Neutron Star Matter Superfluidity: from BCS to QMC

Le gap retrouvé

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Outline

Problem: $^1S_0$ neutron superfluidity at intermediate densities

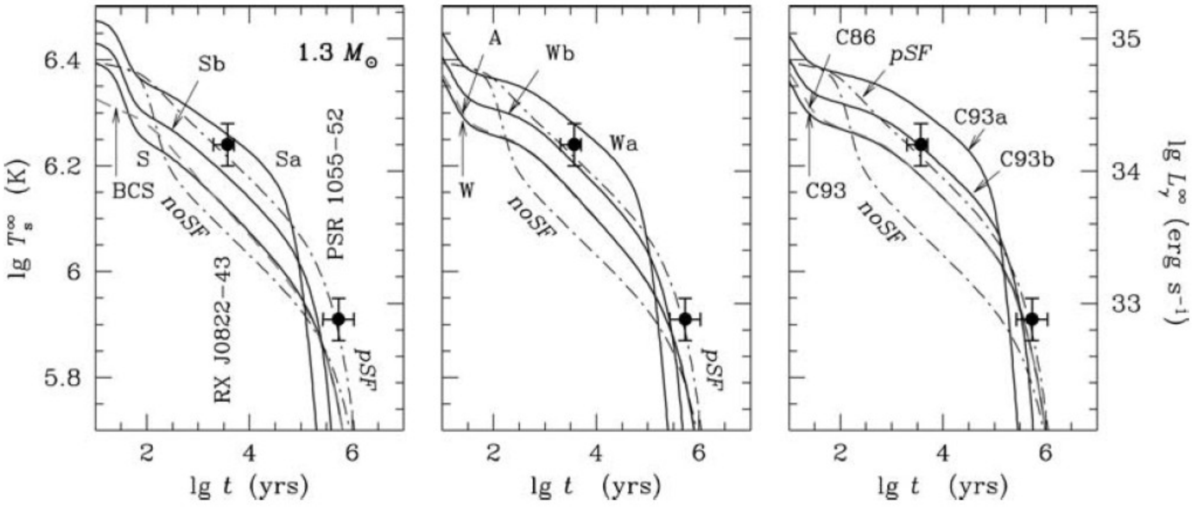
Solution:

- Bardeen-Cooper-Schrieffer (BCS) theory
- Quantum Monte Carlo (QMC) method
- Comparison of QMC with BCS

(a) Isolated low-mass slowly-cooling neutron stars

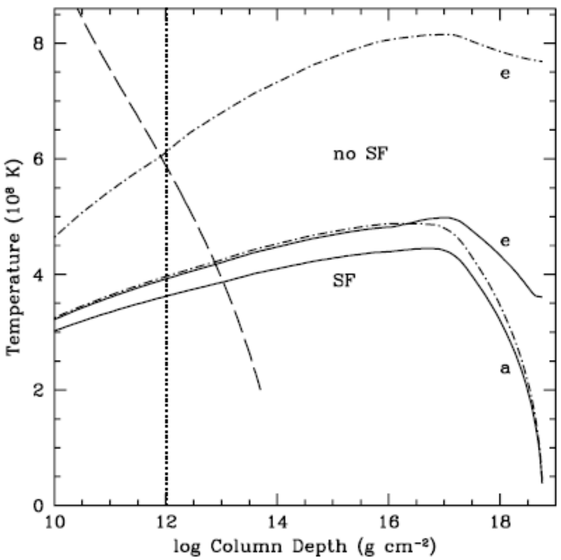
(b) Superburst ignition conditions

(a)



(D.G. Yakovlev, C.J. Pethick, Annu. Rev. Astron. Astrophys. 42:169 (2004).)

(b)

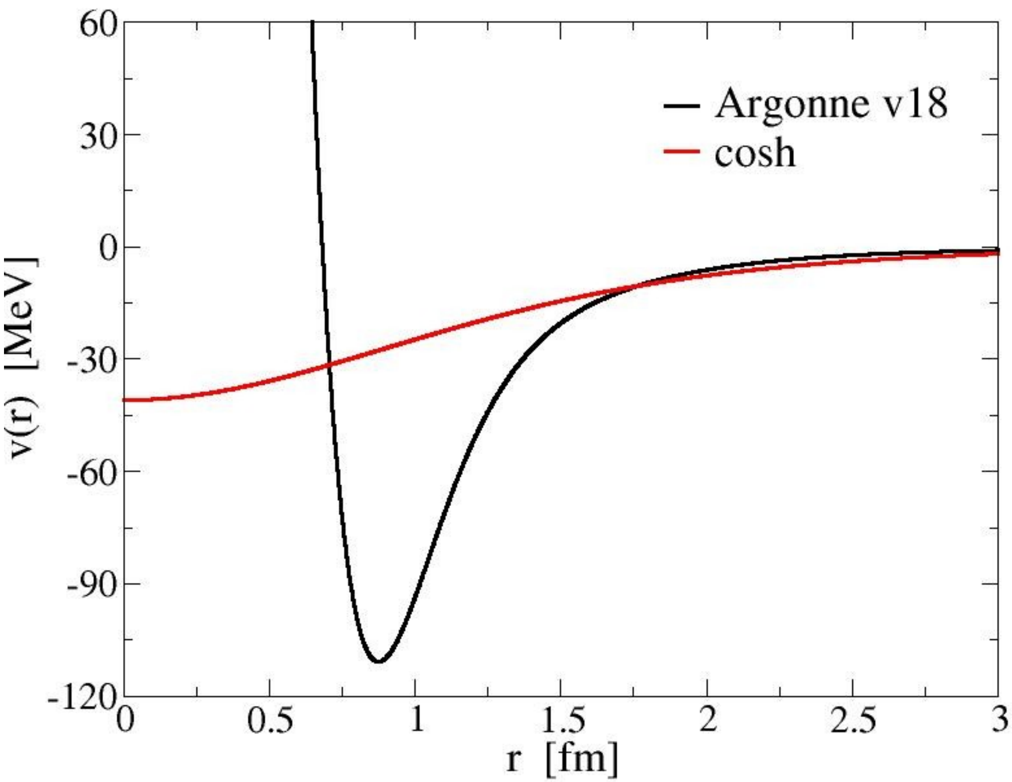


(A.Cumming, J. Macbeth, J. J. in't Zand, Dany Page, 2006, ApJ, 646, 429.)

Hamiltonian

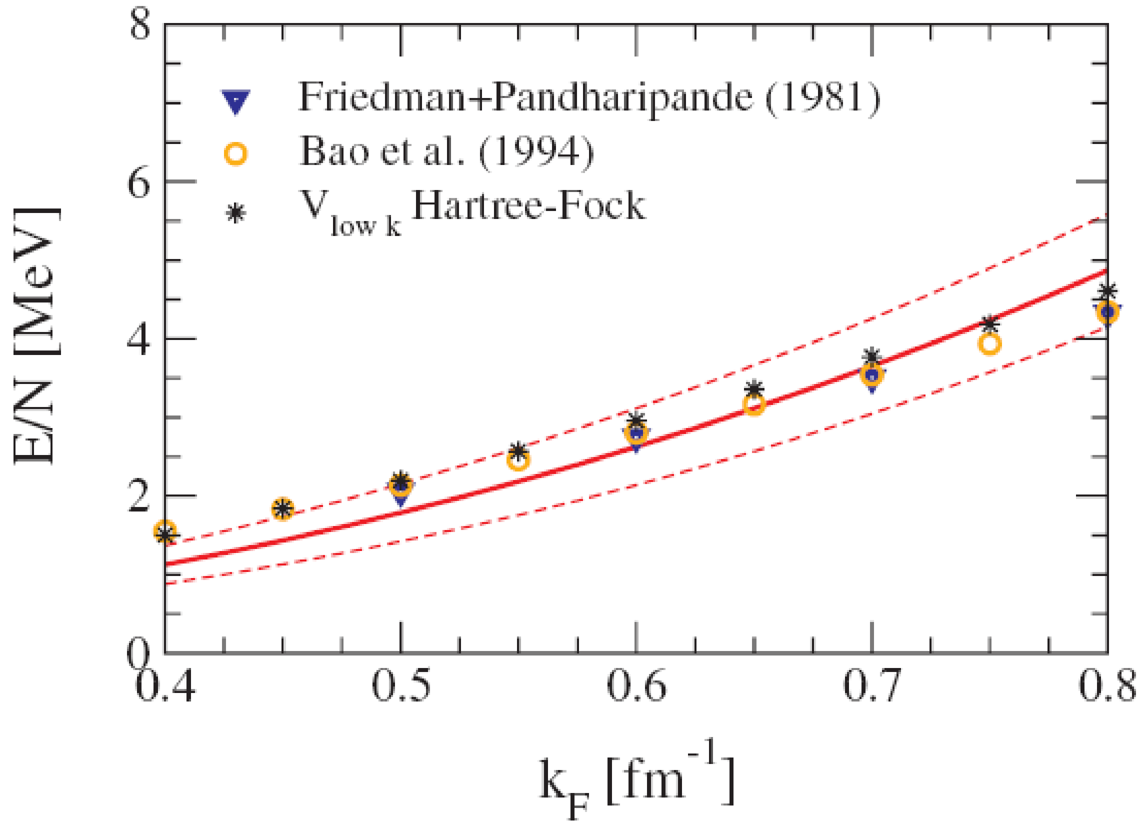
$$\mathcal{H} = \sum_{k=1}^{A}\left(-\frac{\hbar^2}{2m_k}\nabla_k^2\right) + \sum_{i<j'} v(r_{ij'})$$

