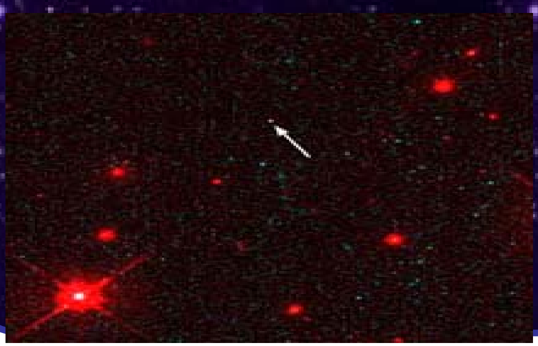
THE NEUTRON STAR CRUST AND SURFACE WORKSHOP

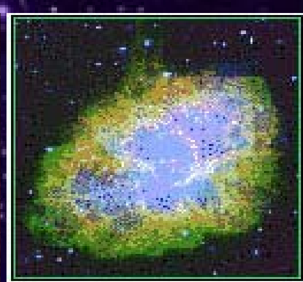
Seattle 25-29 June 2007

Quantum calculation of nucleus-vortex interaction in the inner crust of neutron stars

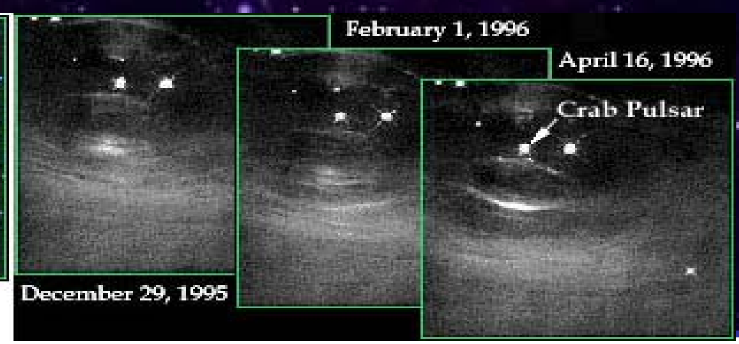
P. Avogadro, F.Barranco, R.A.Broglia, E.Vigezzi

Phys. Rev. C 75 012085 (2007)

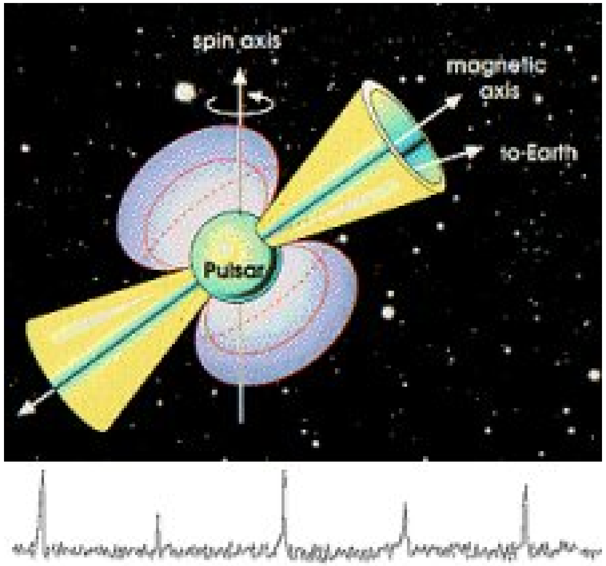




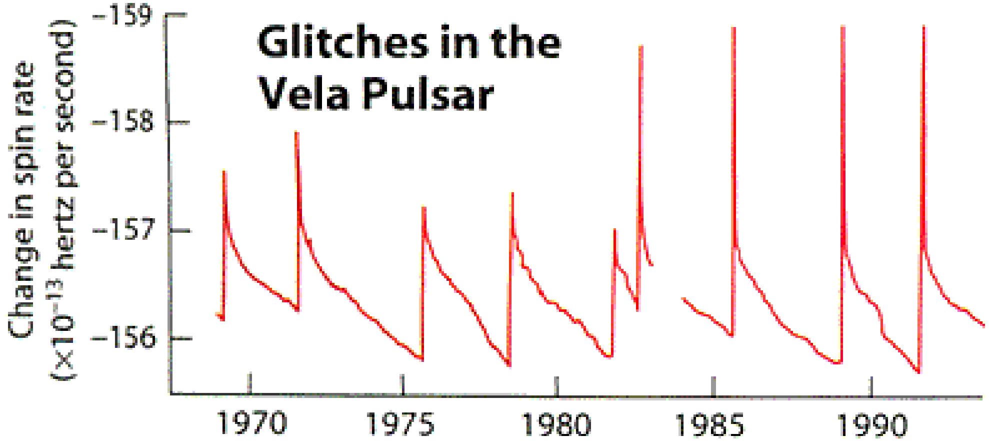
Crab Nebula



Glitches



As a rule, rotational period of a neutron star slowly increases because the system loses energy emitting electromagnetic radiation.



Sudden spin ups are measured

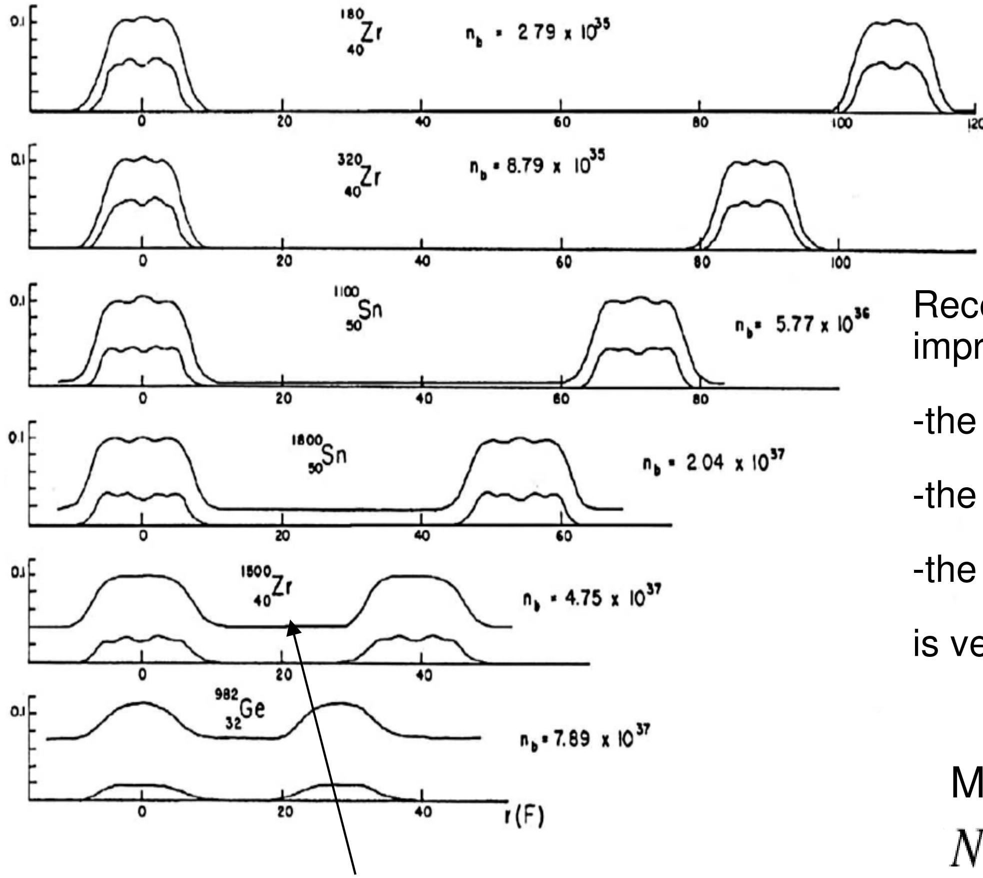
One of the proposed explanations → Superfluid nature of nucleons in the inner crust

P.W. Anderson and N.Itoh, Nature 256(1975)25

The inner crust of a neutron star

Lattice of heavy nuclei surrounded by

a sea of superfluid neutrons.



neutron gas density

Recently the work by Negele & Vautherin has been improved including the effects of pairing coorelations:

-the size of the cell and number of protons changes.

-the overall picture is mantained.

-the energy differences between different configurations

is very small.

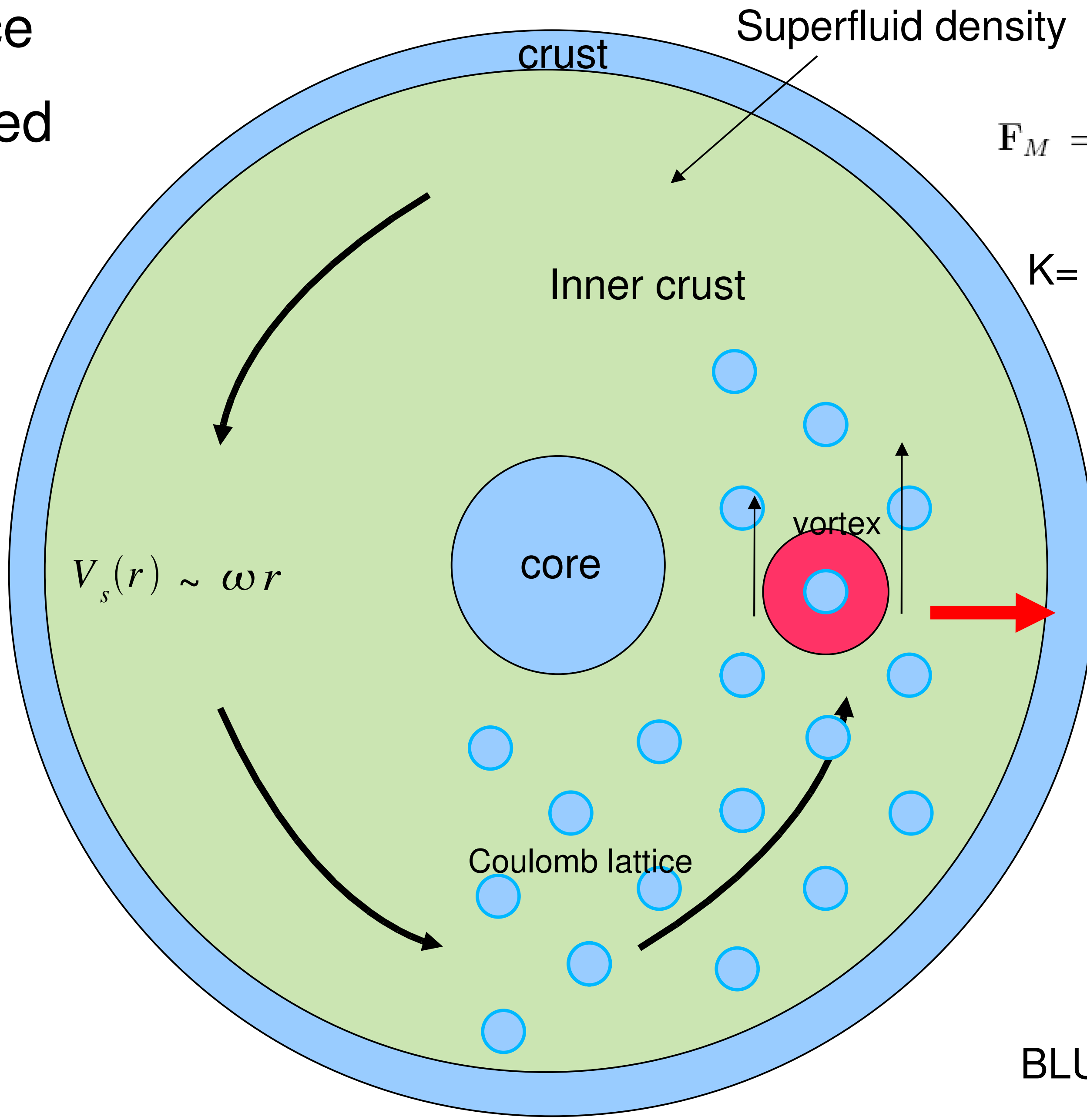
M. Baldo, E.E. Saperstein, S.V. Tolokonnikov
Nuclear Physics A 749 (2005) 42c–52c

The thickness of the inner crust is about 1km.

The density range is from $4 \text{x} 10^{11} g/cm^3$ to $1.6 \text{x} 10^{14} g/cm^3$

In the deeper layers of the inner crust nuclei start to deform.

