Status of lattice calculations of V_{us} (and possibly other CKM elements)

An informal review, based on results of FNAL, MILC, HPQCD, NPLQCD and RBC/UKQCD collaborations

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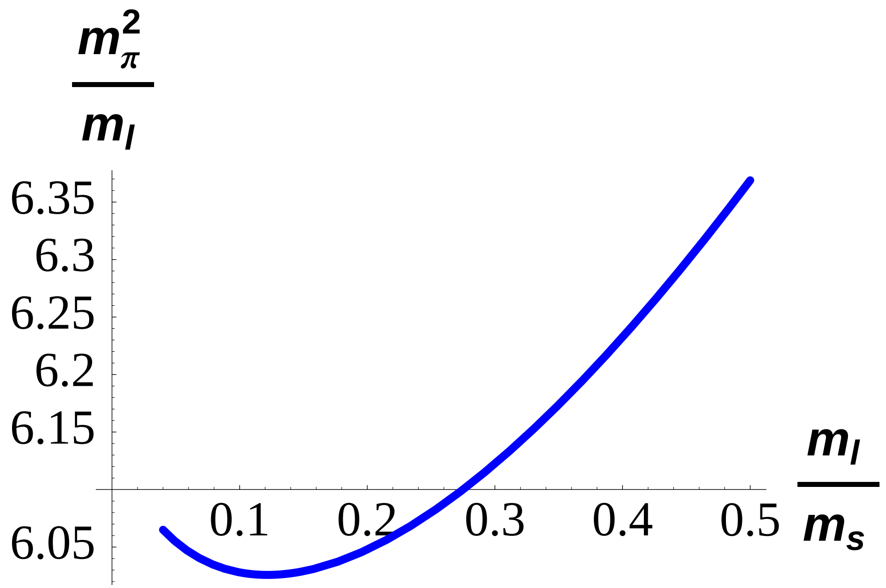
Outline

- Basics of method (on board)
- Lattice systematics—general considerations (on board)
- Choices of fermion discretization
  - MILC improved staggered
  - Domain Wall fermion (DWF)
- Results: present and future
- Constraints on other CKM elements from lattice calculations

Why need $m_\ell^{\rm lat} \approx m_s^{\rm phys}/10$?

For percent-level accuracy (e.g. in f_π) require [MILC/FNAL/HPQCD improved staggered results]

$$m_\ell^{\rm lat} \sim \frac{m_s^{\rm phys}}{10} \approx 3 m_\ell^{\rm phys}$$



Fermion discretizations

Precision results relevant to V_{us} primarily from two lattice fermion actions

- Improved staggered fermions ("asqtad" [MILC,Lepage])
  - \+ Computationally fast
  - − Rooting trick ("ugly"? [SS Lattice 2006 review])
  - − Complicated "staggered chiral perturbation theory" fits needed to take chiral-continuum limits [SS & Lee, Aubin & Bernard]
- Domain-wall fermions (DWF) [Kaplan,Shamir]
  - − 10-20 times slower than staggered
  - \+ Almost exact chiral symmetry
  - \+ Correct number of fermions
- Hybrid DWF valence on staggered sea quarks also used [NPLQCD]

Recent advance:

- RHMC provides $\gtrsim 6$ speed up for both fermion types [Clark & Kennedy]

Other fermion types will also be used in future

- Wilson-like, twisted mass, overlap

MILC staggered ensemble: **past** and present

MILC0: completed 2003-4. Coarse and Fine lattices

a (fm)	am'_ℓ / am'_s	L(fm)	dims.	# lats.	$m_\pi L$
≈ 0.12	0.03 / 0.05	2.4	$20^3 \times 64$	564	7.6
≈ 0.12	0.02 / 0.05	2.4	$20^3 \times 64$	484	6.2
≈ 0.12	0.01 / 0.05	2.4	$20^3 \times 64$	658	4.5
≈ 0.12	0.01 / 0.05	3.4	$28^3 \times 64$	241	6.3
≈ 0.12	0.007 / 0.05	2.4	$20^3 \times 64$	493	3.8
≈ 0.12	0.005 / 0.05	2.9	$24^3 \times 64$	527	3.8
≈ 0.09	0.0124 / 0.031	2.5	$28^3 \times 96$	531	5.8
≈ 0.09	0.0062 / 0.031	2.5	$28^3 \times 96$	583	4.1