- a = -18.5 fm, r_e = 2.7 fm
- $v(r)$ between opposite-spin particles
- Use $^1S_0$ channel of AV18
- Or a cosh potential tuned accordingly



Motivation: Equation of State

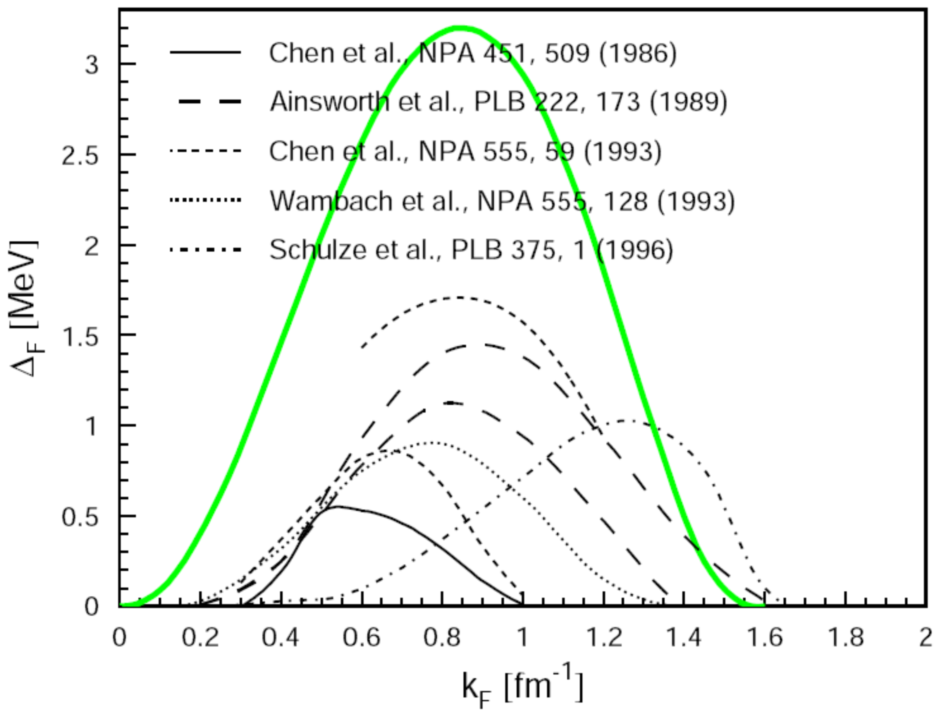
At subnuclear densities: EoS relatively well-known



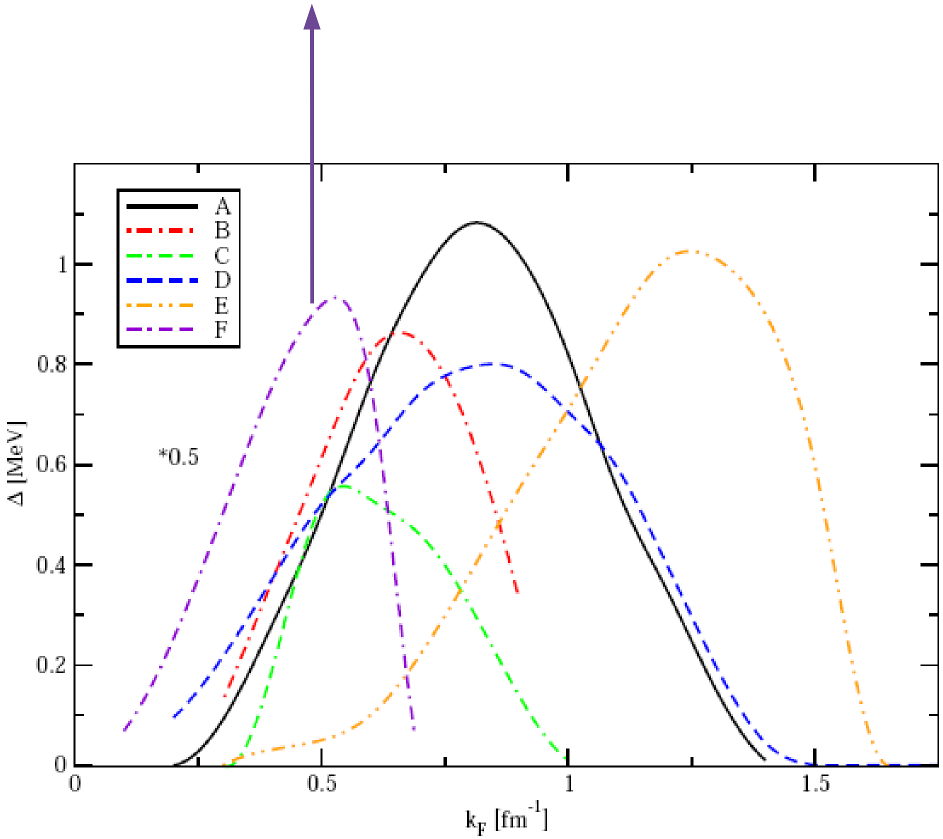
(A. Schwenk, C. J. Pethick, Phys. Rev. Lett. **95**, 160401 (2005).)

Motivation: Pairing Gap

At non-infinitesimal densities: gap not well-known



(U. Lombardo, H.-J. Schulze, LNP **578**, 30 (2001).)



(A. Sedrakian, and J.W. Clark, nucl-th/0607028).)

