Observational Constraints on the Neutron Star Crust and Their Implications for the Dense Matter Equation of State

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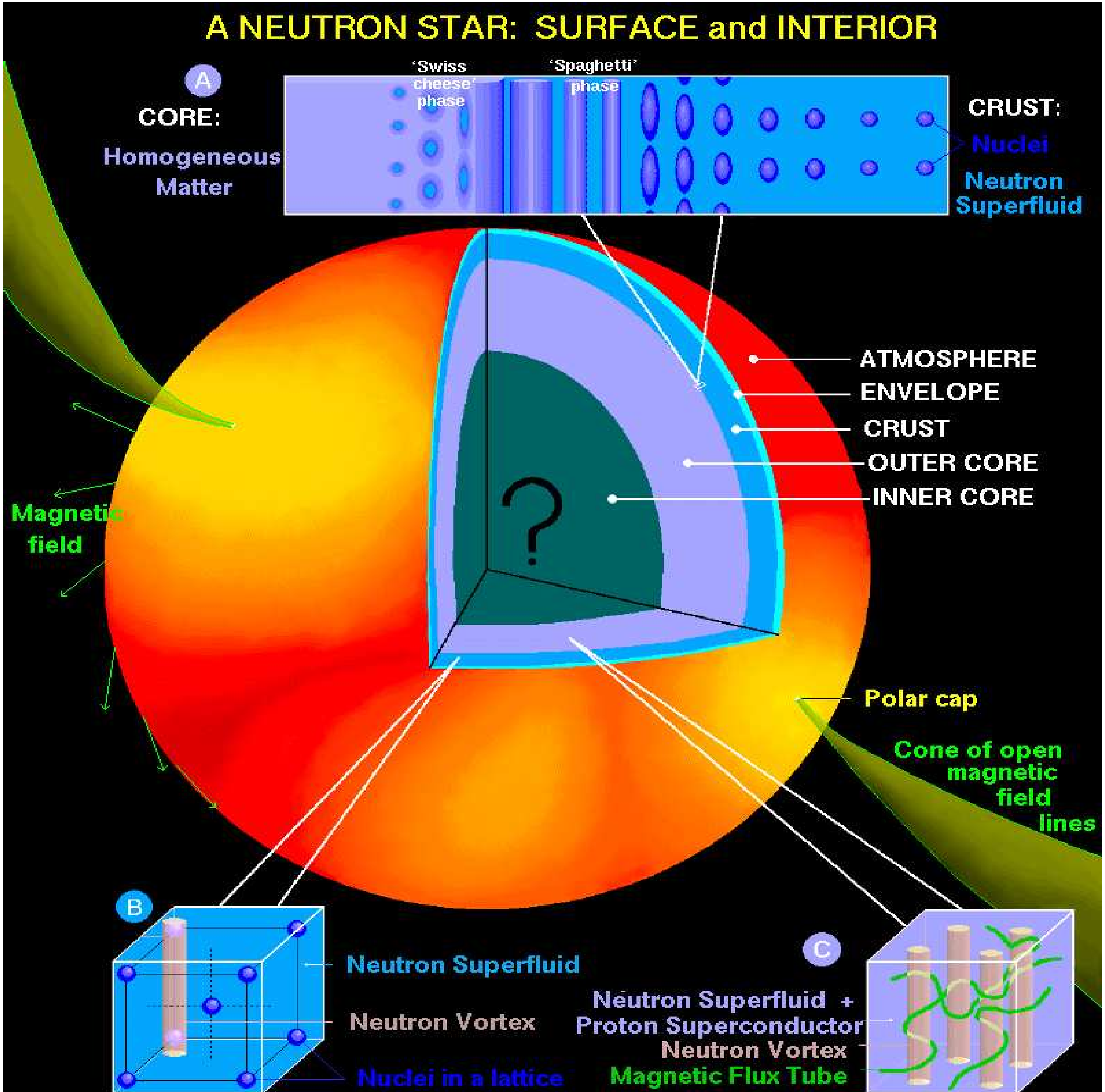
Outline

- Definitions
- Useful Approximations for the Crust
- Pasta Phases?
- Crust Composition and Dependence on Symmetry Energy
- Other Constraints: Masses, Rotation and Thermal Emission
- Experimental Constraints on Symmetry Energy
- Neutron Star Seismology
- Moments of Inertia



Credit: Dany Page, UNAM

The Neutron Star Crust

- Inner edge: transition between phases with nuclei and uniform matter
- Transition density and pressure: $\epsilon_t, \quad P_t$
- Crustal extent: $\Delta \equiv \Delta R, \quad \Delta I$

Beta equilibrium, $M(r) \simeq M$, $p(r) << \epsilon(r)$, $4\pi R^3 p(r) << Mc^2$

$$\frac{dP}{nm_b} = \frac{d\mu}{m_b} \simeq -\frac{GM}{r^2 - 2GMr/c^2} dr$$

$$\ln \mathcal{H} \equiv 2\frac{\mu_t - \mu_0}{m_b c^2} \simeq \ln \left[\frac{R_t(R - 2GM/c^2)}{R(R_t - 2GM/c^2)} \right]$$

$$\Delta \simeq R \frac{\mathcal{H} - 1}{\mathcal{H}(1 - 2GM/Rc^2)^{-1} - 1} = R_\infty \frac{\mathcal{H} - 1}{\mathcal{H} - 1 + 2GM/Rc^2}$$

Moment of Inertia

$$I = -\frac{2c^2}{3G}\int_0^R r^2\omega(r)\frac{dj(r)}{dr}dr$$
$$= \frac{8\pi}{3c^2}\int_0^R r^4[\epsilon(r)+p(r)/c^2]e^{\lambda(r)}j(r)\omega(r)dr$$

$j = e^{-(\nu+\lambda)/2}$, ω is the rotational drag

$$\frac{d\nu(r)}{dr} = -2G\left[\frac{M(r)+p(r)/c^2}{r(r-2GM(r)/c^2})\right], \qquad \ln\lambda(r) = 1 - \frac{2GM(r)}{rc^2}$$

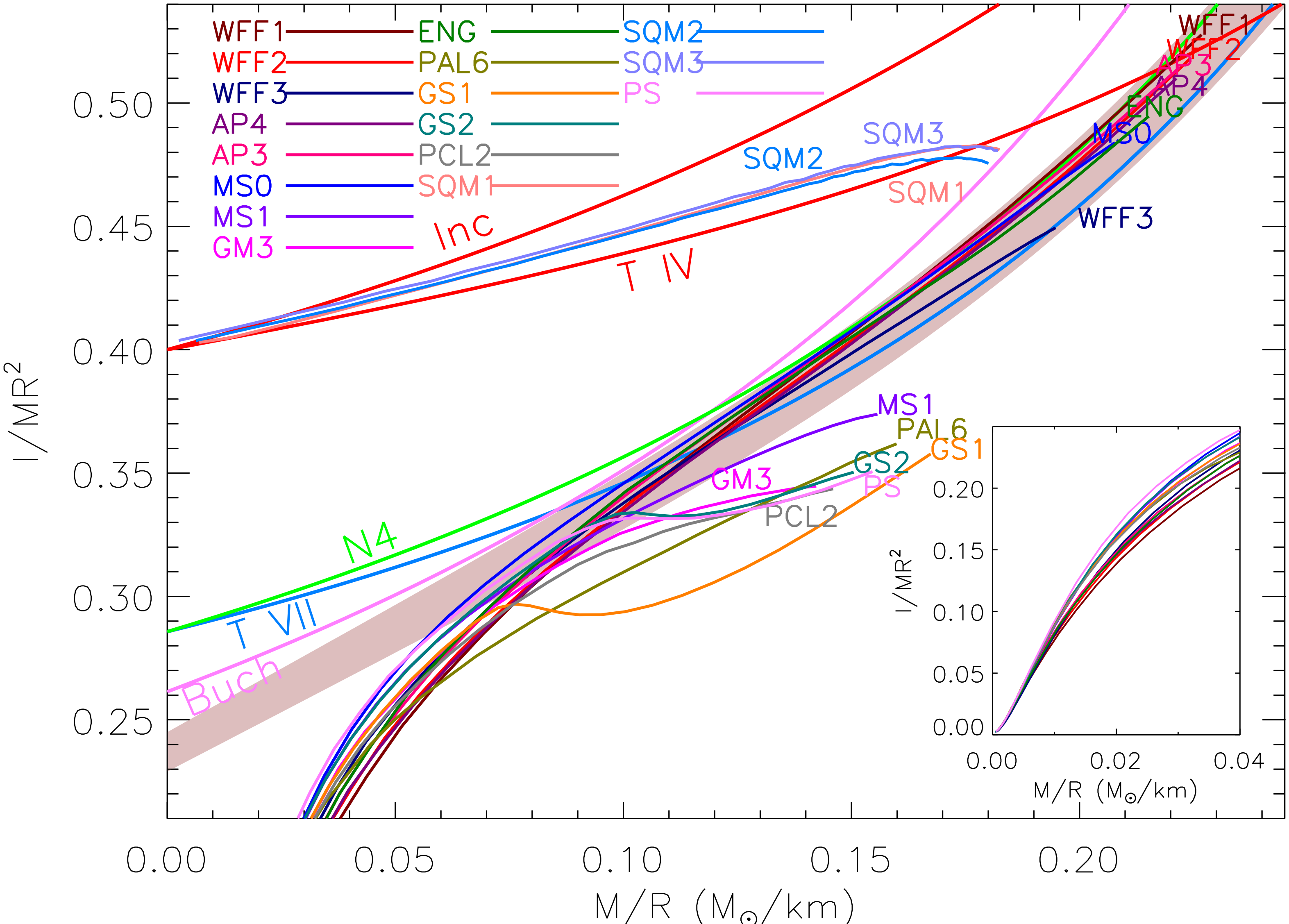
$$\frac{d}{dr}\left[r^4 j(r)\frac{d\omega(r)}{dr}\right] = -4r^3\omega(r)\frac{dj(r)}{dr}, \quad \frac{d\nu(0)}{dr} = \frac{d\lambda(0)}{dr} = \frac{d\omega(0)}{dr} = \frac{dj(0)}{dr} = 0,$$

$$e^{\nu(R)} = e^{-\lambda(R)} = 1 - 2GM/Rc^2, \quad j(R) = 1, \quad \frac{dj(R)}{dr} = 0, \quad \omega(R) = 1 - \frac{2GI}{R^3c^2},$$

$$I = \frac{c^2}{6G}R^4\frac{d\omega(R)}{dr},$$

$$\Delta I \simeq \frac{8\pi\omega(R)}{3Mc^2}\int_{R-\Delta}^R r^6 dP \simeq \frac{8\pi\omega(R)}{3Mc^2}R^6 p_t \exp\left[-\frac{6\gamma}{(2\gamma-1)}\frac{\Delta}{R}\right] \qquad \text{if } \mathrm{P} \propto \epsilon^\gamma$$