MILC staggered ensemble: past and **present**

MILC1: completed FY07. Extra-fine, Medium-coarse, and extended **Coarse**/**Fine**

a (fm)	am'_ℓ / am'_s	L(fm)	dims.	# lats.	$m_\pi L$
≈ 0.15	0.0290 / 0.0484	2.4	$16^3 \times 48$	600	6.6
≈ 0.15	0.0194 / 0.0484	2.4	$16^3 \times 48$	600	5.5
≈ 0.15	0.0097 / 0.0484	2.4	$16^3 \times 48$	600	3.9
≈ 0.15	0.00484 / 0.0484	2.4	$20^3 \times 48$	600	3.5
≈ 0.12	0.03 / 0.05	2.4	$20^3 \times 64$	564	7.6
≈ 0.12	0.02 / 0.05	2.4	$20^3 \times 64$	484	6.2
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≈ 0.12	0.005 / 0.05	2.9	$24^3 \times 64$	527	3.8
≈ 0.12	**0.03 / 0.03**	**2.4**	**$20^3 \times 64$**	**350**	**7.6**
≈ 0.12	**0.01 / 0.03**	**2.4**	**$20^3 \times 64$**	**349**	**4.5**
≈ 0.09	0.0124 / 0.031	2.5	$28^3 \times 96$	531	5.8
≈ 0.09	0.0062 / 0.031	2.5	$28^3 \times 96$	583	4.1
≈ 0.09	**0.0031 / 0.031**	**3.6**	**$40^3 \times 96$**	**473**	**4.4**
≈ 0.06	0.0072 / 0.018	2.9	$48^3 \times 144$	550	6.3
≈ 0.06	0.0036 / 0.018	2.9	$48^3 \times 144$	350^*	4.5

DWF ensemble: **present** and possible future

FY06-07: Coarse lattice at two volumes; Fine lattices underway†

a (fm)	m_ℓ/m_s	Size	L_5	L (fm)	MC traj.	TF-Yr	Label
0.12	0.59	$16^3 \times 32$	16	2.0	4050		DWF0
0.12	0.33	$16^3 \times 32$	16	2.0	4000		DWF0
0.12	0.3	$24^3 \times 64$	16	3.0	9000 †	0.7	DWF1
0.12	0.19	$24^3 \times 64$	16	3.0	9000 †	0.8	DWF1
0.09	0.20	$32^3 \times 64$	16	3.0	4500 †	1.3	DWF1

Speed-up due to RHMC algorithm $\gtrsim 6$

DWF ensemble: present and possible **future**

By FY09: **Major milestone**—two lattice spacings with light quark masses

a (fm)	m_ℓ/m_s	Size	L_5	L (fm)	MC traj.	TF-Yr	Label
0.12	0.59	$16^3 \times 32$	16	2.0	4050		DWF0
0.12	0.33	$16^3 \times 32$	16	2.0	4000		DWF0
0.12	0.3	$24^3 \times 64$	16	3.0	9000	0.7	DWF1
0.12	0.19	$24^3 \times 64$	16	3.0	9000	0.8	DWF1
0.09	0.20	$32^3 \times 64$	16	3.0	4500	1.3	DWF1
0.09	**0.136**	$\mathbf{32^3 \times 64}$	**16**	**3.0**	**4500**	**1.4**	**DWF2**
0.09	**0.136**	$\mathbf{48^3 \times 64}$	**16**	**4.4**	**5000**	**7.0**	**DWF2**
0.09	0.065	$48^3 \times 64$	16	4.4	5000	8.6	DWF3
0.09	$1/27$	$64^3 \times 128$	24	5.9	10000	230	DWF5
0.06	0.144	$48^3 \times 64$	16	3.0	10000	18	DWF3
0.06	0.084	$64^3 \times 128$	16	4.0	10000	130	DWF4
0.06	$1/27$	$96^3 \times 128$	16	5.9	10000	680	DWF6

