THEORETICAL ISSUES IN EXTRACTION V_{ud} FROM NUCLEAR DECAYS

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- V_{ud} from nuclear decays
  - radiative corrections
  - isospin-symmetry breaking corrections
- V_{us} from kaon decays
  - SU(3)-symmetry breaking corrections
- Top row test of CKM unitarity

MASTER EQUATIONS

$$\text{CVC}: \ \mathcal{F}t = ft(1+\delta'_R)(1-(\delta_C-\delta_{NS})) = \text{constant}$$

$$V_{ud}^2 = \frac{K}{2G_F^2\overline{\mathcal{F}t}(1+\Delta_R)} \qquad \frac{K}{(\hbar c)^6} = \frac{2\pi^3\hbar\ln 2}{(m_e c^2)^5}$$

where

ft = experimental nuclear ft values.

$\overline{\mathcal{F}t}$ = average corrected ft values (13 cases).

G_F = weak interaction coupling constant (from muon lifetime).

$\left.\begin{matrix}\Delta_R\\ \delta'_R\\ \delta_{NS}\end{matrix}\right\}$ = calculated radiative correction.

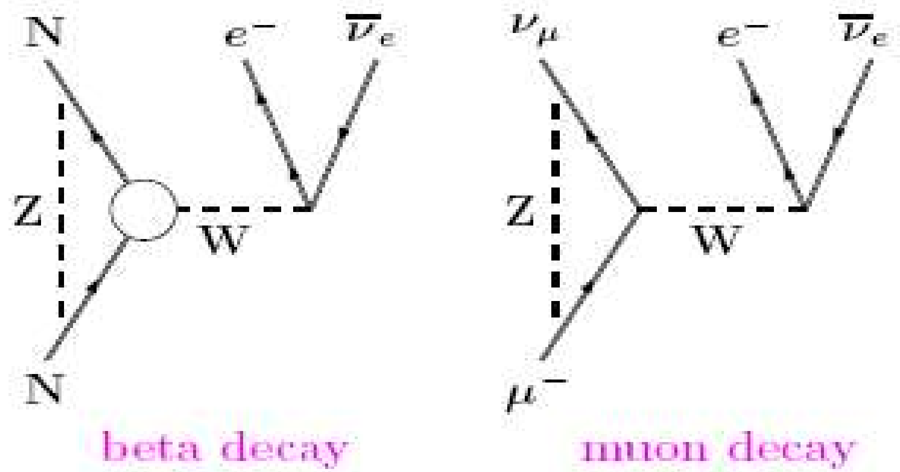
δ_C = calculated isospin symmetry breaking correction.

Note:

$$V_{ud}^2 = \frac{\text{beta decay}}{\text{muon decay}}$$

Any radiative correction that is common to both beta decay and muon decay is called universal, cancels in ratio – not included in calculation.

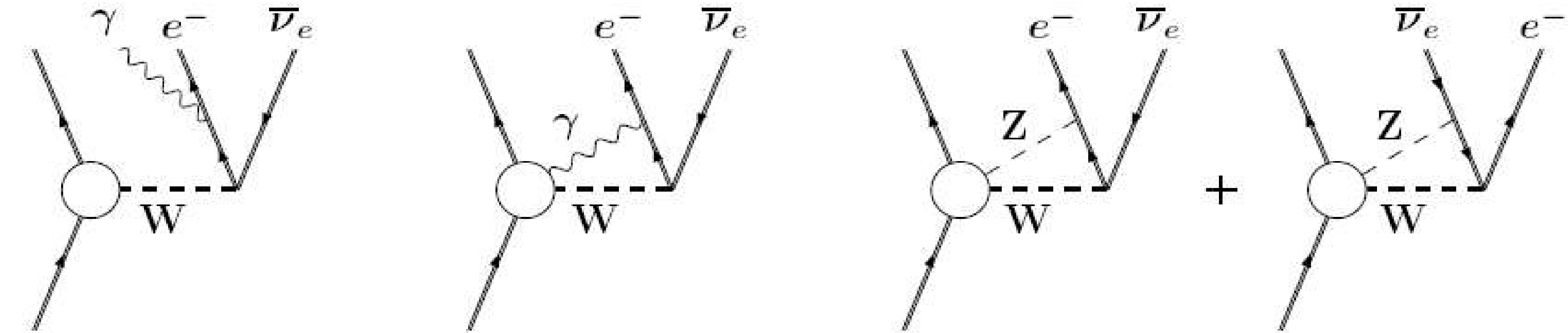
Example:



universal in limit: $\frac{m_h^2}{m_Z^2} \rightarrow 0$

m_h = hadron mass.

RADIATIVE CORRECTION TO ORDER α



bremsstrahlung W-box Z-box

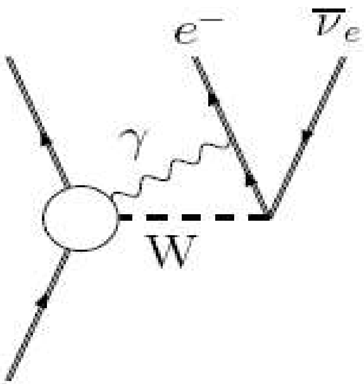
W-box : { long distance (low energies) : sensitive to nucleon structure
short distance (high energies) : only "see" quarks

$$\text{Brem}+W\text{-box}(\text{V},\text{LD}) \;=\; \frac{\alpha}{4\pi}\,\overline{g}(E_m) \overset{\text{Large } E_m}{\longrightarrow} \frac{\alpha}{4\pi}\left[3\ln\left(\frac{m_p}{2E_m}\right)+\frac{81}{10}-\frac{4\pi^2}{3}\right]$$

$$W\text{-box}(\text{A}) \;=\; ?$$

$$W\text{-box}(\text{V},\text{SD})+Z\text{-box} \;=\; \frac{\alpha}{4\pi}\left[3\ln\left(\frac{m_W}{m_p}\right)-4\ln\left(\frac{m_W}{m_Z}\right)\right]$$

THE GAMOW-TELLER PIECE



$$\text{W-Box(A)} = \frac{\alpha}{8\pi}\int_0^\infty \frac{m_W^2}{Q^2+m_W^2} F(Q^2)\, dQ^2$$

Break integration into short and long-distance regimes

a) Short distance: $m_A^2 \leq Q^2 \leq \infty$

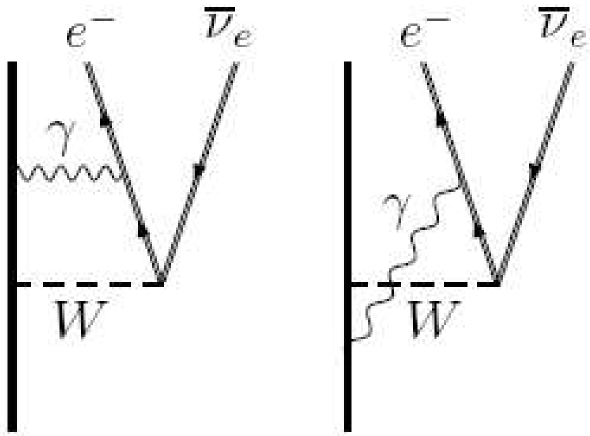
$$F(Q^2) \xrightarrow[Q^2\to\infty]{} \frac{1}{Q^2}\left[1-\frac{\alpha_S(Q^2)}{\pi}\right]$$

$$\text{W-Box(A, SD)} = \frac{\alpha}{4\pi}\left[\ln\left(\frac{m_W}{m_A}\right)+\mathcal{A}_g\right] \qquad \mathcal{A}_g = -0.34$$

QCD loop correction

b) Long distance: $0 \leq Q^2 \leq m_A^2$

$$\mathbf{W\text{-}Box(A, LD)} =$$



Born graphs

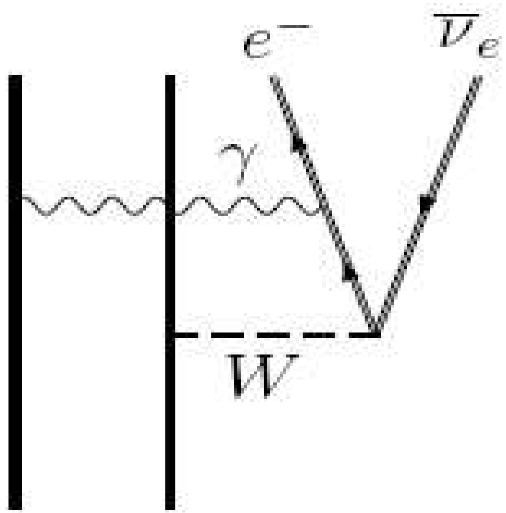
$$= \frac{\alpha}{2\pi}\left[\, C_{Born} \,\right] = \frac{\alpha}{2\pi}\left[\, 0.881 \,\right]$$

Choose m_A ?

