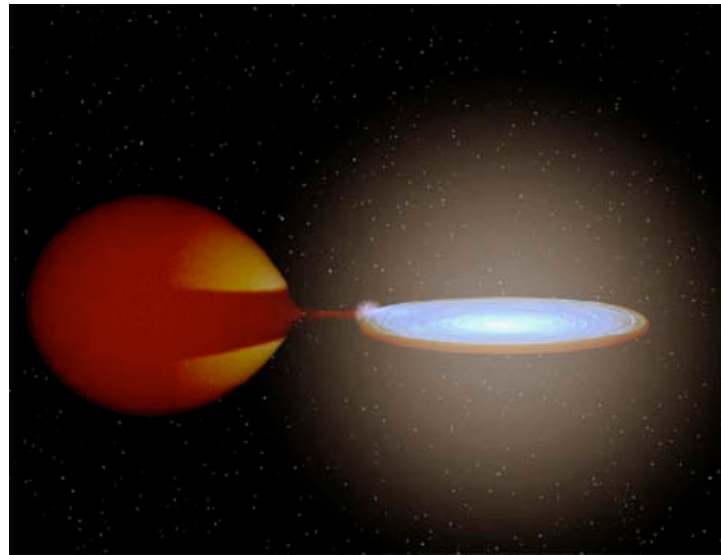


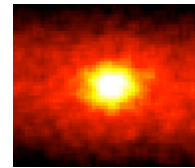
# Neutron Rich Matter and Neutron Star Crusts



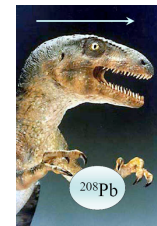
C. J. Horowitz, Indiana University  
INT Workshop, June 2007

# Neutron Rich Matter and Neutron Star Crusts

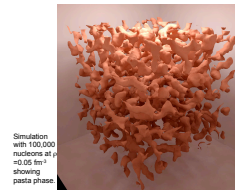
I) Neutrinosphere in supernovae and the viral expansion.



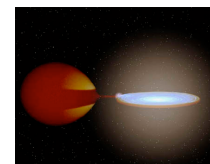
II) Crust properties and the density dependence of the symmetry energy. Update of Pb radius experiment (PREX).



III) Nuclear pasta phases and molecular dynamics simulations.

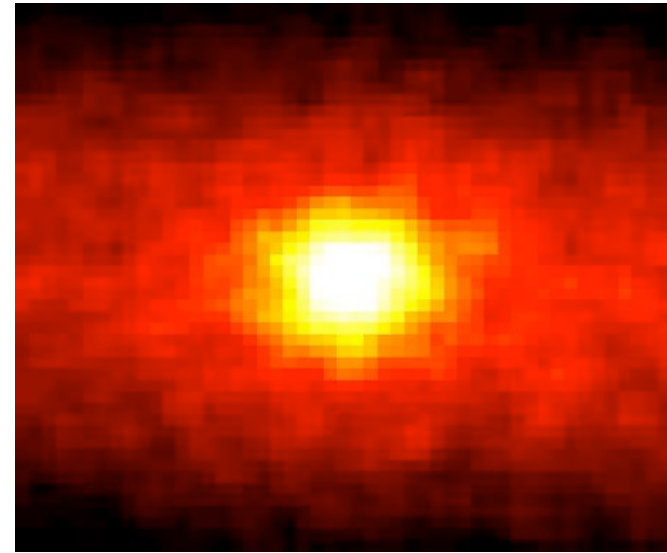


IV) Crust of accreting neutron stars and chemical separation.



# Neutrinosphere of a Supernova

- View sun in neutrinos, (see angular resolution of  $\nu$ -e scattering in Super-K).
- View SN in neutrinos, see surface of last scattering called the neutrinosphere. What is neutrinosphere like?
- Conditions at neutrinosphere:
  - Temperature  $\sim 4$  MeV from 20 SNI987a events.
  - Mean free path  $\lambda = 1/\sigma\rho \sim R$
  - $\sigma \sim G_F^2 E_\nu^2$  and  $E_\nu \sim 3T$
  - $\rho \sim 10^{11}$  g/cm<sup>3</sup> [ $10^{-4}$  fm<sup>-3</sup> or 1/1000 nuclear density]



- What is the composition, equation of state, and neutrino response of the neutrinosphere?
- *At low neutrinosphere density, Virial expansion gives model independent answers!*

# Virial Expansion

with A. Schwenk

- Assume (1) system in gas phase and has not undergone a phase transition with increasing density or decreasing temp. (2) fugacity  $z=e^{\mu/T}$  is small ( $\mu$  chemical pot).
- Expand pressure in powers of  $z$  :

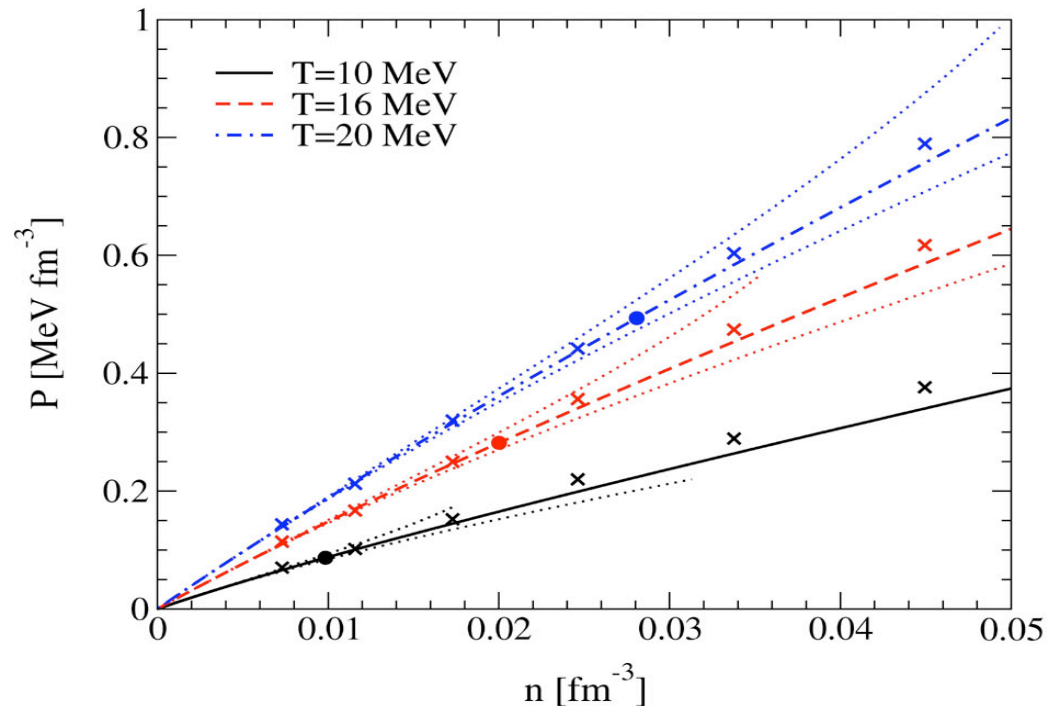
$$P=2T/\lambda^3[z+b_2z^2+b_3z^3+\dots],$$

Here  $\lambda$ =thermal wavelength= $(2\pi/mT)^{1/2}$

- 2<sup>nd</sup> virial coef.  $b_2(T)$  from 2 particle partition function which depends on density of states determined from phase shifts:

$$b_2 = 2^{1/2} \sum_B e^{E_B/T} + \frac{2^{1/2}}{\pi} \int_0^\infty dk e^{-E_k/2T} \sum_l (2l+1) d\delta_l(k)/dk \pm 2^{-5/2}$$

# Neutron matter Equation of State



Error bars (dotted)  
from estimate of  $b_3$

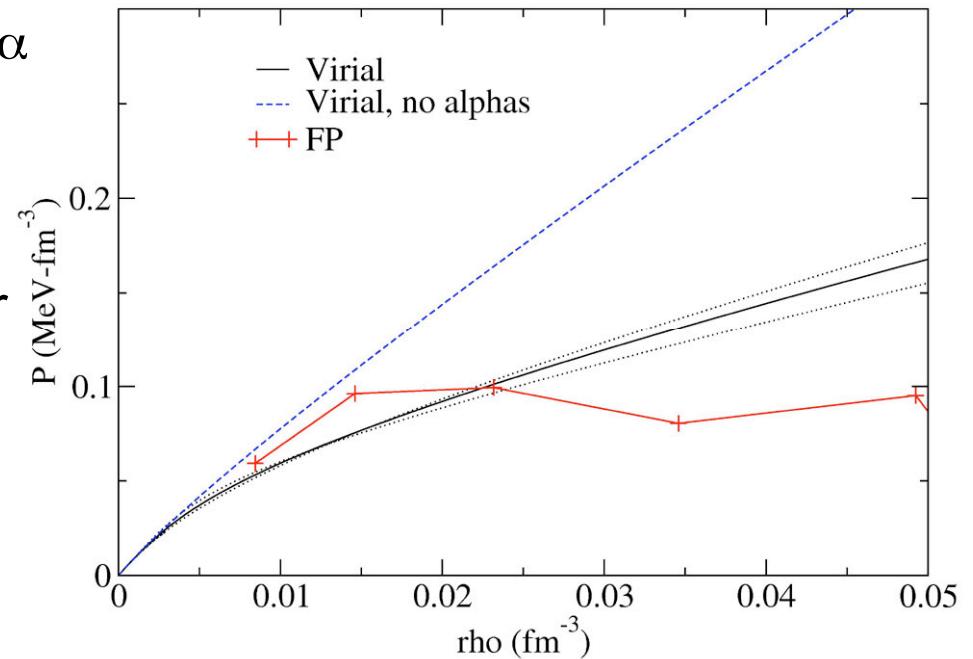
Crosses from  
microscopic FHNC  
calc. by Friedman +  
Pandharipande

Neutron matter is related to the universal *Unitary Gas* that can be studied with cold atoms in laboratory traps. Unitary Gas, with infinite scattering length, has no length scales associated with interactions. As a result the EOS scales:  $P$  only a function of  $n/T^{3/2}$ . We find neutron matter EOS also scales.

