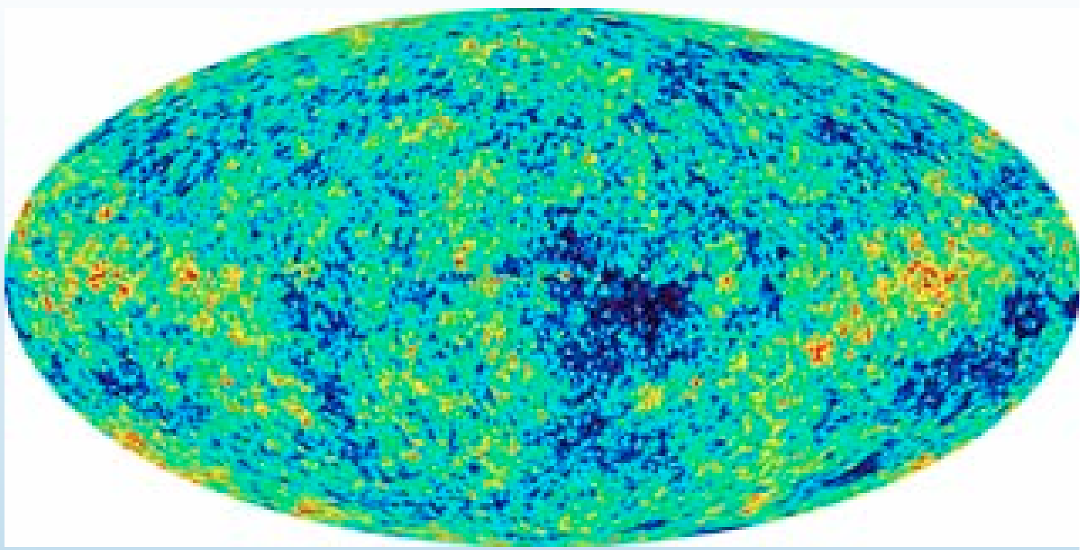
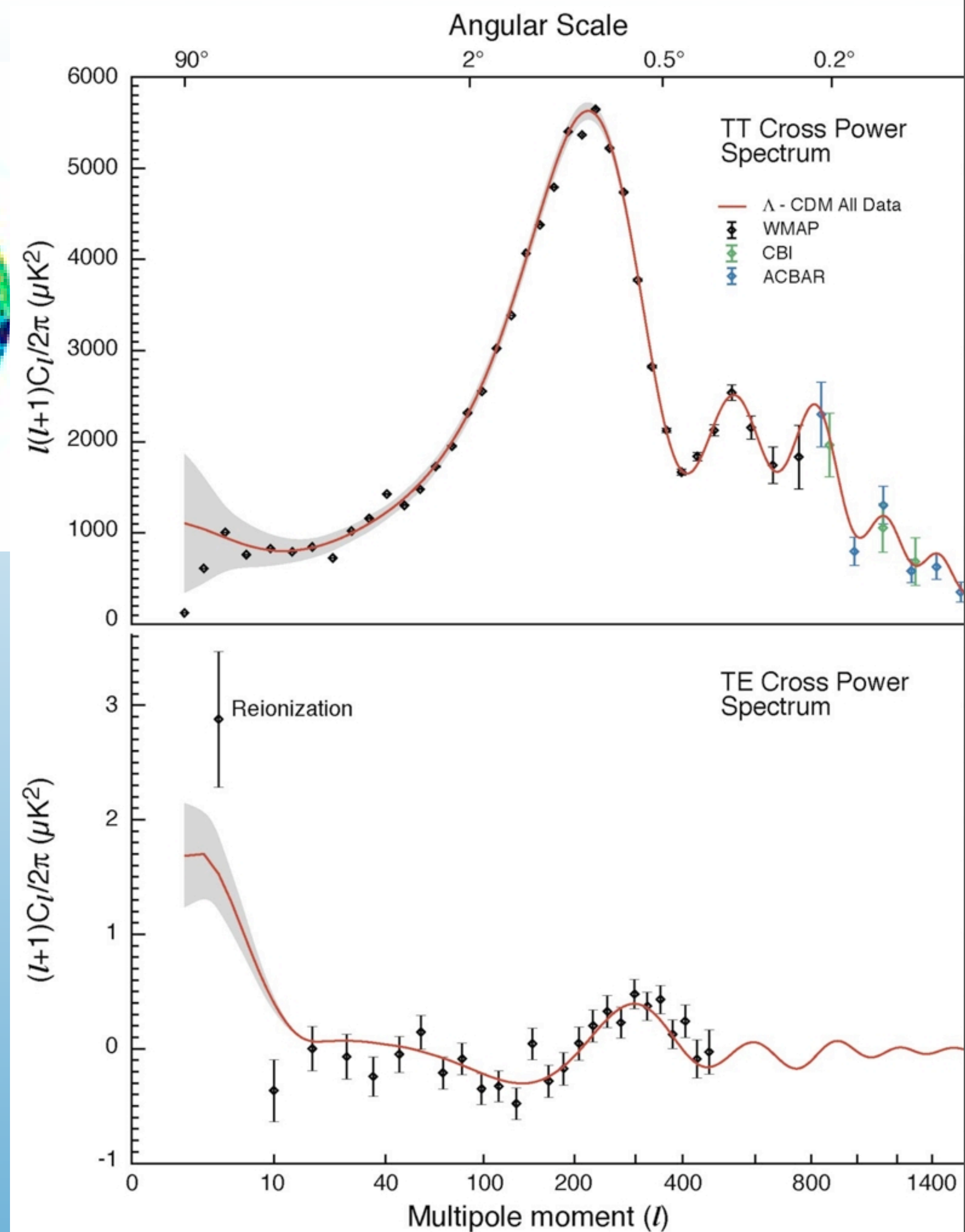
Big Bang Nucleosynthesis and Constraints on the Variation of Fundamental Couplings

- BBN and the WMAP determination of η, $\Omega_B h^2$
- Observations and Comparison with Theory
  - D/H - ^{4}He - ^{7}Li
- Cosmic-ray nucleosynthesis
  - ^{6,7}L - BeB
- Variations of Fundamental parameters
- Sensitivity to BBN
  - Δm_N - τ_n B_D





WMAP best fit (WMAPext + 2dFGRS + Lyman α +running sp. index)

$$\Omega_B h^2 = 0.0223 \pm 0.0009$$

$$\eta_{10} = 6.12 \pm 0.25$$

Conditions in the Early Universe:

$$T \gtrsim 1 \text{ MeV}$$

$$\rho = \frac{\pi^2}{30}(2 + \frac{7}{2} + \frac{7}{4}N_\nu)T^4$$

$$\eta = n_B/n_\gamma \sim 10^{-10}$$

β-Equilibrium maintained by weak interactions

Freeze-out at ~ 1 MeV determined by the competition of expansion rate $H \sim T^2/M_p$ and the weak interaction rate $\Gamma \sim G_F^2 T^5$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

At freezeout n/p fixed modulo free neutron decay, $(n/p) \simeq 1/6 \rightarrow 1/7$

Nucleosynthesis Delayed (Deuterium Bottleneck)

$p + n \rightarrow \mathbf{D} + \gamma$ $\qquad \Gamma_p \sim n_B \sigma$

$p + n \leftarrow \mathbf{D} + \gamma$ $\qquad \Gamma_d \sim n_\gamma \sigma e^{-E_B/T}$