Reference frame fixed with the Coulomb lattice



ρ_s

$$\mathbf{F}_M = \rho_s \mathbf{K} \times (\mathbf{v}_V - \mathbf{v}_s),$$

K= vortex circulation

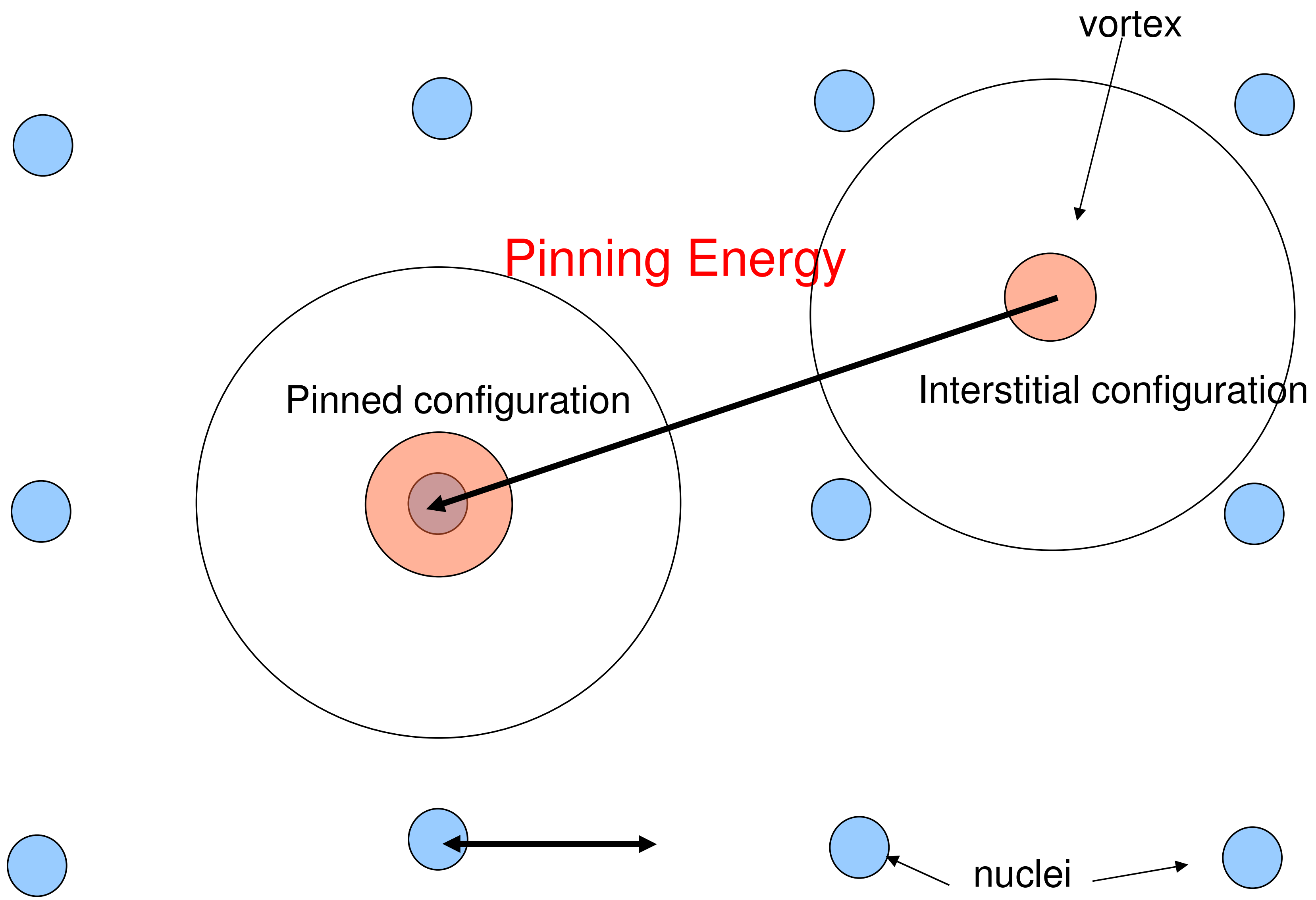
The Magnus force pushes the vortex outwards!

BLUE MOVES AS A RIGID BODY!!

The crust has been exaggerated!!!

We assume that we can neglect the interaction with neighboring nuclei.

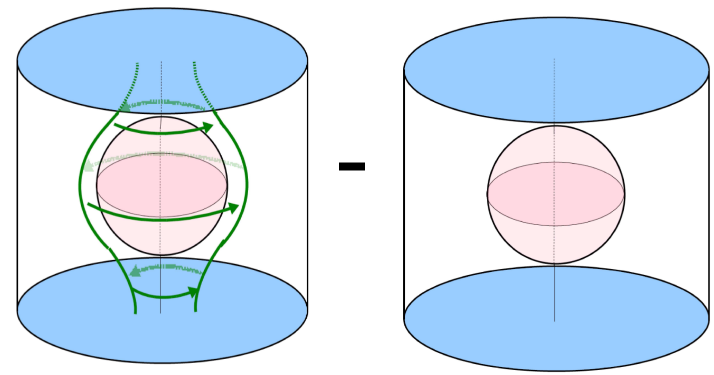
(Wigner-Seitz radius larger than the vortex core).



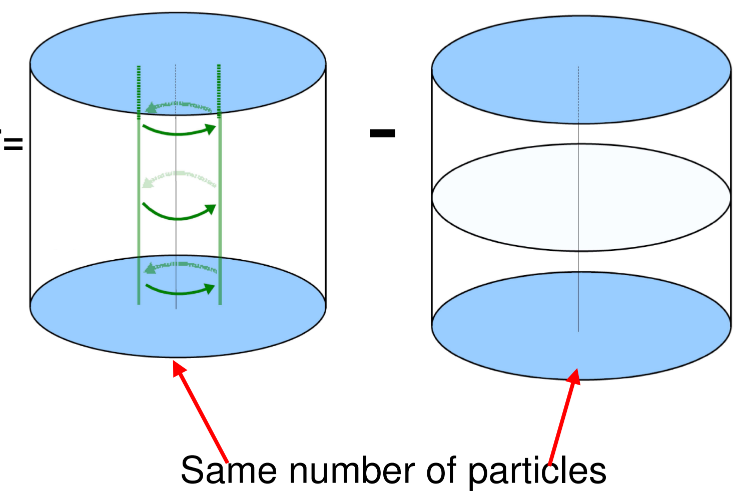
PINNING ENERGY

Pinning Energy= Energy cost to build a vortex on a nucleus - Energy cost to build a vortex in uniform matter.

Energy cost to build a vortex on a nucleus=



Energy cost to build a vortex in uniform matter=



Contributions to the total energy:

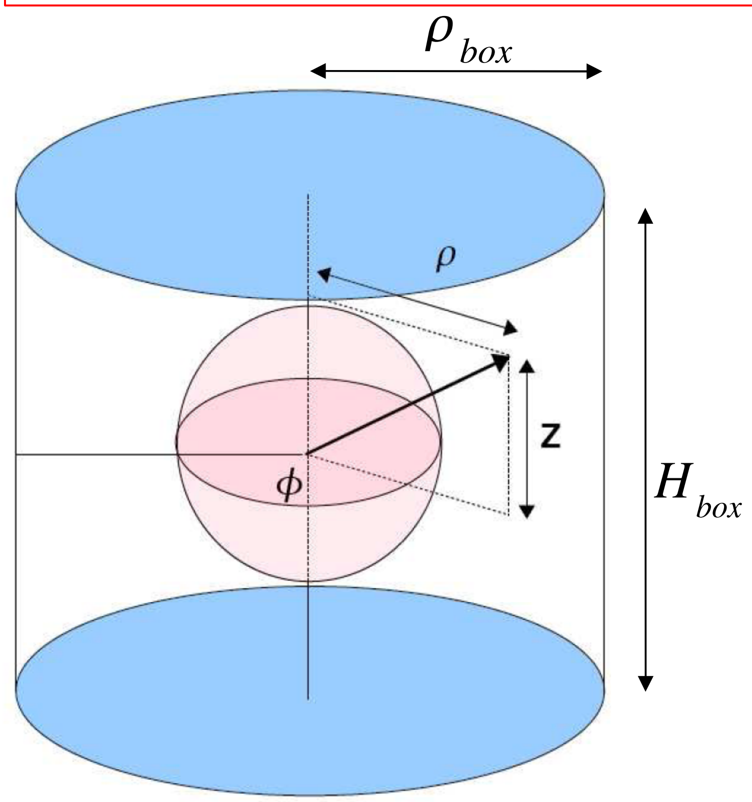
Kinetic, Potential, Pairing

<u>All the cells must have the same asymptotic neutron density.</u>

pinning energy<0 : vortex attracted

pinning energy >0 :vortex repelled by nucleus

We solve the HFB (De Gennes) equations expanding on a single-particle basis in cylindrical coordinates



- The equations are solved self-consistently
- we used SII, SLy4, Skm* and SGII Skyrme interactions for the single particle levels
- we constrained protons to keep spherical symmetry
- we neglected spin-orbit interaction

$$\begin{pmatrix} \varepsilon_i - \lambda & \Delta \\ \Delta & -(\varepsilon_i - \lambda) \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix} = E_i \begin{pmatrix} U_i \\ V_i \end{pmatrix}$$

The solution of the HFB equations expanded on the single particle basis read:

$$u_{qm}(\rho, z, \phi) = \Sigma_{nk} U_{nk}^{qm} J_{nm}(\rho) \sin(kz) e^{im\phi}$$

$$v_{qm}(\rho, z, \phi) = \Sigma_{nk} V_{nk}^{qm} J_{n(m-\nu)}(\rho) \sin(kz) e^{i(m-\nu)\phi}$$

Using a zero range pairing interaction only local quantities are needed:

When ν is different from 0 there is a superfluid velocity field:

Velocity field

$$Vel = -\frac{i\hbar}{m\rho n(\rho,z)} \sum_{qm} v_{qm}^{*}(\rho,z,\phi) \frac{\partial v_{qm}(\rho,z,\phi)}{\partial \phi}$$

Pairing Gap

$$\Delta(\rho,z,\phi) = 481 \left(1 - 0.7 \left(\frac{n(\rho,z)}{0.08}\right)^{0.45}\right) \tilde{n}(\rho,z,\phi)$$

Abnormal density

$$\tilde{n}(\rho,z,\phi) = \sum_{qm} u^{qm}(\rho,z,\phi)(v^{qm}(\rho,z,\phi))^{*}$$

Neutron density

$$n(\rho,z) = 2\sum_{qm} |v^{qm}(\rho,z,\phi)|^{2}$$

m is the angular momentum

q is the quasi-particle index,

Microscopic quantum calculation of the vortex-nucleus system

The ansatz for the study of a vortex is $\Delta(\rho, z, \phi) = \Delta(\rho, z) e^{i\nu\phi}$

where $\Delta(\rho, z)$ is a real function and $\nu = 0, 1, 2, ..$ is the number of quanta of angular momentum carried by each Cooper pair.

Because of the symmetry of the problem $\Delta(\rho, z) = \Delta(\rho, -z)$. The pairing gap is an eigenstate of the parity operator with eigenvalue $(-1)^{\nu}$

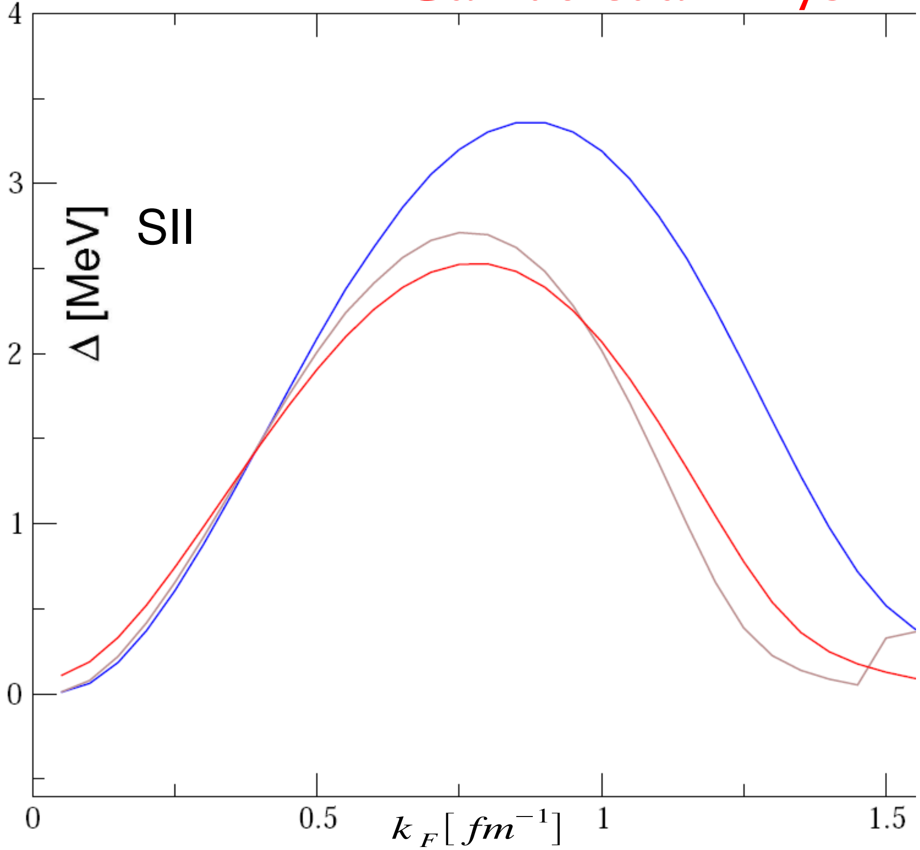
We have considered vortices associated with $\nu = 1$. In this case the Cooper pairs carry one unit of angular momentum and must be formed of single particle levels of opposite parity.