Sirlin recommended: $\frac{1}{2}\, m_{a_1} \leq m_A \leq 2\, m_{a_1}$

This range is largest contributor to error in radiative correction

For finite nuclei (but not neutron decay) there is a two-body contribution from the Born graphs:



Requires a shell-model calculation for its evaluation.

This is the ONLY piece of the radiative correction that depends on a nuclear-structure calculation and it is SMALL.

Typical values:

$$\mathrm{T_z} = -1: \quad \delta_{\mathrm{NS}}(^{10}\mathrm{C}) = -0.36\% \quad \delta_{\mathrm{NS}}(^{14}\mathrm{O}) = -0.25\% \quad \delta_{\mathrm{NS}}(^{34}\mathrm{Ar}) = -0.18\%$$

$$\mathrm{T_z} = 0: \quad \delta_{\mathrm{NS}}(^{26}\mathrm{Al}) = 0.01\% \quad \delta_{\mathrm{NS}}(^{46}\mathrm{V}) = -0.04\% \quad \delta_{\mathrm{NS}}(^{74}\mathrm{Rb}) = -0.06\%$$

Marciano-Sirlin (PL 96, 032002 (2006)) revision

Break integration into three regimes

a) Short distance: $(1.5\ \mathrm{GeV})^2 \leq Q^2 \leq \infty$

$$F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_S(Q^2)}{\pi} - C_2\left(\frac{\alpha_S(Q^2)}{\pi}\right)^2 - C_3\left(\frac{\alpha_S(Q^2)}{\pi}\right)^3\right]$$

QCD corrections to third order; C_2 and C_3 related to Bjorken sum rule for polarized electroproduction.

b) Intermediate distance: $(0.823\ \mathrm{GeV})^2 \leq Q^2 \leq (1.5\ \mathrm{GeV})^2$

$$F(Q^2) = \frac{D_1}{Q^2 + m_\rho^2} + \frac{D_2}{Q^2 + m_A^2} + \frac{D_3}{Q^2 + m_{\rho'}^2}$$

interpolation function parameterized by meson dominance

D_1, D_2, D_3 fixed by matching and other constraints

c) Long distance: $0 \leq Q^2 \leq (0.823\ \mathrm{GeV})^2$

Born graphs: Change in integration range reduces value slightly

$$C_{\mathrm{Born}}:\ 0.881 \longrightarrow 0.829$$

Allow 10% uncertainty in C_{Born}; 100% uncertainty in interpolator

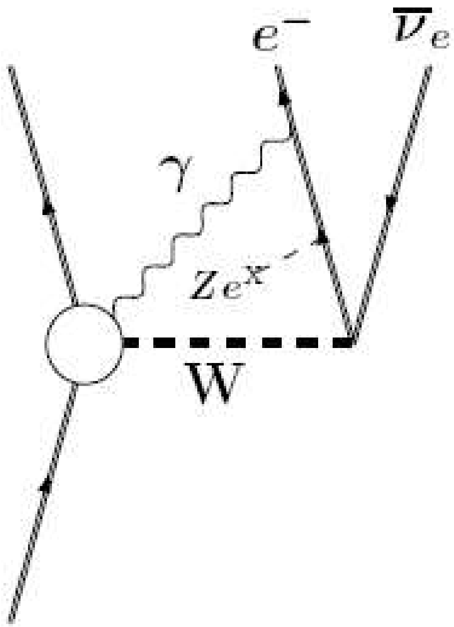
Result:

a) factor of 2 reduction in error assigned to the radiative correction

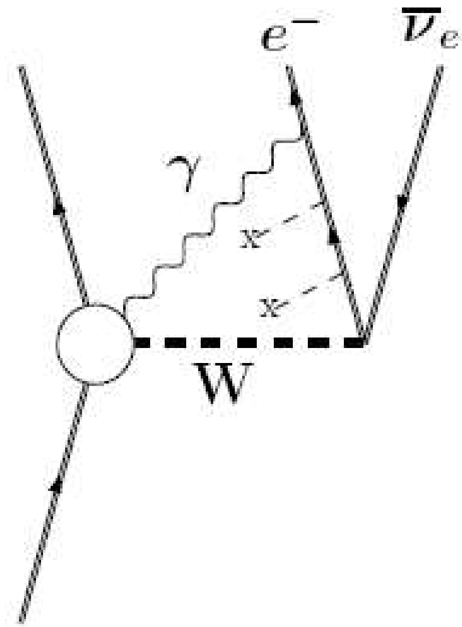
b) little change in magnitude of radiative correction

BEYOND ORDER α

1. QED Corrections



$\mathcal{O}(Z\alpha^2)$



$\mathcal{O}(Z^2\alpha^3)$

Sirlin, Zucchini, PRL **57**, 1994 (1986)

Jaus, Rasche, NP A**143**, 202 (1970); Aust.J.Phys. **39**,1 (1986)

Sirlin, PR D**35**, 3423 (1987)

2. Leading Log corrections, $\alpha^n \ln^n(m_Z/m_p)$

Czarnecki, Marciano, Sirlin PR D70, 093006 (2004)

$$\text{SD}: \quad 1 + \frac{\alpha}{2\pi}\left[4 \ln \frac{m_Z}{m_p}\right] \;\rightarrow\; S(m_p, m_Z) = 1.02248$$

$$\text{LD}: \quad 1 + \frac{\alpha}{2\pi}\left[3 \ln \frac{m_p}{2E_m}\right] \;\rightarrow\; L(2E_m, m_p) = 1.02673 \left[1 - \frac{2\alpha(m_e)}{3\pi} \ln \frac{2E_m}{m_e}\right]^{9/4}$$

where $S(m_p, m_Z)$ and $L(2E_m, m_p)$ are renormalization group summation of leading log.

$$S(m_p, m_Z) = \left(\frac{\alpha(m_c)}{\alpha(m_p)}\right)^{3/4} \left(\frac{\alpha(m_\tau)}{\alpha(m_c)}\right)^{9/16} \left(\frac{\alpha(m_b)}{\alpha(m_\tau)}\right)^{9/19} \left(\frac{\alpha(m_W)}{\alpha(m_b)}\right)^{9/20} \left(\frac{\alpha(m_Z)}{\alpha(m_W)}\right)^{36/17}$$

$\alpha^{-1}(0) = 137$, $\alpha^{-1}(m_e) = 137.089$, $\alpha^{-1}(m_p) \simeq 134$, $\alpha^{-1}(m_Z) \simeq 127.6$

SUMMARY

$$1 + \mathrm{RC} = (1 + \delta_{\mathrm{NS}}) \quad \times \quad (1 + \delta'_{\mathrm{R}}) \quad \times \quad (1 + \Delta_{\mathrm{R}}^{\mathrm{V}})$$

nuclear-structure dependent 2-body Born graphs

$\delta_{\mathrm{NS}} \simeq -0.04\%$

nucleus dependent trivially:

Z, Em

Em = maximum electron energy

$\delta'_{\mathrm{R}} \simeq 1.46\%$

nucleus independent

$\Delta_{\mathrm{R}}^{\mathrm{V}} \simeq 2.36\%$

$$1 + \delta'_{\mathrm{R}} = \left\{1 + \frac{\alpha}{2\pi}\left[\overline{\mathrm{g}}(\mathrm{E_m}) - 3\ln\frac{\mathrm{m_p}}{2\mathrm{E_m}}\right]\right\} \times \left\{\mathrm{L}(2\mathrm{E_m}, \mathrm{m_p}) + \frac{\alpha}{2\pi}[\delta_2 + \delta_3]\right\}$$

$$1 + \Delta_{\mathrm{R}}^{\mathrm{V}} = \mathrm{S}(\mathrm{m_p}, \mathrm{m_Z}) + \frac{\alpha}{\pi}\mathrm{C_{Born}} + \frac{\alpha(\mathrm{m_p})}{2\pi}\left[\ln\frac{\mathrm{m_p}}{\mathrm{m_A}} + \mathcal{A}_{\mathrm{g}}\right] + \mathrm{NLL}$$

Czarnecki, Marciano and Sirlin, PR D70, 093006 (2005).