# Nuclear Matter with p, n, and alphas

$$\frac{P}{T} = \frac{2}{\lambda^3} [z_p + z_n + (z_p^2 + z_n^2)b_n + 2z_p z_n (b_{nuc} - b_n)] + \frac{1}{\lambda_\alpha^3} [z_\alpha + z_\alpha^2 b_\alpha + z_\alpha (z_p + z_n) b_{n\alpha}]$$

- Four virial coefficients from NN, N $\alpha$  and  $\alpha\alpha$  elastic scattering phase shifts.
- Conventional microscopic approaches fail because of cluster (alpha) formation.
- Variational wave-function  $\Psi = \prod_{i < j} f(r_{ij}) \Phi$  can only describe a single cluster. --> FP calc. just numerical noise.



Pressure of symmetric nuclear matter at a temperature of T=10MeV.

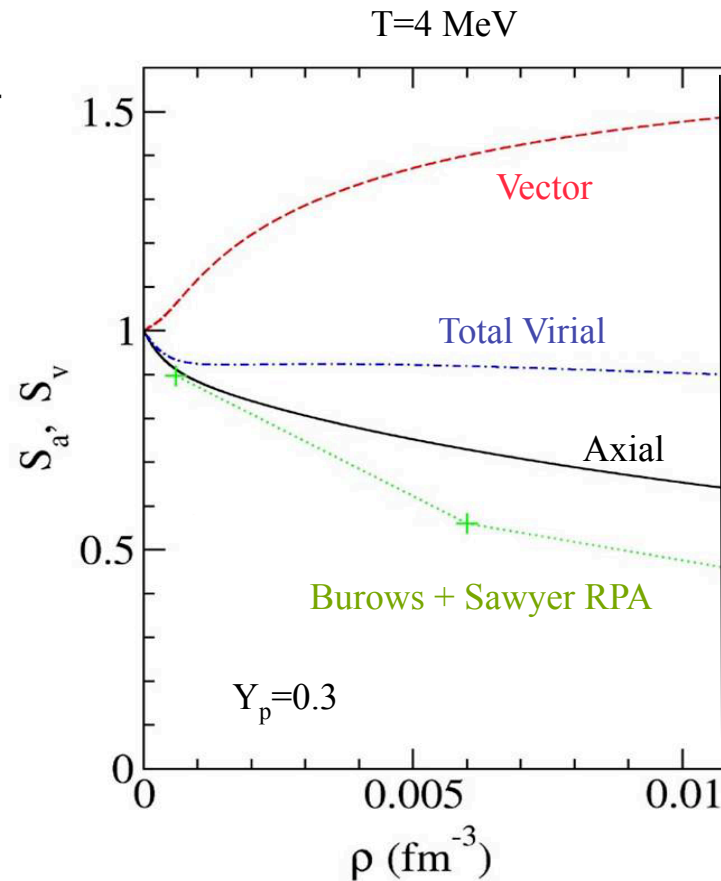
# Neutrino Response

- $\nu$  neutral current cross section:  

$$d\sigma/d\Omega = (G^2 E_\nu^2 / 16\pi^2) [(1 + \cos\theta) S_v + g_a^2 (3 - \cos\theta) S_a]$$
- Vector response is static structure factor  $S_v = S(q)$  as  $q \rightarrow 0$ :  

$$S(0) = T / (dP/dn)$$
- Axial or spin response from spin polarized matter:  

$$S_a = (1/n) d/dz_a (n_+ - n_-) |_{n_+ = n_-}$$
- Typical RPA calculations neglect alpha particles.
- *Virial expansion provides model independent results for EOS, composition, and  $\nu$  response of low density neutron rich matter.*

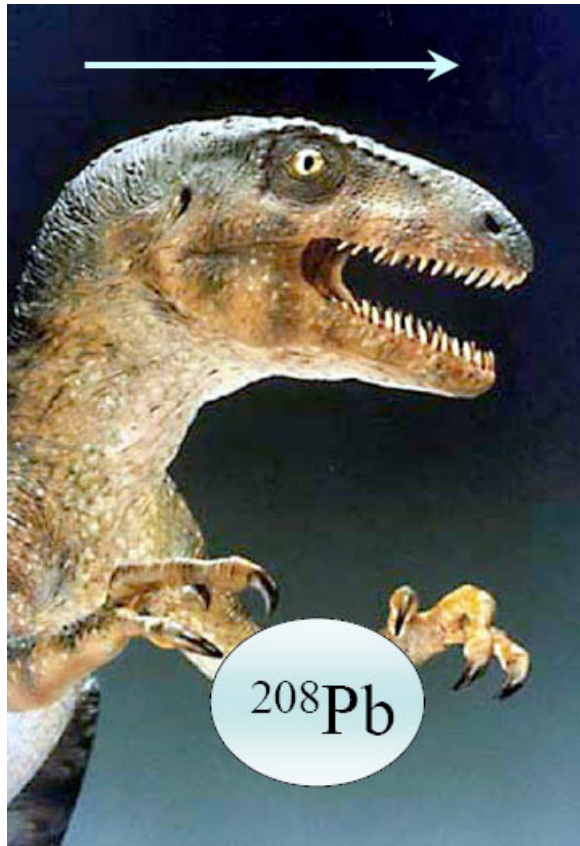


# Neutron Star Crusts and Density Dependence of Symmetry Energy

- Symmetry energy  $S$  describes how  $E$  of nuclear matter rises as one goes away from equal numbers of neutrons and protons:  
$$(E/A)_{\text{neutron}} = (E/A)_{\text{nuclear}} + S$$
- Pressure depends on derivative of  $E$  wrt density.  $P$  of nuclear matter small and largely known.  ***$P$  of neutron matter depends on density dependence of  $S$ .***



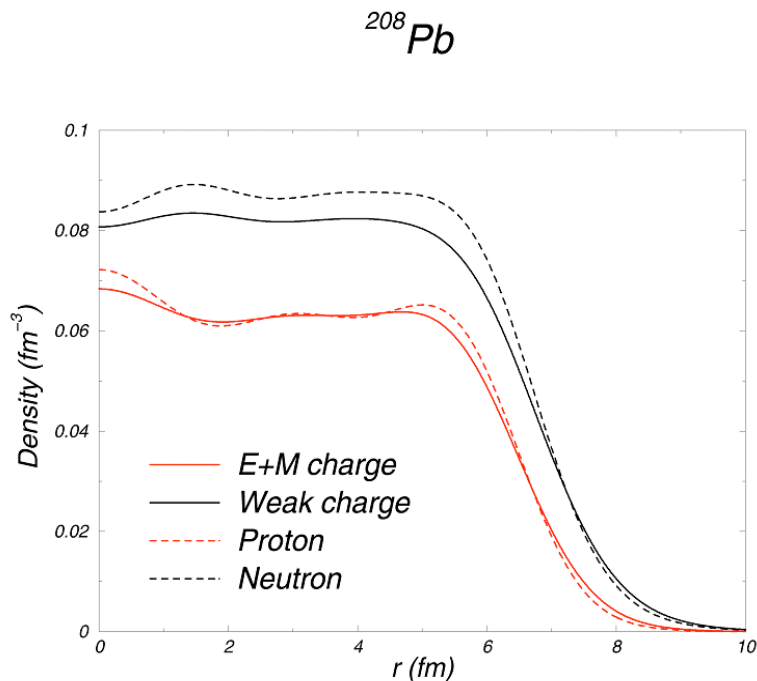
# Pb Radius Experiment (PREX)



- Uses parity violating electron scattering to measure the rms radius of the neutron density in  $^{208}\text{Pb}$ .
- This has many implications for neutron stars and their crusts.



# Probing Skin of $^{208}\text{Pb}$



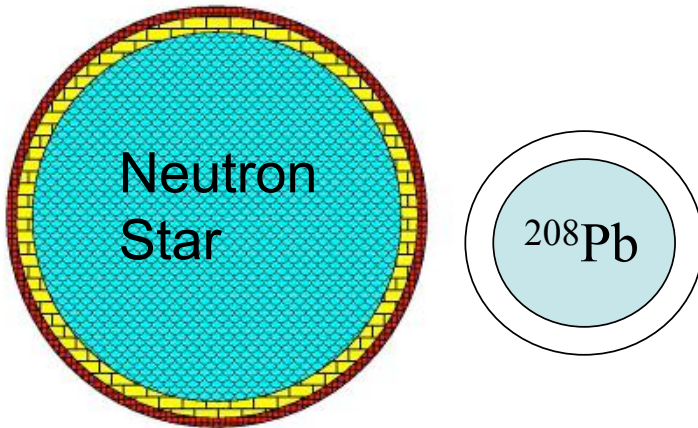
Heavy nucleus has neutron rich skin. Thickness of skin depends on pressure of neutron matter as n are pushed out against surface tension.

