# Taming the Savage Dineutron & The Miller's Correlation Tail

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Work supported by U.S. Department of Energy, Office of Nuclear Physics

## TAMING THE SAVAGE DINEUTRON

## OR DEPENDENCE OF NUCLEAR BINDING ON HADRONIC MASS VARIATION

Could fundamental "constants" have varied over the history of the universe?

Theories unifying gravity with other interactions suggest the possibility of temporal and spatial variations of physical "constants" in the expanding universe.

Some evidence for variations in the fine structure constant  $\alpha$ , strength of the strong interaction, and particle masses has been inferred from studies of big bang nucleosynthesis, quasar absorption spectra, and the Oklo natural nuclear reactor.

More generally, we might ask how much could "constants" change and still give us a universe similar to our own?

Program for studying the universe's dependence on the quark mass:

- Study how hadron masses depend on current-quark mass
- Evaluate how nuclear binding depends on hadron masses
- Study consequences for big bang and stellar nucleosynthesis

Unanticipated application: we may be able to make useful comments on how hadron mass variation affects the extrapolation of nuclear physics results from Lattice QCD calculations.

#### HADRON MASS DEPENDENCE ON CURRENT-QUARK MASS

Prediction from a Dyson-Schwinger equation study of the sigma terms of light-quark hadrons: V.V.Flambaum, A.Höll, P.Jaikumar, C.D.Roberts and S.V.Wright [Few-Body Syst. **38**, 31 (2006)]

$\frac{\delta m_H}{m_H}$	$= \frac{\sigma_H}{m_H} \frac{\delta m_q}{m_q}$		$m_q$ =	$= (m_u + $	$(n_u + m_d)/2$		
	$\pi$	ρ	ω	N	$\Delta$		
$rac{\sigma_H}{m_H}$	0.498	0.030	0.043	0.064	0.041		

But other models are possible ...

### NUCLEAR HAMILTONIAN AND HADRON MASS

To study the effect of  $\delta m_H$  on nuclear systems, we consider Hamiltonians of the form:

$$H = \sum_{i} K_{i} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

We examine three interaction models:

- Argonne  $v_{28}$  (AV28): coupled-channels OPE with explicit  $\Delta$ 's fit to 1981 phase shifts
- Argonne  $v_{14}$  (AV14): nucleons-only with approximate TPE, phase-equivalent to AV28
- Argonne  $v_{18}$  (AV18): updated AV14 with charge-independence-breaking fit to 1993 data, weaker  $f_{\pi NN}$ , deeper well, stiffer core; supplement with Urbana IX (UIX)  $V_{ijk}$

These potentials all have the operator structure:

$$v_{ij} = v_{\gamma}(r_{ij}) + \sum_{p} [v_{\pi}^{p}(r_{ij}) + v_{I}^{p}(r_{ij}) + v_{S}^{p}(r_{ij})]O_{ij}^{p}$$

The number of operators  $O_{ij}^p$  is 28, 14, or 18 according to AVxx.

The NN one-pion-exchange (OPE) in all cases is:

$$v_{\pi}(NN \to NN) = \frac{f_{\pi NN}^2}{4\pi} \left(\frac{m_{\pi}}{m_s}\right)^2 \frac{m_{\pi}c^2}{3} [Y(m_{\pi}r)\sigma_1 \cdot \sigma_2 + T(m_{\pi}r)S_{12}](\tau_1 \cdot \tau_2)$$

$$Y(mr) = \frac{e^{-\mu r}}{\mu r} \xi(r) \qquad T(mr) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) Y(mr)\xi(r)$$

where  $m_s$  is a scaling mass and  $\xi(r)$  is a short-range form factor.

