Transport Properties in The Neutron Star Crust

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The Neutron Star Crust and Surface
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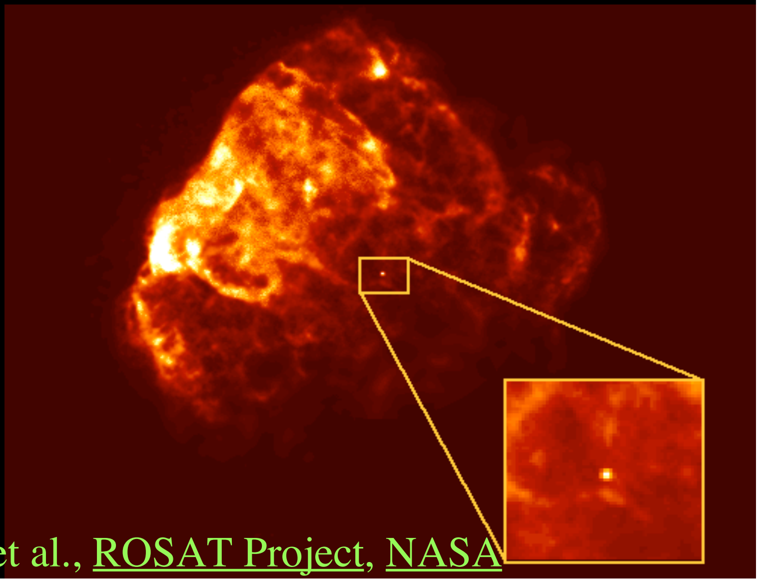
Transport Properties

- Thermal Conductivity
  - Cooling Times
- Diffusion Coefficient
  - Sedimentation
- Shear Viscosity
  - r-modes

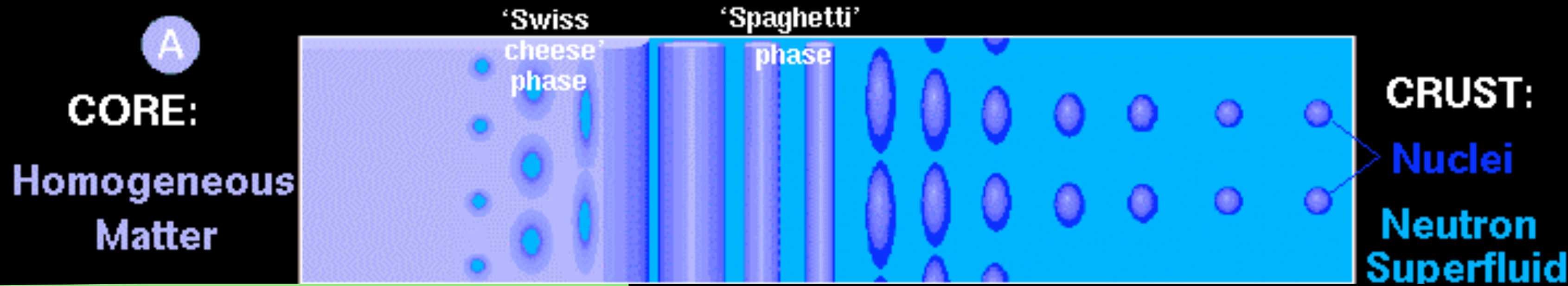


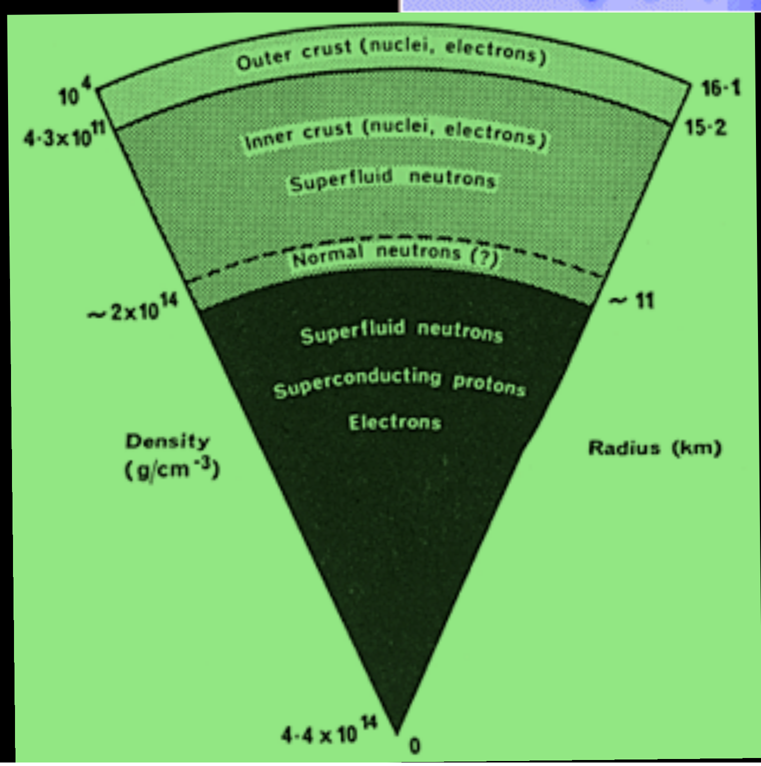
Supernova Remnant and Neutron Star
Credit: S. Snowden, R. Petre (LHEA/GSFC), C. Becker (MIT) et al., ROSAT Project, NASA