Moments of Inertia



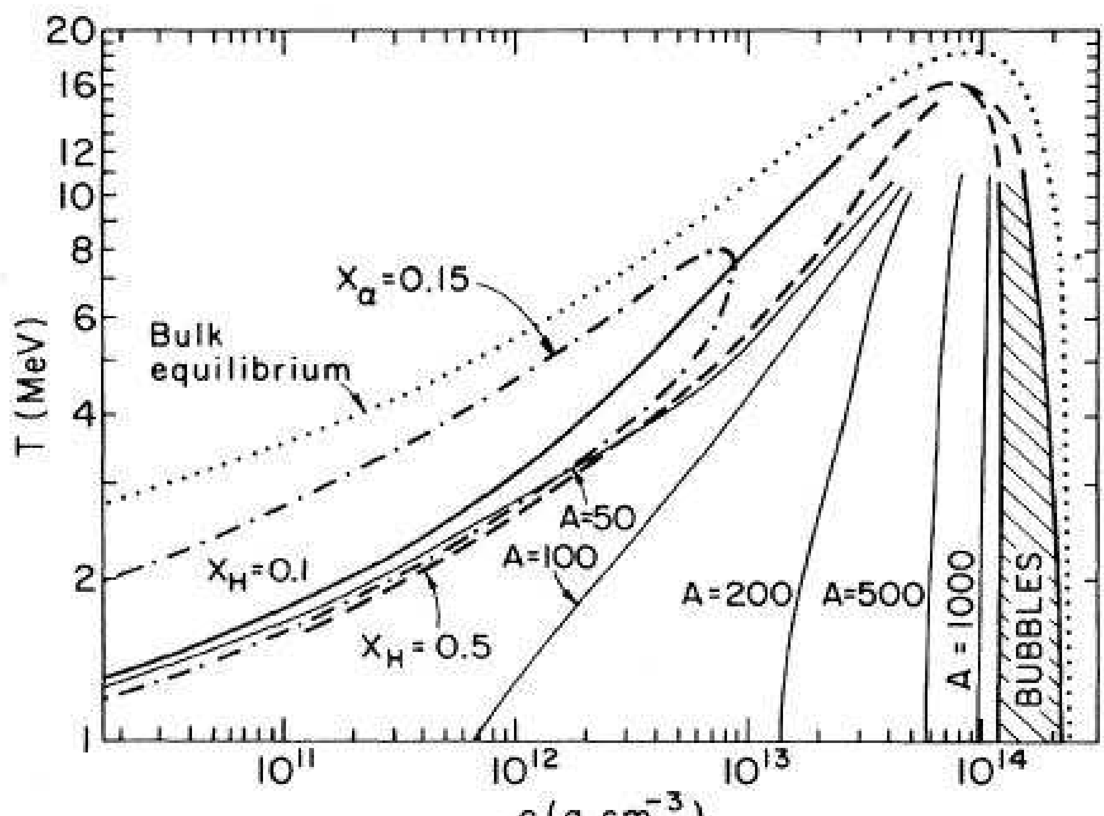
$$I \simeq (0.237{\pm}0.08)MR^2\left[1 + 2.84\frac{GM}{Rc^2} + 18.9\left(\frac{GM}{Rc^2}\right)^4\right] \mathrm{M}_\odot\ \mathrm{km}^2$$

Pasta Phase?

Coulomb energy of spherical nucleus in a charge-neutral Wigner-Seitz cell, uniform electron density:

$$E_C = \frac{3}{5}\frac{Z^2e^2}{R}\left[1 - \frac{3}{2}u^{1/3} + \frac{1}{2}u\right], \qquad u = \left(\frac{R}{R_c}\right)^3$$

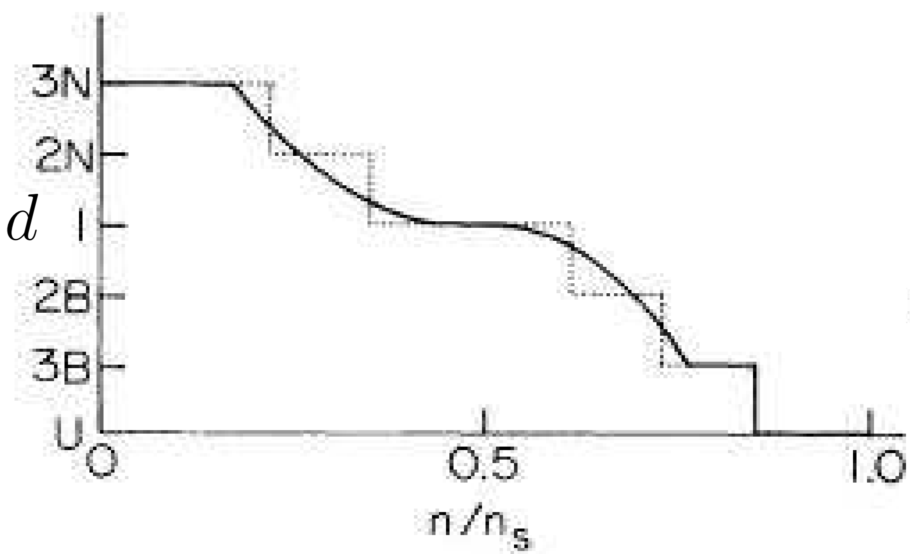
"Inside-out" nuclei have identical surface and Coulomb energies when $u = 1/2$ (LLPR 1978)

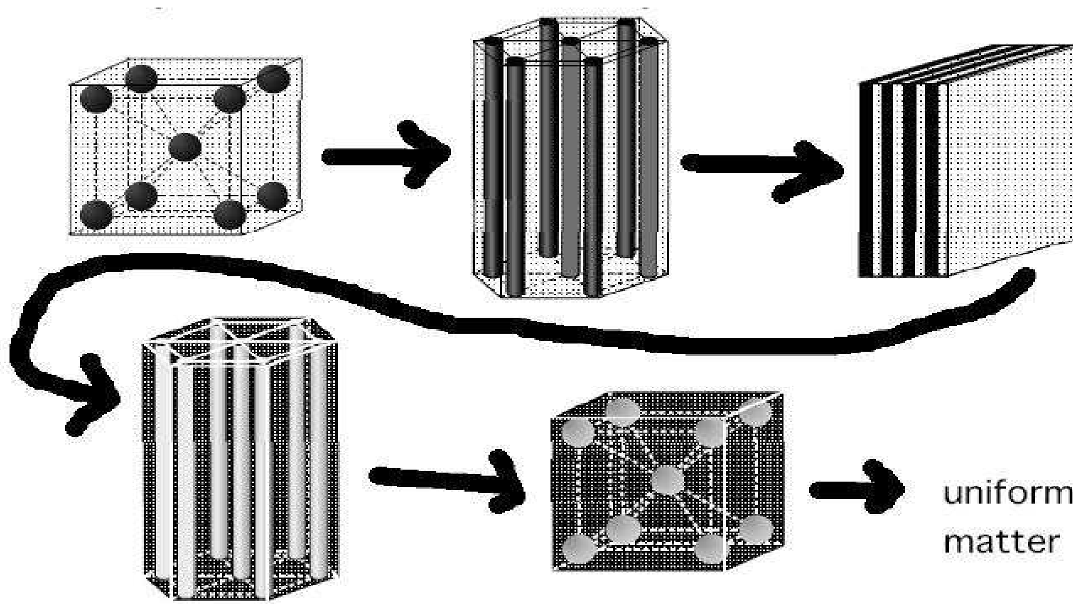


Ravenhall, Pethick & Wilson (1983) proposed a "dimensionality" d to describe surface and Coulomb droplet energies per unit volume:

$$e_s = u\sigma\frac{d}{r}, \qquad e_C = u\frac{4\pi(n_i x_i e R)^2}{d+2}\left[\frac{1}{d-2}\left(1-\frac{d}{2}u^{1-2/d}\right)+\frac{u}{2}\right]$$

When $d=2$, $\qquad e_C = -u(1+\ln u)\pi(n_i x_i e R)^2/2$





K.Oyamatsu, NPA561, 431 (199

Fission and Clustering Instabilites

Fission instability to deformation for isolated nucleus:

$$2e_s = e_{C0}$$

BBP condition for optimum nucleus size in W-S cell:

$$e_s = 2e_{C0}F(u)$$

These are coincident when

$$1 - \frac{3}{2}u^{1/3} + \frac{u}{2} = \frac{1}{4}, \quad \text{or} \quad u \simeq 0.173$$

NO DEPENDENCE on nuclear force parameters

Clustering instability:

$$du = -pdv - \hat{\mu}dq : -\frac{\partial p}{\partial v}\bigg|_q - \frac{\partial p}{\partial q}\bigg|_v \frac{\partial q}{\partial v}\bigg|_{\hat{\mu}} > 0, \ -\frac{\partial \hat{\mu}}{\partial q}\bigg|_v > 0$$

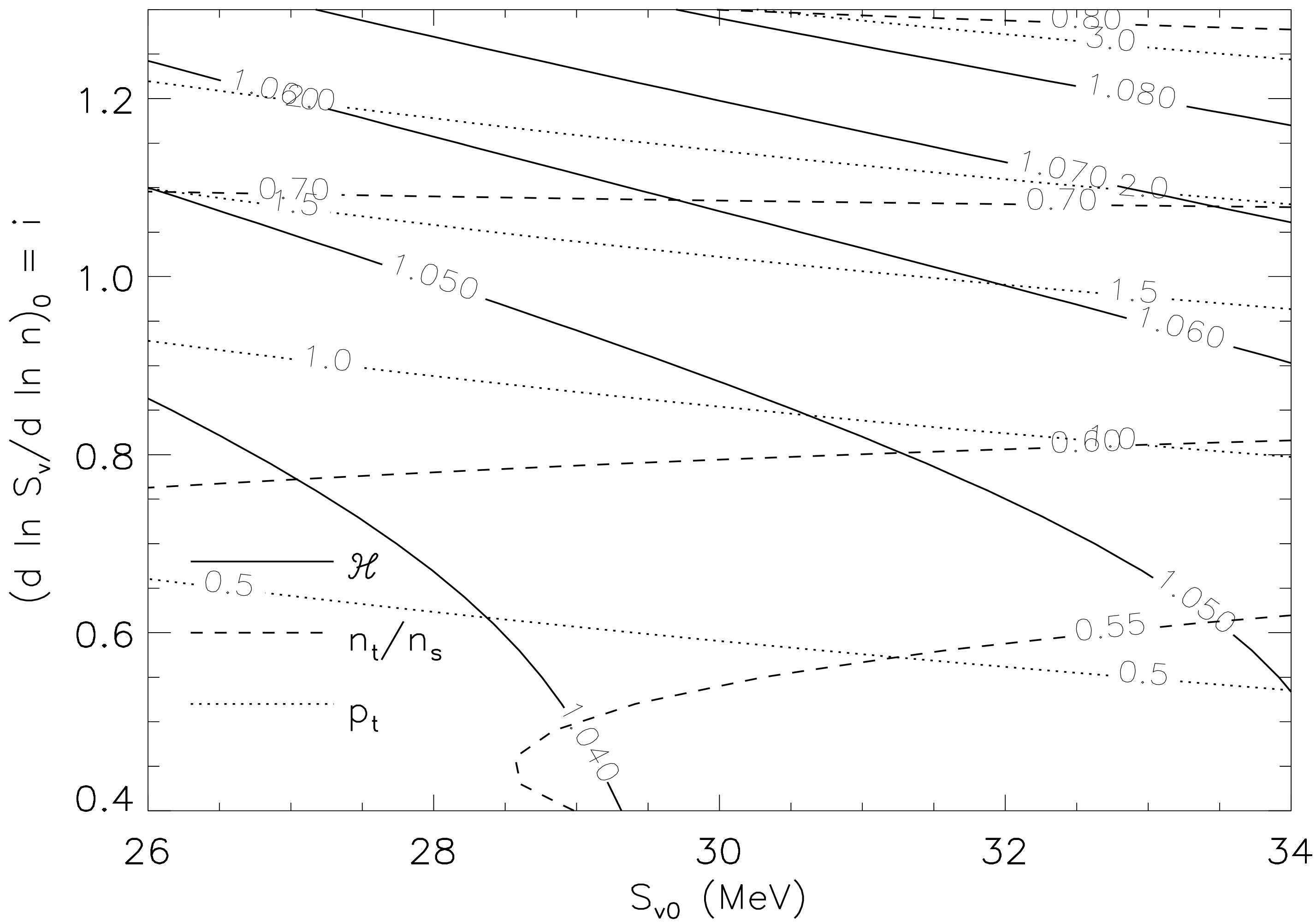
If $E_{sym} \approx S_v + L(u-1)/3 \qquad \text{with} \ \ u = n/n_0$,

$$\frac{K}{9}(3u-2)\left[1 + \frac{L}{3S_v}(1-u)\right] = \frac{2L}{3}\left(1 - \frac{L}{3S_v}\right)$$