The AV28 model adds transition terms to states with intermediate  $\Delta$ 's

$$v_{\pi}(NN \to N\Delta) = \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \left(\frac{m_{\pi}}{m_s}\right)^2 \frac{m_{\pi} c^2}{3} [Y(m_{\pi} r)\sigma_1 \cdot \mathbf{S}_2 + T(m_{\pi} r)S_{12}^{II}](\tau_1 \cdot \mathbf{T}_2)$$
$$v_{\pi}(NN \to \Delta\Delta) = \frac{f_{\pi N\Delta}^2}{4\pi} \left(\frac{m_{\pi}}{m_s}\right)^2 \frac{m_{\pi} c^2}{3} [Y(m_{\pi} r)\mathbf{S}_1 \cdot \mathbf{S}_2 + T(m_{\pi} r)S_{12}^{III}](\mathbf{T}_1 \cdot \mathbf{T}_2)$$

with  $\mathbf{S}_i$  ( $\mathbf{T}_i$ ) the transition spin (isospin) operator that connects spin (isospin)  $\frac{1}{2}$  and  $\frac{3}{2}$  states. AV28 also includes  $V_{N\Delta \to \Delta N}$ ,  $V_{N\Delta \to N\Delta}$ ,  $V_{N\Delta \to \Delta \Delta}$ , and  $V_{\Delta \Delta \to \Delta \Delta}$  terms. Smith & Pandharipande [Nucl. Phys. A256, 327 (1976)] showed the transition potentials effectively represent the time-ordered diagrams for a nonrelativistic interaction Lagrangian, with box diagrams approximating the 2nd Born terms to  $\sim 10\%$ , while cross-box diagrams largely cancel.



The intermediate- and short-range parts of the potentials are given by:

$$v_I^p(r_{ij}) = I^p T^2(m_\pi r)$$
  $v_S^p(r_{ij}) = (S^p + Q^p r + R^p r^2) W(r)$ 

W(r) is a Woods-Saxon function and the CI operators are

$$O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L} \cdot \mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j]$$

The intermediate-range function  $T^2(m_{\pi}r)$  gives the AV14 & AV18 models an approximate TPE character. This can be seen by using the transition potentials in a closure approximation:

$$v^{2\pi} = v_{1\Delta}^{2\pi} + v_{2\Delta}^{2\pi}$$

$$v_{1\Delta}^{2\pi} = \begin{bmatrix} X_{ij}^{II\dagger} \tau_i \cdot \mathbf{T}_j^{\dagger} \end{bmatrix} \frac{-1}{\bar{E}_1 + (m_{\Delta} - m_N)} \begin{bmatrix} X_{ij}^{II} \tau_i \cdot \mathbf{T}_j \end{bmatrix} + (i \leftrightarrow j)$$

$$v_{2\Delta}^{2\pi} = \begin{bmatrix} X_{ij}^{III\dagger} \mathbf{T}_i^{\dagger} \cdot \mathbf{T}_j^{\dagger} \end{bmatrix} \frac{-1}{\bar{E}_2 + 2(m_{\Delta} - m_N)} \begin{bmatrix} X_{ij}^{III} \mathbf{T}_i \cdot \mathbf{T}_j \end{bmatrix}$$

$$X_{ij}^{II} = \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \left(\frac{m_{\pi}}{m_s}\right)^2 \frac{m_{\pi} c^2}{3} \begin{bmatrix} Y(m_{\pi} r) \sigma_i \cdot \mathbf{S}_j + T(m_{\pi} r) S_{ij}^{II} \end{bmatrix}$$

$$X_{ij}^{III} = \frac{f_{\pi N\Delta}}{4\pi} \left(\frac{m_{\pi}}{m_s}\right)^2 \frac{m_{\pi} c^2}{3} \begin{bmatrix} Y(m_{\pi} r) \mathbf{S}_i \cdot \mathbf{S}_j + T(m_{\pi} r) S_{ij}^{III} \end{bmatrix}$$

