Pairing Gaps in low-density neutron matter and in cold atoms

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Original work w/ K. Schmidt (ASU), V. Pandharipande, S.Y. Chang (Ill)

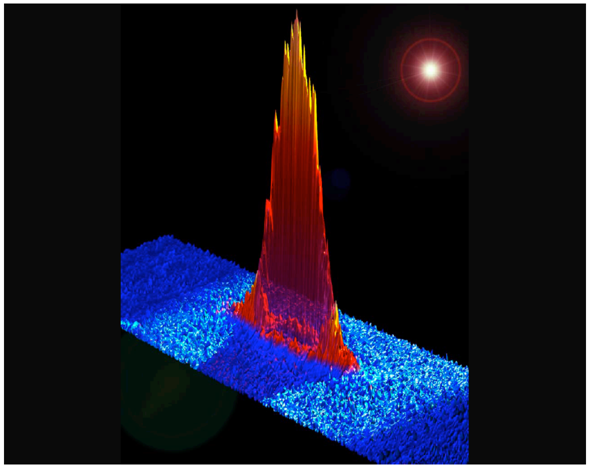


Image from Randy Hulet

length scale: micrometer
temp./Energy: nanokelvin

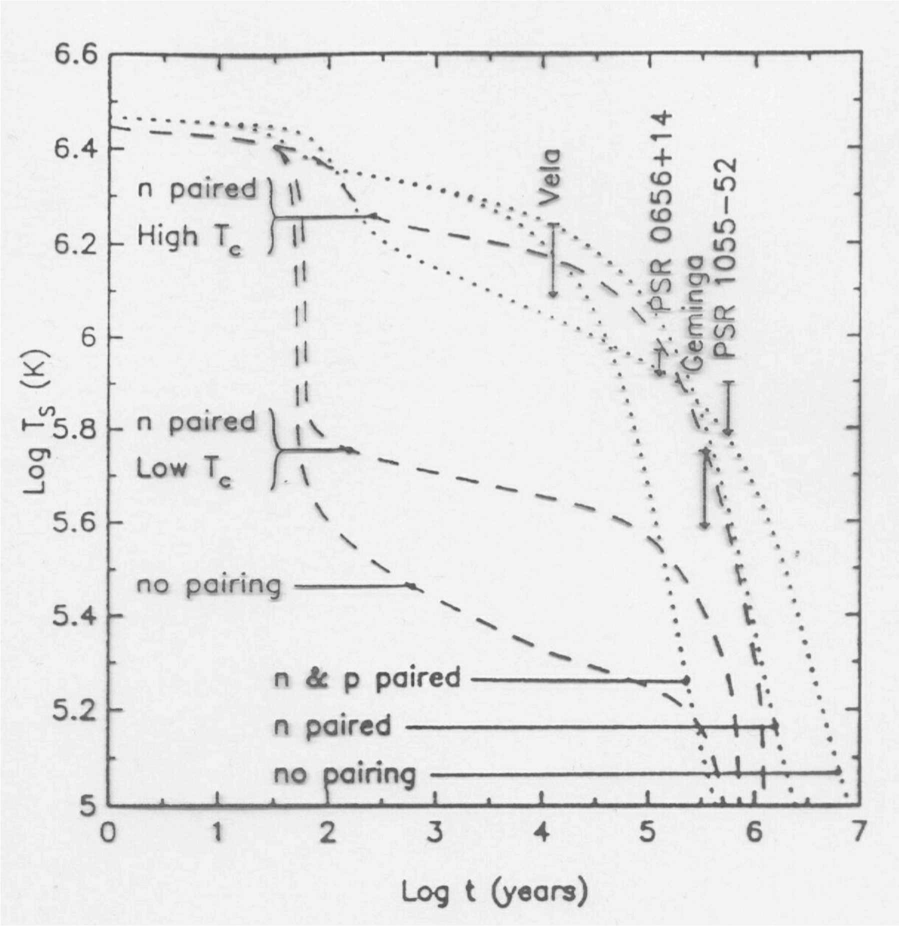
length scale: fermi
temperature/energy: MeV

Even if no new phases, parameters including Superfluid gap Δ are important

Superfluid gap for low-density neutron matter affects cooling

Benchmark for pairing in the strong-coupling QCD

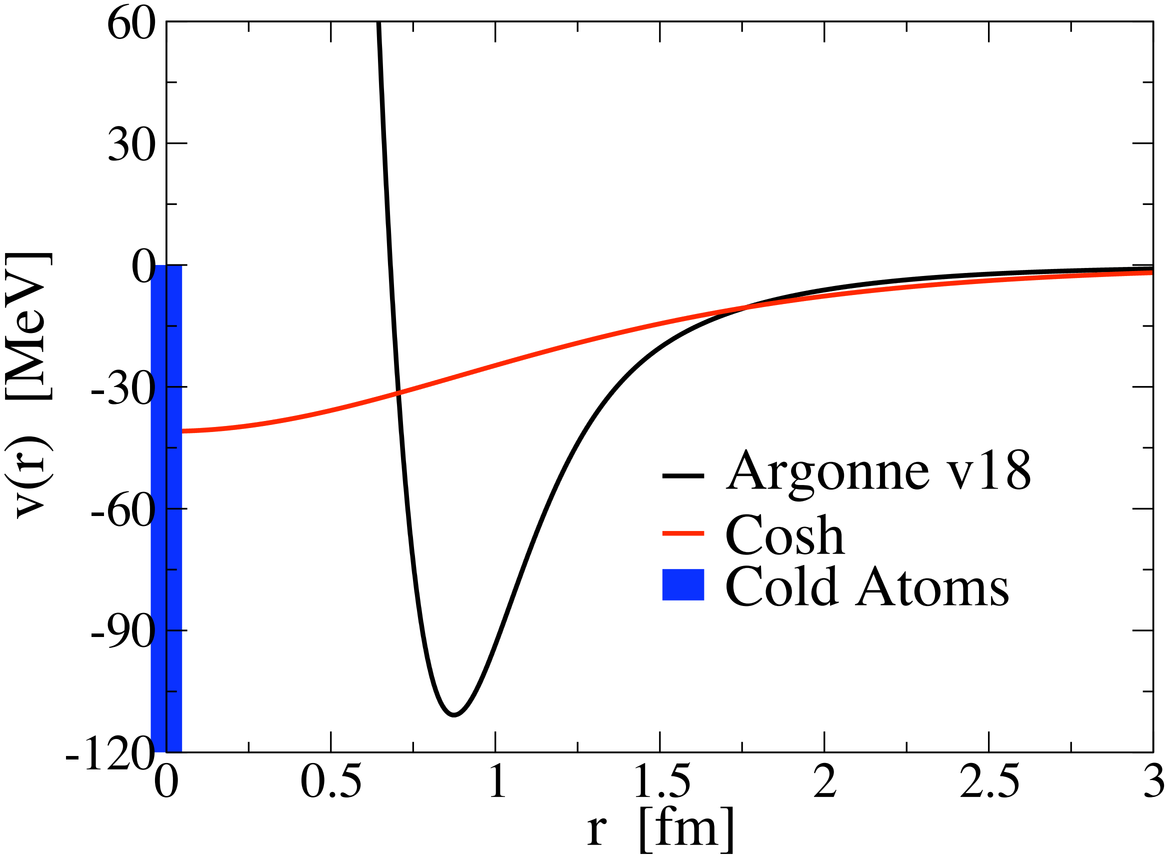
QCD at high densities



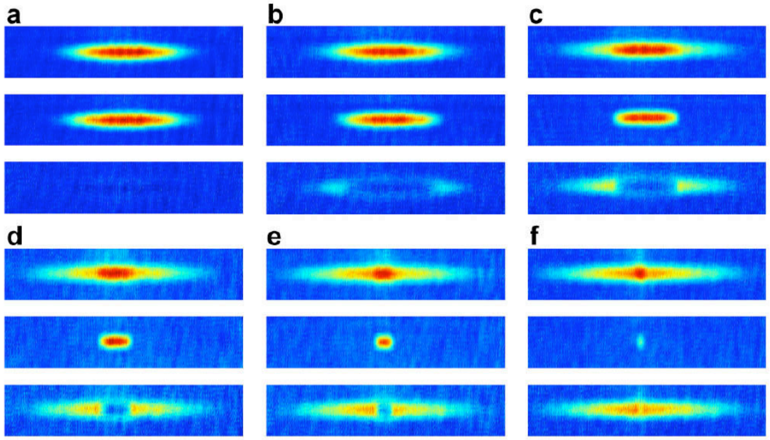
Neutron star cooling curves

Fundamental problem in quantum many-body physics: transition from weak (BCS) to strong (BEC) pairing and `exotic' states of matter

$$\mathcal{H} = \sum_{k=1}^{A}\left(-\frac{\hbar^2}{2m_k}\nabla_k^2\right) + \sum_{i<j} v(r_{ij})$$