$L = 0, \ S_v, \ 2S_v \Rightarrow u = 2/3, \simeq 1/2, \simeq 0.35$

STRONG DEPENDENCE on symmetry energy

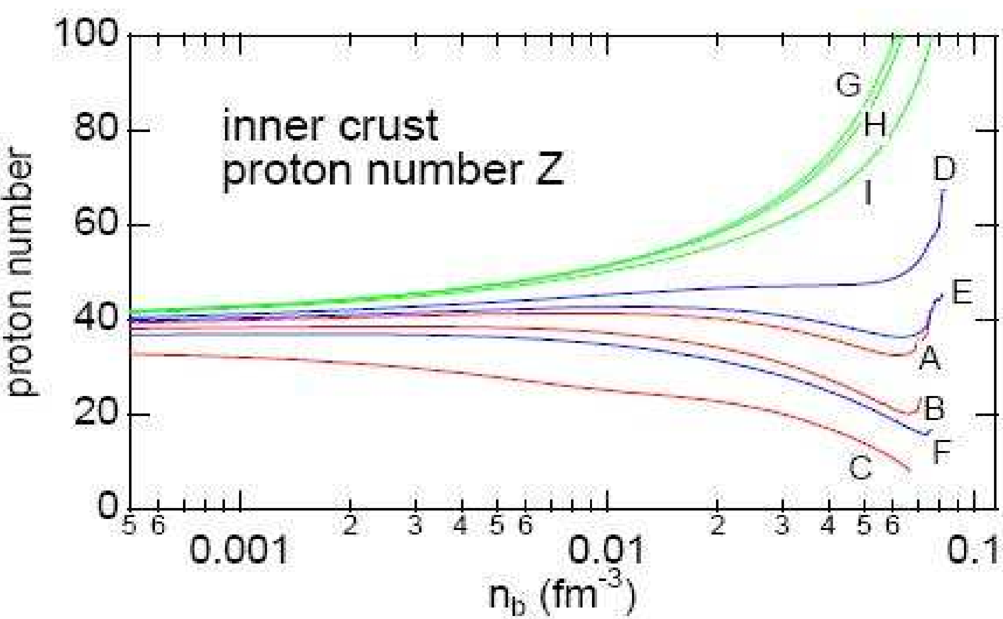
Transition Properties

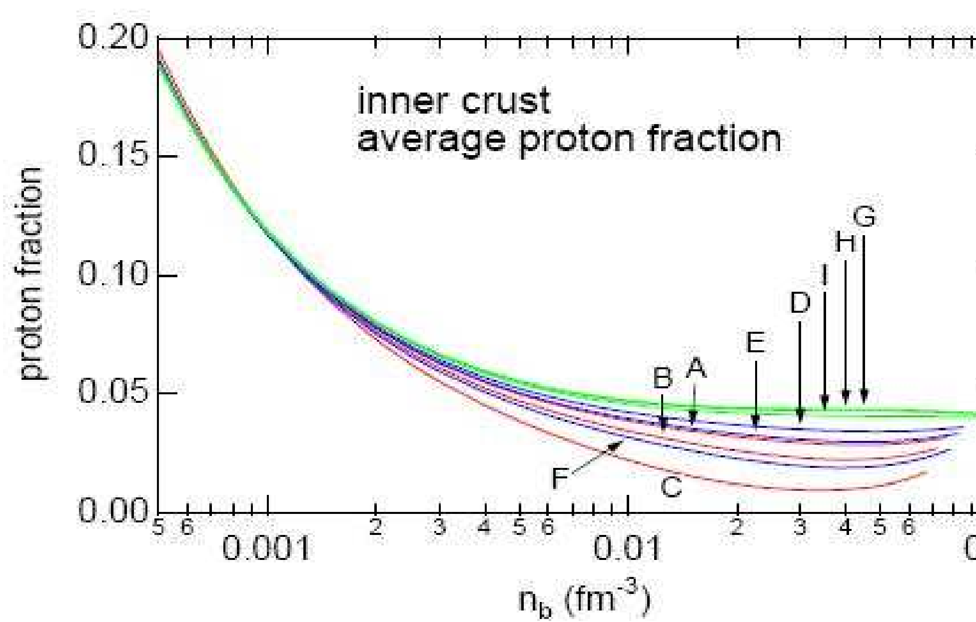


Lattimer & Prakash (2007)

Composition is also symmetry dependent

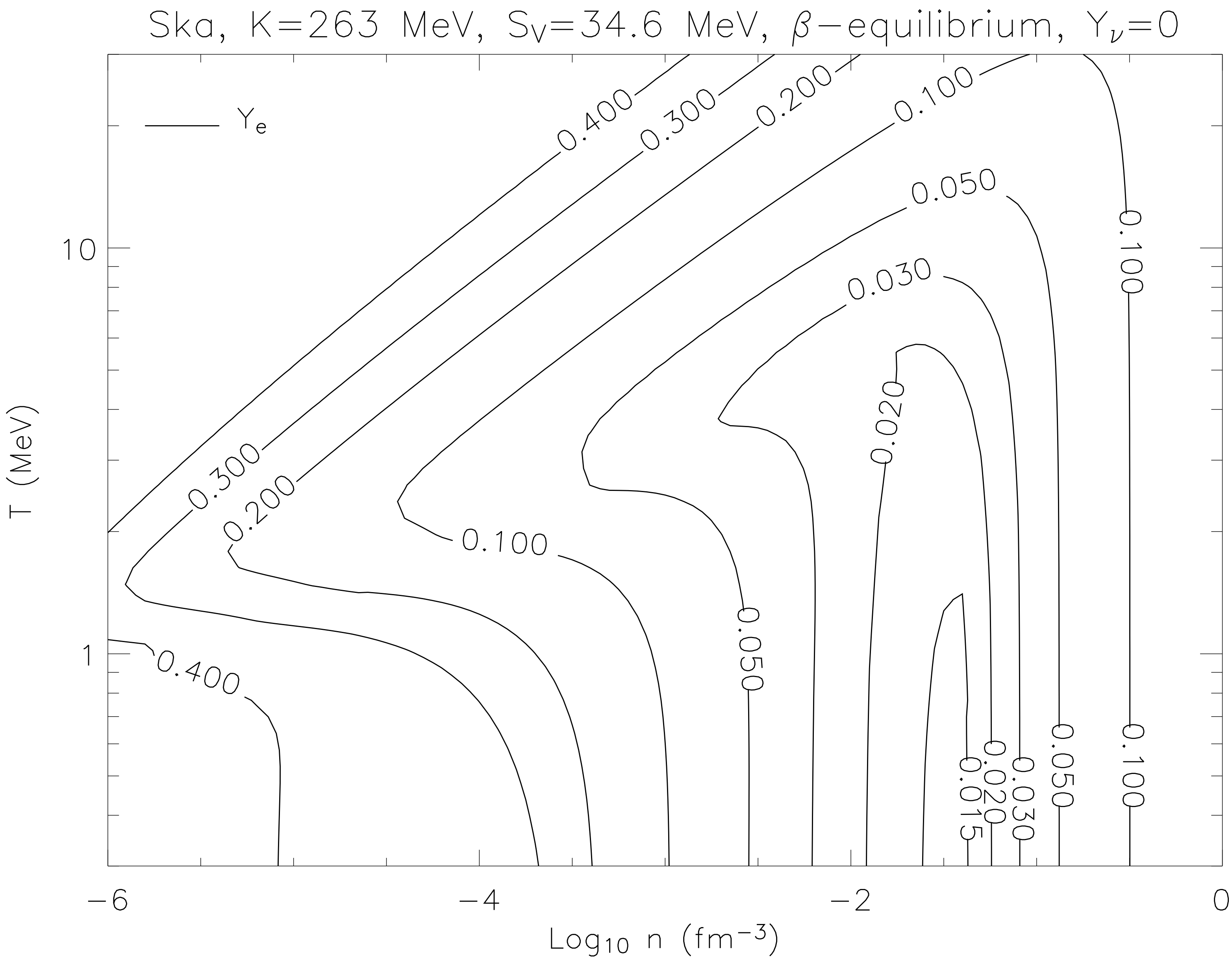
- Charge number of nucleus
- Proton fraction (and nuclear mass fraction)