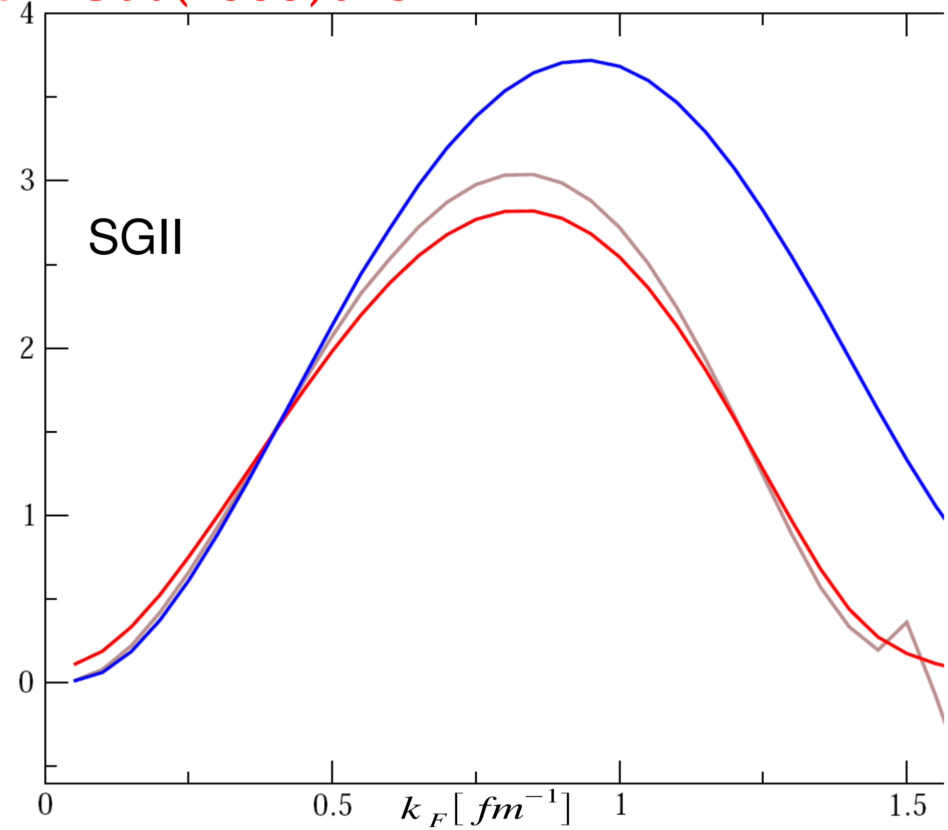
PAIRING INTERACTION

Pairing gap at the Fermi energy in uniform neutron matter obtained with single particle levels calculated with Skyrme type forces (left SII, right SGII) and the density dependent pairing interaction (red) (with a cutoff 60 MeV), and Gogny(blue) and Argonne (brown) interactions in the pairing channel.

$$V = -481\left(1 - 0.7\left(n(x)/n_0\right)^{0.45}\right)\delta(r_1 - r_2) \quad MeV \quad fm^3$$

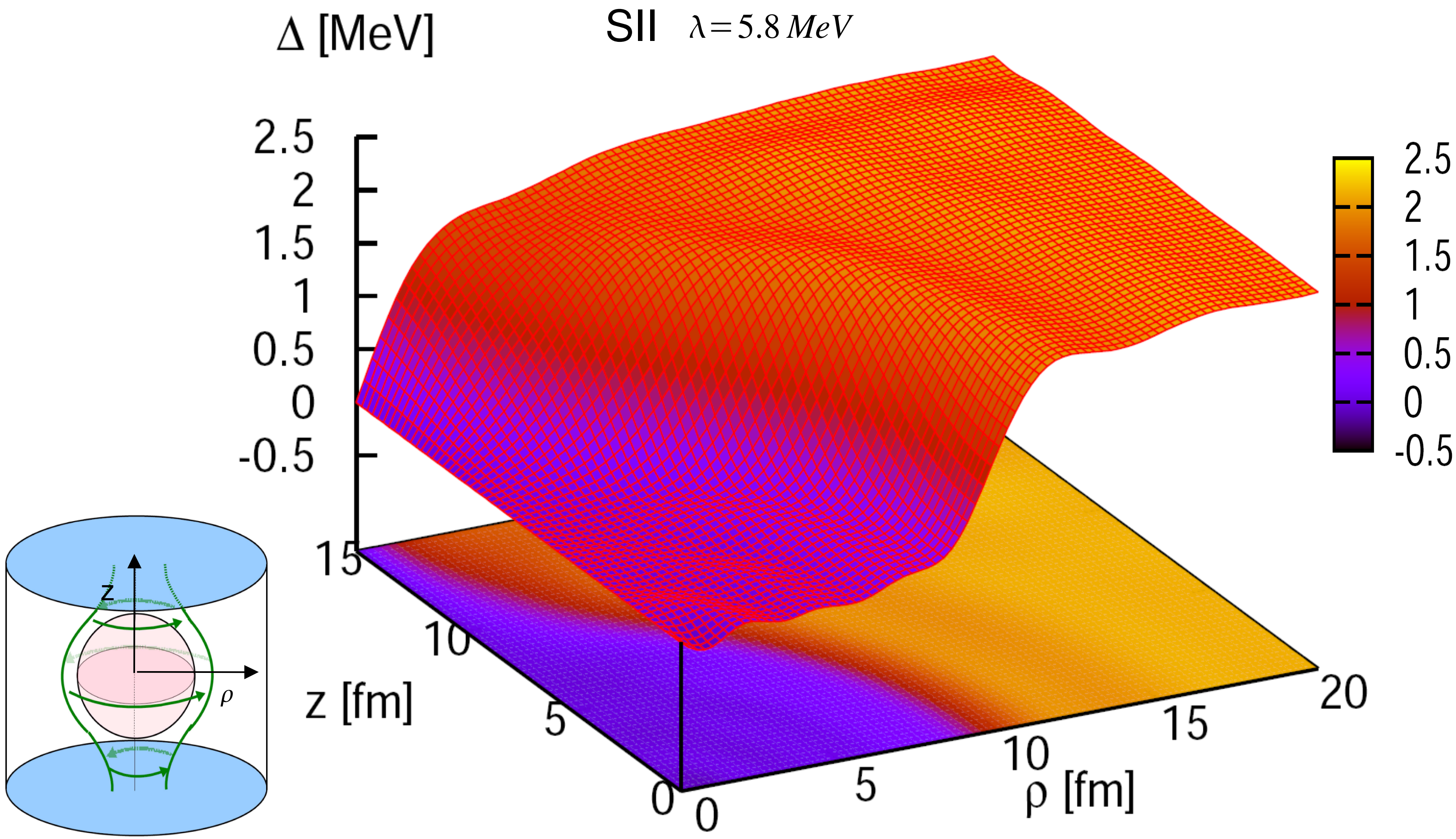
E. Garrido et al. Phys. Rev. C60(1999)64312





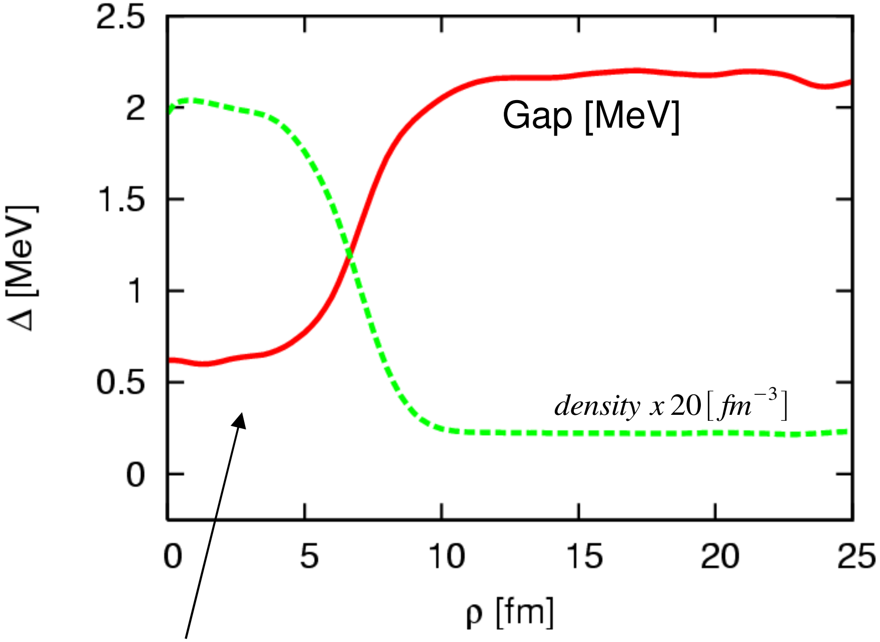
The pairing gap is suppressed in the nuclear region and also in the surface of the nucleus; in the same region also the superfluid flow vanishes.

