Stellar and terrestrial observations from the mean field QCD model

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Aspects of QCD :

Asymptotic Freedom (AF)
 Confinement
 Chiral Symmetry Restoration

Modeling QCD Properties:

We can build a phenomenological model representing QCD properties.

In that case, we can represent quark-quark interactions by a phenomenological potential which incorporates AF and confinement into its structure.

Richardson Potential

$$V(q) = -\frac{N_c + 1}{2N_c} \frac{12\pi}{33 - 2N_f} \frac{1}{q^2 \ln(1 + \frac{q^2}{\Lambda^2})}$$
$$V(r) = -\frac{N_c + 1}{2N_c} \frac{6\pi}{33 - 2N_f} \left[\Lambda^2 r - f(\Lambda r)\right]$$
$$f(t) = 1 - 4 \int_{1}^{\alpha} \frac{dx}{x} \frac{\exp(-xt)}{\{\ln(x^2 - 1)\}^2 + \pi^2}$$

 $\Lambda \sim 400 \ MeV$

J. Richardson, Phys. Lett. B 82 (1979) 272 H. W. Crater, P. Van Alstine, PRL 53 (1984) 1527 Dey et al, Phys. Rev. D 34 (1986) 2104 Dey et al, PLB 438 (1998) 123 $\Lambda \sim 100 MeV$

Modified Richardson Potential



Test of modified Richardson Potential in baryonic sector :

Bagchi et al, NPA 740 (2004) 109.

Bagchi et al, Europhys. Lett. 75 (2006) 548.

 $\Lambda = 100 MeV; \Lambda' = 350 MeV$

Test of modified Richardson Potential in baryonic sector : $\phi_q(r) = \left[\frac{1}{4\pi}\right]^{\frac{1}{2}} \begin{pmatrix} iG(\vec{r})\chi_m \\ \vec{\sigma}.\hat{r}F(\vec{r})\chi_m \end{pmatrix}; \quad q = u, d, s$

 $\phi_q(r)$ is the quark wave function, χ_m is the Pauli spinor

$$G(r) = \sum_{n} C_{n} R_{n0}(r)$$

$$F(r) = \sum_{m} D_{m} R_{m1}(r)$$

$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+\frac{3}{2})}} r^{l} exp(-\frac{1}{2}r^{2}) L_{n}^{l+\frac{1}{2}}(r^{2})$$

 $L_n^{l+\frac{1}{2}}(r^2)$ are Associated Laguerre polynomials

Test of modified Richardson Potential in baryonic sector :

$$H = \sum_{i=1}^{3} t_{i} + \sum_{i < j} V(r_{ij}) \qquad i = 1, 2, 3; \quad j = 1, 2, 3; \quad t_{i} = \vec{\alpha} \cdot \vec{p}_{i} + \beta m_{i}$$

 $E = \langle \Psi_B | H | \Psi_B \rangle \qquad [t_i + \omega_i(r_i)] \phi_i(r_1) = \epsilon_i \phi_i(r_1 i)$

 \mathcal{D}

$$\omega_1(r_1) = \int \phi_2^{\dagger}(r_2) V(r_{12}) \phi_2(r_2) r_2^2 dr_2 + \int \phi_3^{\dagger}(r_3) V(r_{13}) \phi_3(r_3) r_3^2 dr_3$$

$$\begin{aligned} \frac{dG_i(r_i)}{dr_i} - (m_i - \omega_i(r_i) + \epsilon_i) F_i(r_i) &= 0\\ \frac{dF_i(r_i)}{dr_i} + \left(\frac{2}{r_i}\right) F_i(r_i) + (\epsilon_i - \omega_i(r_i) - m_i) G_i(r_i) &= 0\\ E &= \sum_i \epsilon_i - \frac{1}{2} \sum_i \int \phi_i^{\dagger}(r_i) \omega_i(r_i) \phi_i(r_i) r_i^2 dr_i\\ M &= E - T_{CM} \end{aligned}$$

Test of modified Richardson Potential in baryonic sector :

$$\mu_q = \frac{e_q}{2} \int \left(\vec{r} \times \vec{j}\right) d^3r \qquad \vec{j} = \phi_q \vec{\alpha} \phi_q$$
$$\mu_{q\uparrow(\downarrow)} = -(+)e_q \frac{2}{3} \int_0^\infty G(r)F(r)r^3 dr$$