BCS: Weak coupling

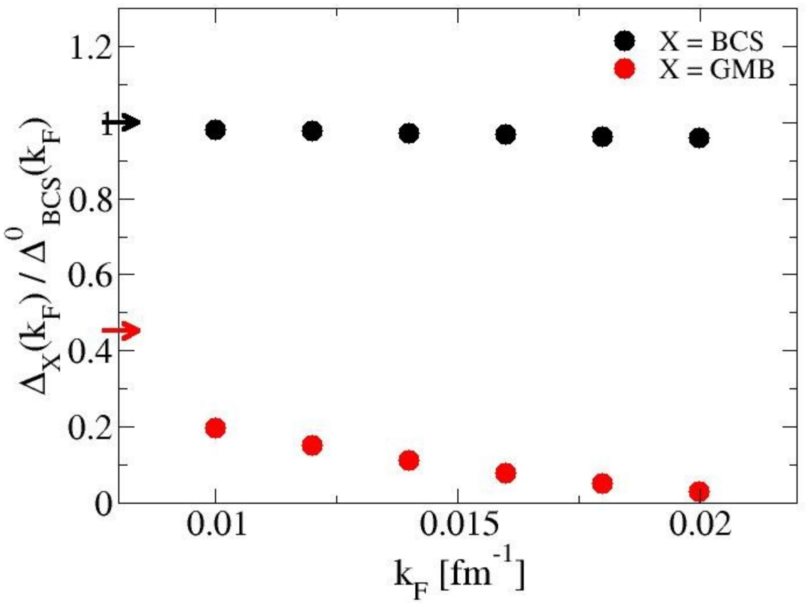
$$\Delta(k) = -\frac{1}{\pi}\int_0^\infty dk'\, k'^2 \frac{\Gamma(k,k')}{\sqrt{\epsilon(k')^2+\Delta(k')^2}}\Delta(k') \quad \text{and} \quad \rho = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2\left(1-\frac{\epsilon(k)}{\sqrt{\epsilon(k)^2+\Delta(k)^2}}\right)$$

- Can be decoupled only if $\frac{\Delta}{\mu}$ is small.
- Mootable for intermediate densities: $\frac{\Delta}{\mu} \sim 0.5$
- Obviously legitimate for infinitesimal densities
  - Bare BCS ($\epsilon(k) = \frac{\hbar^2 k^2}{2m} - \mu$, $\Gamma(k,k') = V(k,k')$)

$$\Delta_{BCS}(k_F) \xrightarrow{k_F \to 0} \Delta^0_{BCS}(k_F) = \frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2ak_F}\right)$$

  - Gorkov/Melik-Barkhudarov

$$\Delta_{GMB}(k_F) \xrightarrow{k_F \to 0} \Delta^0_{GMB}(k_F) = \frac{1}{(4e)^{1/3}}\frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2ak_F}\right)$$

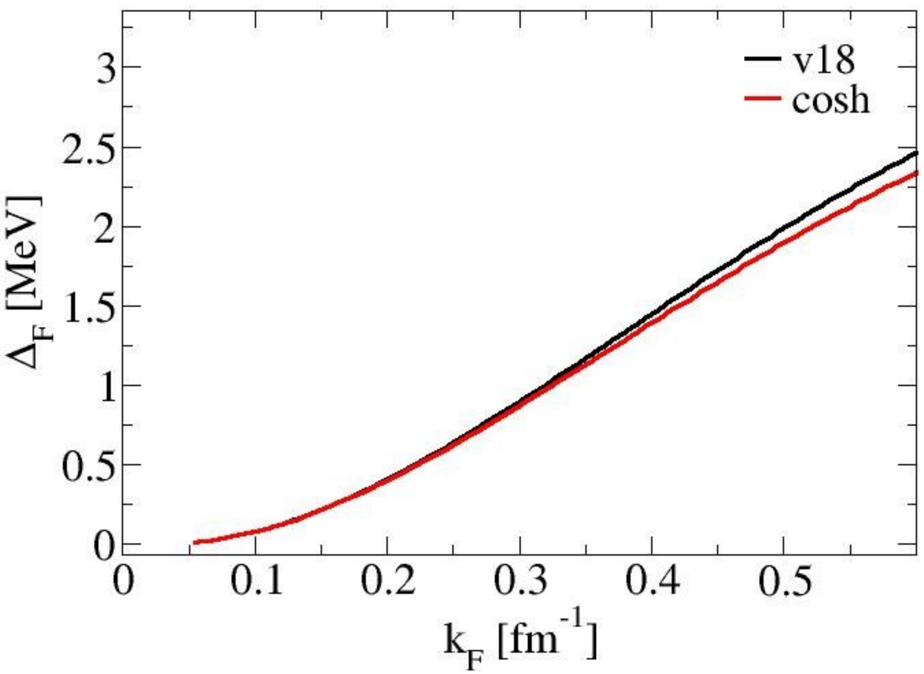


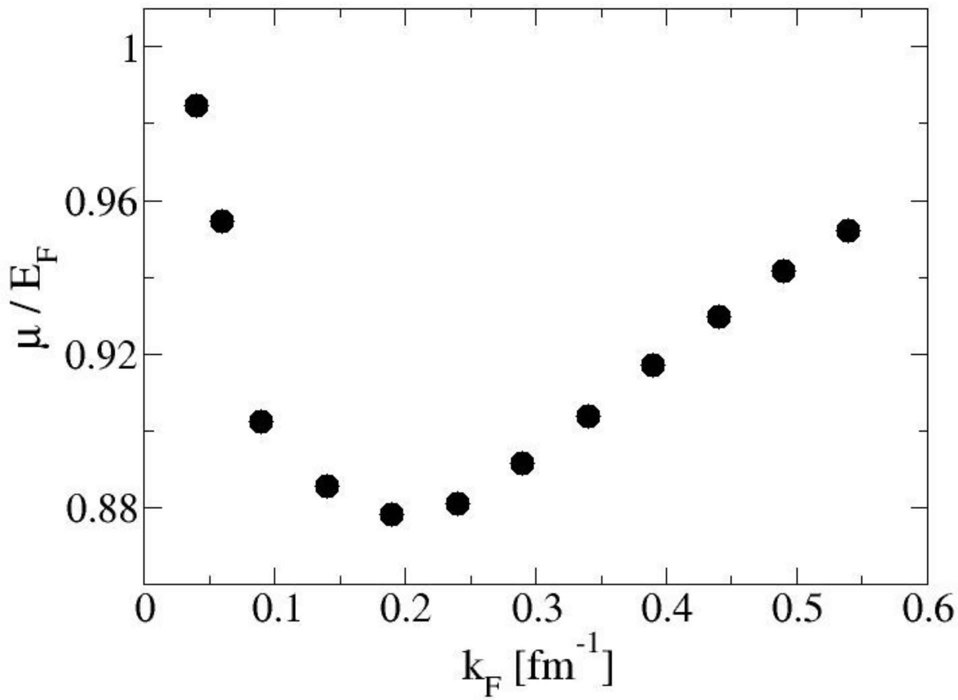
BCS: Strong coupling

$$\Delta(k) = -\frac{1}{\pi}\int_0^\infty dk'\, k'^2 \frac{\Gamma(k,k')}{\sqrt{\epsilon(k')^2 + \Delta(k')^2}}\Delta(k')$$

and

$$\rho = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 \left(1 - \frac{\epsilon(k)}{\sqrt{\epsilon(k)^2 + \Delta(k)^2}}\right)$$