Neutron Star Structure

ρ

10^{14} g/cm^3 ← 10^9 g/cm^3





Model

- Classical Approximation $\lambda_B << r_{ij}$

- Ions interact via screened Coulomb potential

$$V(r_{ij}) = \frac{Z_i Z_j e^2}{4\pi r_{ij}} \exp\left(-\frac{r_{ij}}{\lambda_e}\right)$$

λ_e electron screening

$$\lambda_e = \frac{\pi}{ek_F}$$

r_{ij} distance between Ions

Molecular Dynamics Simulation

- Verlet Algorithm
- $r_i(t), \ i=1,N$

$$\vec{a}_i(t+\Delta t) = -\frac{1}{m_i}\nabla V\left(\vec{r}(t+\Delta t)\right)$$

- Periodic Boundary Conditions
- N=1000 ions (or more)
- $t_{total}=1.6\times10^7$ fm/c, $\Delta t=50$ fm/c
- One-Component Plasma (Pure): $A_i=\langle A\rangle$, $Z_i=\langle Z\rangle$ for all ions
- MultiComponent (Mixed): Ion Distribution

Ion Distributions

Statistical Model

Grand Canonical version of the Statistical Multifragmentation Model.

- Experimental Binding Energies.
- Omits most Interactions between nuclei.

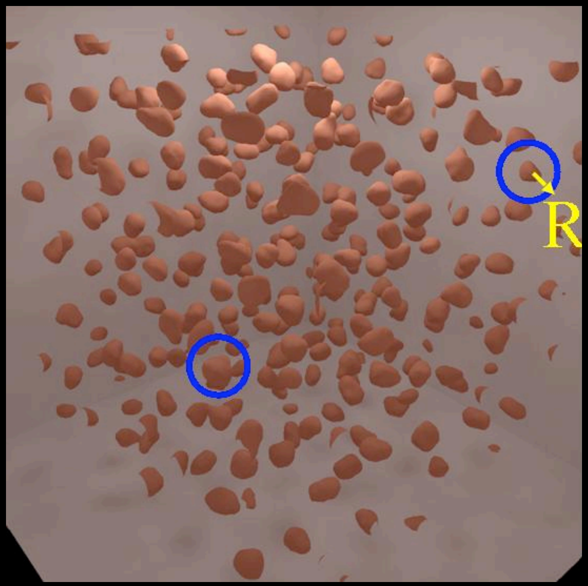
(A.S. Botvina and I.N. Mishustin, Phys. Lett. B 584 (2004) 233.)

Pasta Model

Semi classical Microscopic Model.

- Nuclear interaction reproduces binding energies and nuclear matter saturation density.
- Uses Nucleons as degrees of freedom.

(C.J. Horowitz et. al. Phys.Rev. C 70, 0.65806(2004))



40,000 nucleons, ρ =0.01 fm^{-3}

Ranges

$$\Gamma = \frac{Z^2 e^2}{a k_B T} \leq 175$$

$$a = \left(\frac{3}{4\pi}\rho_i\right)^{1/3}$$

$$10^9\,\mathrm{g/cm^3} < \rho < 10^{13}\,\mathrm{g/cm^3}$$

$$T = 1,\ 10\ \mathrm{MeV}$$

$$0.34 < \Gamma < 180$$

Thermal Conductivity

Itoh, Kohyama (1992)

