

Investigating nucleon polarizabilities in Compton scattering on the proton and the deuteron

DANIEL PHILLIPS

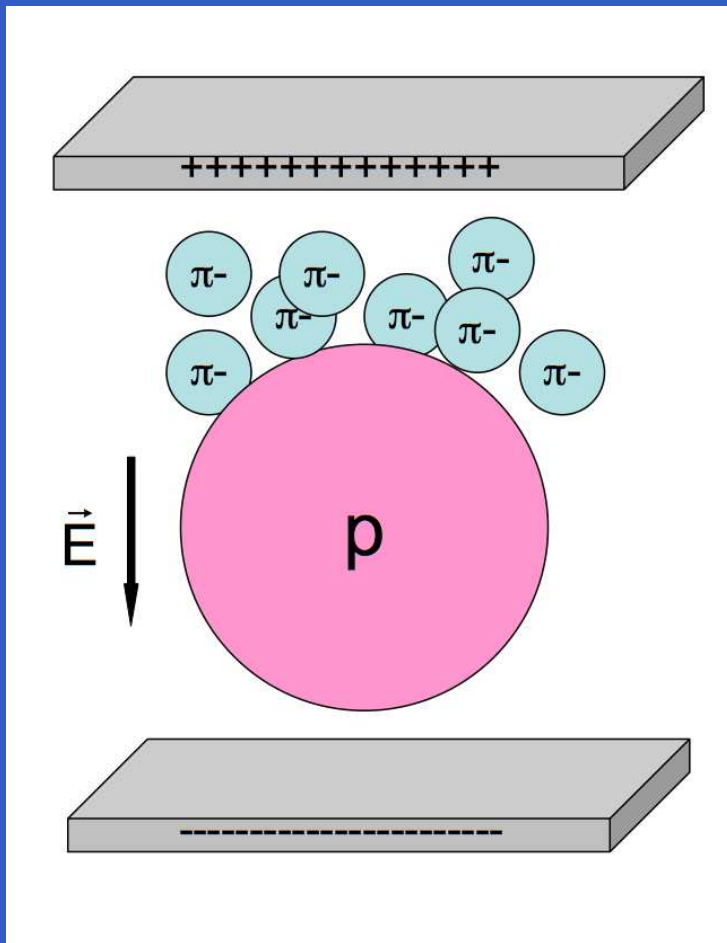
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Outline

- Polarizabilities: the promise and the problem
- Compton scattering on the proton in chiral perturbation theory for $\omega \sim m_\pi$
- Compton scattering on the deuteron: motivation and a first χ PT calculation [$O(e^2 P)$]
- Improving on the $O(e^2 P)$ χ PT calculation: $O(e^2 P^2)$, effects of the Delta(1232), and striving for wave-function independence
- Conclusion and future work

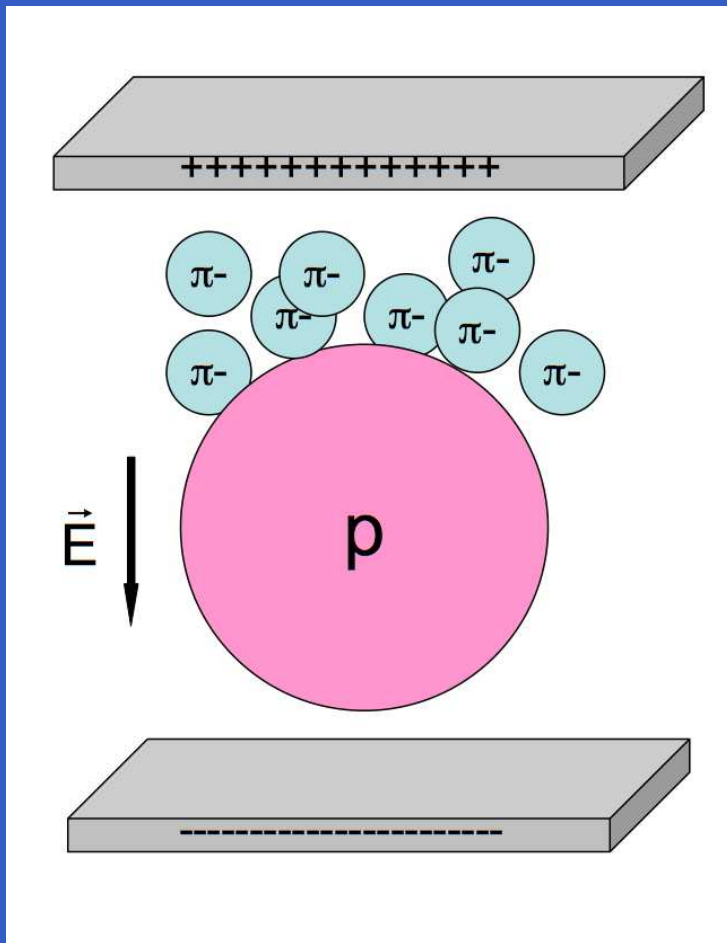
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- EM moments that encode nucleon-structure information;

Figure courtesy R. Miskimen

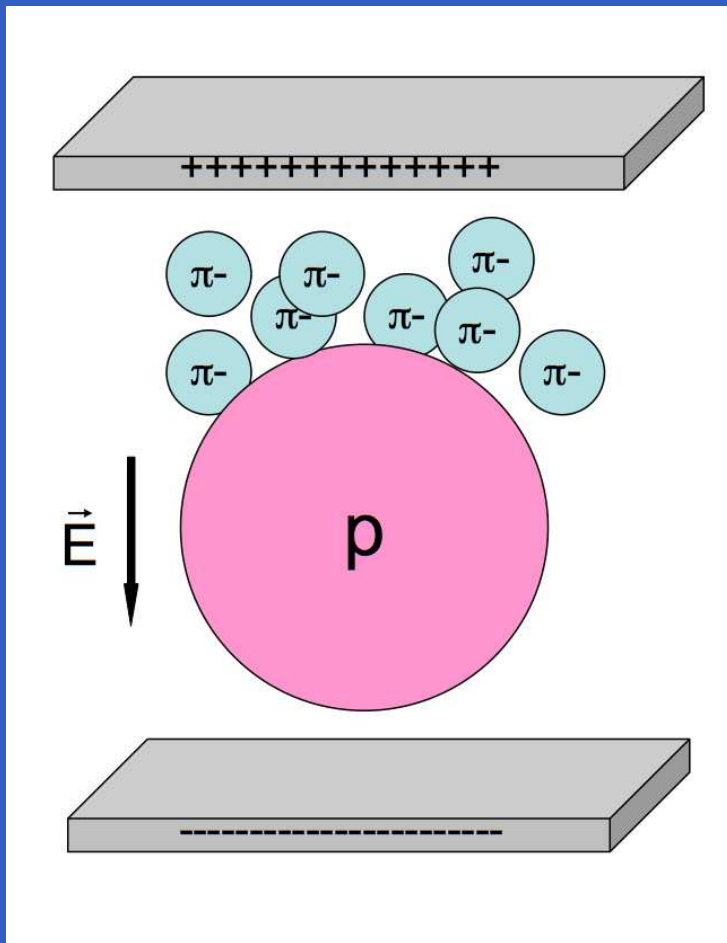
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- EM moments that encode nucleon-structure information;
- Mix long- ($r \gtrsim 1/m_\pi$) and short-distance physics in interesting way;
- Probe pattern of breaking of QCD's chiral symmetry below pion-production threshold.

Figure courtesy R. Miskimen

Polarizabilities: promise and problem

$$\begin{aligned} \mathbf{d} &= 4\pi\alpha_N\mathbf{E}; & \mu &= 4\pi\beta_N\mathbf{B} \\ \Rightarrow H &= \frac{(\mathbf{p} - Ze\mathbf{A})^2}{2M} - 2\pi\alpha_N\mathbf{E}^2 - 2\pi\beta_N\mathbf{B}^2 \end{aligned}$$

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The problem: α_N and β_N are defined at $\omega = 0$, but their influence grows with ω .

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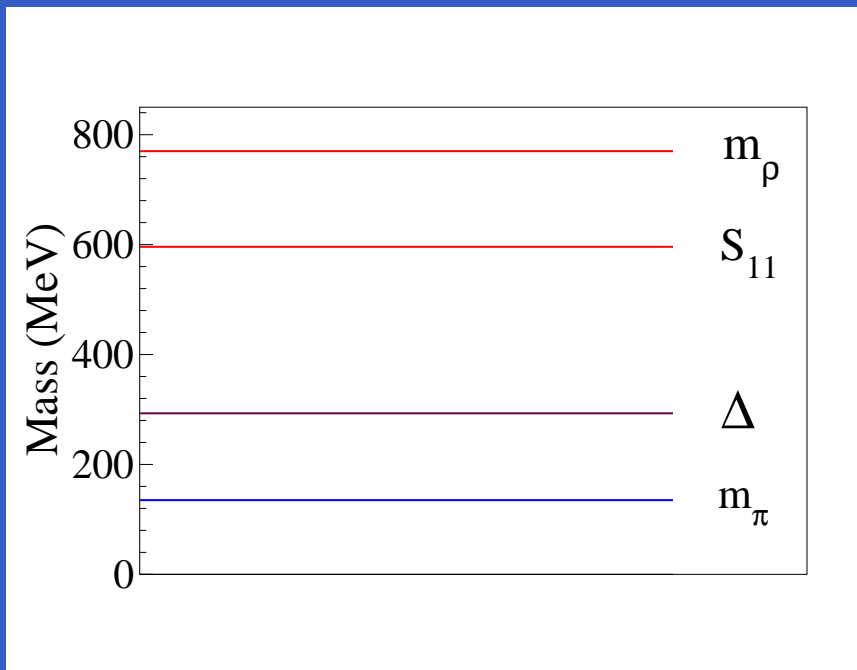
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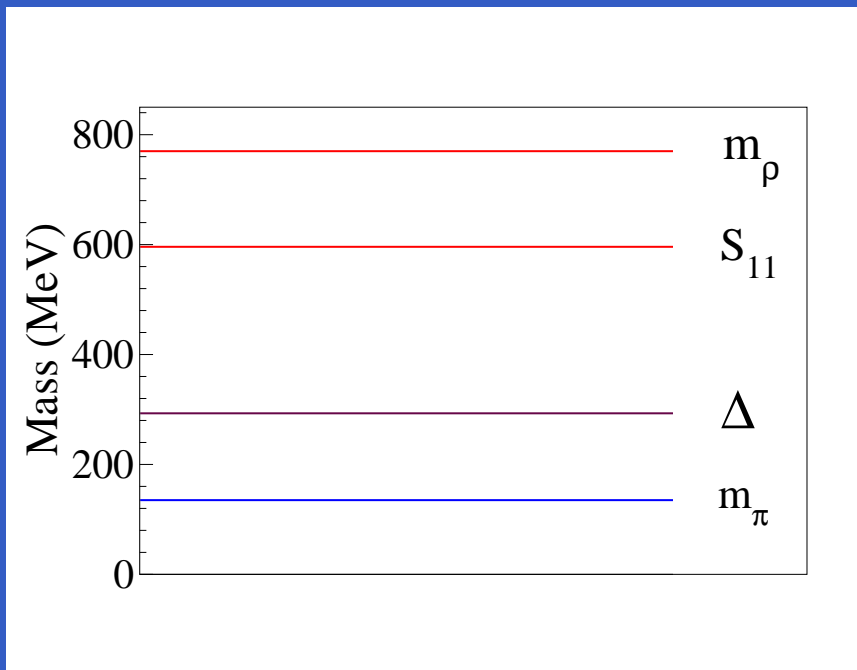
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- Whole $\mathcal{A}_{\gamma N}$ test of low-energy QCD dynamics

EFTs and low-energy scales



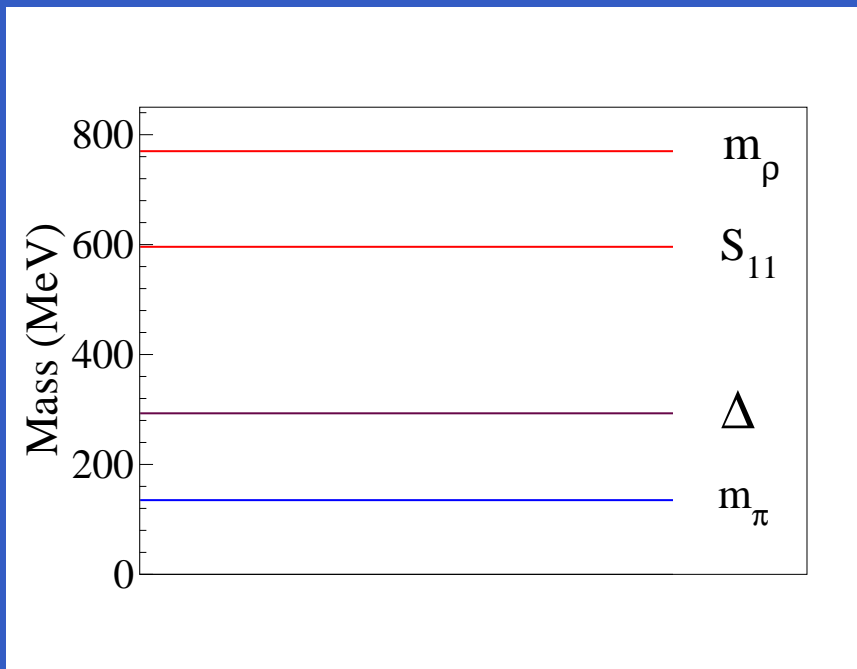
EFTs and low-energy scales



Three possible EFTs:

- EFT(π): $\omega < m_\pi$;
- χ PT (Δ): $\omega \sim m_\pi < \Delta$;
- χ PT + Δ : $\omega \sim \Delta < m_\rho$

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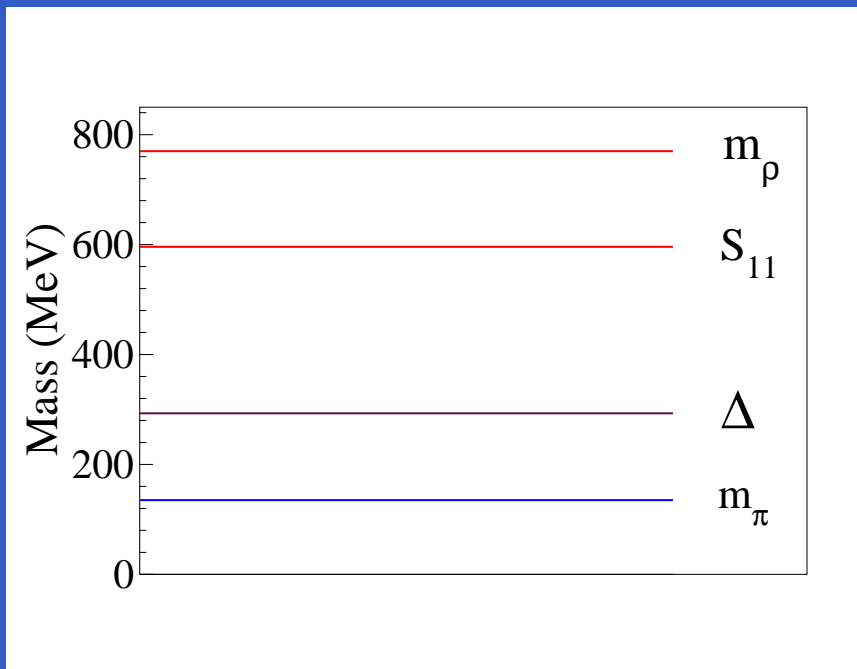


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Each can be applied in A=1 AND A=2 (and A=3 ...)

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$$P \equiv \frac{p, m_\pi}{m_\rho, 4\pi f_\pi}$$

p/M expansion employed: (usually) useful, not essential.

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χ PT without explicit $\Delta \Rightarrow \omega, |\mathbf{q}| < \Delta$

Power counting in χ PT(Δ)

- P^n for a vertex with n powers of p or m_π : $\mathcal{L}^{(n)}$;
- P^{-2} for each pion propagator: $\frac{1}{q^2 - m_\pi^2}$;
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- P^4 for each loop: $\int d^4k$;

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Power counting for loops as well as for \mathcal{L}

$$\Rightarrow \mathcal{A}_{\gamma N} = \sum_n \mathcal{F}_n \left(\frac{p}{m_\pi} \right) P^n,$$

\mathcal{F}_n has non-analytic pieces from pion loops, and constant pieces from “short-distance physics” (Δ , ρ , M-branes, ...)

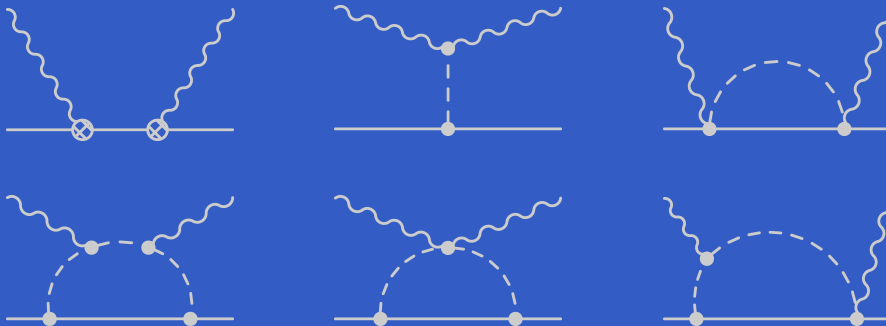
Nucleon Compton Scattering in χ PT

$O(e^2)$:



$$\frac{-e^2}{M} \epsilon' \cdot \epsilon$$

$O(e^2 P)$:



Powell X-Sn +
non-analyticity
from loops

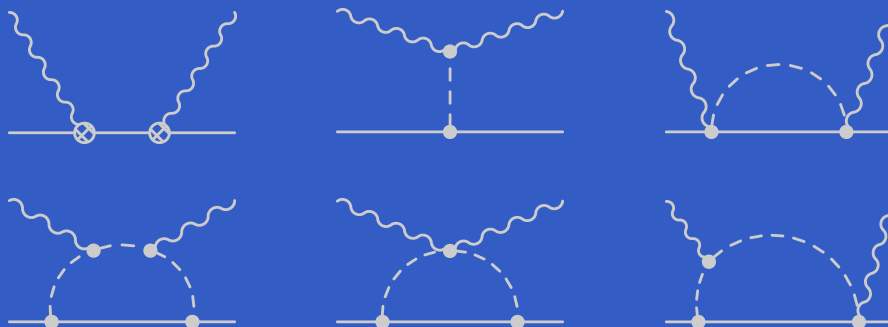
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Matching to a polynomial in ω yields

$$\alpha_N = \frac{5e^2 g_A^2}{384\pi^2 f_\pi^2 m_\pi} = 12.2 \times 10^{-4} \text{ fm}^3; \quad \beta_N = 1.2 \times 10^{-4} \text{ fm}^3.$$

Bernard, Kaiser, Meißner (1992)

PDG average:

$$\alpha_p = (12.0 \pm 0.7) \times 10^{-4} \text{ fm}^3;$$

$$\beta_p = (1.6 \pm 0.6) \times 10^{-4} \text{ fm}^3.$$

N²LO: $O(e^2 P^2)$

γ N amplitude at $O(e^2 P^2)$



J. McGovern, Phys. Rev. C 63, 064608 (2001)

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Short-distance pieces of polarizabilities should be fit:

$$4\pi\alpha_{\text{high}}\mathbf{E}^2, 4\pi\beta_{\text{high}}\mathbf{B}^2 \quad \sim \omega^2 e^2$$

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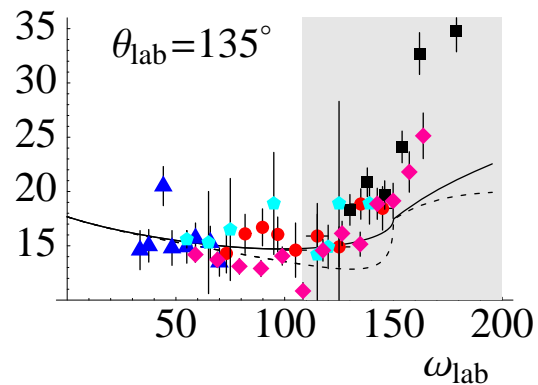
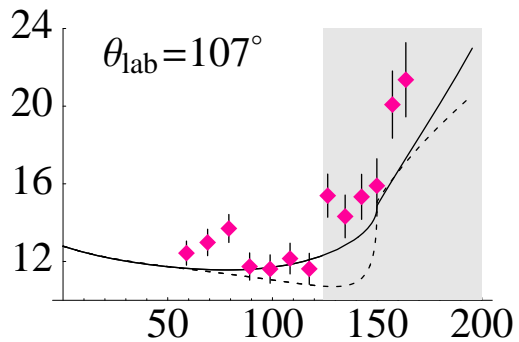
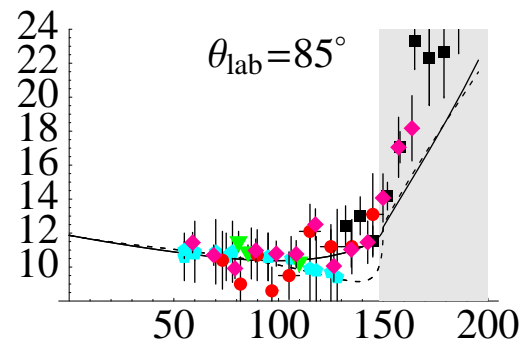
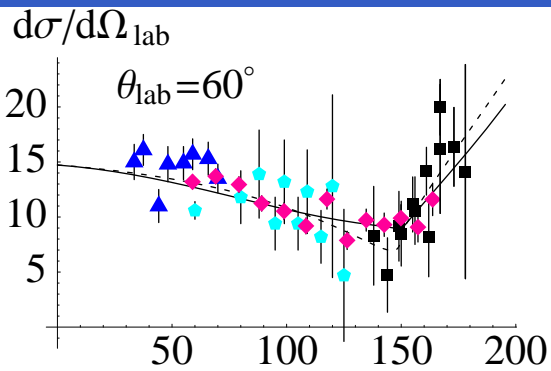
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Experiments: SAL/Illinois, LEGS, MAMI,

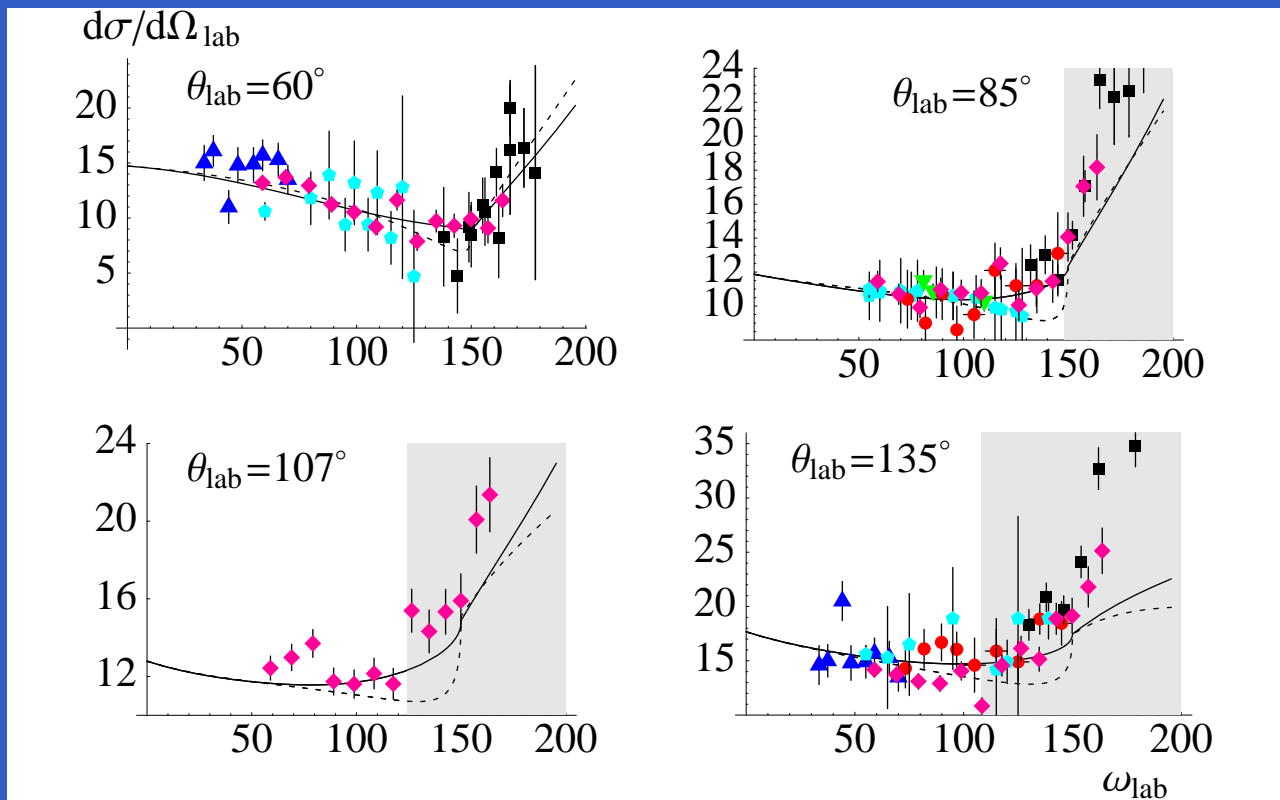
Kinematic restriction (Δ -less χ PT): $\omega, \sqrt{|t|} \leq 180$ MeV.

Results



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$$\alpha_p = (12.1 \pm 1.1)_{-0.5}^{+0.5} \times 10^{-4} \text{ fm}^3$$

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S. R. Beane, J. McGovern, M. Malheiro, D. P., U. van Kolck, PLB, 567, 200 (2003).

Going higher for γp

Breakdown set by first omitted mass scale: $\Delta \equiv M_\Delta - M_N$.