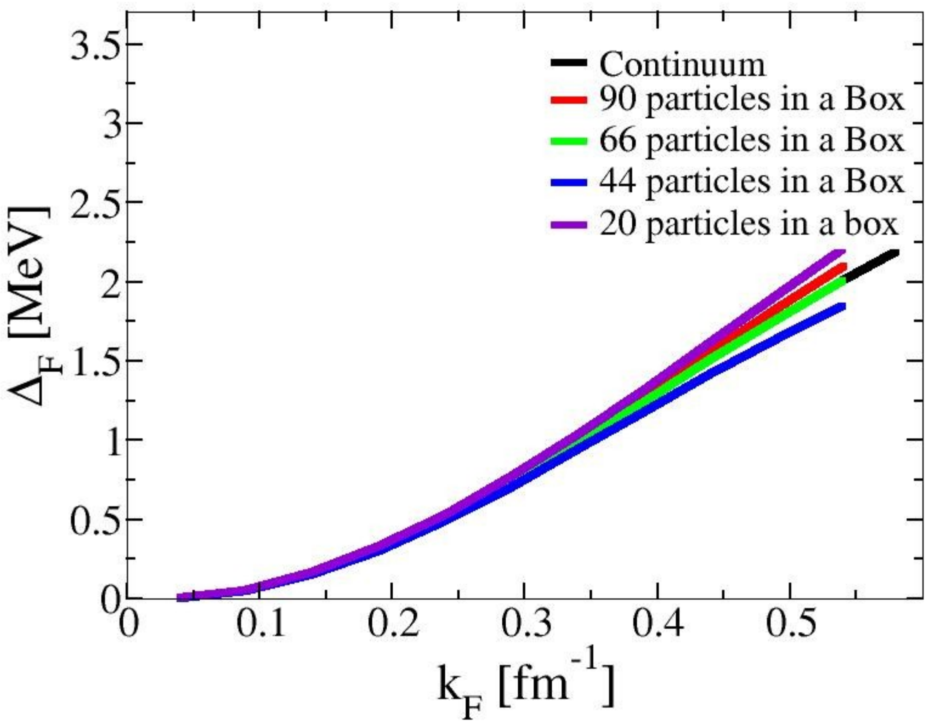


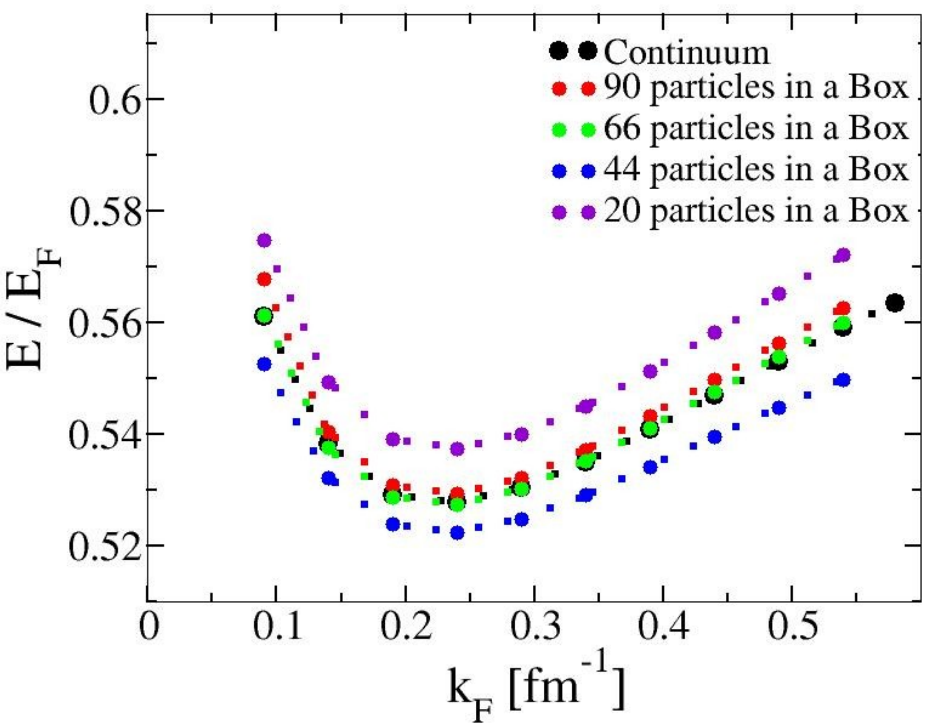


BCS: Problem in a box

Born-von Karman periodic boundary conditions: $\mathbf{k_n} = \frac{2\pi}{L}(n_x\hat{x} + n_y\hat{y} + n_z\hat{z})$

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\epsilon(\mathbf{k}')^2 + \Delta(\mathbf{k}')^2}} \quad \text{and} \quad A = \sum_{\mathbf{k}} \left[1 - \frac{\epsilon(\mathbf{k})}{\sqrt{\epsilon(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}\right]$$





QMC: Wave function I

$$\Psi_V(\mathbf{R}) = \prod_{i,j'} f(r_{ij'})\Phi_{BCS}(\mathbf{R})$$

BCS part $\Phi_{BCS}(\boldsymbol{R})$

- The main concept in QMC
- Reflects Cooper pairing
- Contains the pairing function $\phi(r)$

$$\phi(r) = \sum_{\mathbf{n}} \alpha_n e^{i\mathbf{k_n}\cdot\mathbf{r}}$$

- Has variable nodal surfaces

- Can be written as a determinant
  - for an *even* number of particles

$$\Phi_{BCS}(\mathbf{R}) = \mathcal{A}[\phi(r_{11'})\phi(r_{22'})...\phi(r_{\frac{A}{2}\frac{A}{2}'})]$$

  - for an *odd* number of particles

$$\Phi_{BCS}(\mathbf{R}) = \mathcal{A}\left\{\left[\phi(r_{11'})...\phi(r_{\frac{A}{2}\frac{A}{2}'})\right]\psi_{\mathbf{k}_u}(\mathbf{r})\right\}$$

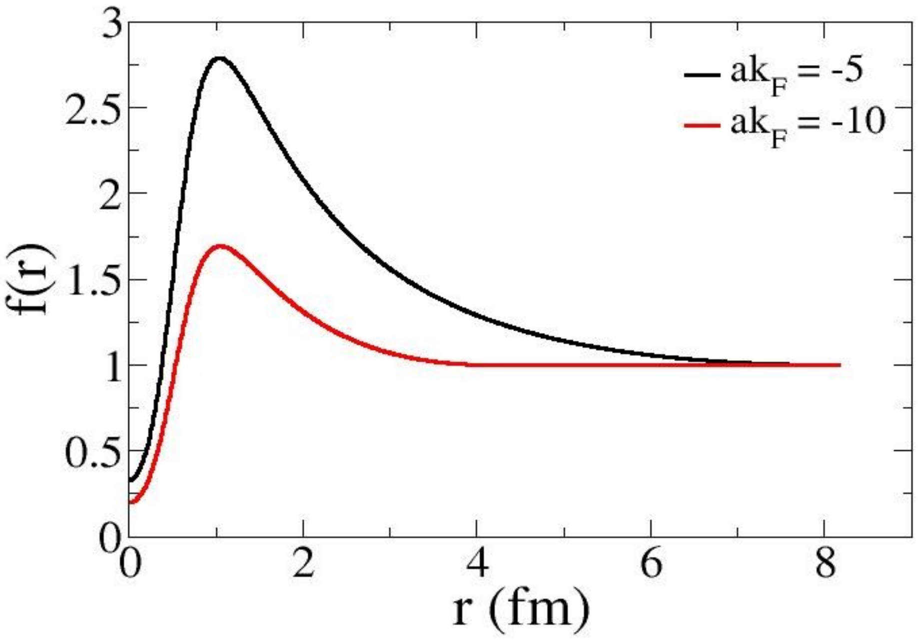
QMC: Wave function II

$$\Psi_V(\mathbf{R}) = \prod_{i,j'} f(r_{ij'})\Phi_{BCS}(\mathbf{R})$$

Jastrow part $f(r)$

- **Not** fundamental in the QMC calculation
- Contains short-range corrections
- Calculated from the LOCV method:

$$-\frac{\hbar^2}{m}\nabla^2 f(r) + v(r)f(r) = \lambda f(r)$$



Quantum Monte Carlo (QMC)

Variational Monte Carlo (VMC)

- Central limit theorem:

$$\langle \mathcal{H} \rangle_{VMC} = \frac{\int d\mathbf{R} \Psi_V(\mathbf{R})^\dagger \mathcal{H} \Psi_V(\mathbf{R})}{\int d\mathbf{R} \, |\Psi_V(\mathbf{R})|^2} =$$

$$= \lim_{N_c \to \infty} \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{\mathcal{H} \Psi_V(\mathbf{R}_i)}{\Psi_V(\mathbf{R}_i)}$$

- Use Metropolis algorithm for the sum

Green's Function Monte Carlo (GFMC)

- Project out:

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

$$= \lim_{\tau \to \infty} \sum_i \alpha_i e^{-(E_i - E_T)\tau} \Psi_i \longrightarrow \alpha_0 e^{-(E_0 - E_T)\tau} \Psi_0$$