$$\kappa = 2.363\times 10^{7} T_8 \rho_6 \left(\sum_i \frac{X_i}{A_i}\right) \frac{\langle Z\rangle^2}{\langle(\Delta Z)^2\rangle} \frac{1}{\left\{1+1.018\left[\sum_i \frac{Z_i X_i}{A_i}\right]^{2/3} \rho_6^{2/3}\right\}\langle S\rangle} \frac{\text{ergs}}{\text{cm s K}}$$

$$T_8 = T(10^8 K)$$

$$\rho_6 = \rho(10^6 \text{g/cm}^3)$$

$$\langle S\rangle = \langle S_{-1}\rangle - \frac{1.018\left[\sum_i \frac{Z_i X_i}{A_i}^{2/3} \rho_6^{2/3}\right]}{1+1.018\left[\sum_i \frac{Z_i X_i}{A_i}^{2/3} \rho_6^{2/3}\right]}\langle S_{+1}\rangle$$

$$\langle S_{-1}\rangle = \int_0^{2k_F} dk \frac{S(k)}{k\varepsilon(k/2k_F,0)^2}$$

$$\langle S_{+1}\rangle = \frac{1}{2k_F}\int_0^{2k_F} dk \frac{kS(k)}{\varepsilon(k/2k_F,0)^2}$$

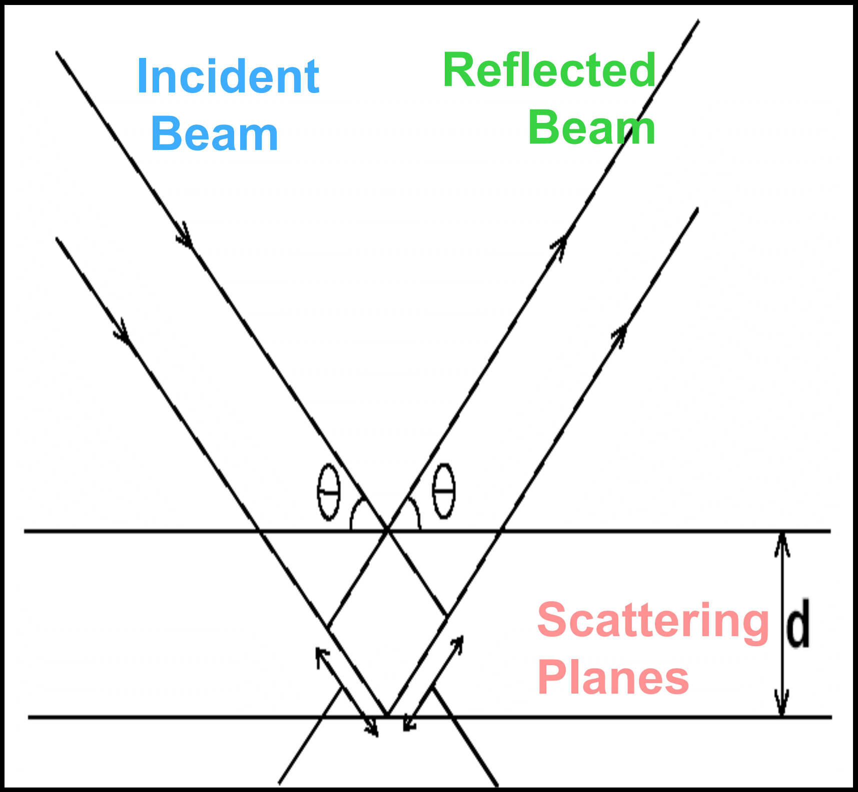
Structure Factor

Static Dielectric Function

Structure Factor

$$S(\vec{q}) = \frac{1}{\langle N \rangle^2 N_{ion}} \left(\langle \Psi_0 | \hat{\rho}^+(\vec{q}) \hat{\rho}(\vec{q}) | \Psi_0 \rangle - \left| \langle \Psi_0 | \hat{\rho}(\vec{q}) | \Psi_0 \rangle^2 \right| \right)$$

