Hot Dilute Neutron Matter

Gautam Rupak

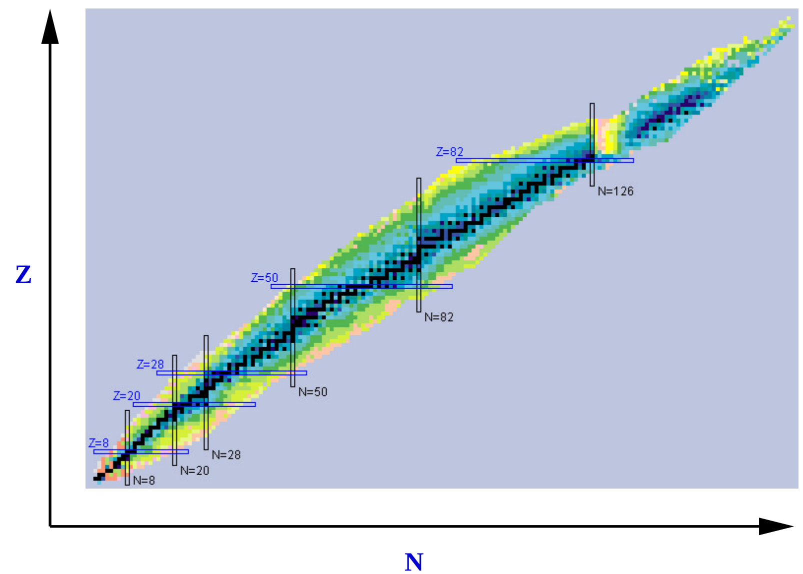
Department of Physics

North Carolina State University

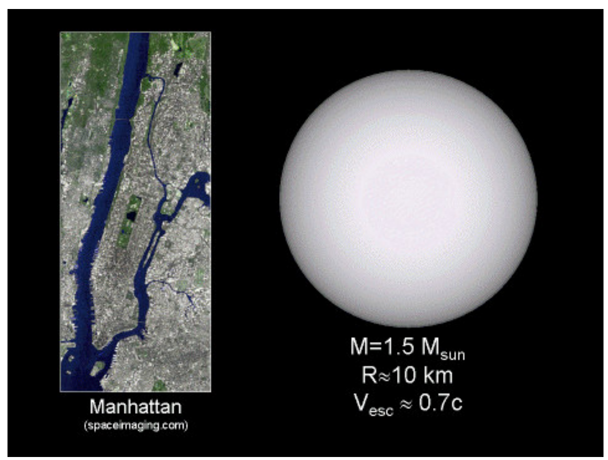
Raleigh, NC 27695

Motivation

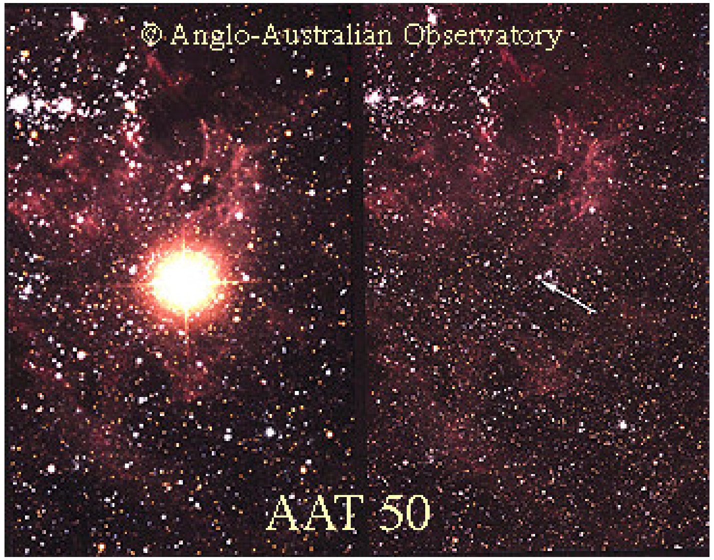
- Origin of elements

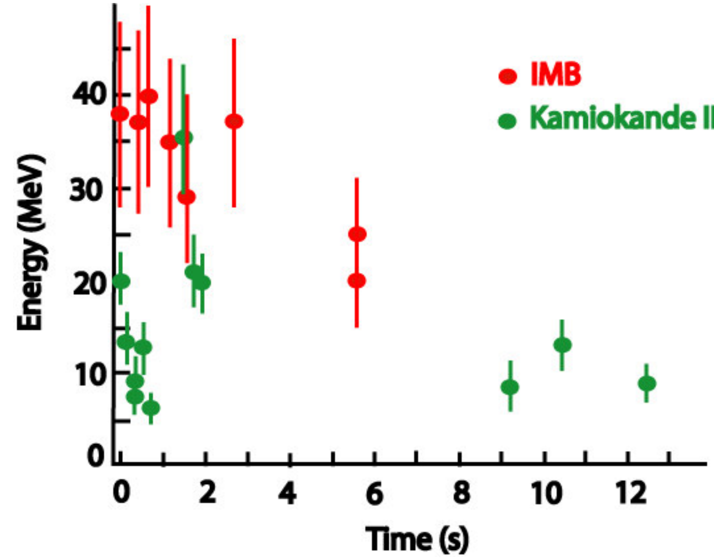


- Giant nuclei



Neutrinosphere



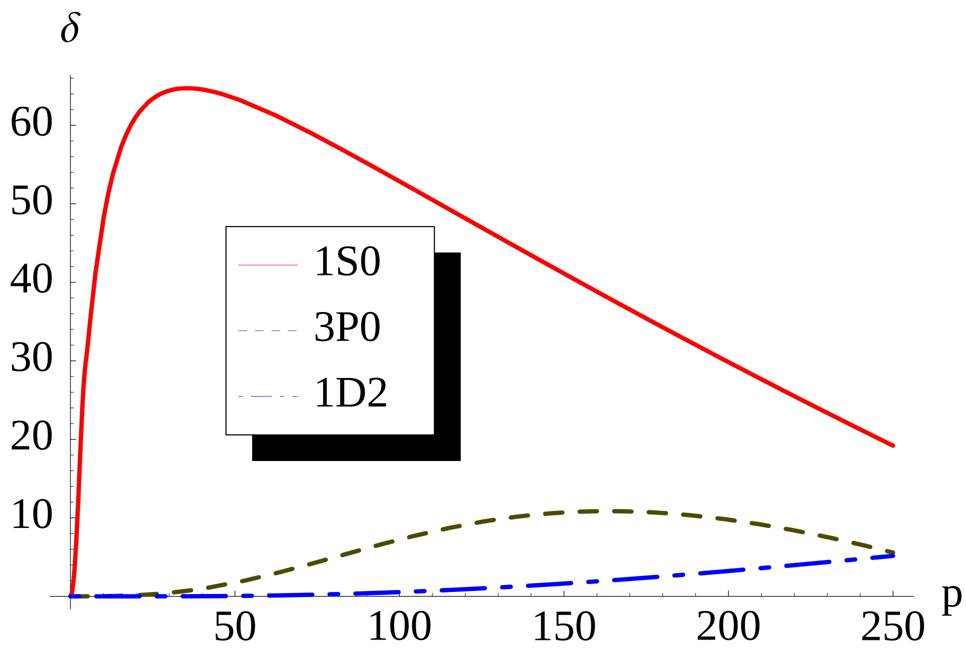


- $T \sim 10 - 20$ MeV.

- Cross section: $n \sim 1/(G_F^2 E_\nu^2 R) \sim 10^{-4}\ \mathrm{fm}^{-3}$.

Dilute $1/n^{1/3} \gg 1/m_\pi$ and cold $\sqrt{2\pi/(MT)} \gg 1/m_\pi$ on nuclear scale: perturbation.