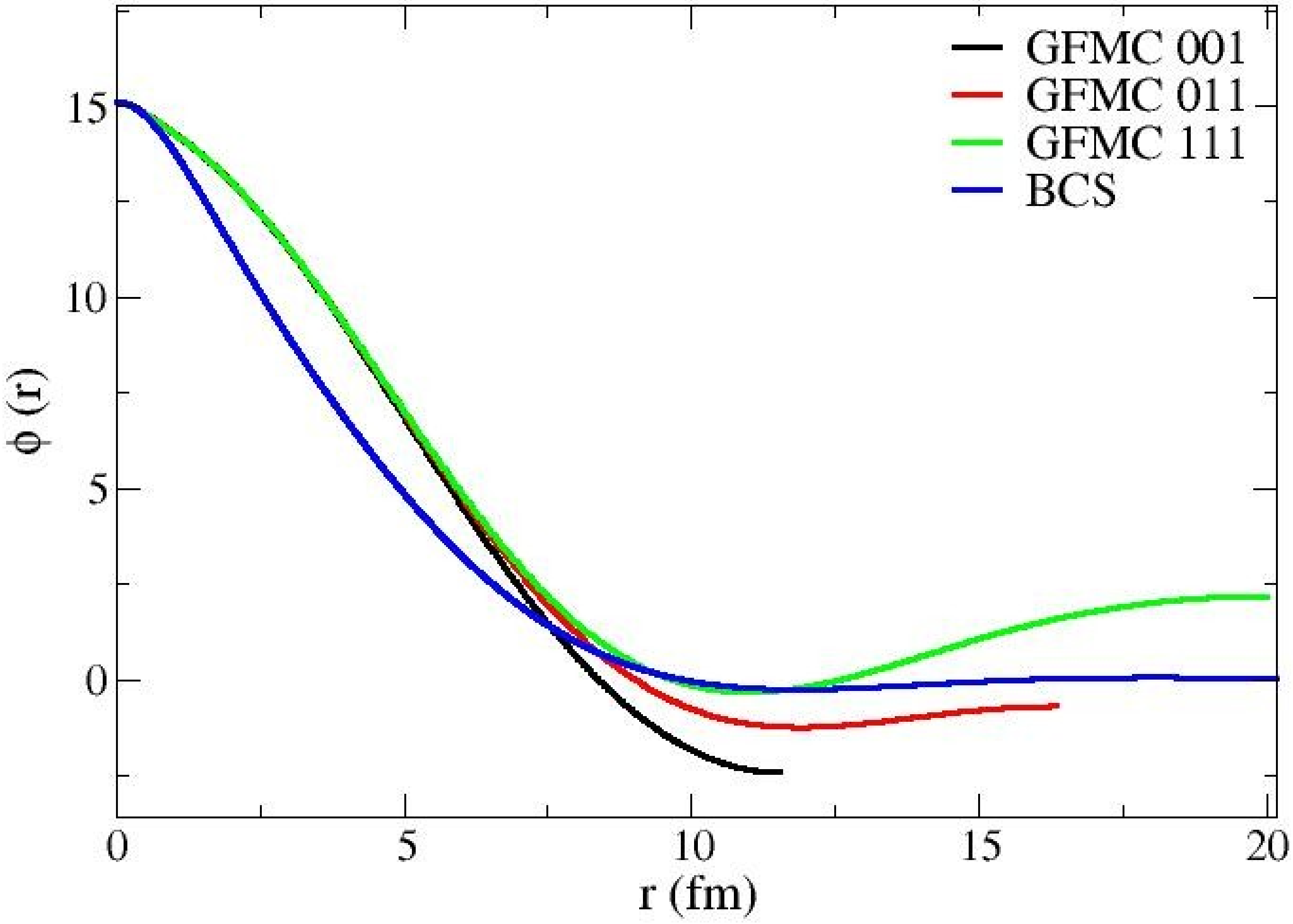
- Mixed estimate:

$$\langle \mathcal{H} \rangle_{mix} = \frac{\langle \Psi_V | \mathcal{H} | \Psi(\tau \to \infty) \rangle}{\langle \Psi_V | \Psi(\tau \to \infty) \rangle}$$