Sum over Transition spin and isospin operators using rules analogous to standard Pauli spin operators:

$$\sigma \cdot \sigma = 3 \quad ; \quad \sigma \times \sigma = 2i\sigma \quad ; \quad \sigma \cdot \mathbf{A} \ \sigma \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{B} + i\sigma \cdot \mathbf{A} \times \mathbf{B} ;$$
$$\mathbf{S}^{\dagger} \cdot \mathbf{S} = 2 \quad ; \quad \mathbf{S}^{\dagger} \times \mathbf{S} = -\frac{2}{3}i\sigma \quad ; \quad \mathbf{S}^{\dagger} \cdot \mathbf{A} \ \mathbf{S} \cdot \mathbf{B} = \frac{2}{3}\mathbf{A} \cdot \mathbf{B} - \frac{1}{3}i\sigma \cdot \mathbf{A} \times \mathbf{B} .$$

$$v^{2\pi} = (8\chi_1 + \frac{32}{9}\chi_2)[T^2(m_\pi(r) + \frac{1}{2}Y^2(m_\pi r)] + (\frac{8}{3}\chi_1 - \frac{16}{27}\chi_2)[T^2(m_\pi(r) + \frac{1}{2}Y^2(m_\pi r)](\tau_i \cdot \tau_j) - (\frac{4}{3}\chi_1 - \frac{8}{27}\chi_2)[T^2(m_\pi r) - Y^2(m_\pi r)](\sigma_i \cdot \sigma_j) - (\frac{4}{9}\chi_1 + \frac{4}{81}\chi_2)[T^2(m_\pi r) - Y^2(m_\pi r)](\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) + (\frac{4}{3}\chi_1 - \frac{8}{27}\chi_2)[T^2(m_\pi r) - Y(m_\pi r)T(m_\pi r)]S_{ij} + (\frac{4}{9}\chi_1 + \frac{4}{81}\chi_2)[T^2(m_\pi r) - Y(m_\pi r)T(m_\pi r)]S_{ij}(\tau_i \cdot \tau_j)$$

$$\chi_{1} = \frac{-2}{\bar{E}_{1} + (m_{\Delta} - m_{N})} \left(\frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \frac{m_{\pi} c^{2}}{3}\right)^{2}$$
  
$$\chi_{2} = \frac{-1}{\bar{E}_{2} + 2(m_{\Delta} - m_{N})} \left(\frac{f_{\pi N\Delta}^{2}}{4\pi} \frac{m_{\pi} c^{2}}{3}\right)^{2}$$

One could adjust  $f_{\pi N\Delta}$ ,  $\bar{E}_1$ , and  $\bar{E}_2$  to fit data when constructing a potential, or just fit the entire coefficient  $I_{ij}^p$  of each  $O_{ij}^p$ , which is the procedure for AV14 and AV18.

Transition potentials with closure approximation can also be used to construct 3N potential terms, including standard Fujita-Miyazawa force. Phenomenological short-range repulsive term added to Urbana models.

$$V_{ijk}^{2\pi} = A_{2\pi}^{PW} \Big( \sum_{cyc} \{X_{ij}, X_{ik}\} \{\tau_i \cdot \tau_j, \tau_i \cdot \tau_k\} + \frac{1}{4} [X_{ij}, X_{ik}] [\tau_i \cdot \tau_j, \tau_i \cdot \tau_k] \\ X_{ij} = Y(m_{\pi}r_{ij})\sigma_i \cdot \sigma_j + T(m_{\pi}r_{ij})S_{ij} \\ A_{2\pi}^{PW} = \frac{-2/9}{\bar{E}_3 + (m_{\Delta} - m_N)} \left( \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \left( \frac{m_{\pi}}{m_s} \right)^2 \frac{m_{\pi}c^2}{3} \right)^2 \\ V_{ijk}^R = A_R \sum_{cyc} T^2(m_{\pi}r_{ij})T^2(m_{\pi}r_{ik})$$

Parameter  $\bar{E}_3$  could be adjusted (along with  $f_{\pi N\Delta}$  from NN potential) but instead the strengths  $A_{2\pi}^{PW}$  and  $A_R$  were adjusted to fit <sup>3</sup>H, <sup>4</sup>He binding and nuclear matter saturation density in many-body calculations when used with given NN potential, e.g., AV14+UVII or AV18+UIX.