$$\rho(\vec{q}) = \sum_{i=1}^{N} N_i \exp(i\vec{q} \cdot \vec{r}_i)$$

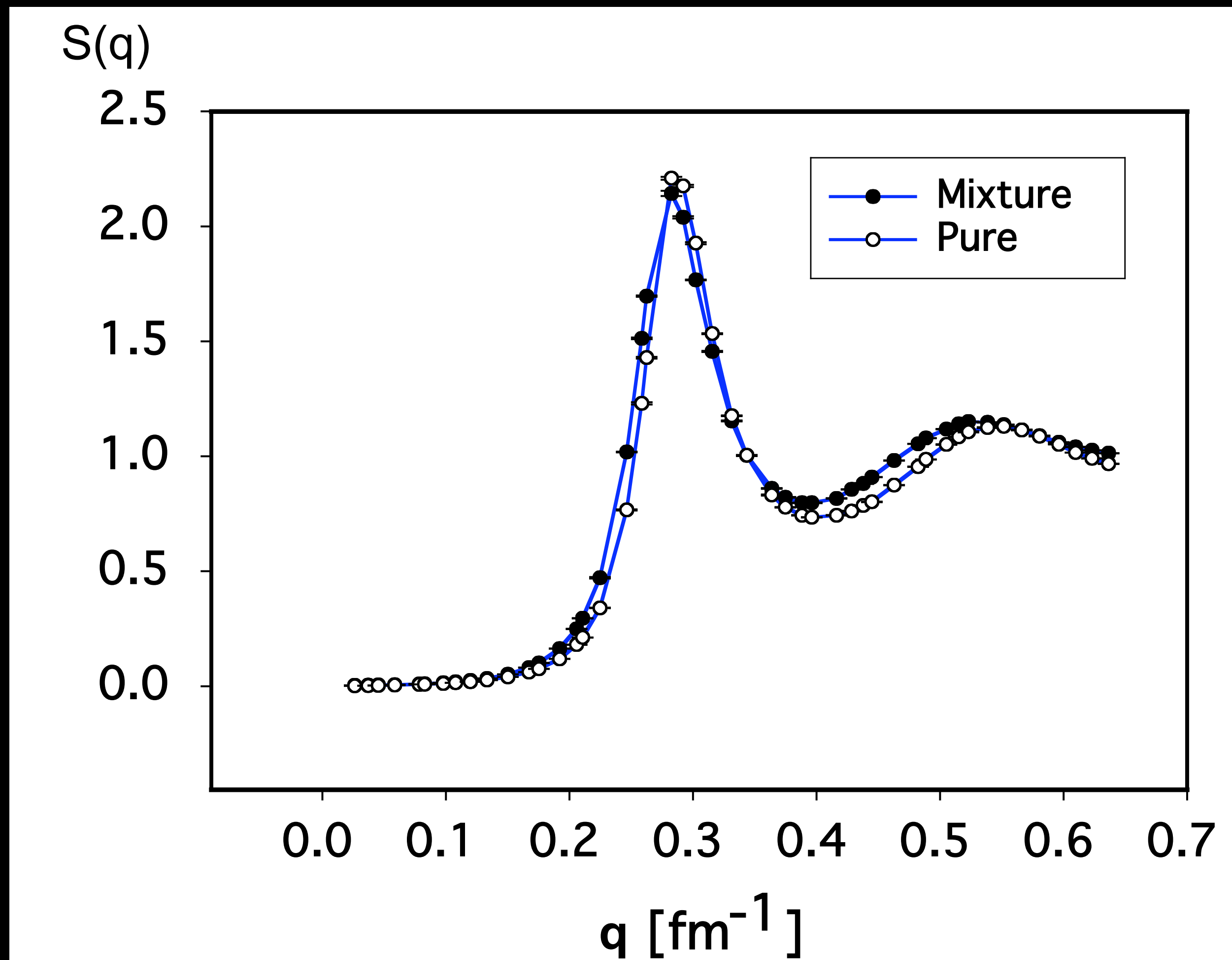


Structure Factor

T=1 MeV, Y_p=0.2