ISOSPIN-SYMMETRY BREAKING CORRECTION

Beta decay in nuclei described by one-body operator

$$\mathbf{F} = \sum_{\alpha,\beta} \langle \alpha | \tau_+ | \beta \rangle \, \hat{a}^\dagger_\alpha \, \hat{a}_\beta$$

Matrix element in many-body system

$$\langle M_F \rangle = \sum_{\alpha,\beta} \langle f | \hat{a}^\dagger_\alpha \, \hat{a}_\beta | i \rangle \, \langle \alpha | \tau_+ | \beta \rangle$$

shell-model one-body density matrix elements evaluated in many-body states

single-particle matrix elements

$$\Omega_\alpha = \delta_{\alpha,\beta} \int_0^\infty R^{\text{proton}}_{n_\alpha l_\alpha} \, R^{\text{neutron}}_{n_\beta l_\beta} \, r^2 \, dr$$

Define:

$$\langle M_F \rangle^2 = 2\,(1 - \delta_C) \quad ; \quad \delta_C = \delta_{C1} + \delta_{C2}$$

Isospin Mixing ~0.1%

Radial Overlap ~0.4%

Radial Overlap: contribution constrained by:

asymptotic radial function for proton matched to proton separation energy, S_p, in decaying nucleus

$$R(r) \sim e^{-\alpha r} \qquad \alpha^2 = \frac{2mS}{\hbar^2}$$

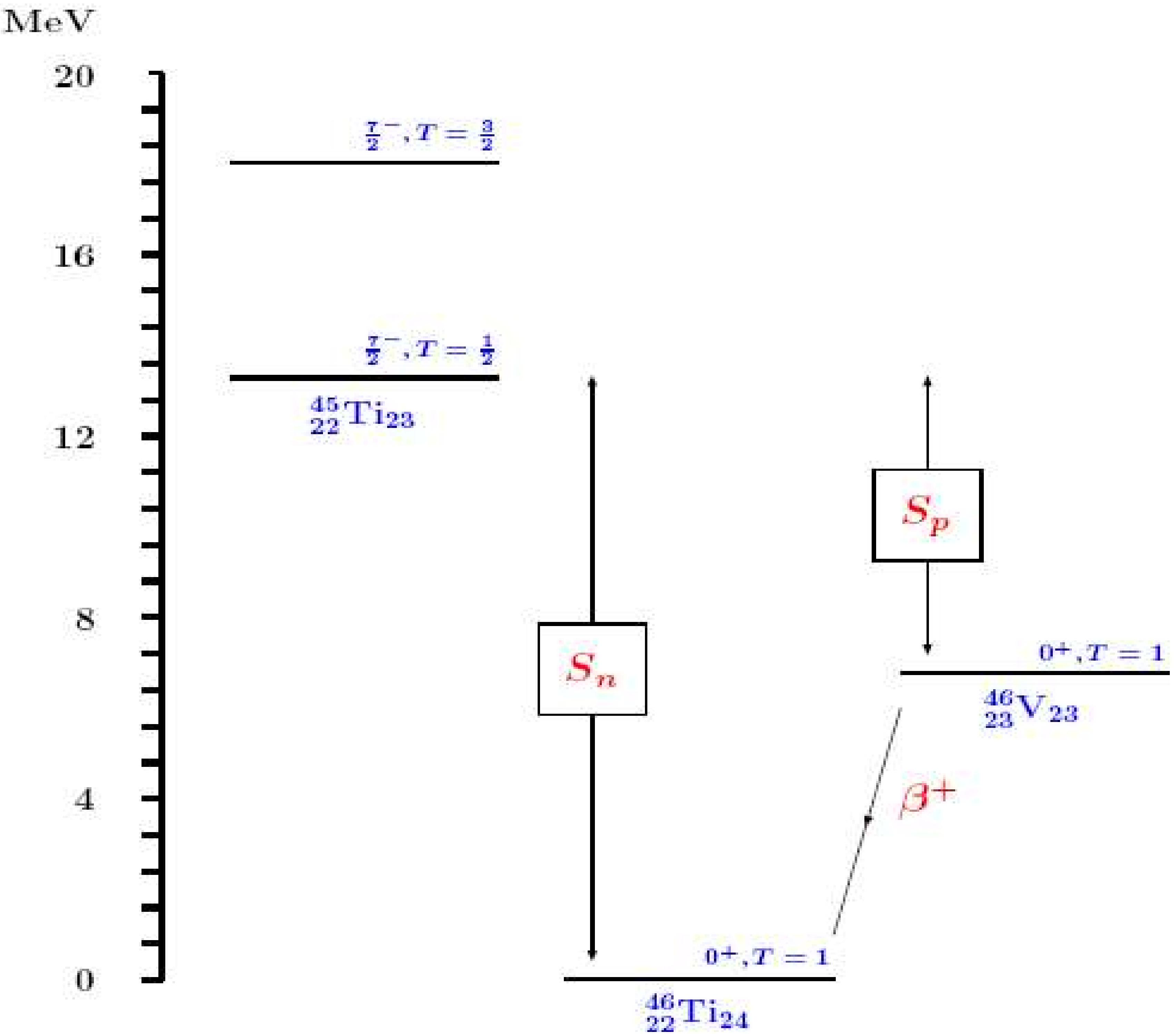
ditto neutron, matched to neutron separation energy, S_n, in daughter nucleus

Towner-Hardy: used Saxon-Woods functions PR C66, 035501 (2002)

Ormand-Brown: used Hartree-Fock functions PR C52, 2455 (1995)

$$\Omega_\alpha = \delta_{\alpha,\beta} \int_0^\infty R^{\mathrm{proton}}_{n_\alpha l_\alpha} R^{\mathrm{neutron}}_{n_\beta l_\beta} r^2 \, dr$$

Radial integral departs from the value of unity because proton and neutron radial functions are matched to different separation energies. Further these separation energies depend on the parentage expansions.


MeV
20
16
12
8
4
0
$\frac{7}{2}^-, T = \frac{3}{2}$
$\frac{7}{2}^-, T = \frac{1}{2}$
$^{45}_{22}Ti_{23}$
S_n
S_p
$0^+, T = 1$
$^{46}_{23}V_{23}$
β^+
$0^+, T = 1$
$^{46}_{22}Ti_{24}$

Radial Overlap (continued)

$$\langle M_F \rangle^2 = 2(1 - \delta_{C_2})$$

$$\langle M_F \rangle = \sum_{\alpha,\beta} \langle f | \hat{a}^\dagger_\alpha \, \hat{a}_\beta | i \rangle \, \langle \alpha | \tau_+ | \beta \rangle$$

$$= \sum_{\alpha,\pi} \langle f | \hat{a}^\dagger_\alpha | \pi \rangle \, \langle \pi | \hat{a}_\alpha | i \rangle \, \Omega^\pi_\alpha$$

$$\delta_{\alpha,\beta} \, \Omega_\alpha$$

Use shell model to calculate these parentage coefficients. Consider:

$$| \, j^n_\alpha; J = 0, T = 1 \rangle \qquad | \, j^{n-1}_\alpha; J = j_\alpha, T = 1/2 \text{ or } 3/2 \rangle$$

$$\delta_{C2} \simeq \frac{n+4}{3}(1 - \Omega^<_\alpha) - \frac{n-1}{3}(1 - \Omega^>_\alpha)$$

