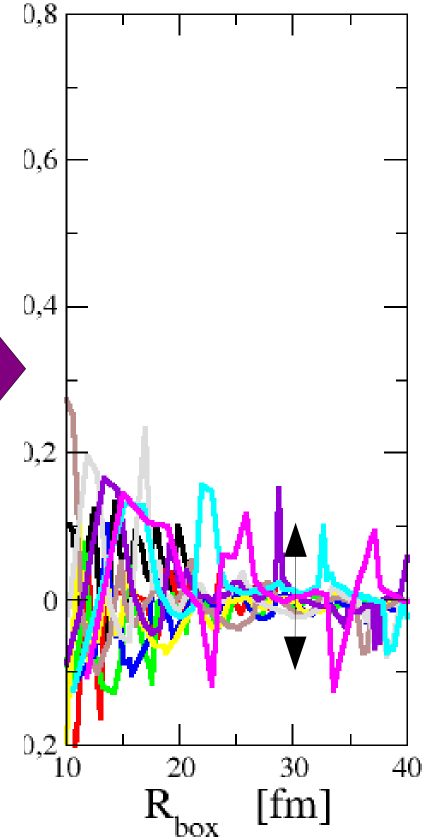
$$89.05 \left(\frac{\rho}{\rho_0} \right)^{0.1425} R_{box}^{-2}$$

Smooth and residual shell effects

$B_{hom.}(\rho) - B_{box-hom.}(R_{box}, \rho)$



after substraction



residual component:

$\epsilon \sim 50\,keV$

The finite size effect is fitted with

$$89.05 \left(\frac{\rho}{\rho_0} \right)^{0.1425} R_{box}^{-2}$$

Irreducible error of the model

$\epsilon \sim 50\,keV$

Back to inhomogeneous matter

unbound neutrons fills the boundary conditions

→ induces spurious shell effects in the continuum

"Corrected" binding energy (exact in homogeneous matter) :