Vortex pinned on a nucleus

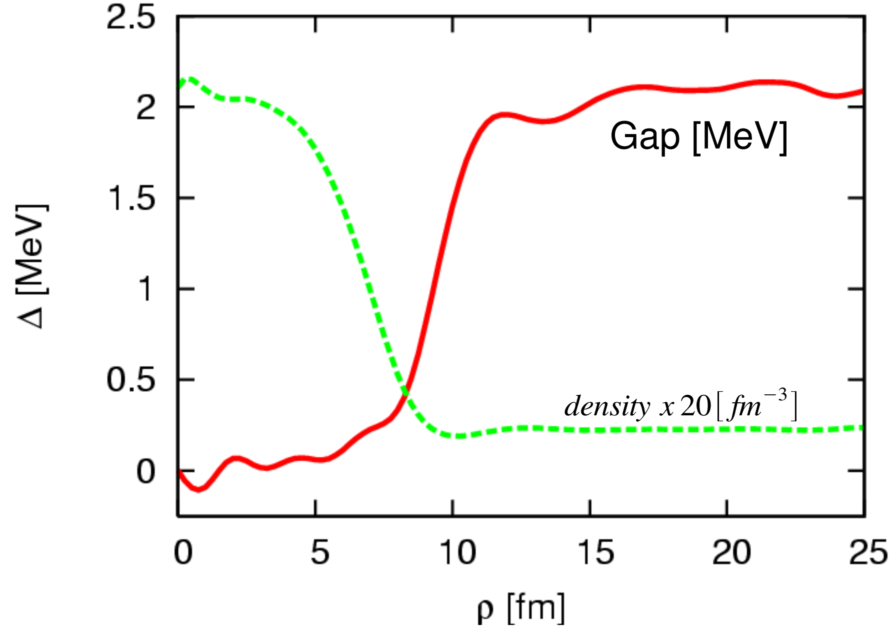


Sly4 5.8MeV

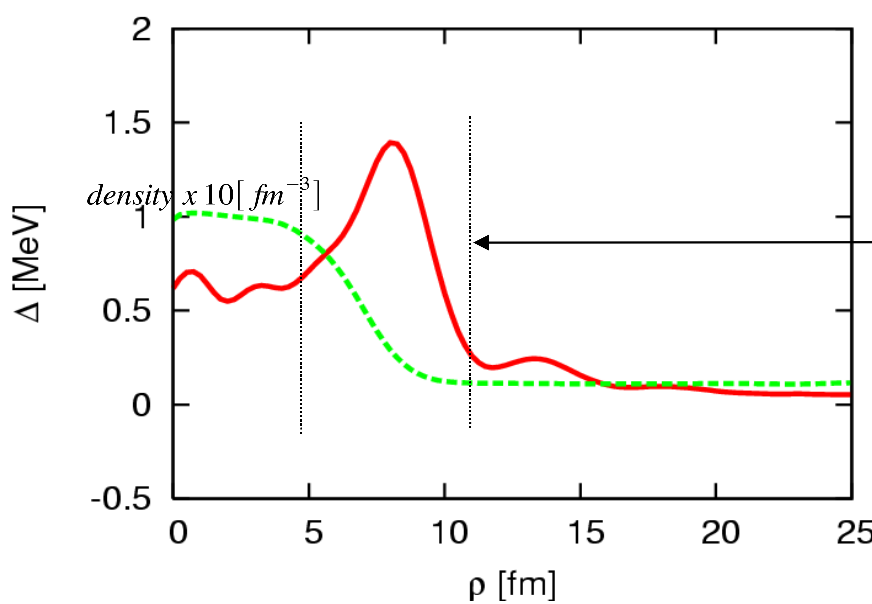
nucleus



vortex pinned on a nucleus

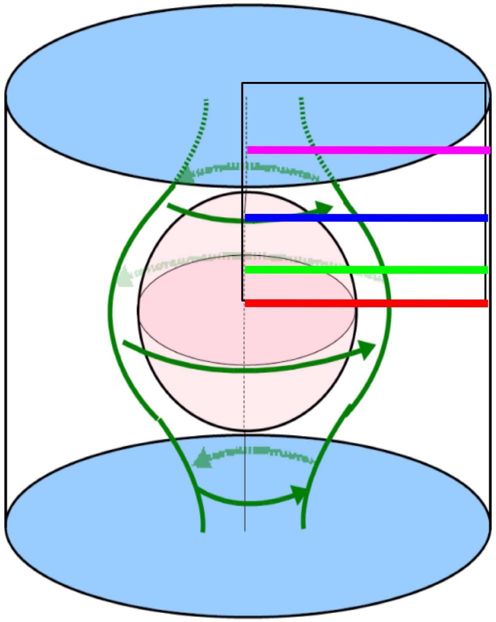


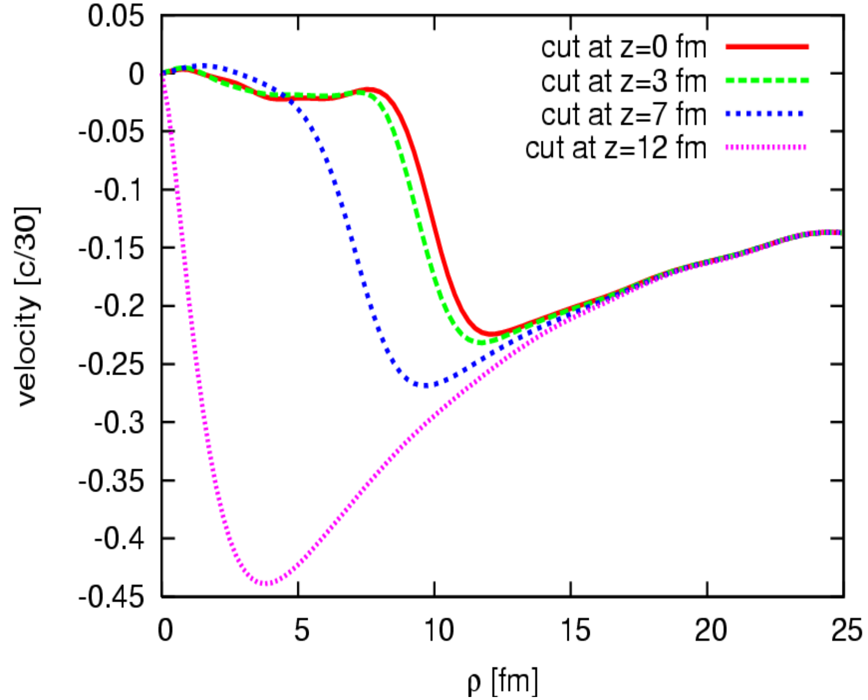
In this region the gap is not completely suppressed



the gap of a nucleus minus the gap of a vortex on a nucleus: on the surface there is the highest reduction of the gap.

The velocity field is suppressed in the nuclear region

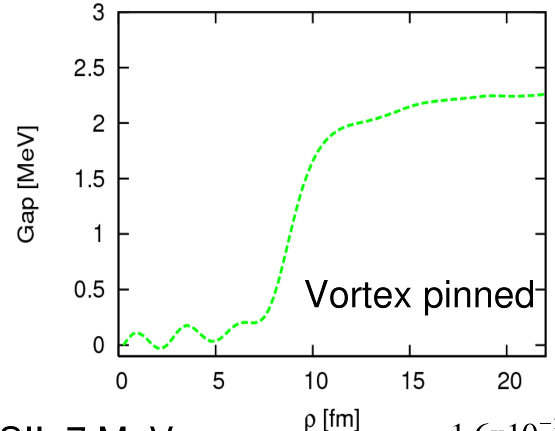




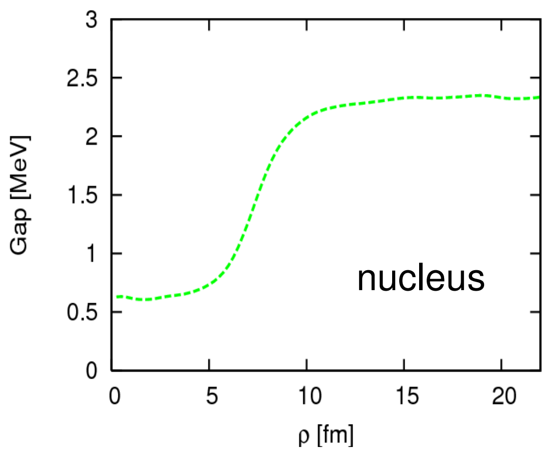
$$Vel = -\frac{i\hbar}{m\rho n(\rho, z)} \sum_{qm} v^*_{qm}(\rho, z, \phi) \frac{\partial v_{qm}(\rho, z, \phi)}{\partial \phi}$$

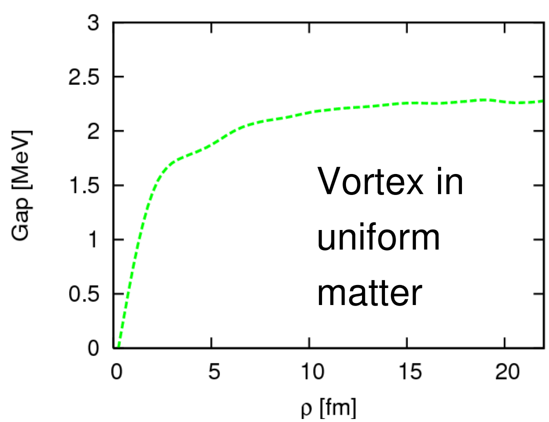
The superfluid flow is destroyed in the nuclear volume.

microscopic calculations

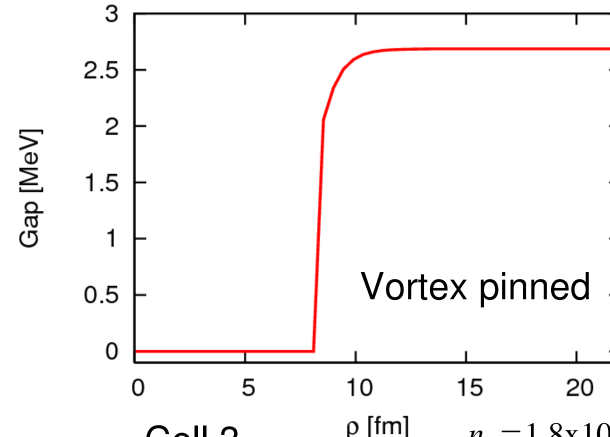


SII 7 MeV $n_g = 1.6\text{x}10^{-2}\ fm^{-3}$

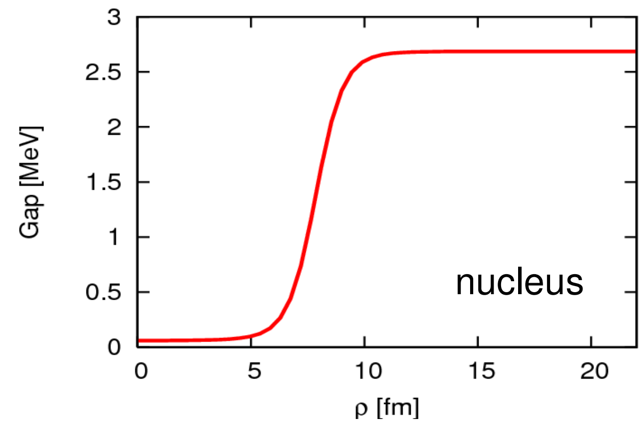


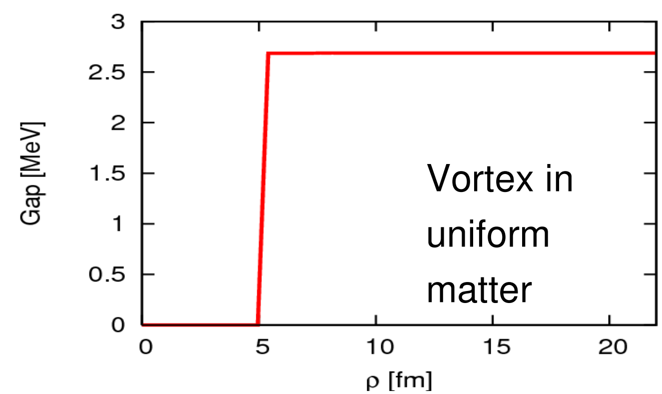


Semiclassical calculations



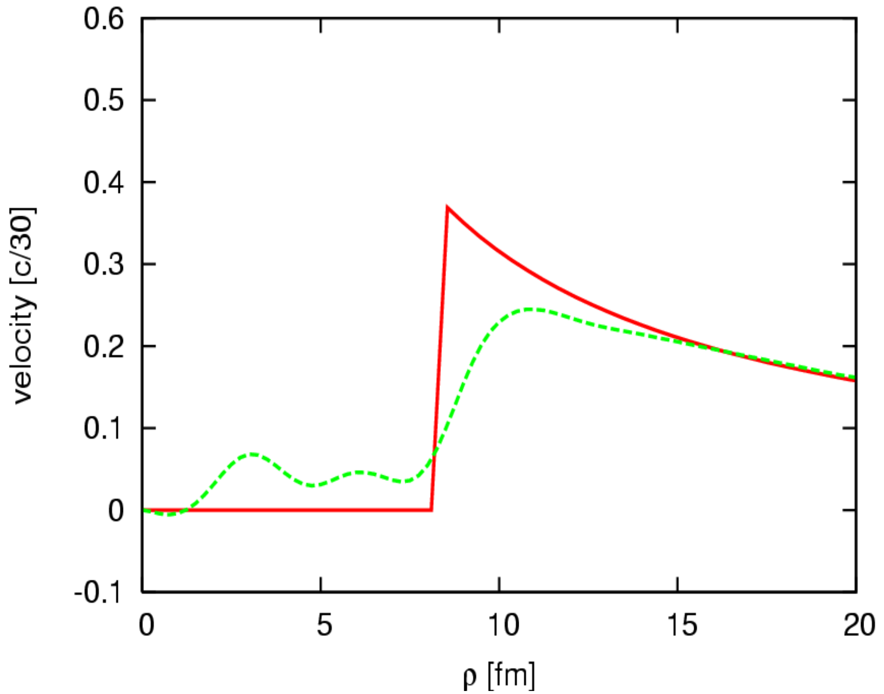
Cell 3 $n_g = 1.8\text{x}10^{-2}\ fm^{-3}$