Nucleosynthesis begins when $\Gamma_p \sim \Gamma_d$

$\frac{n_\gamma}{n_B} e^{-E_B/T} \sim 1$ $\qquad$ **@** $T \sim 0.1$ **MeV**

All neutrons $\rightarrow$ ^{4}**He**

$$Y_p = \frac{2(n/p)}{1 + (n/p)} \simeq 25\%$$

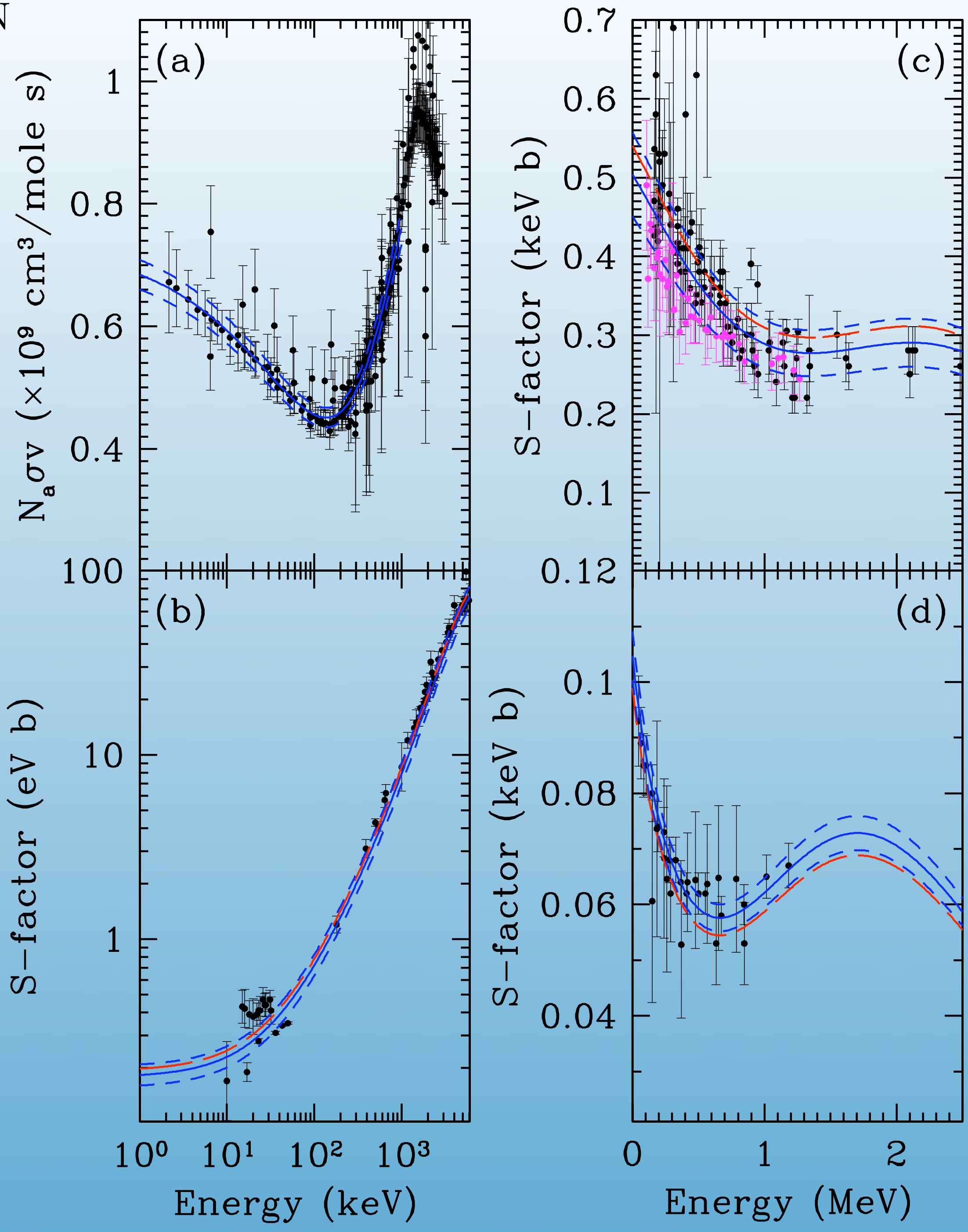
Remainder:

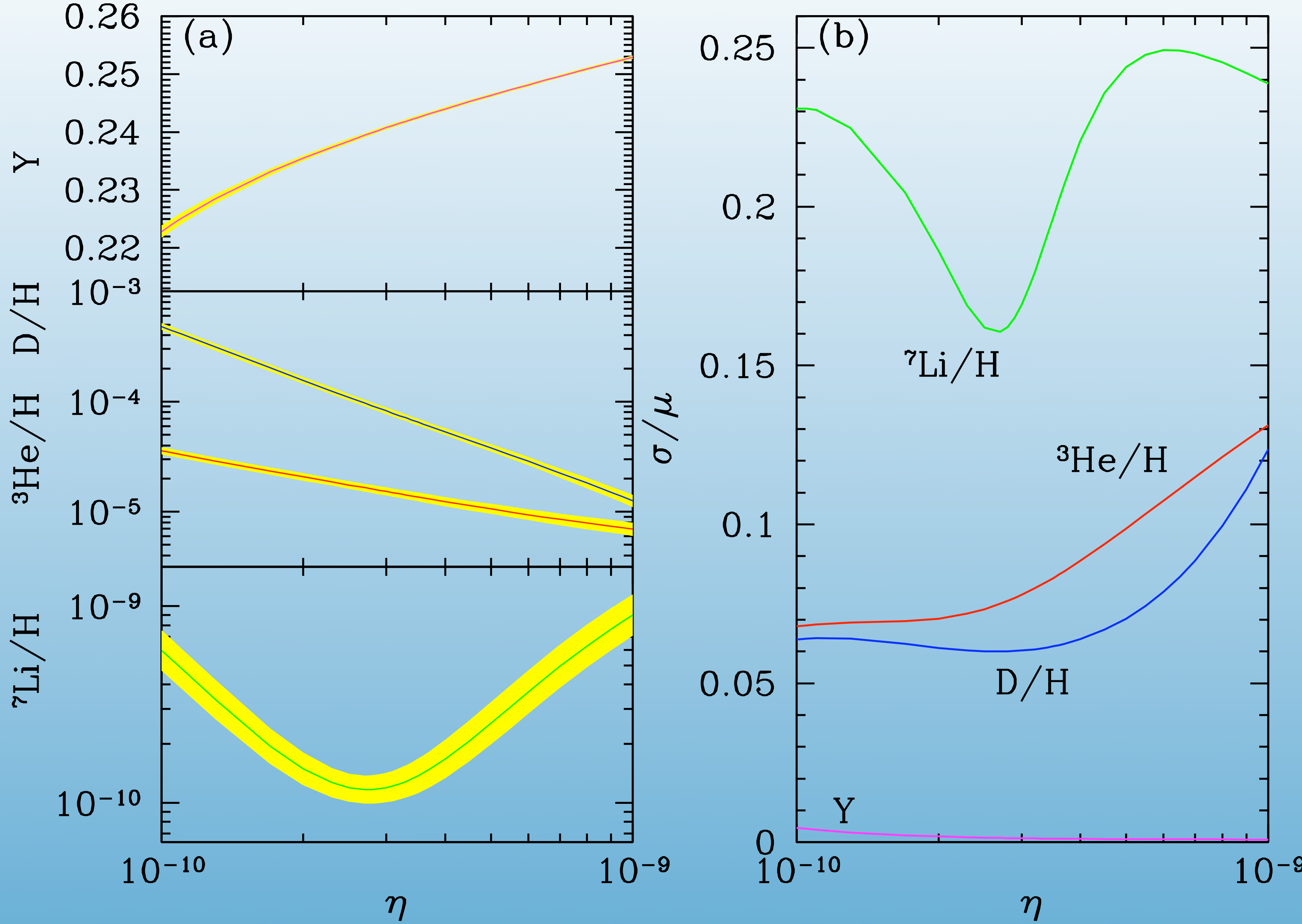
D, ^{3}**He** $\sim 10^{-5}$ **and** ^{7}**Li** $\sim 10^{-10}$ **by number**