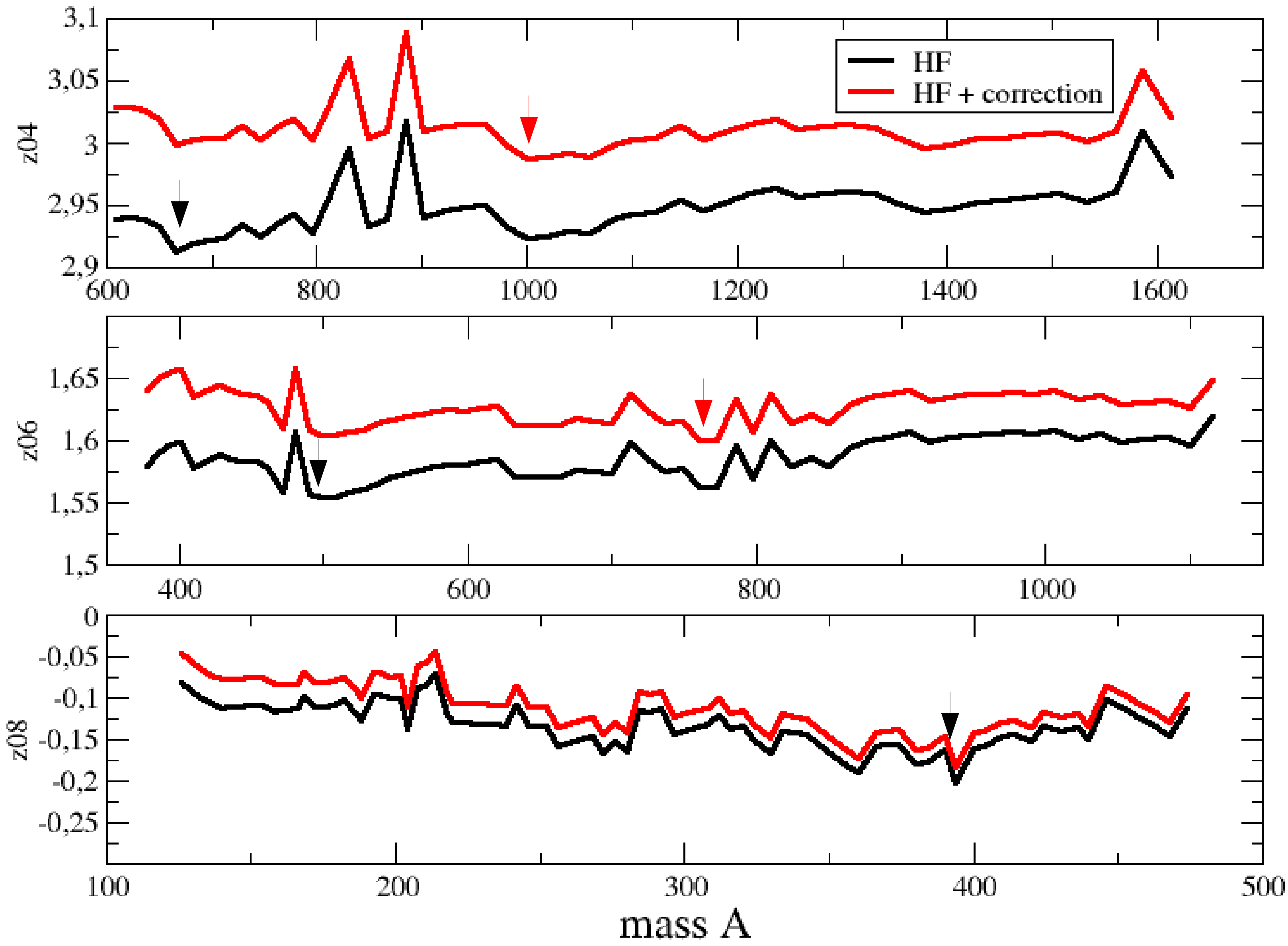
$$B(\rho, R_{box}) \approx B_{box-inhom.}(\rho, R_{box}) + \left[B_{hom.}(\rho_{unbound}) - B_{box-hom.}(\rho_{unbound}, R_{box}) \right]$$

↓

$$B(\rho, R_{box}) \approx B_{box-inhom.}(\rho, R_{box}) + 89.05 \left(\frac{\rho}{\rho_0} \right)^{0.1425} R_{box}^{-2}$$