Baryonic magnetic moments (μ_B) are found by combining μ_q s with the help of baryonic wave function $\Psi_{spin} \times \Psi_{flavor}$

Test of modified Richardson Potential in baryonic sector :

Baryons	Experimental Mass (MeV)	Theoretical Mass (MeV)
Δ 's	1232	1251
Σ^* 's	$1383 \\ 1384 \\ 1387$	1361
Ξ*'s	$ \begin{array}{r} 1532 \\ 1535 \end{array} $	1455
Ω^{-}	1672	1556
N	939	938
Σ 's	$ \begin{array}{r} 1189 \\ 1193 \\ 1197 \end{array} $	1188
Λ^0	1116	1098
Ξ's	$ \begin{array}{r} 1315 \\ 1321 \end{array} $	1282

Test of modified Richardson Potential in baryonic sector :

	Δ^{++}	Δ^+	Δ^0	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	[]*	Ω^{-}
Ours	+5.77	+2.88	0.03	-2.86	+2.81	+0.17	-2.46	+0.30	-2.17	-1.92
Expt.	+6.14	+2.70	-	-	-	-	-	-	-	-2.02
QCDSR	+6.14	+3.02	0.0	-3.07	+1.90	-0.07	-2.03	+0.80	-2.71	-2.02
Iattice	+6.09	+3.05	0.0	-3.05	+3.16	+0.33	-2.5	+0.58	-2.08	-1.73
χpt	+4.0	+2.1	-0.17	-2.25	+2.0	-0.07	-2.2	+0.10	-2.0	-
$1/N_c$	-	+3.04	0.0	-3.04	+3.35	+0.32	-2.79	+0.64	-2.36	-
Daí fit A	+5.84	-	-	-	-	-	-	-	-	-2.08
Dai fit B	+5.86	-	-	-	-	-	-	-	-	-2.06
		р	n		Σ^+	Σ^0 , Λ^0	Σ^{-}	Ξ	[I]	
Ours		+2.88	-1.91		+2.59	+0.83, -0.71	-0.92	-1.45	-0.62	
Expt.		+2.79	-1.91		+2.46	-0.61	-1.16	-1.25	-0.65	
QCDSR		+3.04	-1.79		+2.73	-0.50	-1.26	-1.32	-0.93	
CDM		+2.79	-2.07		+2.47	-0.71	-1.01	-1.52	-0.61	
Dai fit A		+2.84	-1.87		+2.46		-1.06	-1.28	-0.61	
Dai fit B		+2.80	-1.92		+2.46		-1.23	-1.26	-0.63	

Application of modified Richardson potential for study of strange star properties :

What is a Strange Star?

Strange stars are stars composed of strange quark matter i.e. a very high density strange quark phase consisting of deconfined u, d and s quarks. In our model, strange stars are more compact than neutron stars.

Mean Field Model :

- Modified Richardson potential as interquark potential
- Screening of potential due to medium
 - dependence on density and temperature
- Density dependent quark mass
- Charge neutrality, beta equilibrium condition (hard part)
- Relativistic HF formalism to obtain EOS.
- Finite temperature through Fermi function
- TOV equations are solved to obtain the mass-radius of the stars

fermi function
$$F_i(k_i,T) = \frac{1}{\exp(\varepsilon_i - \varepsilon_i^f/T)}$$

 $n_i = \frac{3}{\pi^2} \int_0^\infty k_i^2 F_i dk_i \quad \varepsilon_i = \frac{3}{\pi^2} \int_0^\infty \phi k_i^2 F_i dk_i$

 ϕ is single particle energy – comes from interquark potential entropies

$$S_{i} = -\frac{3}{\pi^{2}} \int_{0}^{\infty} k_{i}^{2} F_{i}(k_{i}, T) \ln(F_{i}(k_{i}, T)) (1 - F_{i}(k_{i}, T)) \ln(1 - F_{i}(k_{i}, T)) dk_{i}$$

- free energies $f_i = \varepsilon_i TS_i$
 - **pressure** $P_i = n_i \frac{\partial f_i}{\partial n_i} f_i ; P = \sum_i P_i$
 - Bagchi et al, Astron. & Astrophys. 450 (2006) 431.