- Parity violation probes neutrons because weak charge of a  $n \gg p$ .
- Elastic scattering of 850 MeV e from  $^{208}\text{Pb}$  at 6 deg.

$$A = \frac{\frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega} -}{\frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega} -}$$

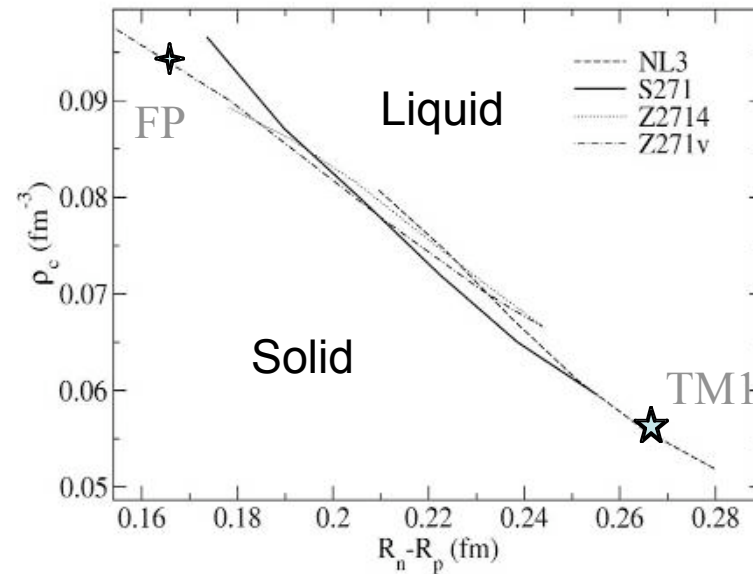
- Measure  $A \sim 0.6$  ppm to 3%. This gives neutron radius to 1%.
- Purely electroweak reaction is model independent
- Spokespersons: P. Souder, R. Michaels, G. Urciuoli

## Neutron Star Crust vs Pb Neutron Skin



- Neutron star has solid crust (yellow) over liquid core (blue).
- Nucleus has neutron skin.
- Both neutron skin and NS crust are made out of neutron rich matter at similar densities.
- **Common unknown is EOS at subnuclear densities.**

## Liquid/Solid Transition Density



- Thicker neutron skin in Pb means energy rises rapidly with density  $\rightarrow$  Quickly favors uniform phase.
- Thick skin in Pb  $\rightarrow$  low transition density in star.

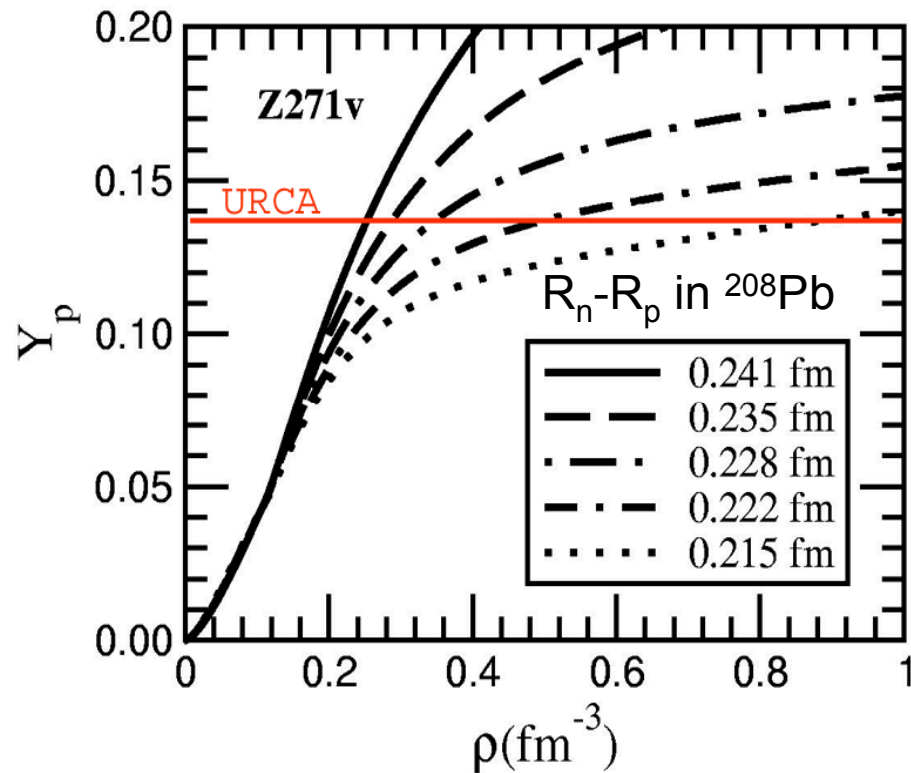
with J. Piekarewicz

# PREX Constrains Rapid Direct URCA Cooling of Neutron Stars

- Proton fraction  $Y_p$  for matter in beta equilibrium depends on symmetry energy  $S$ .  

$$\mu_e = \mu_n - \mu_p \sim S$$
- $R_n$  in Pb determines density dependence of  $S$ .
- The larger  $R_n$  in Pb the lower the threshold mass for direct URCA cooling.
- If  $R_n - R_p < 0.2$  fm all EOS models do not have direct URCA in 1.4 M- stars.
- If  $R_n - R_p > 0.25$  fm all models do have URCA in 1.4 M- stars.

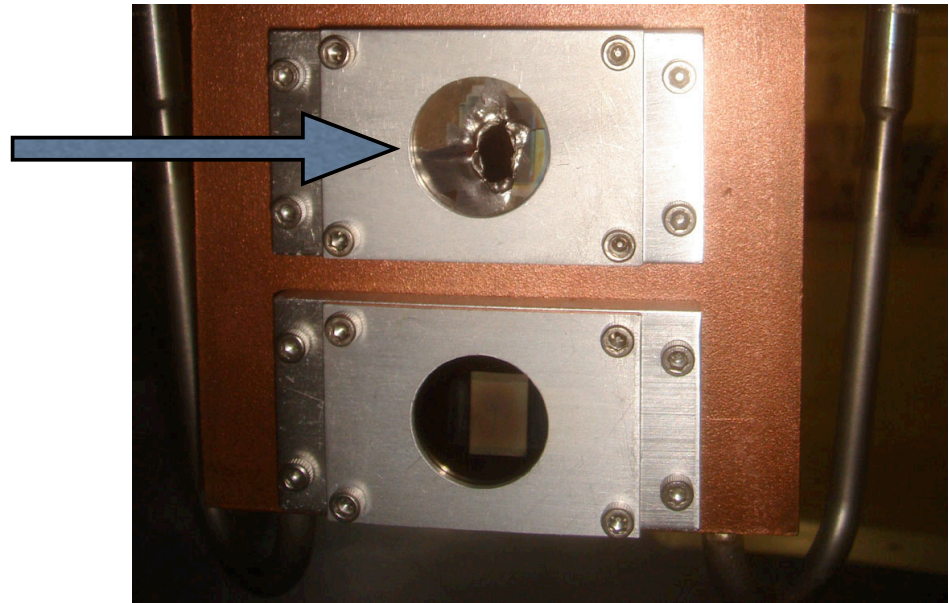
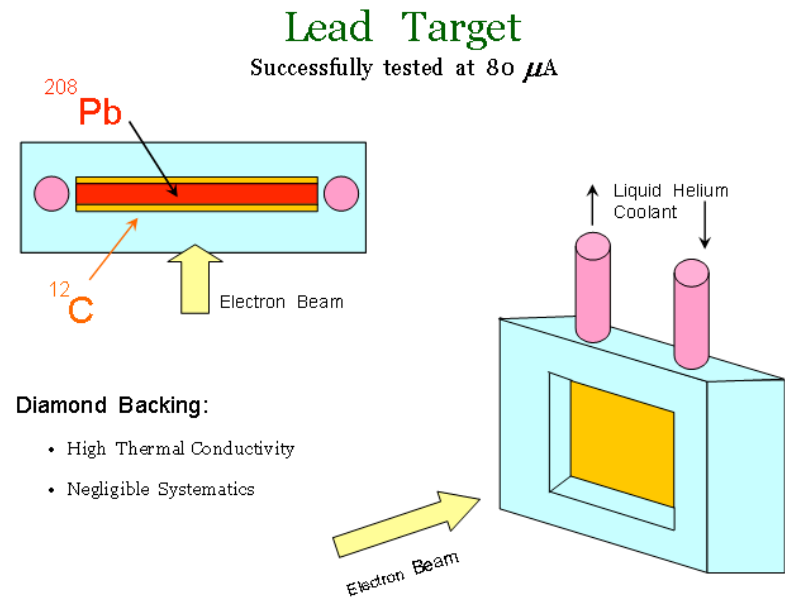
with J. Piekarewicz



If  $Y_p >$  red line NS cools quickly via direct URCA reaction  $n \rightarrow p + e + \bar{\nu}$

# Pb Target Test

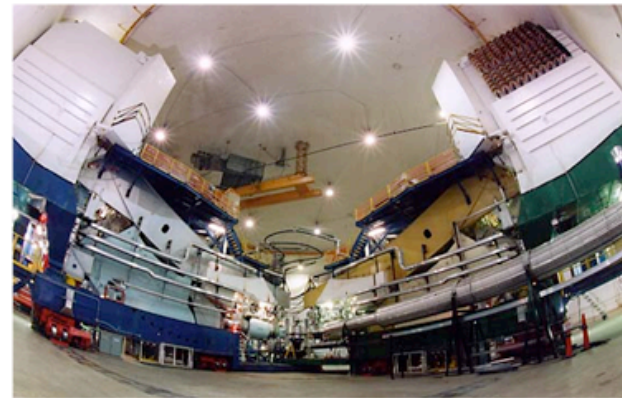
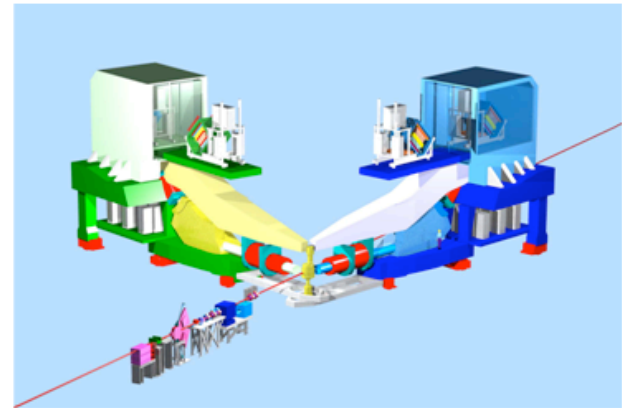
- A thinner version of the Pb-diamond foil target was tested recently by a different group.
- Diamond foils did not suffer radiation damage.
- Target melted near 80 micro-amps!  
Presumably because of poor thermal contact Pb to diamond.  
Target did not use vacuum grease!
- Earlier, shorter, test with grease was fine.