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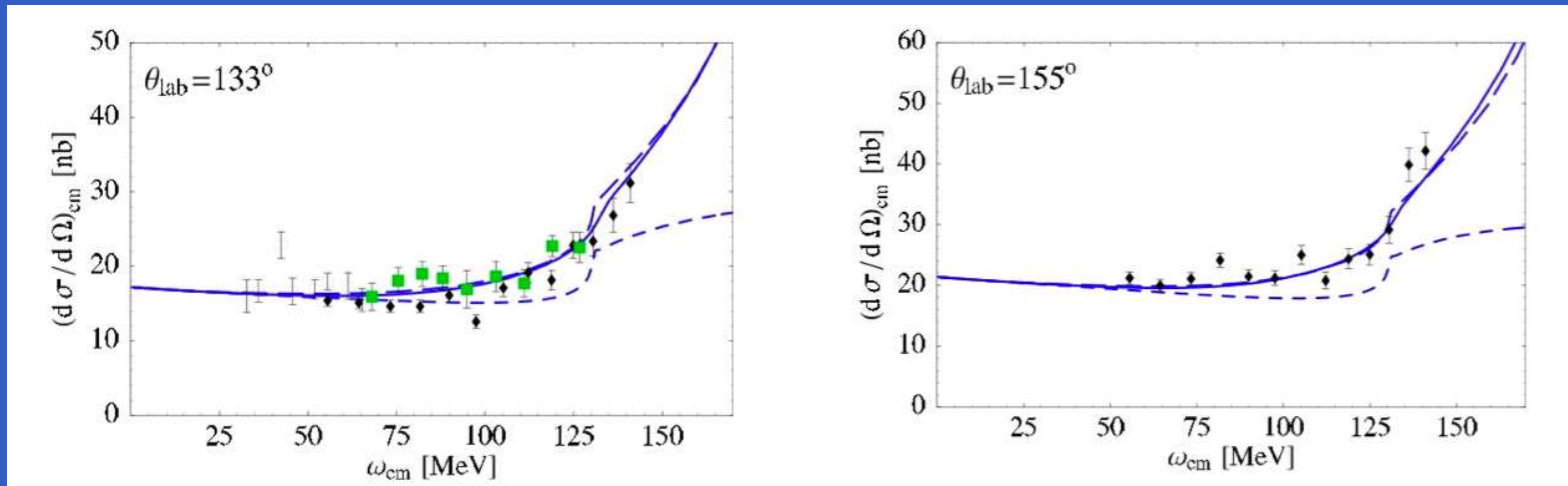
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Also add (isoscalar) counterterms $\delta\alpha_{\text{high}}\mathbf{E}^2 + \delta\beta_{\text{high}}\mathbf{B}^2$

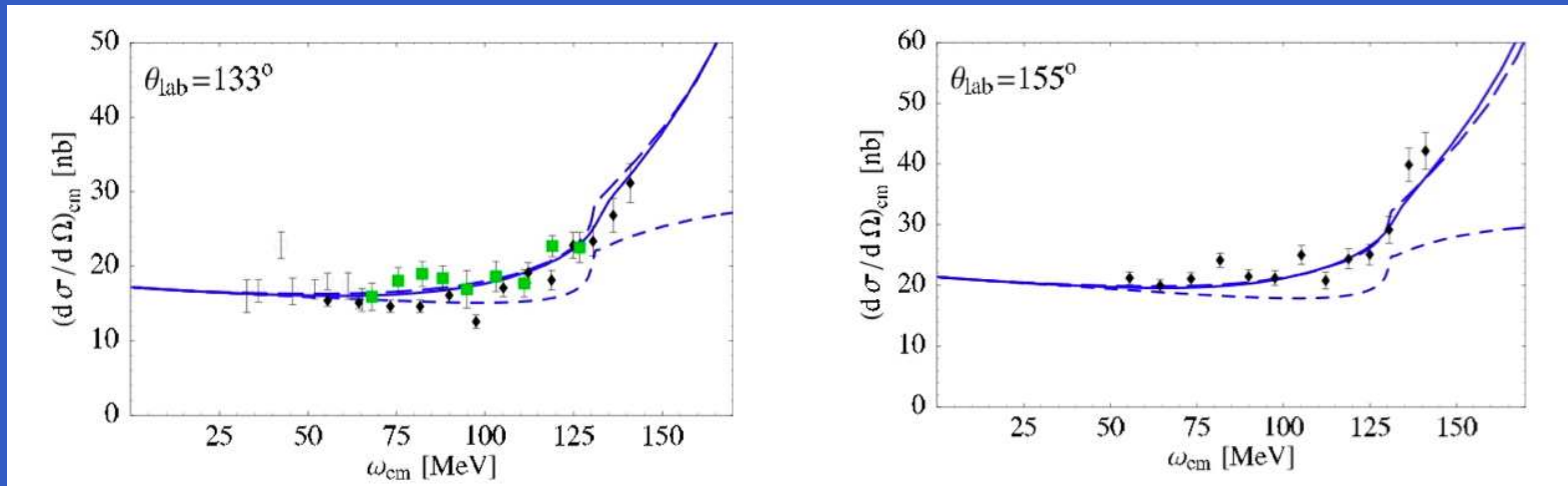
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Fit to γp data with $\omega < 200$ MeV

$$\alpha_p = (11.04 \pm 1.3 \pm 1.0) \times 10^{-4} \text{ fm}^3$$

$$\beta_p = (2.76 \mp 1.3 \pm 1.0) \times 10^{-4} \text{ fm}^3$$

Baldin Sum Rule constraint used here

γp : Conclusions

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Pascalutsa, D. P., 2003

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- Measurement of Σ at $\text{HI}\gamma\text{S} \rightarrow$ information on β_p ?
- Issues with experimental database: can χ PT help?

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- Use elastic cross γd section not quasi-free $\gamma d \rightarrow \gamma np$: exploit quantum coherence!

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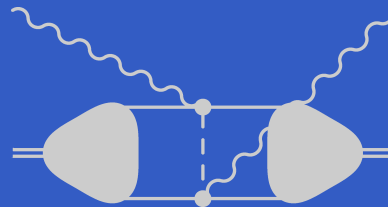
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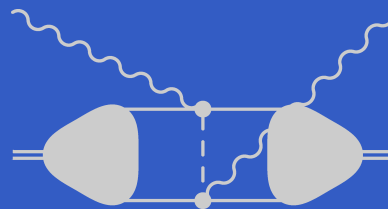


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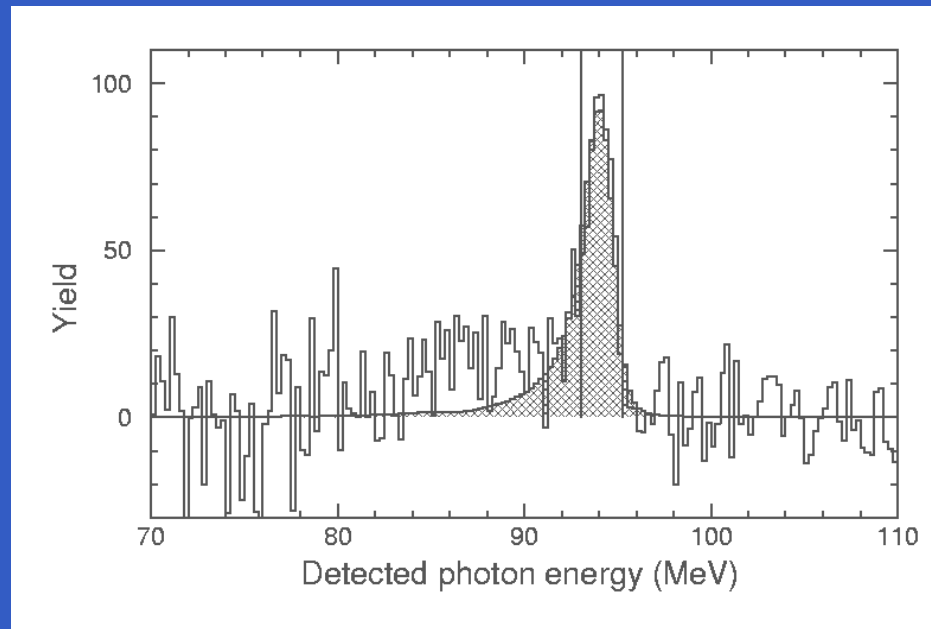
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Possible to extract α_N and β_N from $\gamma d \rightarrow \gamma d$ data,
but need to treat 2B effects **SYSTEMATICALLY**.

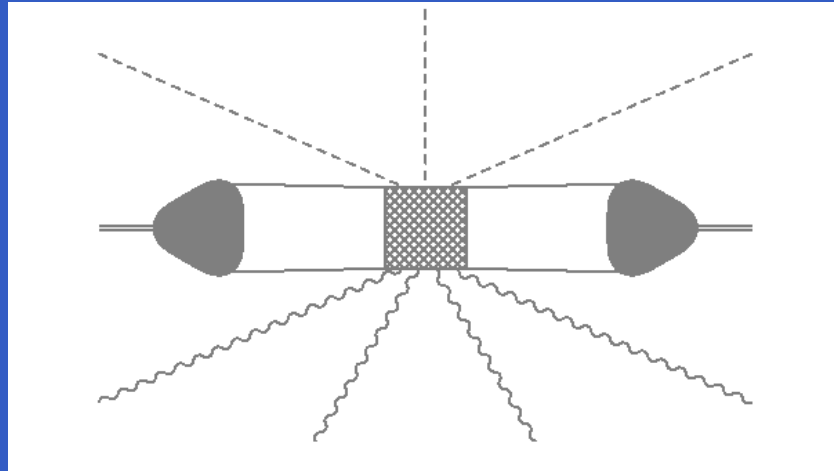
γ d experiments

- Illinois (1994): M. Lucas, Ph.D. thesis, $\omega = 49, 69$ MeV;
- SAL (2000): D. Hornidge et al., PRL 84, 2334 (2000), $\omega = 85 - 105$ MeV;
- Lund (2003): M. Lundin et al., PRL 90, 192501 (2003), $\omega = 55, 65$ MeV.

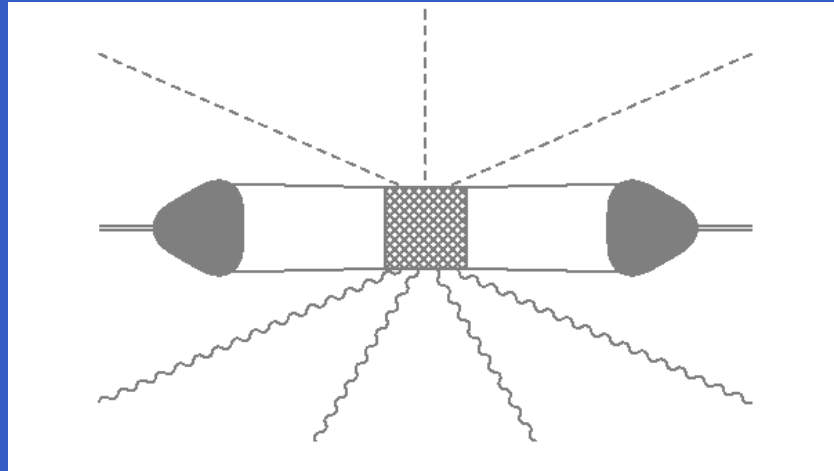


D. Hornidge, PhD thesis (1999)

Reactions on deuterium

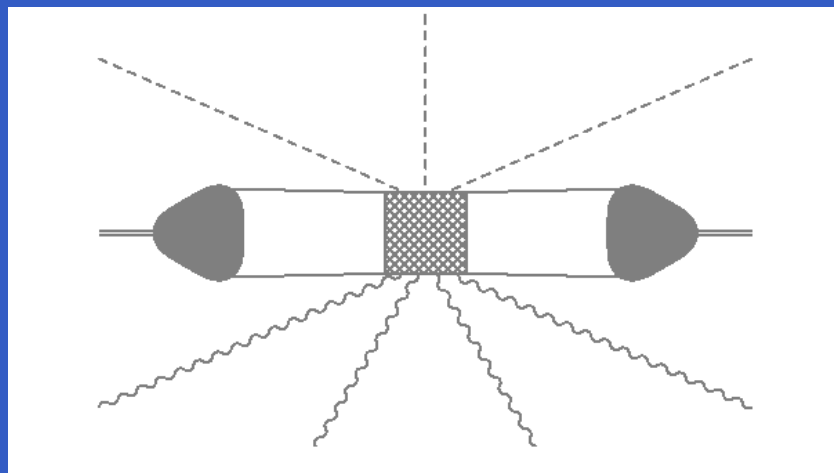


Reactions on deuterium



$$\langle \psi | \hat{O} | \psi \rangle$$

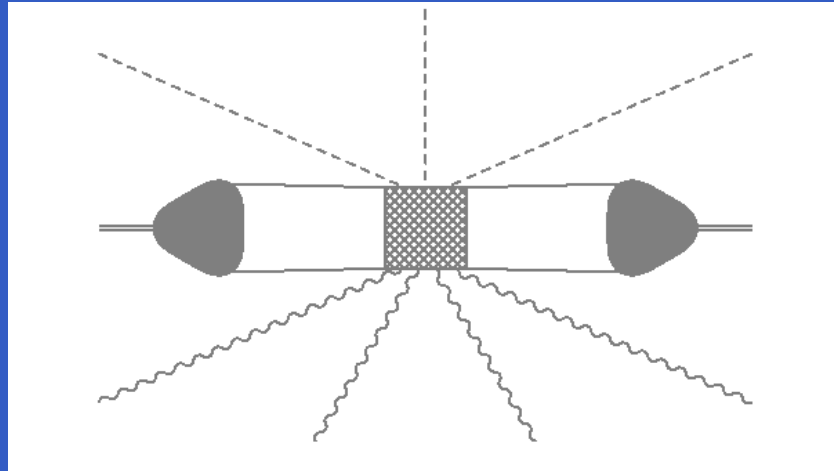
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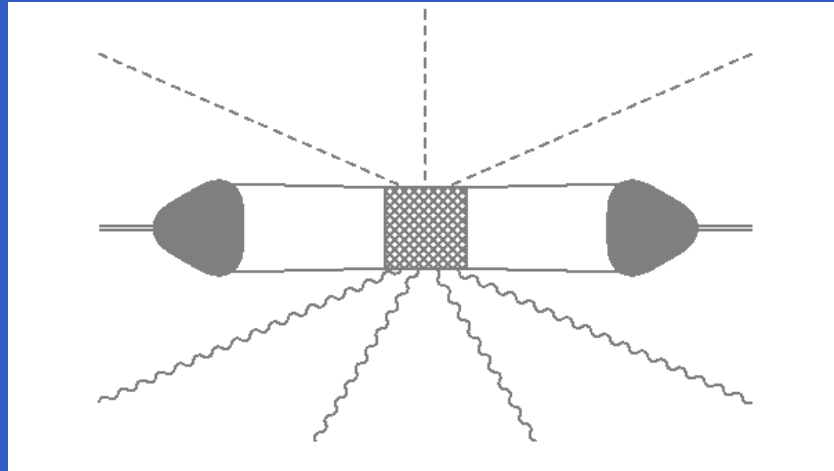
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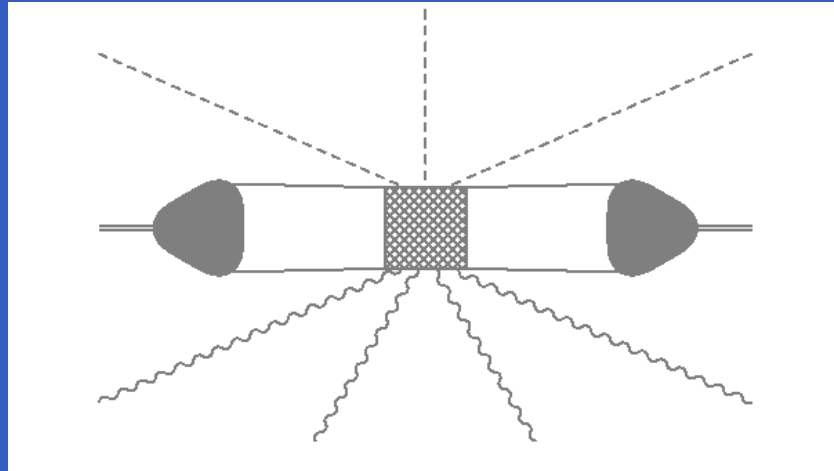