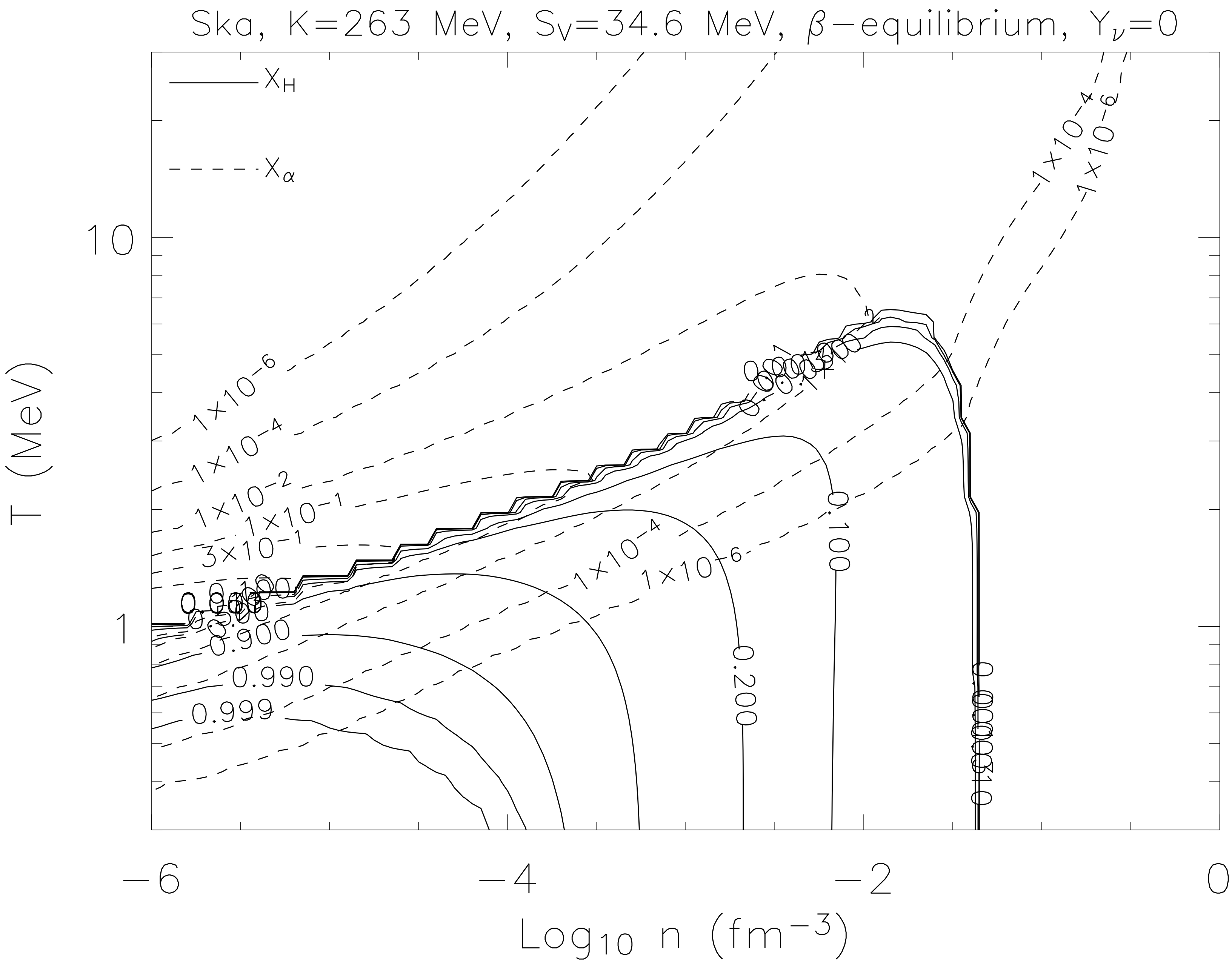


Oyamatsu & Iida (2006)

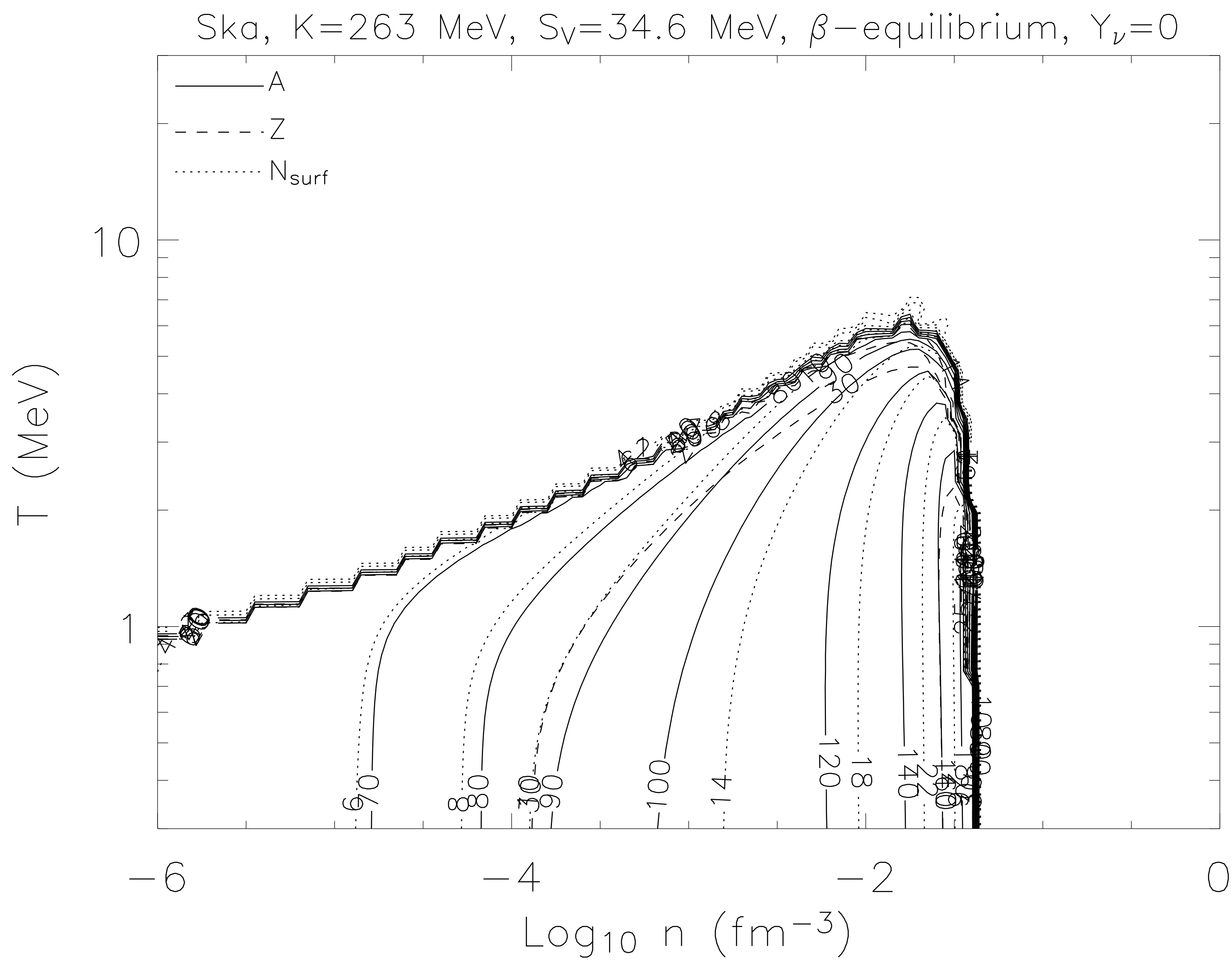
Composition, proton fraction



Composition, heavy mass fraction



Composition, mass and charge



Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\rho_s/\rho_f)^{1/2}\ \mathrm{M}_\odot$

  Rhoades & Ruffini (1974), Hartle (1978)

- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)\ \mathrm{km}$

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)

- $\rho_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2\ \mathrm{g\ cm}^{-3}$

  Lattimer & Prakash (2005)

- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

  Koranda, Stergioulas & Friedman (1997)

- $P_{min} \simeq (0.96 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$
  (empirical) Lattimer & Prakash (2004)

- $\rho_c > 0.44 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2\ \mathrm{g\ cm}^{-3}$ (empirical)

Newtonian Roche model for rotation

(*c.f.*, Shapiro & Teukolsky 1983)

$\rho^{-1}\nabla P = -\nabla(\Phi_G + \Phi_c)$

$\Phi_G \simeq -GM/r, \qquad \Phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$

Bernoulli integral:

$H = h + \Phi_G + \Phi_c = -GM/R_p$

Enthalpy $h = \int_0^P \rho^{-1} dP$

$h = \mu_n(\rho) - \mu_n(0)$ in beta equilibrium

Numerical calculations show R_p is nearly constant for arbitrary rotation

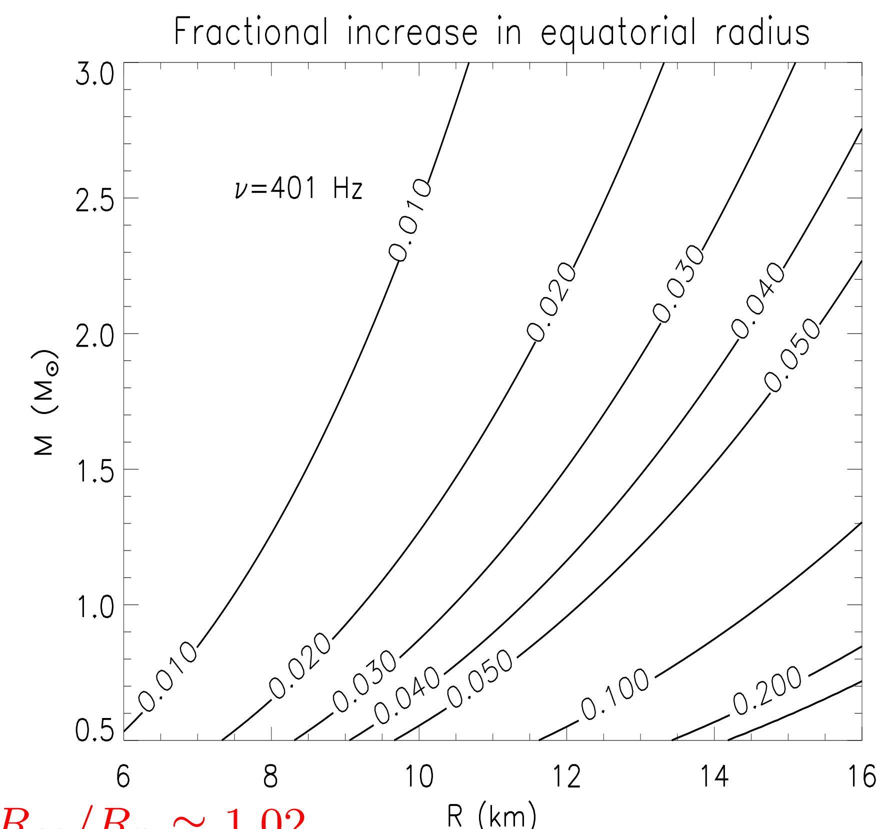
Variation of Bernoulli integral: $R_{shed}/R_p = 3/2$

Cook, Shapiro & Teukolsky (1994): 1.43–1.51

$$\Omega_{shed} = \left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{GM}{R_p^3}} = 0.544\sqrt{\frac{GM}{R_p^3}}$$

Lattimer & Prakash (2005): $0.570 \pm 3\%$

$$\frac{\Omega^2 R_p^3}{2GM} = \left(\frac{R_p}{R_{eq}(\Omega)}\right)^2 \left(1 - \frac{R_p}{R_{eq}(\Omega)}\right)$$



$\nu = 400$ Hz, $M = 1.4$ M$_\odot$, $R_p = 11$ km imply $R_{eq}/R_p \simeq 1.02$