# PREX Status

- Some beam time this fall for full tests.
- Target (with vacuum grease) seems fine. Radiation in hall is ok.
- Plan to build septum magnets (bend 6 deg. scattered electrons to 12 deg. to enter spectrometers).
- Plan to upgrade polarimetry including green laser for Compton e-gamma polarimeter.
- Full run soon: '08, '09?

*JLab Hall A Collaboration*



P. Souder, R. Michaels, G. Urciuoli



# Nuclear Pasta

- Near nuclear densities, there is frustration between comparable attractive nuclear and repulsive Coulomb interactions.
  - Leads to a complex ground state.
  - Can involve round (meat ball), rod (spaghetti), plate (lasagna), or other shapes.

with J. Piekarewicz

# Molecular Dynamics Pasta Model

- Charge neutral system of n, p, e. Electrons form relativistic, degenerate, Fermi gas.
- Thermal wavelength of heavy clusters small compared to inter cluster spacing: semi-classical approx. should be good.
- n, p interact via 2 body potential

$$v(r) = a \text{Exp}[-r^2/\Lambda] + b_{ij} \text{Exp}[-r^2/2\Lambda] + e_i e_j \text{Exp}[-r/\lambda]/r$$

- Parameters fit to binding energy and saturation density of nuclear matter.
- Pasta may follow from any model that includes Coulomb interactions and reproduces nuclear saturation.



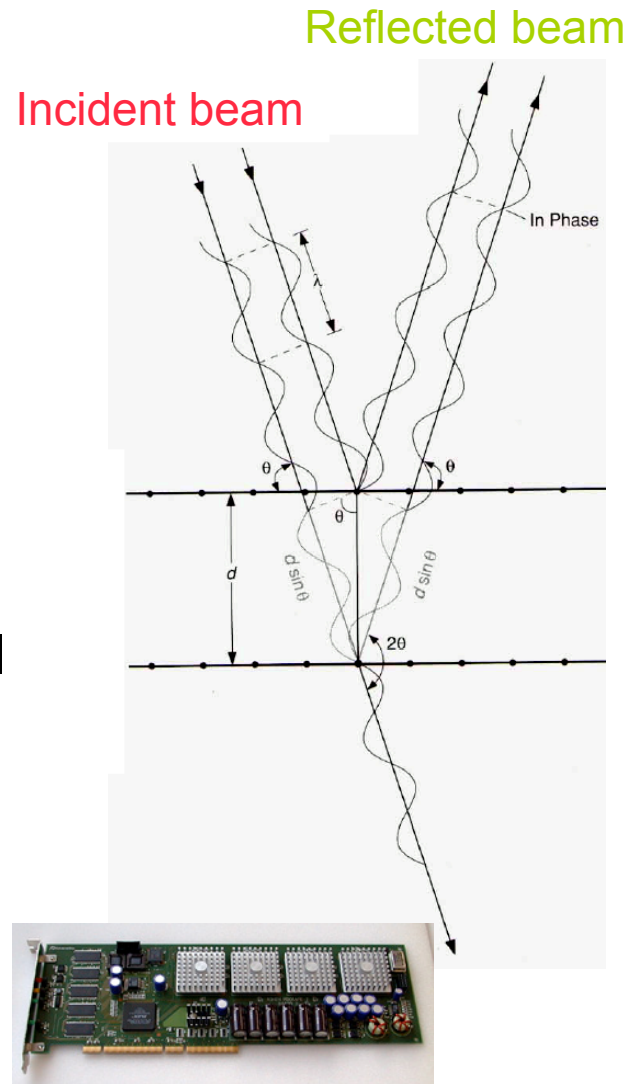
# Neutrino Pasta Scattering

- Core collapse supernovae radiate 99% of their energy in neutrinos.
- SN dynamics very sensitive to how neutrinos interact with the matter.
- Neutrino cross section per nucleon is
- Structure factor  $S$  is sum over all possible reflections.

$$d\sigma/d\Omega = S(q) d\sigma/d\Omega|_{\text{free}}$$

$$S(q) = \sum_{i,j} \exp[iq \cdot r_{ij}]$$

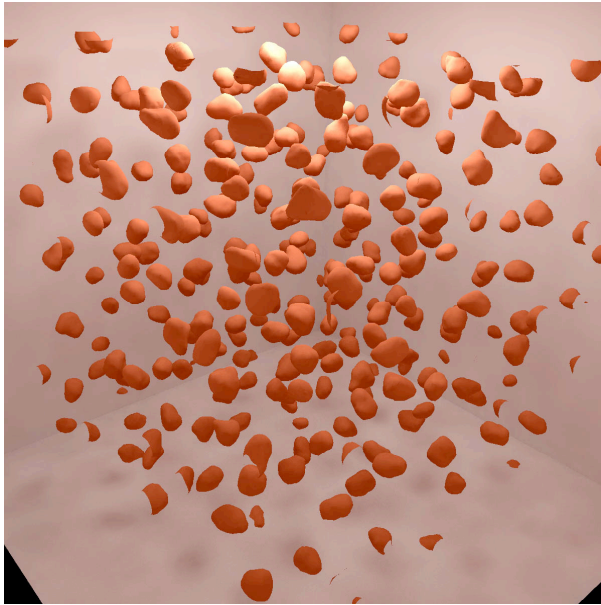
Wavelength of 10 MeV neutrino is 120 fm. Use MDGRAPE-2 hardware to run MD simulations with up to 200,000 nucleons.



Simulation with 40,000 nucleons at  $T=1$  MeV,  $\rho = 0.01 \text{ fm}^{-3}$ ,  $Y_p = 0.2$

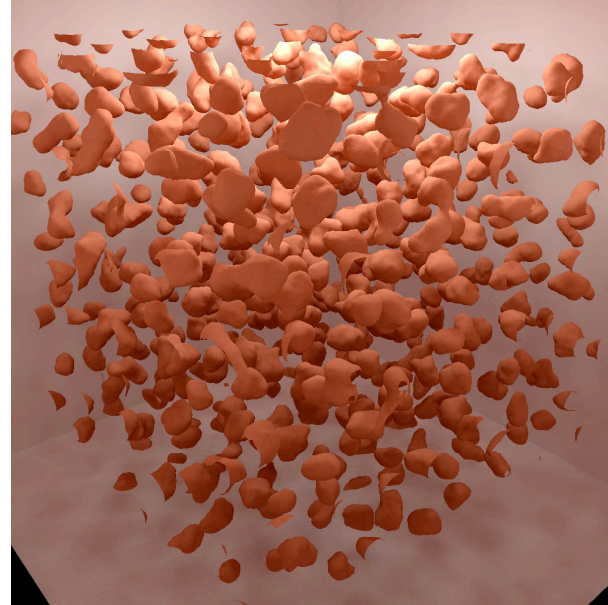
Isospin distillation:  $Y_p \sim 0.3$  in clusters.

$0.03 \text{ fm}^{-3}$  surface of the proton density

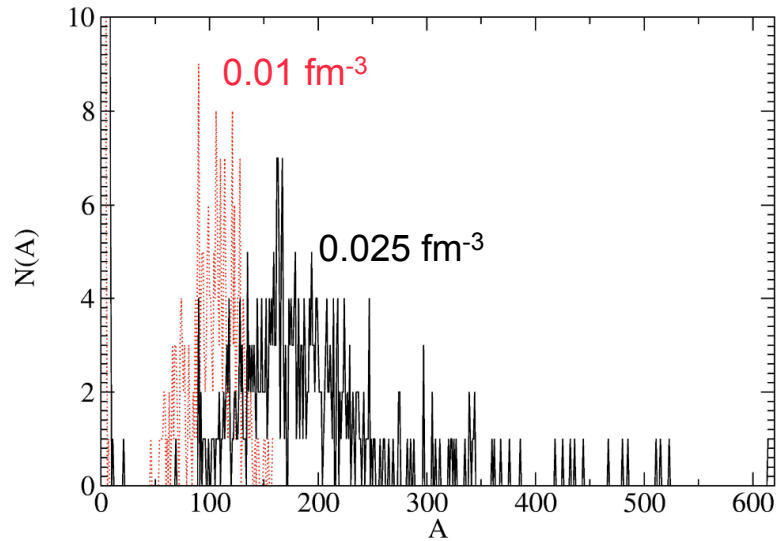


Graphics by Brad Futch FSU

Proton density at  $\rho = 0.025 \text{ fm}^{-3}$

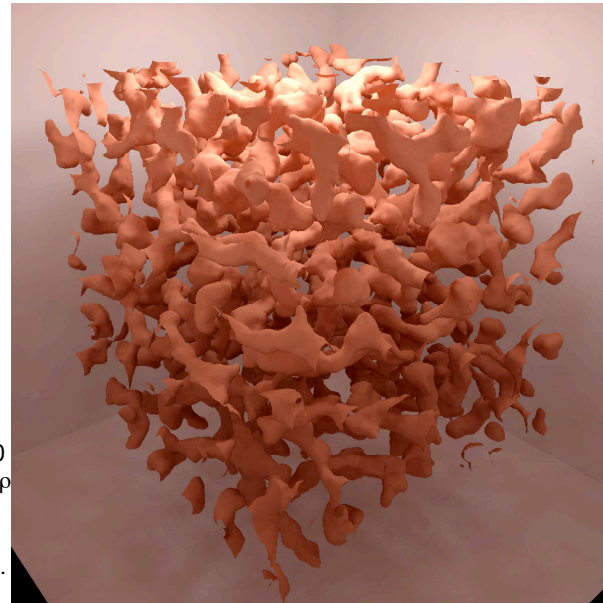


Graphics by Brad Futch FSU



Composition: # of clusters with mass  $A$  at different densities. Rare nuclei with big weak cross sections can be important.