#### BARYON MASS DEPENDENCE

Baryon masses appear in the kinetic energy operator:

$$K_i = -\frac{\hbar^2}{2m_i}\nabla_i^2 + (m_i - m_N)c^2$$

with  $m_i$  being  $m_N$  or  $m_\Delta$  as appropriate.

Also contribute through the  $NN-N\Delta-\Delta\Delta$  coupled channels in AV28, e.g., in the closure approximation

$$\bar{E}_1 \approx \frac{\hbar^2 \bar{k}^2}{2m_N} + \frac{\hbar^2 \bar{k}^2}{2m_\Delta} \qquad \bar{E}_2 \approx 2 \frac{\hbar^2 \bar{k}^2}{2m_\Delta}$$

Approximate this effect in AV14 by altering intermediate-range attraction

$$\bar{I}^p \approx (1 + \delta_N + \delta_\Delta) I^p$$

Fix  $\delta_N$  and  $\delta_\Delta$  by requiring same mass dependence as phase-equivalent AV28.

$$\delta_N = 0.49 \frac{\delta m_N}{m_N} \qquad \qquad \delta_\Delta = -0.57 \frac{\delta m_\Delta}{m_\Delta}$$

Assume same factors are approximately correct modifications for AV18.

#### PION MASS DEPENDENCE

Pion mass-dependence enters through  $m_{\pi}^{3}Y(m_{\pi}r)$  and  $m_{\pi}^{3}T(m_{\pi}r)$  in OPE and TPE, etc. The scaling mass  $m_{s}$  is not changed, and possible variations in the coupling strength are neglected. Same dependence as Beane & Savage [Nucl. Phys. A713, 148 (2003).]



Volume integral of  $m^3 Y(mr)$  = constant with r = 2/m crossing. However, if m increases  $m^3 T(mr)$  decreases for all r and vice versa. Add pion mass dependence in 3-4 steps:

- change  $m_{\pi}$  in OPE only (including generalized OPE for AV28)
- add change in static TPE  $(I^p O_{ij}^p \text{ for } p=1,6)$
- add change in all other TPE  $(I^p O_{ij}^p \text{ for } p=7,14)$
- add change in  $V_{ijk}^{2\pi}$  (no attempt to change  $V_{ijk}^R$ )

#### HEAVY MESON MASS DEPENDENCE

No heavy mesons in AVxx models, so alter short-range Woods-Saxon W(r) to mimic heavy-meson Yukawa  $m_S^3 Y(m_S r)$  by changing range and strength:

$$\frac{\delta r_0}{r_0} = \frac{\delta a}{a} = -\frac{2}{3} \frac{\delta m_S}{m_S}$$
$$1 + 2\delta m_S / m_S$$

$$W(r) = \frac{1 + 20m_S/m_S}{1 + \exp[(r - r_0)/a]}$$



Volume integral of W(r) is almost constant for changes in  $m_S$  with crossing at  $r \approx .75$  fm, instead of  $2/m_S \approx .5$  fm for  $\omega$ - or  $\rho$ -meson.

#### **ENERGY CALCULATIONS**

Study deuteron and  ${}^{1}S_{0}$  virtual bound state by direct solution of two-body equations.