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

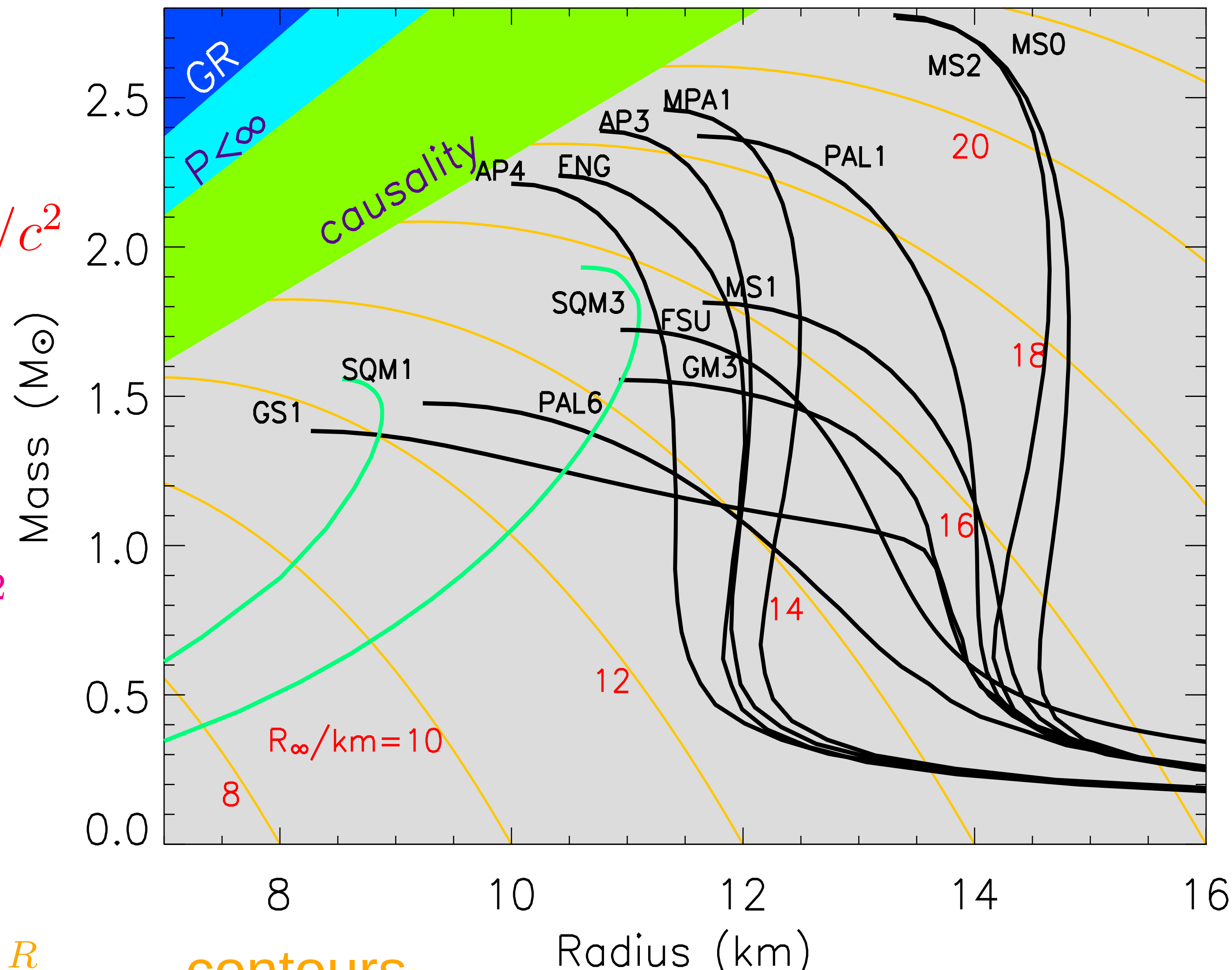
causality:

$R \gtrsim 2.9GM/c^2$

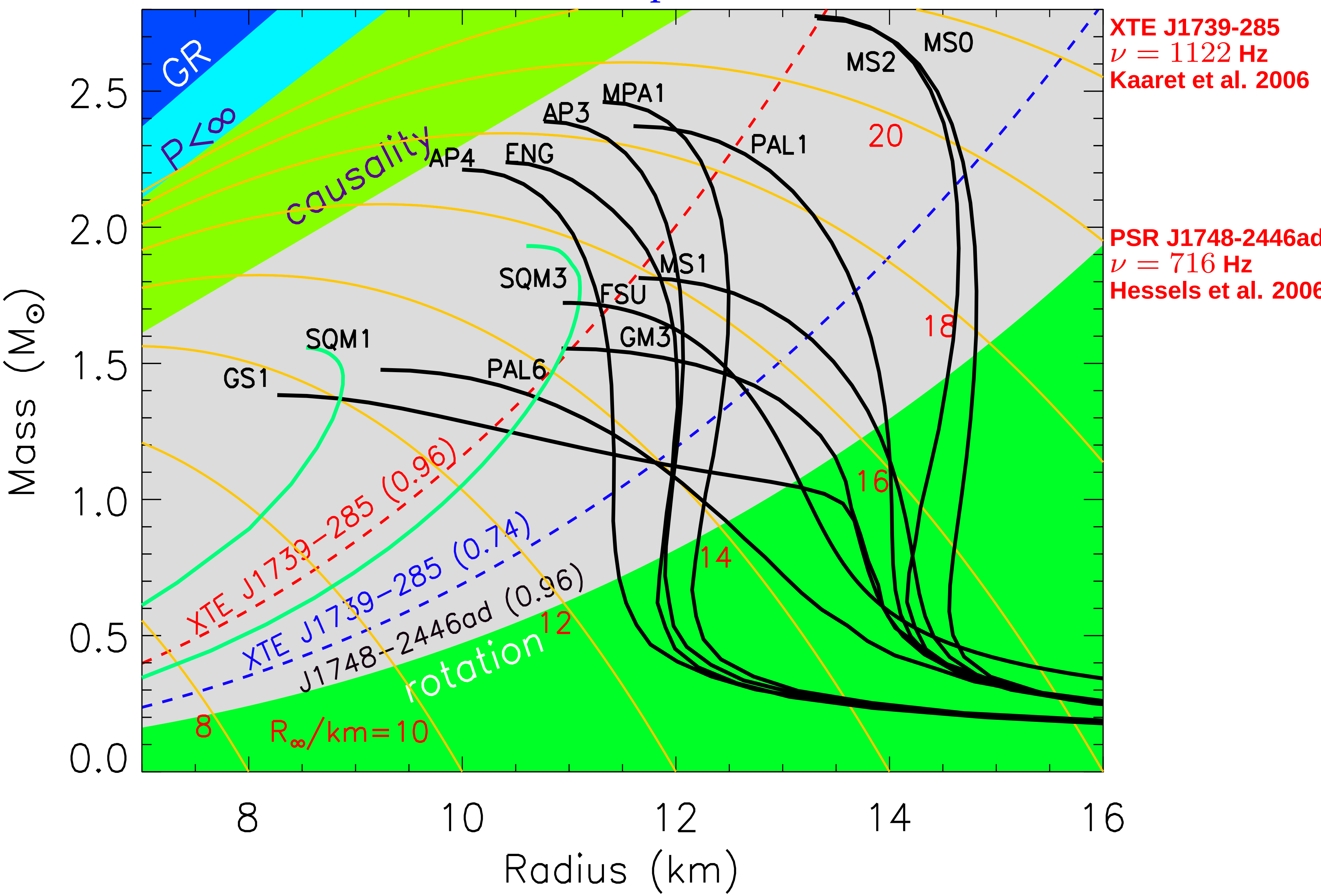
— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



Constraints from Pulsar Spins



Observed Masses

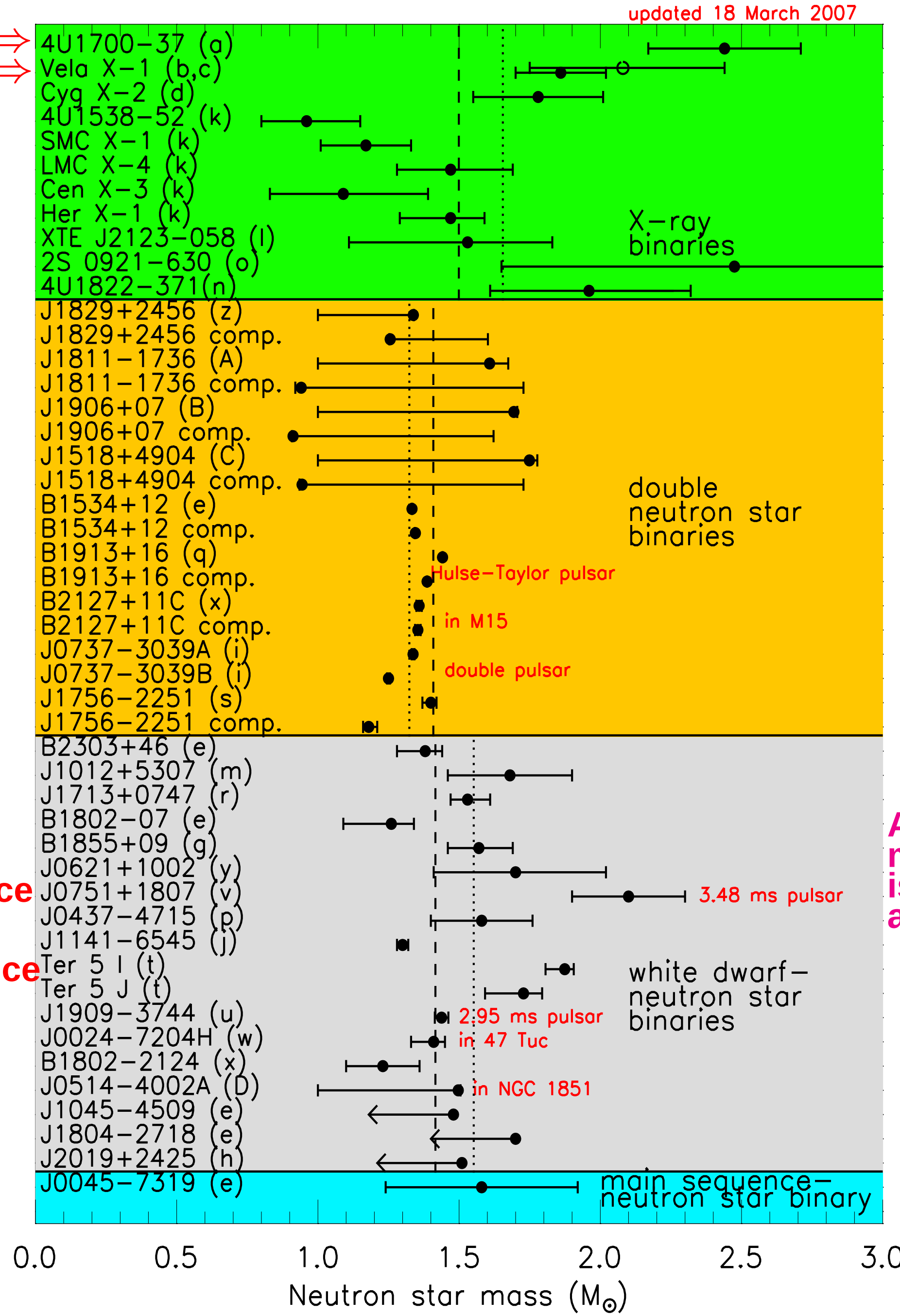
Black hole? ⇒

Firm lower mass limit? ⇒

$M \simeq 1.18$ M$_\odot$

$M > 1.6$ M$_\odot$, 95% confidence

$M > 1.68$ M$_\odot$, 95% confidence



Although simple average mass of w.d. companions is 0.21 M$_\odot$ larger, weighted averages are the same