Nuclear interaction

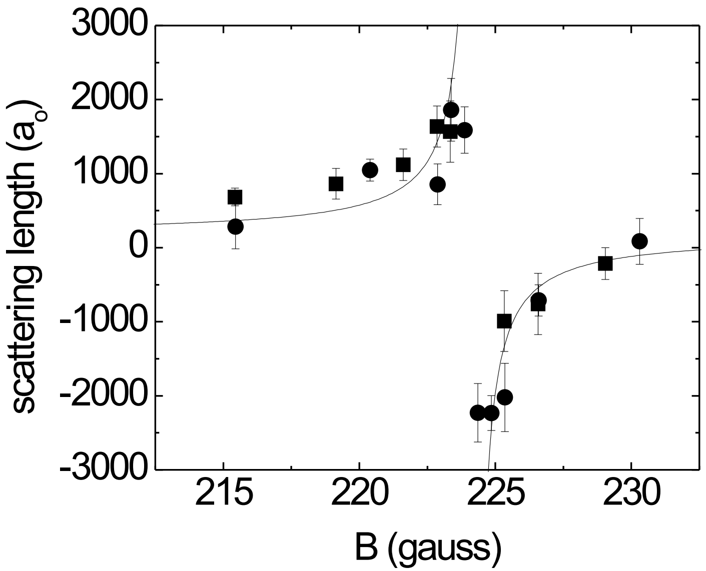


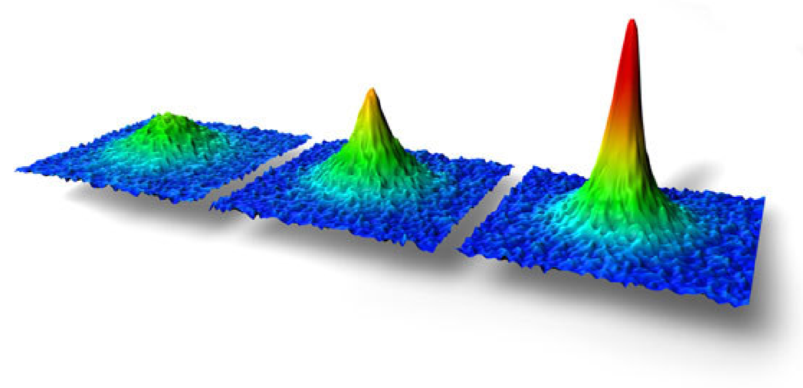
$$i\mathcal{A}(k)_{^1S_0} = \frac{4\pi}{M}\frac{i}{k\cot\delta - ik} \approx \frac{4\pi}{M}\frac{i}{-\frac{1}{a} + \frac{r_0}{2}k^2 - ik + \cdots},$$

- $|a| \sim 19$ fm $\gg 1/m_\pi \sim 1.4$ fm.
- neutron-α P-wave resonance $|a_P| \sim 60\ \mathrm{fm}^3$.
- atomic systems: In ^{4}He gases, $a \sim 100\mathring{A} \gg r_0 \sim 7\mathring{A}$.

From Atoms to Neutron Stars

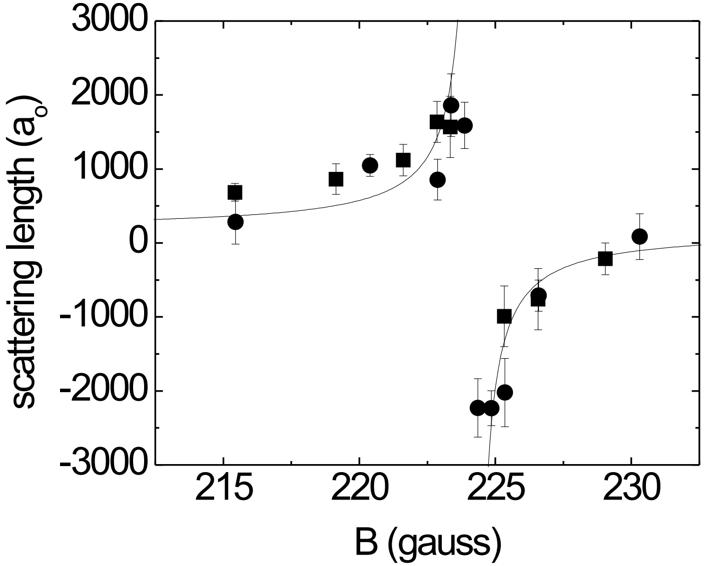
Regal and Jin, 2003



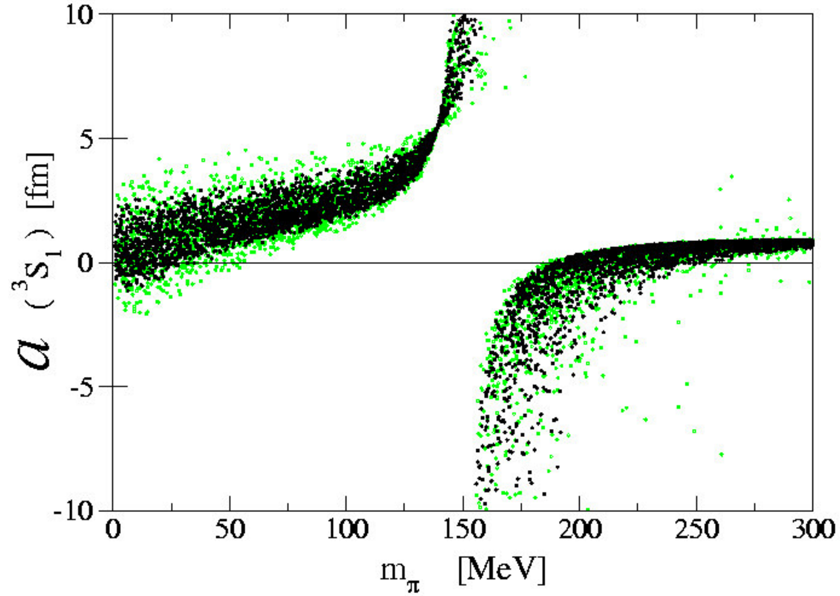


From Atoms to Neutron Stars

Regal and Jin, 2003



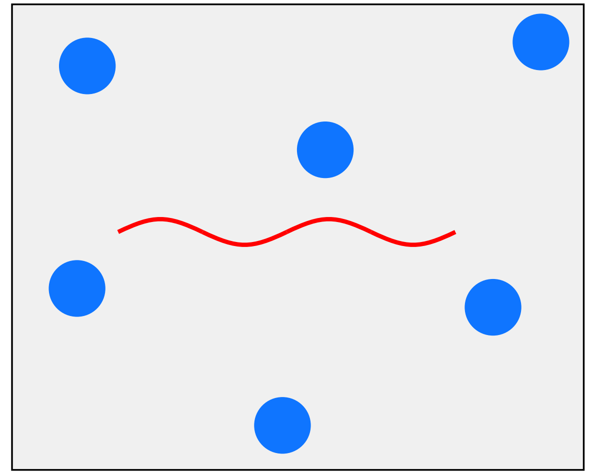
Beane and Savage, 2003



Finite density system

- Consider a dilute system: $R^3 n \ll 1$.
- At temperature T: $\lambda = \sqrt{2\pi/(MT)} \gg R$.

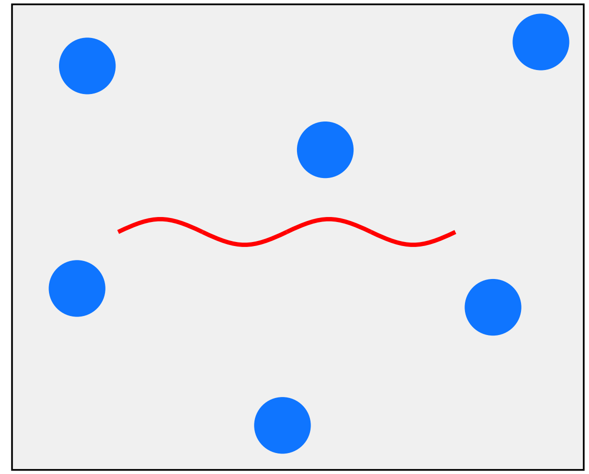
Microscopic physics irrelevant.