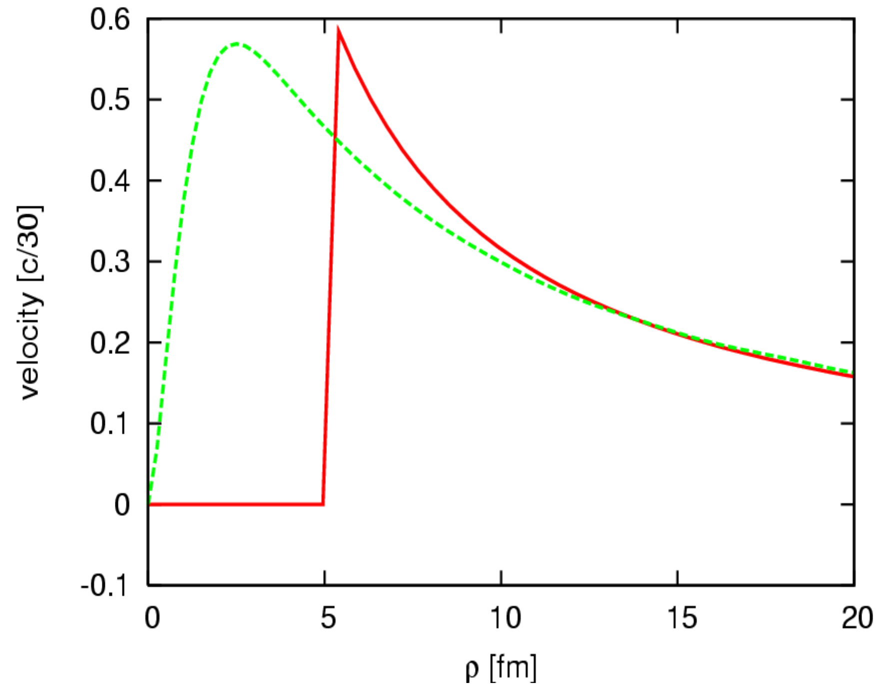


Velocity field: Green microscopic calculations SII 7 MeV

Red: semiclassical calculations cell 3

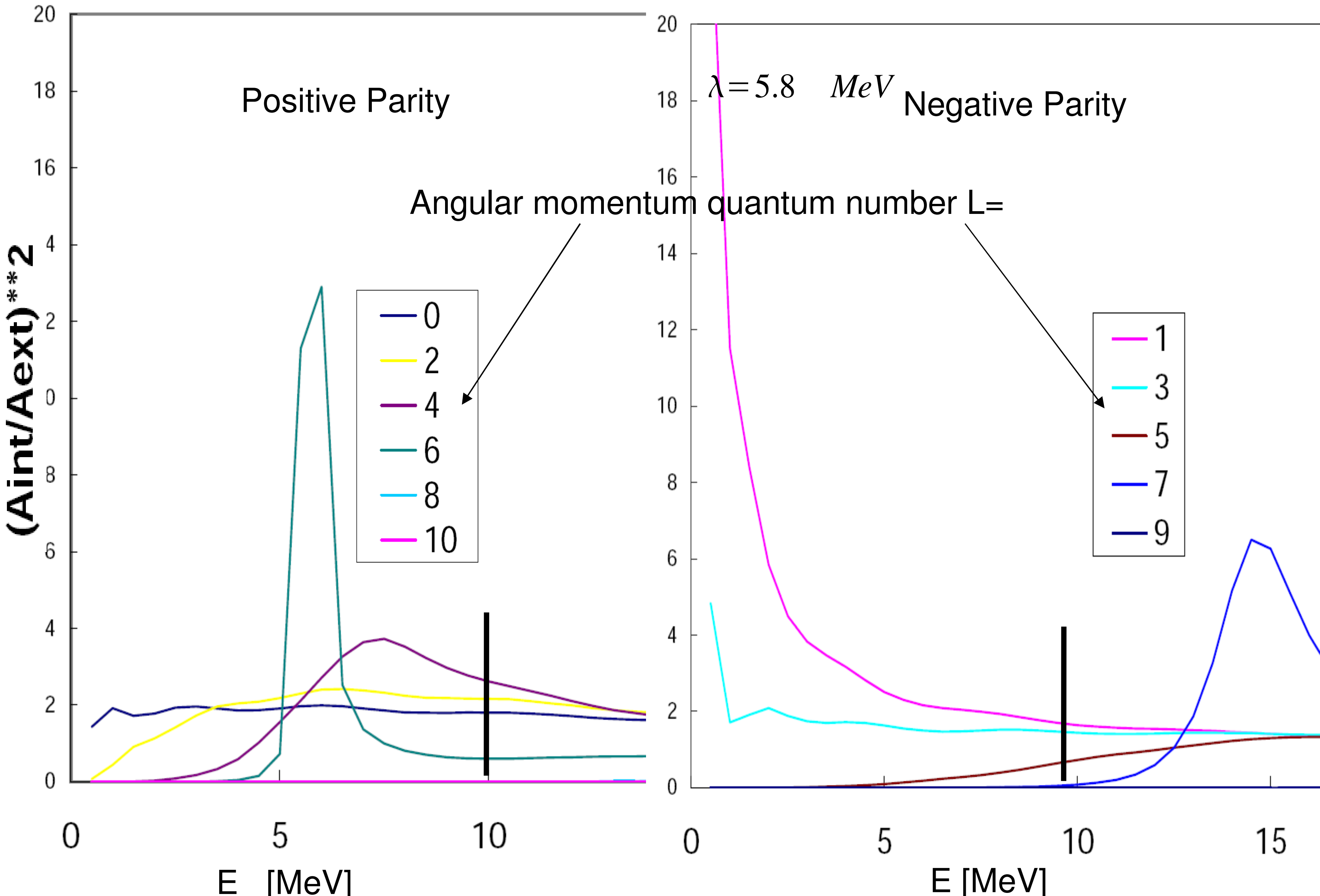


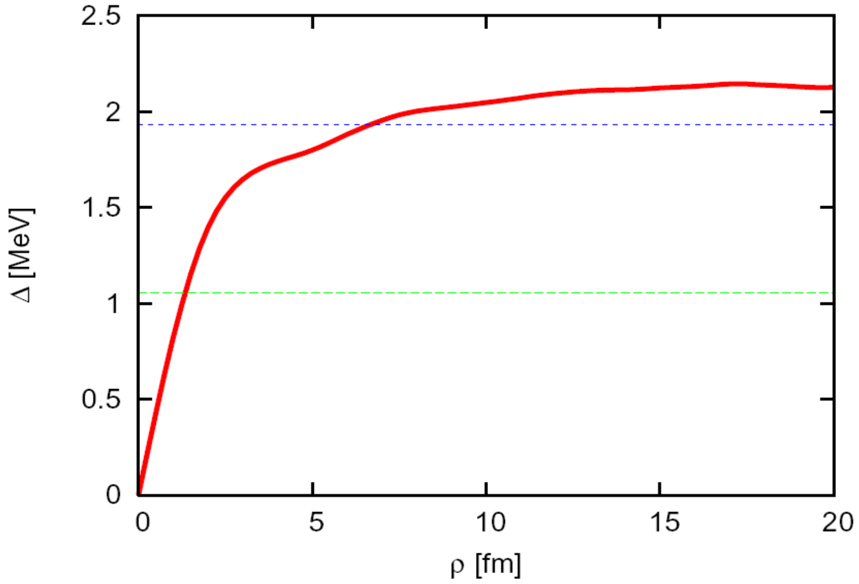
Vortex pinned on a nucleus



Vortex in uniform matter

Single Particle Resonances in the nuclear potential obtained with the SII interaction.

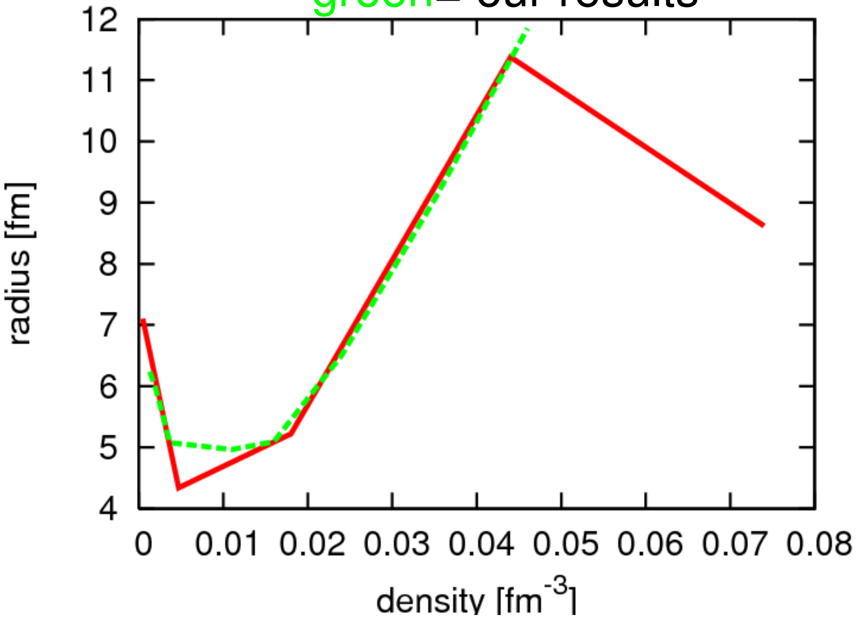




We use as a vortex radius the distance where the pairing gap recovers approx 85% of the asymptotic value.

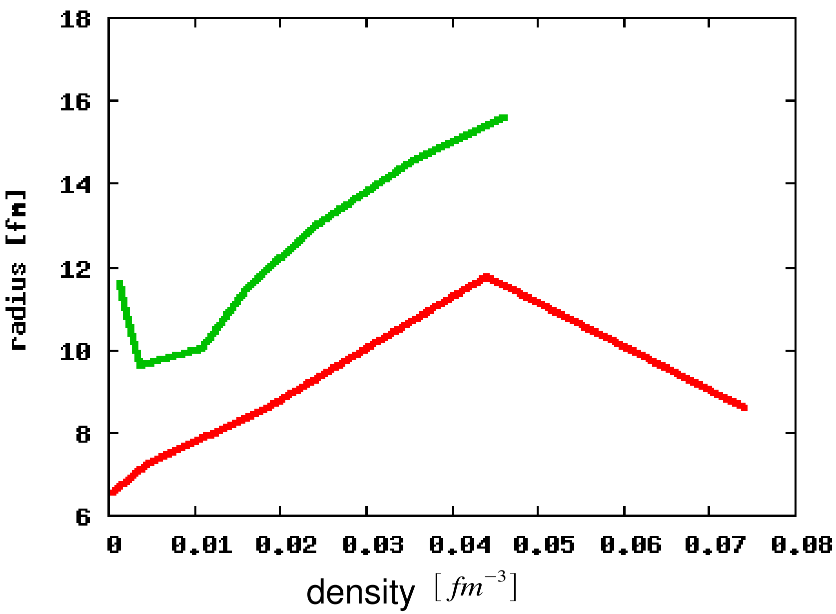
Vortex in uniform matter

green= our results



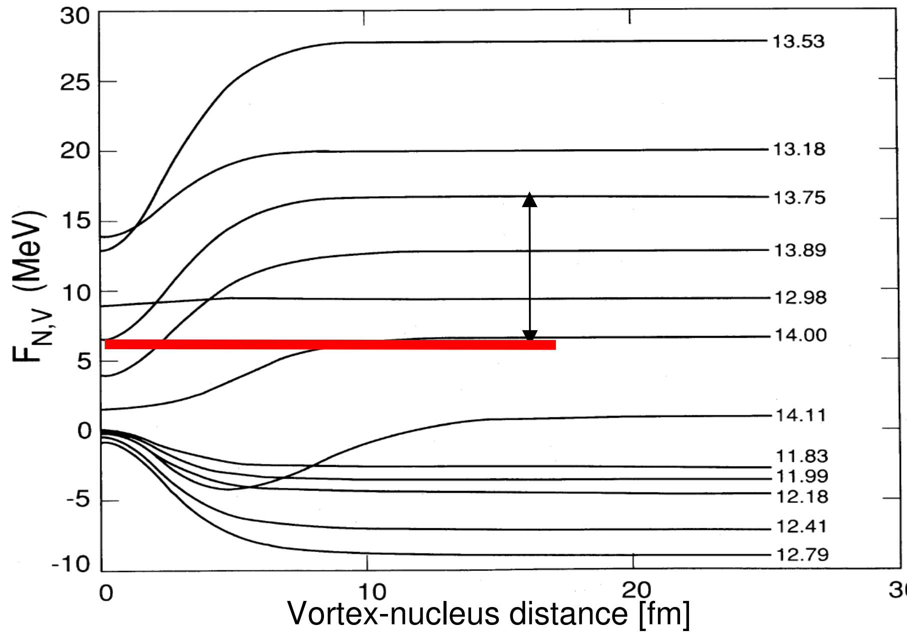
Vortex on a nucleus

red=NPA 742(2004)363

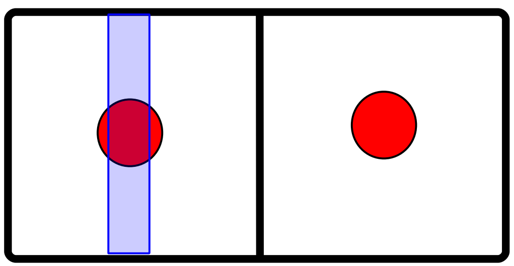


The Pinning Energy was first defined by Epstein & Baym as the difference between the minimum of the free energy of the vortex-nucleus system and the free energy when the nucleus and vortex are distant.

The Astrophysical Journal, 328; 680-690, 1988



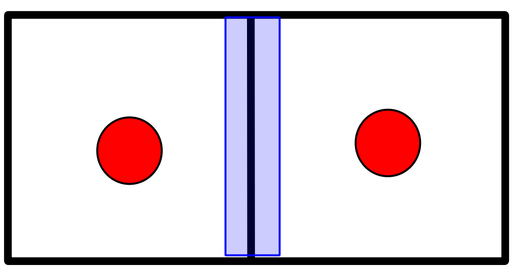
Approach by Donati & Pizzochero:



Pinned configuration

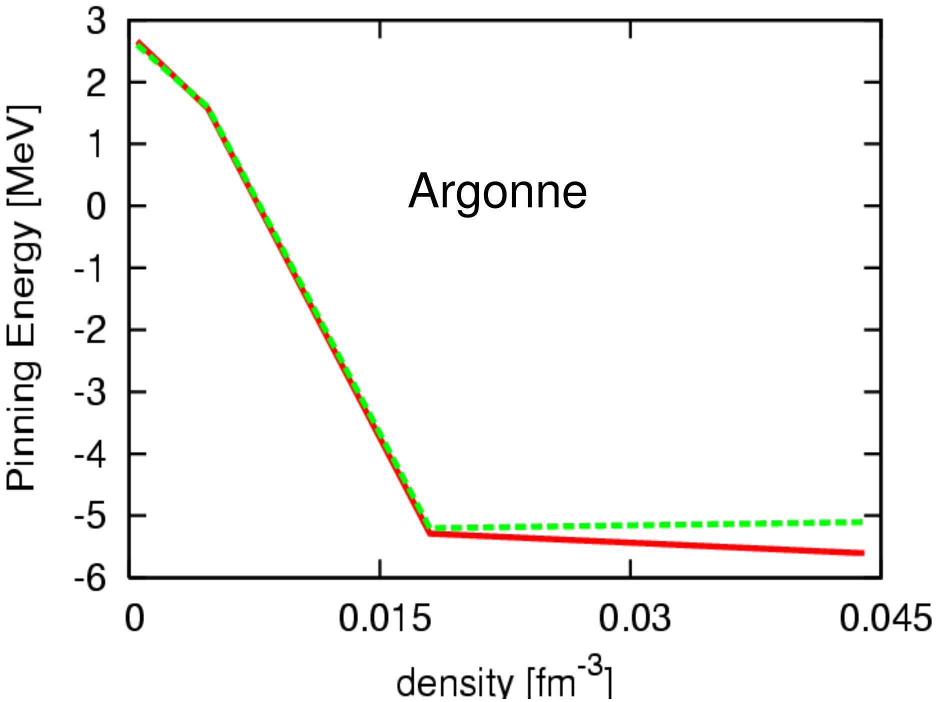
Nuclear Physics A 742 (2004) 363–379

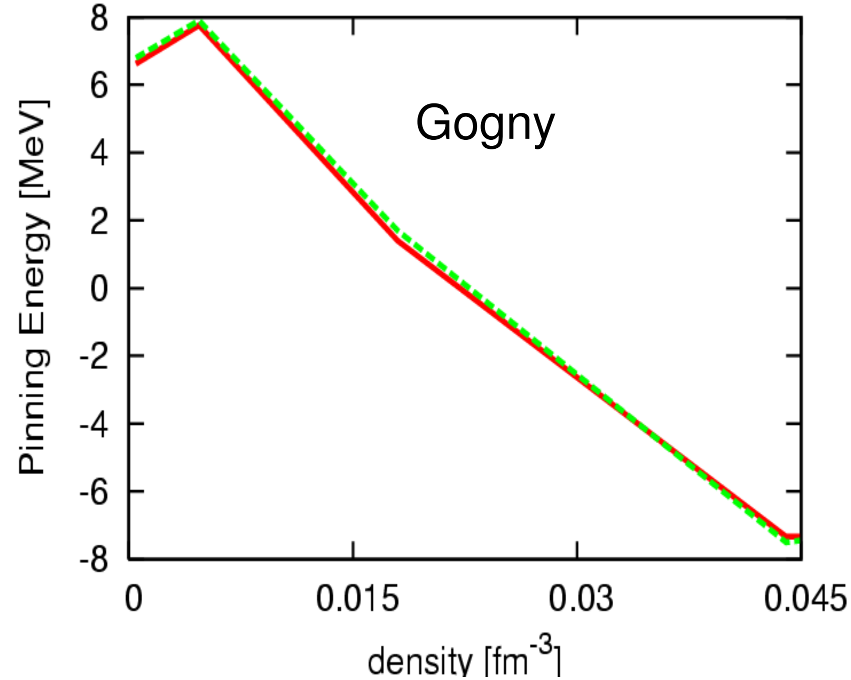
Physics Letters B 640 (2006) 74–81



Interstitial configuration

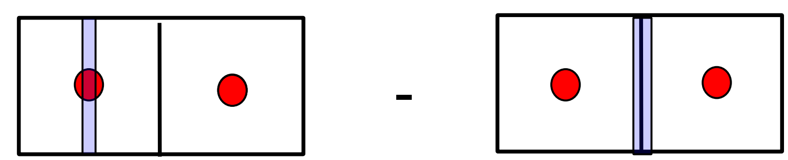
In this approach the Pinning Energy is calculated as the difference between the energy of Pinned and Interstitial configurations.





Semiclassical calculations of

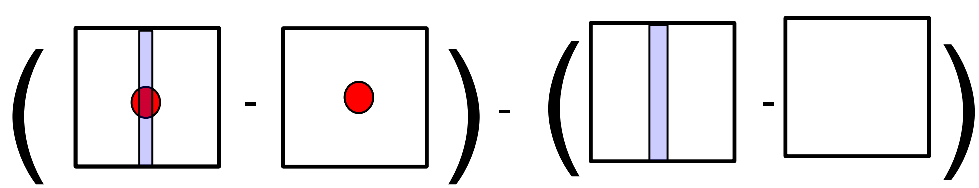
Nuc.Phys A 742 363-379 } Pinning Energy =



The same semiclassical calculations as in

Nuc.Phys A 742 363-379

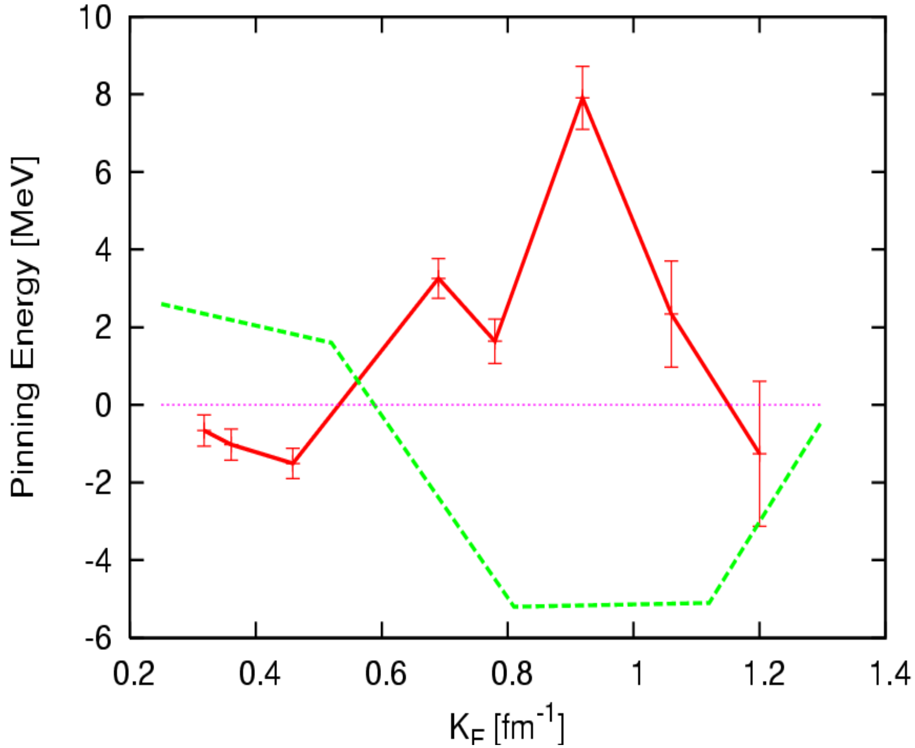
but with our geometry } Pinning Energy =



cost of a vortex on a nucleus

cost of a vortex in uniform matter

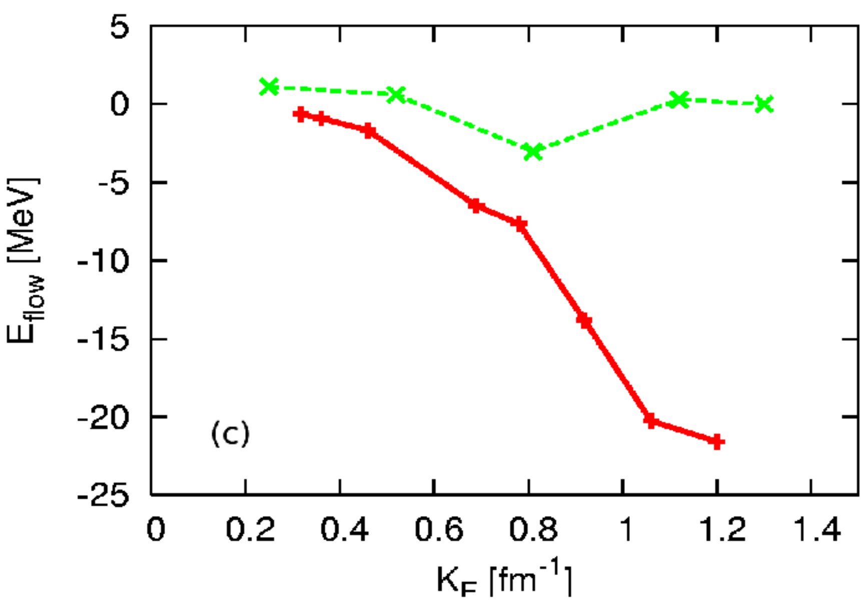
Pinning Energy

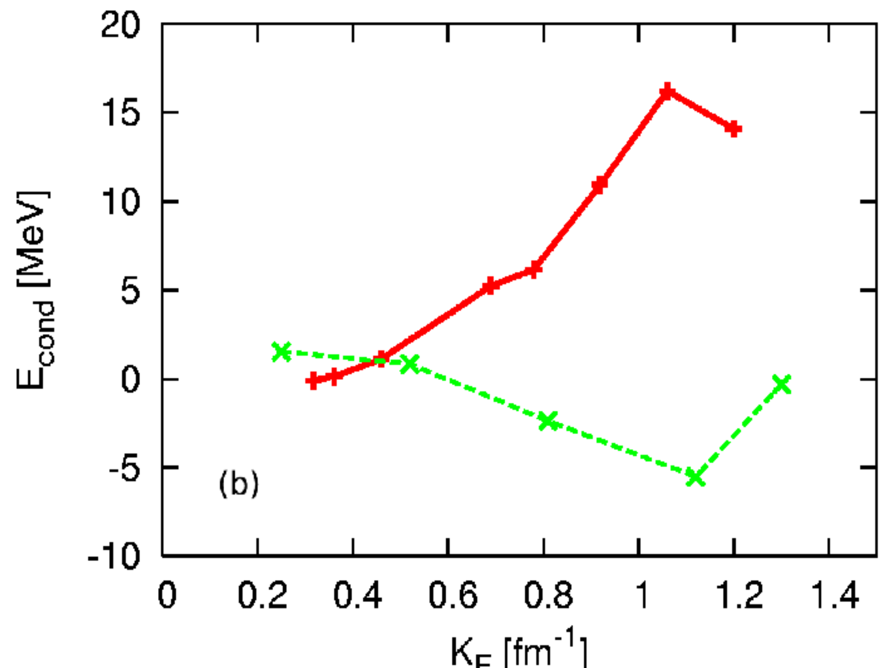


Our results with a SII : RED

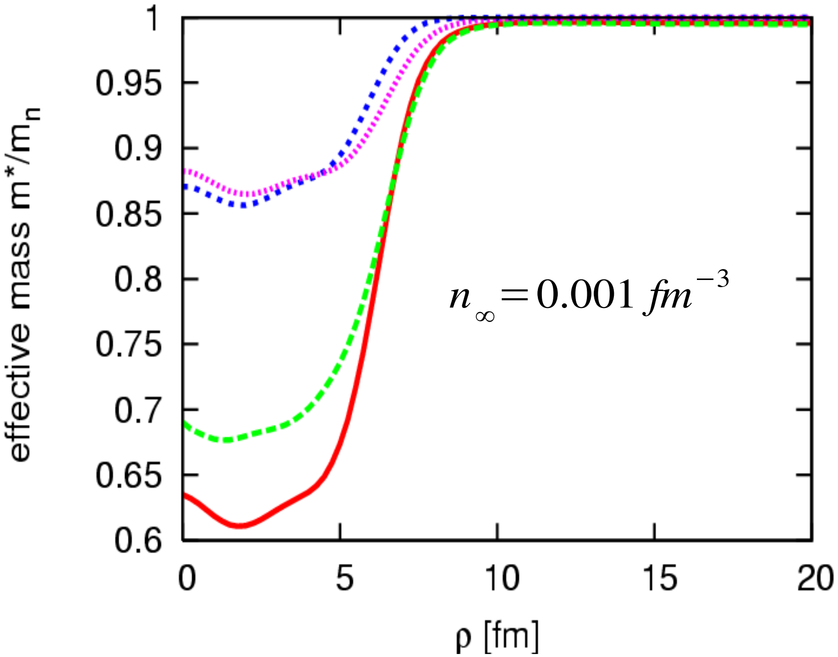
Pizzochero & Donati: GREEN

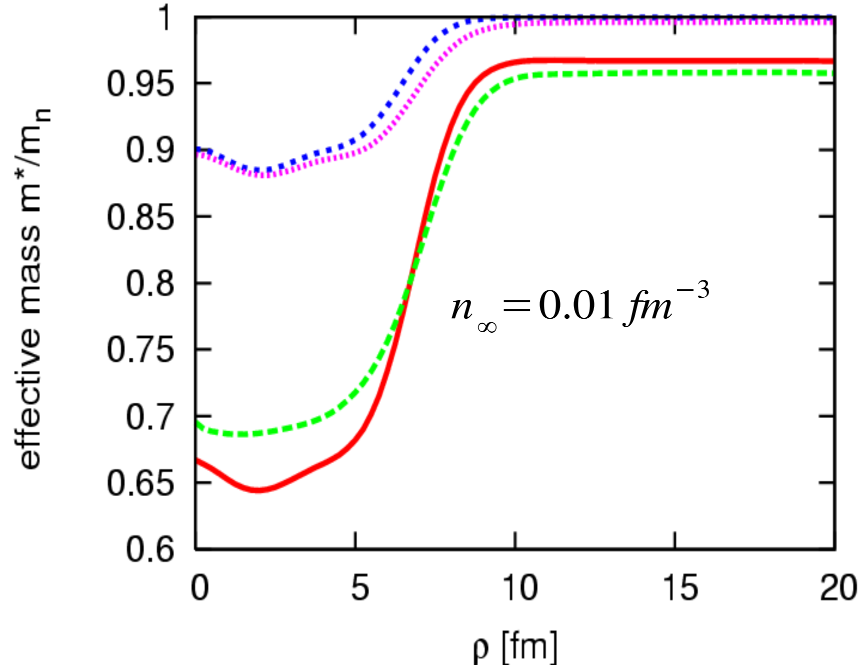
P.M. Pizzochero and P. Donati, Nucl. Phys. A742,363(2004) Semiclassical model with spherical nuclei.