Neutron Star Matter Pressure and the Radius

$P \simeq K\rho^{1+1/n}$

$n^{-1} = d\ln P/d\ln\rho - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto P_*^{1/2} \rho_*^{-1} M^0$

$(1 < \rho_*/\rho_s < 2)$

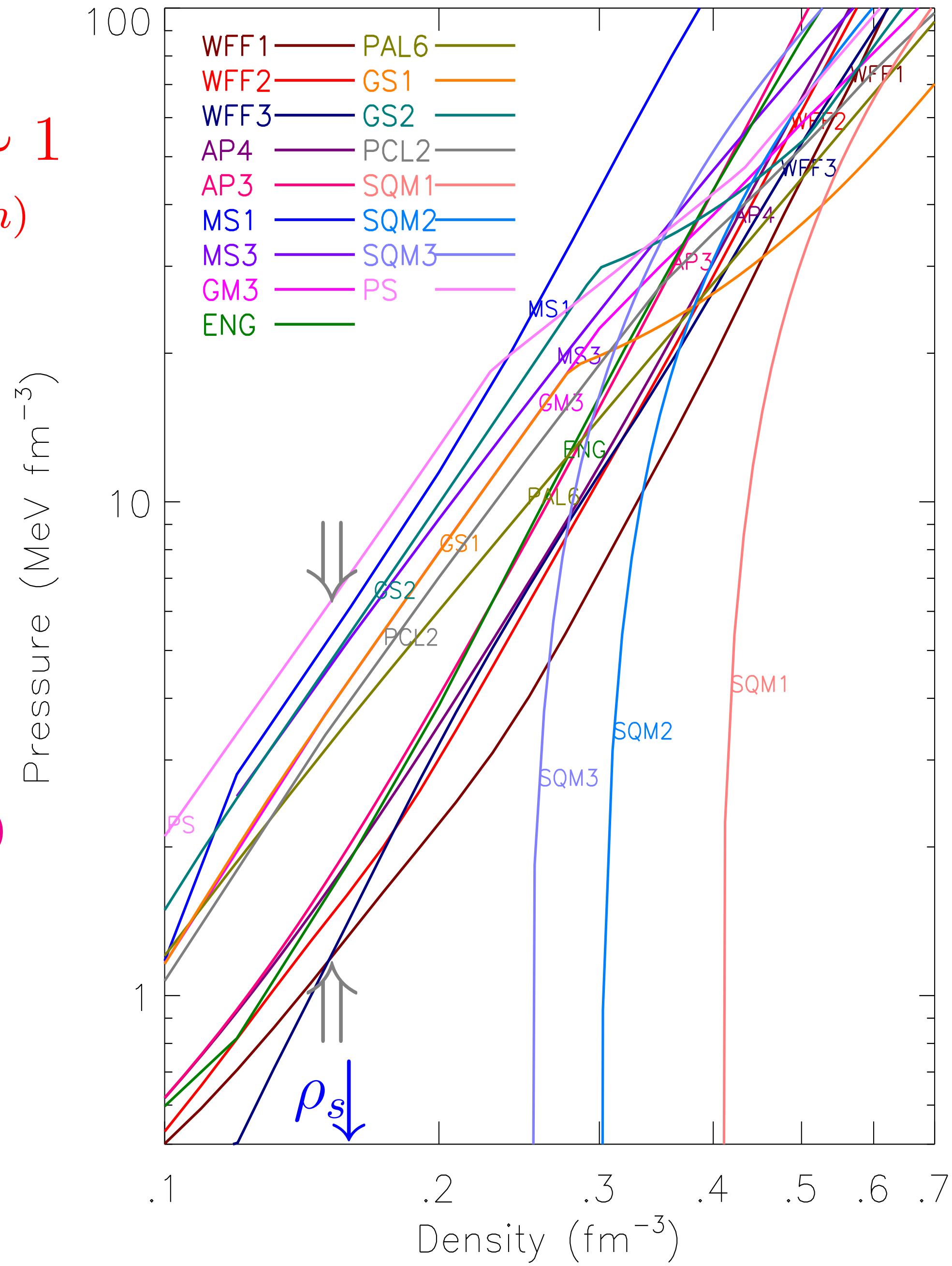
Wide variation:

$1.2 < \frac{P(\rho_s)}{\mathrm{MeV\ fm^{-3}}} < 7$

GR phenomenological result (Lattimer & Prakash 2001)

$R \propto P_*^{1/4} \rho_*^{-1/2}$

$P_* = n^2 dE_{sym}/dn$



Radiation Radius

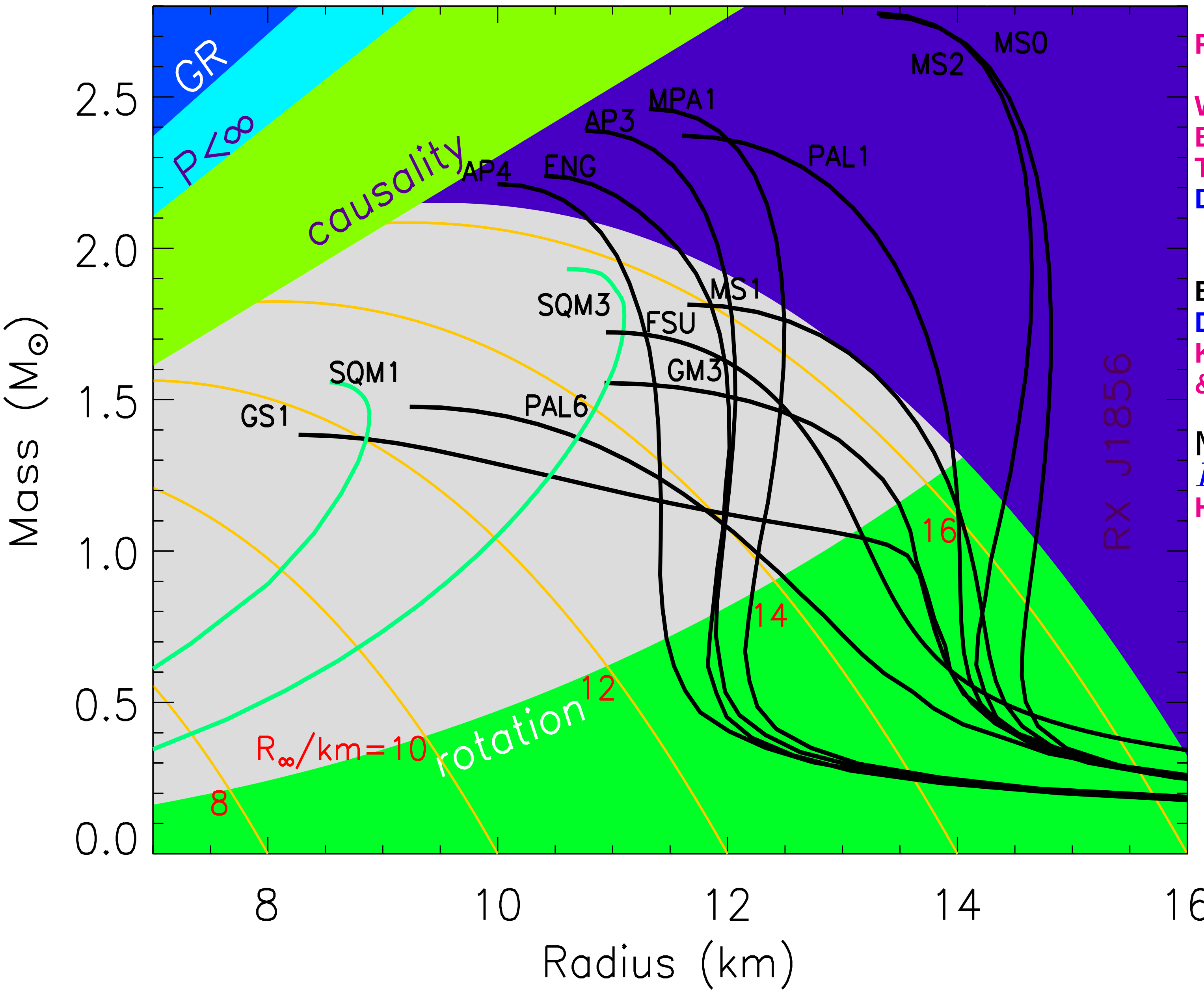
- Combination of flux and temperature measurements yield
$$R_\infty = R/\sqrt{1 - 2GM/Rc^2}$$
- Uncertainties include distance ($R_\infty \propto d$), interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)

Optical flux on Rayleigh-Jeans tail 5-7 times extrapolated X-ray BB and $T_{opt} \sim T_X/2$:

$$F_{opt} = 4\pi \left(\frac{R_{opt}}{d}\right)^2 T_{opt} = 4\pi f \left(\frac{R_X}{d}\right)^2 T_X$$

$$R = \sqrt{R_{opt}^2 + R_X^2} = \sqrt{1 + 2f} R_X$$

Radiation Radius: Nearby Neutron Star



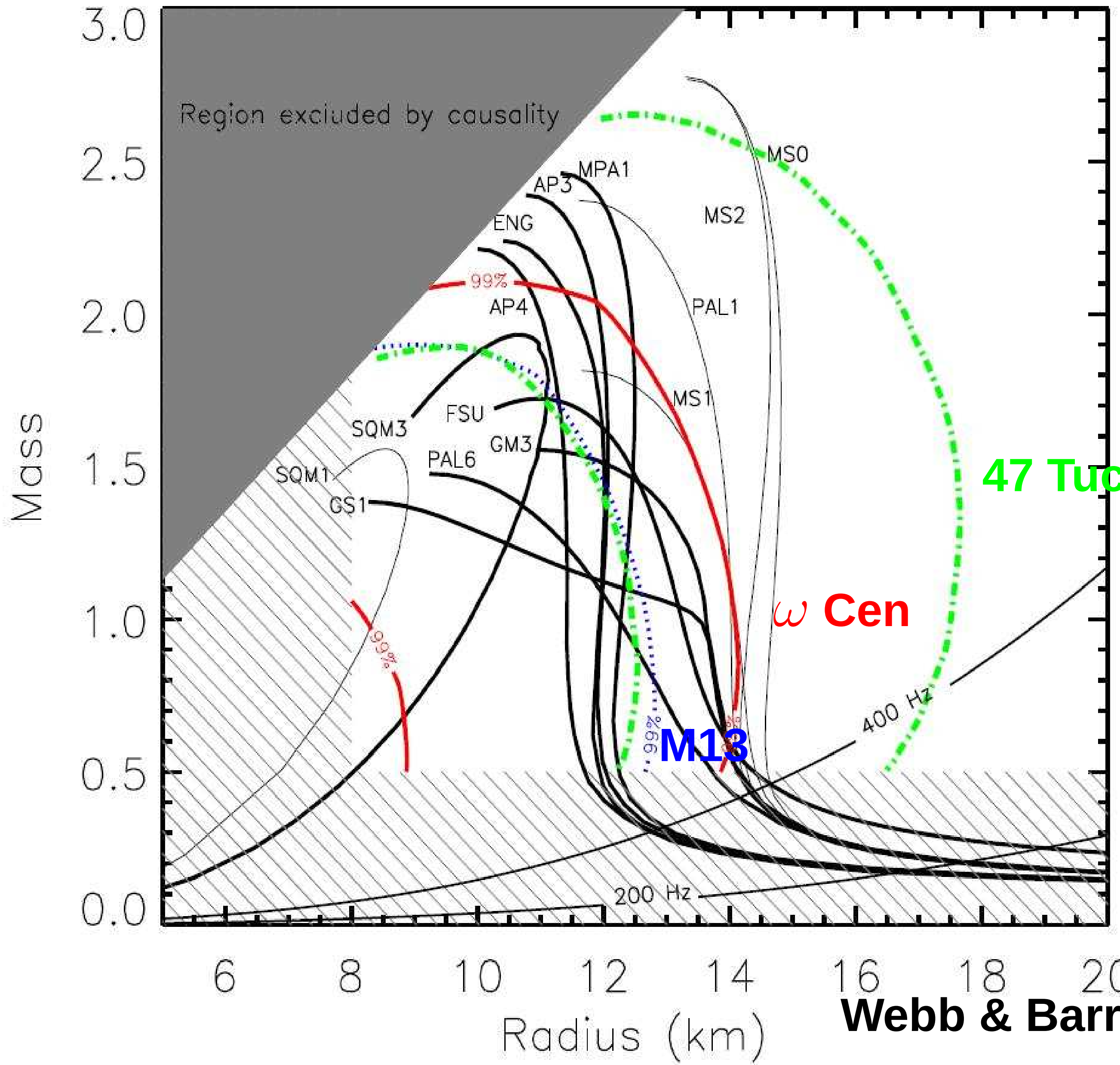
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk
& Anderson 2002