Table 1: Key Nuclear Reactions for BBN

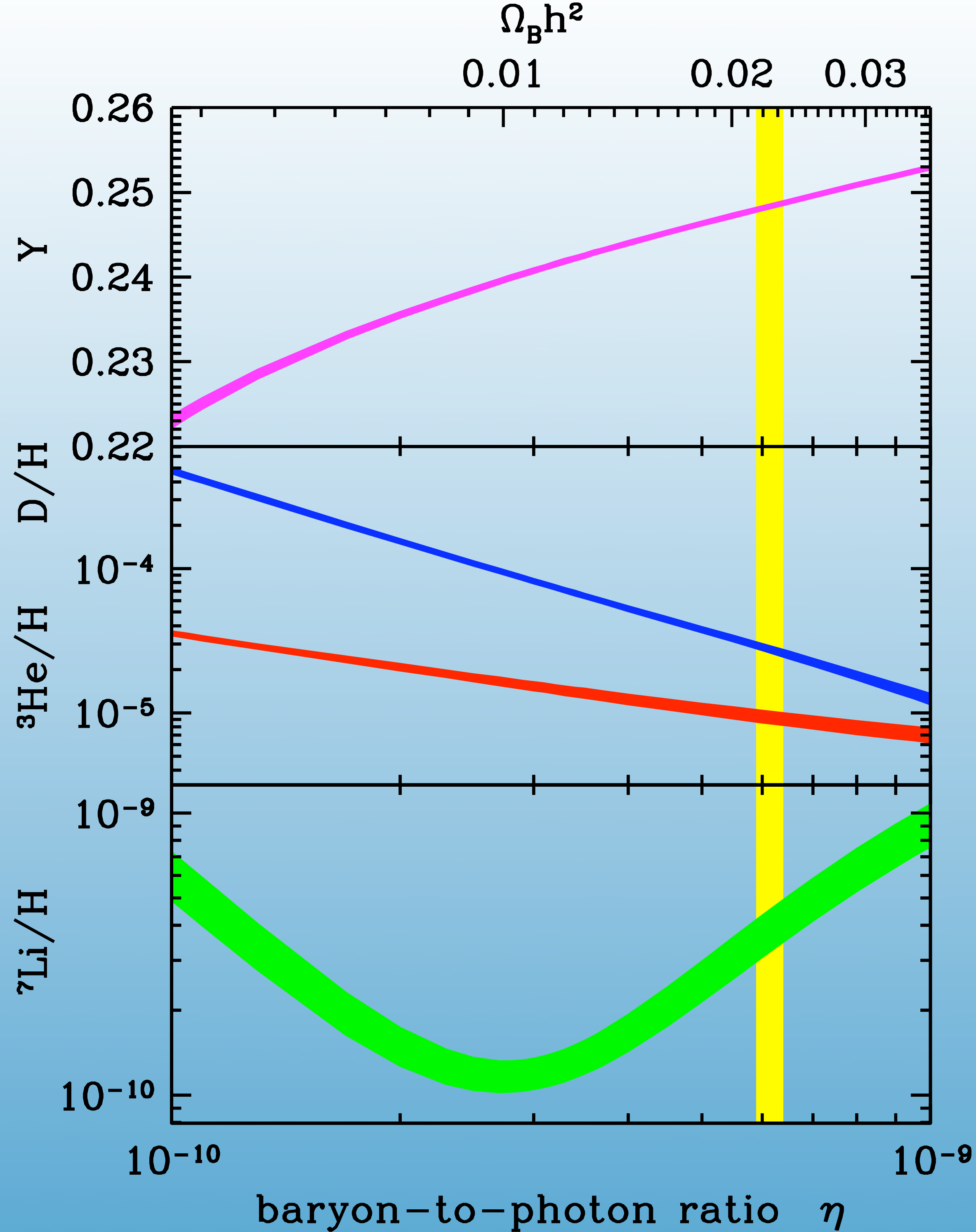
Source	Reactions	
NACRE	$d(p,\gamma)^3\mathrm{He}$	(b)
	$d(d,n)^3\mathrm{He}$	
	$d(d,p)t$	
	$t(d,n)^4\mathrm{He}$	
	$t(\alpha,\gamma)^7\mathrm{Li}$	(d)
	$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$	(c)
	$^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$	
SKM	$p(n,\gamma)d$	
	$^3\mathrm{He}(d,p)^4\mathrm{He}$	
	$^7\mathrm{Be}(n,p)^7\mathrm{Li}$	
This work	$^3\mathrm{He}(n,p)t$	(a)
PDG	τ_n	

NACRE
Cyburt, Fields, KAO
Nollett & Burles
Coc et al.





(a)
Y
0.26
0.25
0.24
0.23
0.22
D/H
^{3}He/H
10^{-3}
10^{-4}
10^{-5}
^{7}Li/H
10^{-9}
10^{-10}
10^{-10}
10^{-9}
η
(b)
σ/μ
0.25
0.2
0.15
0.1
0.05
0
^{7}Li/H
^{3}He/H
D/H
Y
10^{-10}
10^{-9}
η



$\Omega_B h^2$
0.01
0.02
0.03
0.26
0.25
0.24
0.23
0.22
Y
D/H
10^{-4}
^{3}He/H
10^{-5}
10^{-9}
^{7}Li/H
10^{-10}
10^{-10}
10^{-9}
baryon-to-photon ratio η

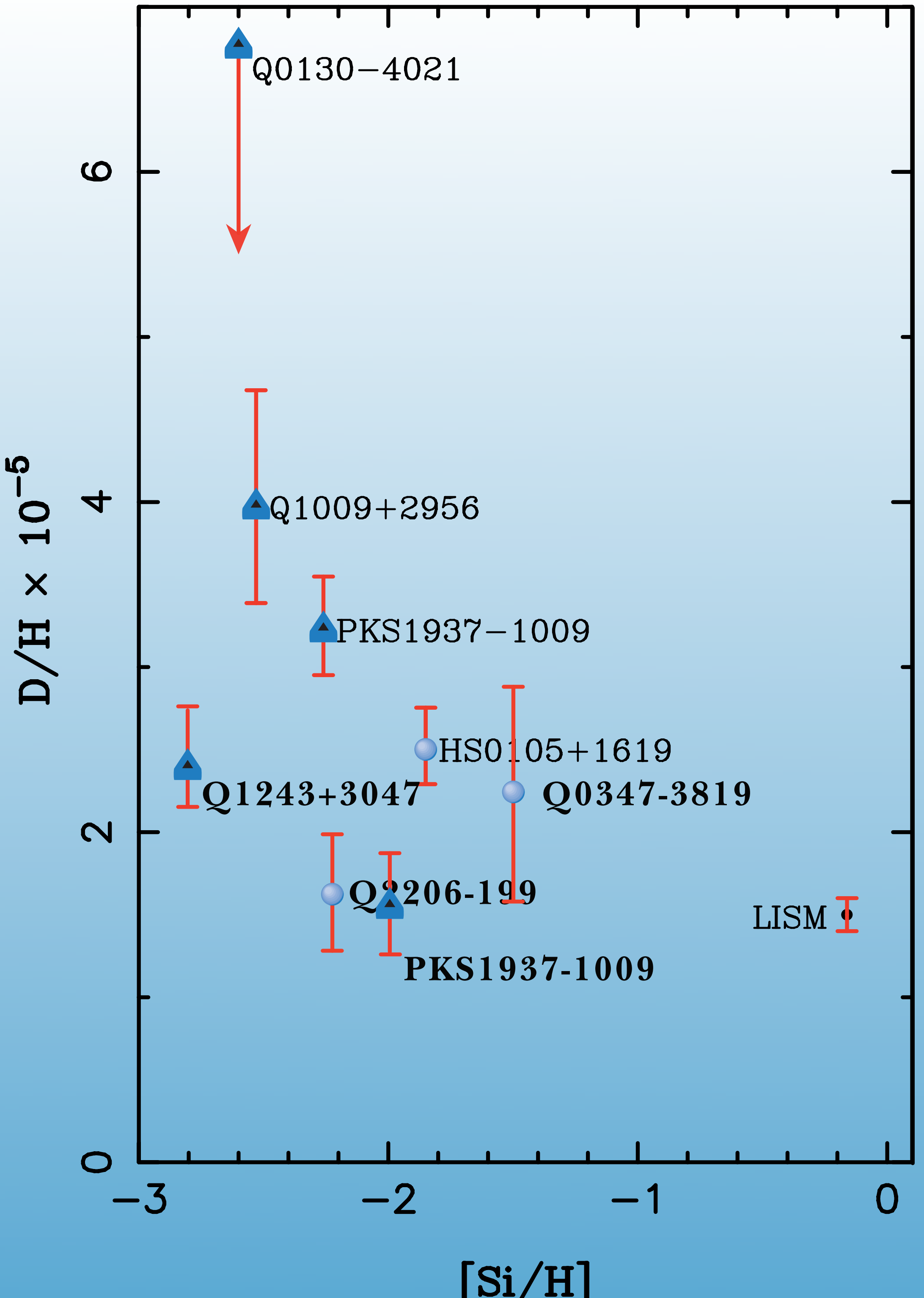
Big Bang Nucleosynthesis

- Production of the Light Elements: D, ^{3}He, ^{4}He, ^{7}Li
  - ^{4}He observed in extragalctic HII regions:
    abundance by mass = 25%
  - ^{7}Li observed in the atmospheres of dwarf halo stars:
    abundance by number = 10^{-10}
  - D observed in quasar absorption systems (and locally):
    abundance by number = 3×10^{-5}
  - ^{3}He in solar wind, in meteorites, and in the ISM:
    abundance by number = 10^{-5}