 $q^2 \rightarrow q^2 + D^{-2}$

$$D^{-1} = mg = \left[\frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_i^f \sqrt{\left(k_i^f\right)^2 + m_i^2}\right]^{1/2} + 7.152\alpha_0 T$$





 $\frac{dP}{dr} = -\frac{G}{c^4} [P(r) + \varepsilon(r)] \left| \frac{M(r)c^2 + 4\pi r^3 P(r)}{r^2 \left(1 - \frac{2GM(r)}{c^2 r}\right)} \right|$

 $\frac{dM}{dr} = \frac{4\pi r^2 \varepsilon(r)}{c^2}$

$$\frac{2}{3}n_u - \frac{1}{3}n_s - \frac{1}{3}n_d - n_e = 0 \qquad n_q = \frac{\left(k_q^f\right)^3}{\pi^2}, \ n_e = \frac{\left(k_e^f\right)^3}{3\pi^2}$$

 $\mu_s = \mu_d$ $\mu_s = \mu_u + \mu_e$

 $s + u \rightleftharpoons d + u$ $s \rightleftharpoons u + e + v$

 $H = \sum_{i} \overrightarrow{\alpha_{i}} \cdot \overrightarrow{p_{i}} + \beta_{i} M_{i} + \sum_{i < i} V_{ij}$

 $dQ = TdS = dE + PdV - \mu dN$

 $P = \left(\sum_{i} \mu_{i} n_{i} - \varepsilon_{i}\right) + \mu_{e} n_{e} - \varepsilon_{e}$

Property of Strange Quark Matter



At high density the strange quark matter is hypothesized to be more favorable energetically to normal matter (Bodmer 1971, Witten 1984) ²³

Detection of X-ray pulsation requires:

- (1) $R < R_0$ (2) $R_0 < R_{co}$



R = radius of the compact star

 \mathbf{R}_0 = radius of the inner edge of the accretion disk

 $\mathbf{R}_{co} = \text{corotation radius}: \mathbf{P}_{orb}(\mathbf{R}_{co}) = \mathbf{P}$

$$R_{co} = \left[\frac{GM}{4\pi^2}P^2\right]^{1/3}$$

Spherical accretion (and dipolar magnetic field)



ξ does not depend on the accretion rate

 $\mathbf{F} = \mathbf{X}$ -ray flux measured with the RXTE $\mathbf{F} = \mathbf{k} \cdot \mathbf{m}$ $\mathbf{k} = \text{const}$

$$R_0 = \xi \left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/7} \left(\frac{B_s^4 R^{12}}{M}\right)^{1/7} k^{-2/7} F^{-2/7} \equiv A F^{-2/7}$$

 $\mathbf{F}_{\min} \leq \mathbf{F} \leq \mathbf{F}_{\max}$

X-ray flux variation observed during april-may 1998

$$R < \frac{A}{F_{\max}^{2/7}} < \frac{A}{F_{\min}^{2/7}} < R_{co} \implies R < \left(\frac{F_{\min}}{F_{\max}}\right)^{2/7} R_{co}$$

upper limit for the radius of the compact object in SAX J1808.4-3658

$$R < (F_{min} / F_{max})^{2/7} (GM_{\odot} / 4\pi^2)^{1/3} P^{2/3} (M / M_{\odot})^{1/3}$$

 F_{min} = X-ray flux measured during the "low state" of the source F_{max} = X-ray flux measured during the "high state" of the source $F_{max} / F_{min} \sim 100$ P = period of the X-ray PSR

X.-D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel, Phys. Rev. Lett. 83, (1999), 3776





X.D. Li, I. Bombaci, M. Dey, J. Dey, E.P.J. Van den Heuvel, Phys. Rev. Lett. 83 (1999) 3776 SS1, SS2: M. Dey, I. Bombaci, J. Dey, S. Ray, B.C. Samanta, Phys. Lett. B438 (1998) 123

Property of Strange Quark Matter



New results

EOS Label	Λ' (MeV)	M_q (MeV)	N	α_0	$(E/A)_{min}$ (MeV)	$M_{max} \ (M_{\odot})$	R (km)
А	350	325	3.0	.55	874.0	1.532	7.411
В	350	325	3.0	.65	907.0	1.470	7.140
С	300	325	2.7	.55	877.0	1.554	7.530
D	300	325	3.0	.55	906.0	1.463	7.110
Е	300	335	3.0	.55	912.0	1.447	7.022







1.095.370.99801.055.200.85Bagchi et al, A & A. 450 (2006) 431



New results

Witten argued that if strange quark matter is the ground state, then strange stars can be born in the early universe around a temperature of 100 MeV. From our model we found that stable star structure is possible up to a temperature of 80 MeV. This closeness to Witten's hypothesis supports our SS model. Furthermore, the Kovtun, Sons and Starinets bound is also saturated at this temperature ArXiv 0705.4645 !