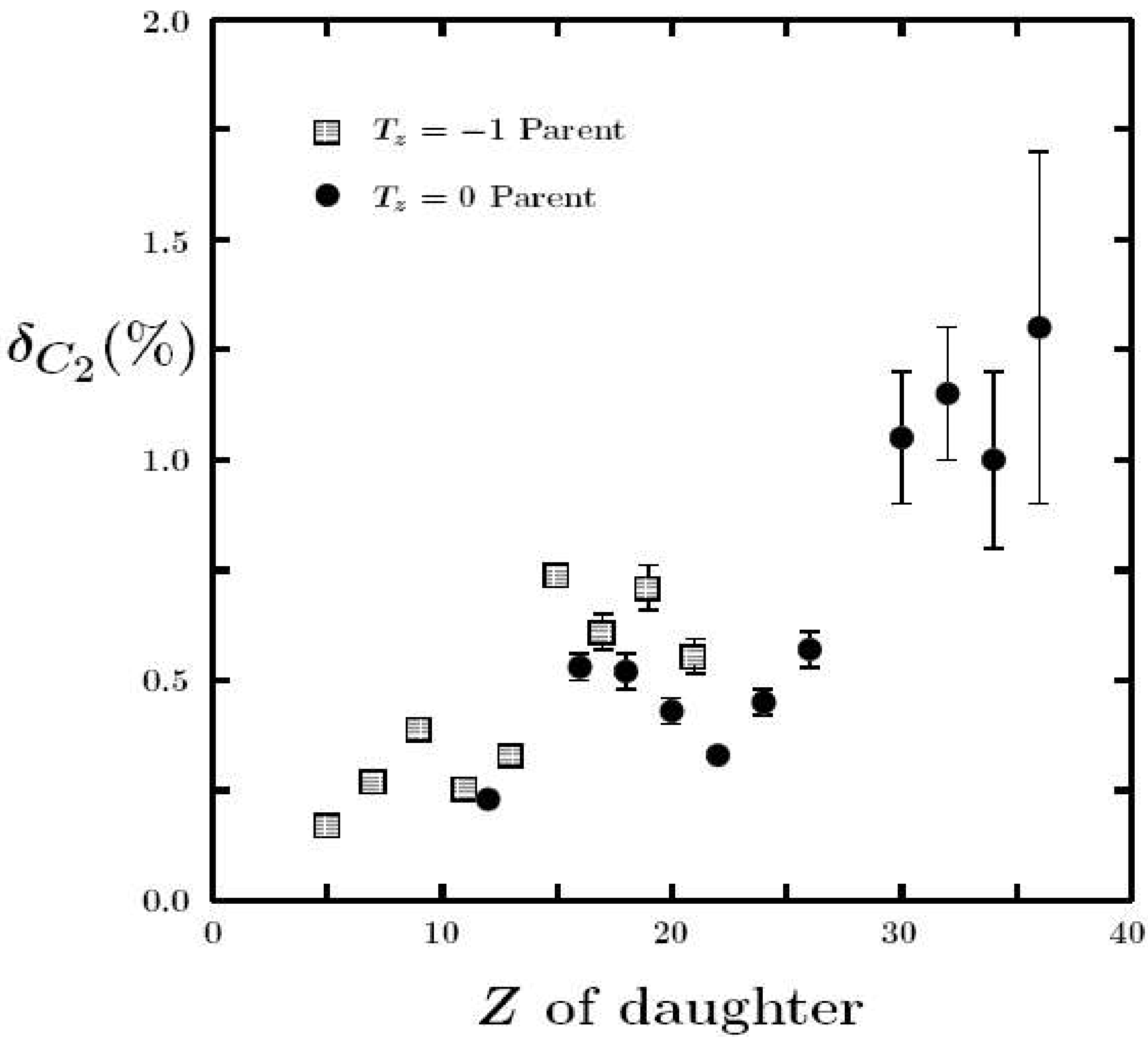
$T = 1/2$ $\qquad$ $T = 3/2$

Also contribution from core orbitals: $| \, j^n_\alpha \, j^{-1}_c; J = j_c, T = 1/2 \text{ or } 3/2 \rangle$

$$\delta_{C2} \simeq \sum_c \frac{4j_c + 2}{3} \left[(1 - \Omega^<_c) - (1 - \Omega^>_c) \right]$$

Core contribution $\rightarrow 0$ as $\Omega^<_c \longrightarrow \Omega^>_c \longrightarrow 1$ as separation energies increase.

Saxon-Woods, TH02



Isospin Mixing:

Introduce charge-dependent terms in shell-model Hamiltonian:

Constrain the calculation to reproduce coefficients of IMME equation

$$M(A, T, T_z) = a + bT_z + cT_z^2$$

Require calculation to fit experimental b and c coefficients

Then compute: $\langle M_F \rangle \Longrightarrow \delta_{C_1}$

With isospin symmetry:

Parent state can only decay to its isospin analogue state

With isospin-symmetry breaking:

Parent state can now decay (weakly) to non-analogue states.

Calculation, besides yielding δ_{C1}, predicts these weak branches.

Subject to experimental test.

Test of calculation of δ_{C1}

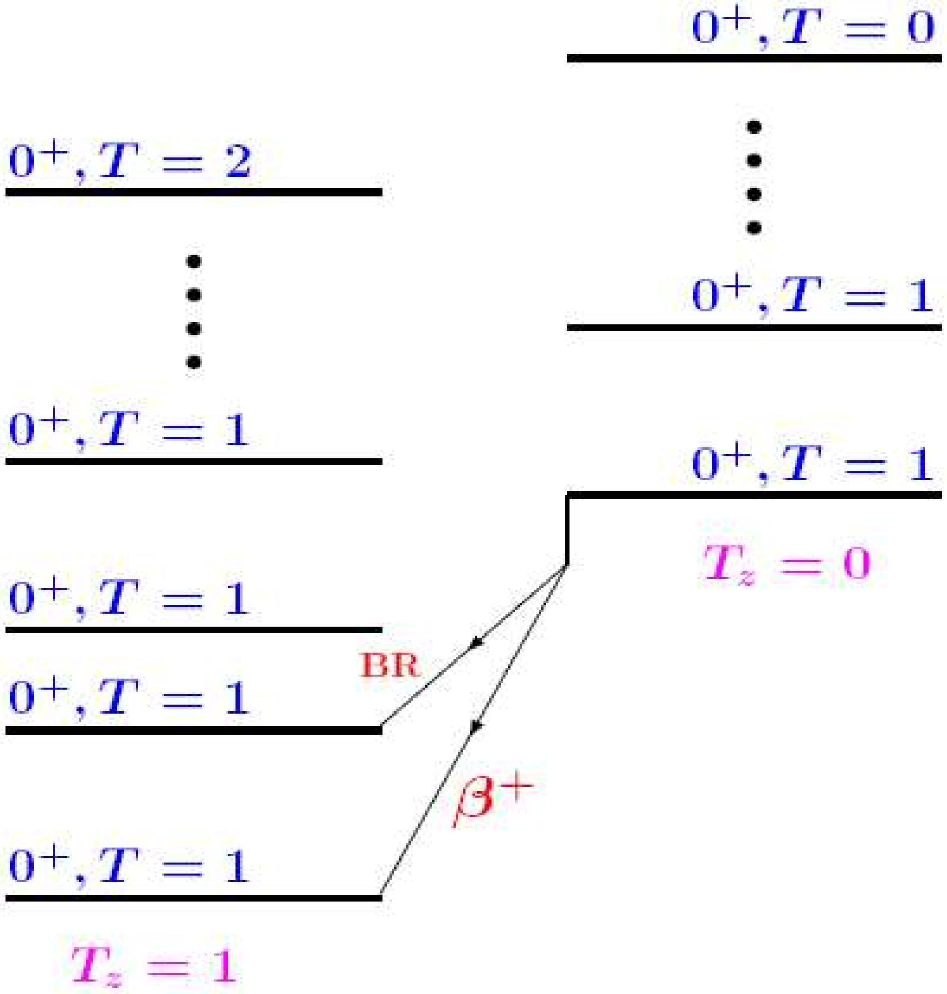
Branches to non-analogue 0+ states

$$\langle \mathrm{M}_0 \rangle^2 = 2\,(1 - \delta_{C_1})$$

$$\langle \mathrm{M}_1 \rangle^2 = 2\delta^1_{C_1}$$

$$\langle \mathrm{M}_2 \rangle^2 = 2\delta^2_{C_1}$$

⋮



$$\sum_{n=1} \delta^n_{C_1} = \delta_{C_1} + \text{corrections due to mixing with } \mathrm{T} = 0, 2, 3 \ldots$$

$$\mathrm{BR} = \frac{\mathrm{f}_1}{\mathrm{f}_0}\, \delta^1_{C_1}$$

Experimental non-analogue branching ratios

	Theory	Experiment		
	$\delta_{C_1}^1$ (%)	$\delta_{C_1}^1$ (%)	BR(ppm)	
^{38}K	0.090(30)	$\le$ 0.280 $\le$ 0.120	$\le$ 19 $\le$ 8	Ha94 prelim
^{42}Sc	0.020(20)	0.040(9)	59(13)	DR85
^{46}V	0.035(15)	0.053(5)	39(4)	Ha94
^{50}Mn	0.045(20)	<0.016	<3	Ha94
^{54}Co	0.040(20)	0.035(5)	45(6)	Ha94
^{62}Ga	0.085(20)	$\le$0.040(15)	$\le$80(30)	Hy06
^{74}Rb	0.050(30)	<0.070	<540	Pi03

Theory is within a factor of two of these small experimental quantities.