D/H

- All Observed D is Primordial!
- Observed in the ISM and inferred from meteoritic samples (also HD in Jupiter)
- D/H observed in Quasar Absorption systems

D/H abundances in Quasar apsorption systems

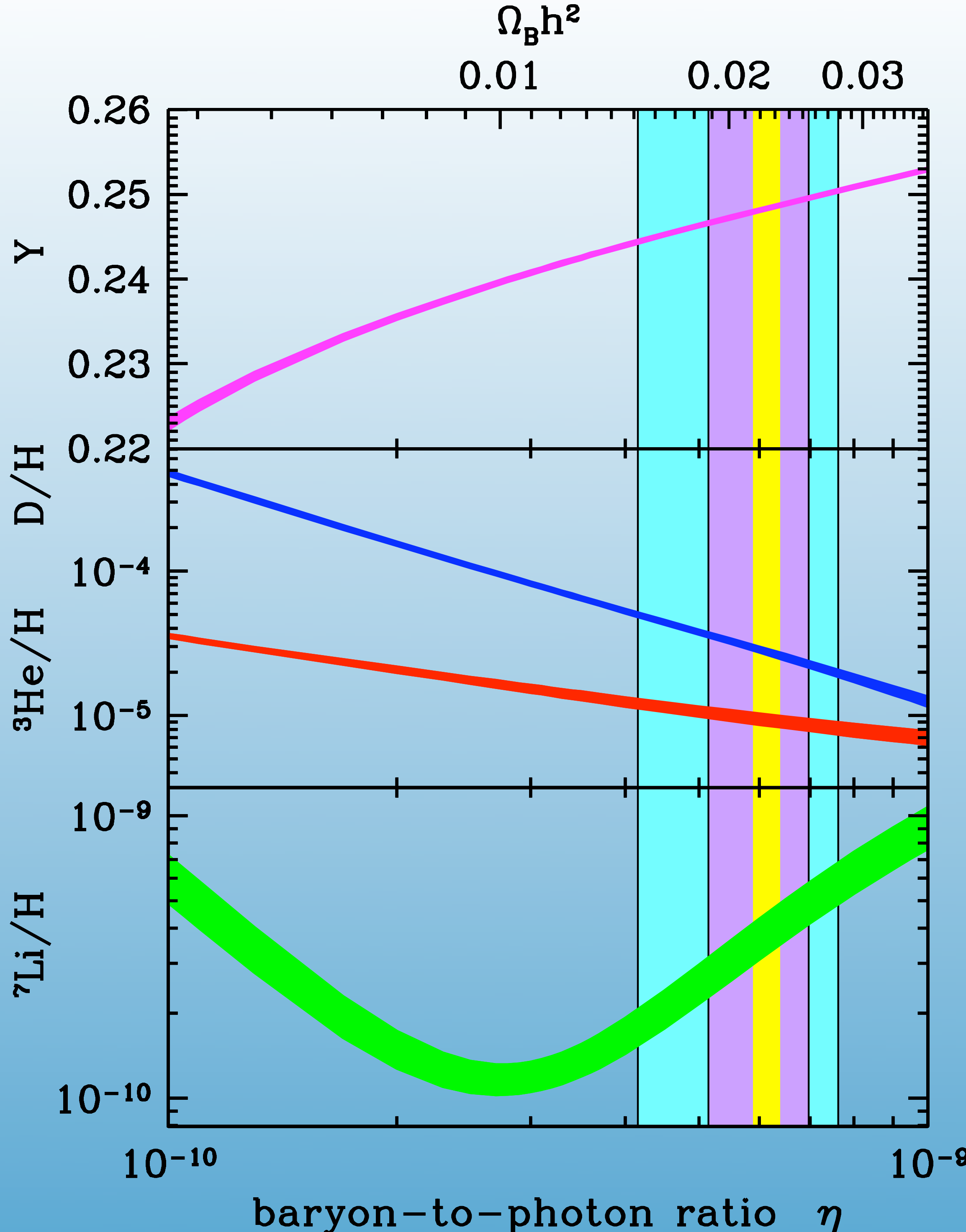


D/H

- D/H observed in Quasar Absorption systems
- Is the dispersion real?
- Is there a correlation with α/H?
- Is there a correlation with density?

Evidence for evolution?

Fields, et al.