Rich Set of Experimental Results in Cold Fermi Atoms: MIT, JILA, Rice, Duke, Innsbruck



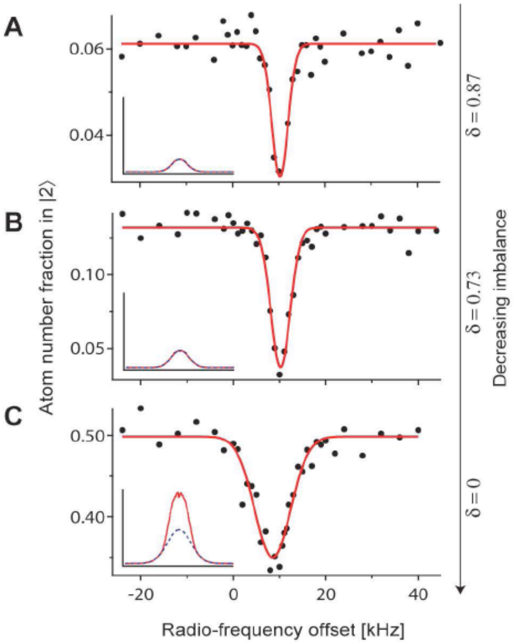
Rice

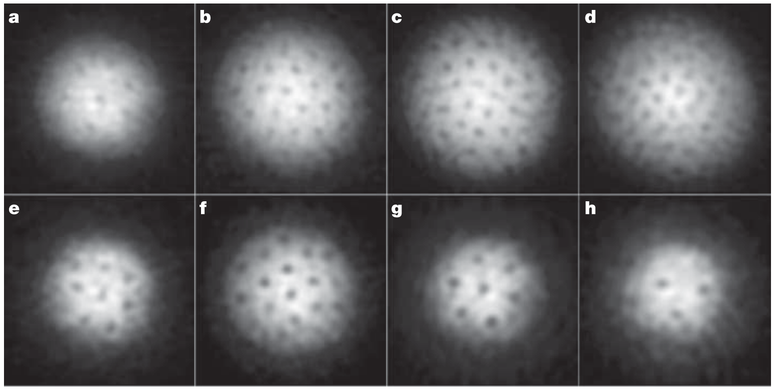
Radial Density

Polarization

Vortices

RF response





MIT

Cold Fermi Atoms Introduction:

Interaction strength adjustable, range essentially zero

At infinite scattering length:

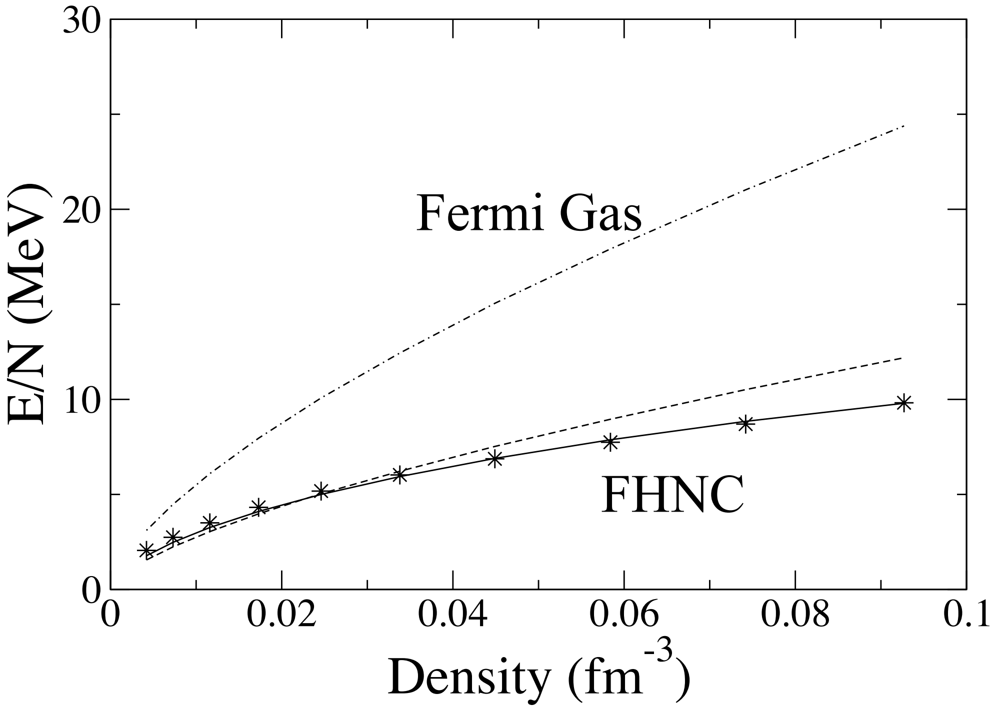
Ground State Energy	0.25(1)
Pairing Gap	0.50(5)
Superfluid transition temperature	0.25(3)
...	