Simulation with 100,000 nucleons at  $\rho = 0.05 \text{ fm}^{-3}$  showing pasta phase.



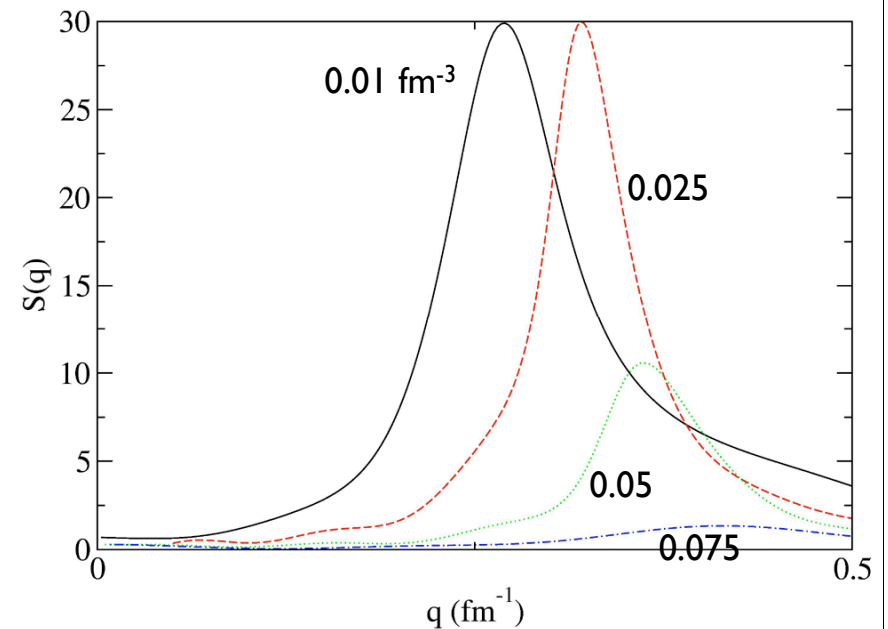
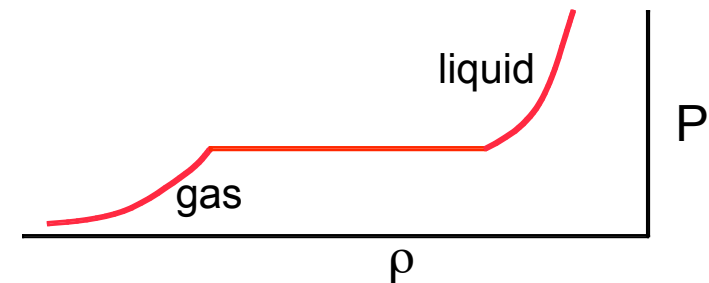
Graphics by Brad Futch FSU

# S(q) and Liquid Vapor Phase Transition

- In a first order phase transition, low density vapor is in equilibrium with high density liquid.
- Large density fluctuations arise as liquid is converted to/ from vapor.
- Static structure factor at  $q=0$  related to density fluctuations.

$$S(0) = kT / (dP/d\rho)$$

- Expect very large  $S(0)$  in two-phase coexistence region of simple first order phase transition -> very short neutrino mean free paths in a supernova [J. Margueron, PRC70 (2004)028801].
- We find no large enhancement of  $S(0)$  -> system is **not** described by simple first order phase transition because ratio of liquid to vapor fixed by charge density of electrons. -> **mixed phase**



# Dynamical Response of Pasta may be important for transport properties

- Dynamical response function  $S(q,w)$

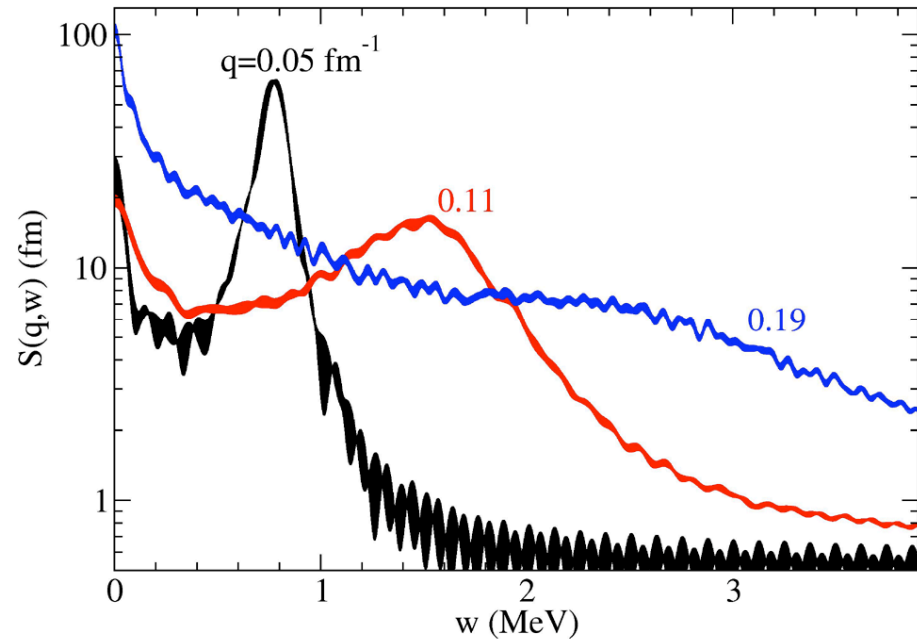
$$S(q,t) = \frac{1}{N_{ion}T_{ave}} \int_0^{T_{ave}} \rho(q,t+s)^* \rho(q,s) ds$$

$$S(q,w) = \frac{1}{\pi} \int_0^{T_{max}} S(q,t) \cos(wt) dt,$$

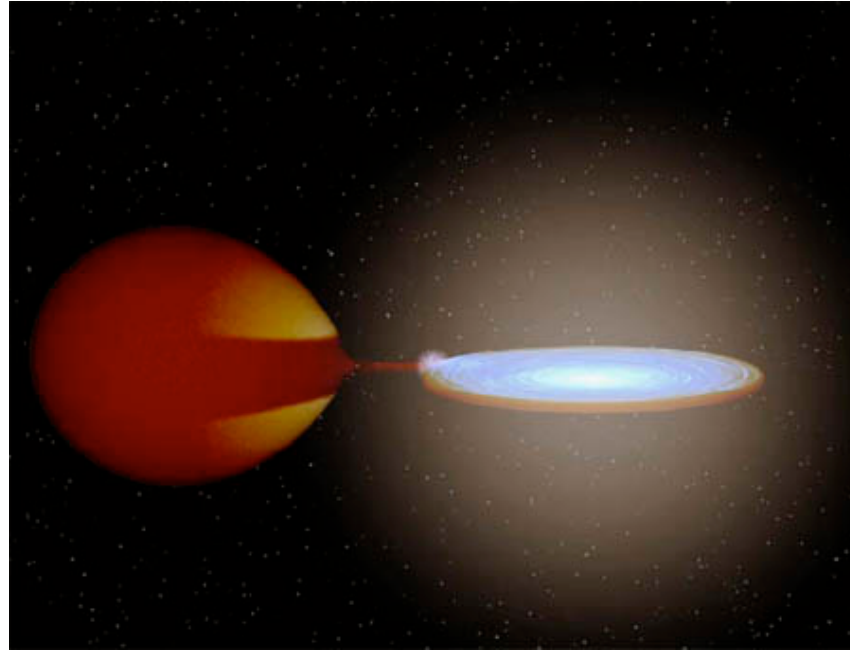
Density in q space:

$$\rho(q,t) = \sum_j e^{iq \cdot r_j(t)}.$$

- Average over  $\sim 10,000$  configurations (10-20GB)  
Simulation: 1-2 weeks on 4 MDGRAPE-2 boards.



$S(q,w)$  at  $\rho=0.05 \text{ fm}^{-3}$  shows plasma oscillation peak at finite  $w$  and concentration fluctuation peak at  $w=0$ .