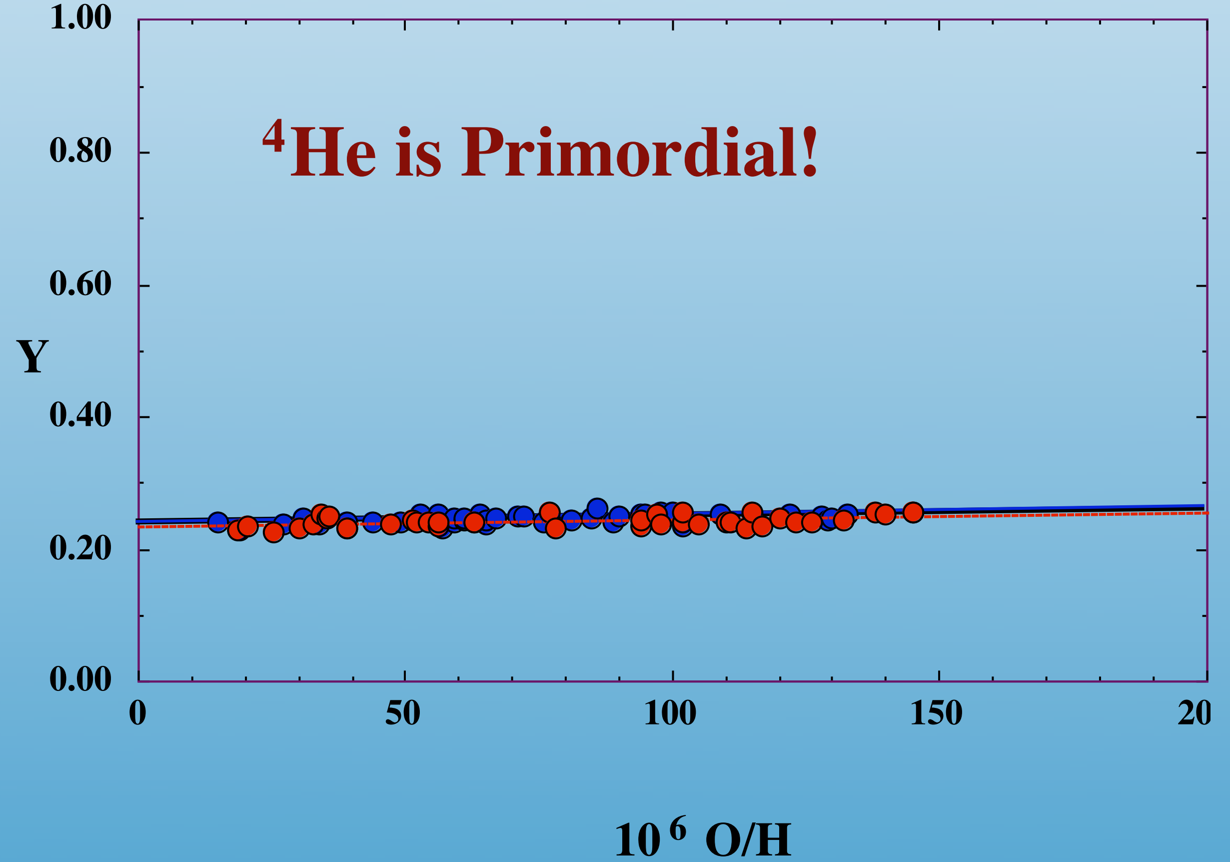


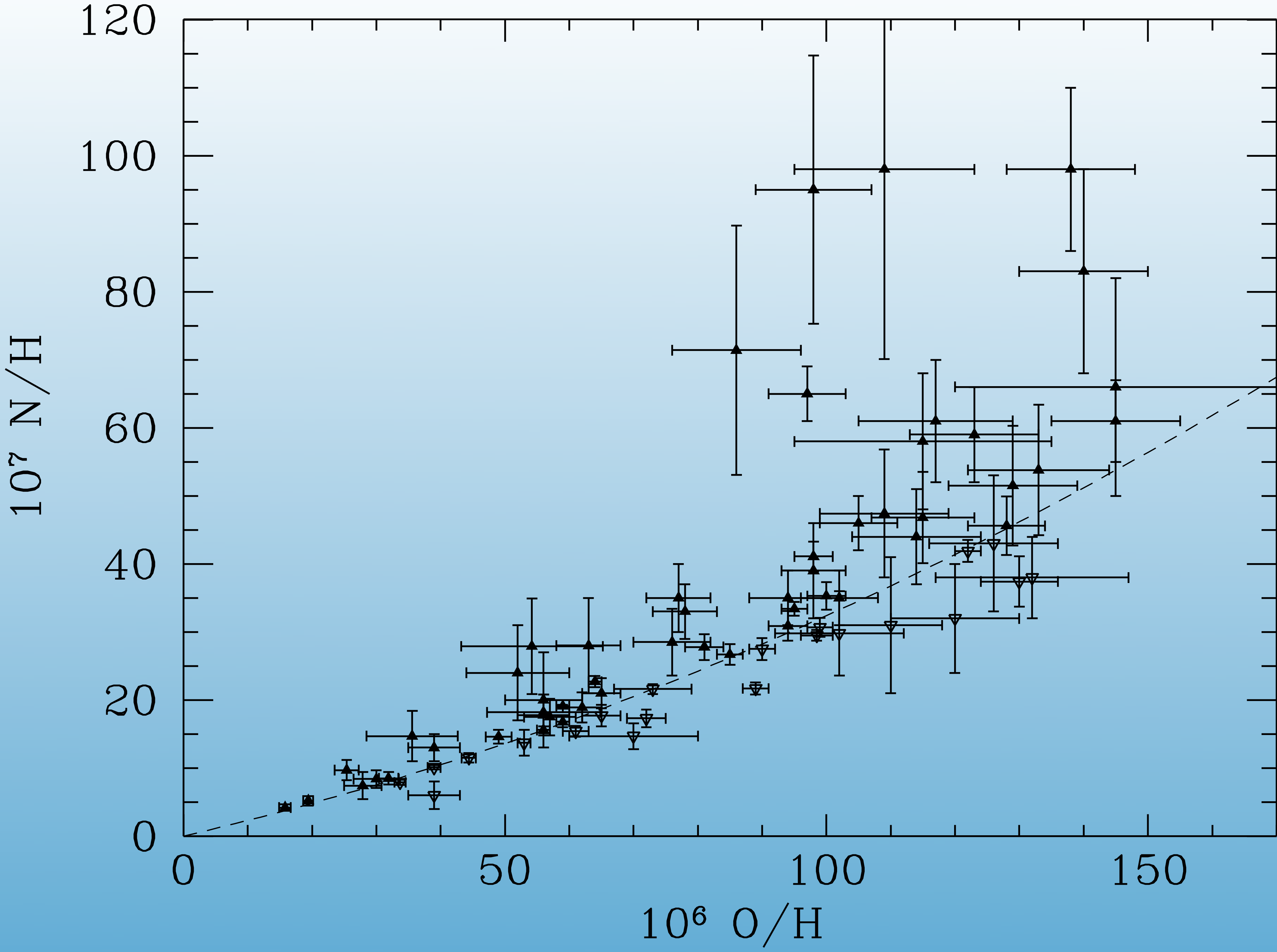
$\Omega_B h^2$
0.01
0.02
0.03
0.26
0.25
0.24
0.23
0.22
Y
D/H
10^{-4}
^{3}He/H
10^{-5}
10^{-9}
^{7}Li/H
10^{-10}
10^{-10}
10^{-9}
baryon-to-photon ratio η

^{4}He

Measured in low metallicity extragalactic HII regions (~100) together with O/H and N/H

$$Y_P = Y(O/H \rightarrow 0)$$





120
100
80
60
40
20
0
10⁷ N/H
0
50
100
150
10⁶ O/H

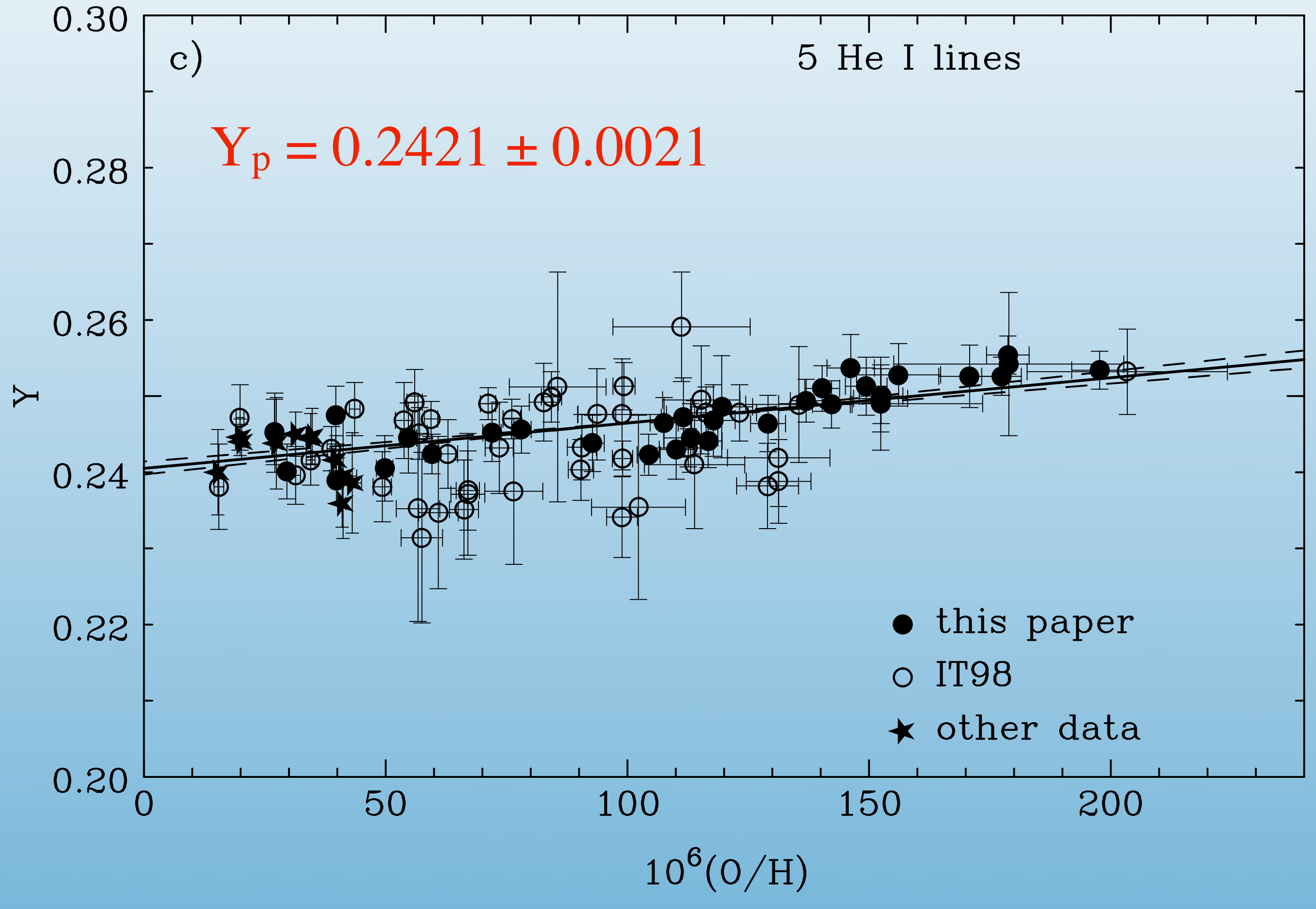
- 0.228 ± 0.005 — Pagel etal
  S II densities
- 0.244 ± 0.002 — Izotov etal
  “self consistent”
- 0.238 ± 0.002 — Fields & KAO
  S II densities
- 0.234 ± 0.003 — Peimbert etal
  “self consistent”