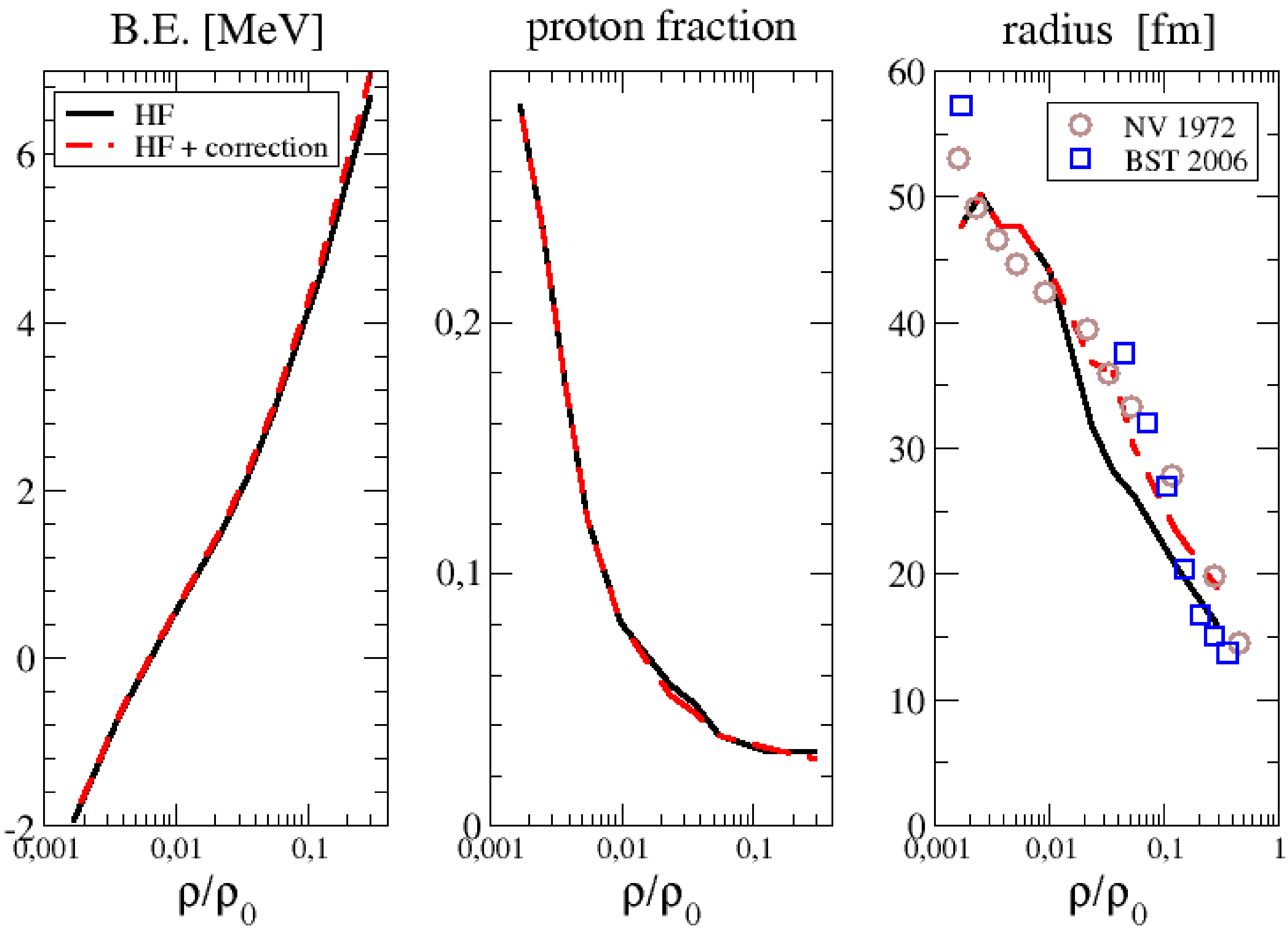
Treatment of the spurious shell effects (HF)