ρ=1.66x10^{13} g/cm^3,
<A>=105.2, <Z>=32.07

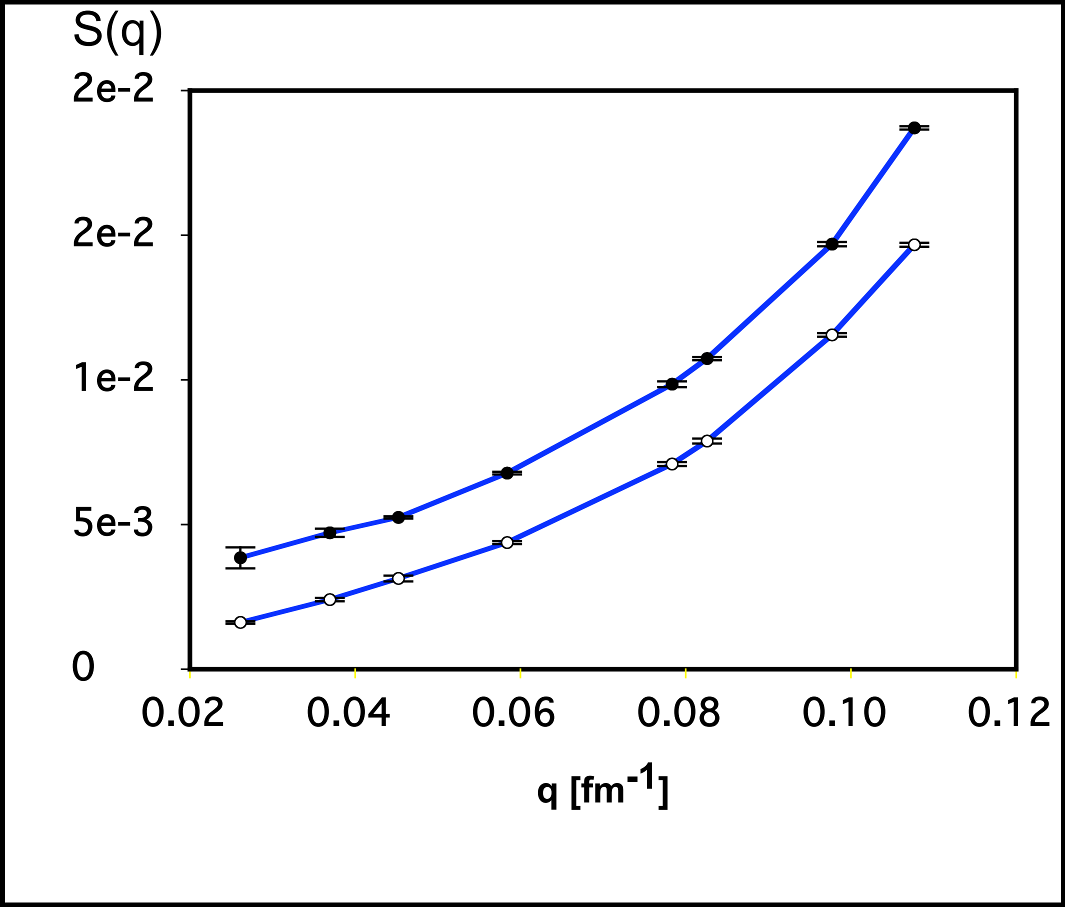
$$\frac{d\sigma}{d\Omega} = S(\vec{q})\frac{d\sigma}{d\Omega}\bigg|_{Free}$$