Surface Tension

Surface tension is the surface energy per unit area. For ordinary fluid, it is the property of the interaction between the media forming an interface and gravitation does not play any significant role.

Defining Surface Tension

SS is a huge drop of strange quark matter, the pressure difference across the surface can be expressed in terms of S. The pressure on top of the surface is zero.

$$|\Delta P|_{r=R} = \frac{2S}{R} \text{ where } |\Delta P|_{r=R} = h \left| \frac{dP}{dr} \right|_{r=R}$$

$$P_{r=R} = 0$$
So
$$S = \frac{hR}{2} \left| \frac{dP}{dr} \right|_{r=R} \frac{dP}{dr}_{r=R} \neq 0$$

What is the relevant thickness h ?? Interaction radius " r_0 " of the quarks.

- surface area of an SS = 4 π R² thickness of a shell of one quark layer = r_0 volume of the shell = 4 π R² r_o quark number inside the shell $n_t = 4 \pi R^2 r_0 n_c$ number density at star surface projection of a quark = πr_0^2 quark number at the surface $n_s = 4 \pi R^2 / \pi r^2 = 4R^2 / r^2$ for densely packed system, $n_s = n_t$ giving $r = \left(\frac{1}{\pi n}\right)^{1/3}$ $r_0 \sim 0.5 \text{ fm} - r_0$ at surface does not
 - depend on star size. But for a particular star, r₀
 increases from centre to surface

"interaction radius" of the quarks

A check of interaction radius

$$\sigma_{qq} = \pi r^2$$

$\sigma_{pp} = 3 \sigma_{qq}$ Heiselberg, Pethick, PR D 48 (1993) 2916.

$\sigma_{pp} \text{=} \text{25 mb} \longrightarrow \text{matches with experiment}$

Estimated value of surface tension

$$S = \frac{R}{2} \left(\frac{1}{\pi n}\right)^{1/3} \left|\frac{dP}{dr}\right|_{r=R} \quad \text{Putting h} = r$$

➤ Large value of surface tension
→ upto 140 MeV fm⁻² (174 MeV³). [Typical values of S used in literature range within 10 – 50 MeV fm⁻²
Heiselberg, Pethick, PRD 48 (1993) 2916 : Iida, Sato, PRC 58 (1998) 2538.]

For us S depends on star size !! Gravitation plays an Important role.

Bagchi et. al, A & A, 440 (2005) L33.



We have used our estimated value of surface tension to

study the properties of surface waves at the surface of a strange star.



 $y = h \sin\left(\frac{2\pi x}{\lambda}\right)$ h -> amplitude, v -> velocity, T -> time period, λ -> wavelength b -> radius of curvature $b = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{d^2 y/dx^2} = \mp \frac{y^2 T^2}{4\pi^2 h}$

- at crest, + at trough

where $\lambda = vT$

correction to velocity due to circular motion of fluid particle:

$$\mathbf{V} \rightarrow \mathbf{V} + \frac{2\pi |\mathbf{b}|}{T} \rightarrow (\text{crest}) \quad \mathbf{V} \rightarrow \mathbf{V} - \frac{2\pi |\mathbf{b}|}{T} \rightarrow (\text{trough})$$

condition for streamline flow-Bernoulli's equation

$$\mathbf{g}\mathbf{z} + \frac{\mathbf{v}^2}{2} + \frac{\mathbf{p}}{\rho} = \mathbf{0}$$

$$z \rightarrow (R \pm h) \rightarrow (R \pm h) \left(1 - \frac{R_s}{R \pm h}\right)^{-1/2}$$
 + at crest, - at trough

where
$$R \to R \left(1 - \frac{Rs}{R}\right)^{-1/2}$$
 $R_s = \frac{2GM}{c^2} \to \text{Schwarzschild Radius}$
 $p \to \frac{S}{b}$ $\rho \to \frac{\varepsilon}{c^2}$ $\frac{p}{\rho} \to \frac{Sc^2}{\varepsilon b}$ 41