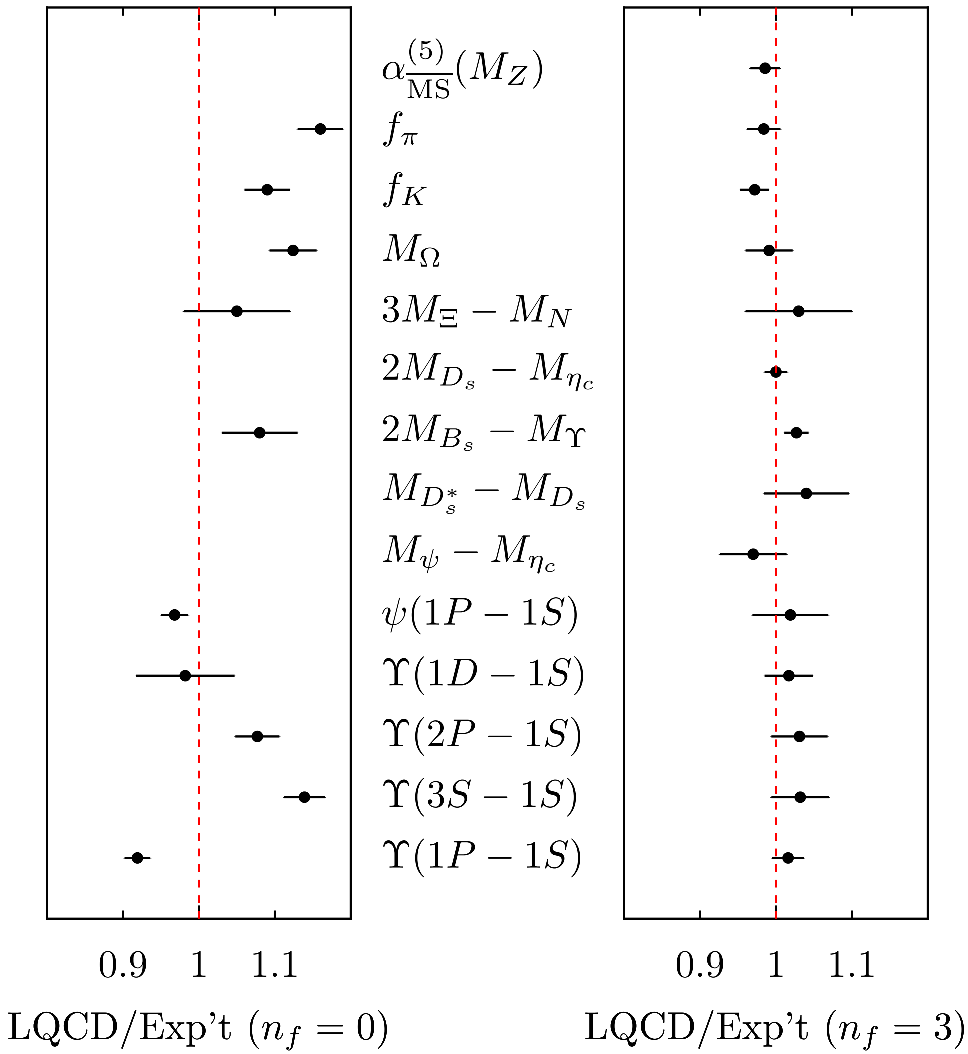
Speed-up due to RHMC algorithm $\gtrsim 6$

Summary of present situation

- Staggered fermions:
  - Ensemble allows all errors to be controlled
  - Successful validation (and predictions of f_D, $D \to K$ form factor, and $m(B_c)$) support validity of rooting trick
- Domain-wall fermions:
  - Present results with only one lattice spacing, and only with $m_\ell / m_s \geq 0.39$
  - Present results do not control all errors
  - Over next 2-3 years, will have ensemble allowing all errors to be controlled