Structure Factor
Low q Behavior

T=1 MeV, Y_p=0.2

ρ=1.66x10^{13} g/cm^3,
<A>=105.2, <Z>=32.07



Thermal Conductivity

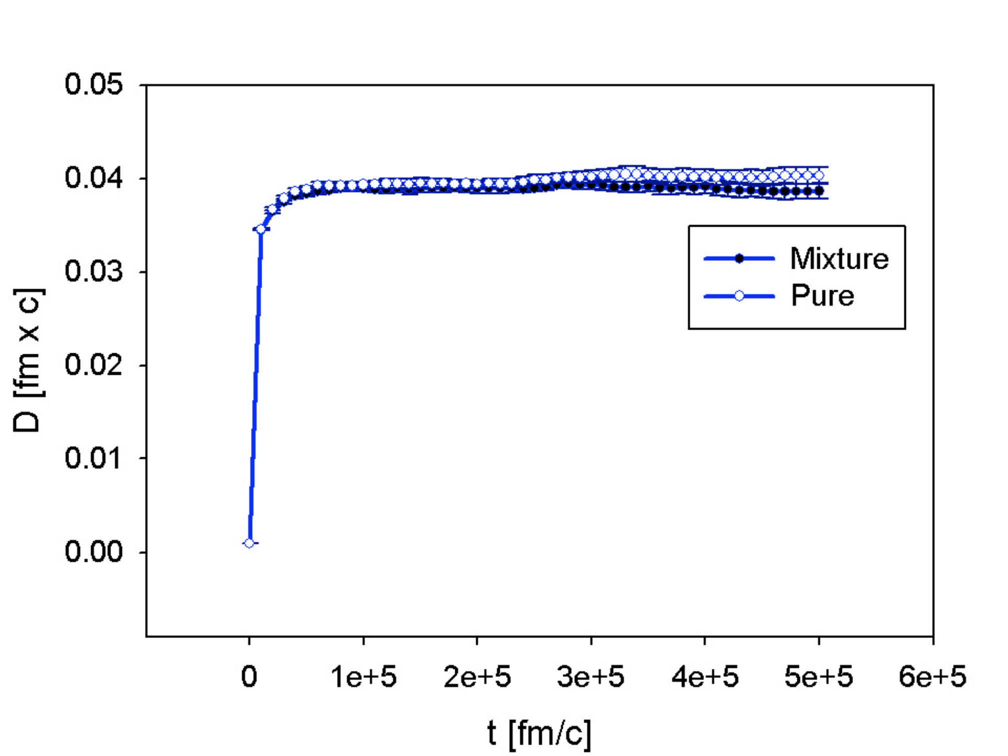
T=1 MeV Y_p=0.5, ε=1.0

ρ (g/cm^3)	Z (OCP)	Γ	$\langle S \rangle_{ItoH}$	$\langle S \rangle_{LC}$
10^{12}	28.3	40.85	0.9418	1.0136(9)
10^{13}	44.21	184.98	1.0641	1.2580(4)

Diffusion Coefficient

$$D = \frac{1}{3N_{ions}} \int_0^\infty \left\langle \sum_{j=1}^{N_{ions}} \vec{v}_j(t)\vec{v}_j(0) \right\rangle dt$$

Y_p= 0.5 T=1 MeV $\rho=10^{13}$g/cm^3	Dn (fm*c)	Dz (fm*c)
MCP	0.003254(6)	0.003256(5)
OCP	0.00313(3)	0.00313(3)
Y_p= 0.2 $\rho=1.66\times10^{13}$ g/cm^3		
MCP	0.00656(2)	0.00654(2)
OCP	0.00659(2)	0.00659(2)

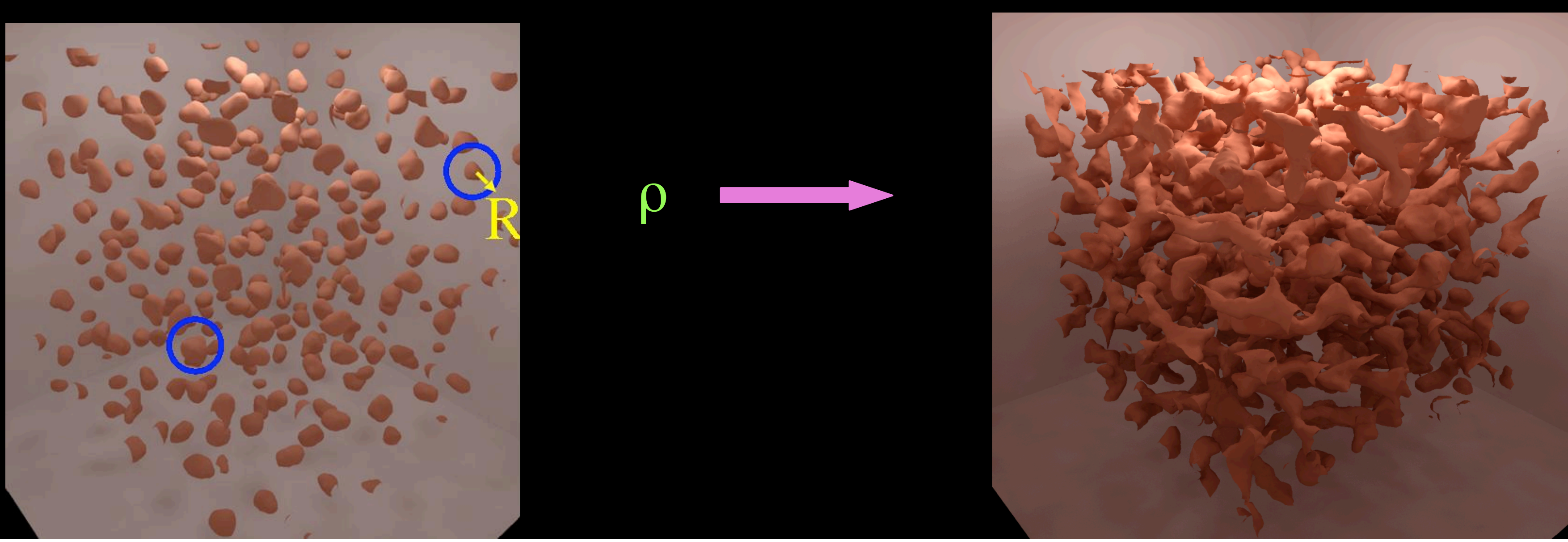


Y_p=0.5, $\rho=10^{12}$ g/cm^3, T=1 MeV

Shear Viscosity

$$\eta = \frac{V}{3k_B T}\int_0^\infty \left\langle \sum_{x<y} P_{xy}(t) P_{xy}(0) \right\rangle dt$$

$$P_{xy} = \frac{1}{V}\left[\sum_j m_j v_{xj} v_{yj} + \frac{1}{2}\sum_{i\neq j} r_{xij} F_{yij}\right]$$