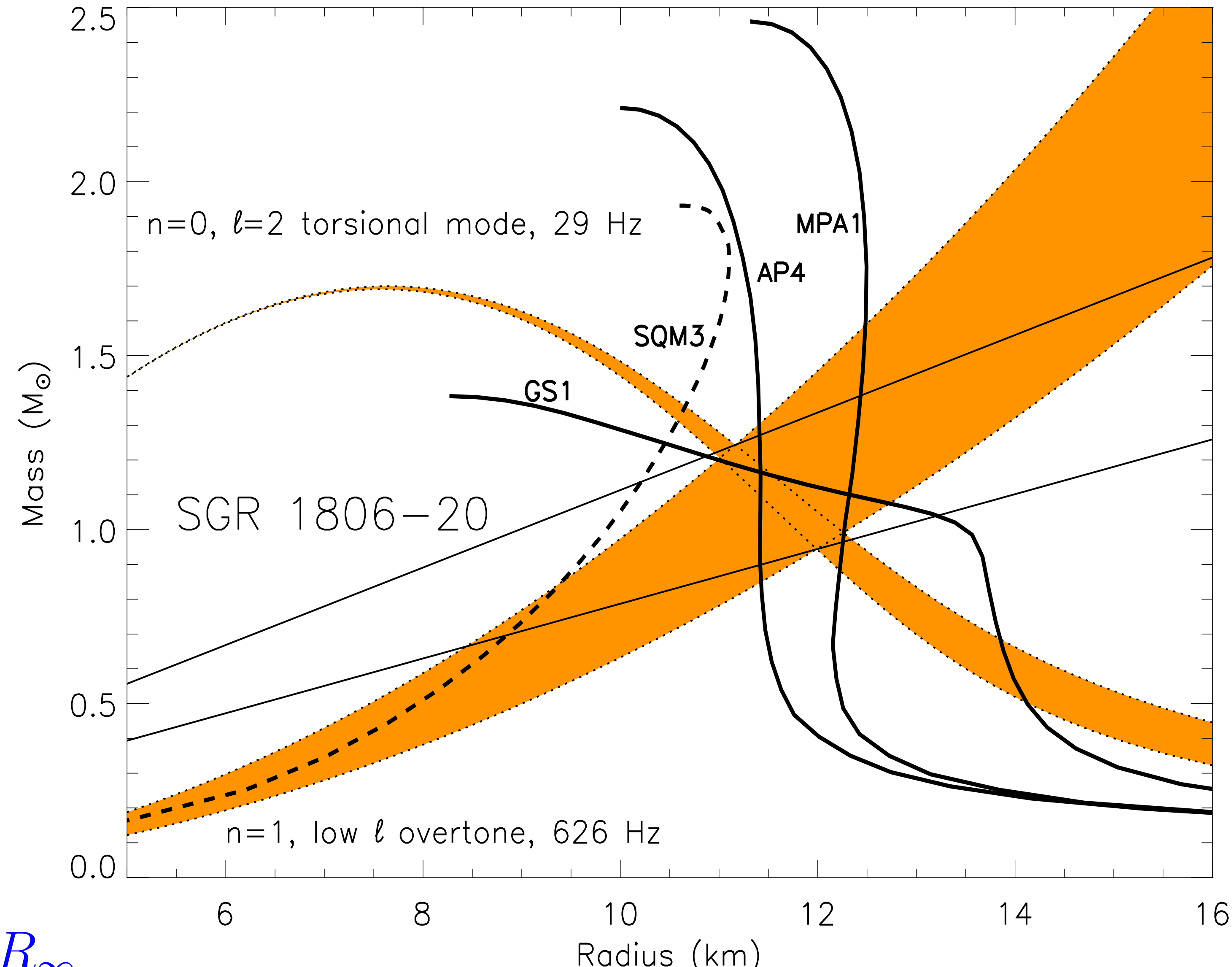
Magnetic H atmosphe
$R_\infty \approx 17$ **km**
Ho et al. 2007

Radiation Radii: Globular Cluster Sources



Webb & Barret 2007

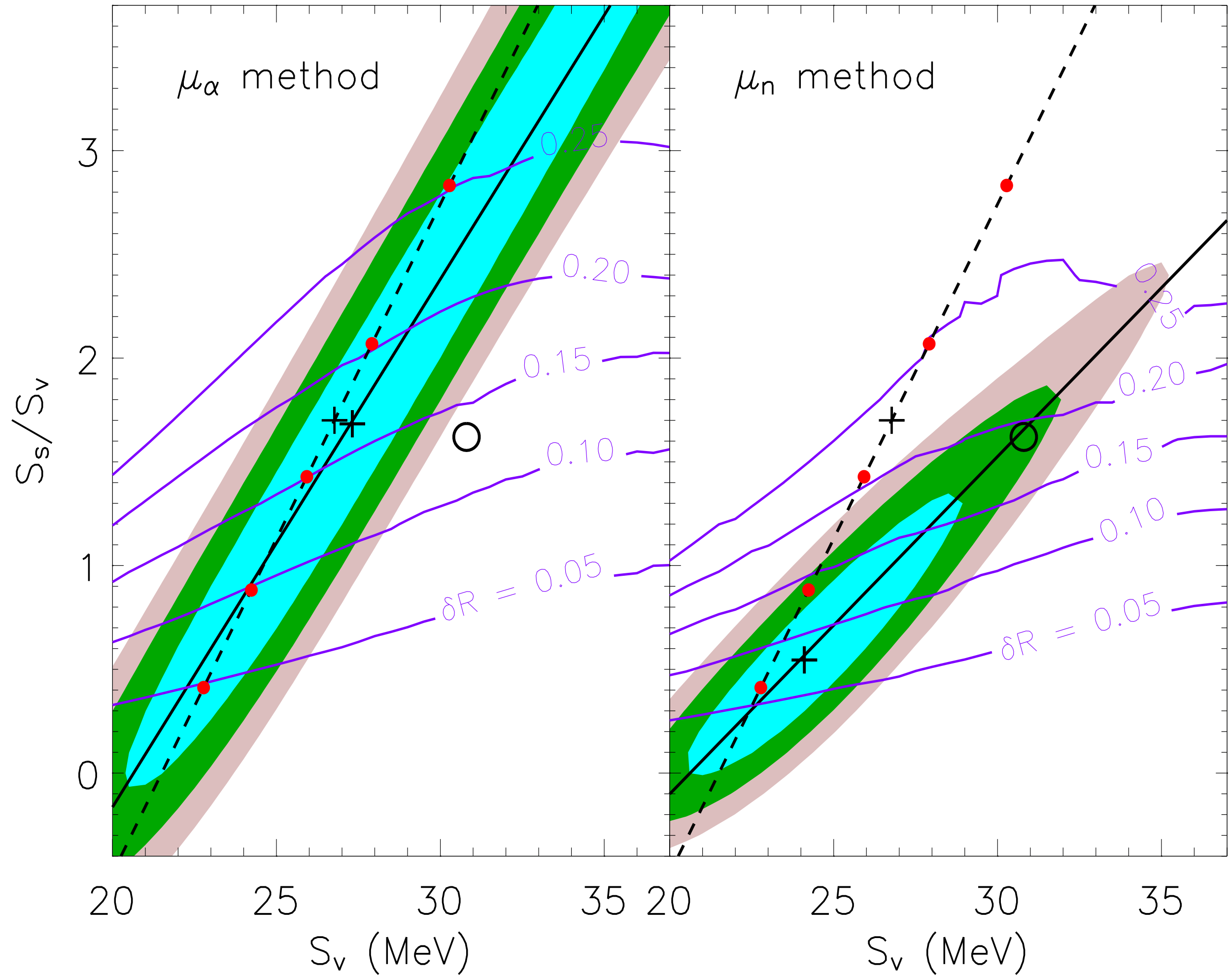
Neutron Star Seismology



$$f_{n=0} \sim v_t / R_\infty$$
$$f_{n>0} \sim v_r (1 - 2GM/Rc^2)/\Delta \propto R^2/M$$

Strohmayer & Watts (2005)
Samuelsson & Andersson (2006)
Lattimer & Prakash (2006)

Fits to Nuclear Masses and Neutron Skin Thickness



Blue: $\Delta E < 0.01$ MeV/b
Green: $\Delta E < 0.02$ MeV/b
Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)
Crosses: Best fits
Dashed: Danielewicz (2004); Solid: Steiner et al. (2005)

$$S_s/S_v \simeq 4\pi r_o^2 \int_0^R n\left[S_v/E_{sym}(n) - 1\right] dr$$

Thomas-Fermi Nuclear Model

$$H = ne_B + \frac{Q}{n}(\nabla n)^2, \qquad e_B = -B + \frac{K}{18}(1-u)^2 + E_{sym}(n)(1-2x)^2$$