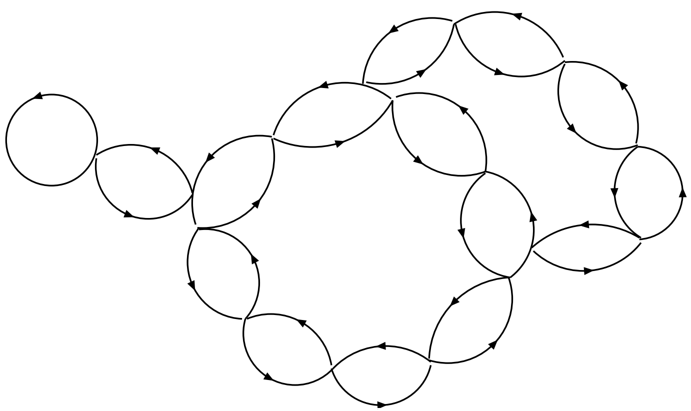
Finite density system

- Consider a dilute system: $R^3 n \ll 1$.
- At temperature T: $\lambda = \sqrt{2\pi/(MT)} \gg R$.

Microscopic physics irrelevant.

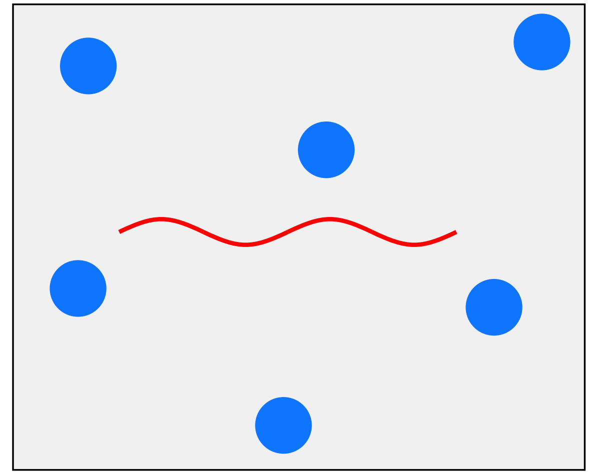


- However, no $a^3 n,\ a\sqrt{MT} \gg 1$ expansion.



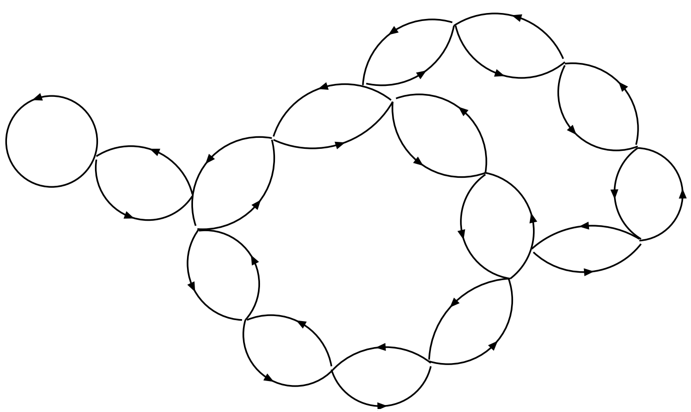
Finite density system

- Consider a dilute system: $R^3 n \ll 1$.
- At temperature T: $\lambda = \sqrt{2\pi/(MT)} \gg R$.



Microscopic physics irrelevant.

- However, no $a^3 n,\ a\sqrt{MT} \gg 1$ expansion.



Systematic calculation for $n\lambda^3 \ll 1$ even as $a^3 n \to \infty$ and $a\sqrt{MT} \to \infty$.

Outline

1. Virial expansion

2. Unitarity

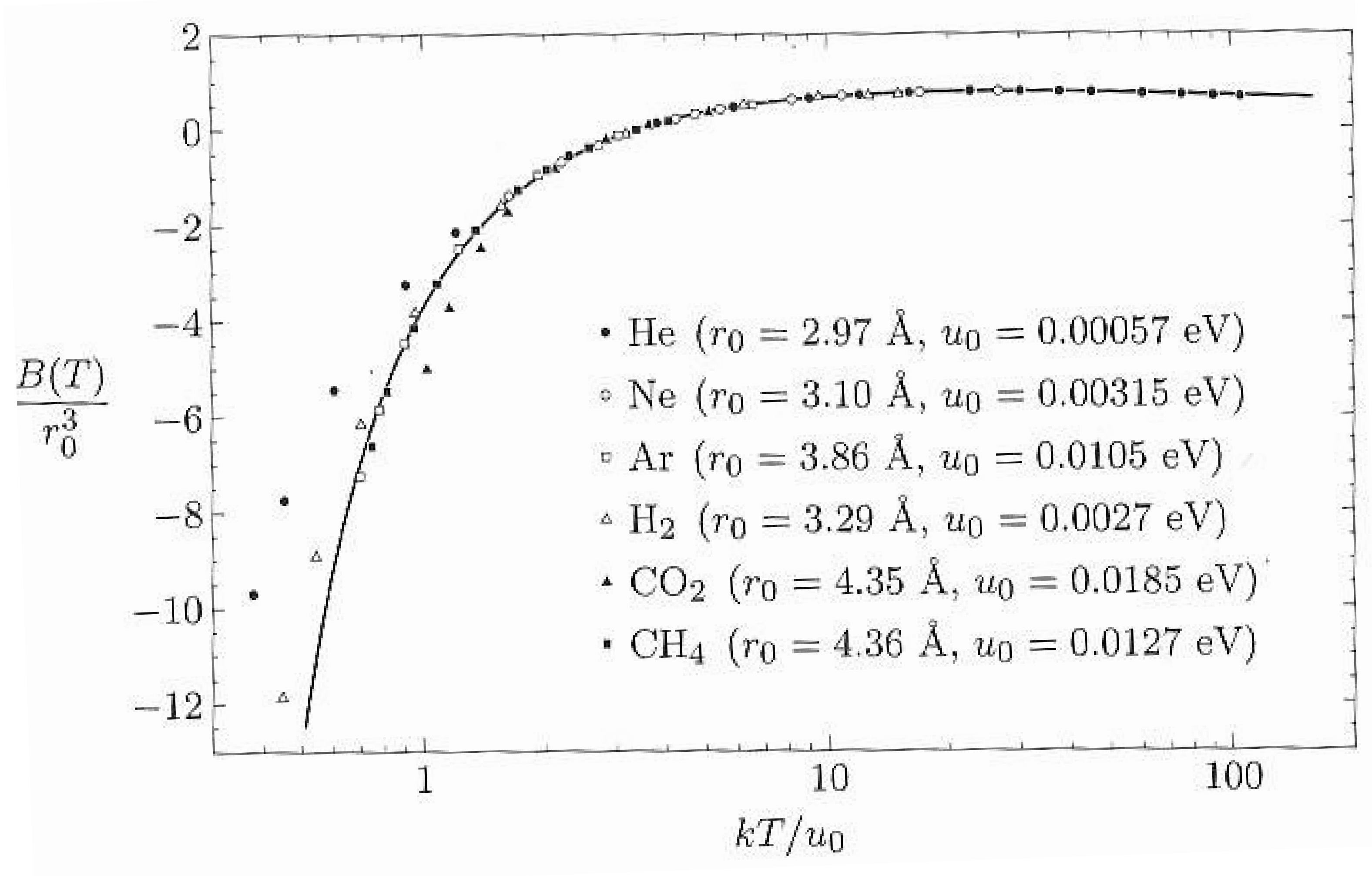
3. Neutron matter: effective range ρ, P-wave, ...