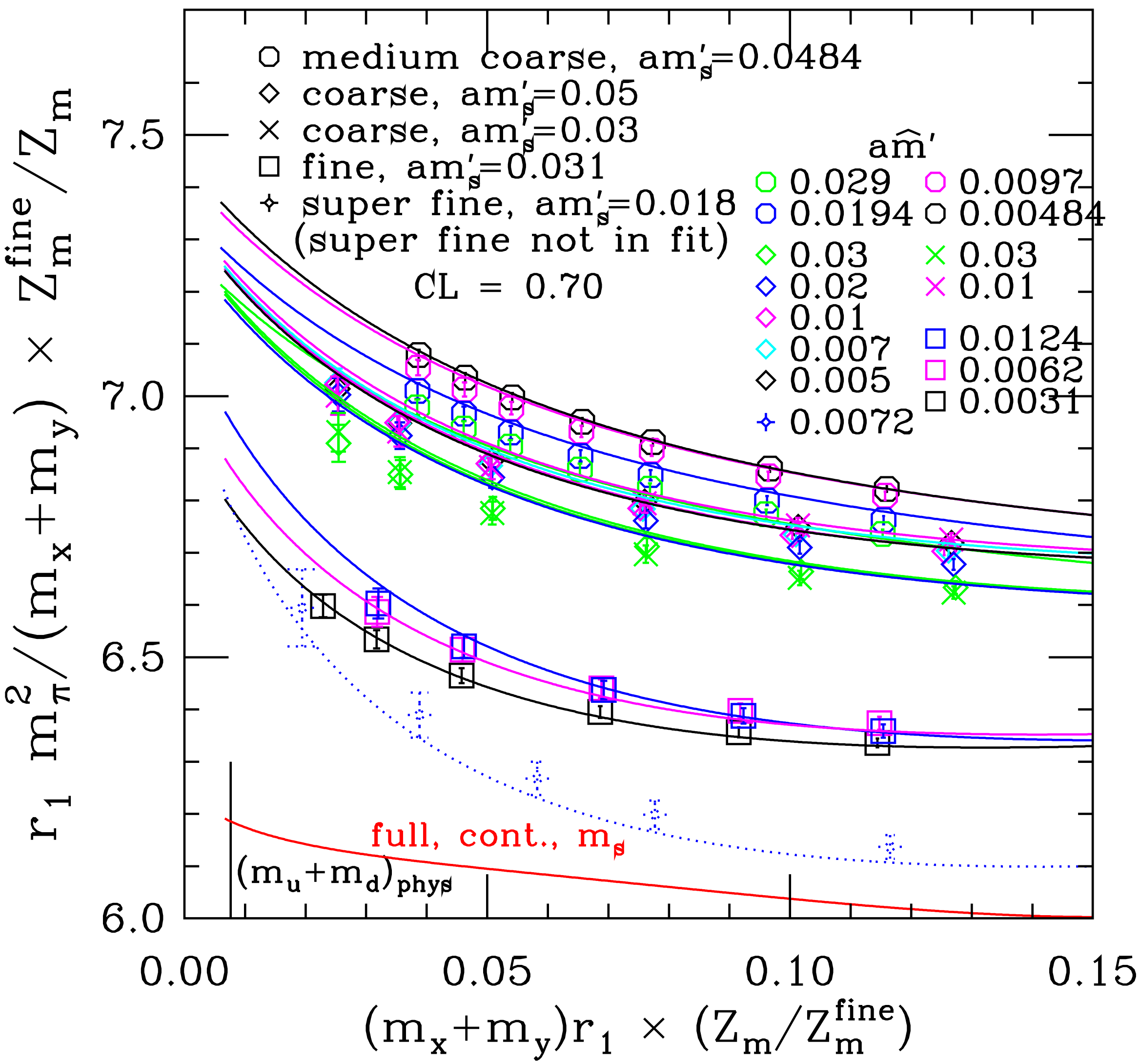
Staggered results: validation

Updated (2006) comparison with experiment for “gold-plated” quantities: [Lepage]