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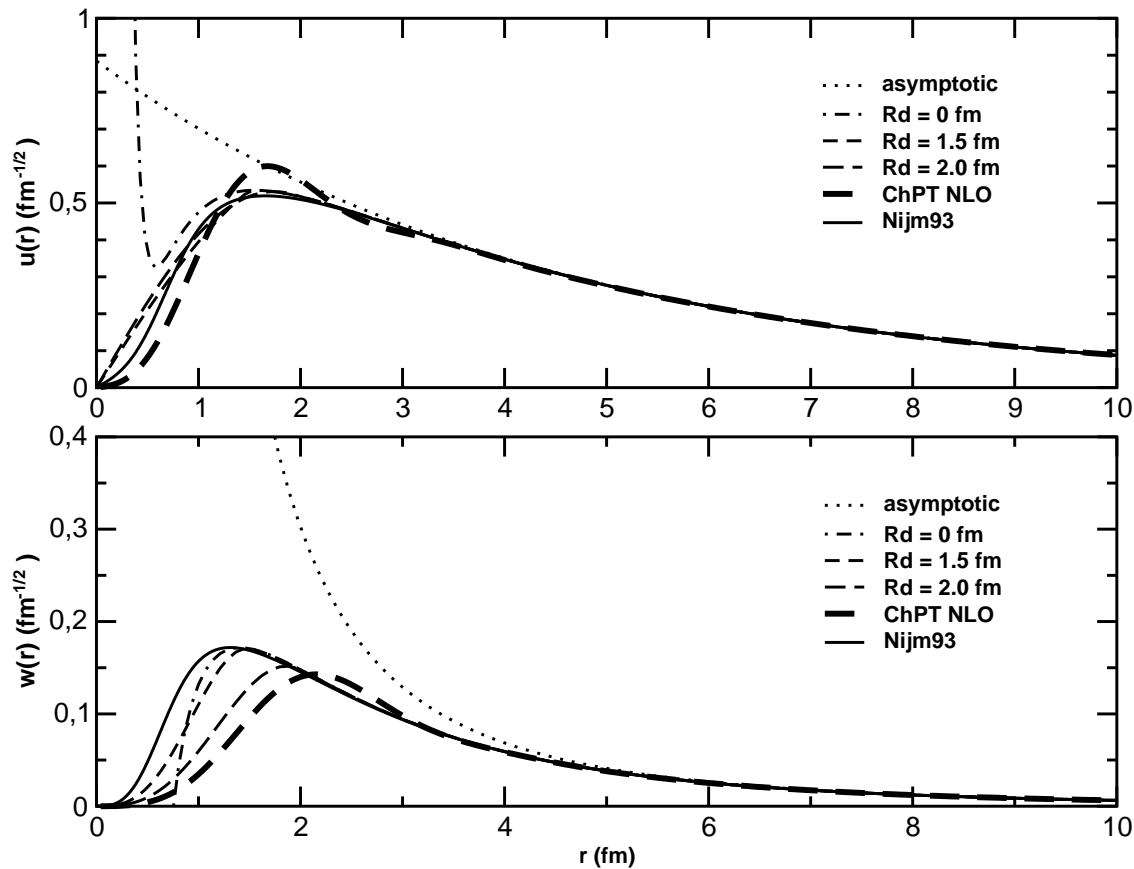
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Should be model independent, systematically improvable,
accurate at low momentum/energy transfer.

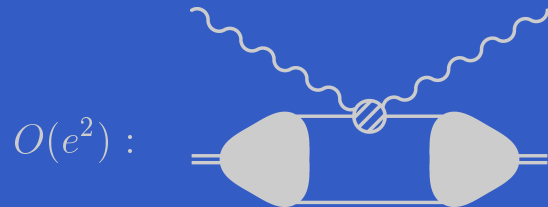
Deuteron wave functions



Same at long range:
 $B, A_S, A_D, f_{\pi NN}, m_{\pi}$.
Some differences at
two-pion range.

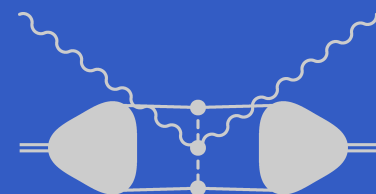
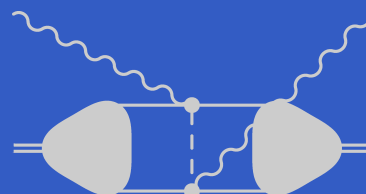
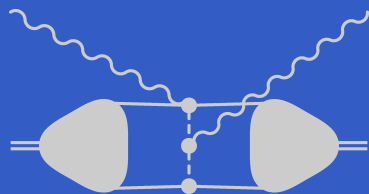
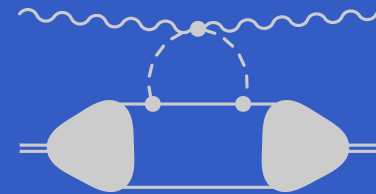
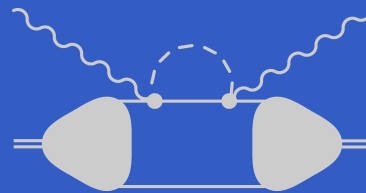
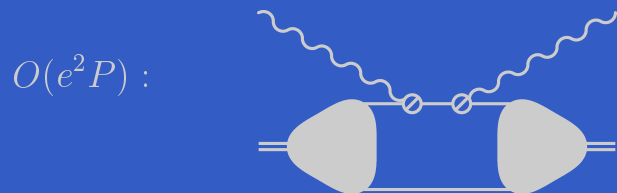
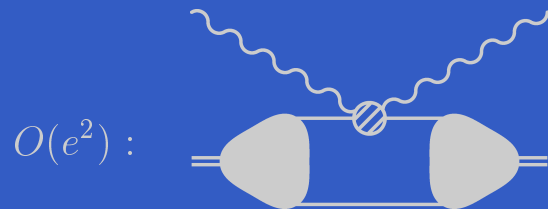
γd in χ PT to $O(e^2 P)$

S. R. Beane, M. Malheiro, D. P., U. van Kolck, Nucl. Phys. **A656**, 367 (1999)



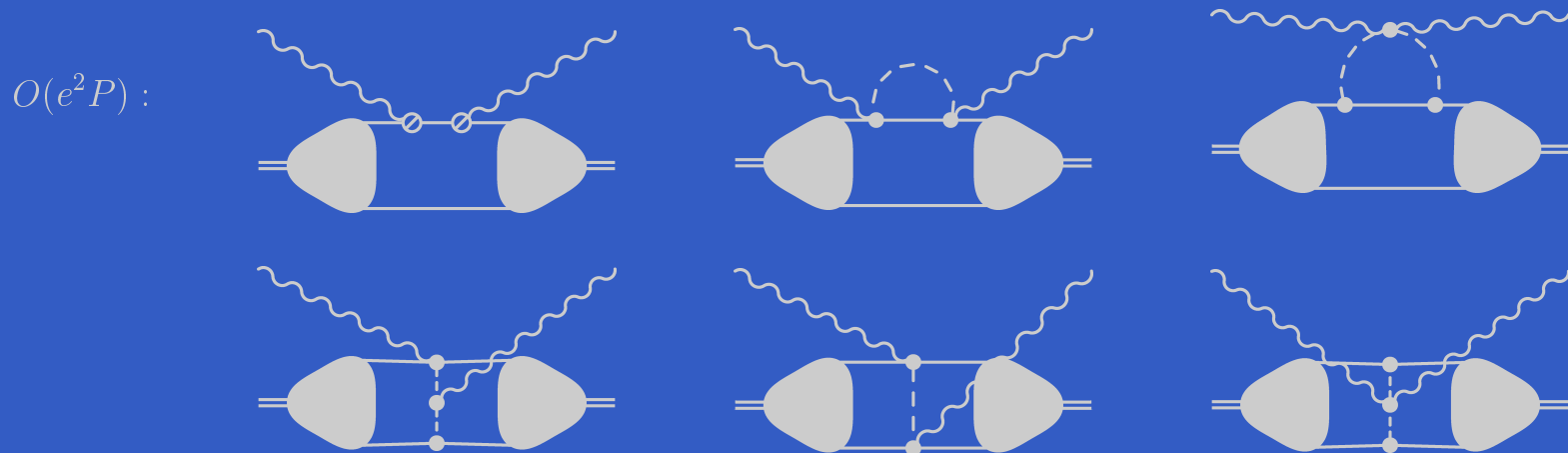
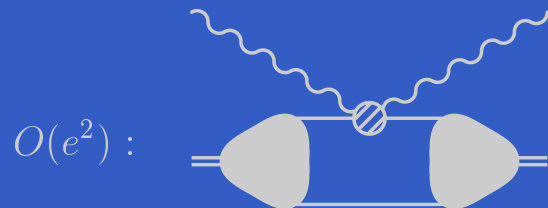
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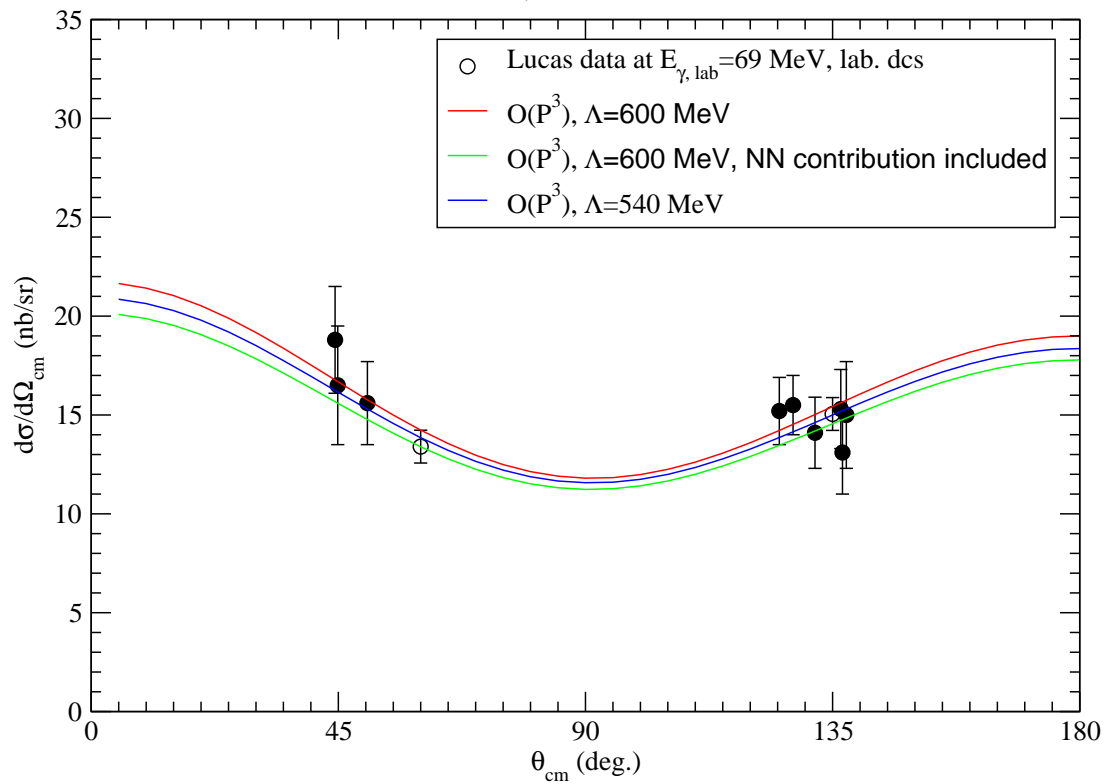