4. Neutrino response

5. Conclusion

Non-ideal gases

$$\frac{P}{T} = n + B(T)n^2 + C(T)n^3 + \cdots \; .$$



Free Theory

Fugacity $z = \exp(\mu/T)$ expansion:

$$n = 2\int \frac{d^3\, k}{(2\pi)^3} \frac{1}{e^{(k^2/(2M)-\mu)/T} + 1},$$
$$\approx \frac{2}{\lambda^3}\left[z - \frac{z^2}{2\sqrt{2}} + \frac{z^3}{3\sqrt{3}} + \cdots\right].$$

Small $n\lambda^3$ implies small z, where $\lambda = \sqrt{2\pi/(MT)}$.

Effective positive pressure

$$\frac{P}{n} = T\left[1 + \frac{\lambda^3 n}{8\sqrt{2}} + \cdots\right].$$

In general

$$n = \frac{2}{\lambda^3}\left[b_1(T)z + 2b_2(T)z^2 + 3b_3(T)z^3 + \cdots\right],$$

$$P = \frac{2T}{\lambda^3}\left[b_1(T)z + b_2(T)z^2 + b_3(T)z^3 + \cdots\right].$$

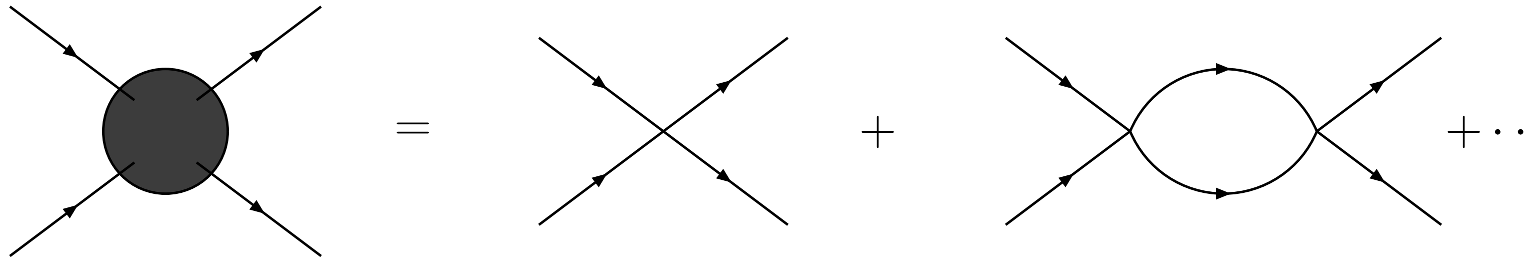
b_l receives contribution at most from l-body physics.

- Density dependence in $z \sim n\lambda^3$
- Interaction in b_l

$$b_l = f(a\sqrt{MT})\left[1 + R\sqrt{MT} + \cdots\right].$$

Interacting Theory

Short range interactions at low densities.



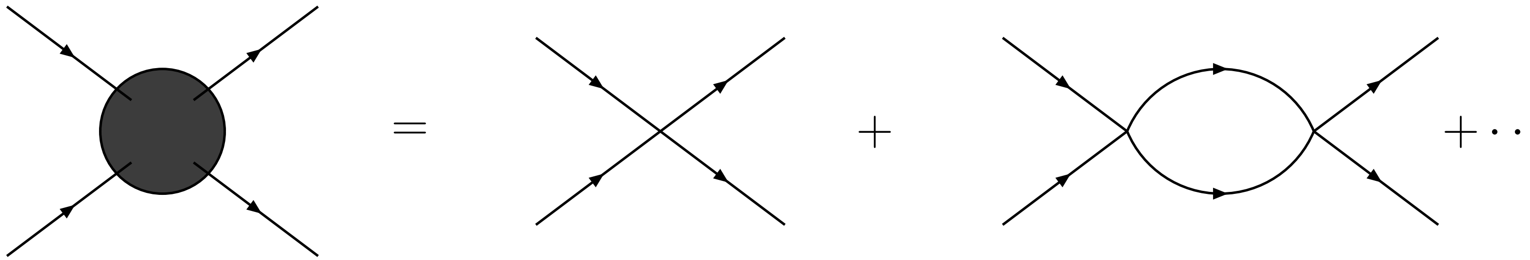
$\mathcal{O}(a)$ $\qquad\qquad$ $\mathcal{O}(a^2p)$

Need to resum for $|a|p \sim |a|\sqrt{MT} \gtrsim 1$

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{-1/a - ip} \, .$$

Interacting Theory

Short range interactions at low densities.



$\mathcal{O}(a)$ $\mathcal{O}(a^2p)$

Need to resum for $|a|p \sim |a|\sqrt{MT} \gtrsim 1$

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{-1/a - ip} \; .$$

The effective range expansion, Schwinger 1947

$$A = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip} \; ,$$

$$p\cot\delta = -1/a + \frac{r_0}{2}p^2 + \cdots \; .$$

Effective Field Theory

$$\mathcal{L} = \sum_n C_n \mathcal{O}^{(n)} \ , \text{ from symmetry}$$

Example:

$$\mathcal{O}^{(n)} = N_\uparrow^\dagger N_\downarrow^\dagger N_\uparrow p^{2n} N_\downarrow \ .$$

Separation of high-momenta Λ and low-momenta p:

$$C_n \sim \frac{1}{M\Lambda^{2n+1}} \ ,$$

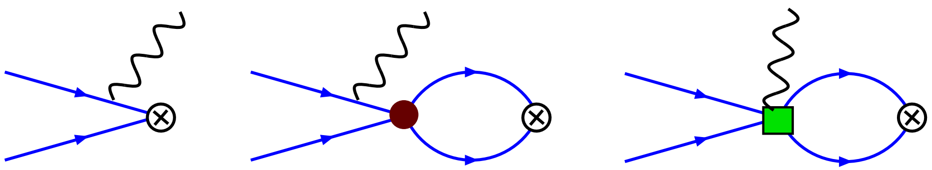
$$\frac{C_1\mathcal{O}^{(1)}}{C_0\mathcal{O}^{(0)}} \sim \frac{p^2}{\Lambda^2} \ , \text{systematic, improvable, optimal}$$

where m_π-Λ_χ, γ-m_π, Halo nuclei

Subtleties: quantum corrections and large scattering length.

Impact on BBN

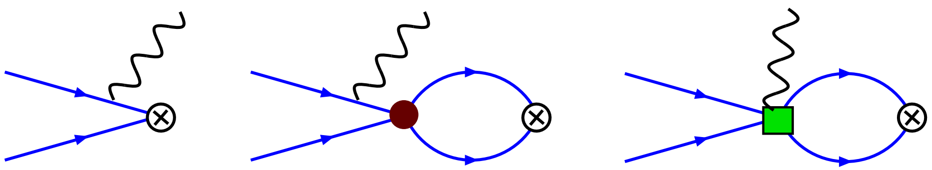
- $n + p \rightarrow d + \gamma$ for light element production in early universe.
- Experimental data scarce at $E \sim 100$ KeV.
- An EFT calculation with 1% uncertainty was performed (Rupak '00).