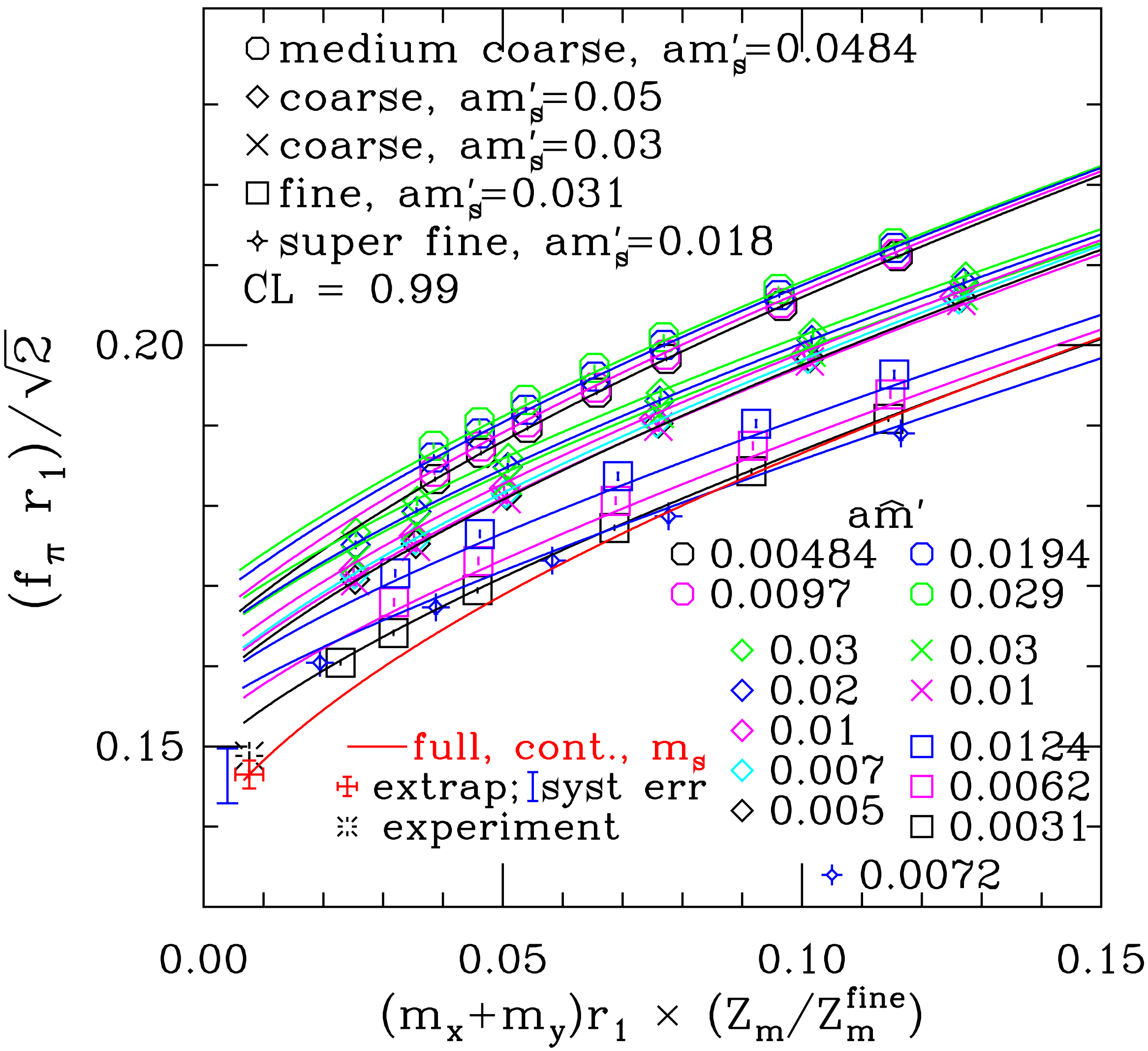


2006 staggered results: spectrum [MILC]

- Partially quenched results for $m_\pi^2/(m_{av}^{val})$
- Part of global fit to PGB properties
- Data shows chiral logarithms
- Super-fine lattice results agree with predictions
- Partially quenched staggered chiral perturbation theory describes data well



2006 staggered results: decay constants [MILC]



Results for V_{us} and implications for unitarity

- Present staggered fermion result [MILC06]

$$f_K/f_\pi = 1.208^{+7}_{-14} \quad \Rightarrow \quad |V_{us}| = 0.2223^{+26}_{-14} \quad \text{[cf. PDG } 0.2257(21)\text{]}$$

  - Error dominated by chiral/continuum extrap, with statistical error 0.2%

- DWF valence on staggered sea *at single lattice spacing* [NPLQCD]
  $f_K/f_\pi = 1.218^{+11}_{-24}$
- Thus f_K/f_π method competitive with that using $K \to \pi\ell\nu$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9977(5)(12)(0) \qquad \text{[cf. } 0.9992(5)(9)(0) \text{ PDG]}$$