preliminary results

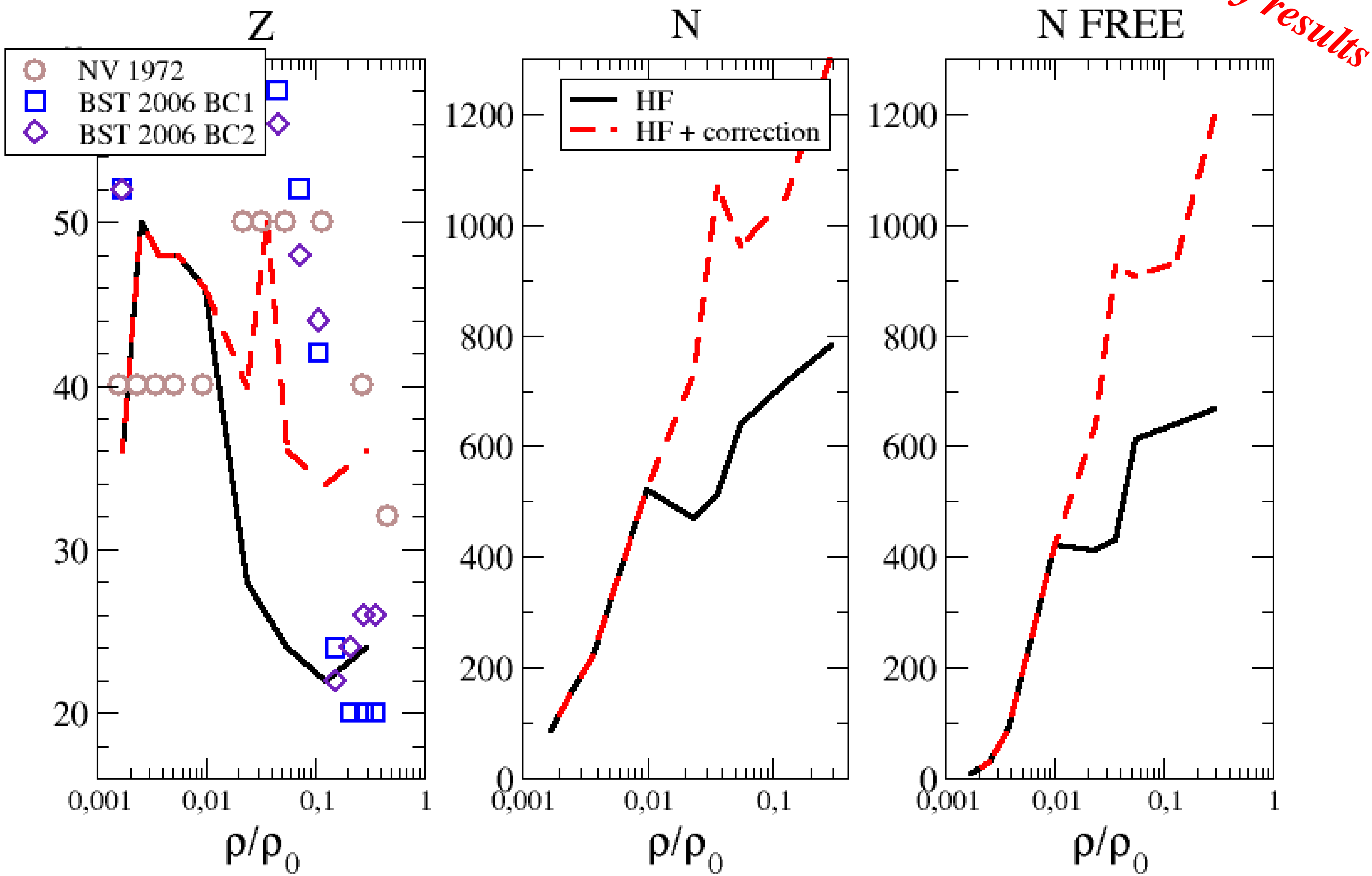


HF: B.E., proton fraction, radius

preliminary results



HF: (Z,N)


preliminary results
Z
N
N FREE
NV 1972
BST 2006 BC1
BST 2006 BC2
HF
HF + correction
20
30
40
50
0
200
400
600
800
1000
1200
0,001
0,01
0,1
1
ρ/ρ0

Conclusions

• Equation of state obtained is based on an unique nuclear interaction (Skyrme)

• The properties of the inner crust are mainly given by unbound neutrons states and electrons.

then :

→ **nuclear clusters** induce **corrections** to homogeneous matter properties

→ It is then important to have the best description of homogeneous neutron gas.

• Spurious shell effects could induce shifts of the energy up to 300 keV (depending of the radius)

We have proposed a method to partly remove spurious shell effects and reduce the fluctuations to about 50 keV.

Outlooks

- Perform the same analysis for system with
  - pairing
  - finite temperature

- The model provides the basic ingredients of macroscopic models ;
- Link between gap in homogeneous matter / non-hom. ;
- Application : cooling process (specific heat). *cf N. Sandulecu*
- Explore the sensitivity of the macroscopic variables on :
  - Nuclear interaction ;
  - Symmetry energy ;
  - Pairing interaction (SP/WP) ;