(the latter is based on a single careful measurement of Y = 0.240 ± 0.002 for the SMC at [O/H] = -.8)

- 0.2384 ± 0.0025 — Peimbert etal
  “self consistent”
- 0.2421 ± 0.0021 — Izotov etal
  “self consistent”
- 0.2491 ± 0.0091 — KAO & Skillman
  “self consistent”

There is clearly some underlying systematics which must be sorted out!

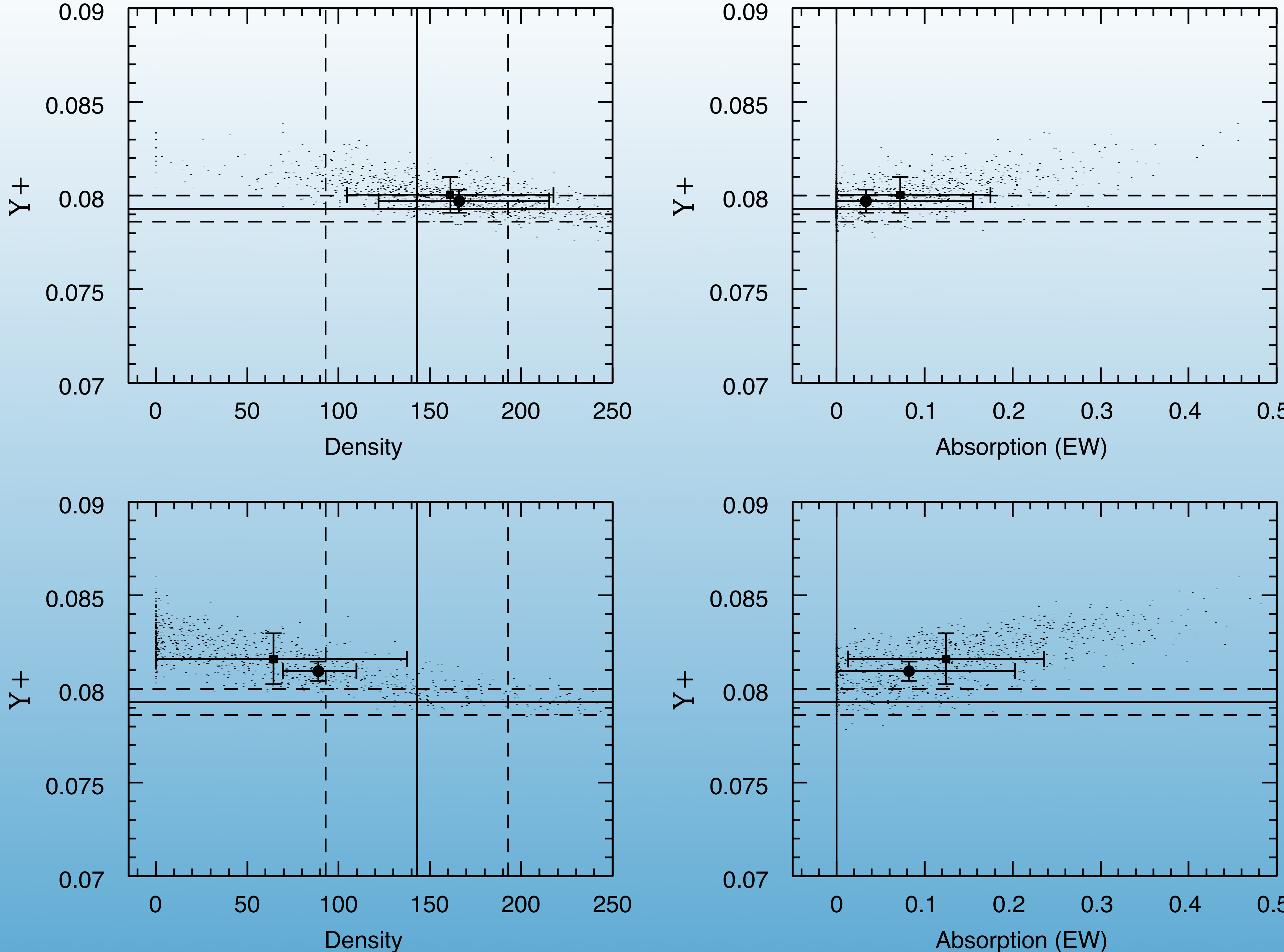
^{4}He



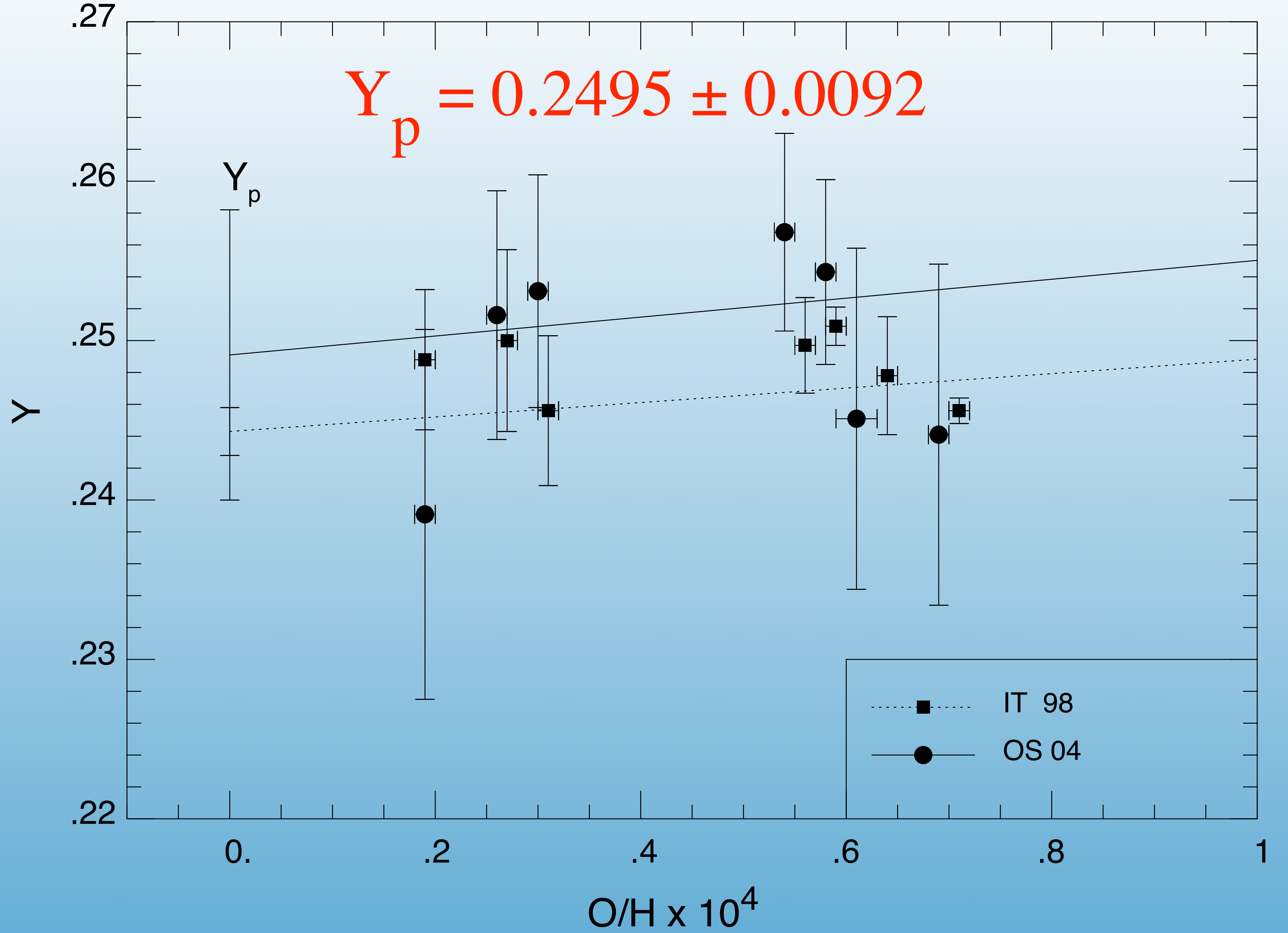
Izotov & Thuan

Method:

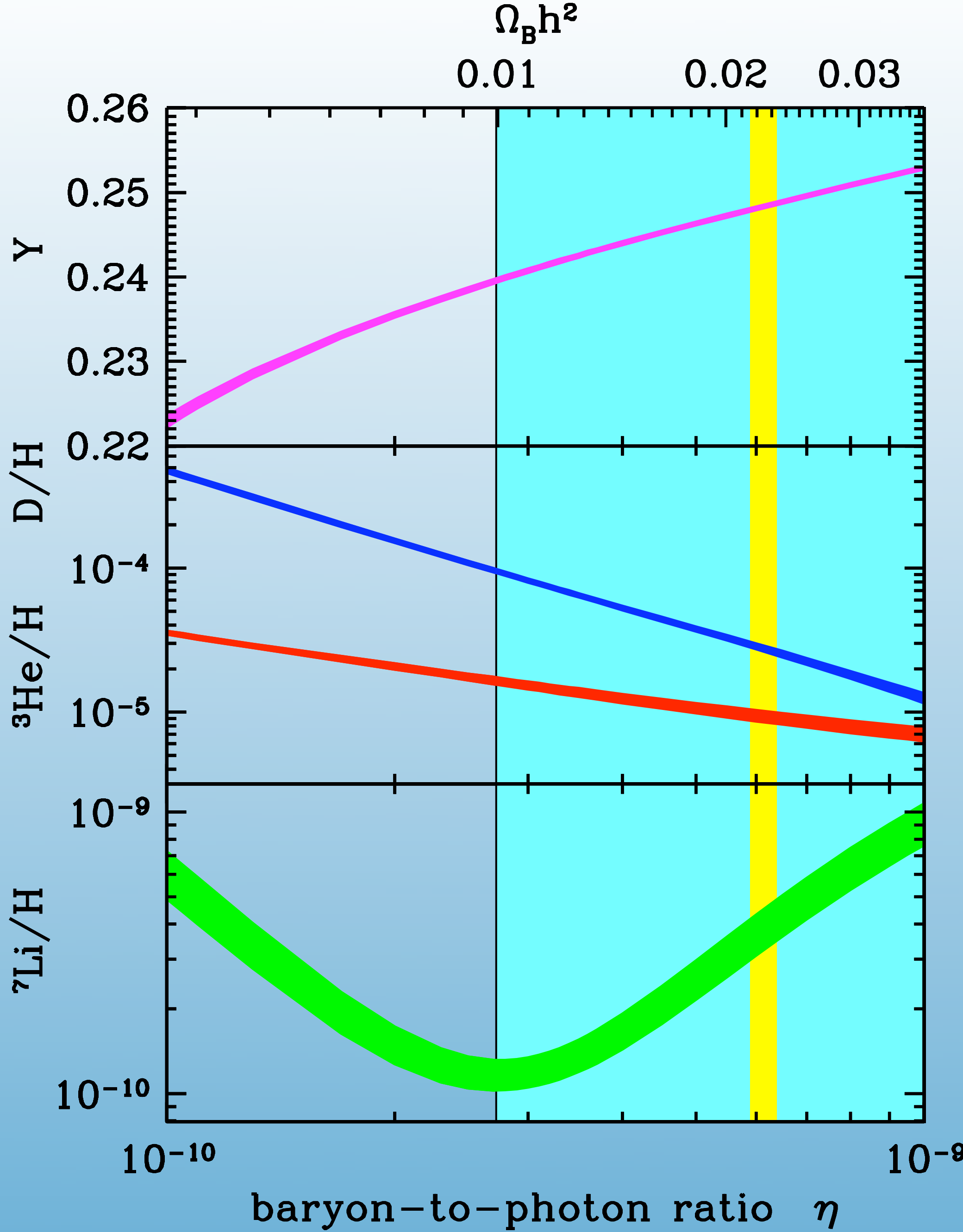
- Intensity and Eq. Width for H and He
- Determine H reddening and underlying absorption
- Use 6 He emission lines to determine physical parameters:
  - denisty, optical depth, temperature, underlying He absorption, ^{4}He abundance
- Severe degeneracies revealed by Monte Carlo anaysis



KAO + Skillman



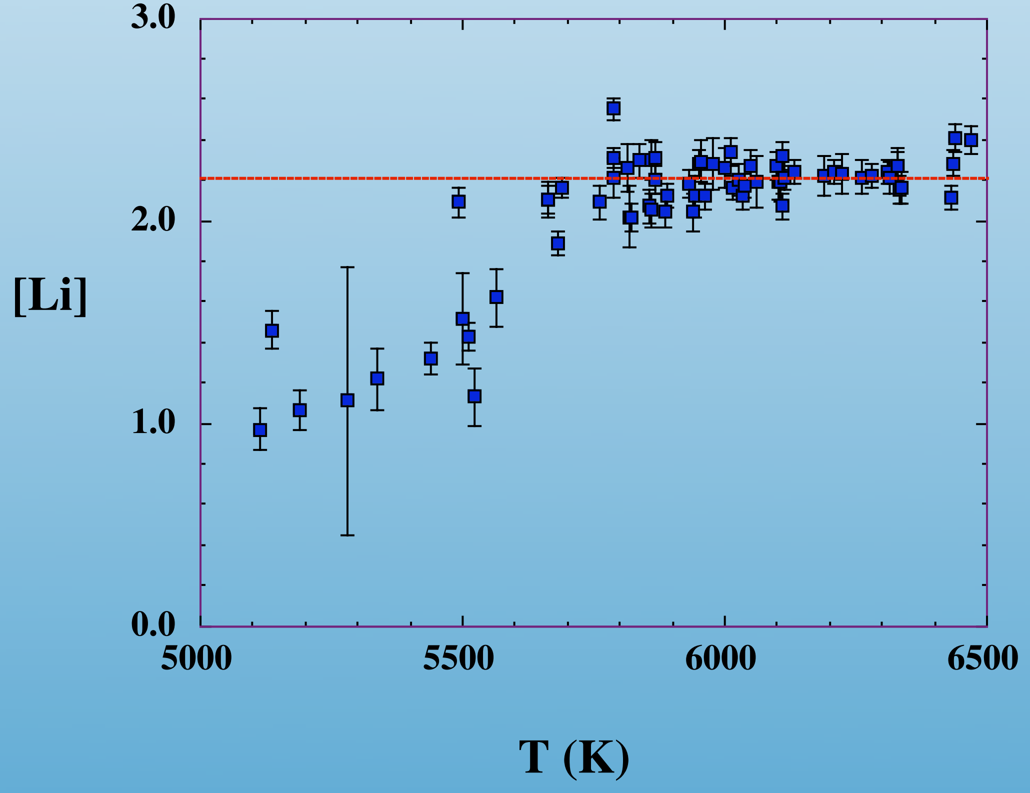
$Y_p = 0.2495 \pm 0.0092$
.27
.26
.25
.24
.23
.22
Y
Y_p
0.
.2
.4
.6
.8
1
O/H x 10^4
IT 98
OS 04