- By end of 2009 expect to halve error in f_K/f_π [LQCD white paper]
- If so, will have more stringent test of unitariy, e.g. *could* be:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9977(5)(6)(0) = 0.9977(8)$$

V_{us} using $K \to \pi$ form factor

- Lattice calculates:

$$\langle \pi(p')|\bar{s}\gamma_\mu u|K(p)\rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)$$

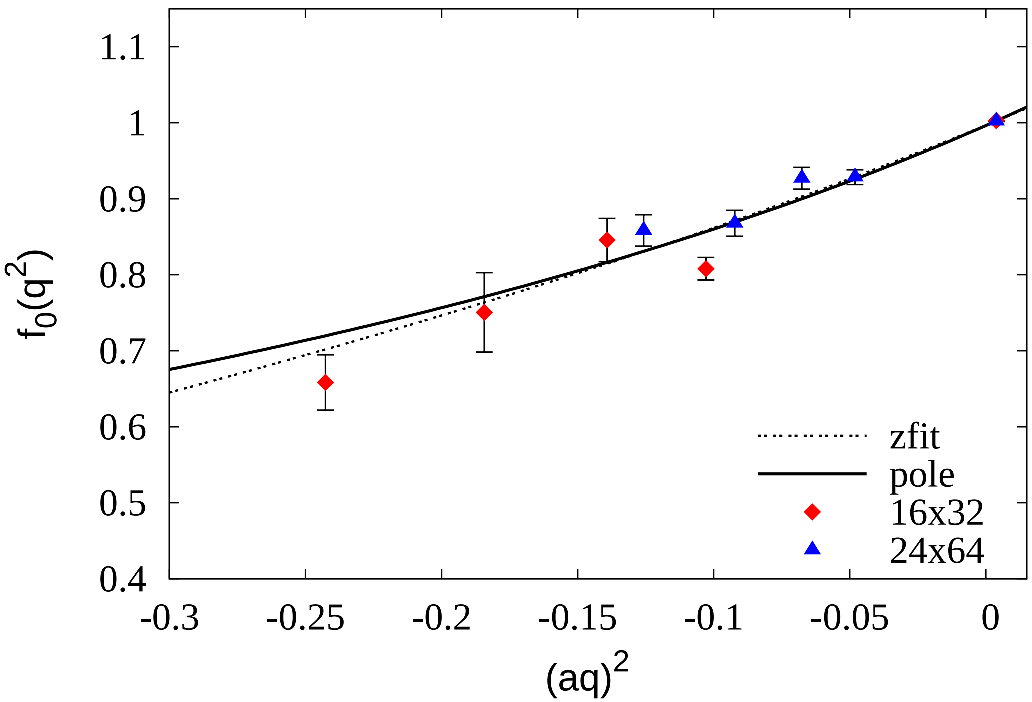
- Measured $\Gamma[K \to \pi \ell \nu]$ determines $|f_+(q^2) V_{us}|^2$
- Experiment gives q^2 dependence; need theory to provide

$$f_+(0) = 1 + f_2 + f_4 + \cdots = 1 - 0.023 + f_4 + \ldots$$

- f_2 is known function of PGB masses (Ademollo-Gatto th'm)
- "Job" of lattice is to calculate $\Delta f = f_+(0) - 1 - f_2$
- PDG uses 1984 estimate [Leutwyler & Roos] : $\Delta f = -0.016(8)$
- Need more reliable calculation
- Full continuum chiral symmetry greatly simplifies calculation:
  $\Rightarrow$ **use DWF**

