Dynamics of vortex pinning

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QUESTION

Vortices are attracted to nuclei in the crust,
<u>but do they pin?</u>

<u>Probably they cannot</u>

Why worry about vortex pinning?

To develop a theory of NS "seismology"

Some observed modes in NSs:

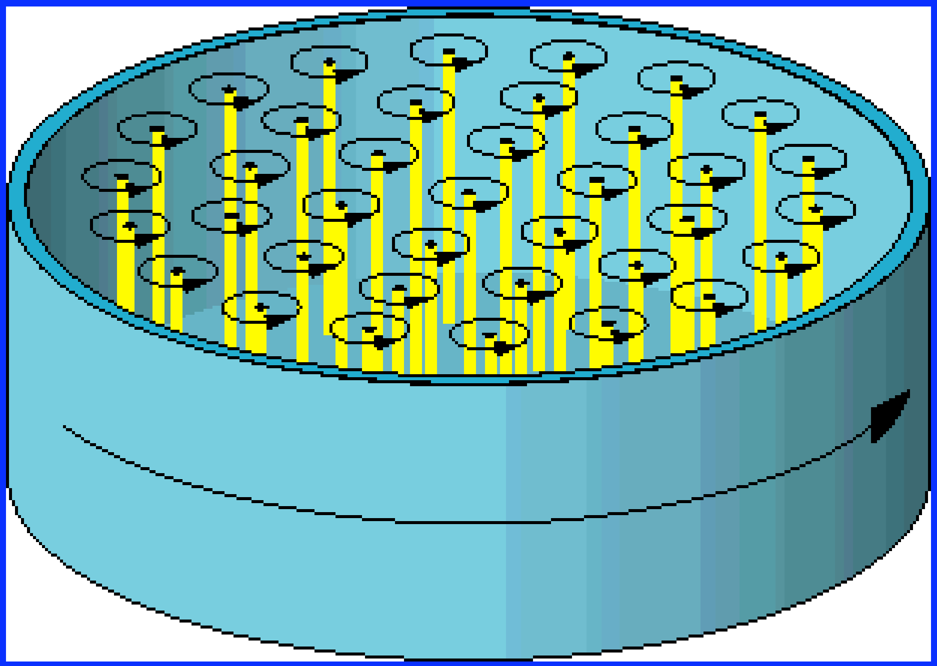
- Spin jumps (glitches).
- Precession ("wobble", nutation). Constrains pairing states of the outer core. (Link 03).
- Stochastic spin variations (timing noise).
- Crust shear modes. Difficult to explain with a strange star. (see Watts & Reddy 07).

Why worry about vortex pinning?

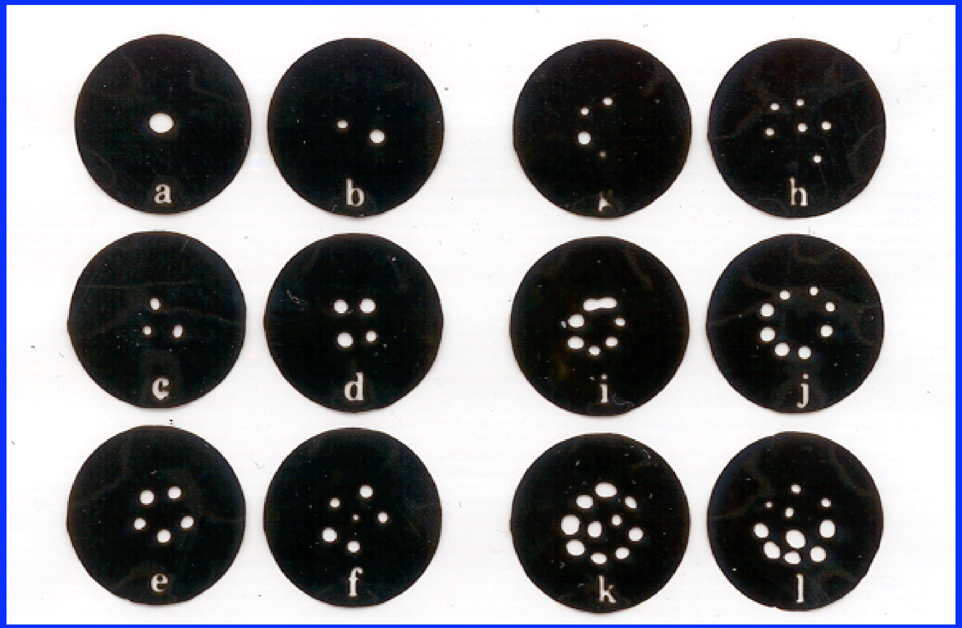
Phenomena to explain/understand:

- Pinning/unpinning might be responsible for observed spin jumps (glitches).
  (Anderson & Itoh 75; Alpar et al. 81; Link & Epstein 96).
- Problem: pinning is inconsistent with observations of long-period NS precession.
  (Shaham 77; Sedrakian et al. 99; Link 03; 06).