are all ‘universal’ constants times $E_f = k_f^2 / 2m$

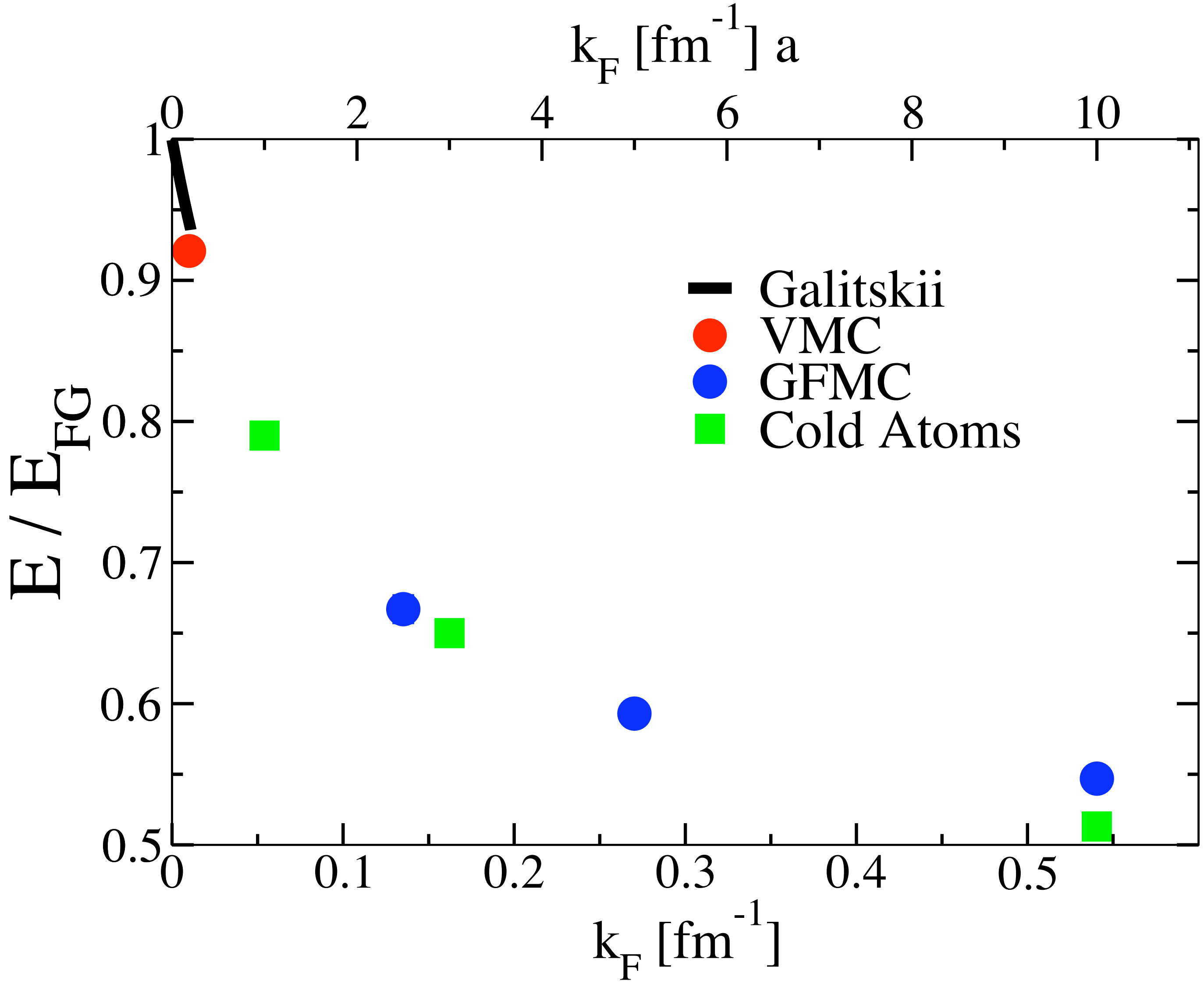
More generally functions of $(k_f\ a) \times E_f$

Neutron Matter Equation of State

Neutron-Neutron interaction - dominantly s-wave (spin 0) at low energy
Large scattering length ~ -18 fm
Modest effective range ~ 2.7 fm



Equation of State: Neutrons vs. Cold Atoms



Method: Diffusion (Green's function) Monte Carlo

Fixed Node - Variational Upper Bound

Vary parameters in nodal surfaces ~ different 'phases' (superfluid or normal)

Transient Estimation

Comparisons to Lattice Methods at Equal Populations

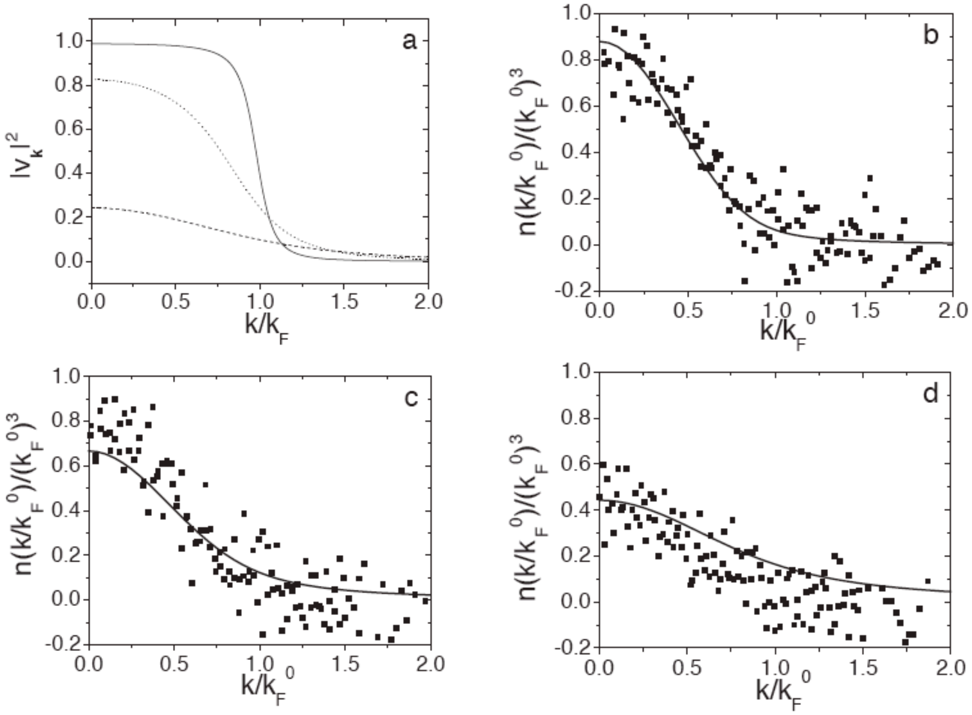
$$\Psi(\tau \rightarrow \infty) = \lim_{\tau \rightarrow \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

Variational wavefunction

$$\Psi_V(\mathbf{R}) = \prod_{i,j'} f(r_{ij'}) \Phi_{BCS}(\mathbf{R})$$

Measurements and EOS at a = infinity

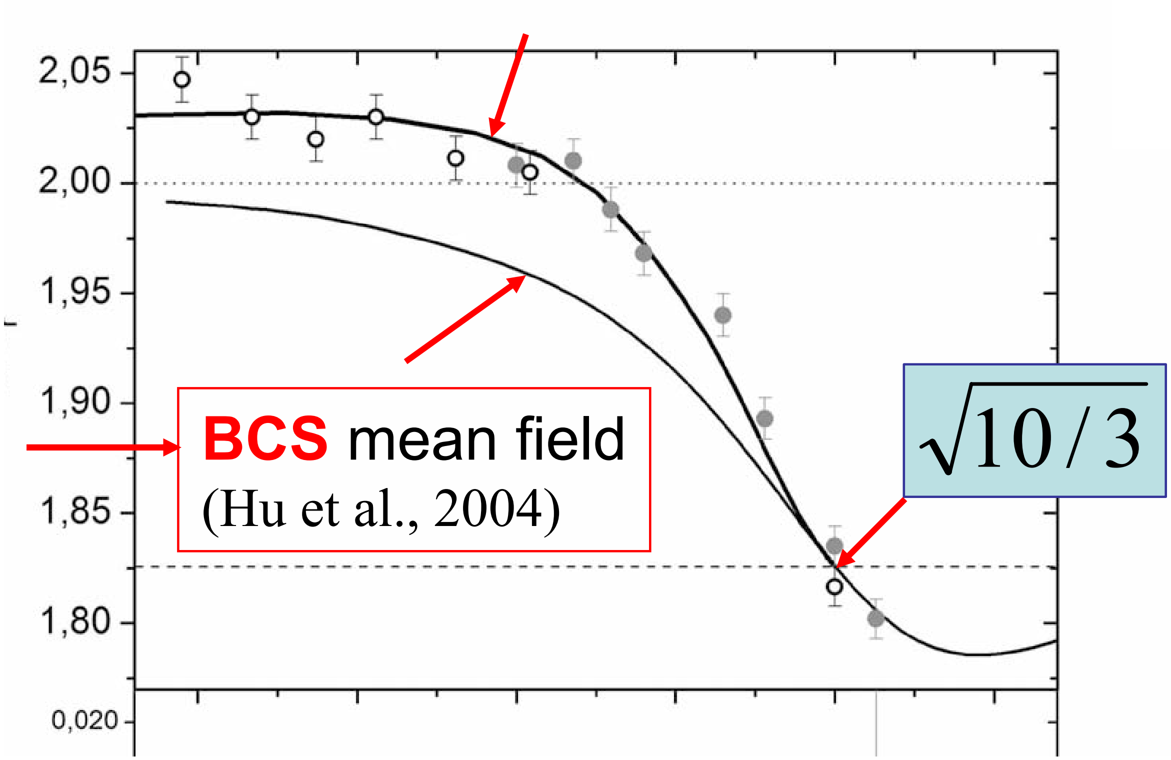
0.51 (4)	Kinast, et al., Science (2005)
0.32 (+.13,-.1)	Bartenstein, et al., PRL (2004)
0.36(15)	Bourdel, et al., PRL (2004)
0.46(5)	Partridge, et al., PRL (2004)
0.45(5)	Stewart, et al., PRL (2006)
0.41(15)	Tarruell, et al., cond-mat/0701181