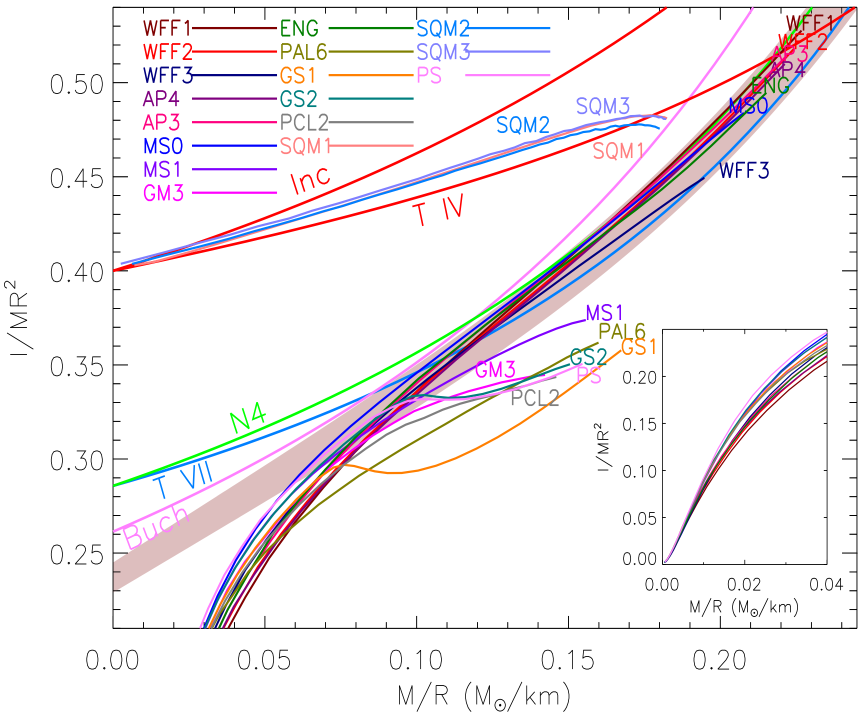
$$n(e_B + B) = \frac{Q}{4n}n'^2, \qquad n = n_0(1+e^{x/a})^{-1}, \qquad a = \sqrt{\frac{9Qn_0}{2K}}$$

$$\omega = \omega_0 + \omega_\delta(1-2x)^2 + \ldots$$

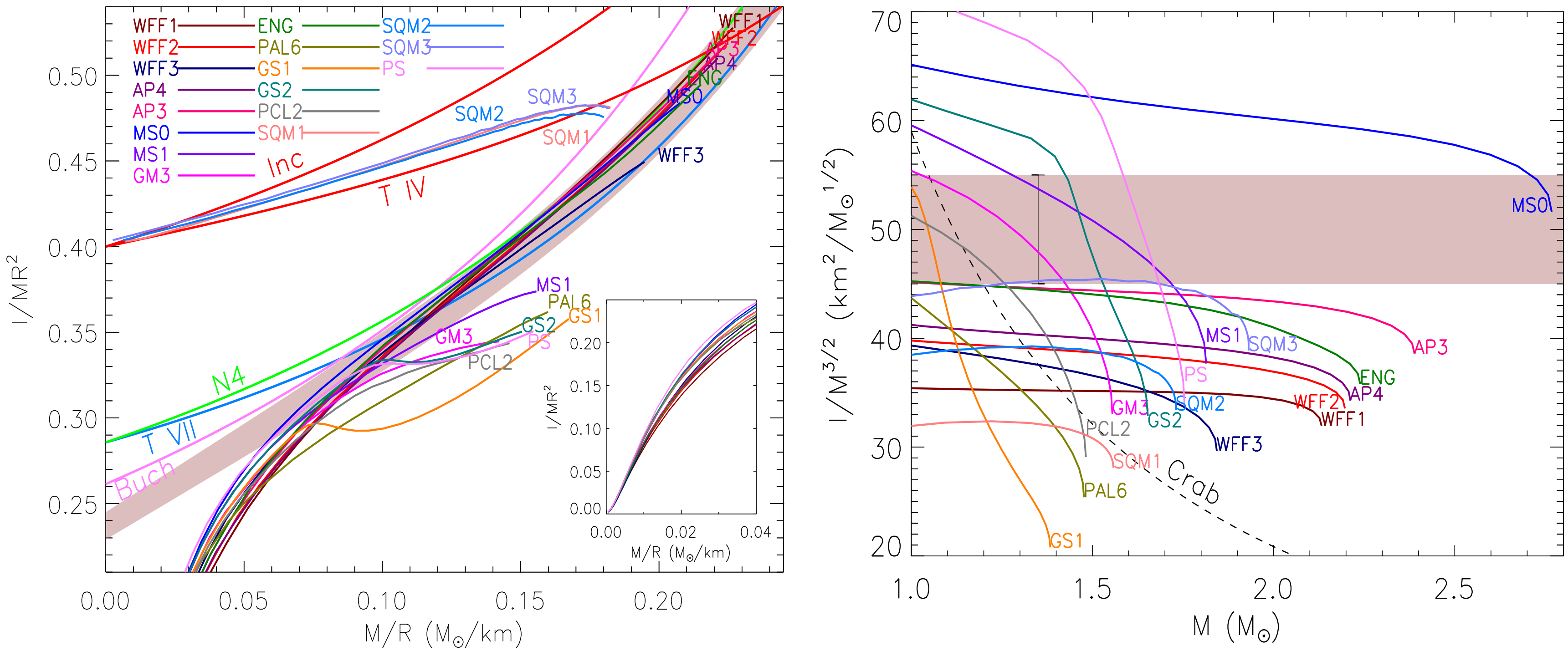
$$\begin{aligned}\omega_0 &= \int [H_0 - nB]dr = 2\int n[e_{B0} + B]dr \\ &= \sqrt{Q}\int\sqrt{e_{B0} + B}\,dn = \frac{Qn_0^2}{12a} = \frac{aKn_0}{54}\end{aligned}$$

$$\begin{aligned}\omega_\delta &= S_v n_0 \sqrt{\frac{9Qn_0}{2K}} \int \left[\frac{1}{1+b(u-1)} - 1\right](1-u)^{-1}du \\ &= -S_v n_0 a \ln(1-b), \qquad b = \frac{L}{3S_v}, \qquad S_s = 4\pi r_o^2 \omega_\delta\end{aligned}$$

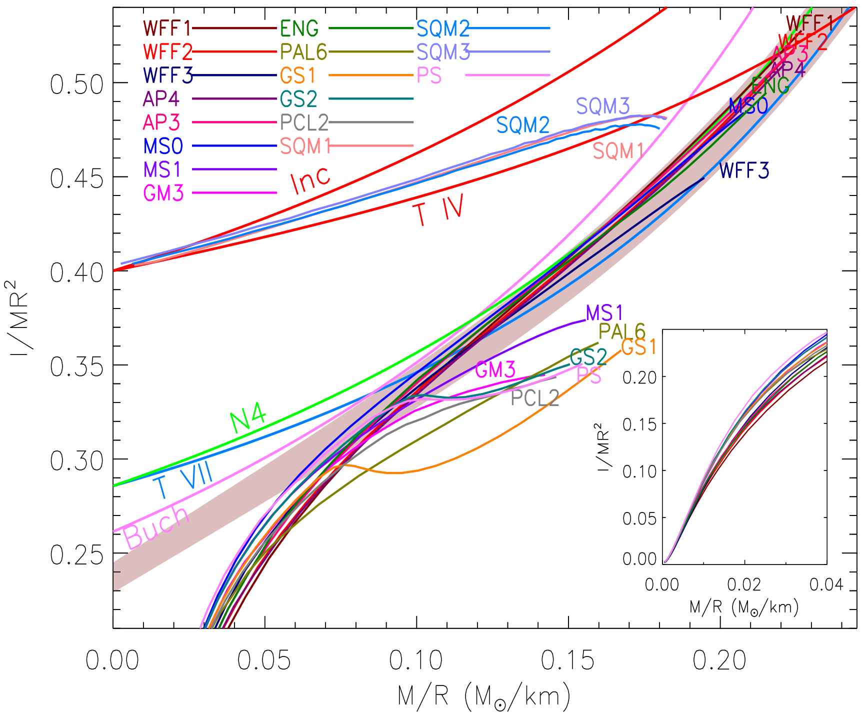
Moments of Inertia

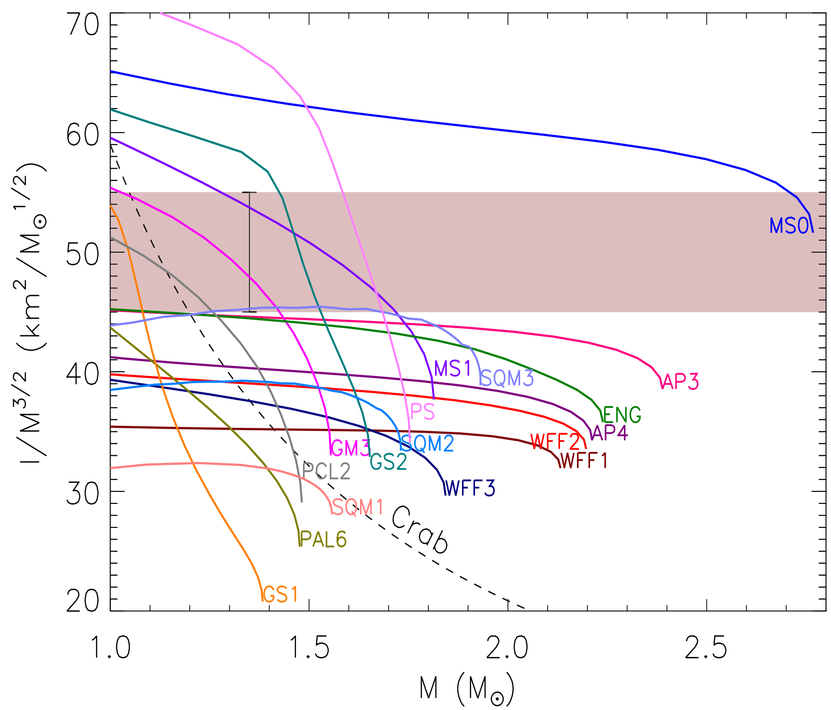


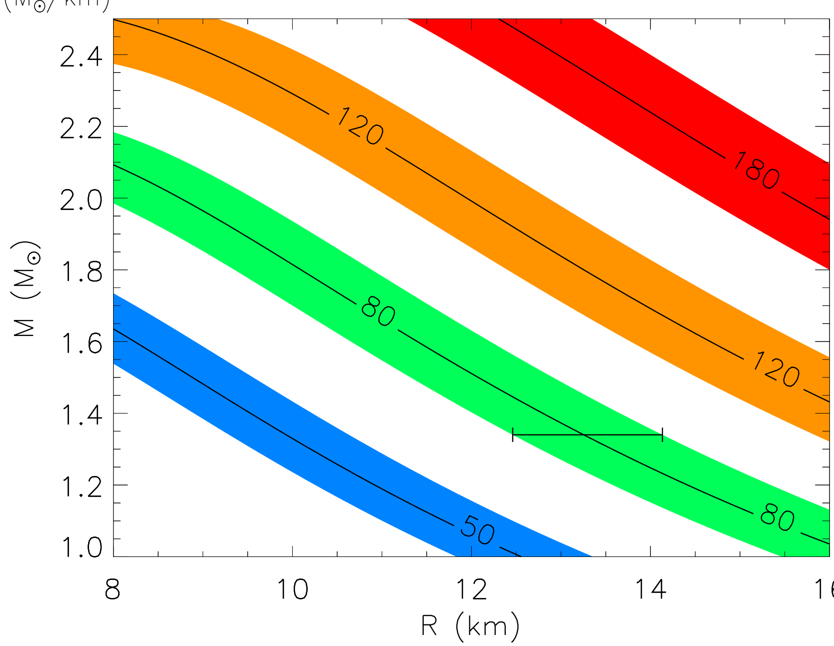
Moments of Inertia



Moments of Inertia







Lattimer & Schutz (2005)