Type I X-ray bursts are observed from 7 LMXB by RXTE.

 rise time t_{rise}~ 1 sec 20 800 15 decay time ~ 10 sec ower 10 C₆₀₀, • a peak in power density spectrum – burst oscillation 345 350 355 360 365 370 375 ³N400 380 Frequency (Hz) ANI geraring new order and the second states of the 200 5 10 15 20 25 30 Time (31.25 ms bins) M. van der Klis, Ann. Rev. Astron. Astrophys. 38 (2000) 717. Strohmayer et al, Astrophys. J. 469 (1996) L9. 43

- The frequency of oscillation
- is not constant, it
- increases
 - becomes constant near
 the burst tail –
- "Asymptotic Frequency" [light curve in white] frequency shift $\Delta f \sim 1.5 HZ$

http://lheawww.gsfc.nasa.gov/users/stroh/



We claim this frequency shift is the onset of a surface wave as the burst proceeds and due to the anticoupling of the wave's power to the burst power.

Bagchi et al, A & A 440 (2005) L33 $\mathbf{v} = \frac{\pi \mathbf{R}}{\mathbf{t}_{rise}} \qquad \mathbf{T} = (\Delta \mathbf{f})^{-1}$



Burst from 4U 1728-34 observed by RXTE on February 16 (1996) ; $t_{rise} = 0.6$ sec,

 $\Delta f = 1.5 HZ$ \rightarrow T = 2/3 sec. v = 36.89 km/sec(R = 7.05 km)graphical solution gives h = 300 cm



The Nambu Jona-Lasinio (NJL) model was proposed by Nambu and Jona-Lasinio in 1961 [2, 3] *i.e.* when QCD or even the quarks were unknown. The original NJL model was therefore a model of interacting nucleons. The problem was to find a mechanism which explains the large nucleon mass without destroying the chiral symmetry. To solve this problem, they introduced a Lagrangian as :

$$\mathcal{L} = \bar{\psi}(i\partial \!\!/ - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$
(4.28)

where ψ is the nucleon field, m is the small bare mass of the nucleon, $\vec{\tau}$ is a Pauli matrix acting on isospin space and G is a dimensionful coupling constant. The self energy induced by the interaction generates an effective mass M which can be considerably larger than m and stays large, even when m is taken zero ("chiral limit").

[2] Y. Nambu, G Jona-Lasinio, Phys. Rev. 122 (1961) 345.

[3] Y. Nambu, G Jona-Lasinio, Phys. Rev. 124 (1961) 246.

$$M^* = m_0 + 4G\left(N_c N_f + \frac{1}{2}\right) M^* \int^{\Lambda} \frac{d^3 p}{(2\pi)^3} \frac{1}{E}$$

$$f_{\pi}^{2} = N_{c} M^{*2} \int^{\Lambda} \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{E^{3}}$$

$$\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -6M^* \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{E}$$

Here the number of color $N_c = 3$ and the number of flavor $N_f = 2$ (for a two flavor SU(2), coloriant)

SU(2) calculation).

S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649.

$$M_{i} = m_{i} + M_{q} \quad sech\left(\frac{n_{B}}{Nn_{0}}\right), \qquad i = u, d, s.$$
$$M^{*} = m_{0} + M_{q} \quad sech\left(\frac{n_{B}}{Nn_{0}}\right),$$

where $m_0 = (m_u + m_d)/2$.





Density f_{π} from different models up to $5n_0$: + corresponds to the nuclear matter model of ZM3, diamonds corresponds to QCDSR results, squares correspond to the SQM.

Concluding Remarks

- Existence of Strange Star is not still well established.
 We hope further study of x-ray and radio astronomy will help to remove the dispute.
- But why we are so much interested to Strange Stars?
- QCD properties are still inaccessible to terrestrial experiments. Experimentalists tried to get signatures of asymptotic freedom and confinement in laboratory – RHIC, but could NOT succeed.
- So Strange Star serves as the lab set up by nature for us to test QCD properties.!!
- Further study of Strange Star properties will be interesting. Connecting RHIC data to SS is possible through viscosity/entropy ratio. ⁵¹

THANK YOU

- I liked Bob Rutledge's question. The take home for astrophysicists is observations are good. The one for Nuclear Model people is – u r doing well.
- For phenomenologists like us the message is : look for discrepancies and try simple QCD models.