Calculations:

0.42 (2)

Radial modes in a trap



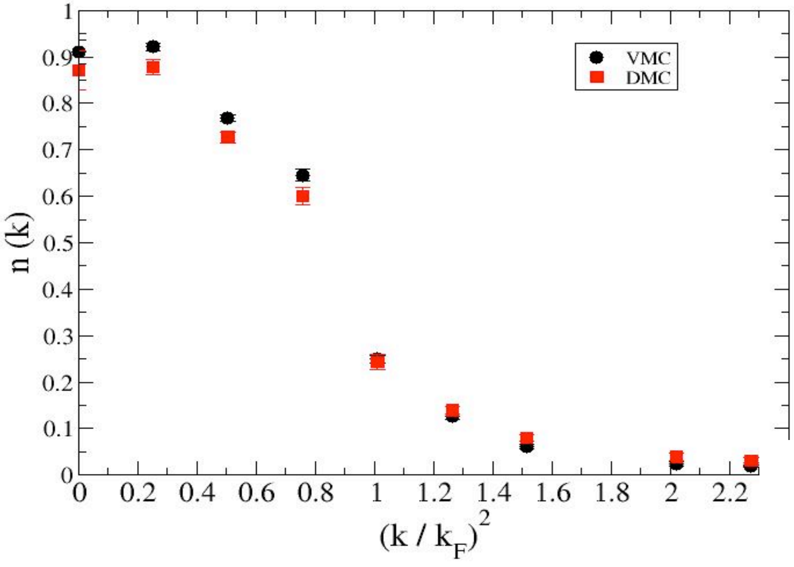
BCS mean field
(Hu et al., 2004)

$\sqrt{10/3}$

Data: Innsbruck 2007

From Stringari (ECT* 2007)