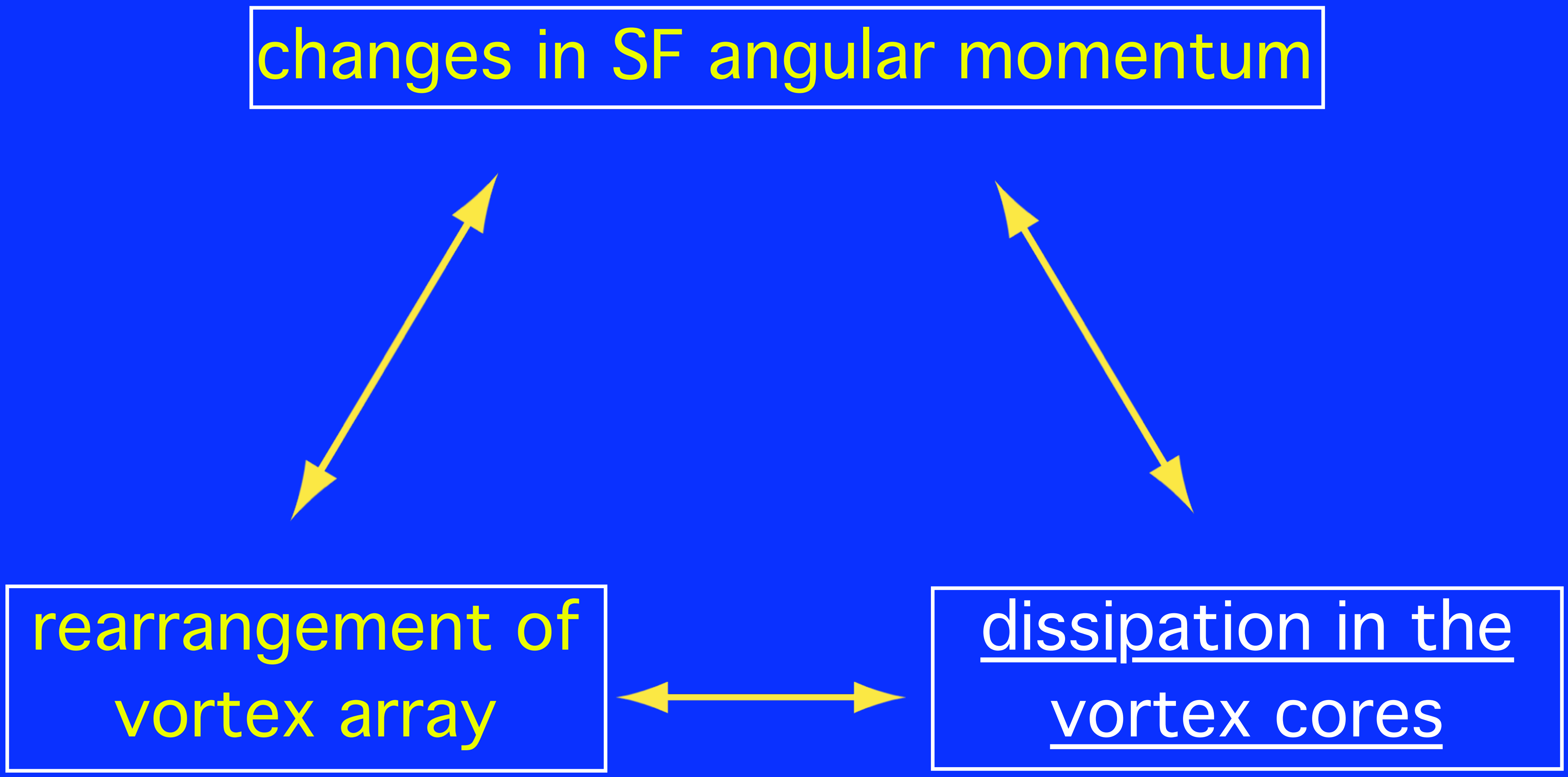
The neutron superfluid's rotation



Rotating superfluid He

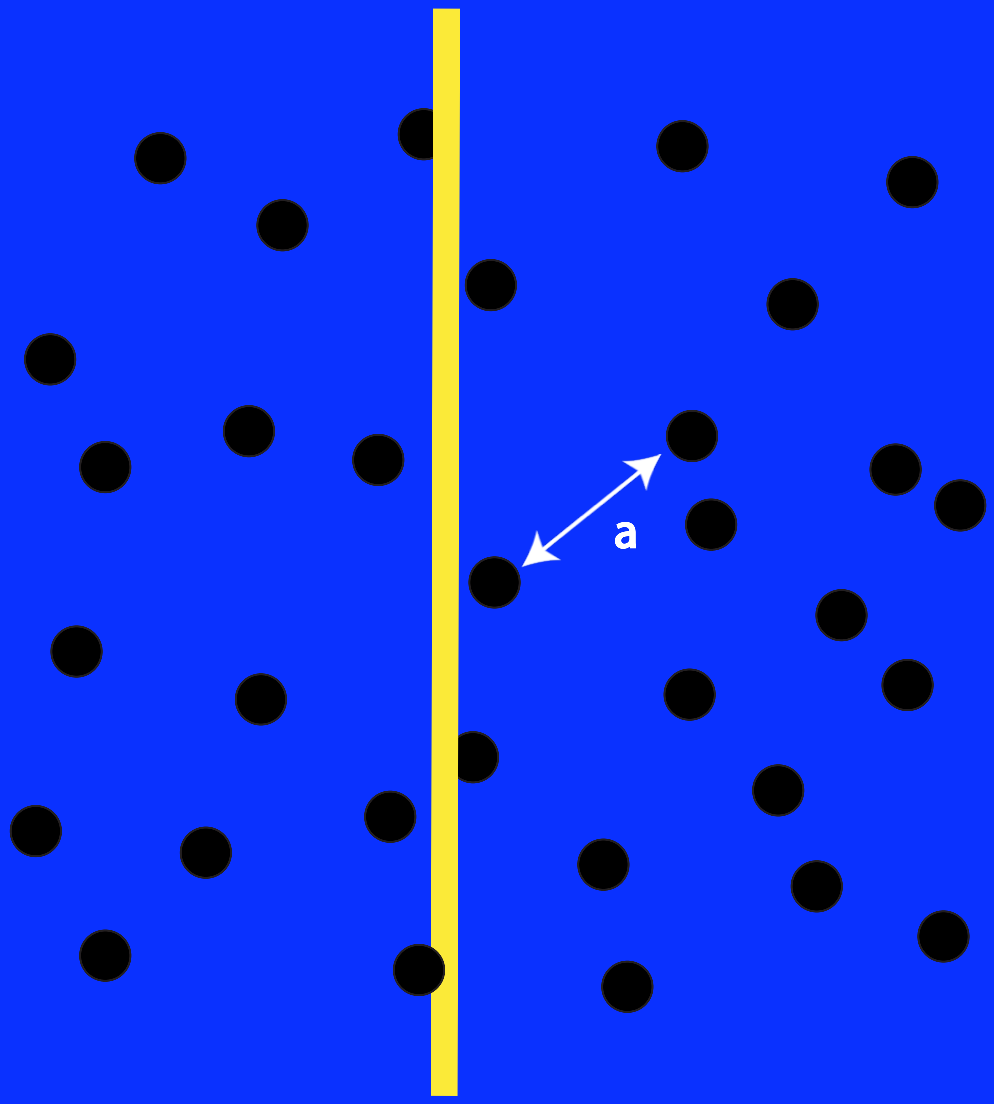


Distribution of vortices determines the fluid’s angular momentum
⇒ mobility of vortices determines torque on container.


changes in SF angular momentum
rearrangement of vortex array
dissipation in the vortex cores

How mobile are vortices in the presence of nuclei?

Vortex pinning geometry

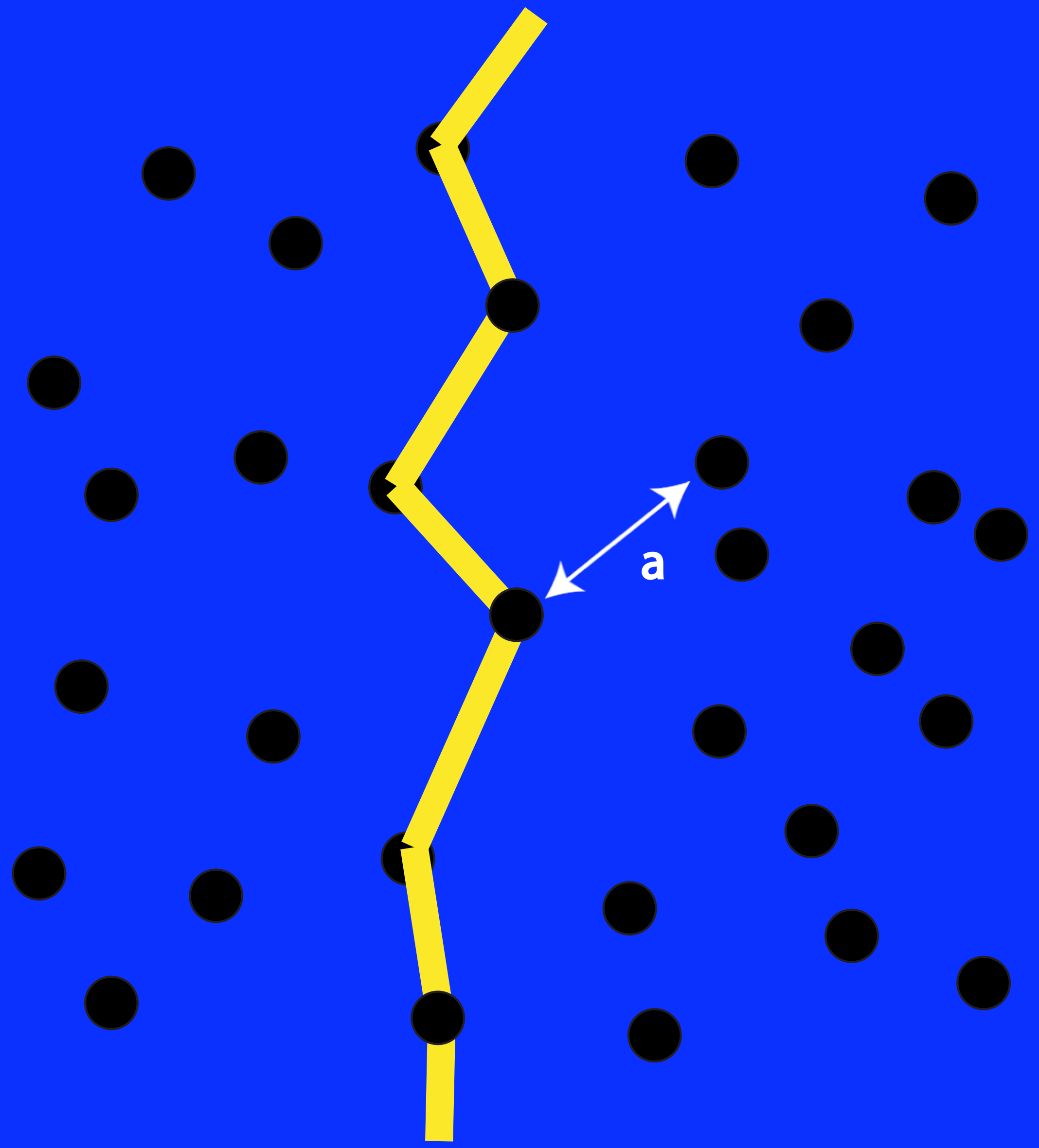


This vortex is <u>not pinned.</u>

Force/length is:

$$\frac{F}{L} = \frac{1}{a}\sum_{n} F_{nv} \simeq 0$$

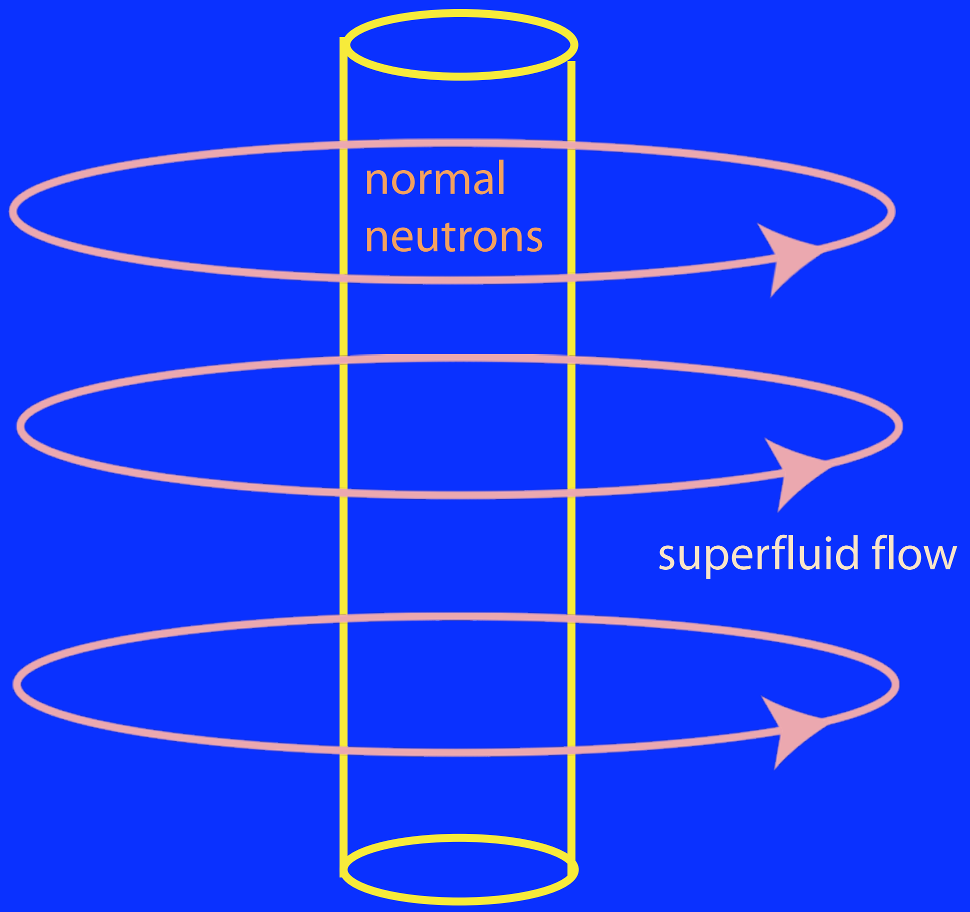
To pin, a vortex must bend



$$\frac{F}{L} \simeq \frac{F_{nv}}{a}$$

But such sharp bends are energetically prohibited.

A vortex has a large self-energy

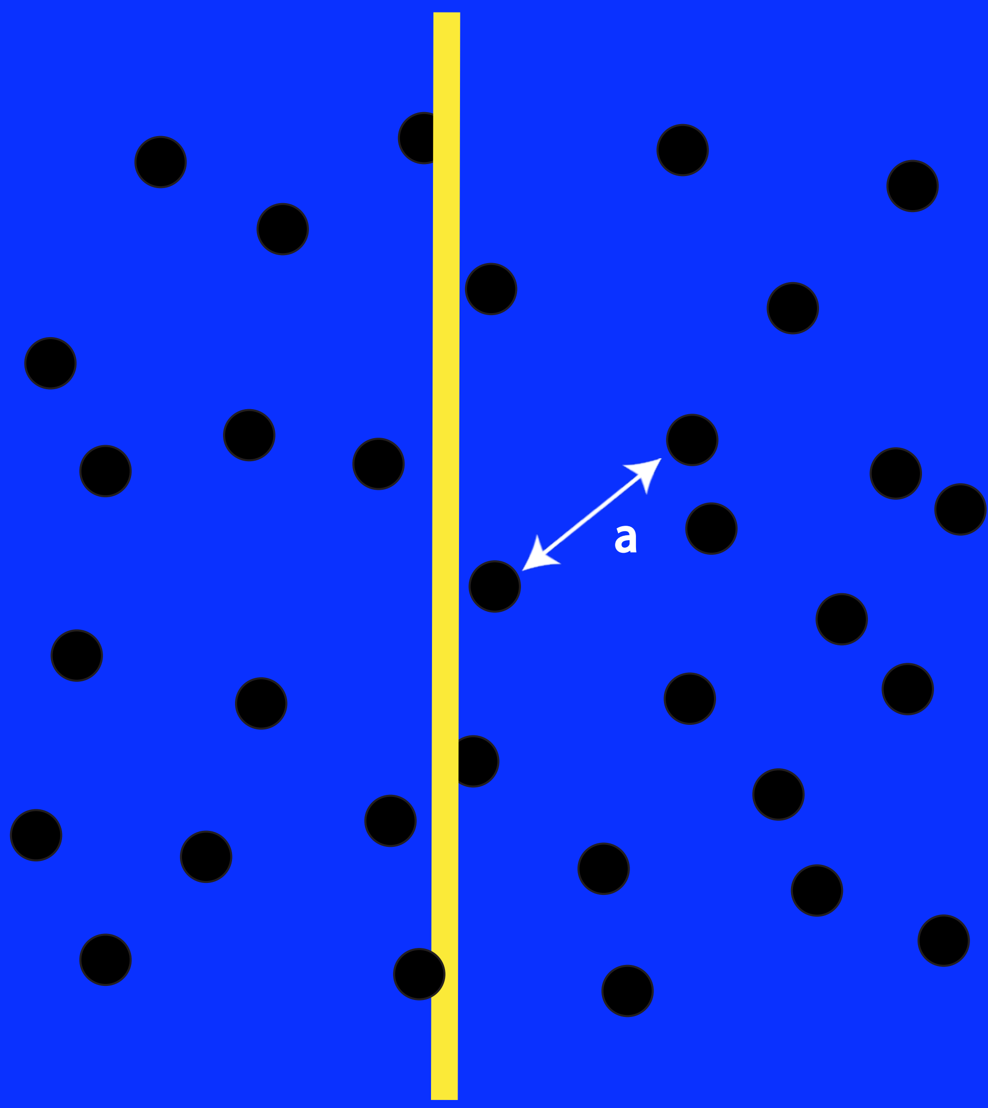


$$\oint v_s \cdot \mathbf{d}l = \kappa \equiv h/2m_n$$

“Tension”