SUMMARY: Isospin-symmetry breaking

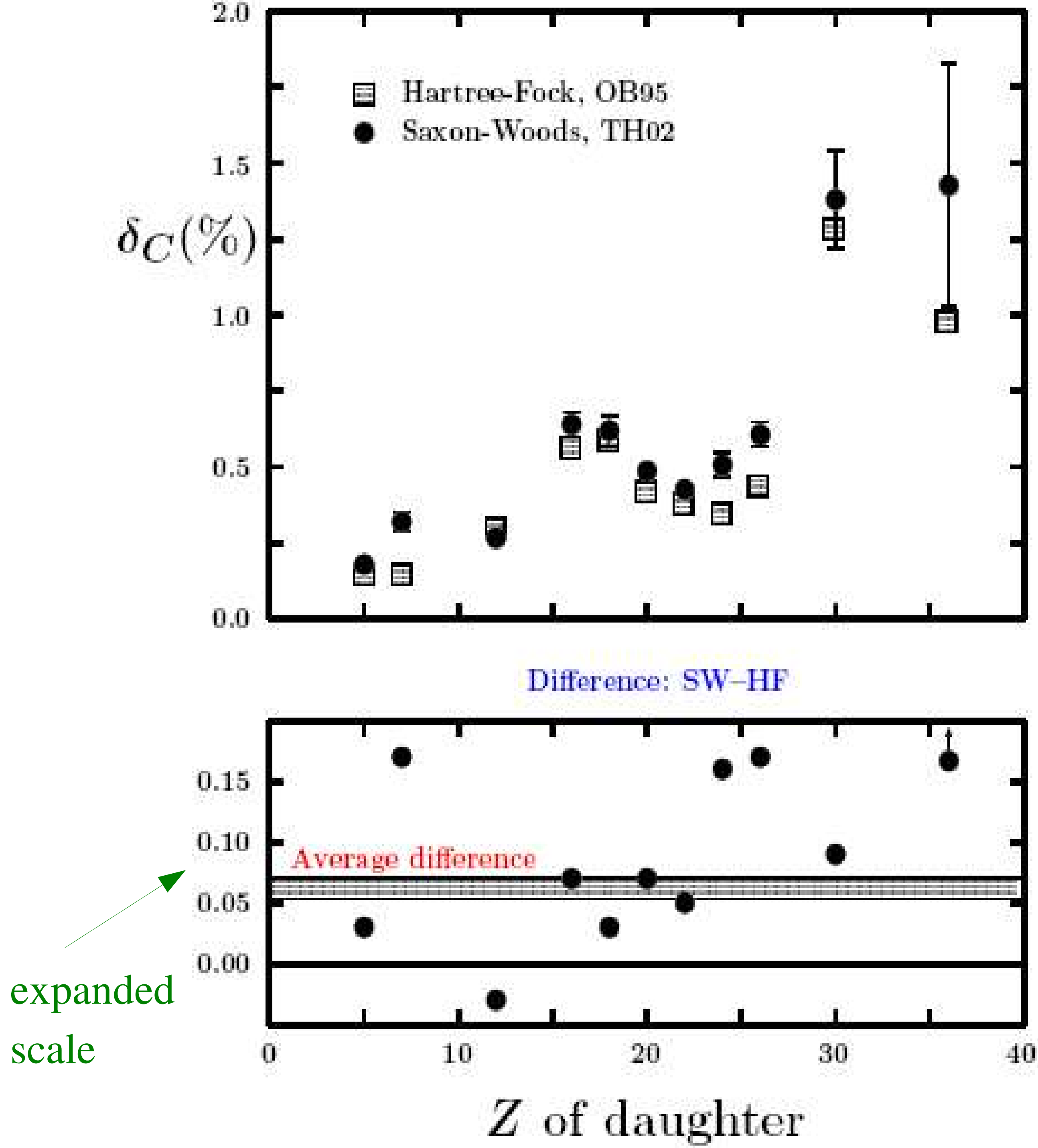
$$\delta_C = \delta_{C1} + \delta_{C2}$$

	isospin mixing	radial overlaps
Typical values (in percent)		
Avg (A = 10 --> 54):	0.05	0.40
Avg (A = 62 --> 74):	0.25	1.10

Two calculations (constrained by separation energies and fits to IMME coefficients):

TH02: Saxon-Woods radial functions — PR C66, 035501 (2002)

OB95: Hartree-Fock radial functions — PR C52, 2455 (1995)

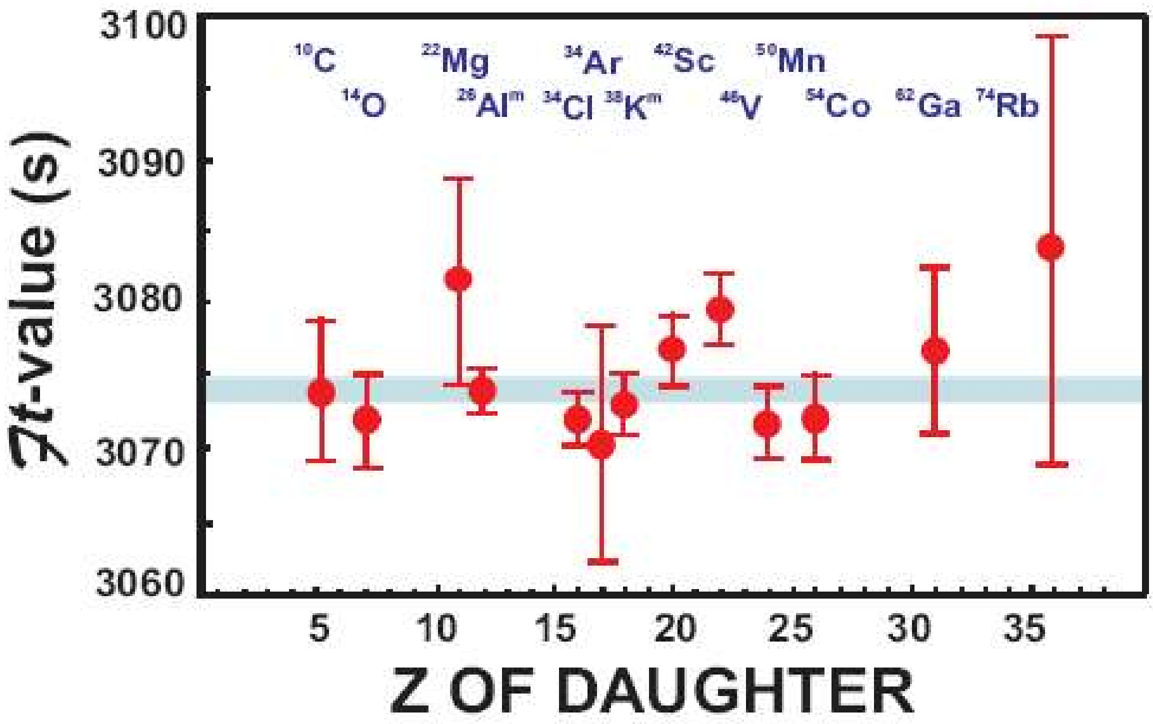


Both calculations produce similar nucleus to nucleus variations.

Results in a systematic uncertainty of 0.9s in Ft of 3070s.