- Fermion-sign problem:
  - Fixed-node approximation
  - Transient estimation

Results at $a^*k_F = -10$, for 66 particles

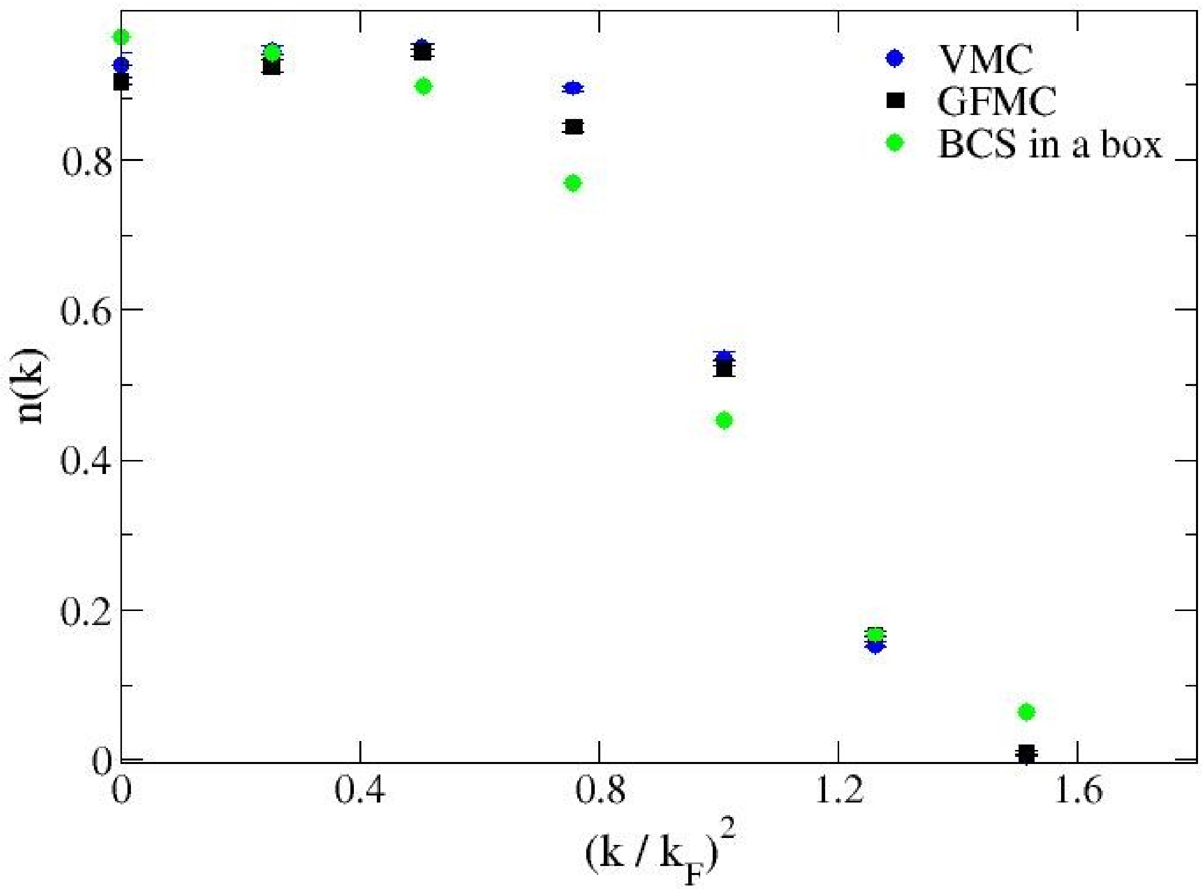
Pairing function



Results at $a^*k_F = -10$, for 66 particles

Momentum distribution

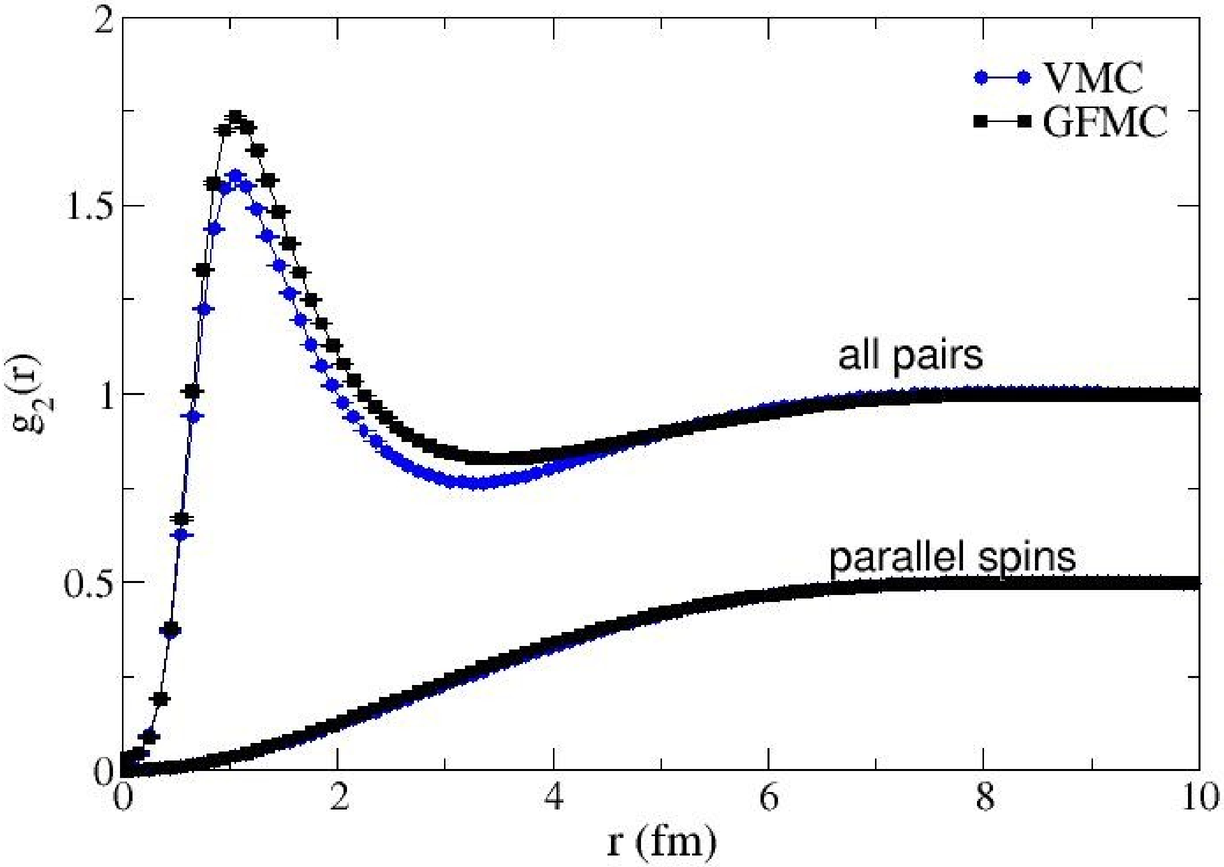
$$n(k) = \int e^{ikr} g_1(r) dr \qquad g_1(r) \sim \langle \Psi_0 | \psi(r)^\dagger \psi(0) | \Psi_0 \rangle$$



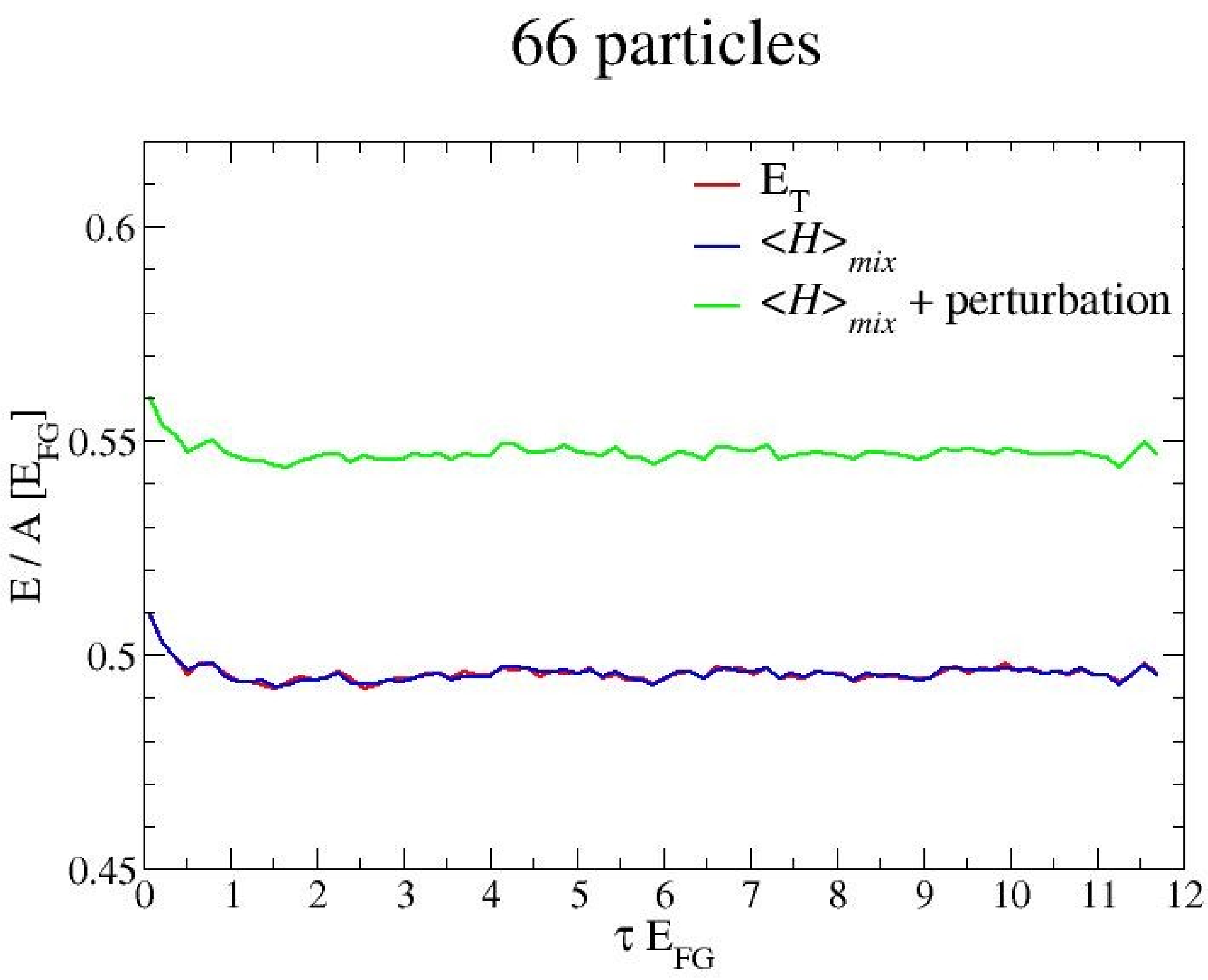
Results at a^*k_F = -10, for 66 particles

Pair distribution function

$$g_2(r) \sim \langle \Psi_0 | \psi(r)^\dagger \psi(0)^\dagger \psi(0) \psi(r) | \Psi_0 \rangle$$