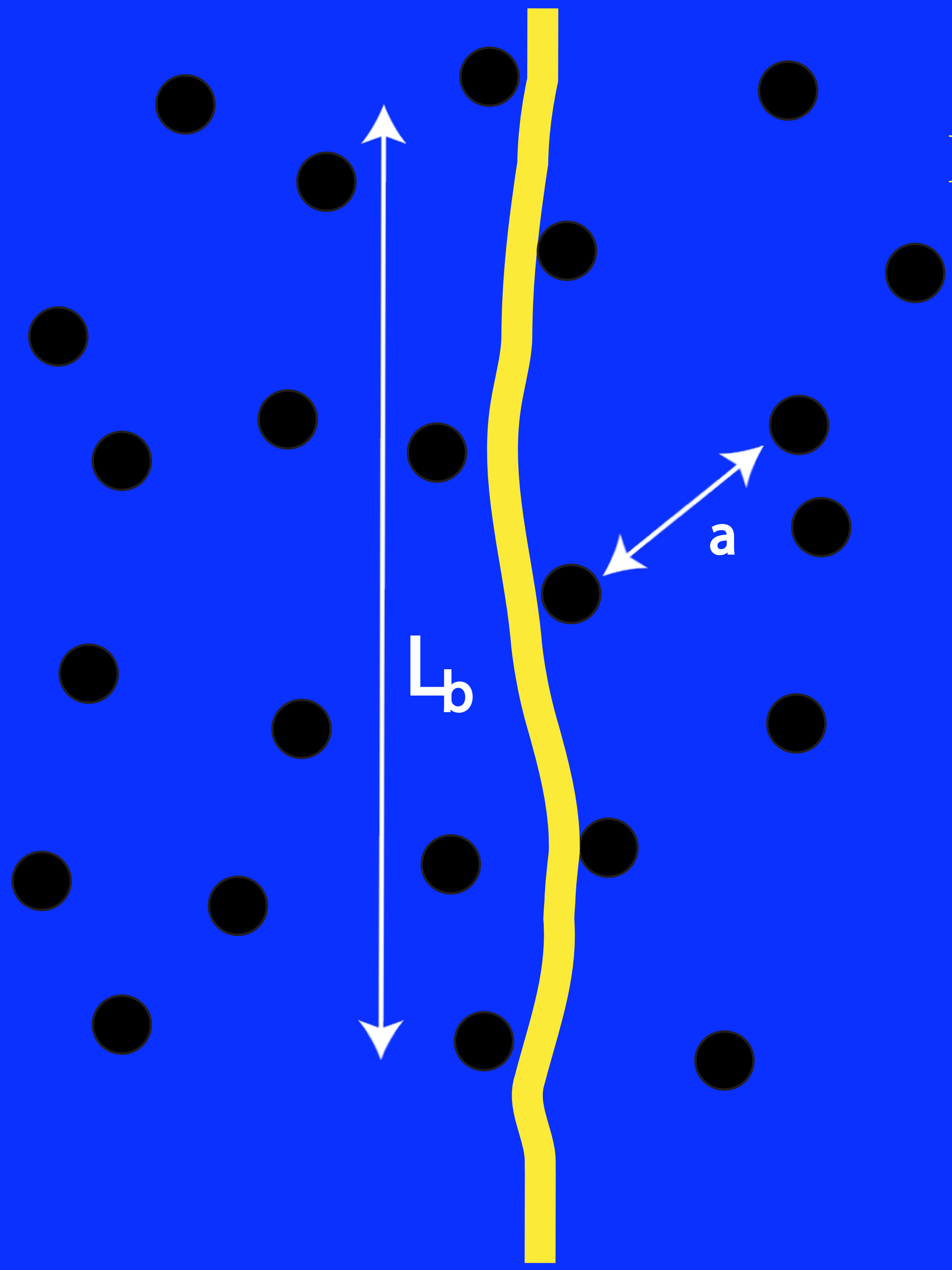
$$\frac{E_k}{L} \equiv T_v \simeq \; 10 \text{ MeV fm}^{-1}$$

Were the vortex self-energy infinite...



...the vortex could never pin

The pinned state



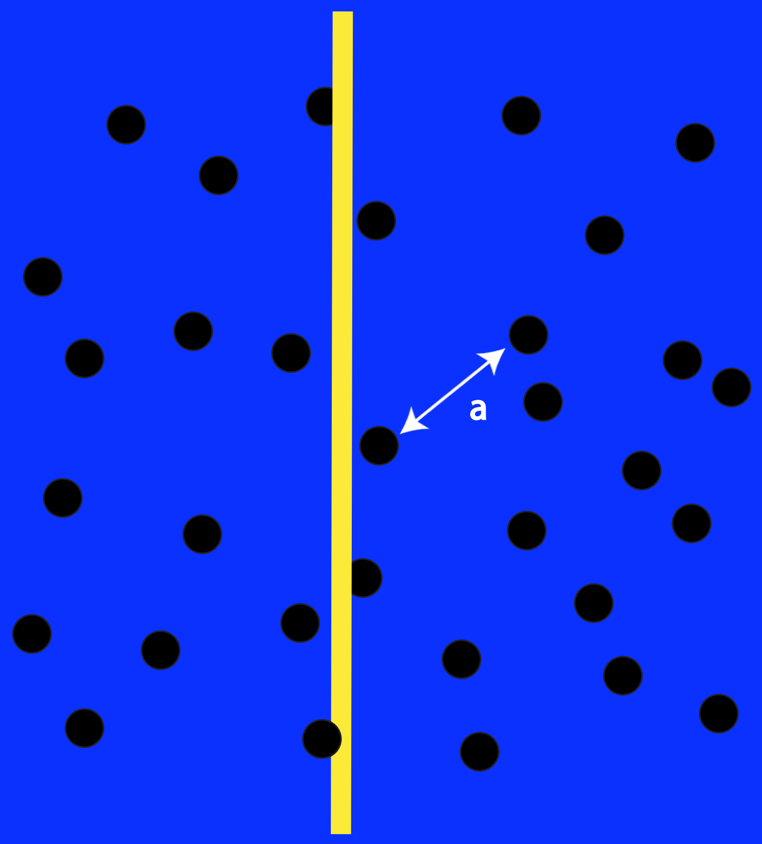
Pinning force per length $\frac{F_{vn}}{L_b} < \frac{F_{vn}}{a}$

$$\frac{E_{vn}}{T_v a} << 1 \Longrightarrow$$

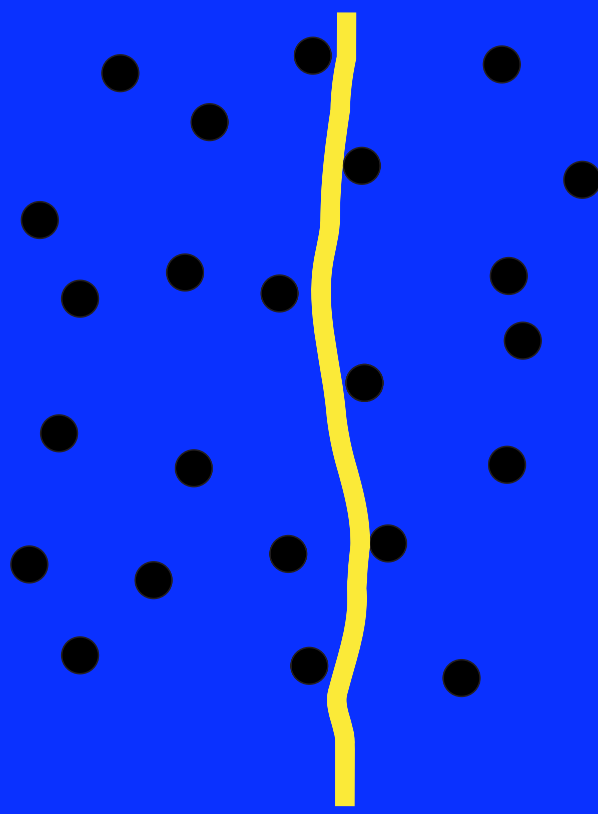
vortices are very stiff
(Link & Cutler 02)

How is this state reached?