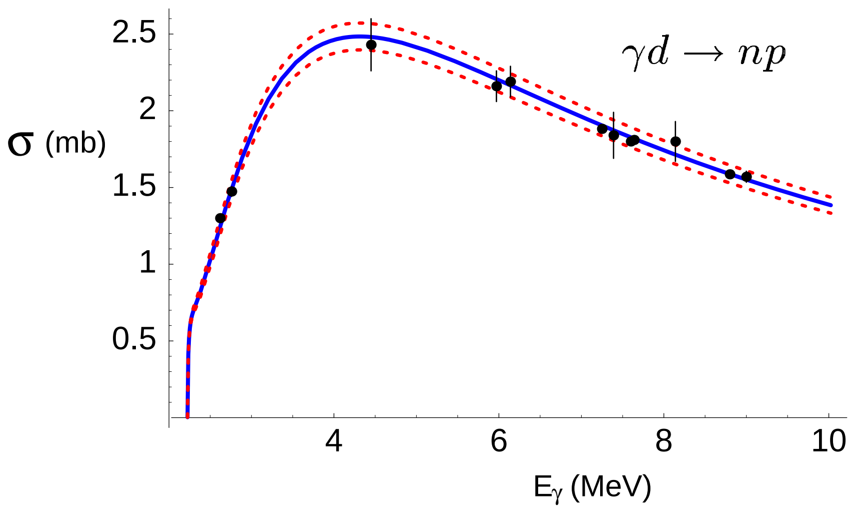
$$\sigma(\underbrace{a, r_0, \gamma, \rho_d, \ldots,}_{\text{N}-\text{N data}} \underbrace{L}_{\text{Not N}-\text{N}})$$

Impact on BBN

- $n + p \to d + \gamma$ for light element production in early universe.
- Experimental data scarce at $E \sim 100$ KeV.
- An EFT calculation with 1% uncertainty was performed (Rupak '00).



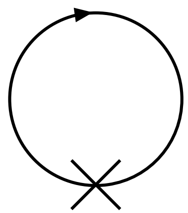
$$\sigma(\underbrace{a, r_0, \gamma, \rho_d, \ldots,}_{\text{N}-\text{N data}} \quad \underbrace{L}_{\text{Not N}-\text{N}} \quad)$$

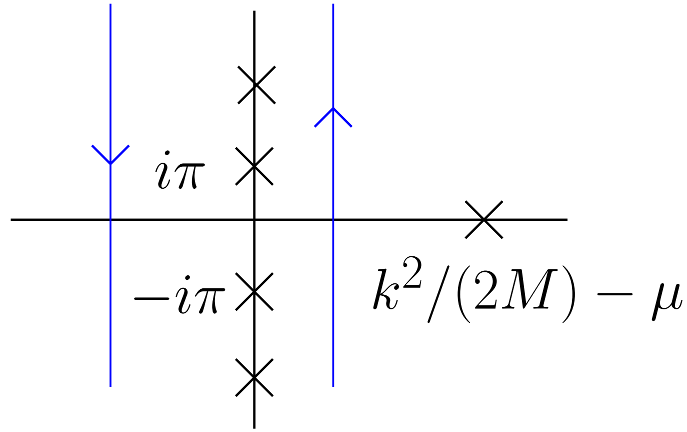


$$\sigma(n + p \to d + \gamma)\big|_{2\ \mathrm{MeV}} =$$
$$0.0218(1 + 0.6389 + 0.0135 - 0.0053$$
$$+ \cdots)\ \mathrm{fm}^2.$$

Analytical expressions.

Free Theory again

$n =$ 



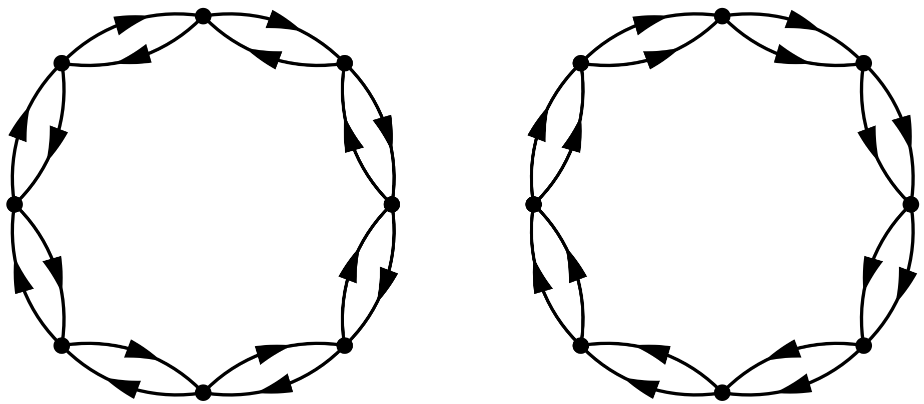
$$
\begin{aligned}
n =& \frac{i}{\pi} \oint d\eta \frac{1}{e^{\eta/T}+1} \int \frac{d^3\,k}{(2\pi)^3} \frac{1}{\eta + \mu - k^2/(2M)} \\
=& 2 \int \frac{d^3\,k}{(2\pi)^3} \frac{1}{z^{-1} e^{k^2/(2MT)} + 1} \approx \frac{2}{\lambda^3} \left[z - \frac{z^2}{2\sqrt{2}} + \cdots \right] .
\end{aligned}
$$

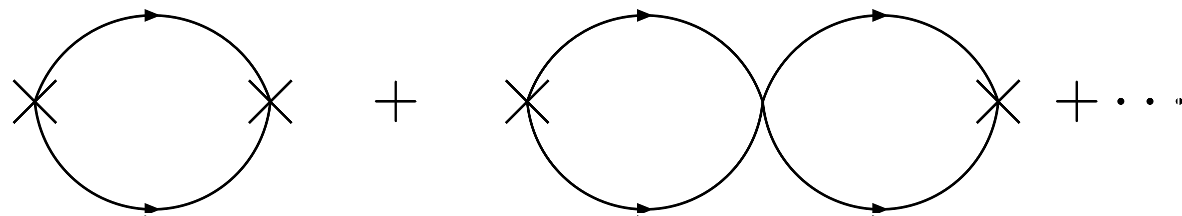
Aside:

$$
\sum_j \langle j | X e^{-\frac{H}{T}} | j \rangle \to \sum_j \langle j | X e^{-itH} | j \rangle \text{ , identify } t = -\frac{i}{T} \text{ ,}
$$

with boundary conditions.

Examples



$\mathcal{D}(p_0, \vec{p}) =$ 

$$\mathcal{D}(p_0, \vec{p}) = \frac{4\pi}{Mg^2} \frac{1}{-1/a + \sqrt{p^2/4 - Mp_0 - 2M\mu}} + \mathcal{O}(z^2).$$

Virial expansion

- Closed particle loop $\mathcal{O}(z)$.
- Closed dimer loop $\mathcal{O}(z^2)$.
- Closed trimer loop $\mathcal{O}(z^3)$, ...

Bedaque and Rupak, 2003

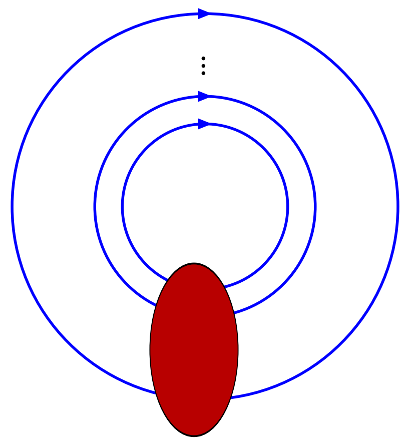
Corollary

- Particle-particle loop $\mathcal{O}(1)$.
- Particle-hole loop $\mathcal{O}(z)$.
- Virial coefficient b_l receives contributions from at most l-body physics. $b_1 = 1$

Remember

$$n = \frac{2}{\lambda^3}\left[b_1(T)z + 2b_2(T)z^2 + 3b_3(T)z^3 + \cdots\right]$$