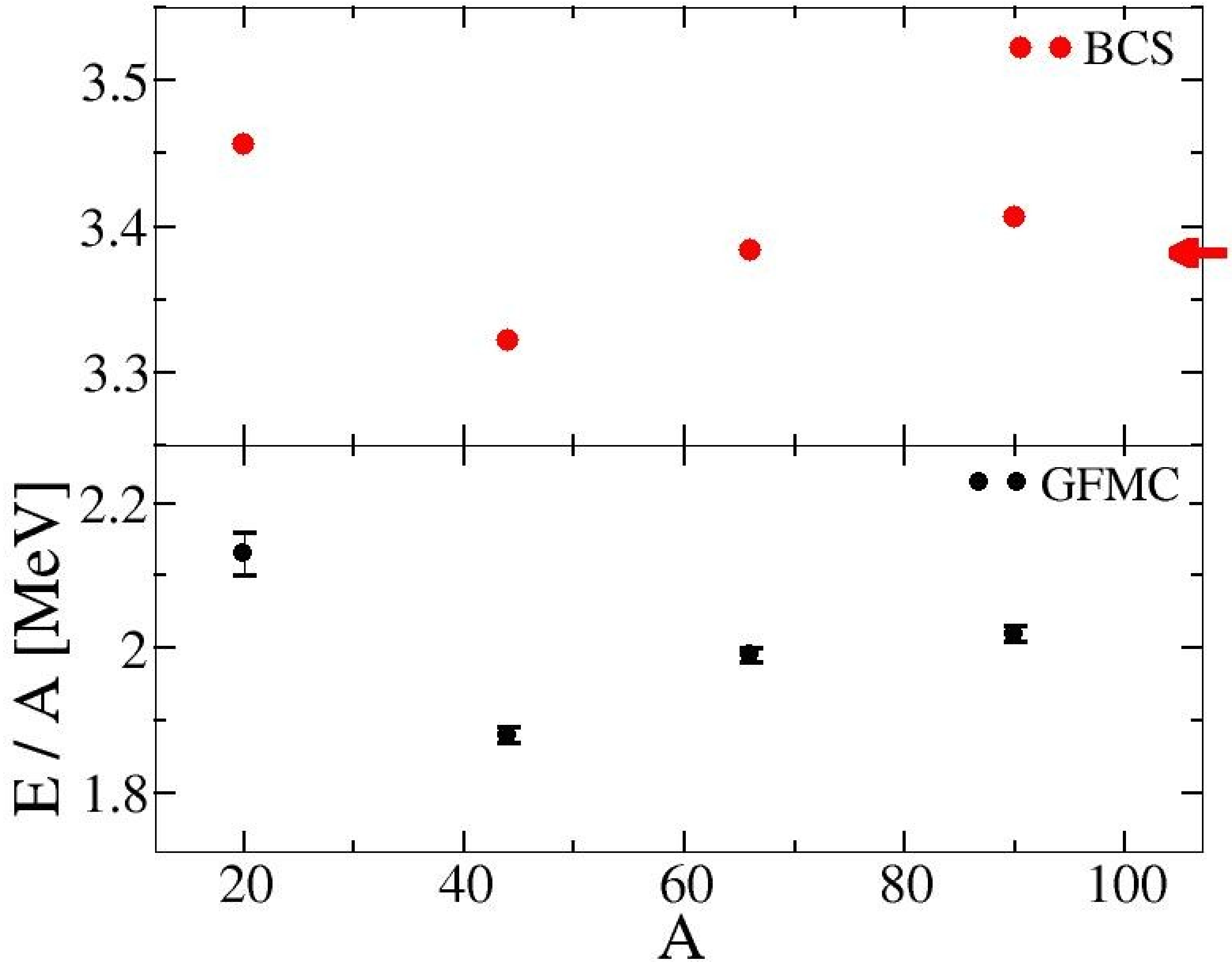


Results at $a^*k_F = -10$: Ground-state energy I

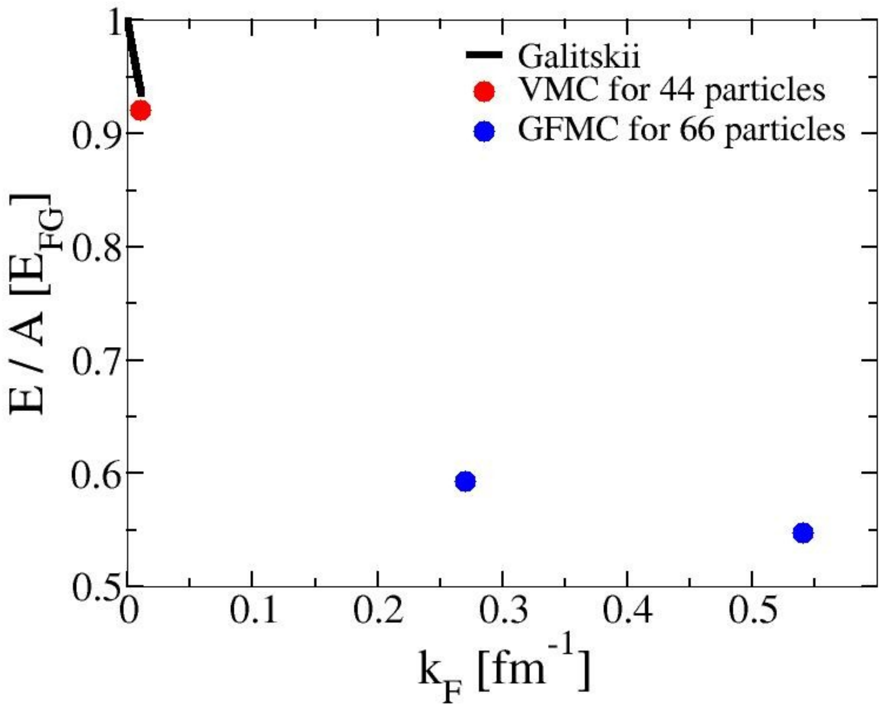


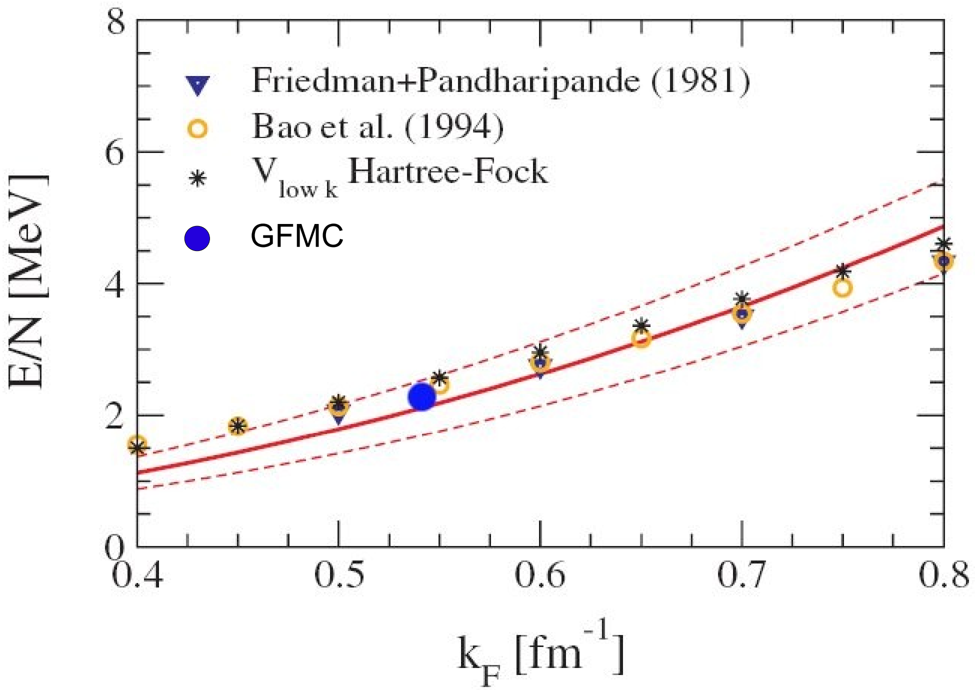
Perturbative correction: Not all of the opposite-spin pairs are in $S = 0$, $M_S = 0$