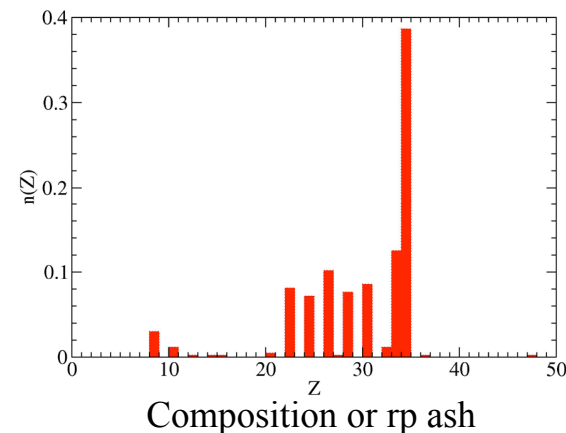
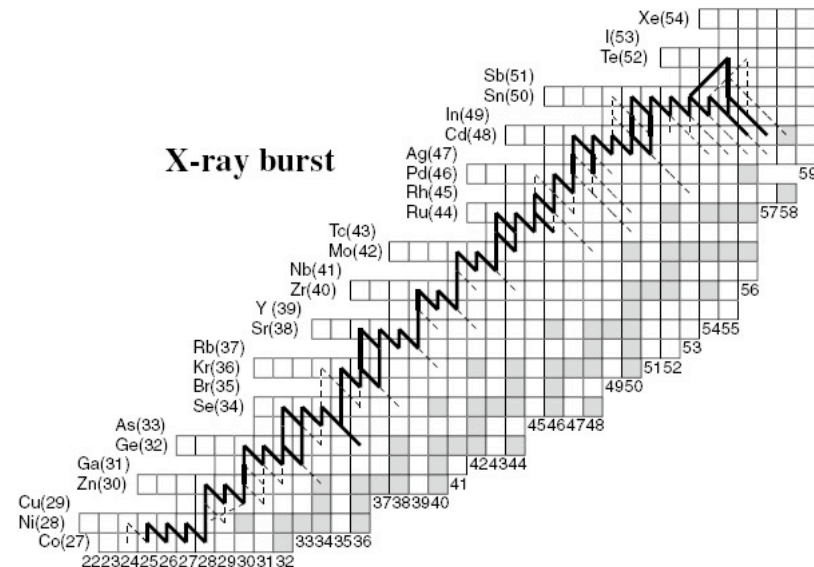
# Chemical Separation in the Crust of Accreting Neutron Stars

# Accreting Neutron Stars

- Type I X-ray bursts: H+He accreting on a NS ignite in thermonuclear flash that repeats in hours to days.
- Rapid proton capture (rp) process: where nuclei quickly capture p, interspersed with beta decays, to build up heavier elements.
- Rp process ash is buried by additional accreting material and compressed, where it solidifies to form the NS crust. During compression material undergoes e capture and pycnonuclear reactions.
- Superbursts: much more energetic bursts that repeat after years. Thought to be unstable carbon burning.

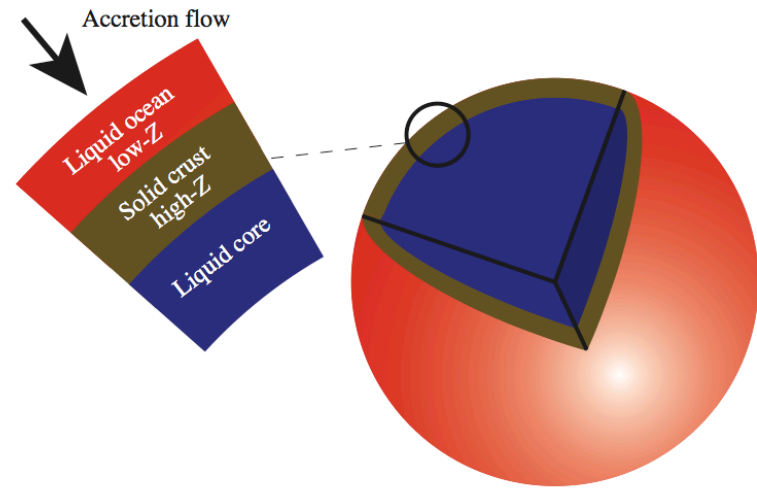
# End point of rp Process

- Schatz et al simulate X-ray bursts with a large reaction network and predict composition of rp ash: PRL**86** (2001)3471.
- May be sensitive to a variety of rxn rates. Example small  $^{15}\text{O}(a,g)^{19}\text{Ne}$  rate helps stabilize H+He burning. Fisker, Gorres, Wiescher, Davids, ApJ**650**(2006) 332.
- Gupta et al includes further electron capture, light particle rxns.
- Important feature is large *dispersion* in Z [ $\sim 50\%$   $Z \sim 34$  and  $50\%$   $8 < Z < 30$ ].

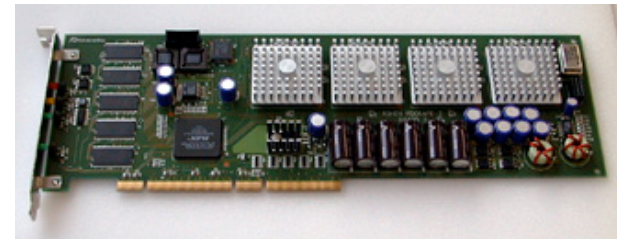


# Rp Ash Freezing Simulations

- Heterogeneous material compressed by further accretion until it freezes.
- We find chemical separation during freezing that **enriches liquid ocean in low Z elements while crust is enriched in high Z material.**
- Electrons form degenerate relativistic Fermi gas, provide screening length  $\lambda$ . Classical ions have screened Coulomb interactions.
- Simulation of freezing can be difficult:  
(1) Compare accurate free energy of liquid to that of solid. Need very high accuracy because free energies parallel.  
(2) Include both liquid and solid regions in simulation volume. Simple and direct and allows treatment of complex compositions. Can have errors from finite size and nonequilibrium effects.



Ash accretes into liquid ocean. Chemical separation takes place as material at bottom of ocean freezes.