Damping to the pinned state

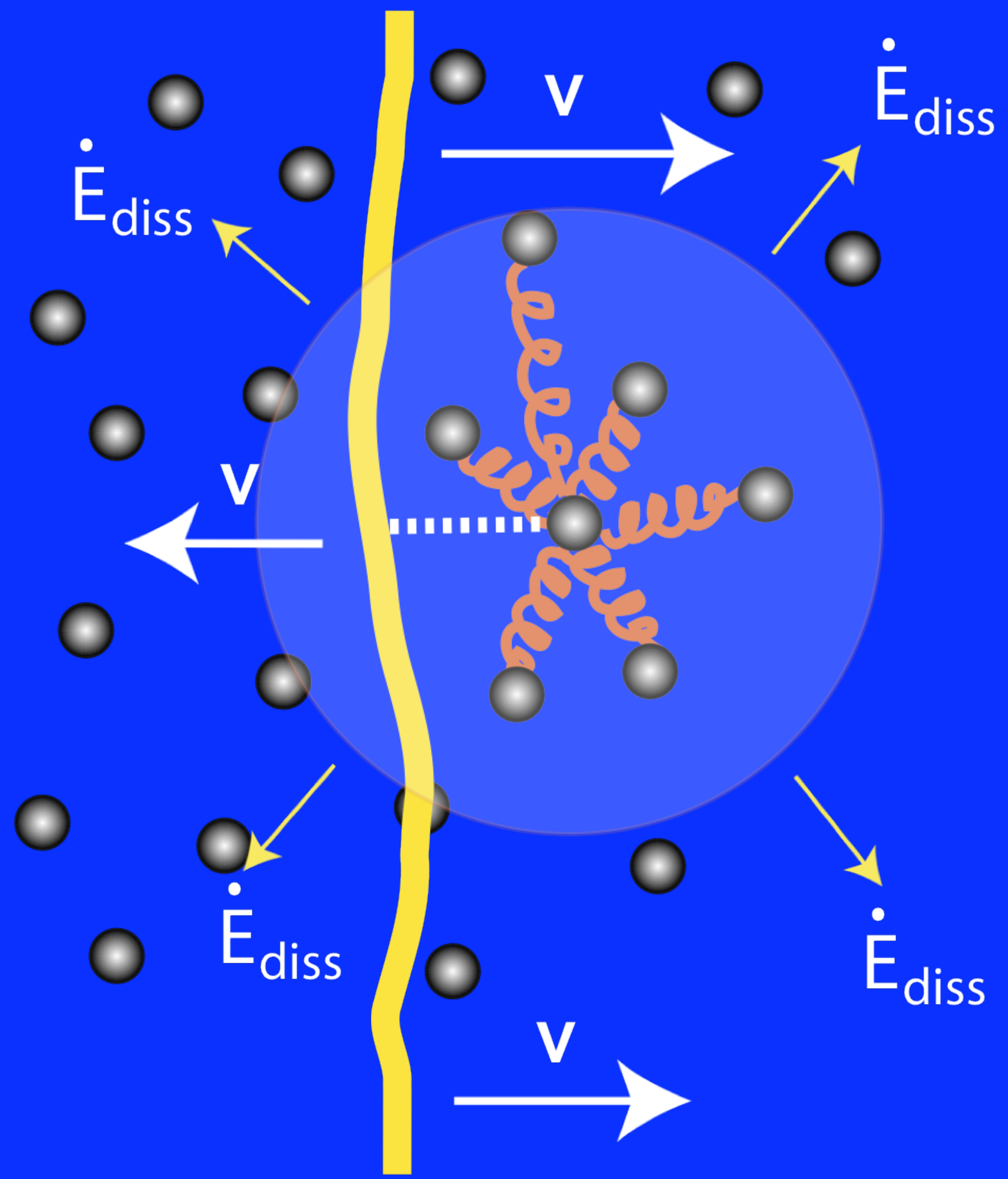


after time t_{pin}



What is t_{pin}?

Energy dissipation creates drag on the vortex



$$\Longrightarrow \mathbf{f}_{drag} = \eta \mathbf{v}$$

$$\eta' \equiv \frac{\eta}{\rho_s \kappa} >> 1 \quad \text{high drag}$$

$$\eta' << 1 \quad \text{low drag}$$

(Sedrakian et al. 99)

Classical calculation of t_{pin}

- Vortex is a (massless) “string” with tension...
- ...which interacts with the nearest nuclei.
- Each nucleus rests in a harmonic potential coupled to nearest neighbors.
- Nuclei are at random positions. (Jones 01)

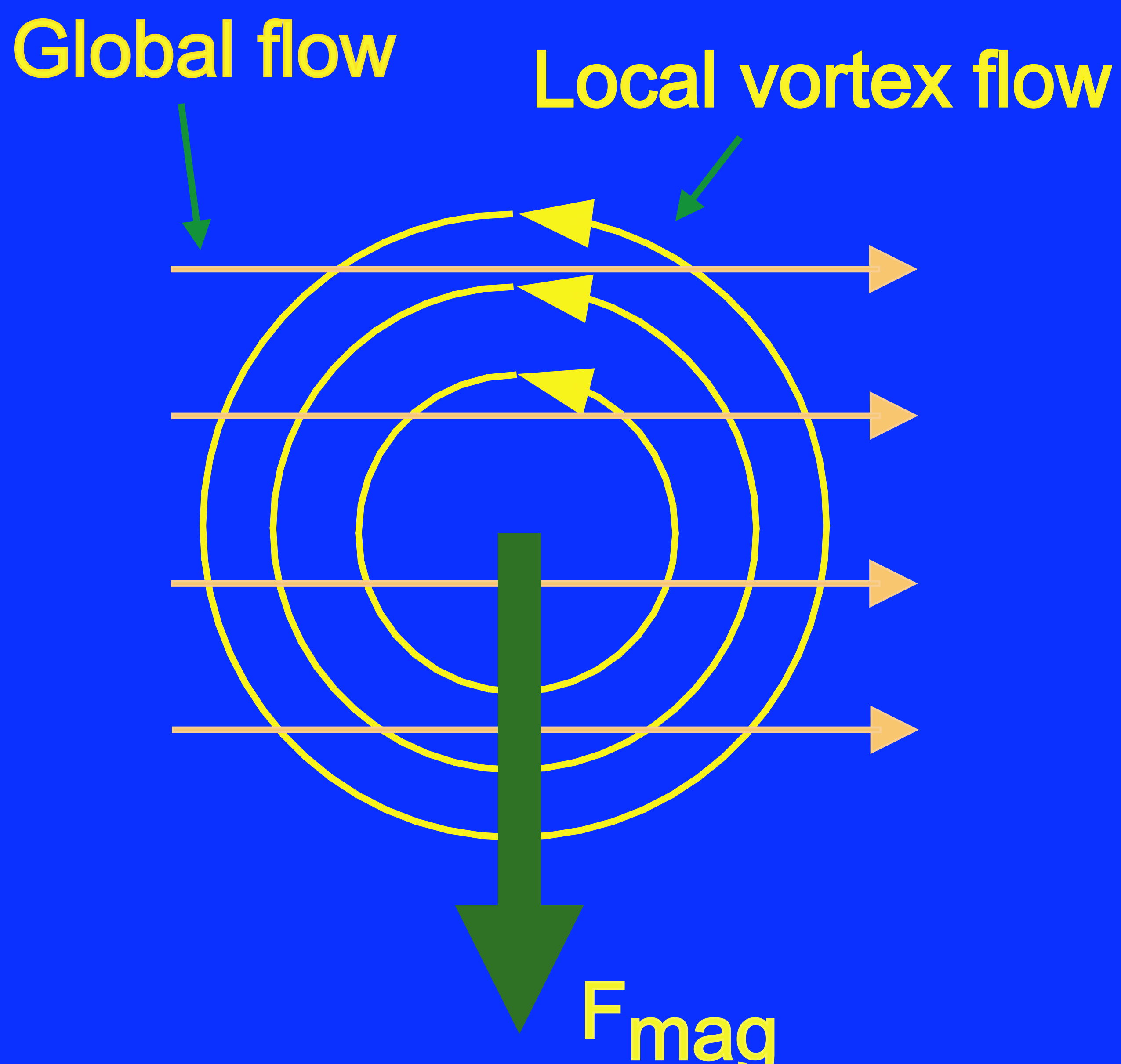
Forces on a nucleus

- Vortex-nucleus attraction
- Coulomb forces from other nuclei

Forces on a vortex

- Vortex-nucleus attraction
- The Magnus force.