Momentum Distribution

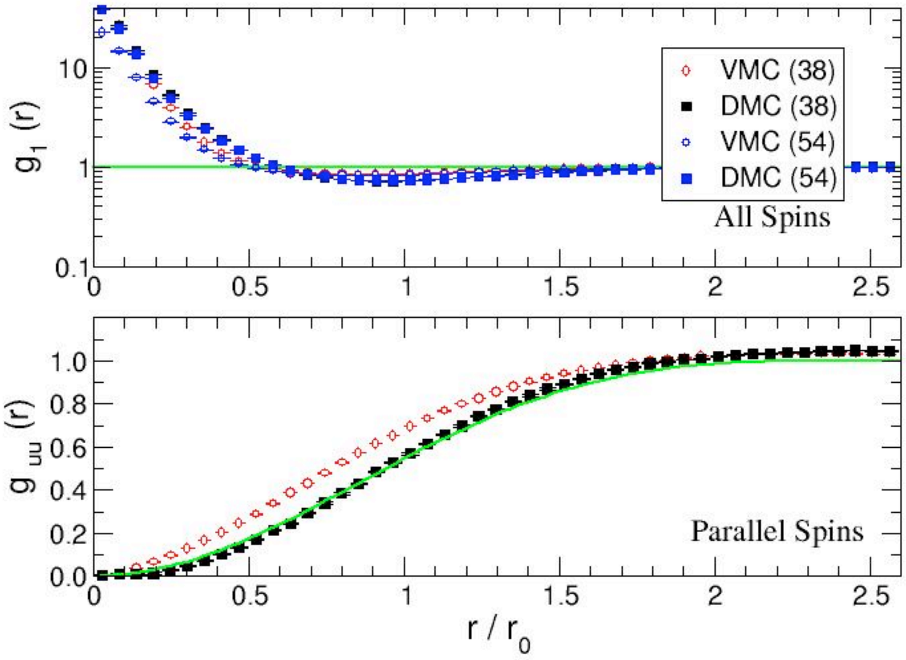


At unitarity

Very different from

Fermi Liquid

Pair Distributions

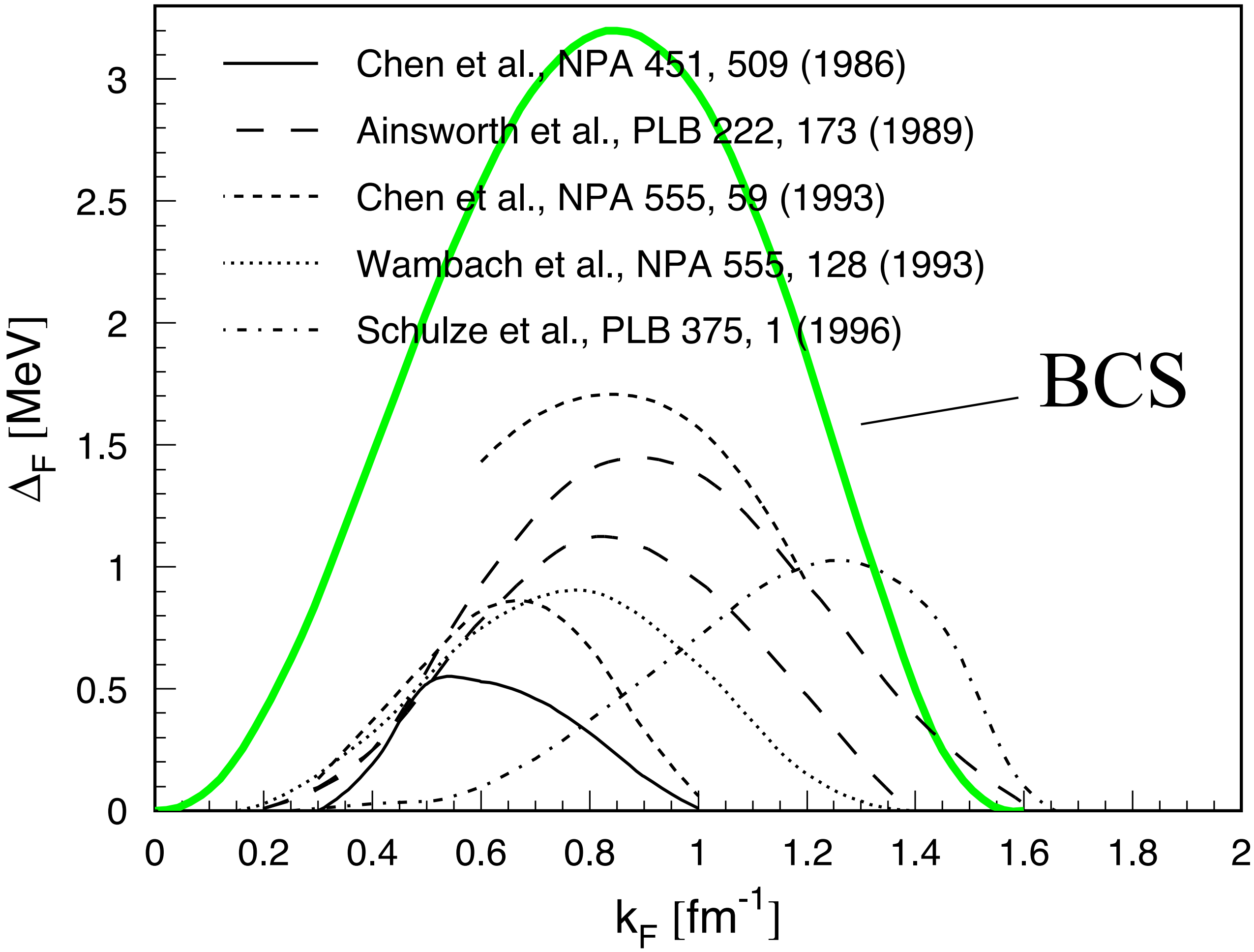


Strongly

Peaked

Pair distribution

Superfluid (Pairing) Gap



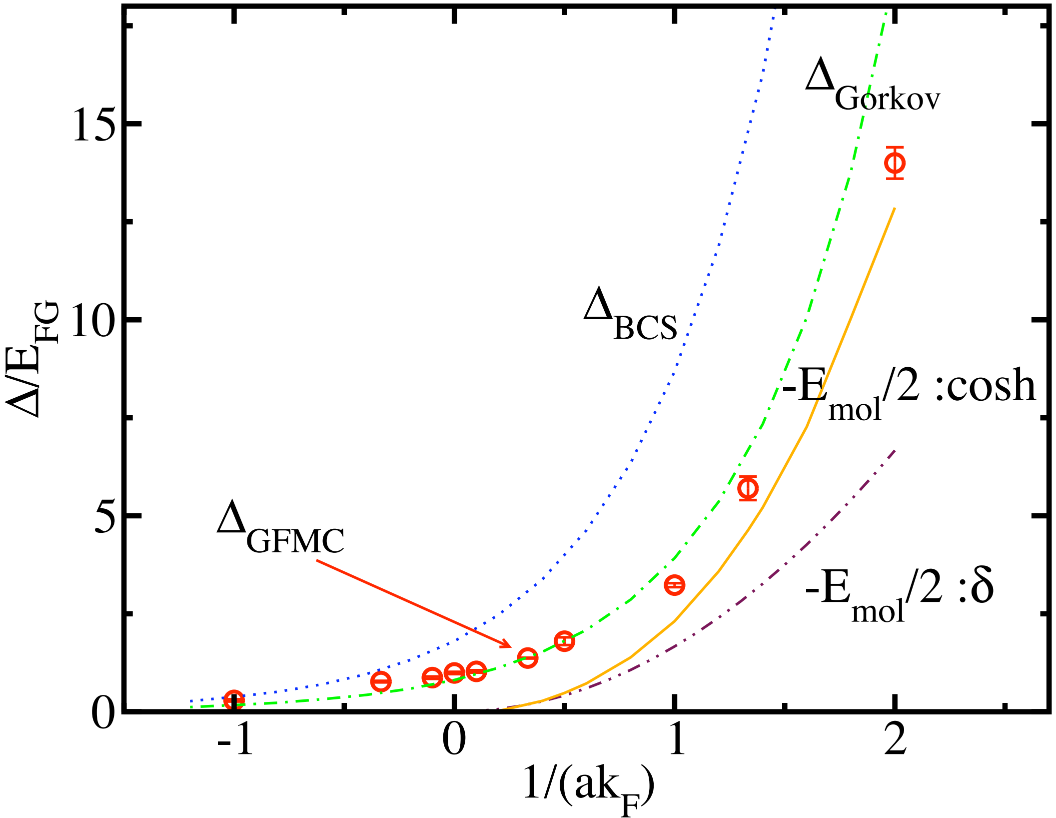
BCS

Dean and
Hjorth-Jenson
RMP (2003)

Pairing Gap (apparently) difficult to get right !
Situation now worse than shown

Pairing in Cold Atoms: gap vs. $k_f\ a$

BCS /
Gorkov



BEC
(Molecules)

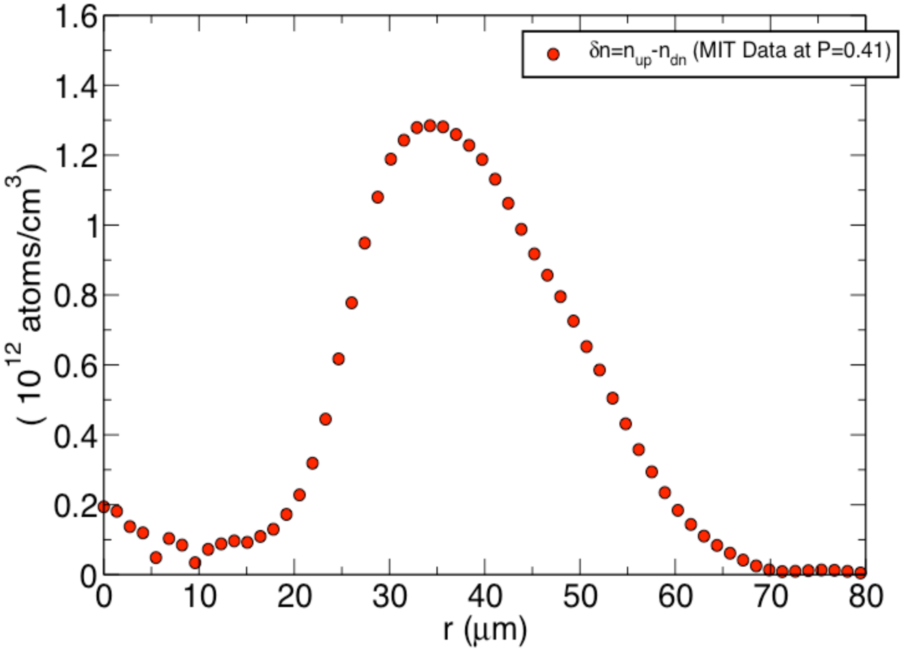
Above: comparison w/ asymptotic formulas