No free parameters at $O(e^2 P) \Rightarrow$ PREDICTION

Results: I

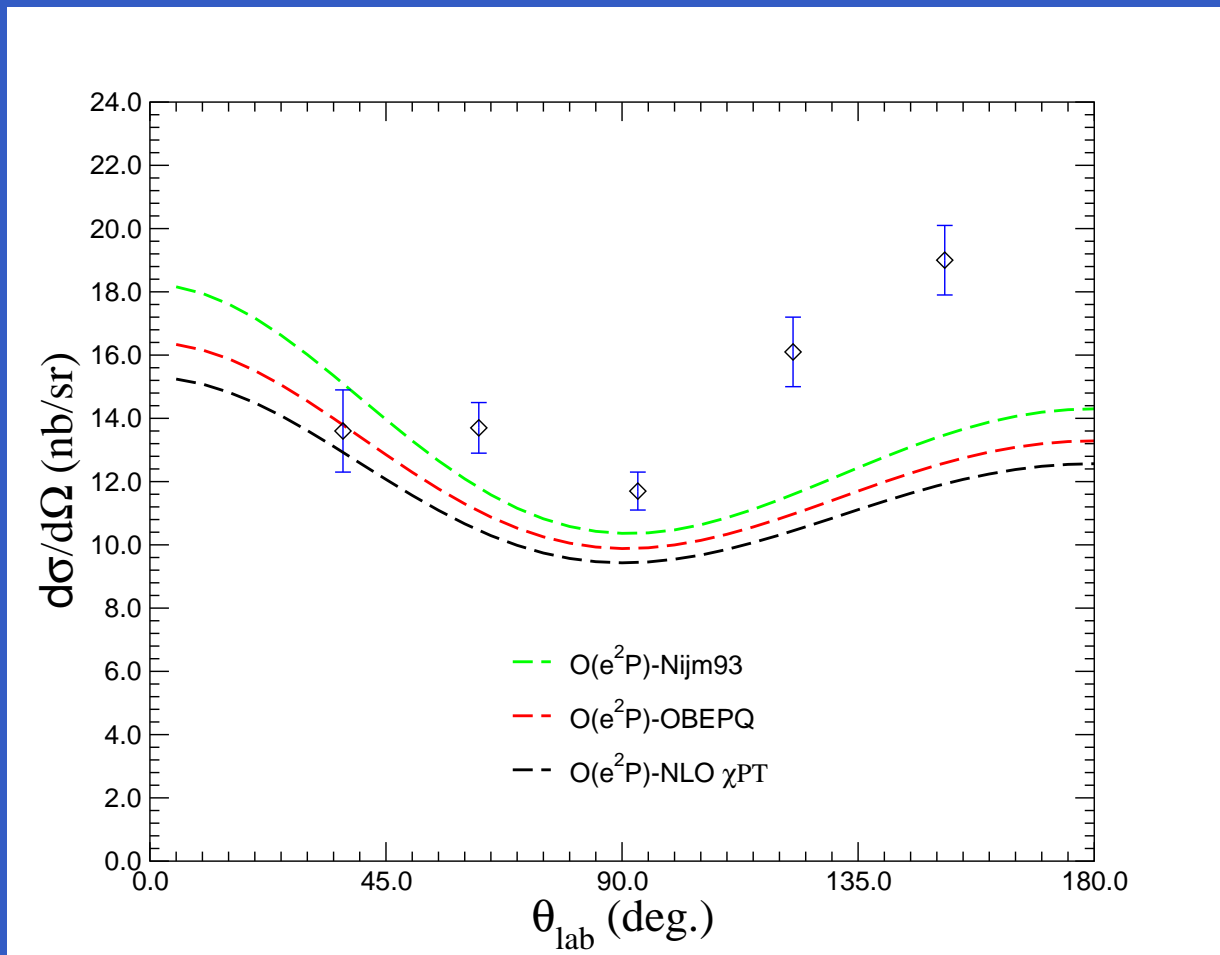
γ -d scattering, data from Lund

$E_{\gamma \text{ cm}} = 66 \text{ MeV}$

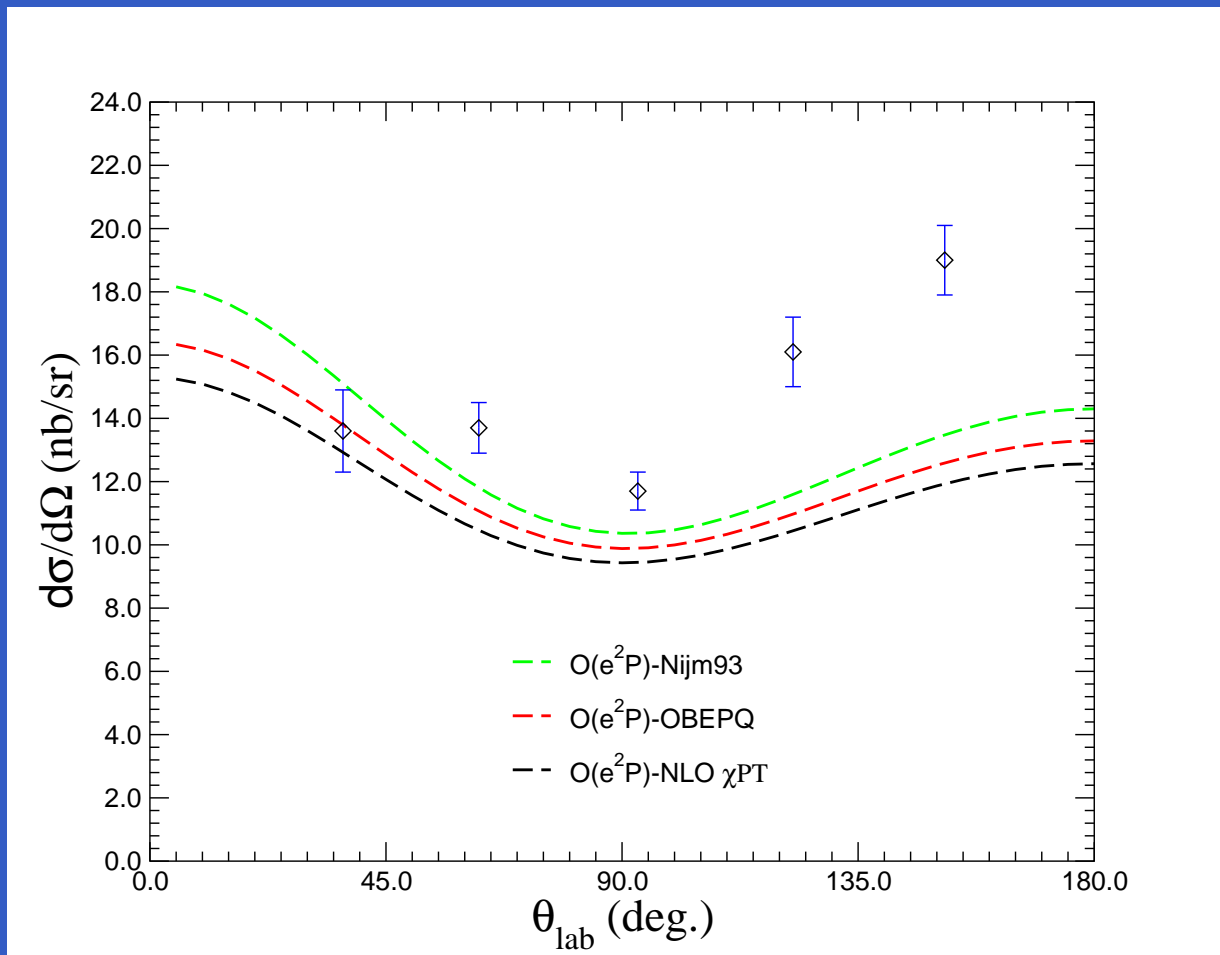


Wave-function
dependence
 \approx theoretical
uncertainty.

A couple of problems



A couple of problems



- Shape at $|q| \approx 150$ MeV
- Wave-function dependence

γ d scattering at $O(e^2 P^2)$

S. R. Beane, M. Malheiro, J. McGovern, D. P., U. van Kolck, PLB (2003) & NPA (2005)

γ d scattering at $O(e^2 P^2)$

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1. γ N amplitude at $O(e^2 P^2)$;

γ d scattering at $O(e^2 P^2)$

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Calculable in terms of f_π , g_A , κ_V , m_π , and M .

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S. R. Beane, M. Malheiro, J. McGovern, D. P., U. van Kolck, PLB (2003) & NPA (2005)

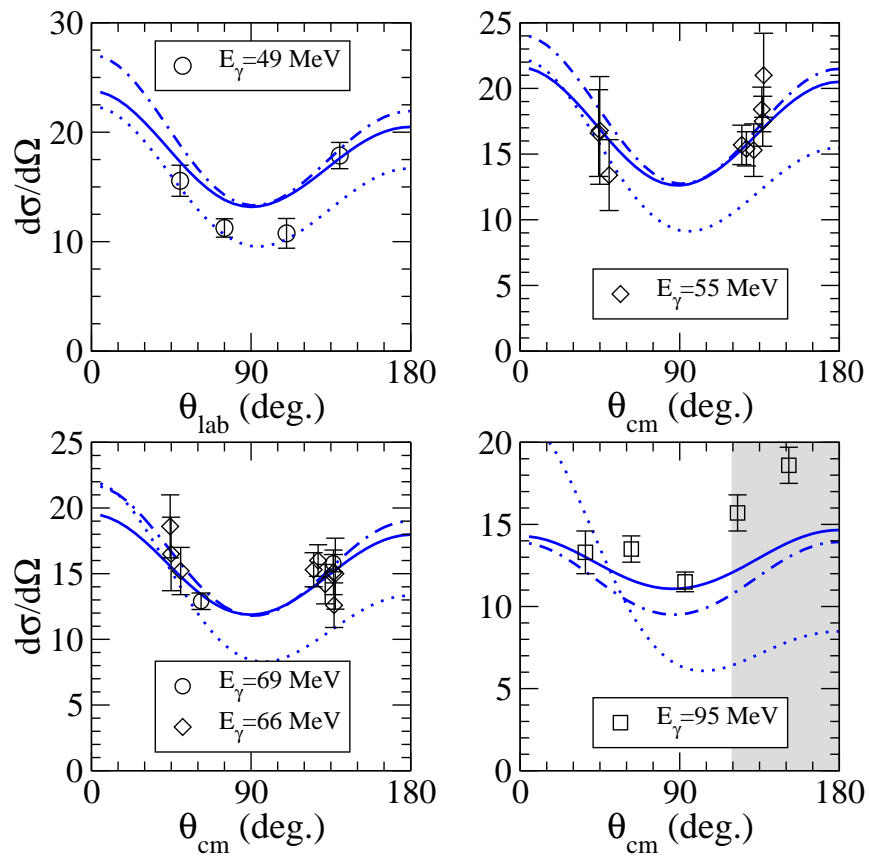
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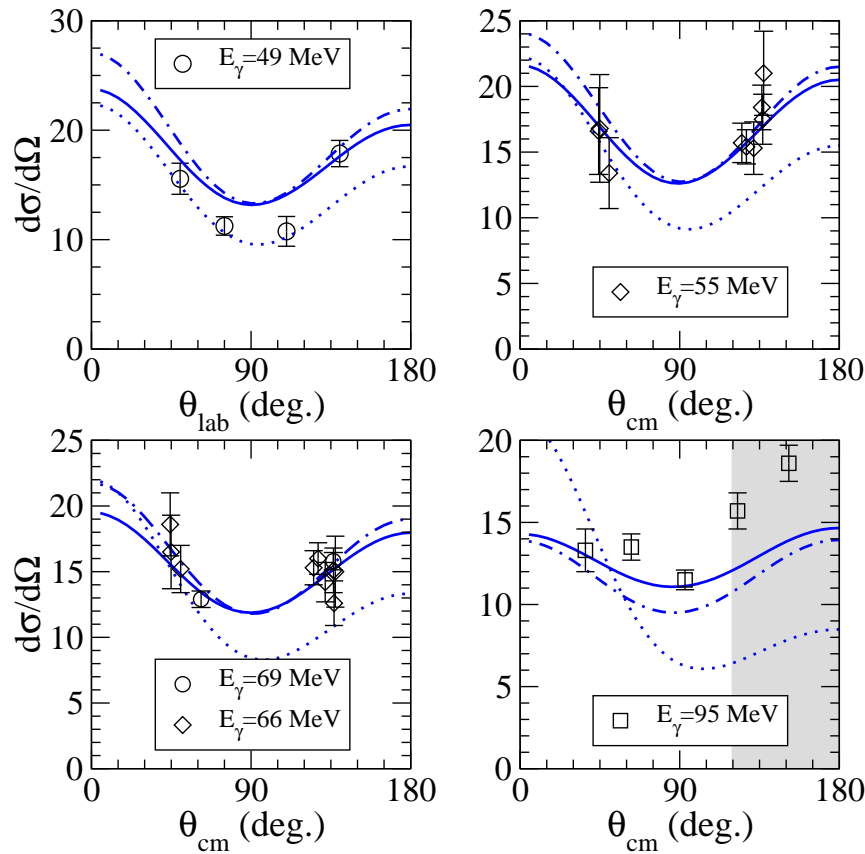
Calculable in terms of f_π , g_A , κ_V , m_π , and M .

Only free parameters are α_N and β_N .