The Magnus force



Relevant rates

- Frequency of vortex waves with ka≈1:

$$\omega_v \simeq 10^{21}\ \mathrm{s}^{-1}$$

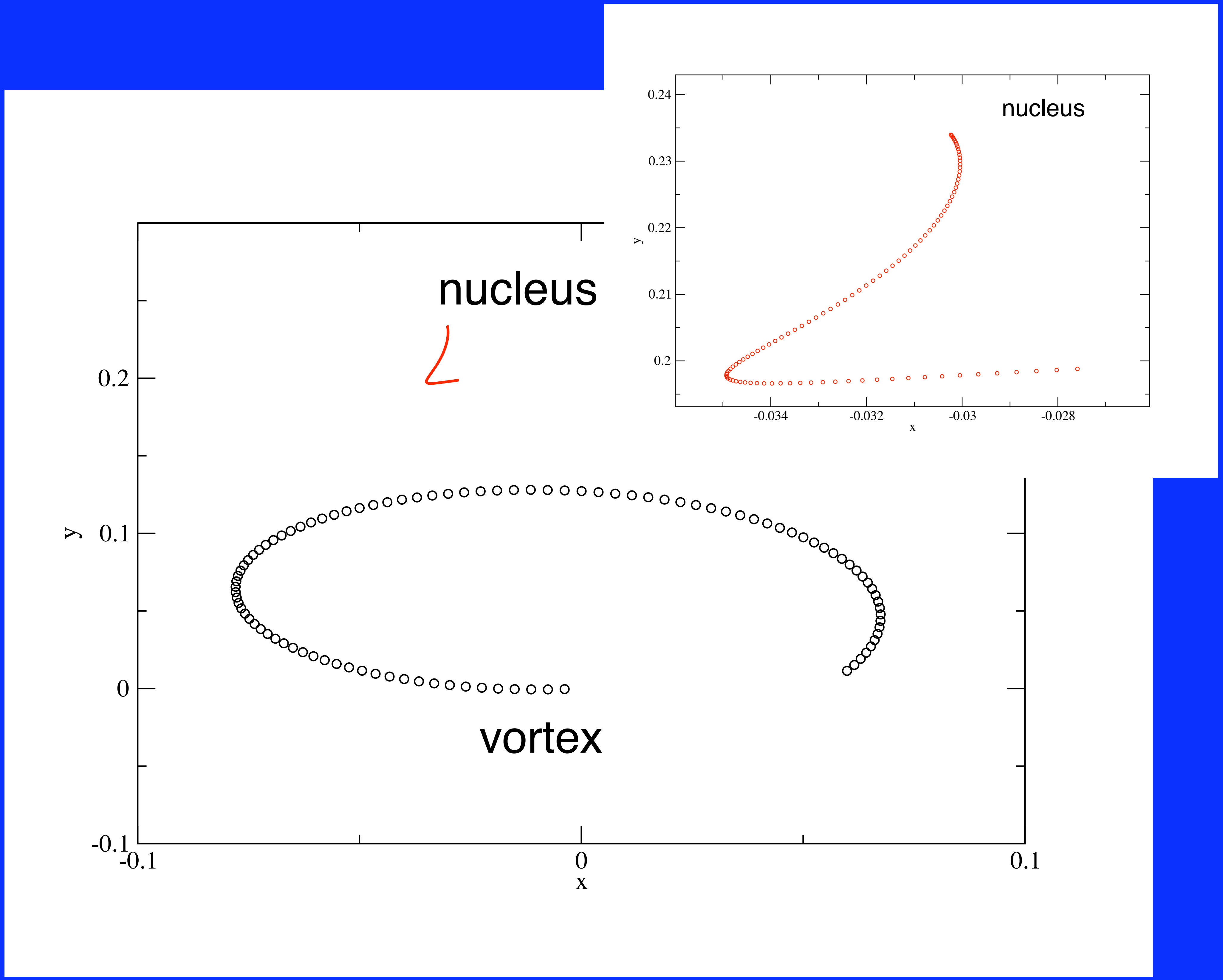
- Plasma frequency of nuclei:

$$\omega_N \simeq \omega_v$$

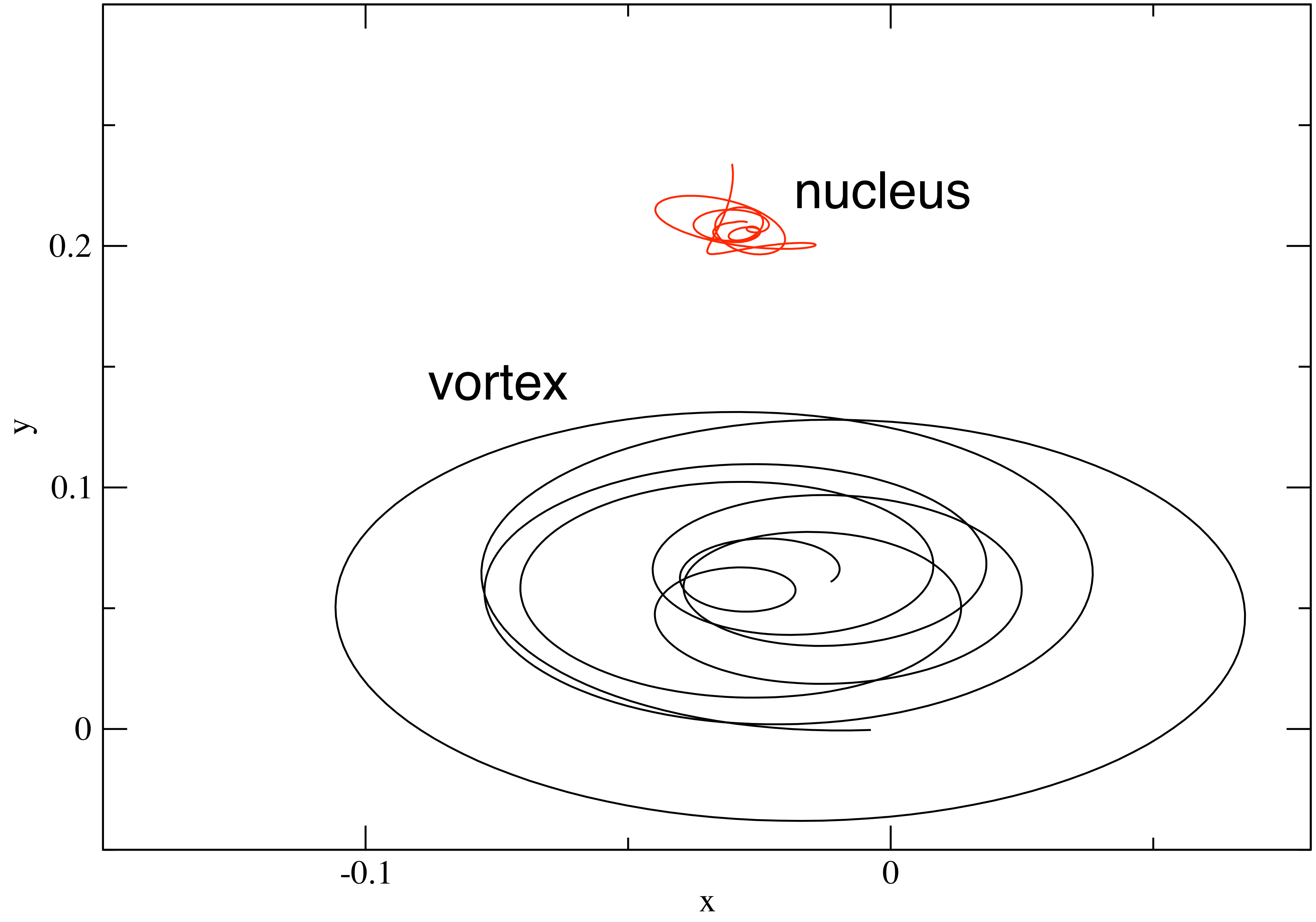
- Damping rate of nuclear motion due to coupling to the phonon field:

$$\gamma_N \simeq 0.03\omega_N$$

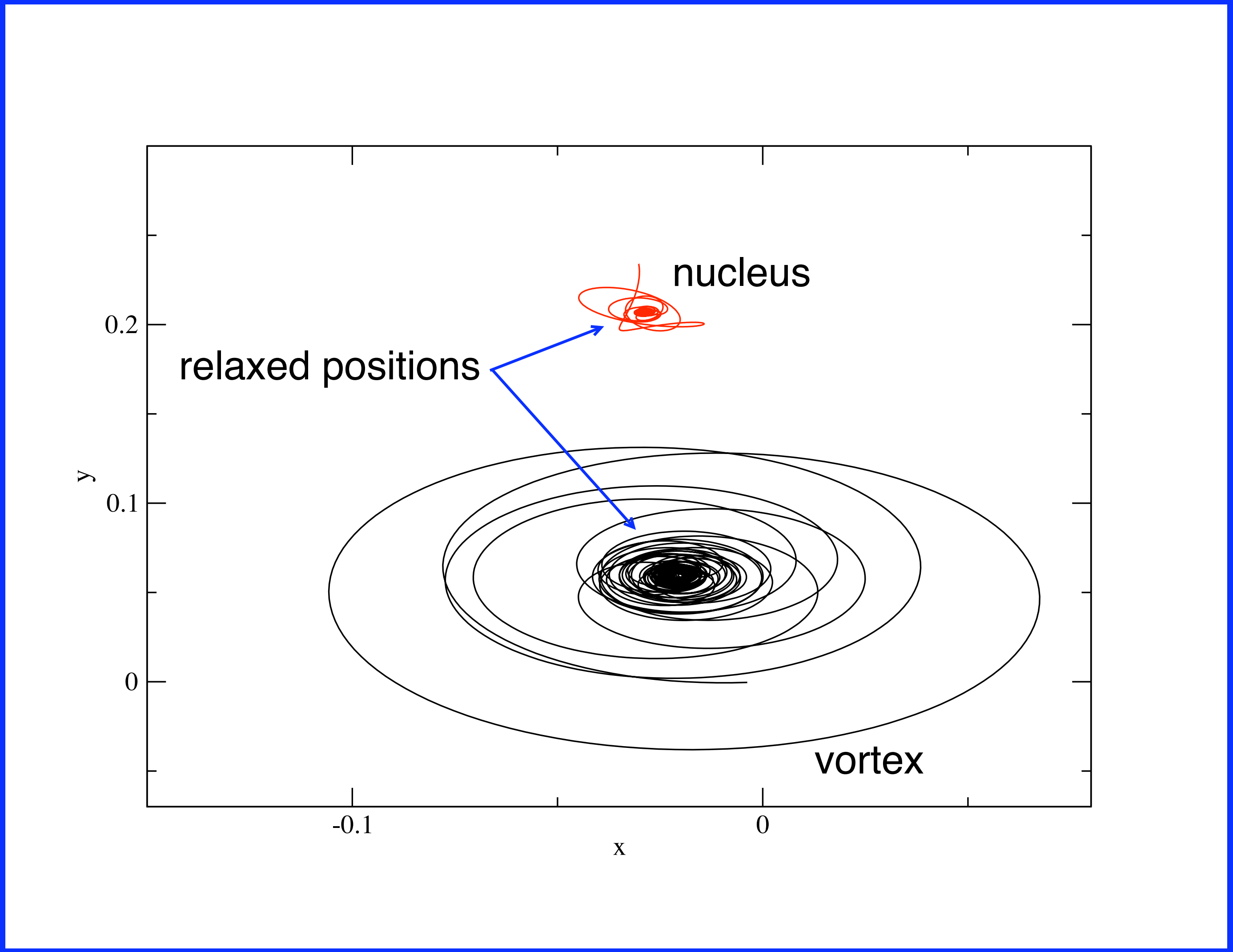
Initial dynamics

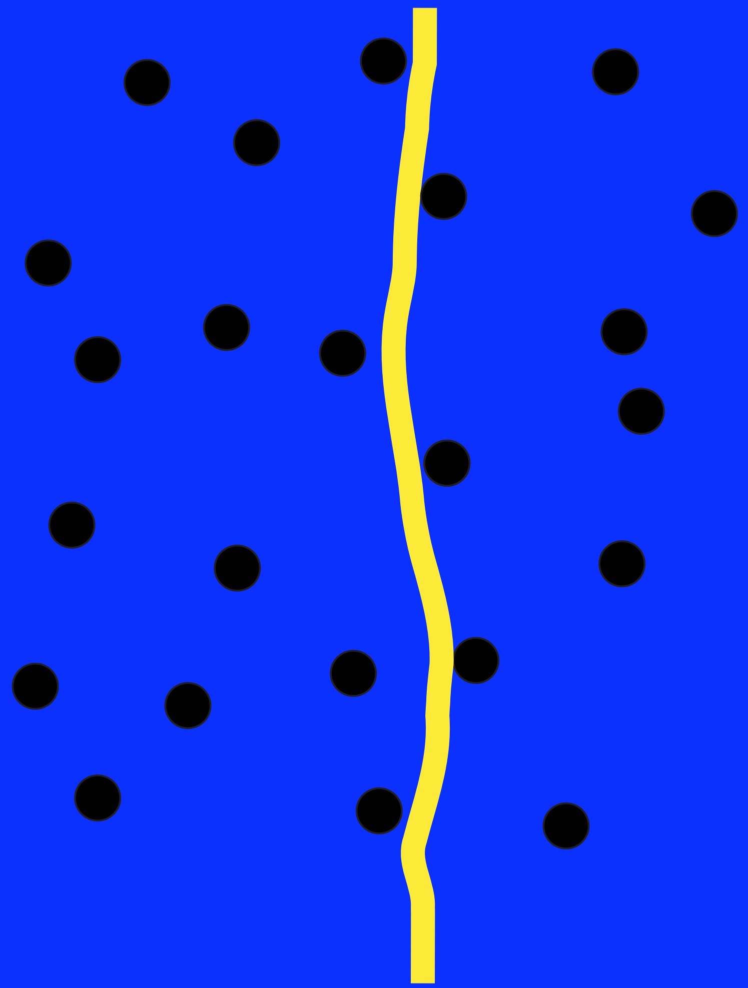


Later...



Relaxation to the pinned state





The pinning time and drag force

$$t_{pin} \propto \frac{T_v \omega_N^4}{E_{vn}^2 \gamma_N}$$

$T_v =$ vortex self-energy (tension)

$\omega_N =$ nucleus plasma frequency

$E_{vn} =$ VN interaction energy

$\gamma_N =$ coupling rate to phonon field

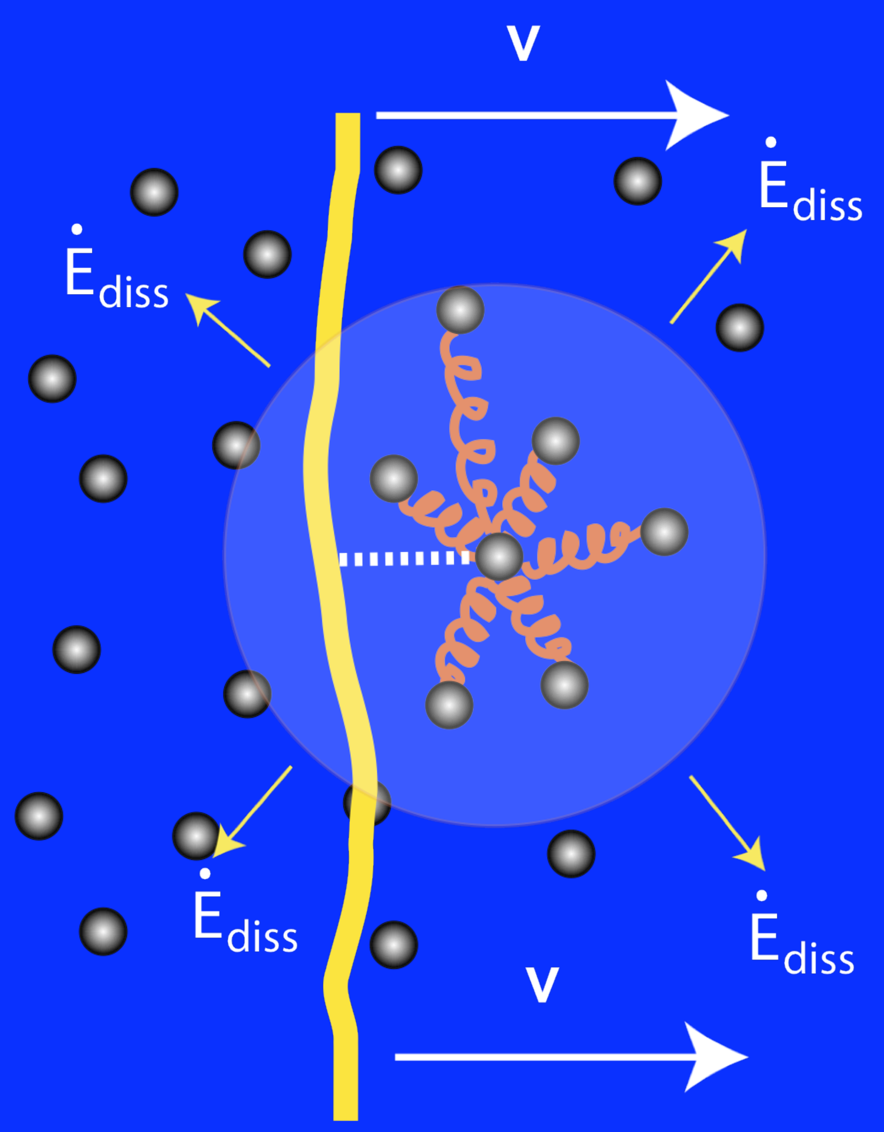
$$\frac{\eta}{\rho_s \kappa} \propto t_{pin}^{-1} \propto \frac{E_{vn}^2 \gamma_N}{T_v \omega_N^4} \simeq 10^{-8} \Longrightarrow \text{ LOW DRAG}$$