BCS equations -> $\Delta = 0.6\ E_f$ at $a = \infty$

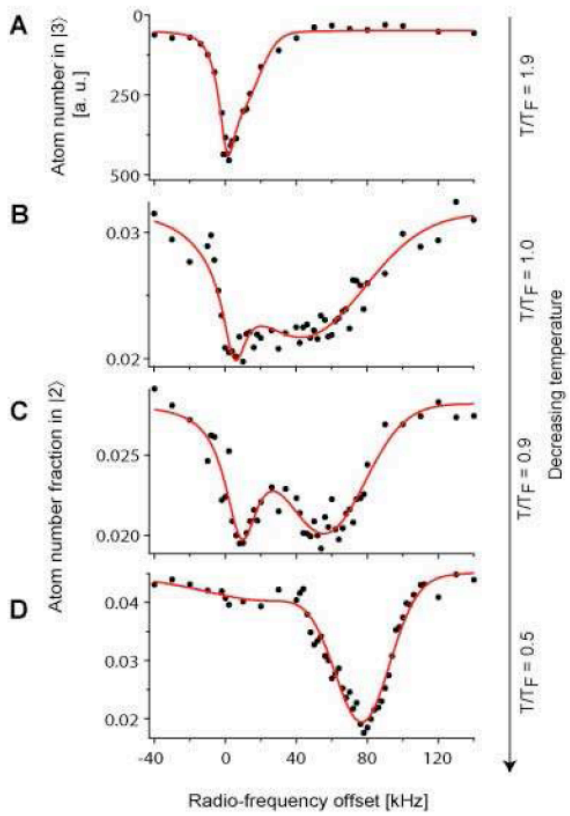
QMC calculations -> $\Delta = 0.5\ E_f$

Experimental Probes of Pairing in Cold Atoms

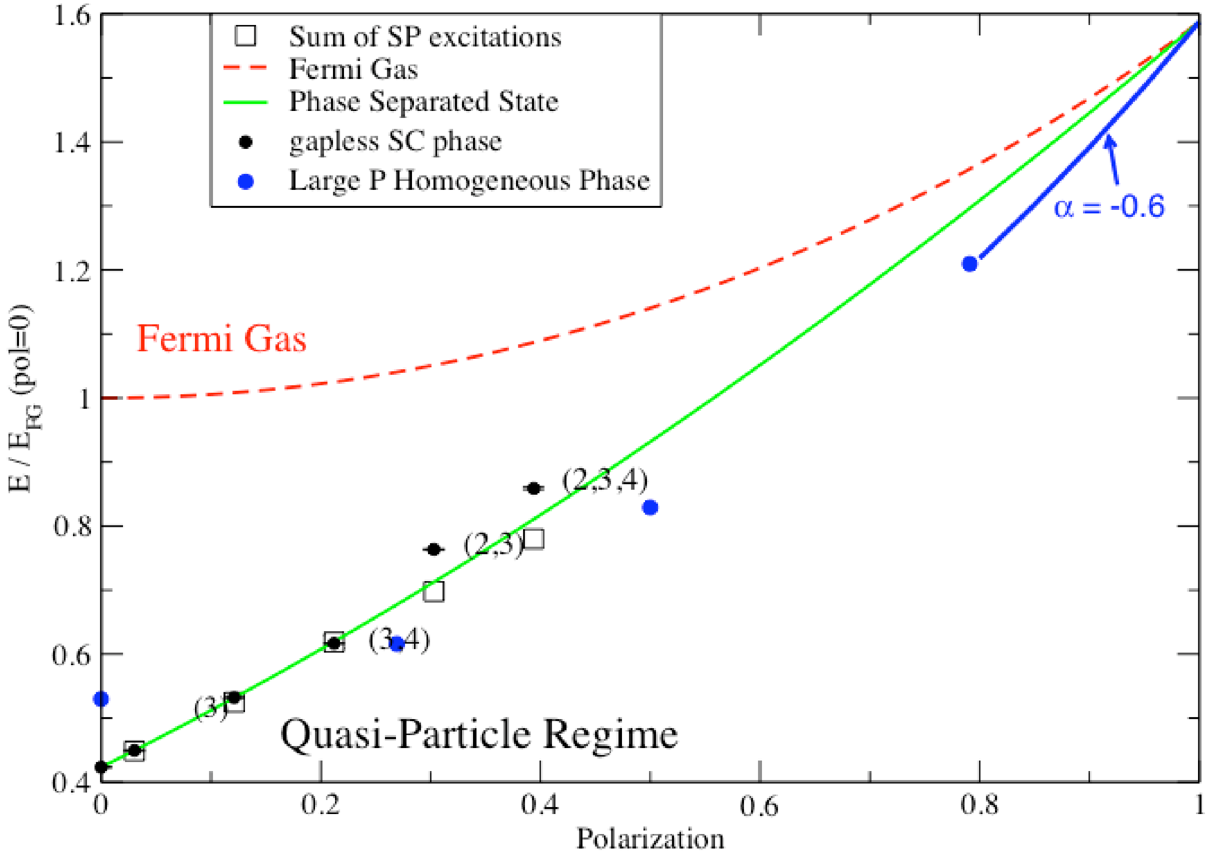
Polarization vs. radius in Polarized Systems

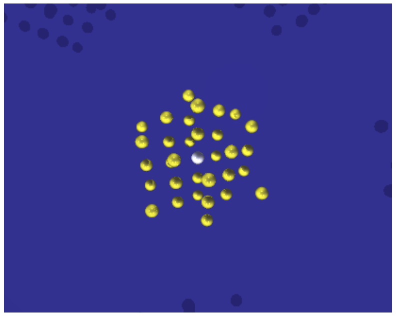


RF response



Polarization at T=0





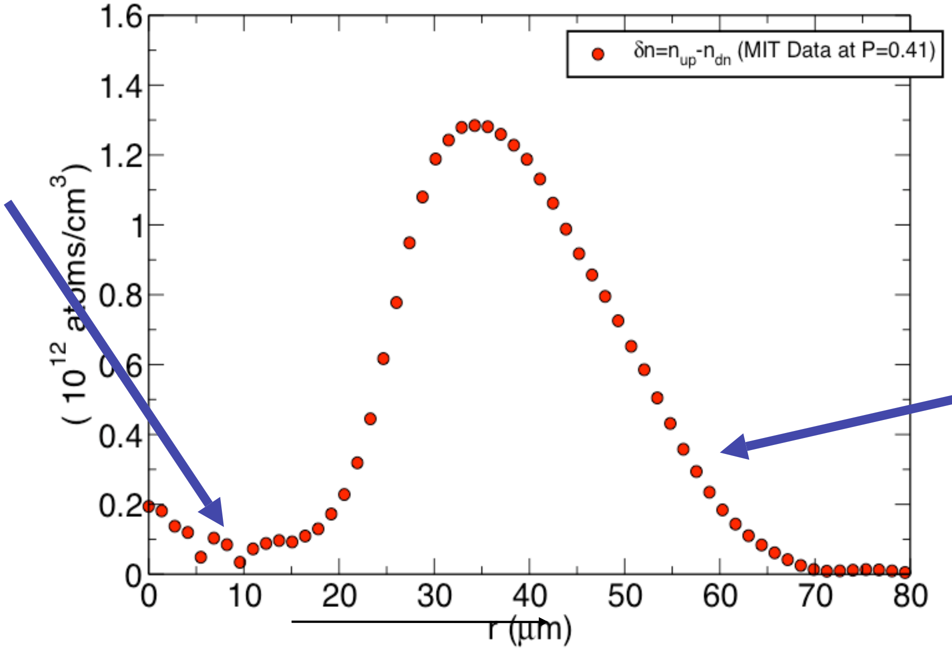
See results by
Lobo, Recati, Giorgini,Stringari
PRL 2006

Polarization vs. Radius : MIT data

MIT data P=0.41

unpolarized
superfluid

N↑- N↓



Fully polarized
Normal state

radius

At T = 0, assume 1st order phase transition
at a local polarization of ~45%

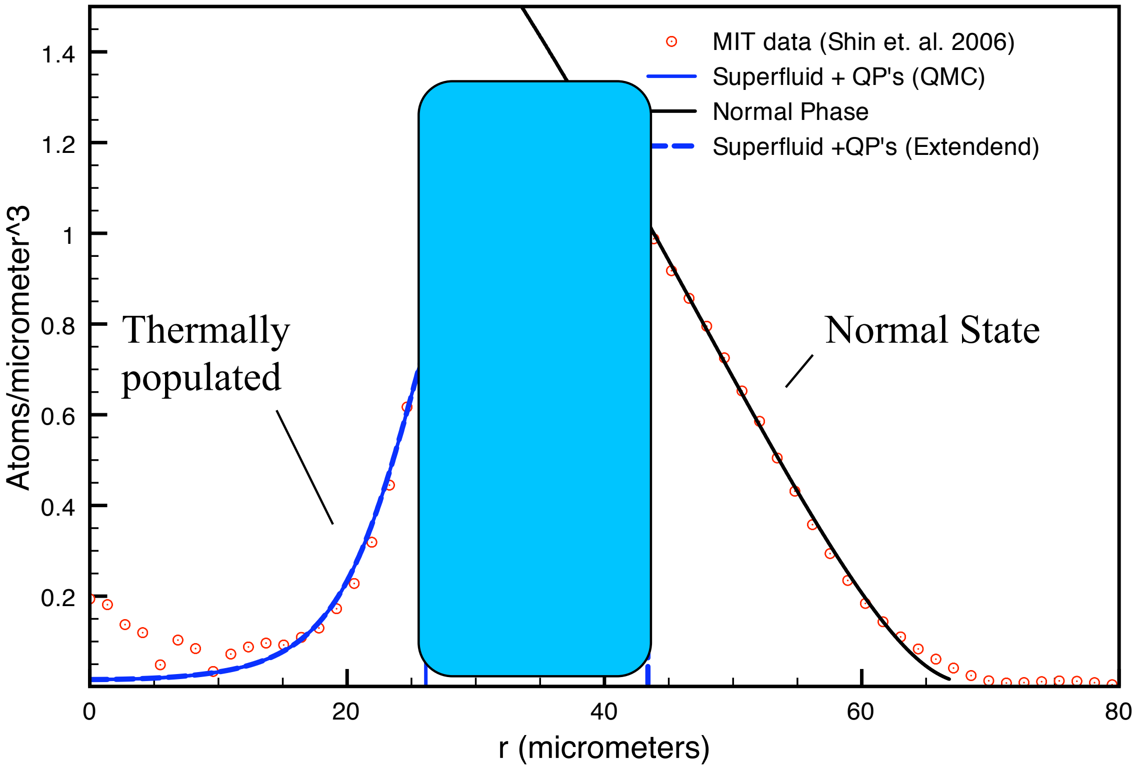
Calculated gap ≈ 0.5 (.05) Ef

If experiments say there is no
polarization in the superfluid at T=0 :