Simulations with up to 27648 ions run on MDGRAPE-2 hardware

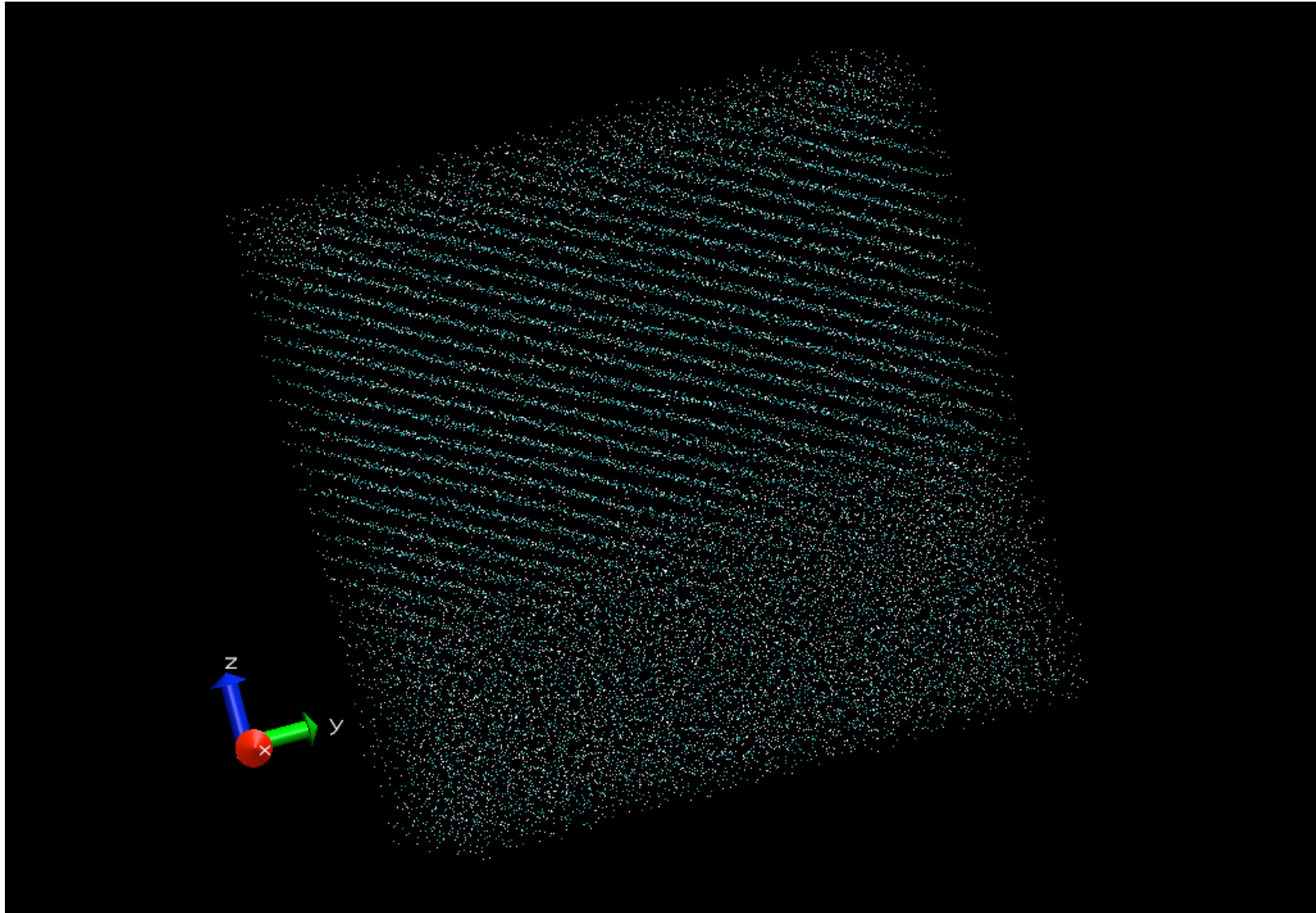
with E. Brown



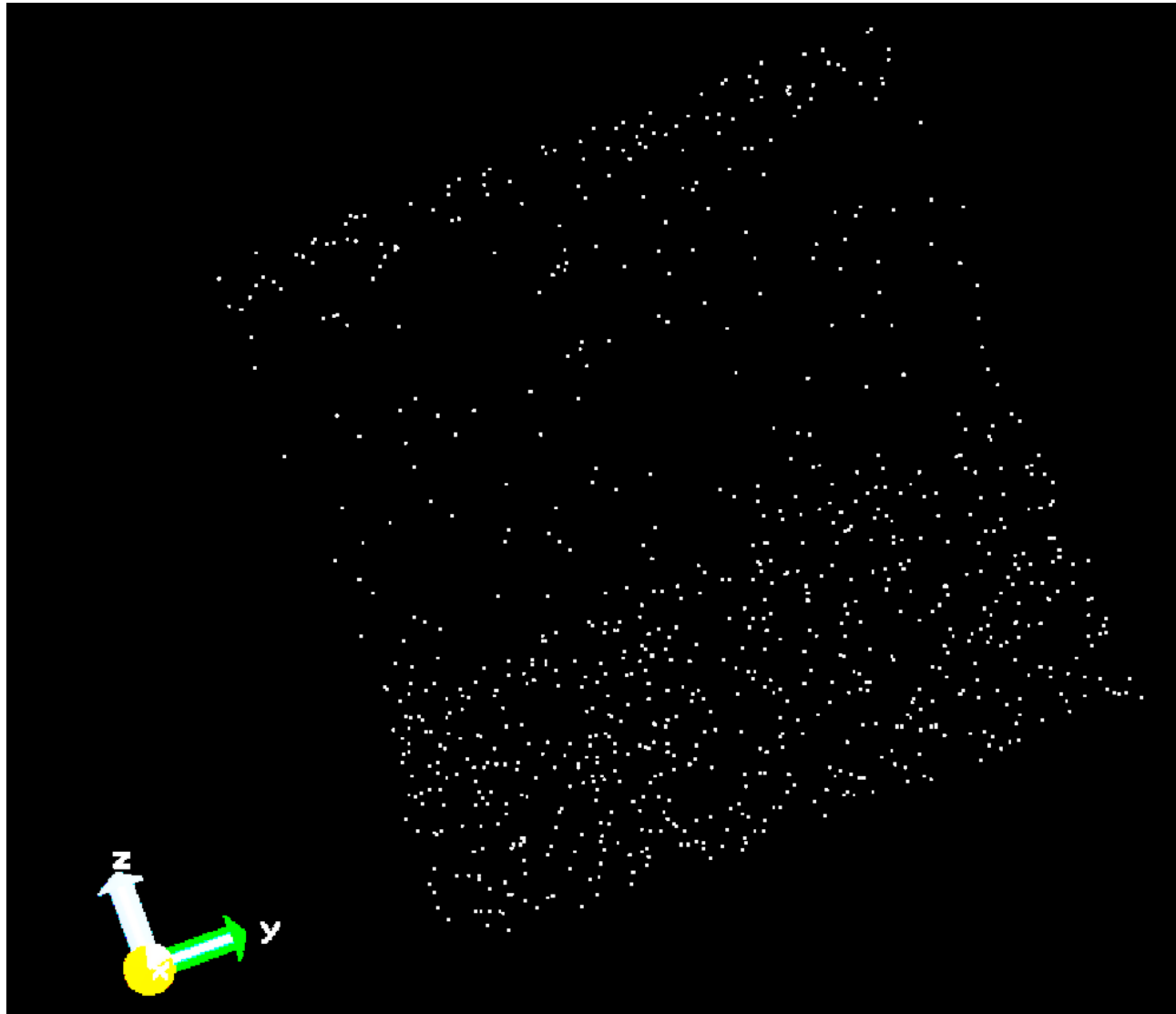
# Simulations

- (1) Single component system of 3456 ions, where each ion has same charge. Melted at  $\Gamma = Z^2 e^2 / akT = 176 \pm 1$  compared to known 175 for OCP. Latent heat per ion 0.77  $kT_m$  compared to 0.78 for OCP.
- (2) rp Ash composition with 27648 ions. Initial configuration: uniform composition, top half solid and bottom half liquid. Total simulation time  $151 \times 10^6$  fm/c. [ $\sim 9$  weeks on a boosted MDGRAPE-2 board.]
- (3) rp Ash composition with 8192 ions. Initial configuration: **uniform composition**. Total time  $2 \times 10^9$  fm/c.
- (4) rp Ash composition with 8192 ions. Initial configuration: **segregated composition**. Solid started with only Z=33,34 material. Liquid had all other Z. Time  $2 \times 10^9$  fm/c.

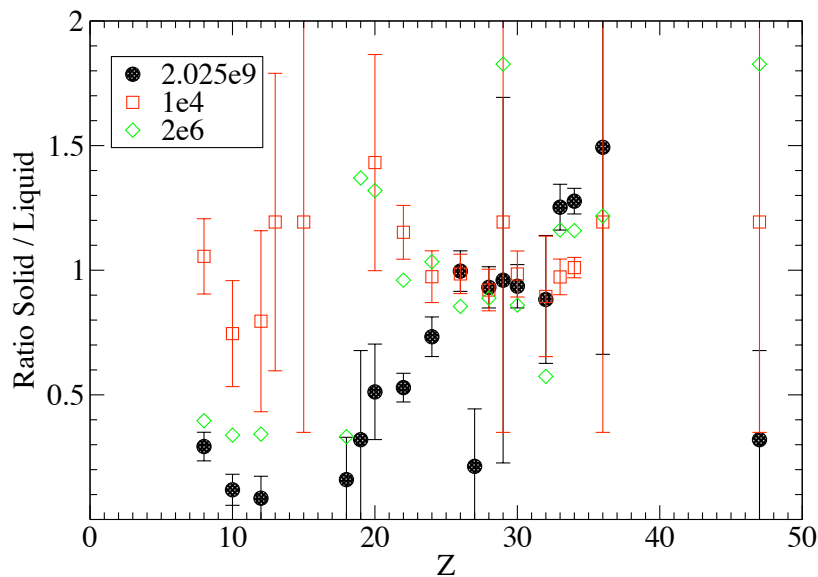
# Final Configuration of 27648 Ions



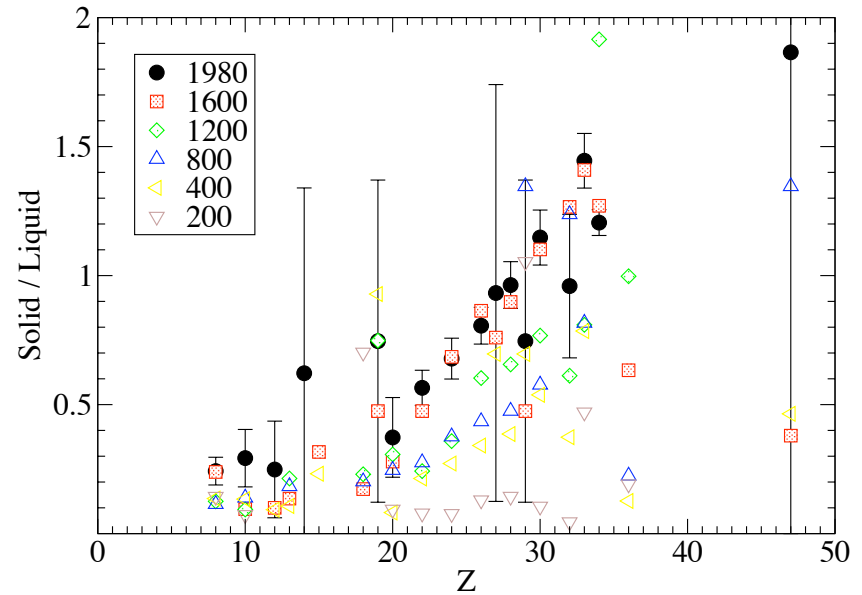
# $^{16}\text{O}$ ions at end of simulation



# Ratio of Solid to Liquid Composition for 8192 ions

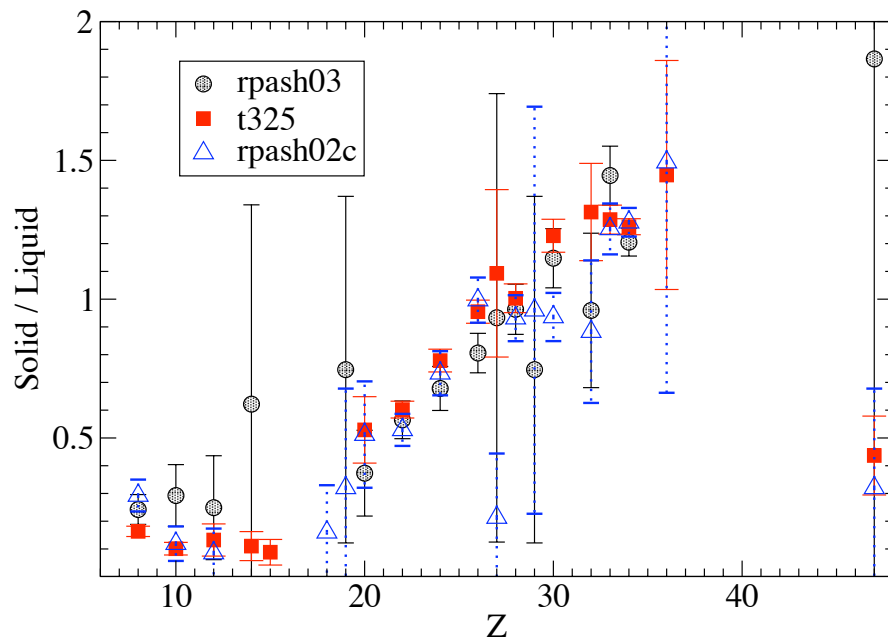


Ratio for uniform initial composition starts at one and quickly evolves to nearly final value. Symbols are for indicated simulation times in fm/c.