Study A = 3 - 8 nuclei using variational Monte Carlo (VMC) method:

Construct suitably parametrized trial wave functions  $\Psi_V$  and evaluate upper bound to ground-state energy:

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \ge E_0$$

Simplified trial wave function:

$$|\Psi_V\rangle = [1 + \sum_{i < j < k} U_{ijk}] [\mathcal{S} \prod_{i < j} (1 + U_{ij})] \prod_{i < j} f_{ij} |\Phi_A(JMTT_3)\rangle$$

 $\Phi_A(JMTT_3)$  is antisymmetric product of single-particle functions coupled to given quantum numbers (translationally invariant with multiple spatial-symmetry components, but NOT harmonic oscillator)  $f_{ij}$  are central (mostly short-ranged repulsion) correlations  $U_{ij}$  are non-commuting 2-body correlations from  $v_{ij}$ 

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p$$

 $U_{ijk}$  are 3-body correlations from  $V_{ijk}$ 

# **ENERGY RESULTS**

VMC energies of light nuclei in MeV for the different Hamiltonians compared to experiment

	${}^{1}S_{0}(np)$	$^{2}\mathrm{H}$	<sup>3</sup> H	<sup>3</sup> He	<sup>4</sup> He	<sup>5</sup> He	<sup>6</sup> Li	<sup>7</sup> Li	<sup>8</sup> Be
AV28	0.0661	-2.2250							
AV14	0.0663	-2.2250	-7.50	-6.88	-23.60	-21.26	-24.31	-28.31	-40.26
AV18+UIX	0.0665	-2.2246	-8.24	-7.49	-27.50	-25.26	-28.22	-33.33	-48.50
Expt.		-2.2246	-8.48	-7.72	-28.30	-27.41	-31.99	-39.24	-56.50

#### **EVALUATING MASS DEPENDENCE**

Change hadron masses  $m_H$  one at a time  $\pm 0.1\%$  and recalculate E to evaluate dimensionless derivatives:

$$\Delta \mathcal{E}(m_H) = \frac{\delta E/E}{\delta m_H/m_H}$$

Results can be combined with any given model for the correlation between hadron and quark masses:

$$E(m_q) = E_0 \left[ 1 + \sum_{m_H} \Delta \mathcal{E}(m_H) \frac{\delta m_H(m_q)}{m_H} \right]$$

## **TWO-NUCLEON RESULTS**

$\Delta C(m_H)$ for the $S_0(m_P)$ virtual bound state $C_0$ and the dediction $Q$									
$m_H$		$\Delta \epsilon_v$		$\Delta Q$					
	AV28	AV14	AV18	AV28	AV14	AV18			
$m_N$	-88.1	-32.7	-33.4	13.06	8.62	8.90			
$m_N + \delta_N$		-91.2	-121.2		13.03	17.82			
$m_\Delta$	63.9			-5.15					
$\delta_\Delta$		68.1	102.2		-5.13	-10.36			
$m_{\pi}$ (OPE)	9.5	-4.1	-3.8	-2.23	-1.55	-1.40			
$m_{\pi}$ (+TPE-s)	24.4	35.6	53.0	-3.63	-4.02	-6.70			
$m_{\pi}$ (+TPE-L)				-4.02	-4.31	-6.74			
$m_S$	-33.5	-56.2	-75.8	4.24	4.07	6.09			

 $\Delta \mathcal{E}(m_H)$  for the <sup>1</sup>S<sub>0</sub>(*np*) virtual bound state  $\epsilon_v$  and the deuteron Q

Dmitriev, Flambaum & Webb [Phys. Rev. D69, 063506 (2004)] find relation between  $\Delta Q$  and  $\Delta \epsilon_v$ :

$\Delta \epsilon_v(m_H)$	$\sim$	$\sqrt{Q}$
$\overline{\Delta Q(m_H)}$	$\sim$ -	$\sqrt{\epsilon_v}$

This is satisfied within factor of 2 for all but two cases:  $m_{\pi}$  (OPE) for AV14 and AV18.



Dotted lines are  $m_{\pi}$  (OPE) only — Solid lines are "full" calculation with DSE  $\delta m_H/m_H$ : AV28 includes  $m_N, m_{\Delta}, m_{\pi}$  (+TPE-L),  $m_S$ ; AV14 & AV18 include  $m_N + \delta_N, \delta_{\Delta}, m_{\pi}$  (+TPE-L),  $m_S$ .