Equilibrium (chemical potentials, pressure)
implies gap > 0.40(.02) Ef

Very close to Sarma phase at unitarity and T=0

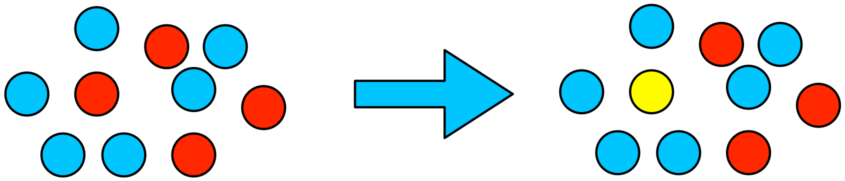
MIT Data (P = 0.41)



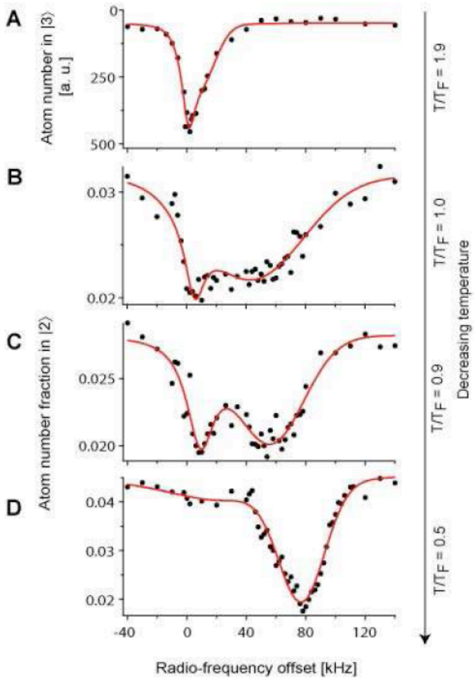
If we assume first order normal/superfluid phase transition and no superfluid polarization at T=0: $\Delta \geq 0.4$ Ef

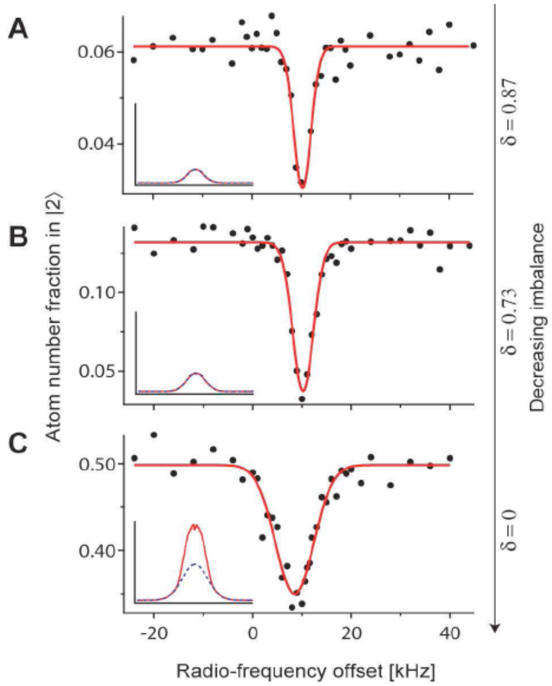
Is this consistent w/ RF response?
measurement of 0.2 Ef claimed

Tune RF to specific transition: flip a minority spin to a 3rd (strongly-interacting) state - zero momentum transfer



Decreasing Temperature





Decreasing Polarization

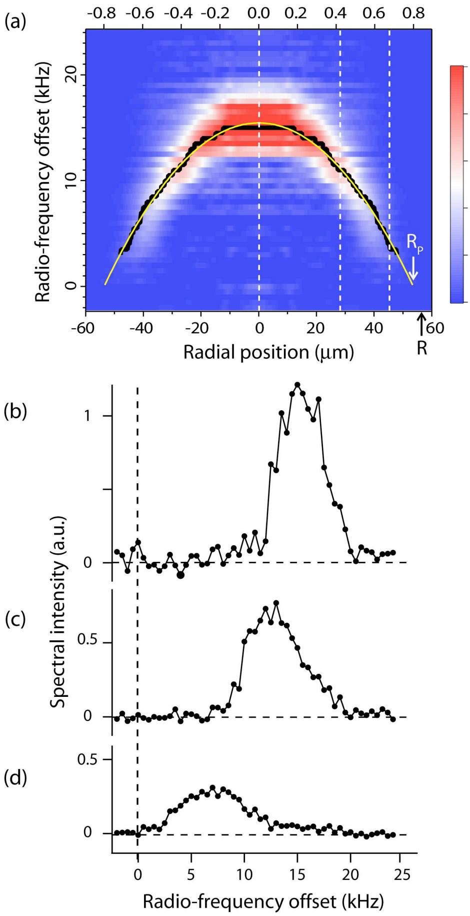
Entire Response Difficult to Calculate:
2 Simple Quantities: Sum Rule and ‘Threshhold

Sum Rule = <V13> - <V12> goes to zero as $a_{13} \Rightarrow a_1$

Threshold = BE (a_{13}) - BE (a_{12}) for normal

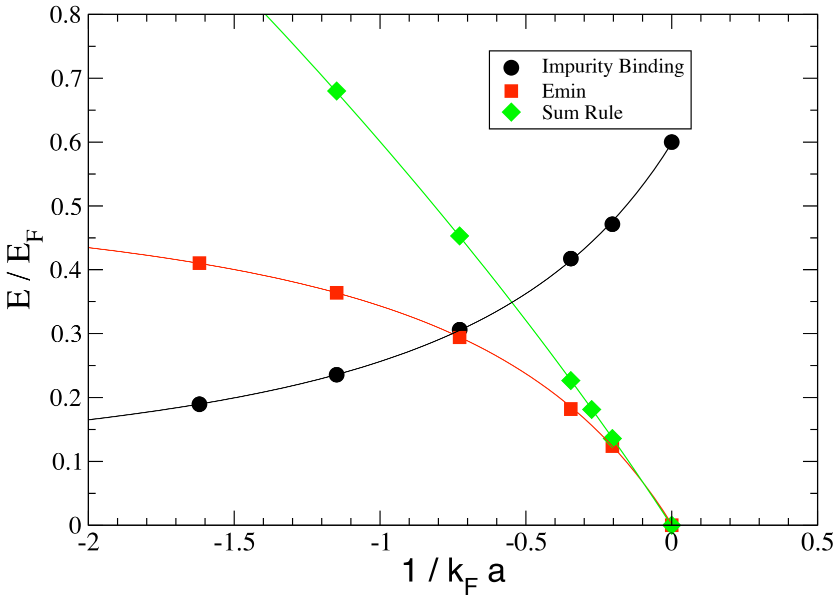
Width decreases as v13 becomes similar to v12