CVC Test: $\mathcal{F}t =$ constant

$$\mathcal{F}t \equiv ft(1+\delta'_R)\left(1-(\delta_C-\delta_{NS})\right) = \frac{K}{2G_F^2V_{ud}^2(1+\Delta_R)}$$



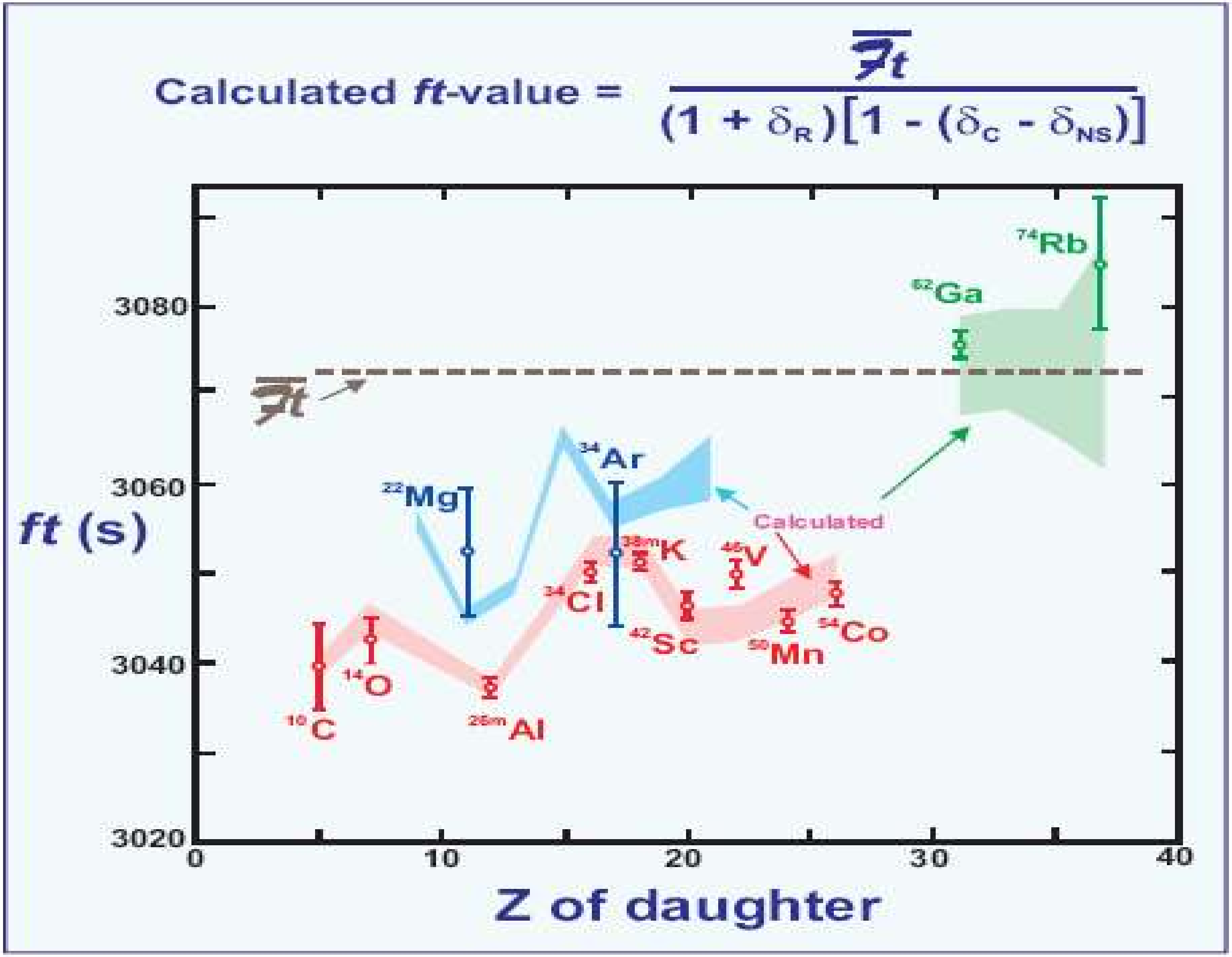
$$\text{Average } \overline{\mathcal{F}t} = 3073.9 \pm 0.8 \pm 0.9 \text{ s}$$

statistical

systematic
difference in δ_C

$$\chi^2/\nu = 0.9$$

Alternative Strategy: Take the CVC test as a given, and use it to probe the nucleus-to-nucleus variations in the corrections.



CKM Unitarity Test

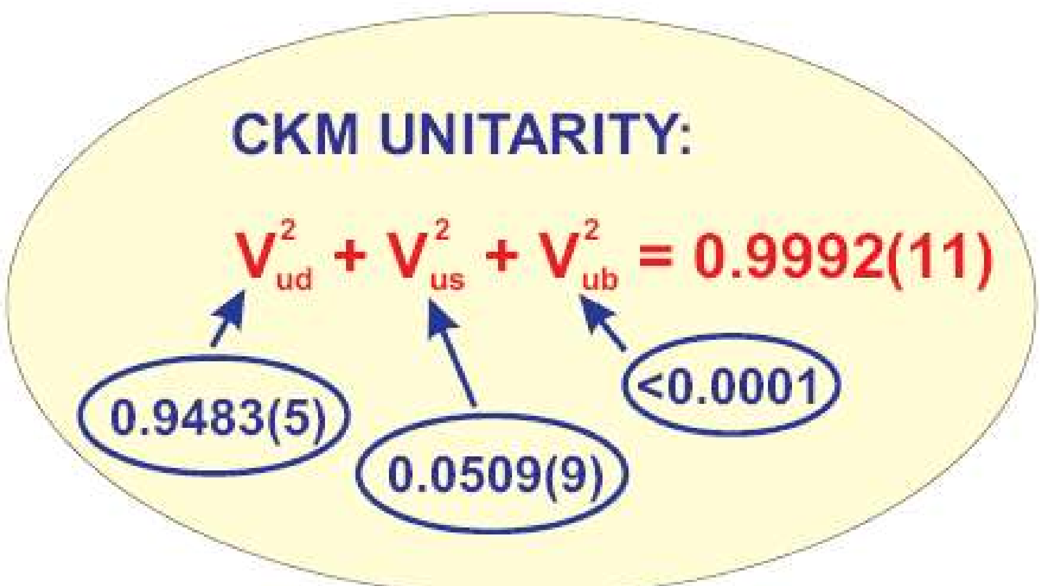
From: $\text{Average } \overline{\mathcal{F}t} = 3073.9 \pm 0.8 \pm 0.9 \text{ s}$

And: $\Delta_R = (2.361 \pm 0.038)\%$

Yields: $V_{ud} = 0.97378(27)$

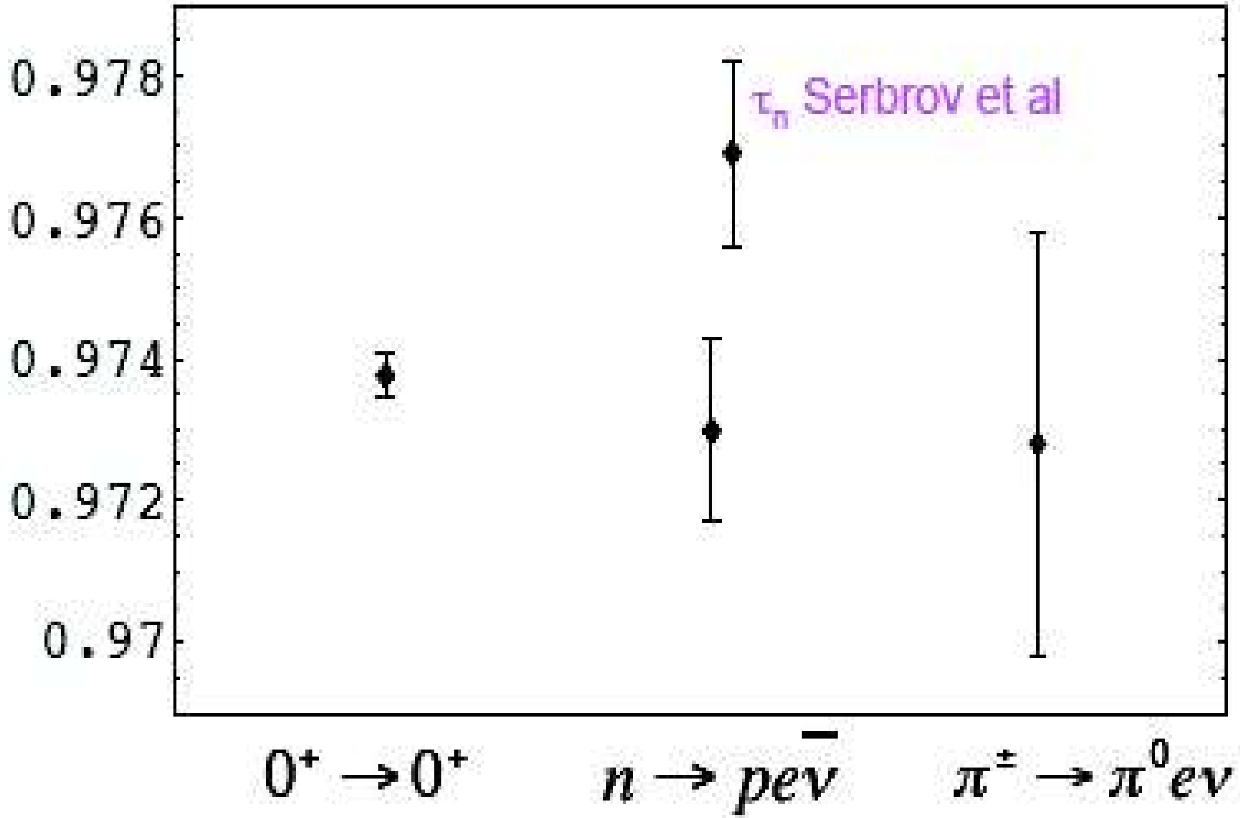
V_{ud} also obtained from neutron and pion decay