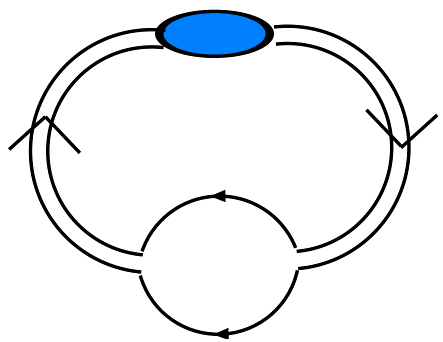
l-body contributions to b_l

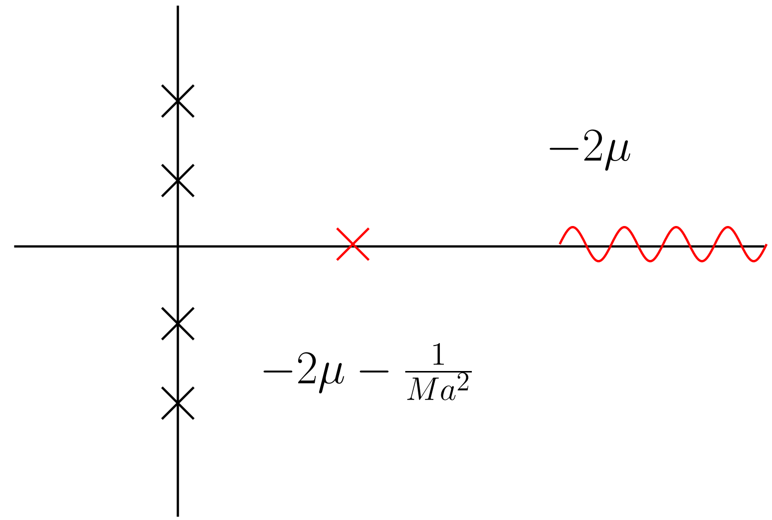
$P =$ 

Calculate

- Leading order l-body propagator (same as in vacuum): T-matrix.
- $T(\eta)$ has cuts, poles at $\eta = -l\mu + \cdots$.
- η integral gives $1/(e^{\eta/T} + 1) \to z^l$.

Warm-up calculation

$P =$ 



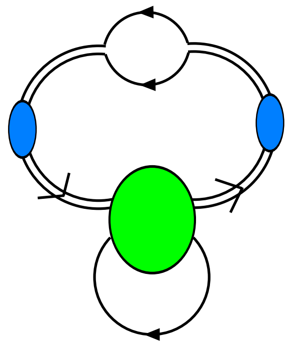
$$P = \frac{i}{2\pi} \oint \frac{d\eta}{e^{\eta/T}+1} \int \frac{d^3q}{(2\pi)^3} \log\left[-1/a + \sqrt{q^2/4 - M(2\mu+\eta)}\right] ,$$

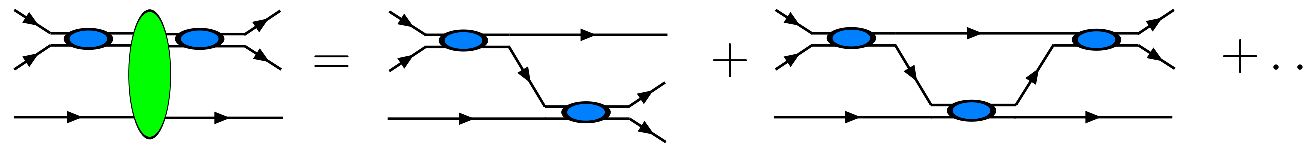
$$b_2(T) = \frac{1}{\sqrt{2}} e^{1/(a^2MT)} \left[1 + \mathrm{Erf}(1/(a\sqrt{MT}))\right] \stackrel{|a|\to\infty}{=} \frac{1}{\sqrt{2}} .$$

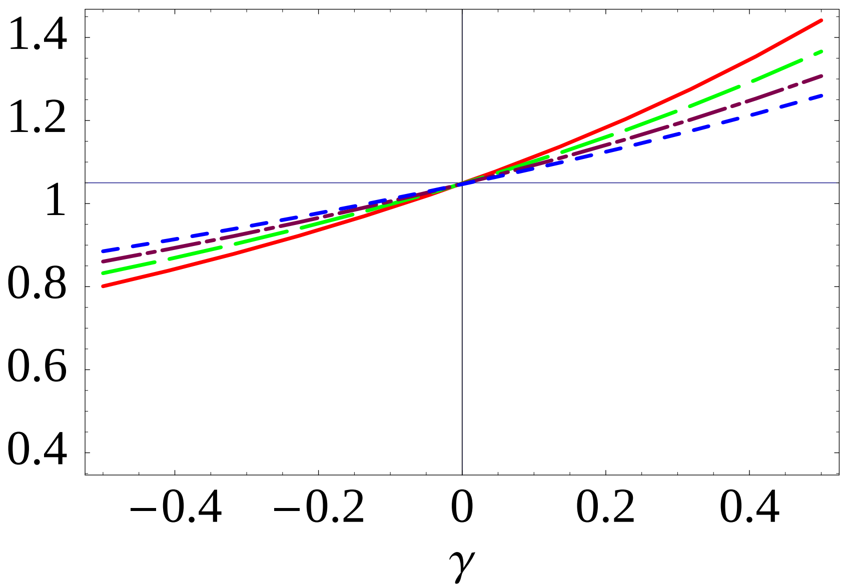
Compare with Beth and Uhlenbeck, 1937

$$b_2(T) = \sqrt{2}\left[\sum e^{B/T} + \frac{1}{\pi}\int dk \partial_k \delta(k) e^{-k^2/(MT)}\right] .$$

$b_3(T)$





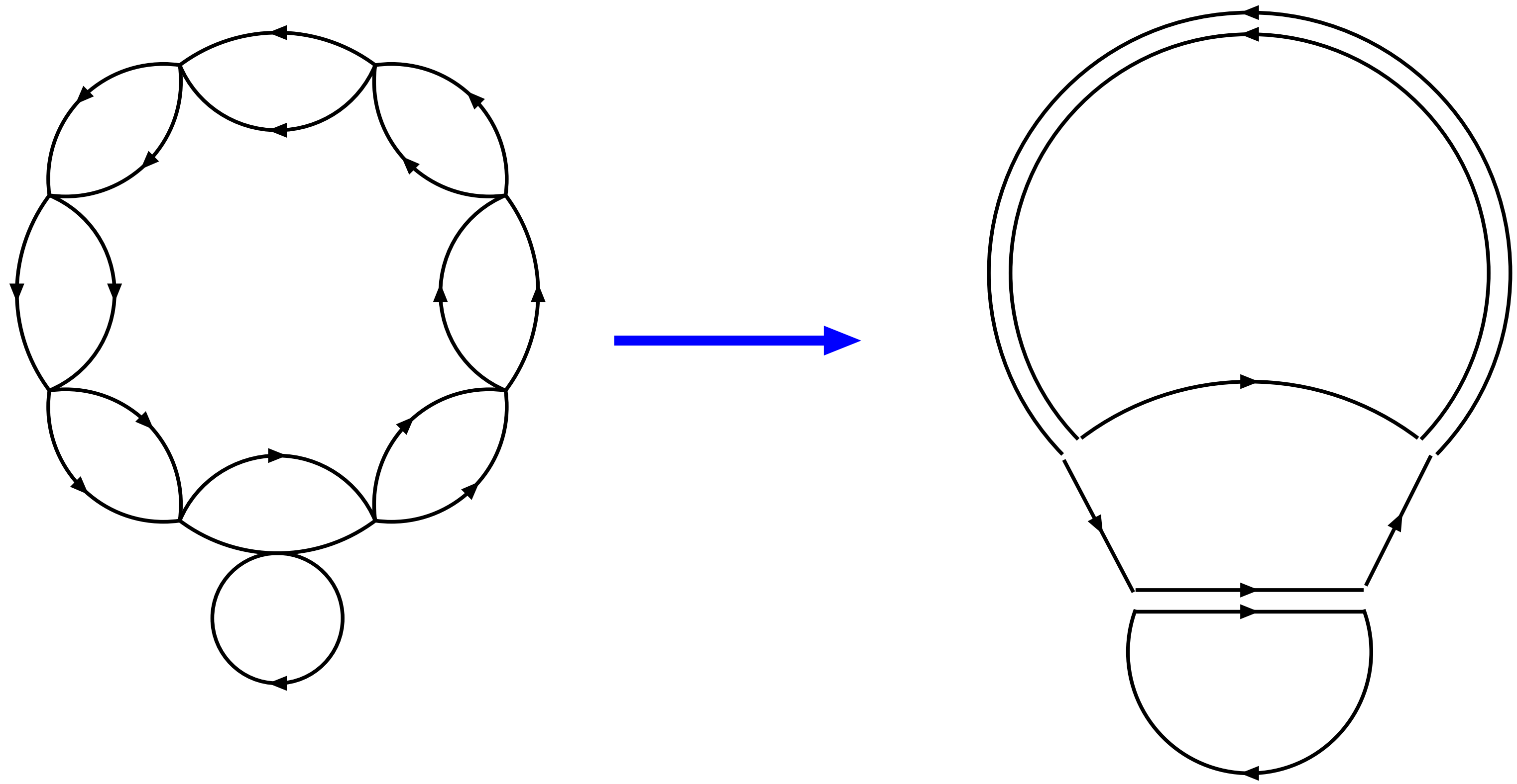


We find $b_3^\infty \approx 1.05$. $b_3(T)$ plotted for T around $6\mu K$ in 40% increments for ^{6}Li.