Best-fit results

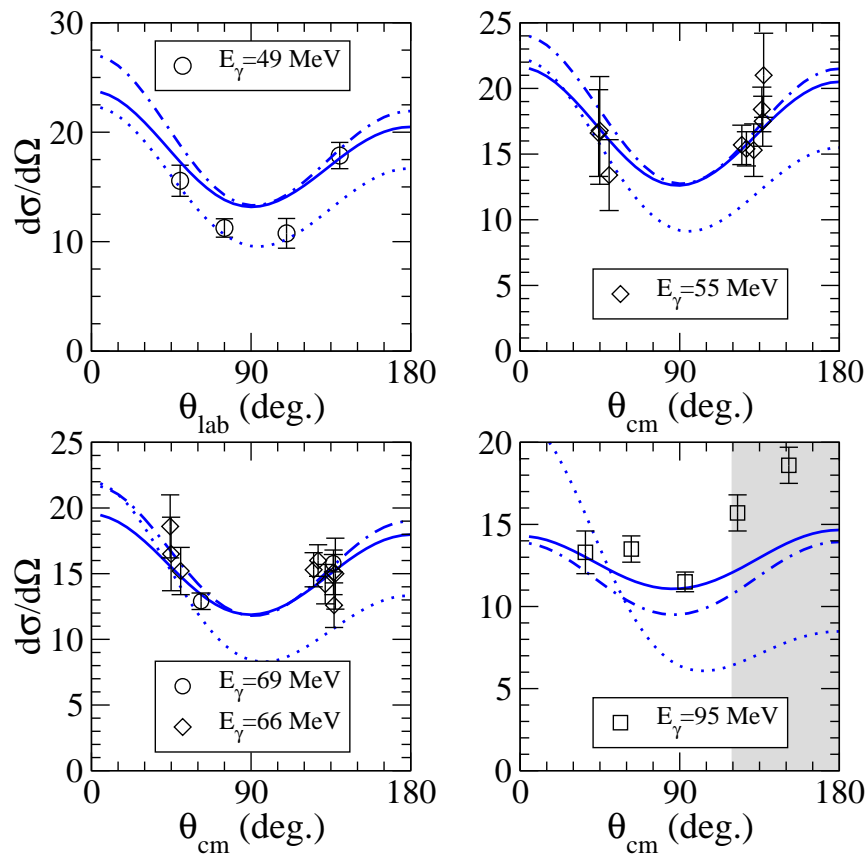


Best-fit results



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Best-fit results

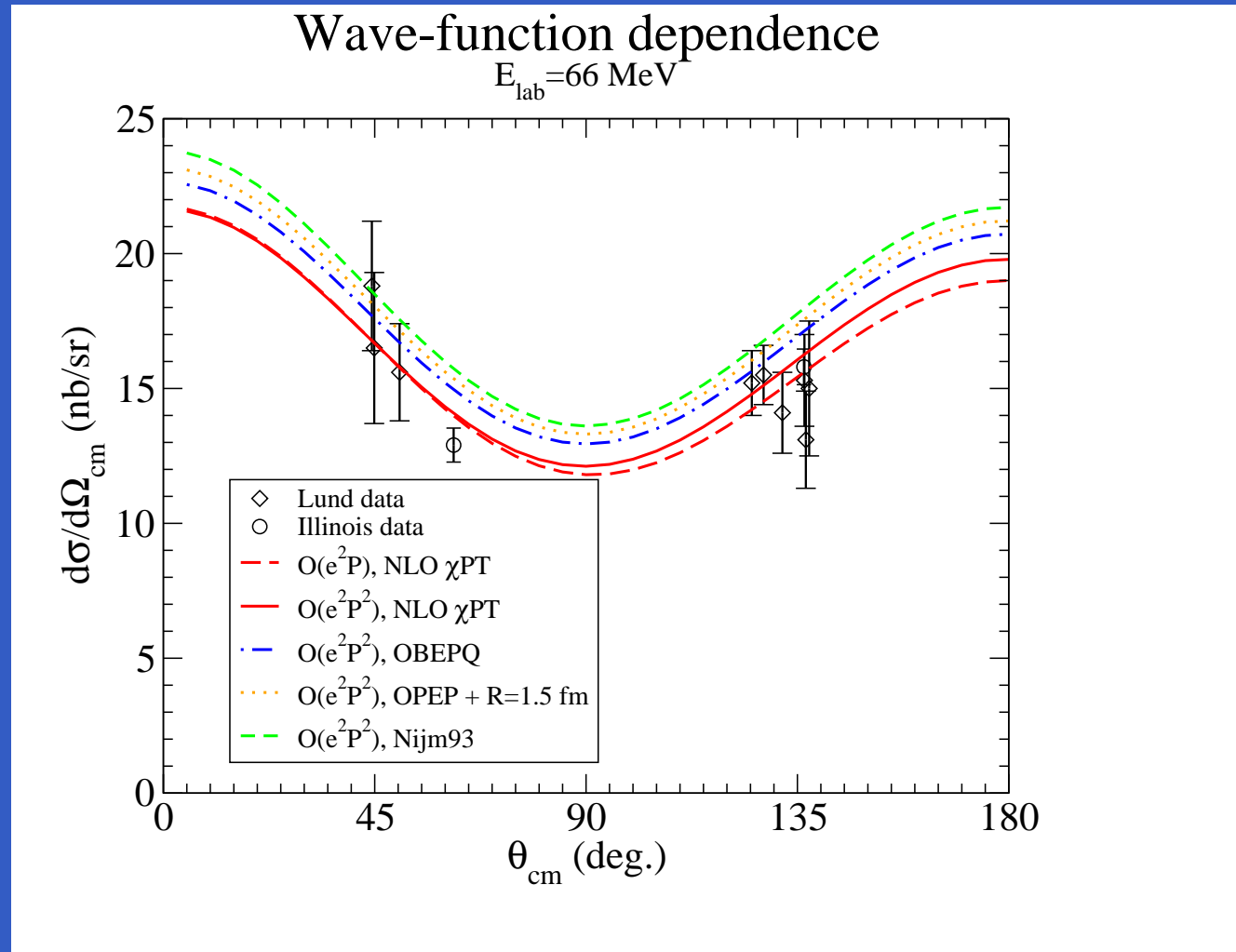


- Convergence good
- $O(e^2P)$ problems persist

$$\alpha_N = (13.0 \pm 1.9)_{-1.5}^{+3.9} \times 10^{-4} \text{ fm}^3$$

$$\beta_N = (-1.8 \pm 1.9)_{-0.9}^{+2.1} \times 10^{-4} \text{ fm}^3$$

Dependence of cross section on $|\psi\rangle$



γd with an explicit Delta(1232)

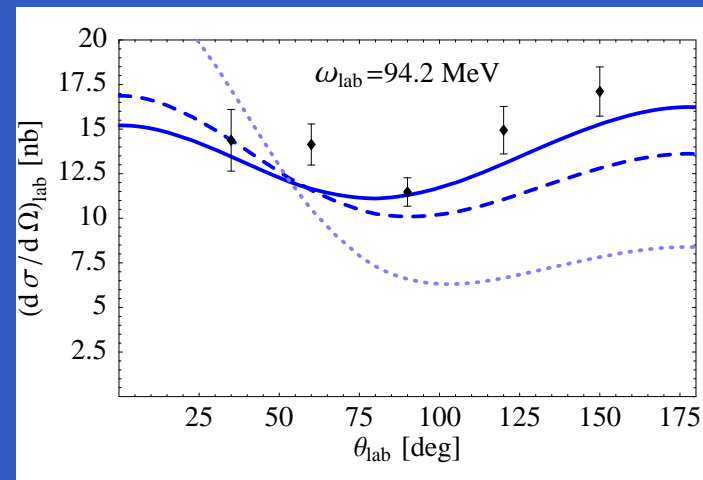
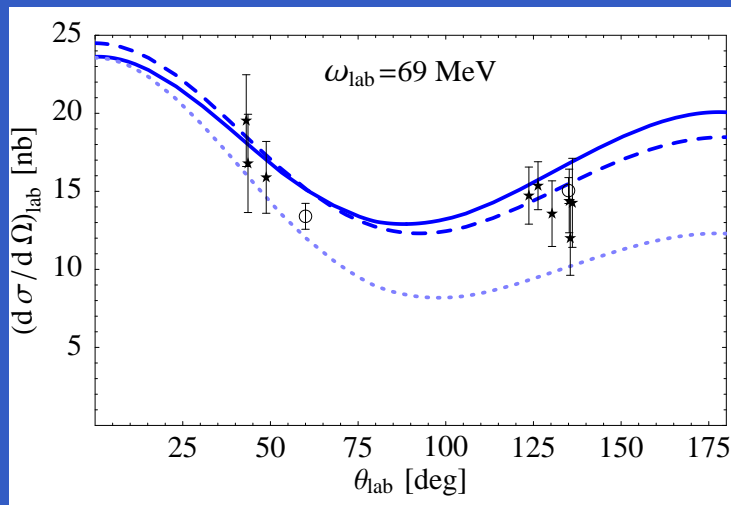
R. Hildebrandt, H. Griesshammer, T. Hemmert, D.P., Nucl. Phys. A (2005)

- Calculation to $O(e^2\epsilon)$ in χ PT with Delta(1232)
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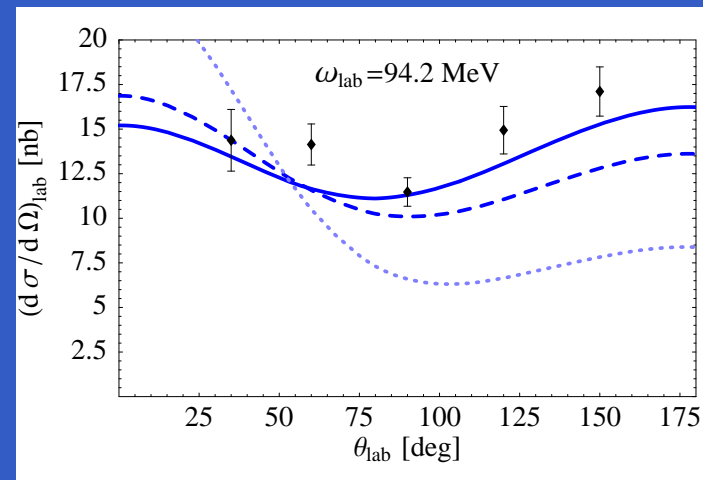
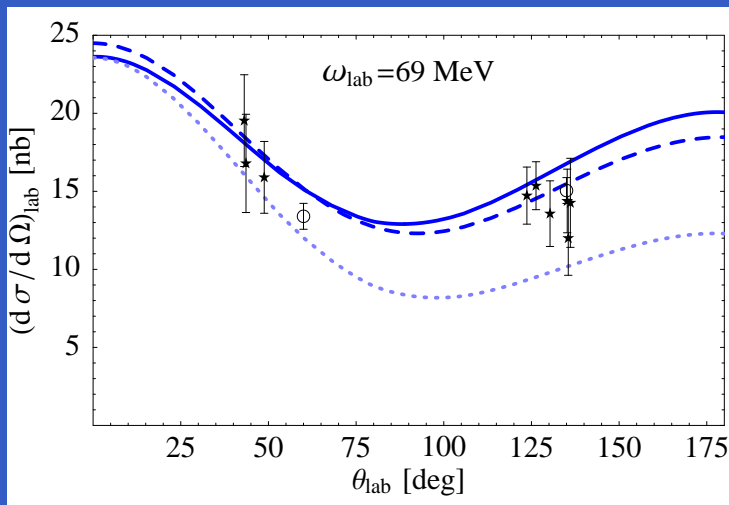
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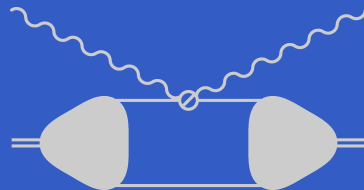
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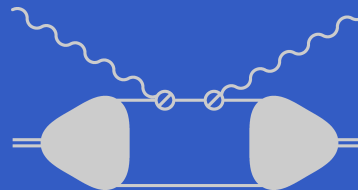


Prediction at $O(e^2\epsilon)$

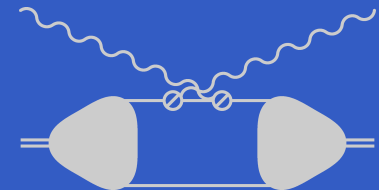
Going to low energies



(a)



(b)

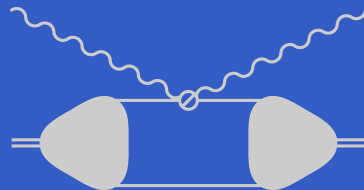


(c)

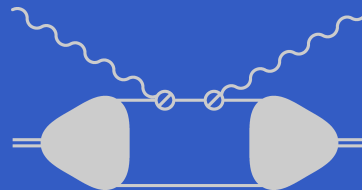
In EFT(π) (b) and (c) crucial for recovery of

$$T(\omega = 0) = -\frac{e^2}{M_d} \epsilon' \cdot \epsilon$$

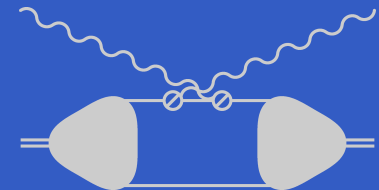
Going to low energies



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(b)



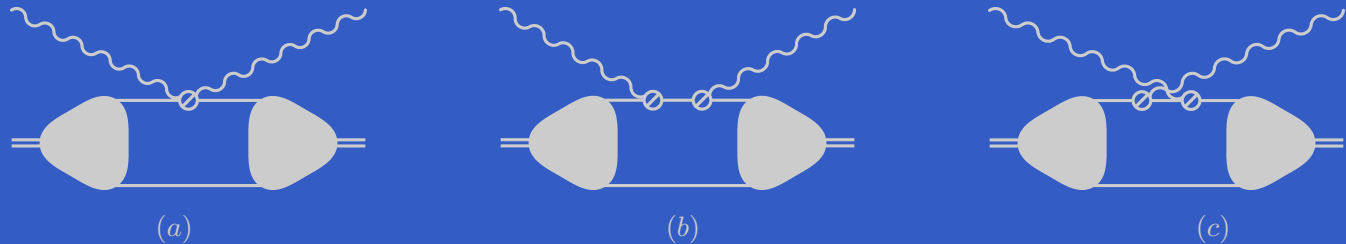
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Also crucial is to use: $\frac{1}{\omega - p^2/M}$ NOT $\frac{1}{\omega}$

Going to low energies



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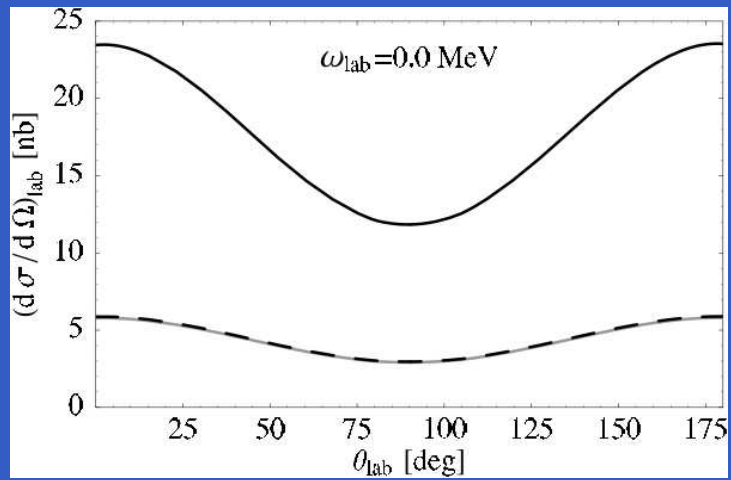
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Also crucial is to use: $\frac{1}{\omega - p^2/M}$ NOT $\frac{1}{\omega}$

- Modification of power-counting needed for $\omega \sim m_\pi^2/M$;
- Estimates \Rightarrow significant at 49 and 55 MeV. Higher order at 95 MeV.