And:



Within the estimated errors: CKM unitarity fully satisfied

Summary on V_{ud}



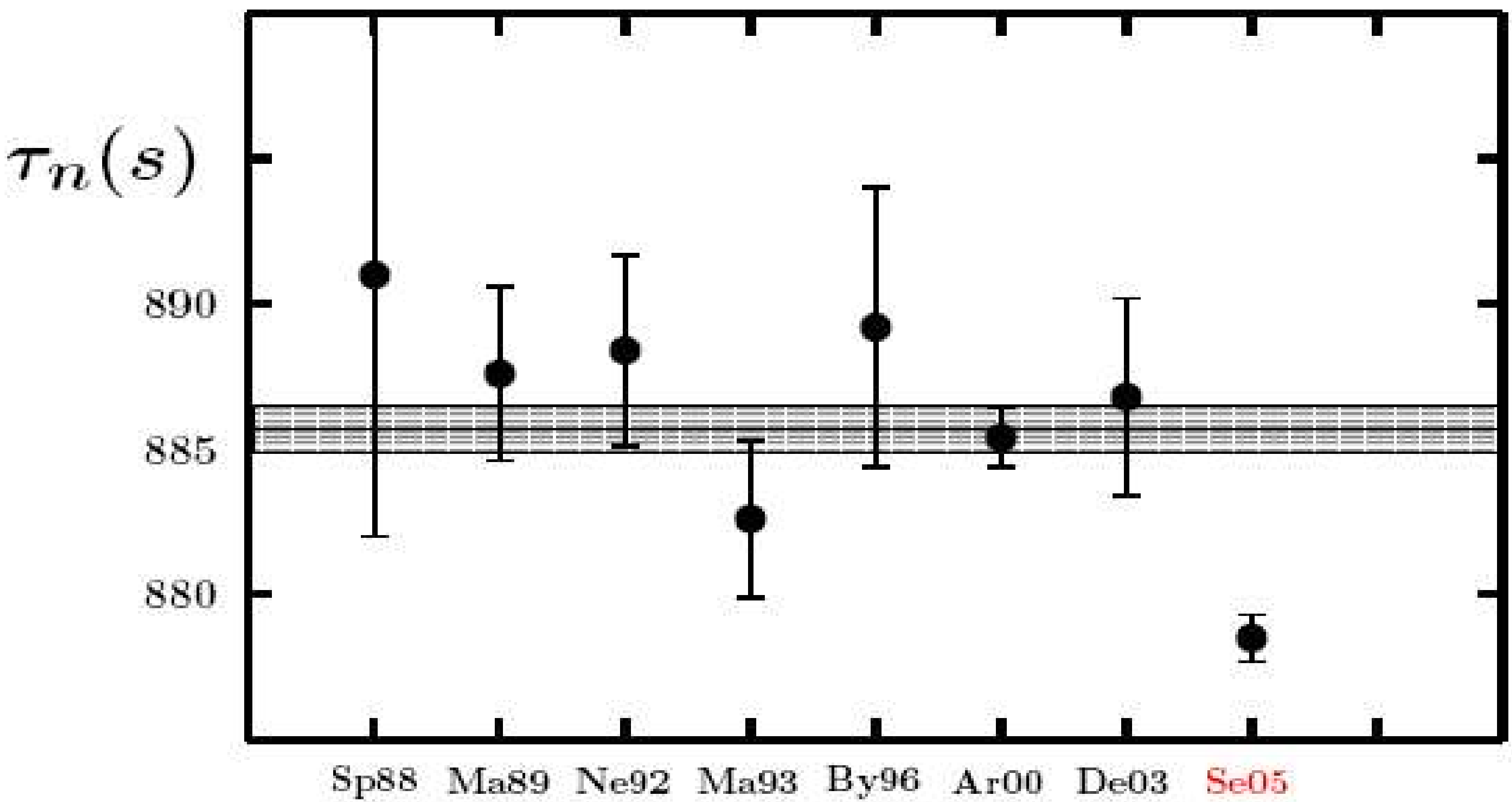
- $0^+ \to 0^+$ provides best determination
- In 3-5 years n-β will catch up: stay tuned !

NEUTRON BETA DECAY

Serious problem with lifetime:

$$\tau_n(s) = \begin{cases} 878.5 \pm 0.7_{\rm stat} \pm 0.3_{\rm syst} & \text{Serebrov et.al. PL B605, 72} \\ 885.7 \pm 0.8 & \text{PDG06} \end{cases}$$

Differs from World Average by 6.5σ.



V_{us} from $K_{\ell 3}$ rates

$$\Gamma(K_{\ell 3}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+(0)|^2 I(\lambda)(1+2\Delta_K^{SU(2)}+2\Delta_K^{EM})$$

$C_K^2 = 1/2$ for K^+, $= 1$ for K^0

S_{EW} = universal short-distance radiative correction

Inputs from theory:

Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

Form-factor correction for $SU(2)$ breaking

Form-factor correction for long-distance EM effects

Inputs from experiment:

Rates with well-determined treatment of radiative decays:

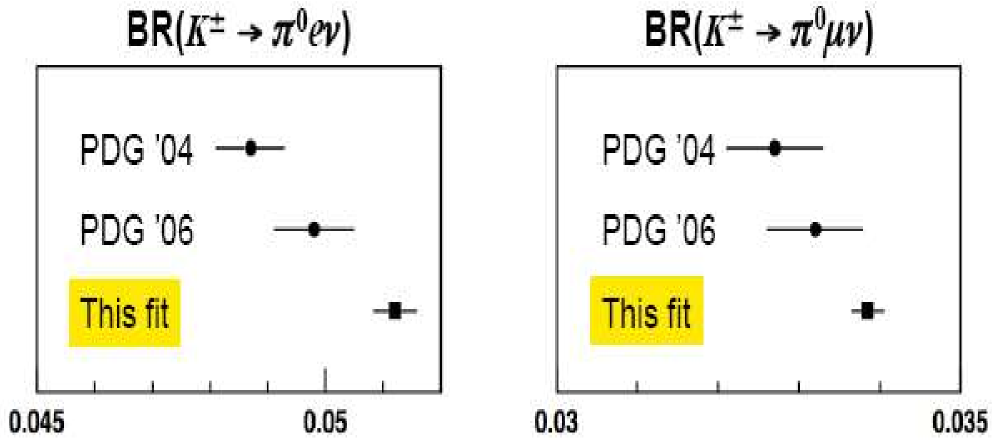
- Branching ratios
- Kaon lifetimes

Integral of form-factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

New fit to Kaon branching ratios

$K^{\pm}$ BR



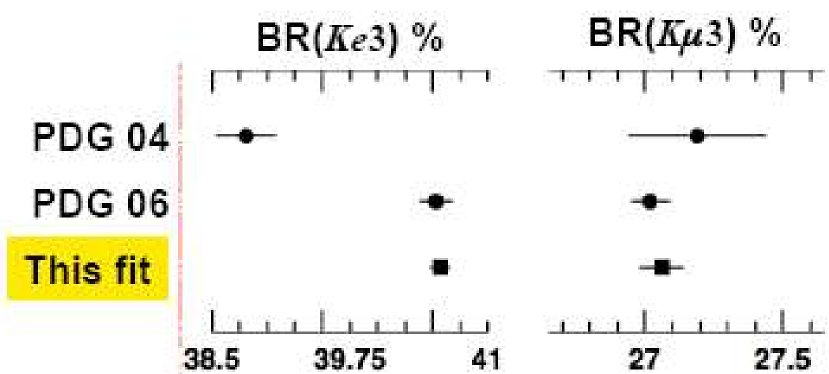
preliminary results from KLOE, ISTRA+, and NA48/2