Rupak, 2006

$1/2\text{eV} \approx 1/4000\text{Å}^{-1}$.

Daisy contributions



Neutron matter: preliminary

Higher partial waves, effective range ρ corrections.

$$b_2(T) = -\frac{1}{2^{5/2}} + \frac{3MC_P}{4\lambda^3} + \frac{1}{\sqrt{2}} e^{\frac{\gamma^2}{MT}} \left(1 + \mathrm{Erf}\left(\frac{\gamma}{\sqrt{MT}}\right)\right) - \frac{\rho}{2\lambda} - \frac{\gamma\rho^2}{4\lambda} \,,$$

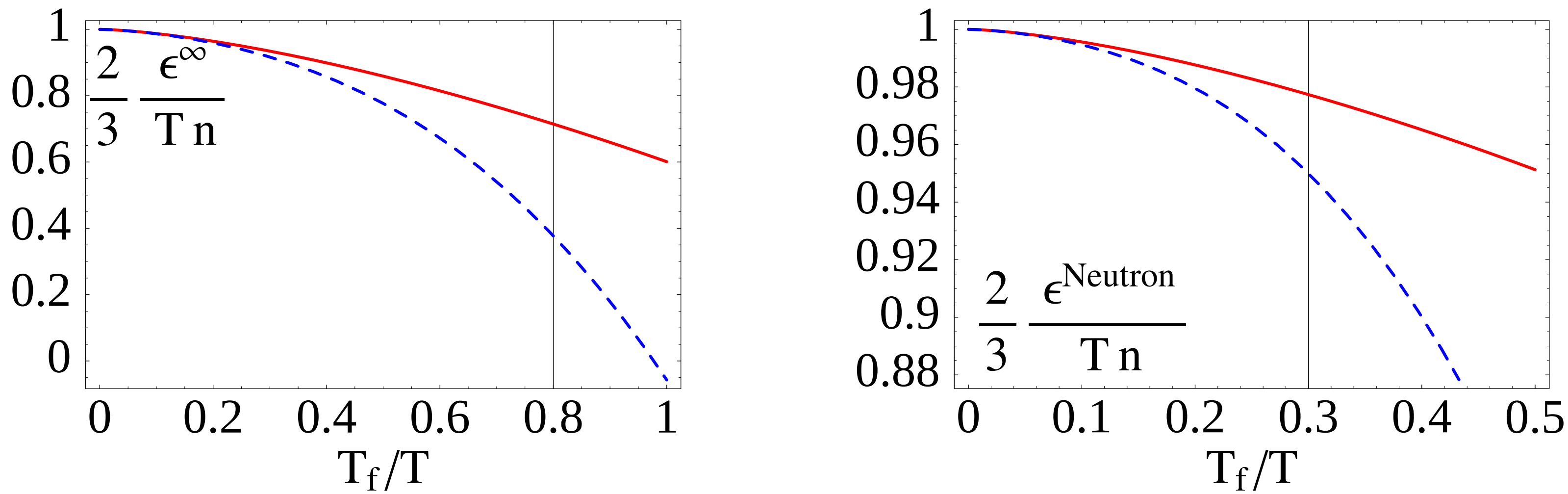
$$b_2(T = 5 \text{ MeV}) = -0.18 + 0.017 + 0.62(1 - 0.30 + 0.016)$$

$$\hat{b}_3(5 \text{ MeV}) \sim 1.05(0.93),$$

$$\hat{b}_3(25 \text{ MeV}) \sim 1.2(0.99),$$

close to unitarity value ≈ 1.05

Equation of state



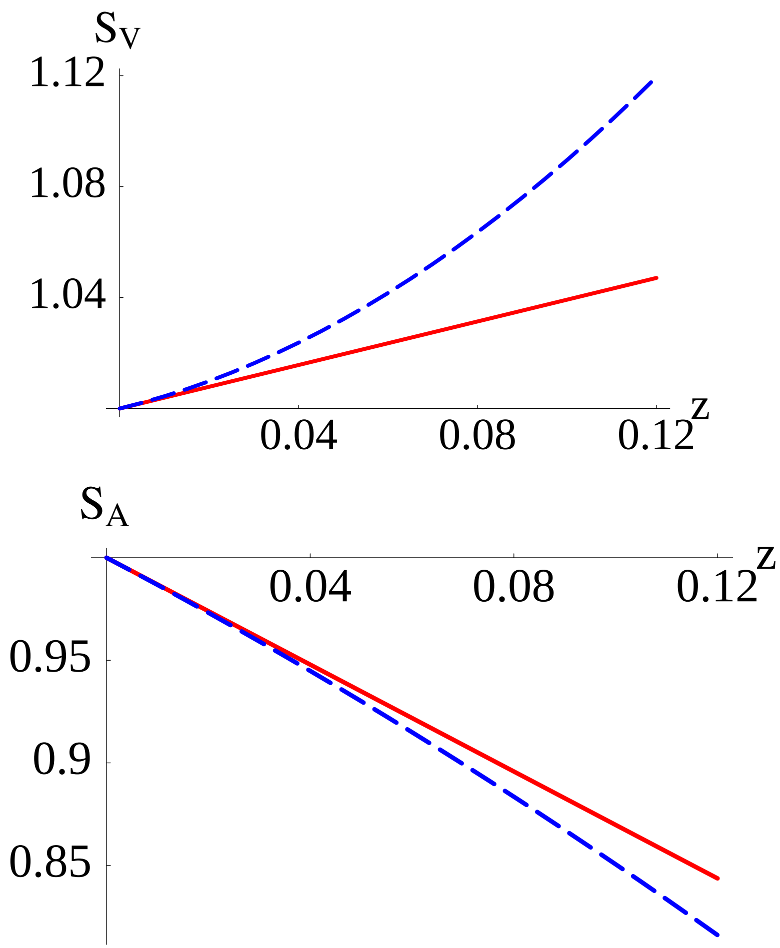
At $T = 5$ MeV, $T_F/T \approx 0.3$ corresponds to $n \approx 10^{-3}\ \mathrm{fm}^{-3}$.