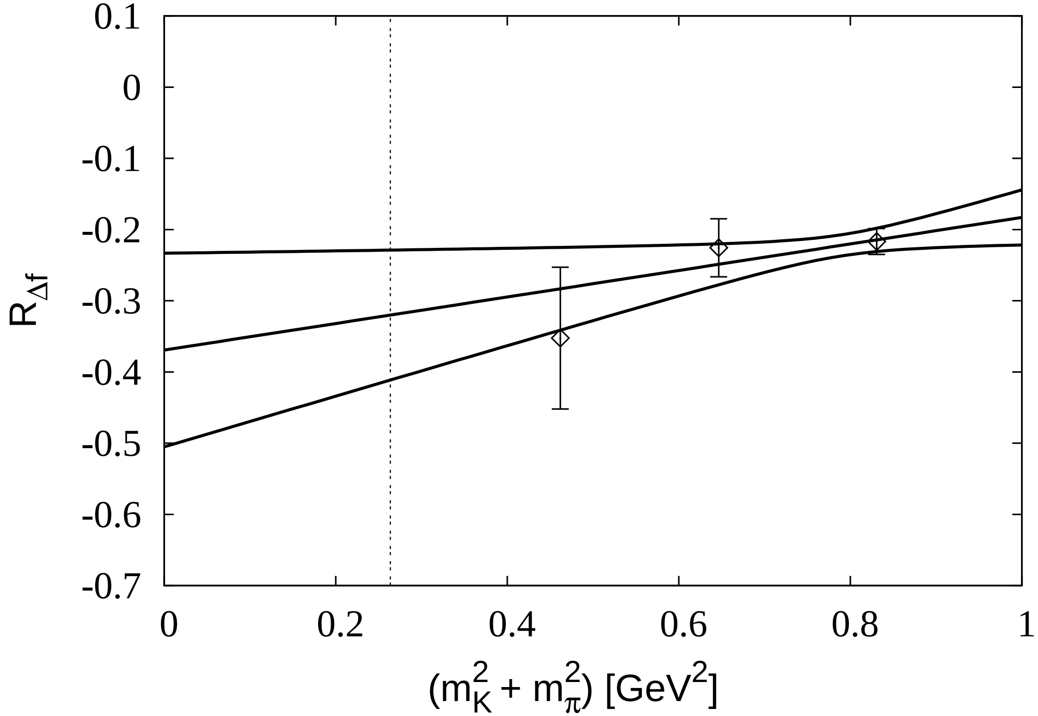
First unquenched results [RBC/UKQCD]

- DWF at $a = 0.12\,\text{fm}$ and $L = 2$ & $3\,\text{fm}$ boxes [hep-lat/0702026]
- Builds on method developed in quenched theory by [Becirevic *et al.*]
- Step 1: determine $f_+(0) - 1 = f_0(0) - 1$ for fixed m_ℓ and m_s
- Very accurate point at $q^2 = q^2_{\rm max} > 0$ anchors extrapolation



First unquenched results (cont.)

- Step 2: extrapolate $R_{\Delta f} = \Delta f/(m_K^2 - m_\pi^2)^2$ to physical masses
- Use approximate NNLO chiral form (analytic terms only)



First unquenched results (final)

- Step 3: estimate systematic errors
- That due to $a \to 0$ extrpolation is (reasonable) guesstimate
- Find $\Delta f = -0.0161(46)(15)(16)(7) = -0.016(5)$
- Errors from statistics, chiral extrap., q^2 extrap, a extrap
- Consistent with Leutwyler-Roos value: $\Delta f = -0.016(8)$

Conclusions:

- Fully controlled result from $K \to \pi$ form factor with errors smaller than Leutwyler-Roos will be available in next couple of years when full DWF ensemble available (which will also allow validation of DWF methodology)
- Errors on $|V_{us}|$ likely comparable to those from f_K/f_π

What else can be achieved by ~ 2009?