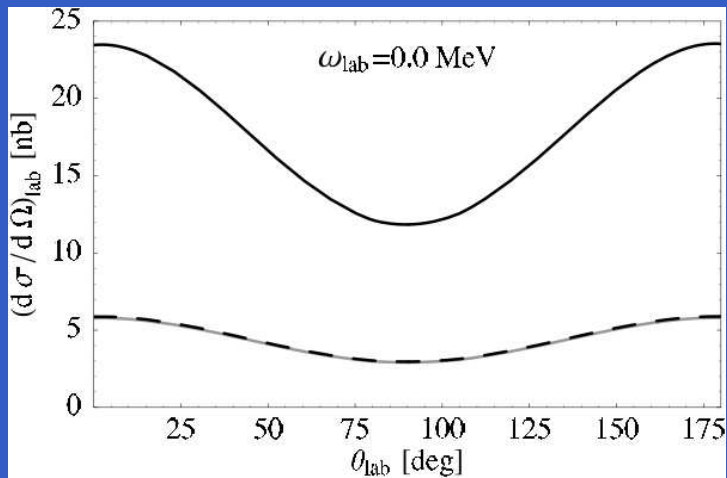
The solution

Hildebrandt, Grieshammer, Hemmert, [nucl-th/0512063](#)

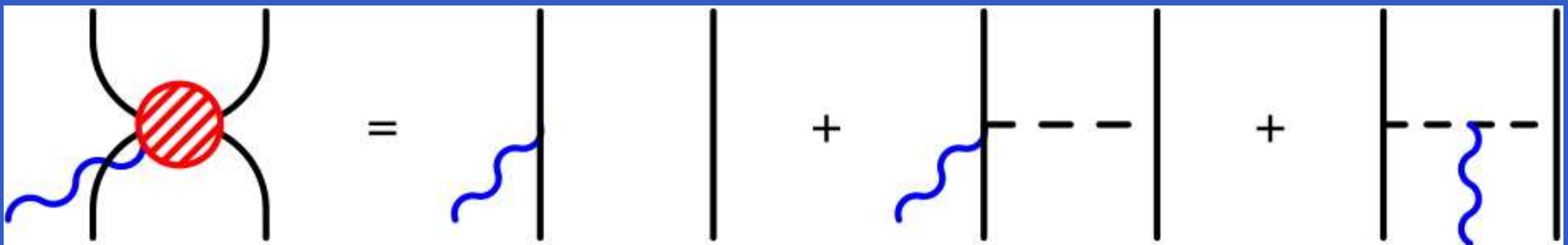
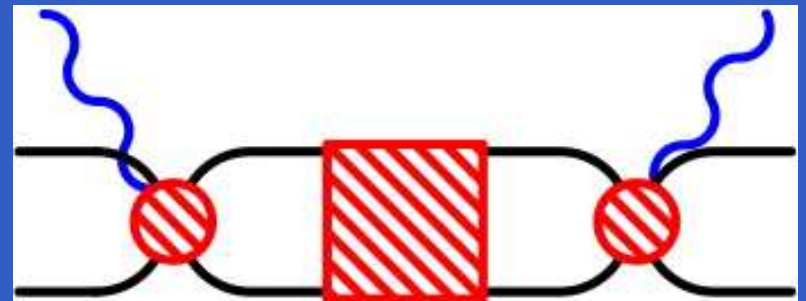


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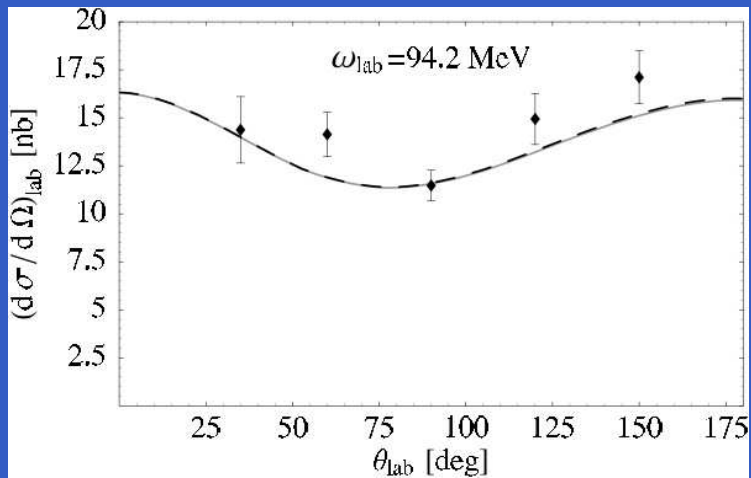
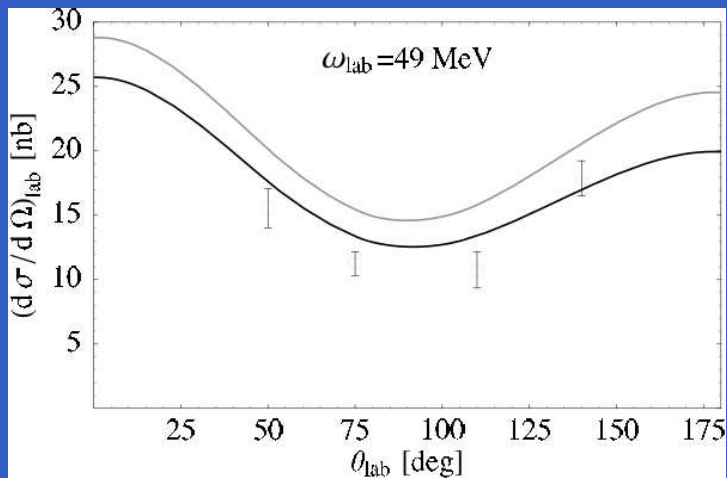
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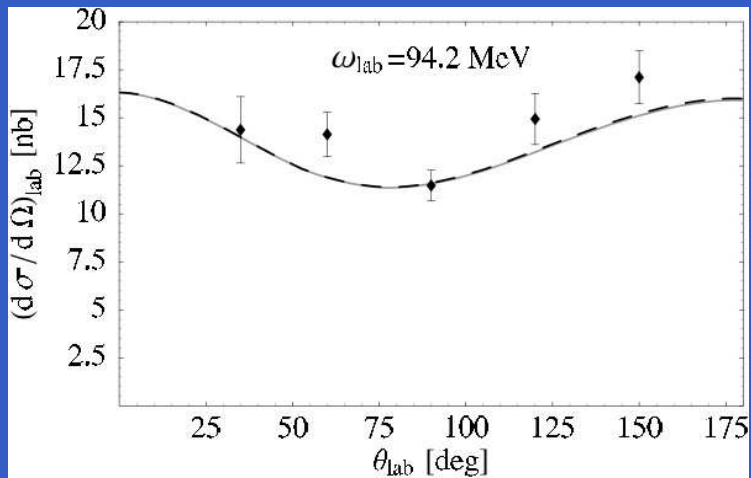
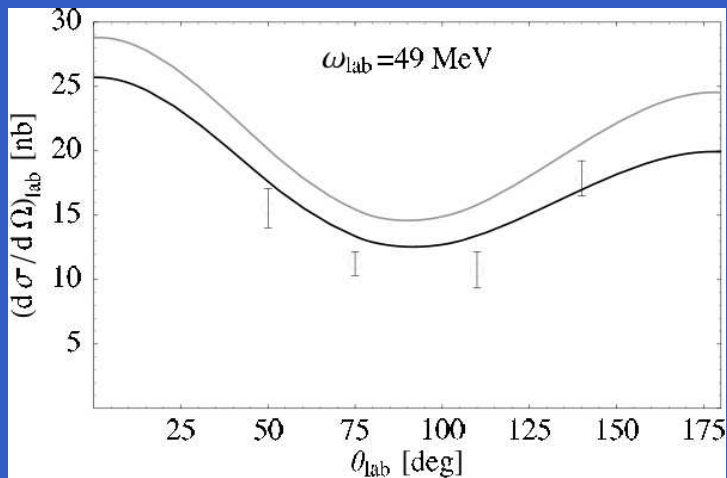
How is Thomson limit repaired?



The benefits of clean living



The benefits of clean living



$$\alpha_N = (11.3 \pm 0.7 \pm 0.6) \times 10^{-4} \text{ fm}^3$$
$$\beta_N = (3.2 \pm 0.7 \mp 0.6) \times 10^{-4} \text{ fm}^3$$

**Theoretical uncertainty
much smaller now**

γ d at MAX-Lab

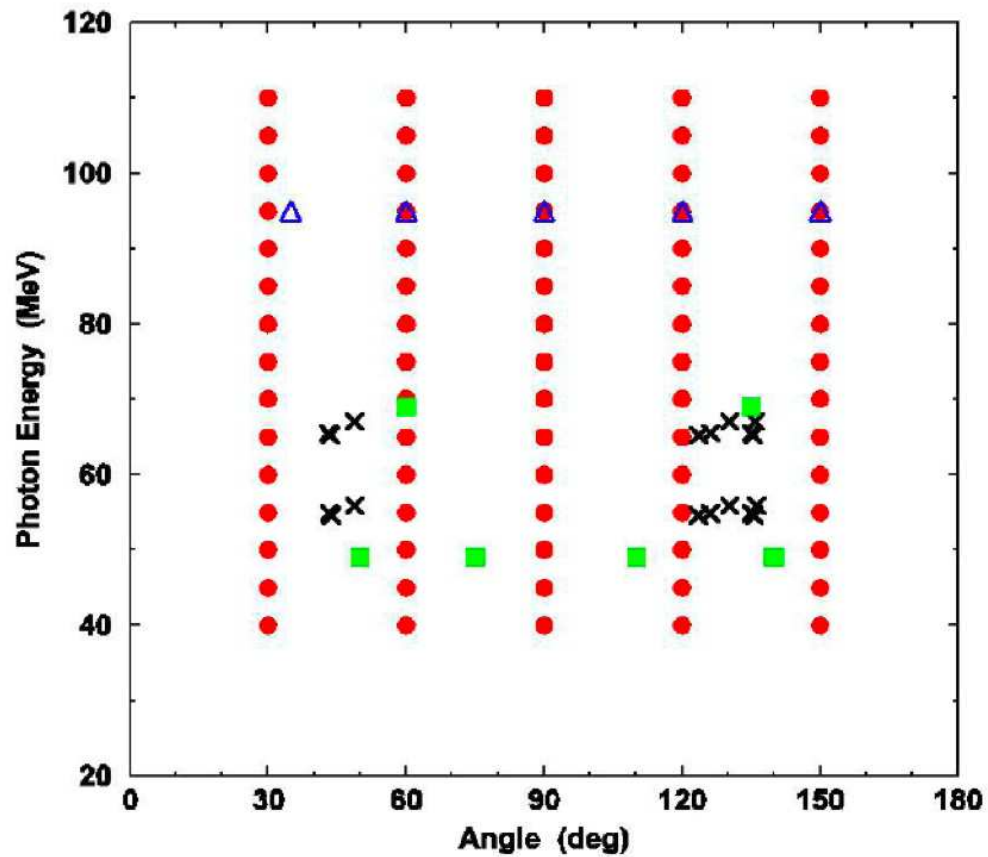


figure courtesy J. Feldman

Conclusions: γd

- $O(e^2 P)$ [NLO]: Parameter-free predictions.

✓ $\omega \leq 80$ MeV.

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Thanks to the U.S. Department of Energy for support.