Results at a*k_F = -10: Ground-state energy II



Comparison with other predictions: Energy



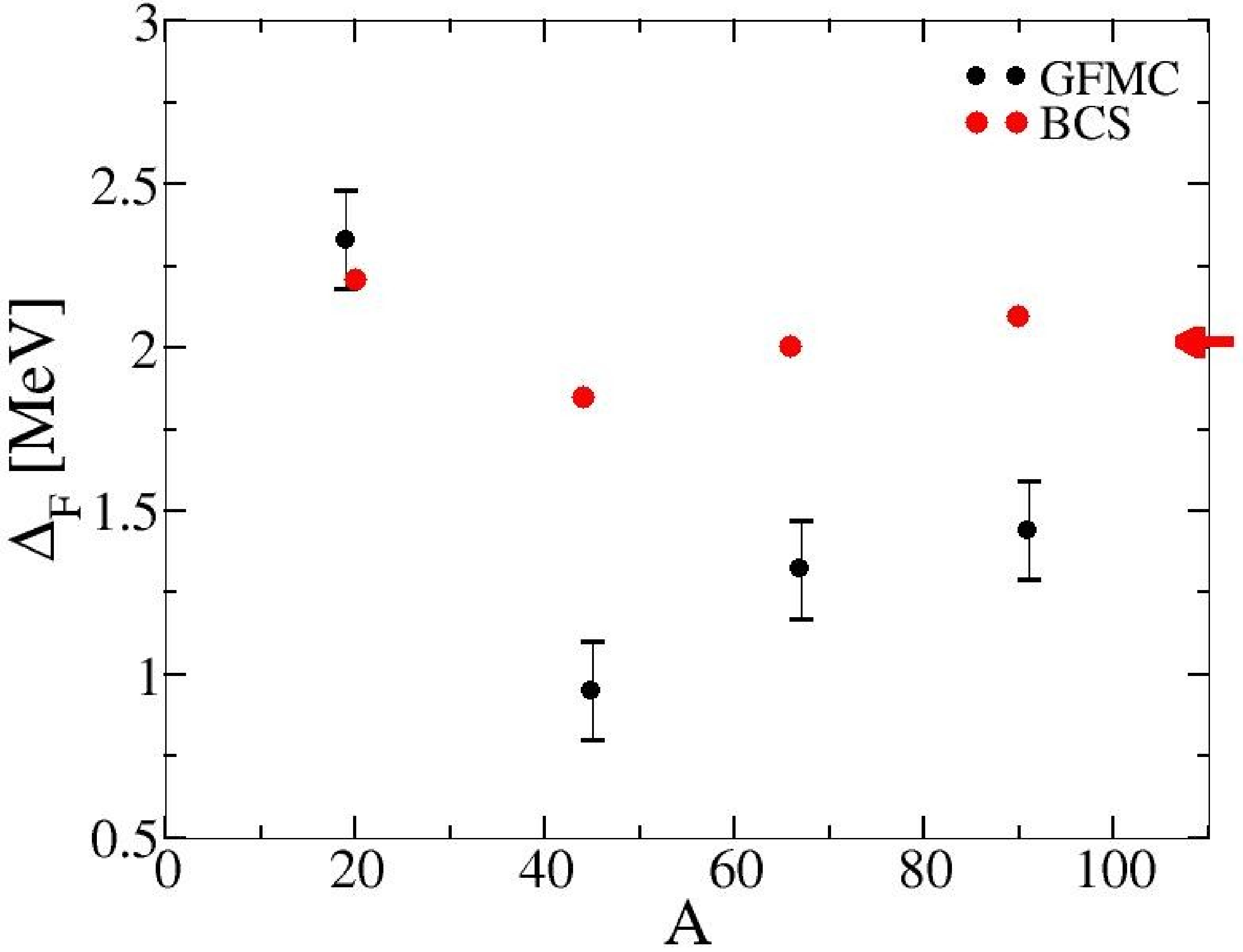


(A. Schwenk, C. J. Pethick, Phys. Rev. Lett. **95**, 160401 (2005).)

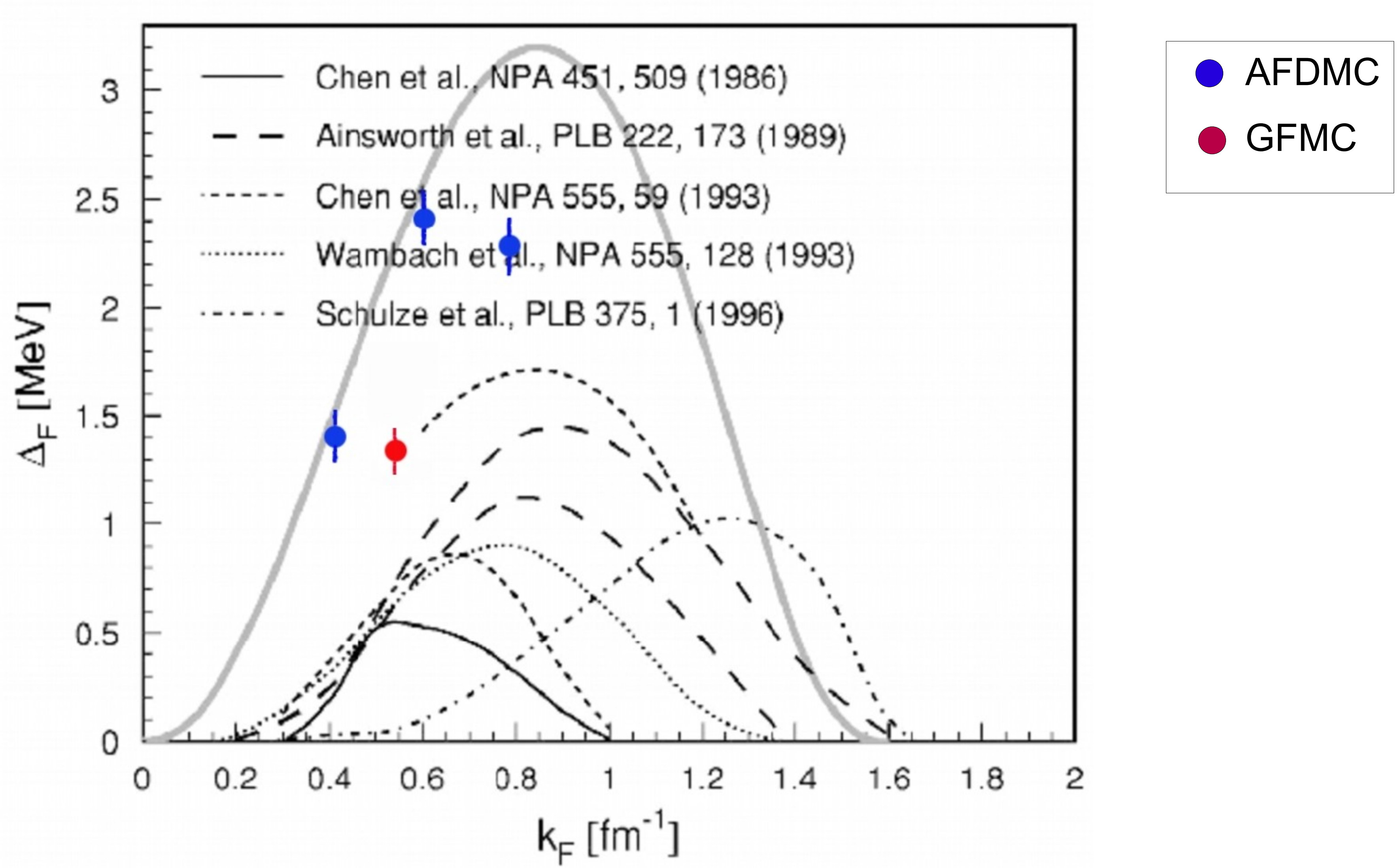
Galitskii: $\frac{E}{AE_{FG}} = 1 + \frac{10}{9\pi} ak_F + \frac{4}{21\pi^2}(11 - 2\ln 2)(ak_F)^2$

Results at a^*k_F = -10: Pairing gap

$$\Delta(N) = E(N) - \frac{1}{2}[E(N+1) + E(N-1)]$$



Comparison with other predictions: Pairing Gap



(U. Lombardo, and H.J. Schulze, LNP **578**, 30 (2001).)
(A. Fabrocini *et al*, Phys. Rev. Lett. **95**, 192501 (2005).)
AFDMC = Auxiliary Field Diffusion Monte Carlo for 12-20 particles

Conclusions / The Present Future

Conclusions

- S-wave interactions ($a + r_e$) describe neutron matter up to surprisingly large densities
- The momentum distribution does not fall off rapidly
- The superfluid gap is large, though smaller than the bare BCS prediction

The Present Future

- Repeat calculation for lower & higher densities
- Neutron drops
- Add a few protons