Sum Rule decreases also



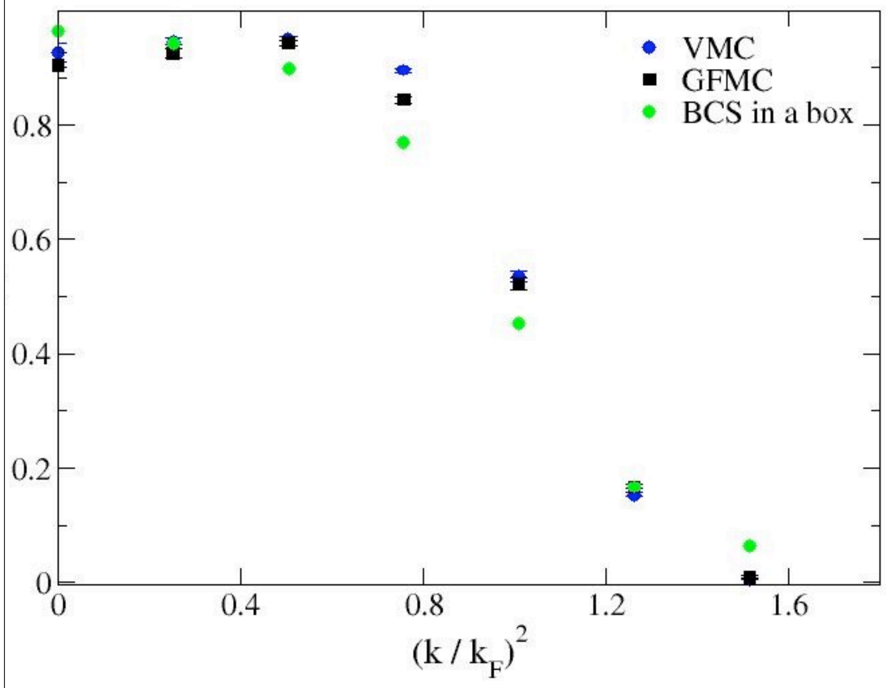
MIT data, 2007

Normal State



Neutron Matter

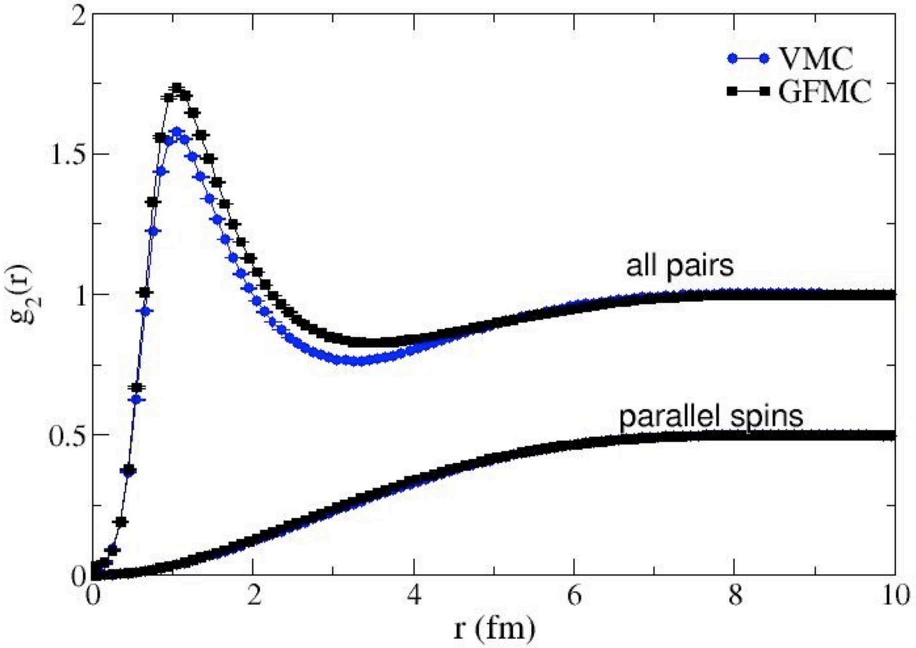
momentum distribution



Qualitatively similar to cold atoms

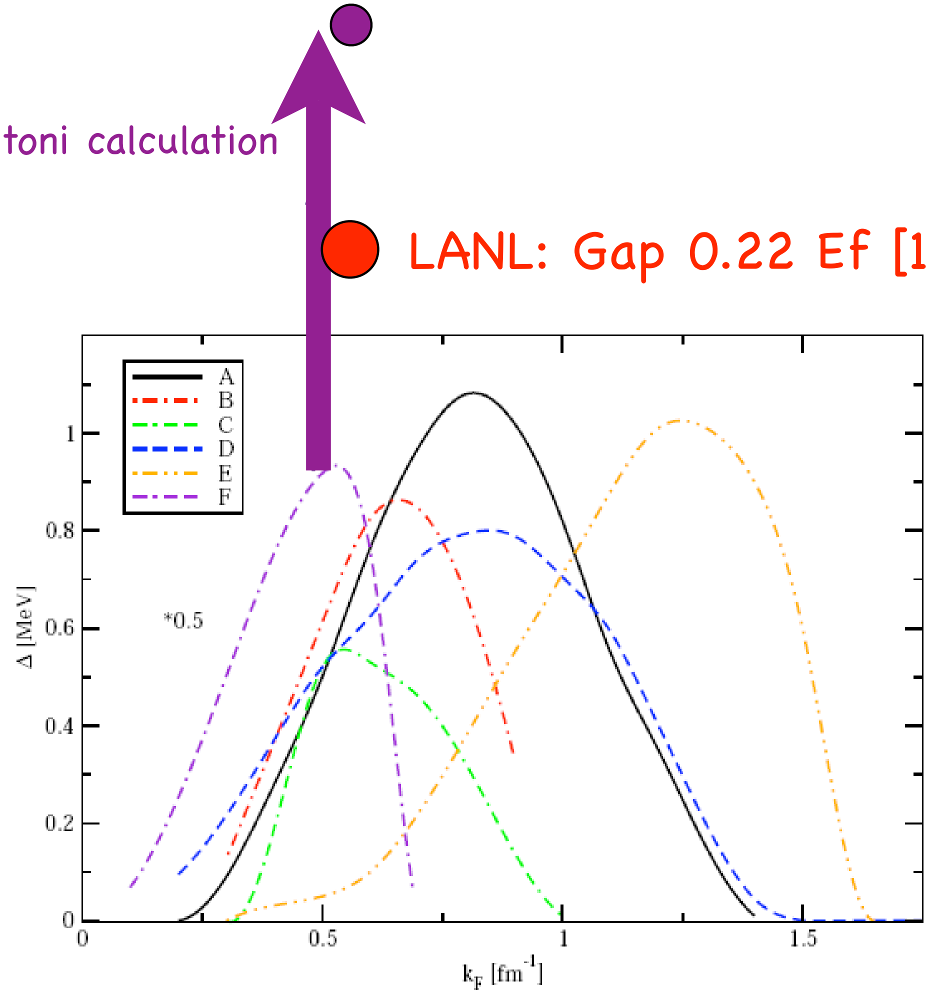
$a\, k_f = 10$ or $k_f \approx 0.54\ \mathrm{fm}^{-1}$

pair distributions



carefully studied size dependence
from A = 22 to 90
compared to BCS

GFMC compared other Results



Schmidt, Fantoni calculation

LANL: Gap 0.22 Ef [1.35(15)] MeV
at kf a = -10

curves from Sedrakian and Clark: nucl/th-0607028

Conclusions / Future Directions

Experimental probes of pairing gap in cold atoms important
to constrain quantum many-body theories.

Gap at unitarity in cold atoms approximately 0.5 Ef

Neutron matter gap significantly larger than typical calculations,
but smaller than BCS theory or cold atoms (finite range)

Experiment:

Experiments which measure both n, n↑ - n↓ vs. r
for different Geometries, Polarizations and Temperatures

Theory

Calculations in different geometries (inhomogeneous, ...)
More accurate calculations of Gap, dispersion, RF response
Calculations of different possible phases