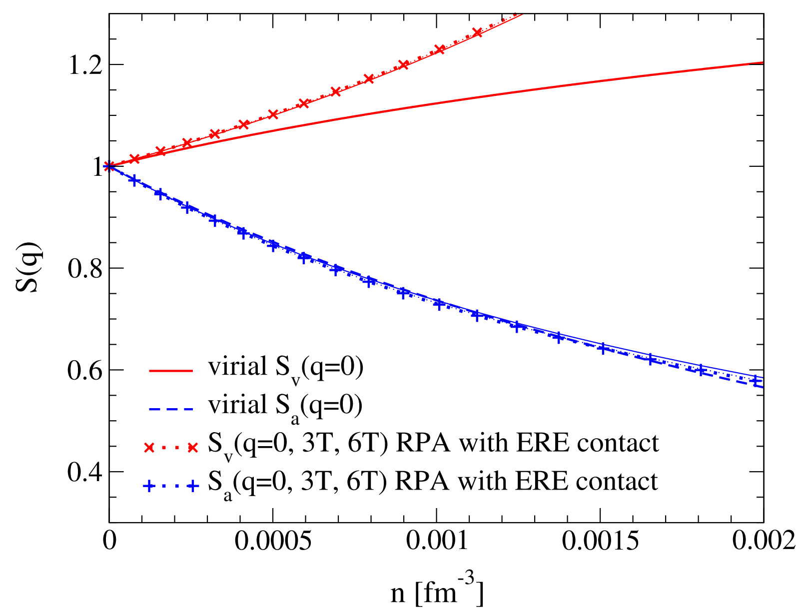
At $T = 25$ MeV, $n \approx 10^{-2}\ \mathrm{fm}^{-3}$.

Neutrino response

$$\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left[C_A^2(3-\cos\theta)S_A(q) + C_V^2(1+\cos\theta)S_V(q)\right]$$

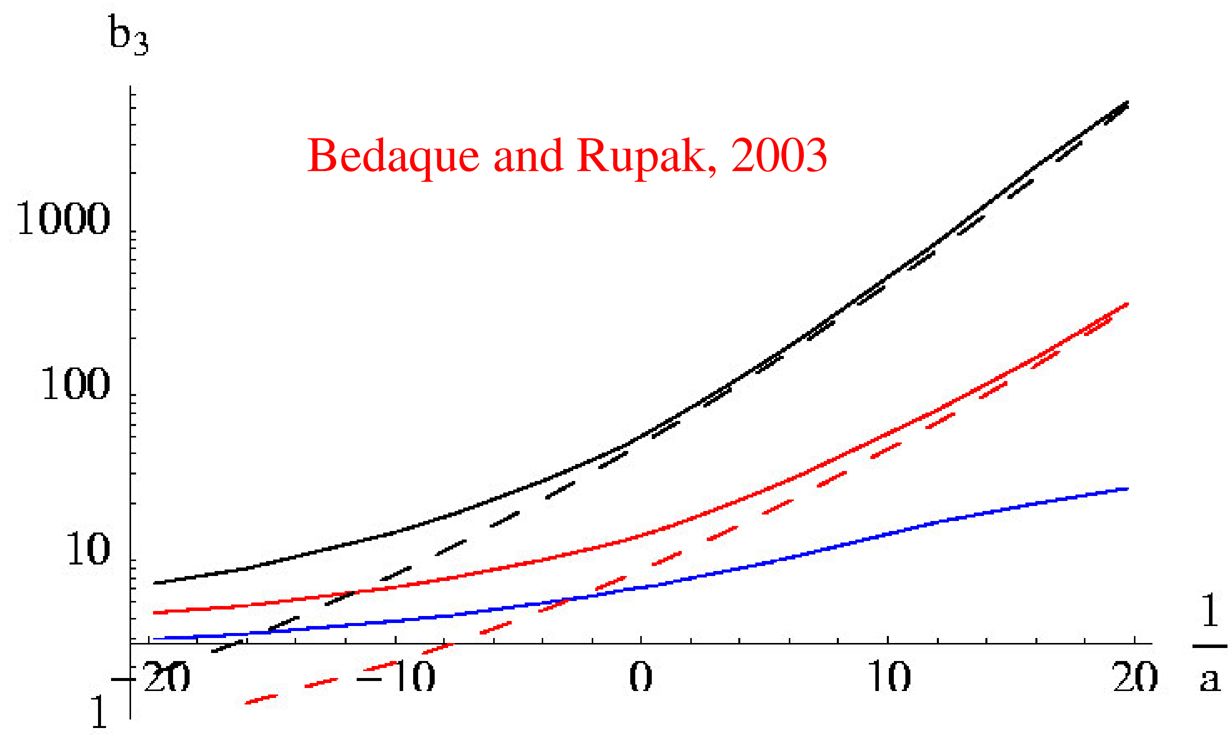
Preliminary at $T = 5$ MeV.





Horowitz, Schwenk ('06). $T = 4$ MeV.

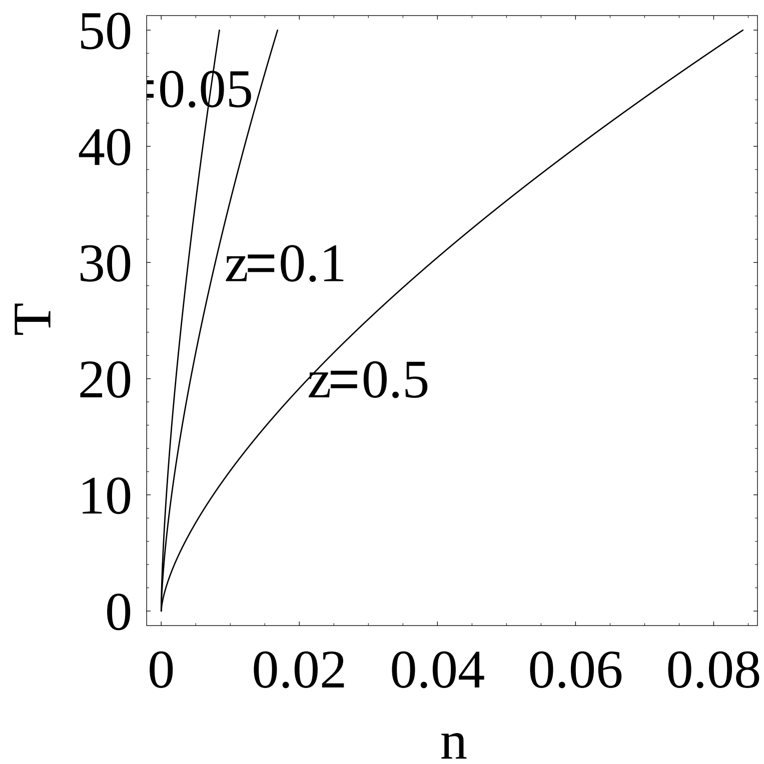
Helium



Is Efimov effect important?

Coulomb effect: $\alpha M \lambda \lesssim 0.3$ for $T \gtrsim 5$ MeV.

Beyond neutrinosphere



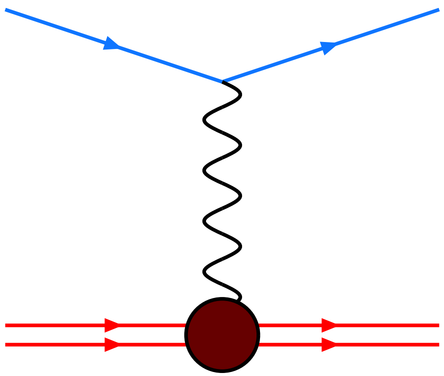
At $T = 50$ MeV ($1/\lambda \sim 90$ MeV, $\sqrt{MT} \sim 200$ MeV), pions using KSW in ${}^1S_0$.

Conclusions

- Systematic expansion, related to T-matrix in vacuum.
- Benchmark neutrino calculations.
- Isospin, heavier nuclei.
- Neutrino interactions away from the static limit.
- Higher densities with pions ...

Thank You

Deuteron properties



$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\Big|_{\mathrm{point}} [A(q^2) + B(q^2)\tan(\frac{\theta}{2})],$$

where

$$A = F_C^2 + \frac{2}{3}\eta F_M^2 + \frac{8}{9}\eta^2 F_Q^2,$$

$$B = \frac{4}{3}\eta(1+\eta)F_M^2,\ \ \eta = -\frac{q^2}{4M_d}.$$

$1 \otimes 1 = 0 \oplus 1 \oplus 2$

Chen, Rupak and Savage ’98, Phillips, Rupak and Savage ’99