	$^{3}\mathrm{H}$	<sup>3</sup> He	<sup>4</sup> He	<sup>5</sup> He	<sup>6</sup> Li	<sup>7</sup> Li	<sup>8</sup> Be
$m_N$	6.00	6.44	3.97	4.58	5.25	5.60	5.10
$m_N + \delta_N$	12.32	13.17	9.03	10.38	11.35	12.74	11.71
$\delta_{\Delta}$	-7.35	-7.82	-5.89	-6.74	-7.10	-8.31	-7.69
$m_{\pi}$ (OPE)	-0.45	-0.50	-0.20	-0.24	-0.36	-0.30	-0.23
$m_{\pi}$ (+TPE-s)	-4.35	-4.66	-3.33	-3.87	-4.19	-4.83	-4.38
$m_{\pi}$ (+TPE-L)	-4.53	-4.85	-3.47	-4.04	-4.40	-5.06	-4.59
$m_S$	5.99	6.39	4.79	5.72	6.21	7.41	6.62

Multi-nucleon  $\Delta \mathcal{E}(m_H)$  for AV14

	$^{3}\mathrm{H}$	<sup>3</sup> He	<sup>4</sup> He	<sup>5</sup> He	<sup>6</sup> Li	<sup>7</sup> Li	<sup>8</sup> Be
$m_N$	6.07	6.54	3.99	4.51	5.12	5.24	4.81
$m_N + \delta_N$	16.56	17.73	11.86	13.31	14.41	15.53	14.36
$\delta_\Delta$	-12.20	-13.02	-9.16	-10.24	-10.80	-11.96	-11.11
$m_{\pi}$ (OPE)	-0.37	-0.42	-0.19	-0.24	-0.36	-0.29	-0.23
$m_{\pi}$ (+TPE-s)	-6.90	-7.38	-5.11	-5.82	-6.33	-6.95	-6.34
$m_{\pi}$ (+TPE-L)	-6.87	-7.36	-5.06	-5.75	-6.24	-6.84	-6.24
$m_{\pi}$ (+TNI)	-6.91	-7.40	-5.12	-5.82	-6.31	-6.91	-6.31
$m_S$	8.46	9.07	6.58	7.61	8.24	9.34	8.36

Multi-nucleon  $\Delta \mathcal{E}(m_H)$  for AV18+UIX



Multi-nucleon  $E(m_q)$  for "full" AV14 calculation with DSE  $\delta m_H/m_H$ 



Multi-nucleon  $E(m_q)$  for "full" AV18+UIX calculation with DSE  $\delta m_H/m_H$ 

### CONCLUSIONS

Dependence of nuclear binding on  $m_{\pi}$ ,  $m_S \approx m_{\rho} \approx m_{\omega}$ ,  $m_N$ ,  $m_{\Delta}$  calculated for  $A \leq 8$  nuclei. Results can be combined with any particular model for  $\delta m_H/m_H$  variation as function of  $m_q$ . Two-pion-exchange contributions are important. With TPE, all nuclei, including <sup>1</sup>S dineutron, show same trends with variations in  $\delta m_H/m_H$ . Instability of <sup>8</sup>Be vs.  $2\alpha$  appears to be greatest "fine-tuning" issue. Consequences for big bang and stellar nucleosynthesis should be explored.

# THE MILLER'S CORRELATION TAIL OR IMPROVED SHORT-RANGE CORRELATIONS FOR NUCLEAR MATRIX ELEMENTS

Miller & Spencer [Ann. Phys. **100**, 562 (1976)] developed a function to parameterize the short-range nucleon-nucleon correlations induced by the repulsive core of the *NN* interaction:

$$f_{MS}(r_{ij}) = 1 - (1 - br_{ij}^2)e^{-ar_{ij}^2}$$

The parametrization vanishes at the origin and goes to unity at some moderate (few fm) distance – typical values for the parameters are  $a = 1.1 \text{fm}^{-2}$  and  $b = 0.68 \text{fm}^{-2}$ .