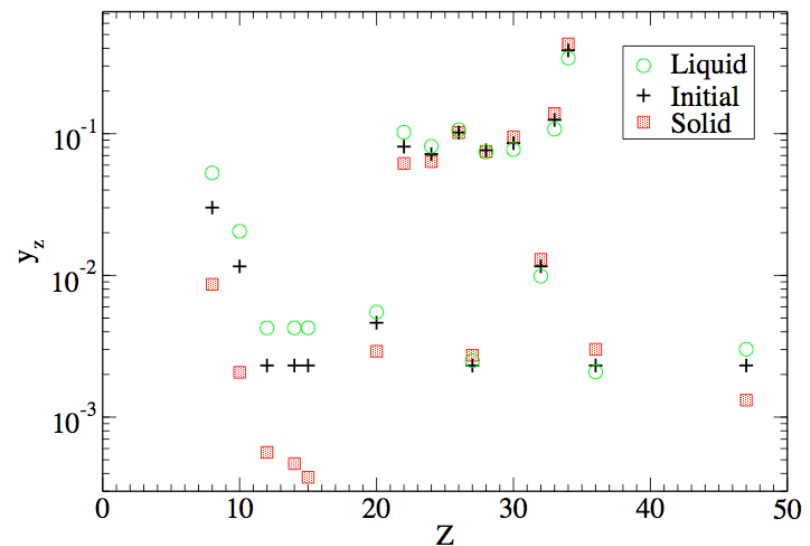


Ratio for segregated initial composition starts small and slowly increases as impurities diffuse into uniform solid. Symbols are for different simulation times in  $10^6$  fm/c

# Final Composition

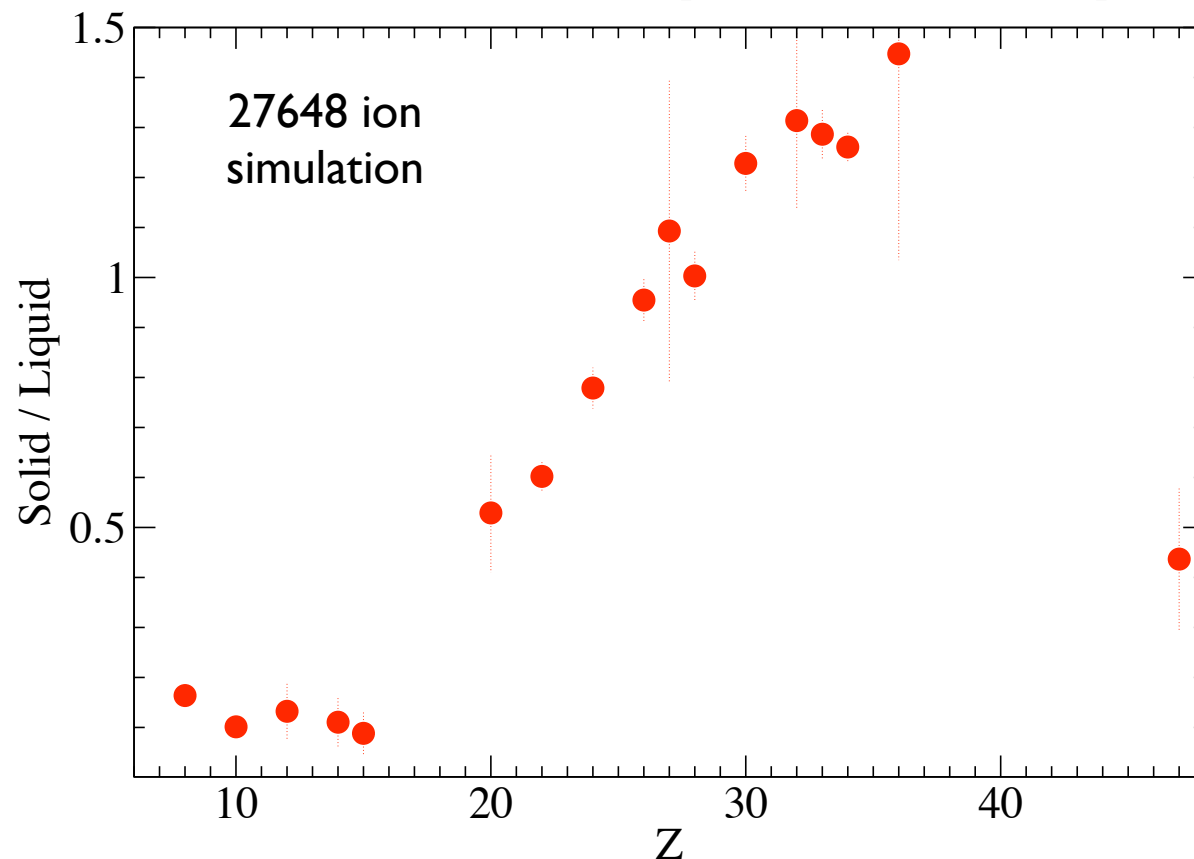


Ratio for 27648 ion simulation (red squares), 8192 ions with segregated initial conditions (gray circles) or uniform initial conditions (blue triangles).



Abundance by number for 27648 ion simulation: Initial (pluses), solid (squares) and liquid (circles)

# Ratio of Solid to Liquid Composition



We find chemical separation upon freezing. The solid phase is greatly depleted in low  $Z$  elements, by up to a factor of 10, compared to the liquid phase.

# Implications for Accreting Stars

- I. Impurities lower the melting temperature by  $\sim 30\%$  or increase the freezing density by up to a factor of 3.

$$\Gamma = \left\langle \frac{Z^2 e^2}{a} \right\rangle > \frac{1}{kT}, \quad a = (3/4\pi n)^{1/3} \text{ ion sphere radius}$$

	OCP	Mixture	Liquid	Solid
$\Gamma$	175	247	233	261

- This increase in freezing density will increase the depth of the liquid ocean, perhaps even melting the entire outer crust of a superbursting star.

$$\rho = 2.1 \times 10^{10} \text{ g/cm}^3 (T/5 \times 10^8 \text{ K})^3 (\Gamma/233)^3$$

- Can we probe thickness of solid crust? Depth of ocean?

## II. Liquid Ocean Greatly Enriched in Low Z Elements.

- Assume steady state accretion onto top of liquid ocean.
- Composition of ocean will be greatly enriched in low Z elements until low Z elements can freeze out at same rate as they are accreting into ocean.
- This allows new crust to be formed with same composition as accreting material.
- If carbon survives to bottom of ocean, it will be greatly enriched. Help superburst ignition?

Accreting Material



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Ocean enriched in low Z elements

Freezing and chemical separation



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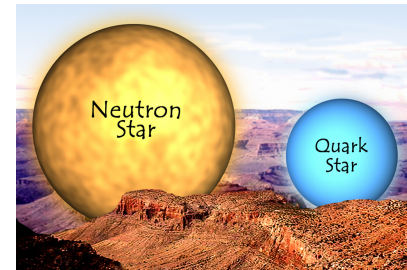
Solid crust, same composition as accreting material



# III. Chemical Separation Could Lead to Formation of Layers

- Chemical separation could lead to the formation of layers of different chemical composition in the crust.
- Example: shut off accretion, star cools and ocean freezes to form low  $Z$  layer.
- Layers impact thermal conductivity, shear modulus, shape, ... of NS.
- Thermal conductivity also depends on impurity parameter  $Q$ . This is reduced because many impurities stay in liquid.

$$Q = \langle Z^2 \rangle - \langle Z \rangle^2 = 53(\text{liquid}), 22(\text{solid})$$



Chandra press release compares size of NS to Grand Canyon



Does NS crust look like layers of Grand Canyon?

# IV. Chemical Separation Could Change shape of NS

- Important for gravitational waves. LIGO has set limits on ellipticity  $(I_1 - I_2)/I_3$  of some pulsars  $< 10^{-6}$ .
- How big a mountain can one forge on a NS?
- This is a metallurgy question! Mountain of iron or steel?
- Is crust regular crystal, amorphous solid, glass...? [We find for rp ash simulation a regular crystal, even with impurities.]
- How many impurities are in solid and how do they impact shear modulus?
- How do impurities impact breaking strain or other fracture properties that may be important for star quakes?

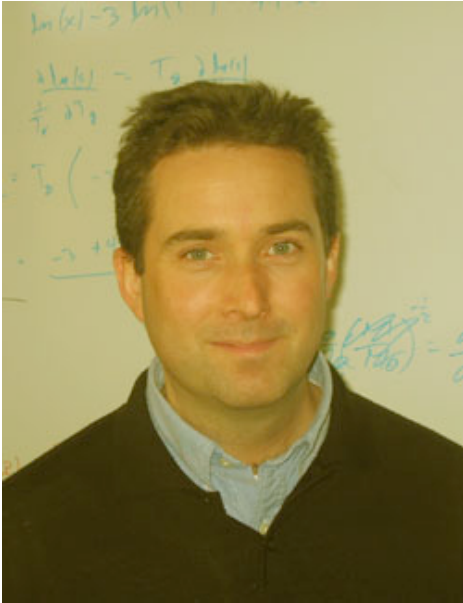
# Future Work

- Calculate chemical separation for other compositions. [Superburst ash running now]
- Calculate nuclear rxns + e capture consistent with chemical separation. How to model star?
- Calculate thermal conductivity and temperature profile with chemical separation. [Thermal cond. straight forward: only need composition and  $S(q)$ .]
- Calculate shear modulus, breaking strain,... from MD simulations. [I have started shear modulus.]
- ...

# Neutron Rich Matter and Neutron Star Crusts

- Virial expansion: **Achim Schwenk** , Eric O'Connor (TRIUMF)
- PReX: **Jorge Piekarewicz** (FSU)
- Nuclear Pasta: **Angeles Perez** (Spain)+JP
- Chemical separation in NS crusts: **Ed Brown** (MSU), **Don Berry** (IU High Performance Computing)
- Graduate students:
  - **Liliana Caballero**
  - **Gang Shen**
  - **Helber Dussan**
- Supported in part by DOE and state of Indiana.





- Bob asks...
- Answer: Astronomers should ask nuclear theorists:
  - Are your results model independent?
  - What are your theoretical error bars?