Differences between the Skyrme interactions



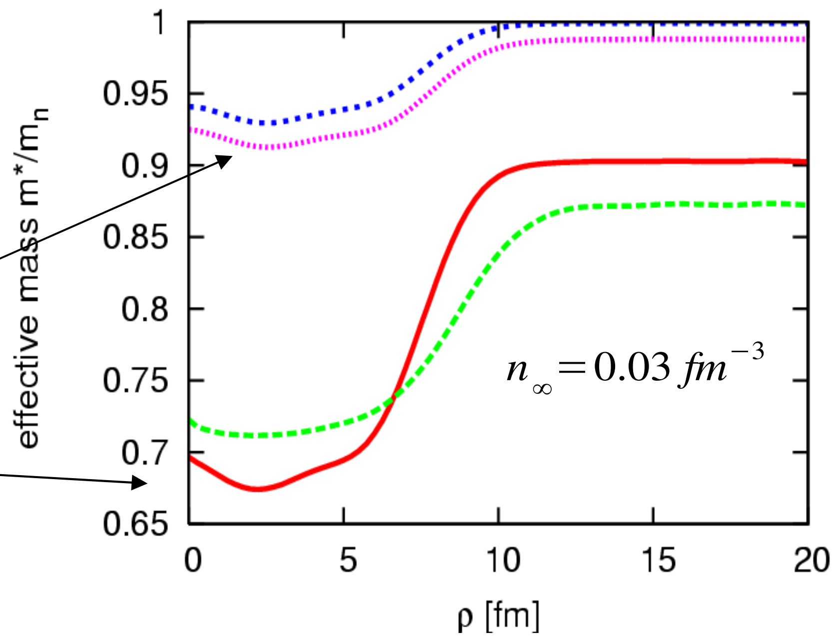


The effective masses can

be divided in two groups:

-Skm* and SGII

-SII and SLy4

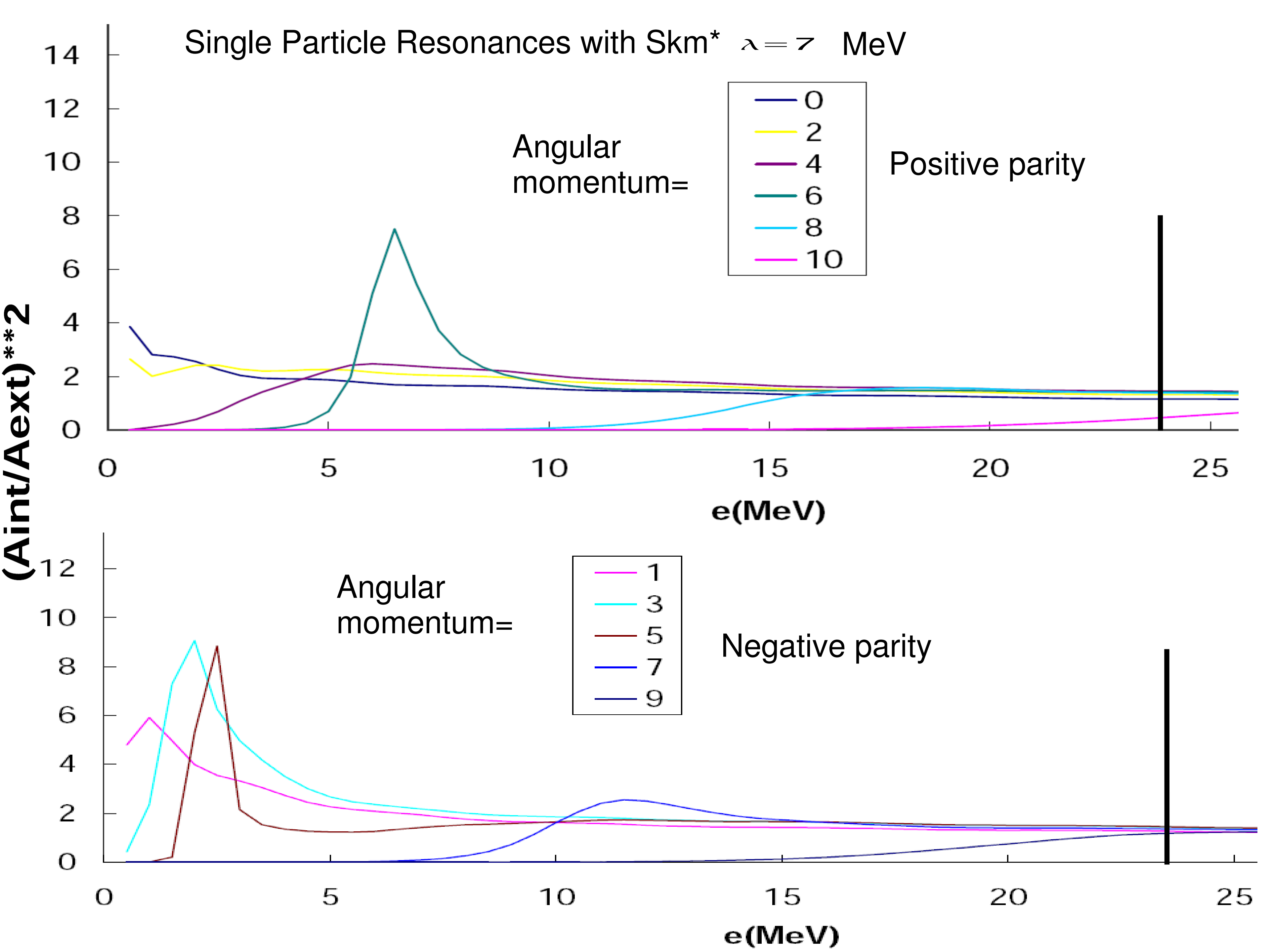


Red =SII

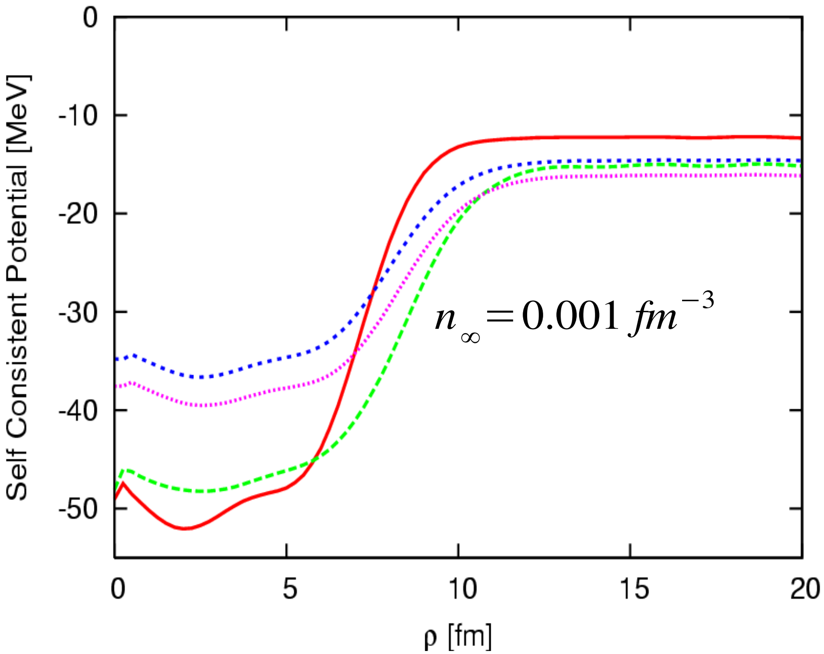
Green=SLy4

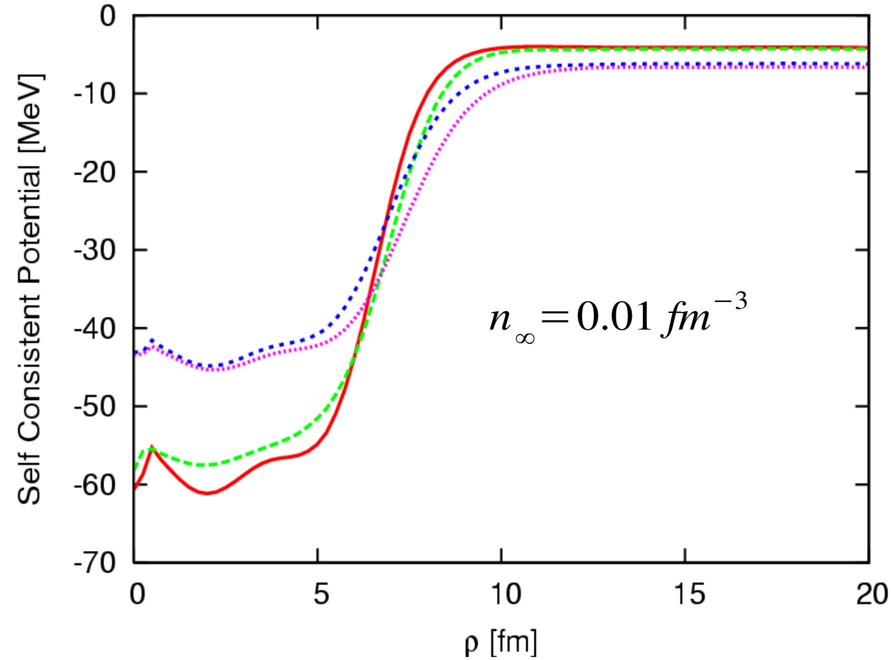
Blue= Skm*

Violet=SGII


Single Particle Resonances with Skm* λ = 7 MeV
(Aint/Aext)**2
Angular momentum=
0
2
4
6
8
10
Positive parity
e(MeV)
Angular momentum=
1
3
5
7
9
Negative parity
e(MeV)