Naive dimensional analysis for \hat{O}

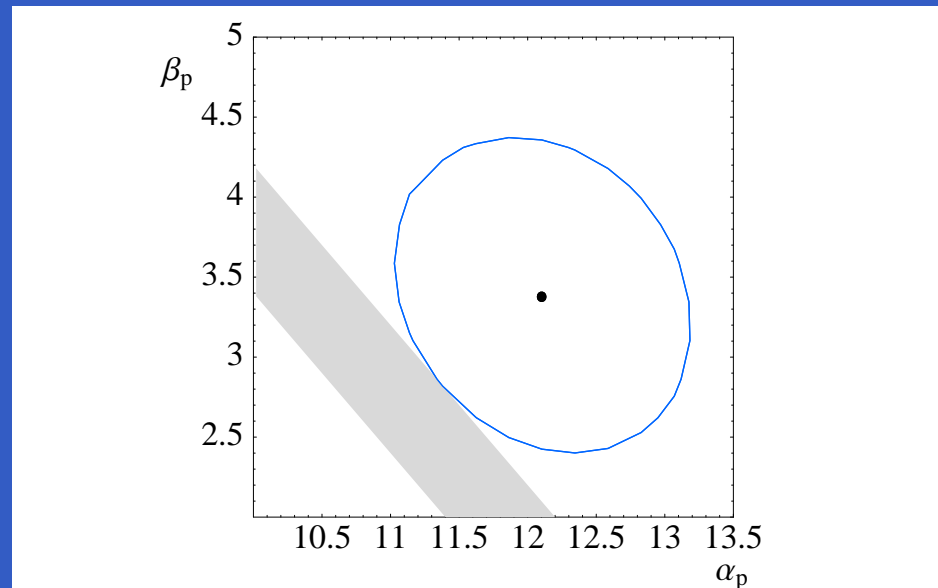
- P^n for a vertex with n powers of p or m_π : $\mathcal{L}^{(n)}$;
- P^{-2} for each pion propagator: $\frac{1}{q^2 - m_\pi^2}$;
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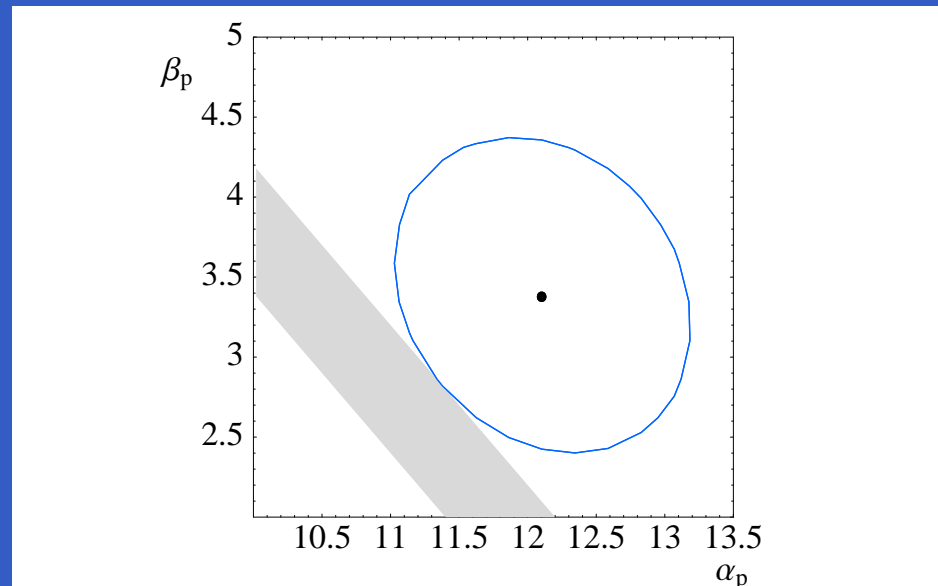
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Loops, many-body effects, and vertices from $\mathcal{L}^{(2,3)}$ etc. suppressed by powers of P .

Baldin Sum Rule



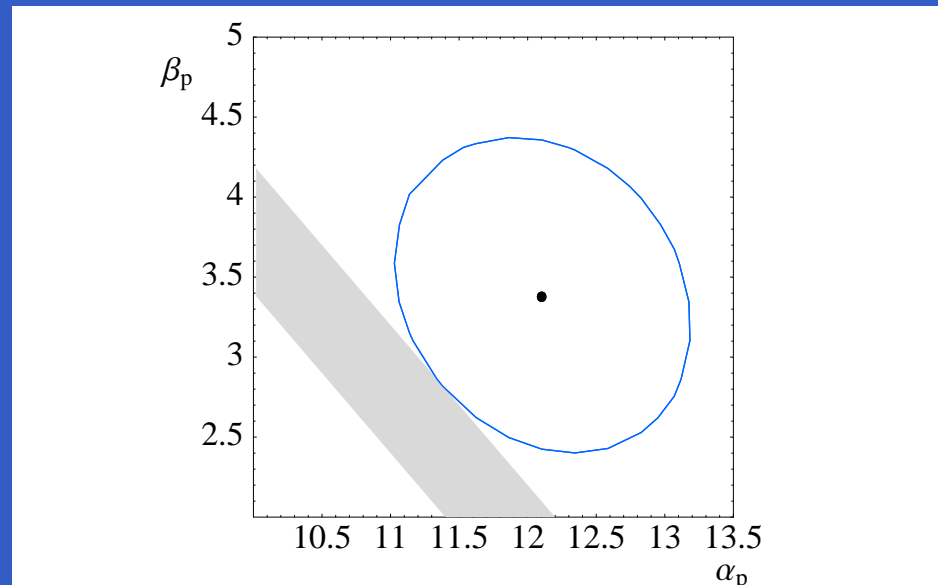
Baldin Sum Rule



With Baldin Sum Rule constraint:

$$\alpha_p + \beta_p = (13.8 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

Baldin Sum Rule



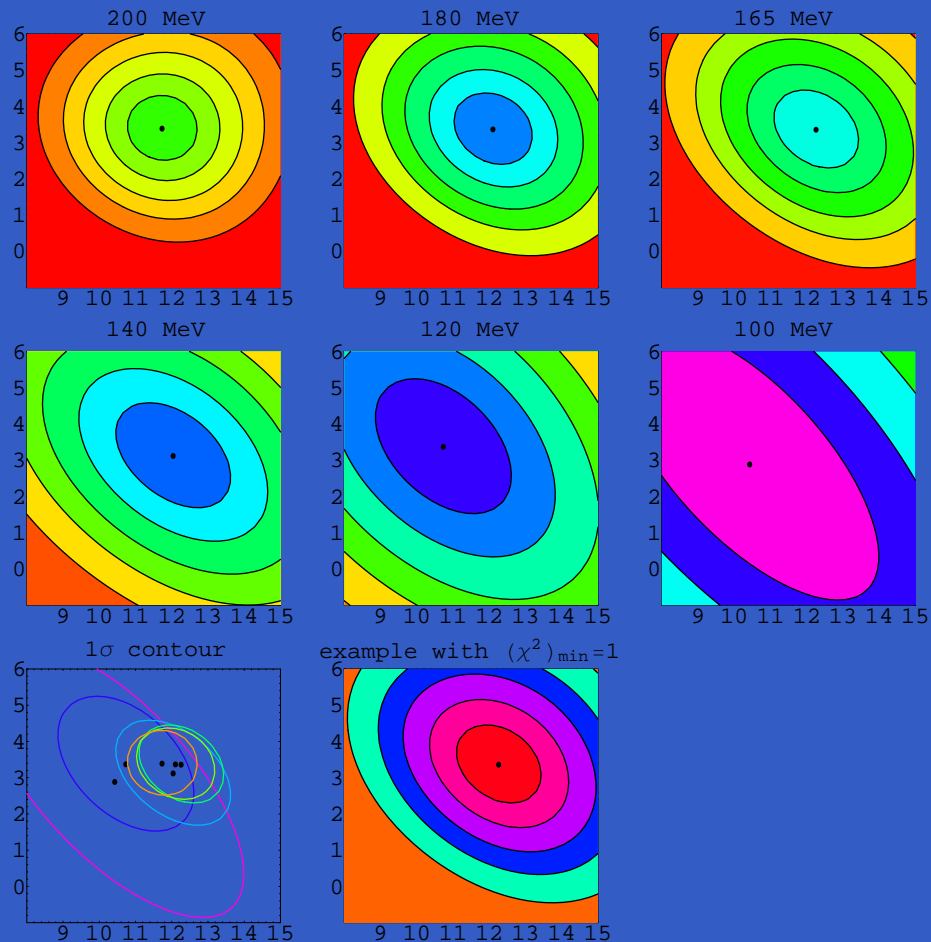
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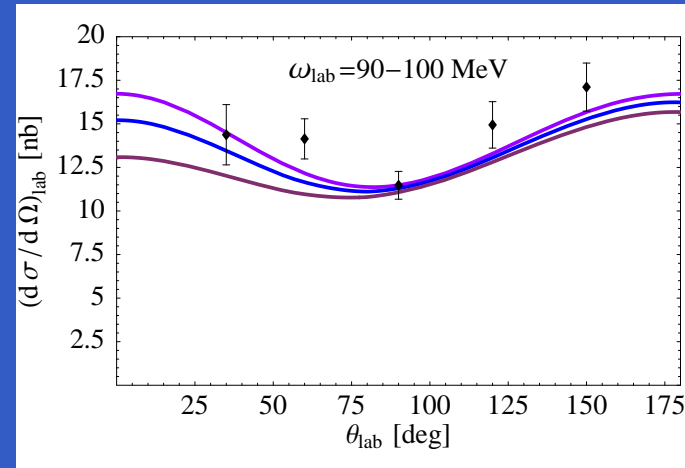
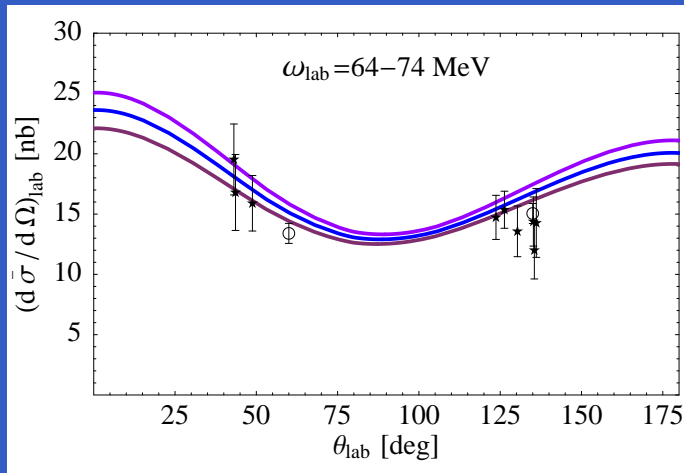
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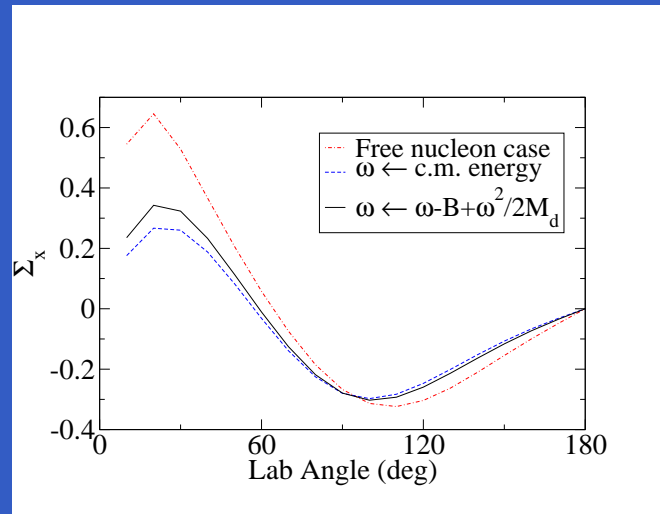
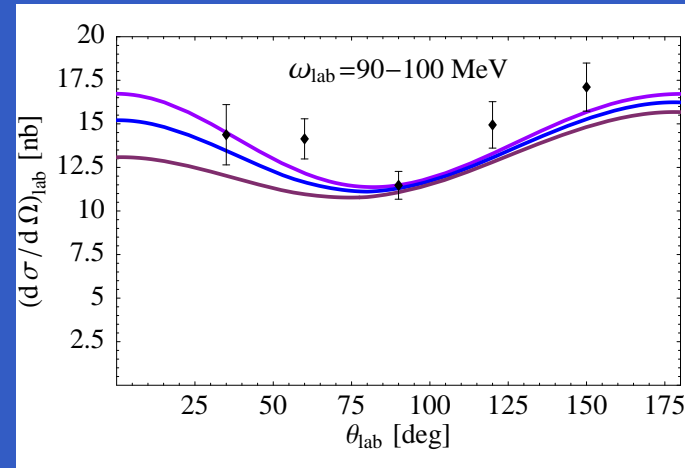
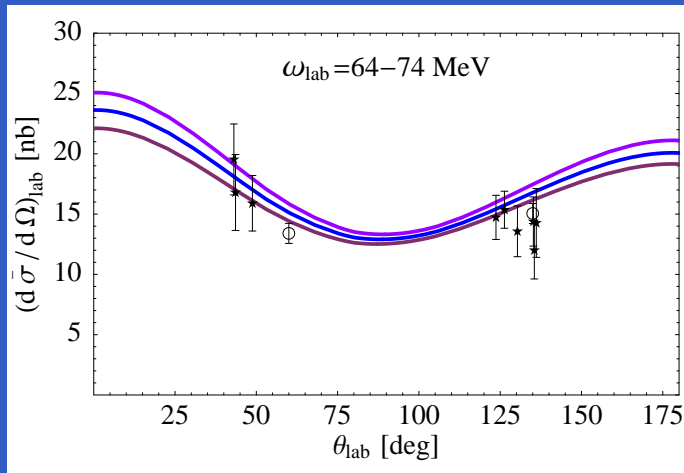
Cutoff dependence of γp fit



Energy dependence of $T_{\gamma N}$?



Energy dependence of $T_{\gamma N}$?



c.f. Lensky, Hanhart, *et al.*