The correlation was motivated by early nuclear matter studies and represents the modification of the Fermi gas wave function by  $v_{ij}$ . The original use was in calculations of pion charge-exchange reactions, but subsequently it has been used in many (> 150) papers for a variety of applications, including parity nonconservation and double beta-decay.

#### **S**TATE-DEPENDENCE OF NUCLEAR CORRELATIONS

Because of the strong spin-, isospin-, and tensor-dependence of nuclear potentials, present-day variational calculations of nuclear matter use correlation operators

$$\mathcal{F}_{ij} = \sum_{p} f_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,6} = [1, \sigma_i \cdot \sigma_j, S_{ij}] \otimes [1, \tau_i \cdot \tau_j]$$

These might be used to evaluate nuclear matrix elements in the same fashion as the Miller-Spencer correlation:

$$\langle \phi | \mathcal{F}_{ij} O_{ij}^X \mathcal{F}_{ij} | \phi \rangle = \langle \phi | G_X(r_{ij}) O_{ij}^X | \phi \rangle$$

Miller [Phys. Rev. C 67, 042501 (2003)] has recently looked at spin-isospin correlations and their effect on parity violation calculations.



AV18+UIX nuclear matter correlation functions ( $\rho = 0.1 \text{fm}^{-3}$ )

For example, for the one-pion-exchange PNC matrix element

$$\mathcal{F}_{ij}O_{ij}^{\pi}\mathcal{F}_{ij} = \mathcal{F}_{ij}(\tau_i \times \tau_j)_z(\sigma_i + \sigma_j) \cdot \hat{r}_{ij}\mathcal{F}_{ij} = G_{\pi}(\tau_i \times \tau_j)_z(\sigma_i + \sigma_j) \cdot \hat{r}_{ij}$$

the distribution function  $G_{\pi}$  is:

$$G_{\pi} = (f_c + f_{\sigma} + 2f_t)^2 - 2(f_c + f_{\sigma} + 2f_t)(f_{\tau} + f_{\sigma\tau} + 2f_{t\tau}) - 3(f_{\tau} + f_{\sigma\tau} + 2f_{t\tau})^2$$

Other operator combinations for PNC meson-exchange:

$$\mathcal{F}_{ij}O_{ij}^{\rho 0}\mathcal{F}_{ij} = \mathcal{F}_{ij}(\tau_i \cdot \tau_j)i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}\mathcal{F}_{ij} = (G_{\rho 0,1} + G_{\rho 0,\tau}\tau_i \cdot \tau_j)i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}$$
$$\mathcal{F}_{ij}O_{ij}^{\rho 1}\mathcal{F}_{ij} = \mathcal{F}_{ij}(\tau_i + \tau_j)_z i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}\mathcal{F}_{ij} = G_{\rho 1}(\tau_i + \tau_j)_z i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}$$
$$\mathcal{F}_{ij}O_{ij}^{\rho 2}\mathcal{F}_{ij} = \mathcal{F}_{ij}(3\tau_{iz}\tau_{jz} - \tau_i \cdot \tau_j)i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}\mathcal{F}_{ij} = G_{\rho 2}(3\tau_{iz}\tau_{jz} - \tau_i \cdot \tau_j)i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}$$
$$\mathcal{F}_{ij}O_{ij}^{\omega 0}\mathcal{F}_{ij} = \mathcal{F}_{ij}i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}\mathcal{F}_{ij} = (G_{\omega 0,1} + G_{\omega 0,\tau}\tau_i \cdot \tau_j)i(\sigma_i \times \sigma_j) \cdot \hat{r}_{ij}$$



PNC distribution functions from nuclear matter correlations

# NO CONCLUSIONS

Work with these improved correlations is just beginning.

Things to be investigated:

- Are any previously evaluated PNC matrix elements noticeably altered?
- Possible double-counting issues when used with shell-model or other wave functions.
- Density dependence of the correlations.
- Any affect on double beta-decay?