K_S BR

BR($K_S \to \pi e\nu$) = 7.046(91) × 10^{-4}

KLOE

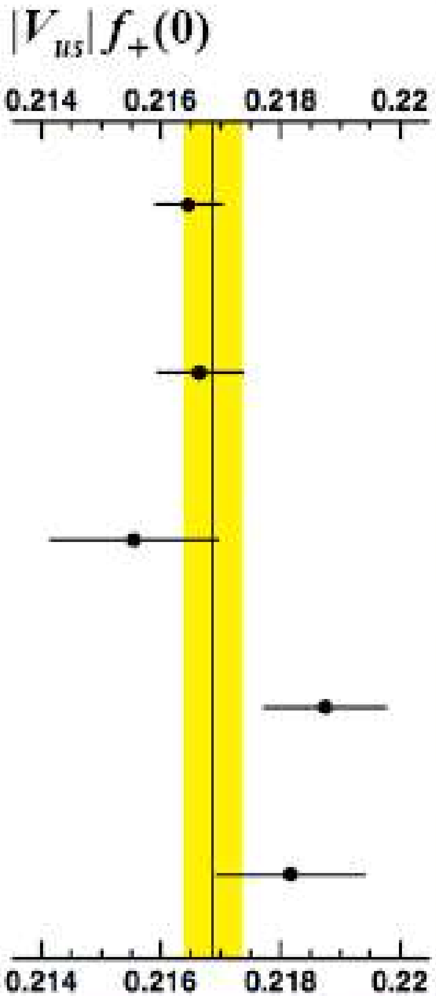
K_L BR



KTeV KLOE

NA48 preliminary

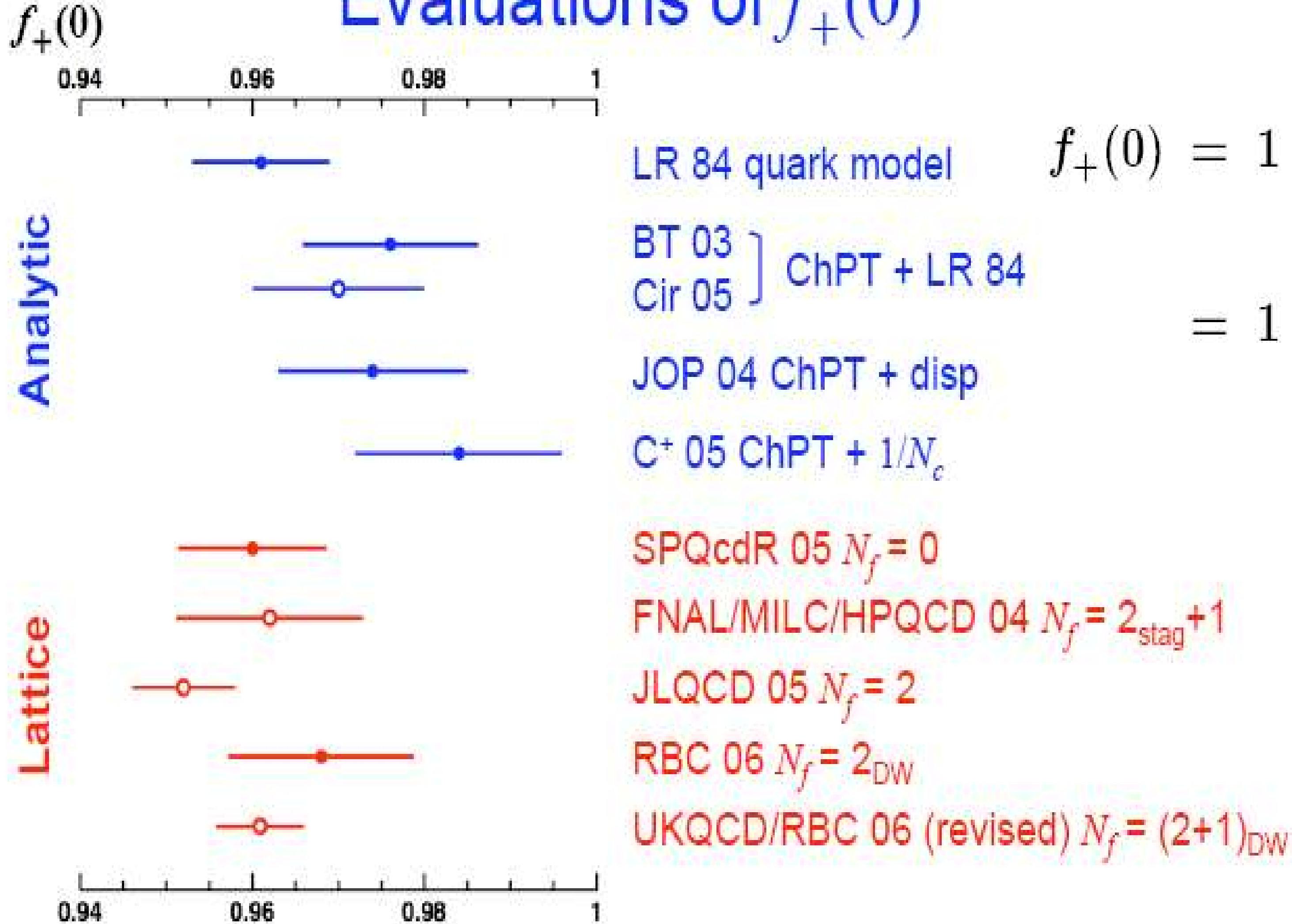
$|V_{us}|\,f_+(0)$ from K_{l3} data



			Approx. contrib. to % err from:			
		% err	BR	τ	Δ	Int
$K_L e3$	0.21646(59)	0.27	0.09	0.19	0.10	0.11
$K_L \mu 3$	0.21665(71)	0.33	0.12	0.19	0.15	0.18
$K_S e3$	0.21555(143)	0.66	0.65	0.02	0.10	0.11
$K^{\pm} e3$	0.21875(104)	0.47	0.37	0.07	0.27	0.11
$K^{\pm} \mu 3$	0.21817(125)	0.57	0.30	0.07	0.45	0.18

Average: $|V_{us}|\,f_+(0)$ = 0.21686(49) χ^2/ndf = 5.0/4 (29.0%)

Evaluations of $f_+(0)$



$$f_+(0) = 1 + f_2 + f_4$$

$$= 1 - 0.023 \pm ?$$

Leutwyler & Roos estimate (LR 84) still widely used: $f_+(0)$ **= 0.961(8)**

Lattice evaluations generally agree well with this value

CONCLUSIONS

- Superallowed β decay currently yields most precise value of V_{ud}, limited by theory uncertainties.

  V_{ud} = 0.97378(27)

- Value of V_{ud} proving to be very robust.

- Neutron and pion decays yield V_{ud} consistent with nuclear result, but with larger experimental errors. This will change in $3 - 5$ years.

- Much activity in nuclear physics focussed on reducing errors still further via tests of structure-dependent corrections.

CONCLUSIONS (continued)

- Experimental uncertainty in $f_+(0)|V_{us}|$ currently at 0.2% with good consistency.

- Dominant uncertainty in V_{us} is still from estimate of $SU(3)$-symmetry breaking: $f_+(0)$.

- With benchmark Leutwyler-Roos value: $V_{us} = 0.2257(20)$

- CKM Unitarity now verified to 0.1% − dominant error from V_{us}.