$$t_{pin}^{-1} \sim 10^{12}\ \mathrm{s}^{-1} << \omega_N,\ \omega_v$$

Correct value of t_{pin}^{-1} (and η) will be even lower

- Quantum mechanics will reduce the pinning rate. $(\hbar\omega_N \simeq 1\ \mathrm{MeV})$.
- A vortex with net translational motion (due to ambient SF flow) will be harder to pin.

Pinning of a vortex with net translational motion



Pinning will occur below a critical v:

$$v < v_{pin} \simeq \frac{a}{t_{pin}} \lesssim 10 \text{ cm s}^{-1}$$

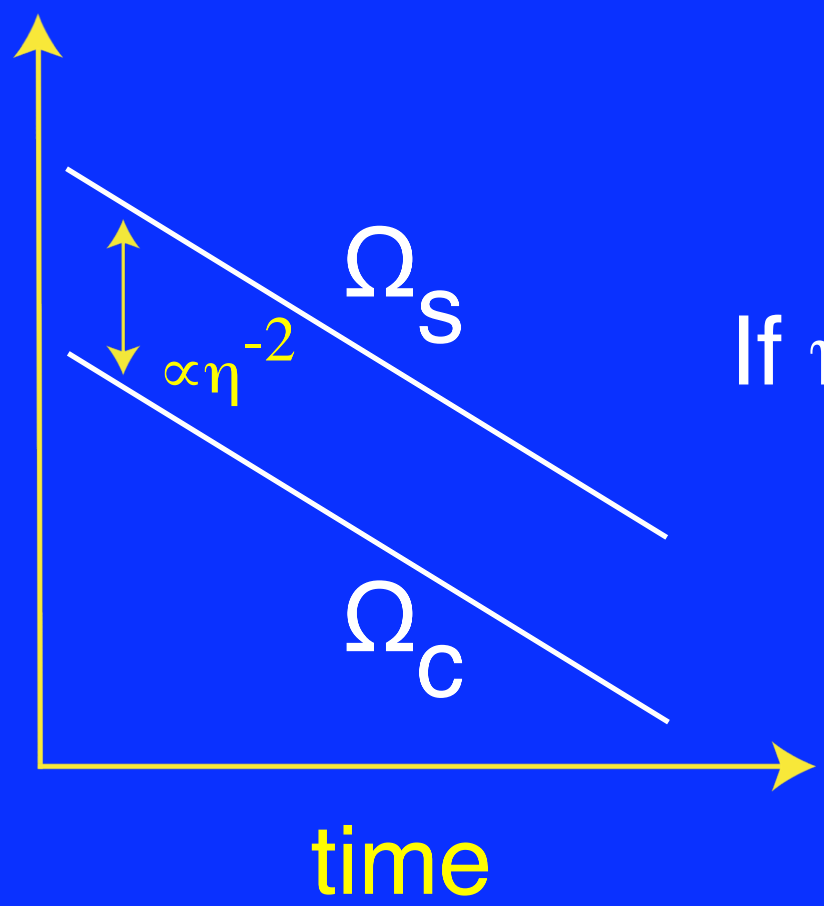
To unpin an <u>already pinned</u> vortex requires:

$$v > v_{crit} \simeq 10^5 \text{ cm s}^{-1}$$

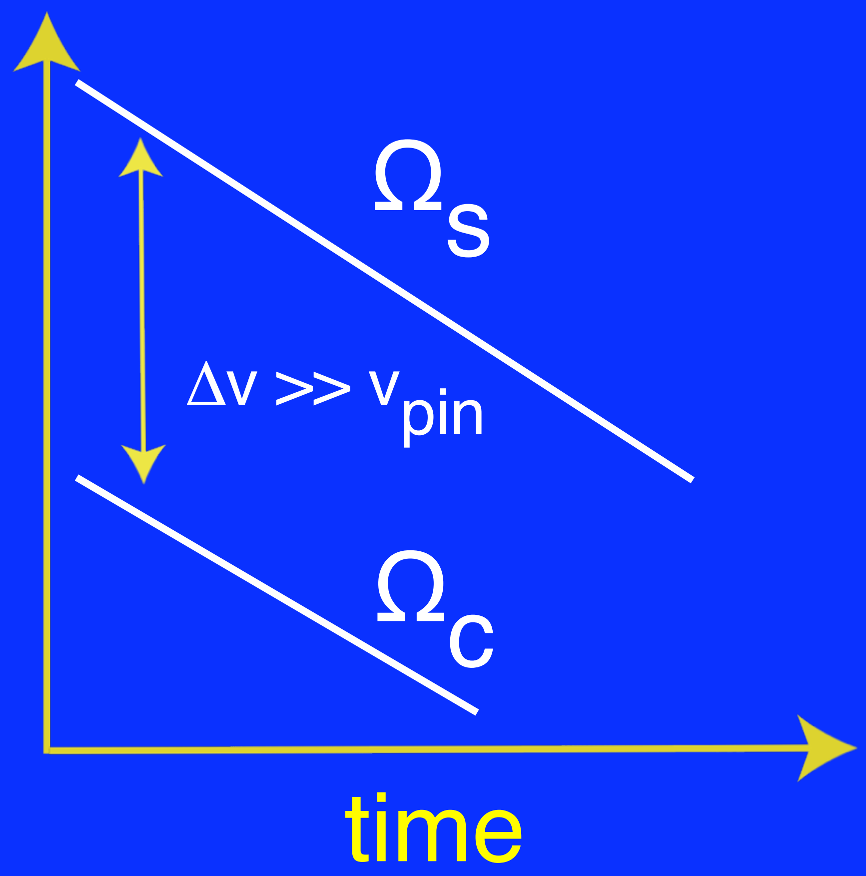
The difference is because:

$$t_{pin}^{-1} \sim 10^{12} \text{ s}^{-1} << \omega_N, \omega_v$$

When is there pinning in a spinning-down NS?



If η is too small ⇒



⇒ NO PINNING

This happens if

$$\eta' \equiv \frac{\eta}{\rho_s \kappa} \lesssim 10^{-7} \left(\frac{t_{age}}{10^4 \text{ yr}} \right)^{-1/2}$$

(In)accessibility of the pinned state

$$\eta' \equiv \frac{\eta}{\rho_s \kappa} \lesssim 10^{-7} \left(\frac{t_{age}}{10^4 \text{ yr}} \right)^{-1/2}$$

Best calculations of E_{vn}

(Avogadro et al. 07; Donati & Pizzochero 06) $\Rightarrow$

$$\eta' \simeq 10^{-7}$$

$\Rightarrow$ <u>pinning is marginal at best.</u>

Conclusions/future directions

- Pinning probably impossible in a spinning-down NS.
- Low-drag motion allows long-period precession (observed).
- Alternatives to vortex pinning/unpinning should be considered to explain glitches.
- Knowledge of $\eta(\rho)$ needed for SF/crust mode calculations.
- Quantum calculation needed to get $\eta(\rho)$.

Wish list

- Observers: identify rotational and seismic modes.
- Nuclear theorists: fully solve the vortex/nucleus interaction problem.