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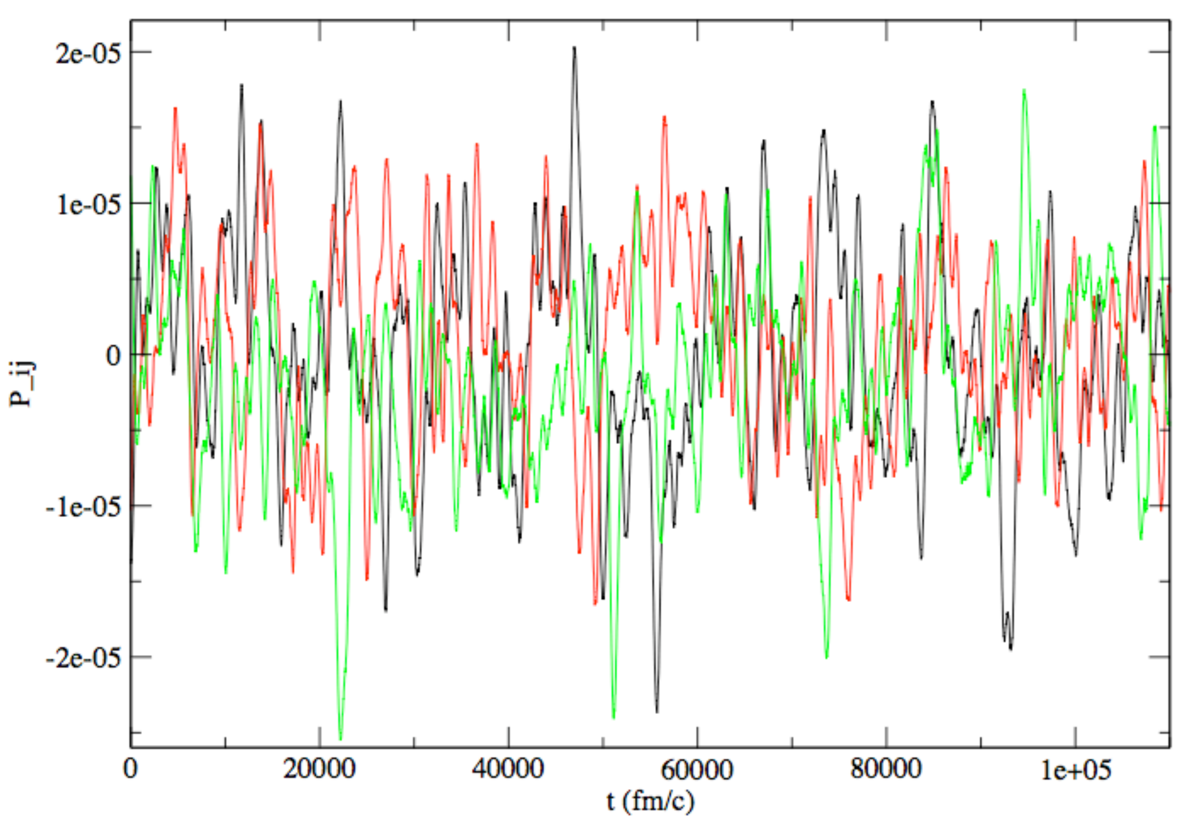
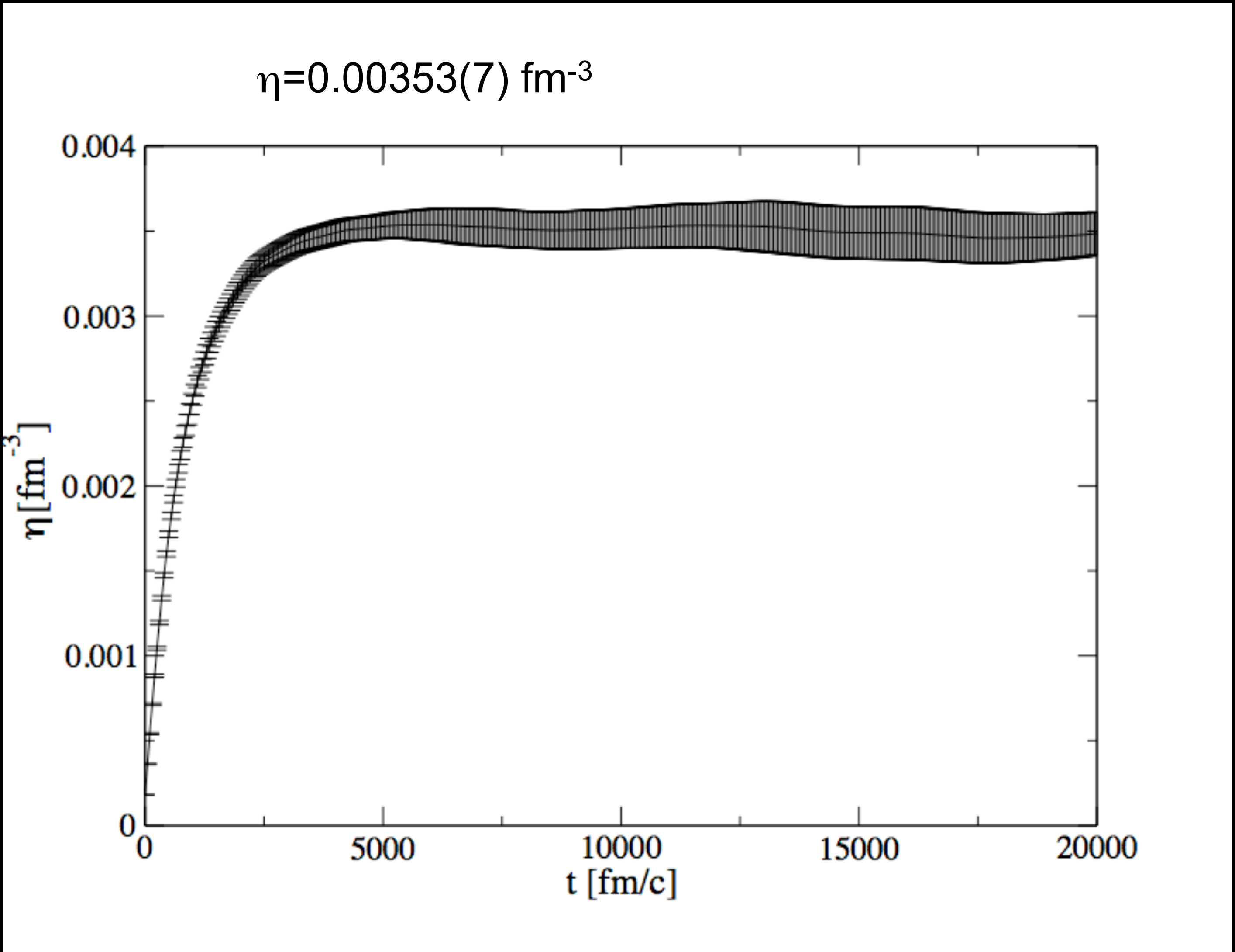


Shear Viscosity

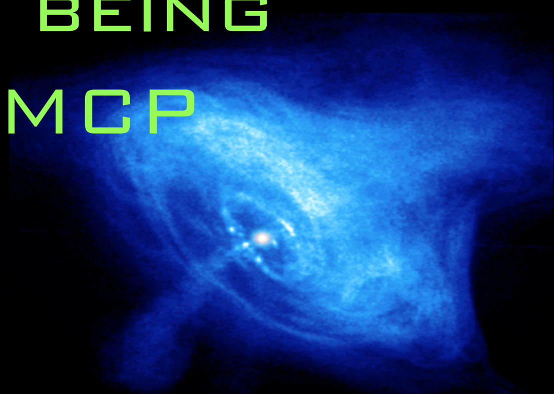
ρ_I=7.18x10^{-5} fm^{-3}, <A>= 88.03 <Z>=29.417 T=1 MeV



Conclusions

- Neutrino → ion+ screening cloud
- At low momentum, fluctuations in the weak charge density, for the mixed medium, may not be screened

  S(q) mixed case > s(q) pure case
- transport properties are being calculated for OCP and MCP



X-Ray image of Crab Nebula Pulsar Credit: NASA/CXC/ASU/J. Hester et al.

THANKS!