Self consistent potentials



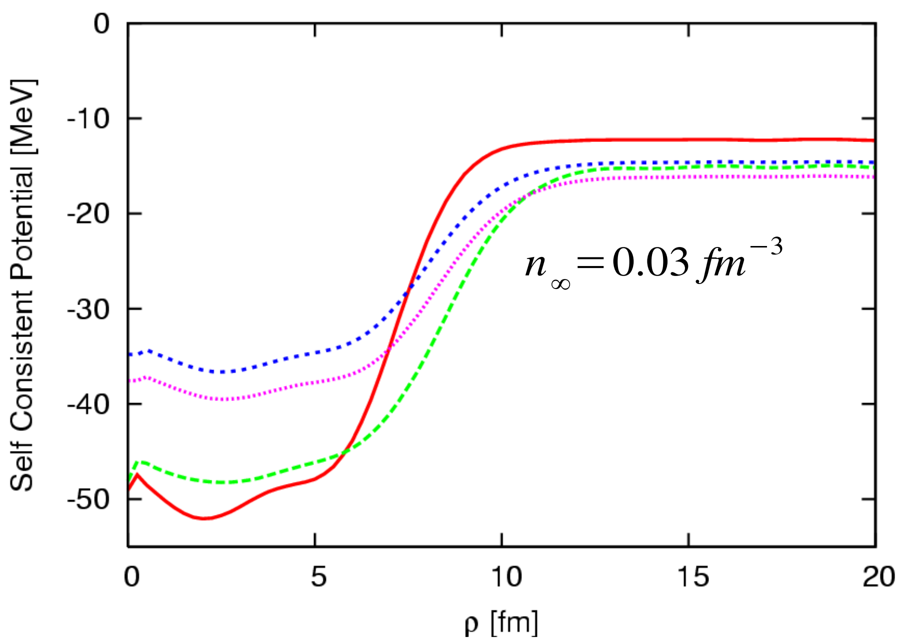


Red= SII

Green= Sly4

Blue= Skm*

Violet= SGII

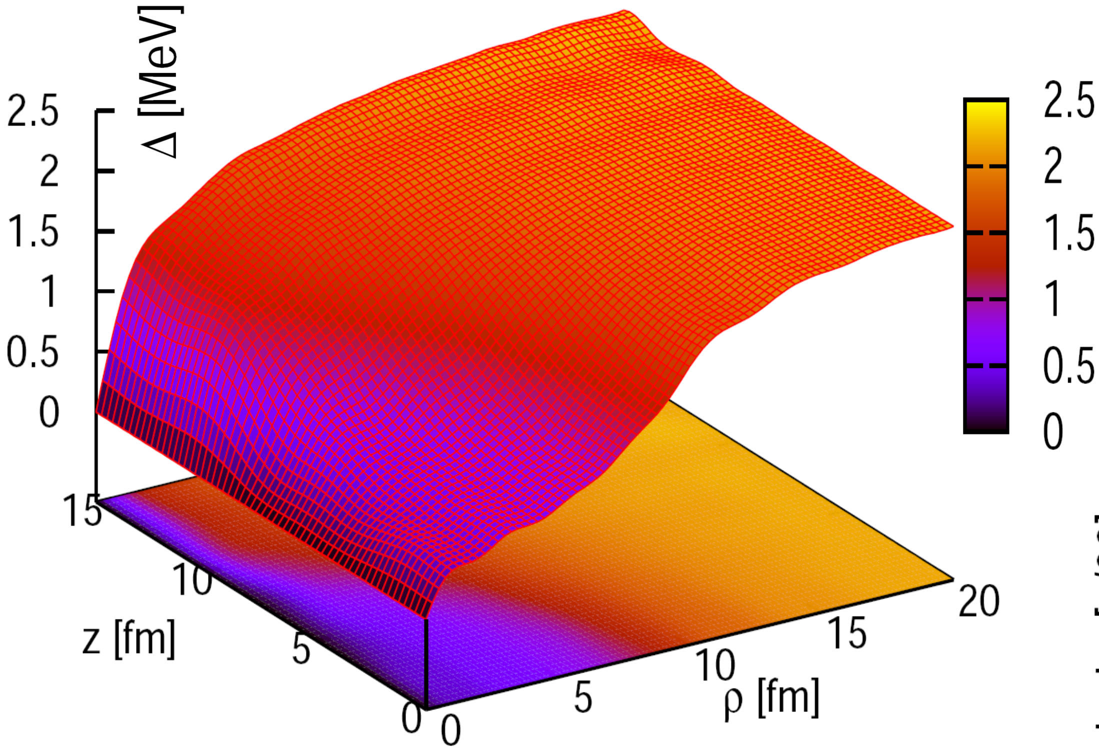


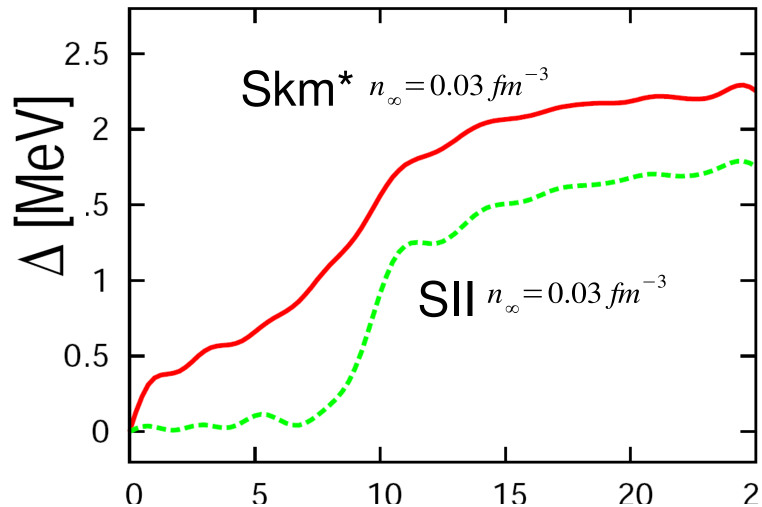
The SII and Sly4 self consistent potentials are deeper than the Skm* and SGII ones.

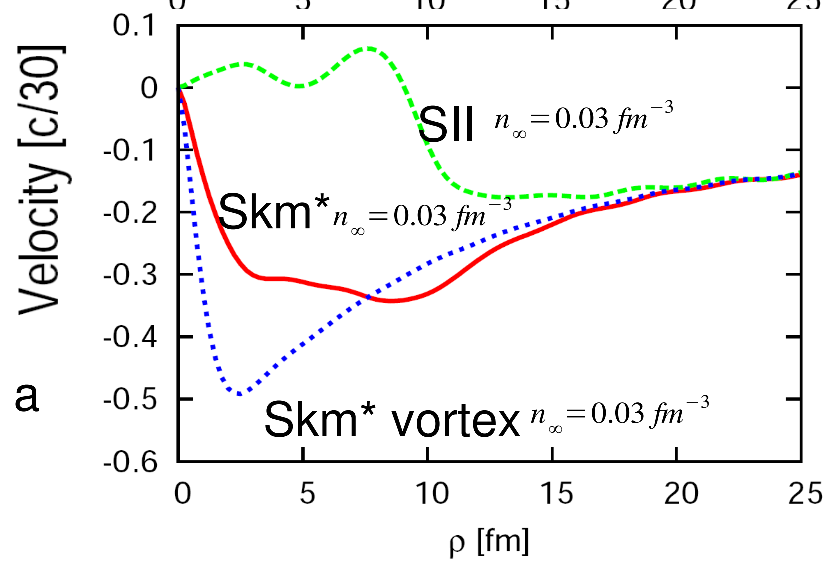
particular features of the Skm* and SGII interactions

The vortex-nucleus calculated with the Skm* and SGII forces at high densities($n_{\infty}=0.03\ fm^{-3}$)

is different from the other interactions, in fact vortex expulsion is no longer displayed.

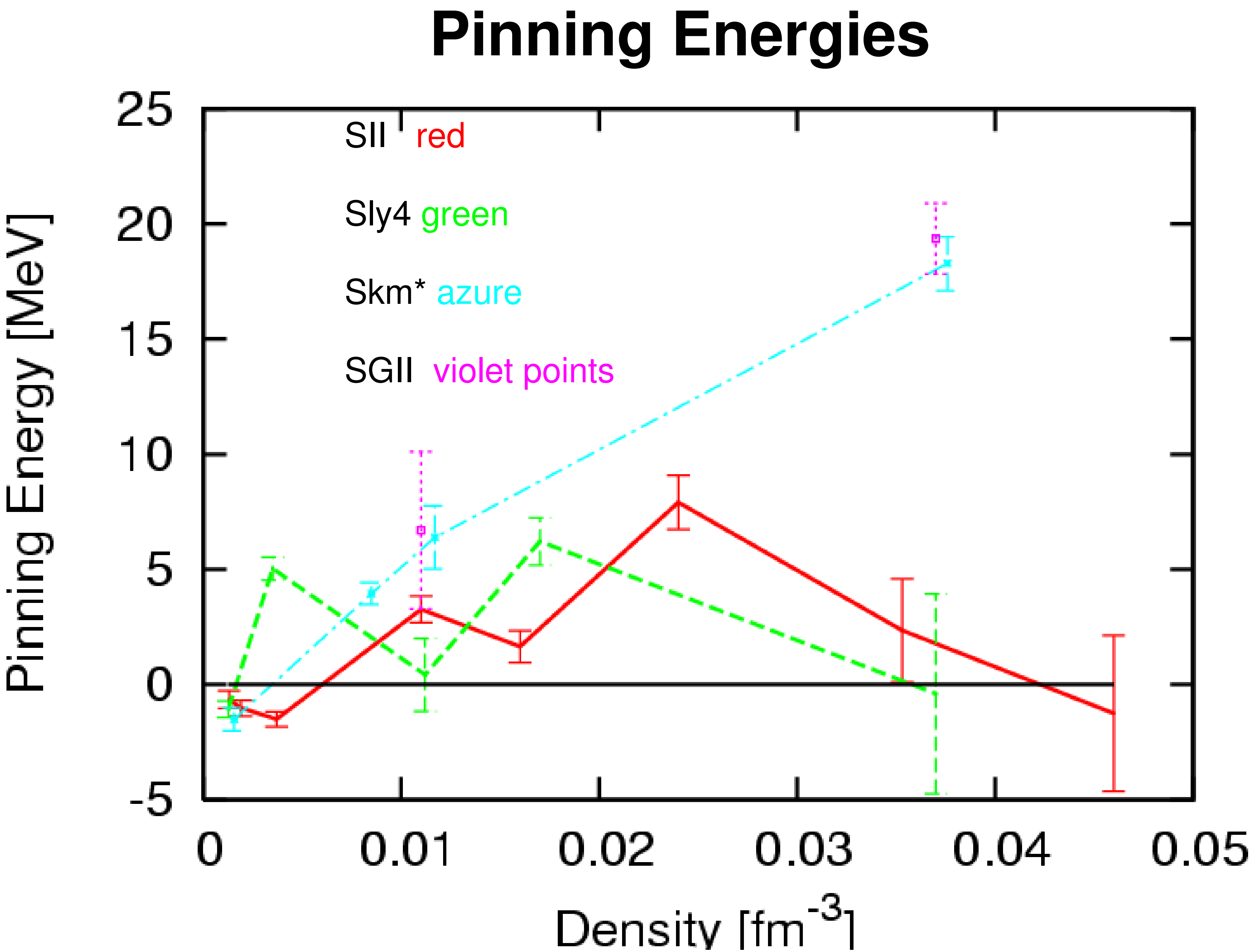






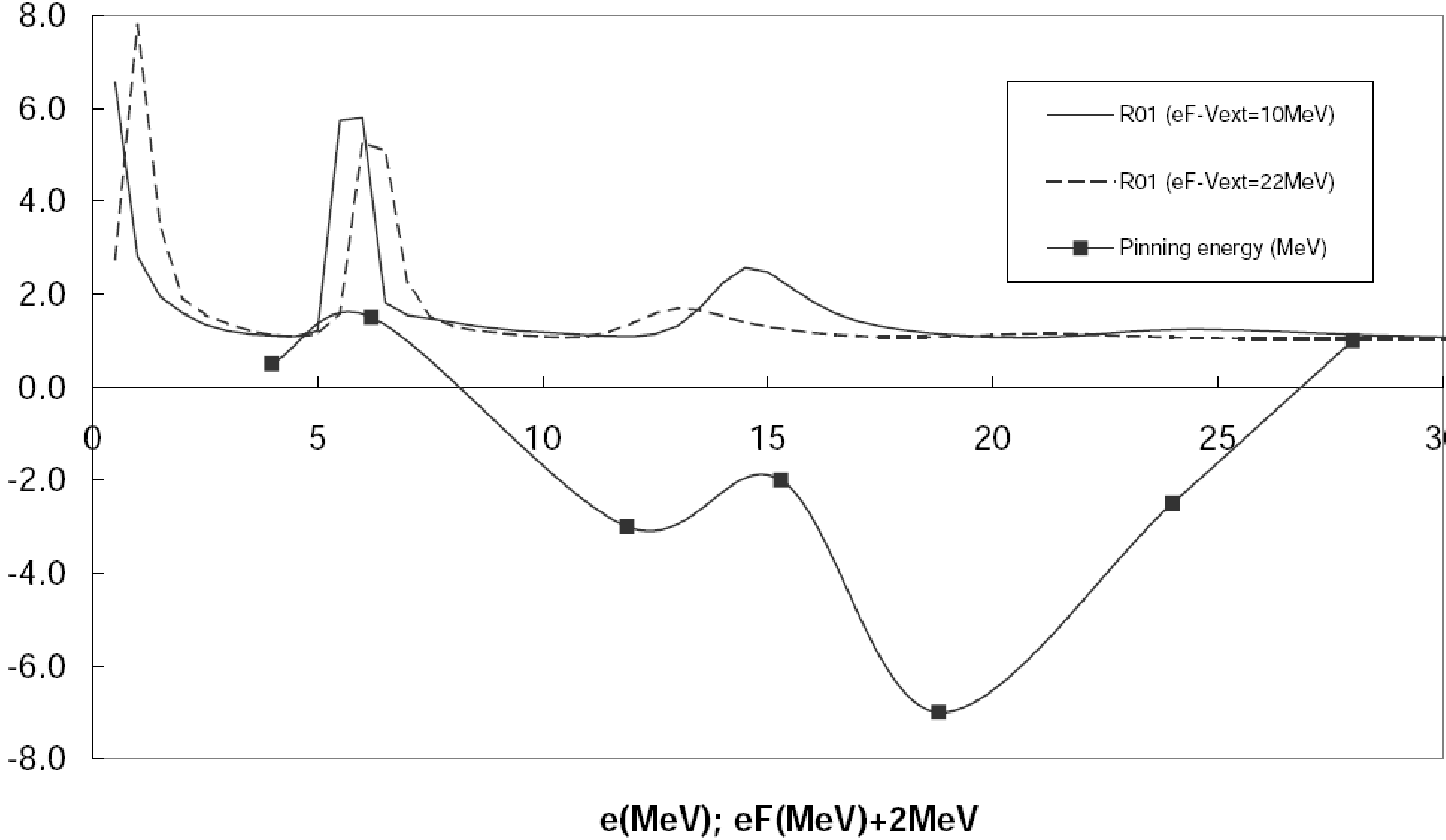
Both the pairing gap and the velocity field are have a

weaker modification respect all the other cases.



Pinning Energies
SII red
Sly4 green
Skm* azure
SGII violet points
Pinning Energy [MeV]
Density [fm^{-3}]
25
20
15
10
5
0
-5
0
0.01
0.02
0.03
0.04
0.05

Relative nu=0/nu=1 inside nucleus presence and associated pinning energy; SII



Conclusions and perspectives

-We have solved the HFB equations for a single vortex in the crust of neutron stars, considering explicitly the presence of a spherical nucleus, generalizing previous studies in uniform matter.

-We have found that finite size effects are important, (v=1) at low and medium density the vortex can form only around the nuclear surface, thus surrounding the nucleus.

-Numerical results at different densities with SII, Sly4, Skm* and SGII interaction indicate that the pinning energy is very small and of the order of a few MeV. In particular pinning is found at low asymptotic neutron density.

-Adoption of the more recent calculations for the inner crust nuclei.
-Which is the role of medium polarization effects for the pairing?
-Consequences on vortex dynamics.