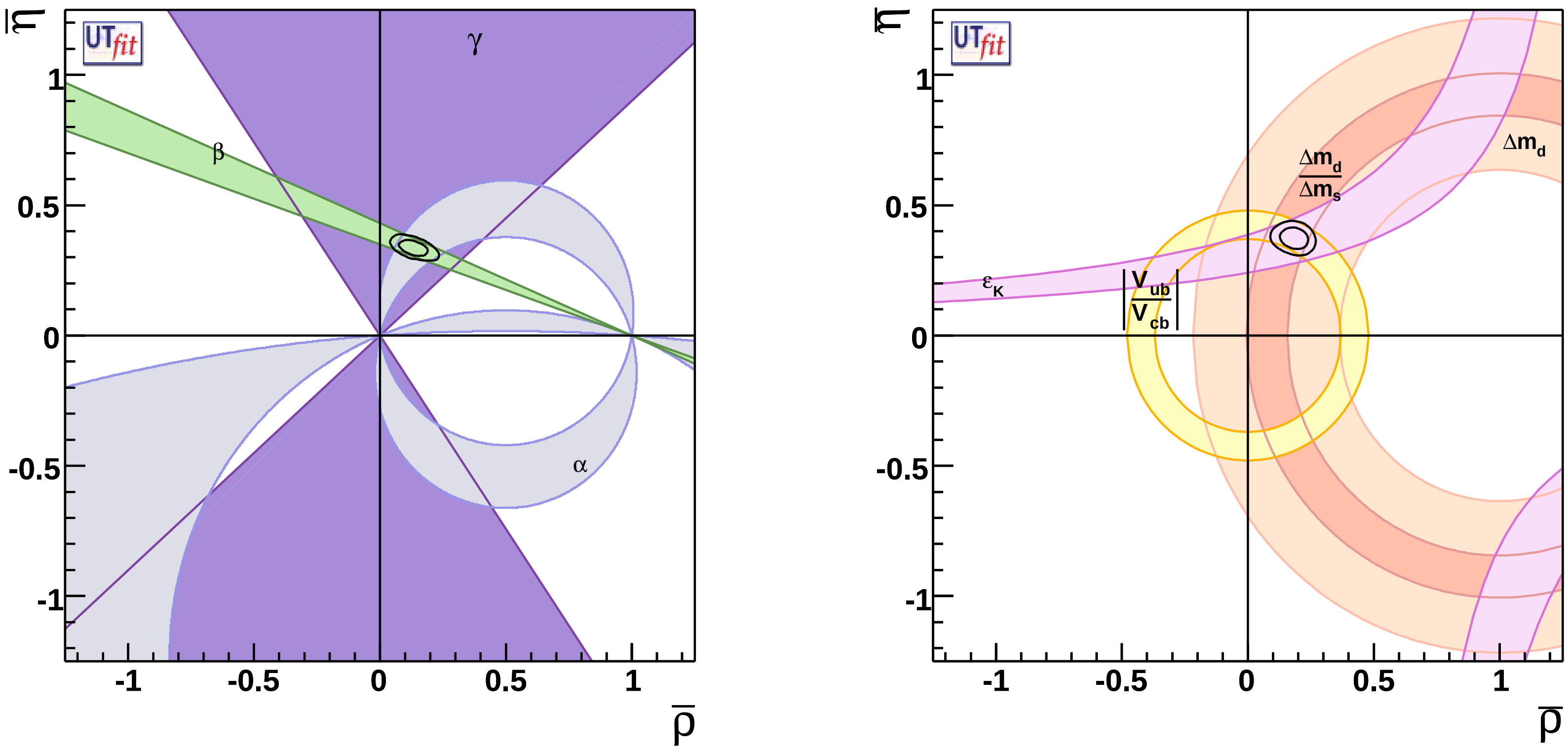
US LQCD project: Calculations of weak matrix elements using both DW and staggered/NRQCD fermions will allow precision tests of SM

Hadronic Matrix Element	Lattice Estimate Current	UTA Result Current	Lattice Errors 10. TF-Yr **2007**	Lattice Errors 50. TF-Yr **2009**
$\widehat{B}_K$	0.77 ± 0.08	0.75 ± 0.09	± 0.05	± 0.03
$f_{B_s}\sqrt{\widehat{B}_{B_s}}$	282 ± 21 MeV	261 ± 6 MeV	± 16 MeV	± 9 MeV
ξ	1.23 ± 0.06	1.24 ± 0.08	± 0.04	± 0.02

- Comparison of UTA (experiment) and lattice tests SM
- **2007:** all lattice errors controlled
- **2009:** all lattice errors at or below experimental errors

Precision tests of SM (cont.)

Present constraints on $\bar{\rho}, \bar{\eta}$ [UTfit]



UTA (experiment) vs lattice-based results

Impressive consistency–will be solidified and made much more precise by this project

Cabibbo-Kobayashi-Maskawa matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{A^2\lambda^5}{2}[1-2(\rho+i\eta)] & 1-\frac{\lambda^2}{2}+\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2+\frac{A\lambda^4}{2}(1-2\rho)-i\eta A\lambda^4 & 1-\frac{A^2\lambda^2}{2} \end{pmatrix} + O(\lambda^6)\,,$$