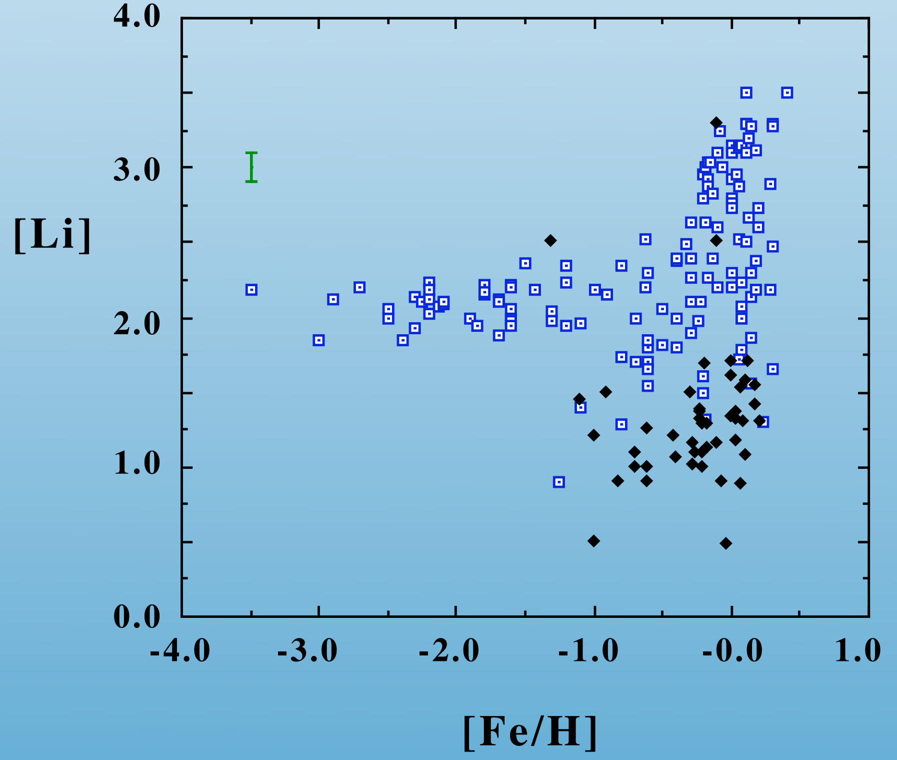


$\Omega_B h^2$
0.01
0.02
0.03
0.26
0.25
0.24
0.23
0.22
Y
D/H
10^{-4}
$^3He/H$
10^{-5}
10^{-9}
$^7Li/H$
10^{-10}
10^{-10}
10^{-9}
baryon−to−photon ratio η

Li/H

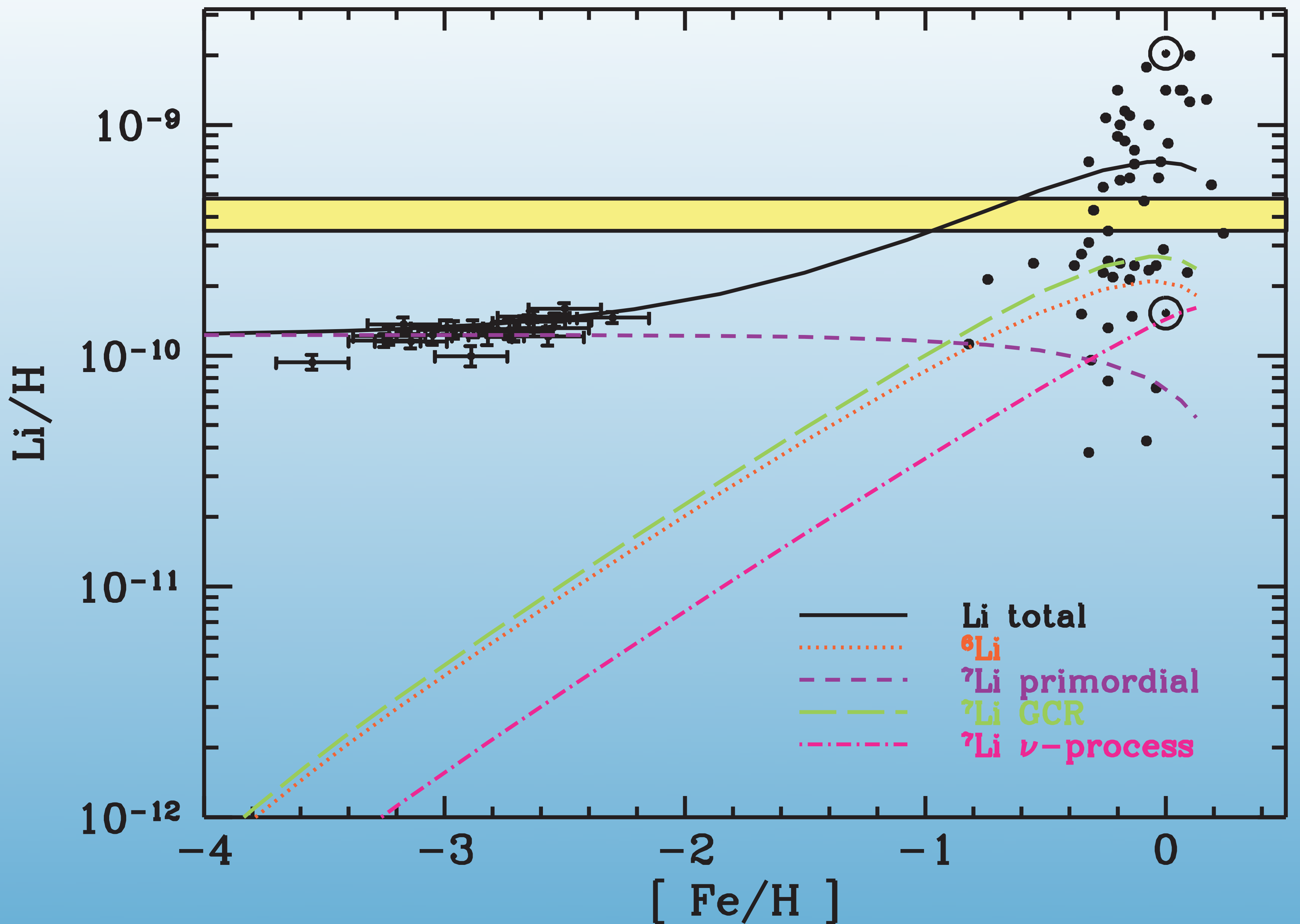
Measured in low metallicity dwarf halo stars (over 100 observed)





Li Woes

- Observations based on
  - "old": Li/H = 1.2×10^{-10} Spite & Spite +
  - Balmer: Li/H = 1.7×10^{-10} Molaro, Primas & Bonifacio
  - IRFM: Li/H = 1.6×10^{-10} Bonifacio & Molaro
  - IRFM: Li/H = 1.2×10^{-10} Ryan, Beers, KAO, Fields, Norris
  - Hα (globular cluster): Li/H = 2.2×10^{-10} Bonifacio et al.
  - Hα (globular cluster): Li/H = 2.3×10^{-10} Bonifacio
  - λ6104: Li/H ~ 3.2×10^{-10} Ford et al.
- Li depends on T, ln g, [Fe/H], depletion, post BBN-processing, ...
- Strong systematics



10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻¹²
Li/H
−4
−3
−2
−1
0
[Fe/H]
Li total
⁶Li
⁷Li primordial
⁷Li GCR
⁷Li ν−process

Possible sources for the discrepancy

- Stellar Depletion
  - lack of dispersion in the data, ^{6}Li abundance
  - standard models (< .05 dex), models (0.2 - 0.4 dex)

Vauclaire & Charbonnel
Pinsonneault et al.
Richard, Michaud, Richer

- Nuclear Rates
  - Restricted by solar neutrino flux

Coc et al.
Cyburt, Fields, KAO

- Stellar parameters

$$\frac{dLi}{dlng} = \frac{.09}{.5} \qquad \frac{dLi}{dT} = \frac{.08}{100K}$$

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- Nuclear Rates
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- Particle Decays

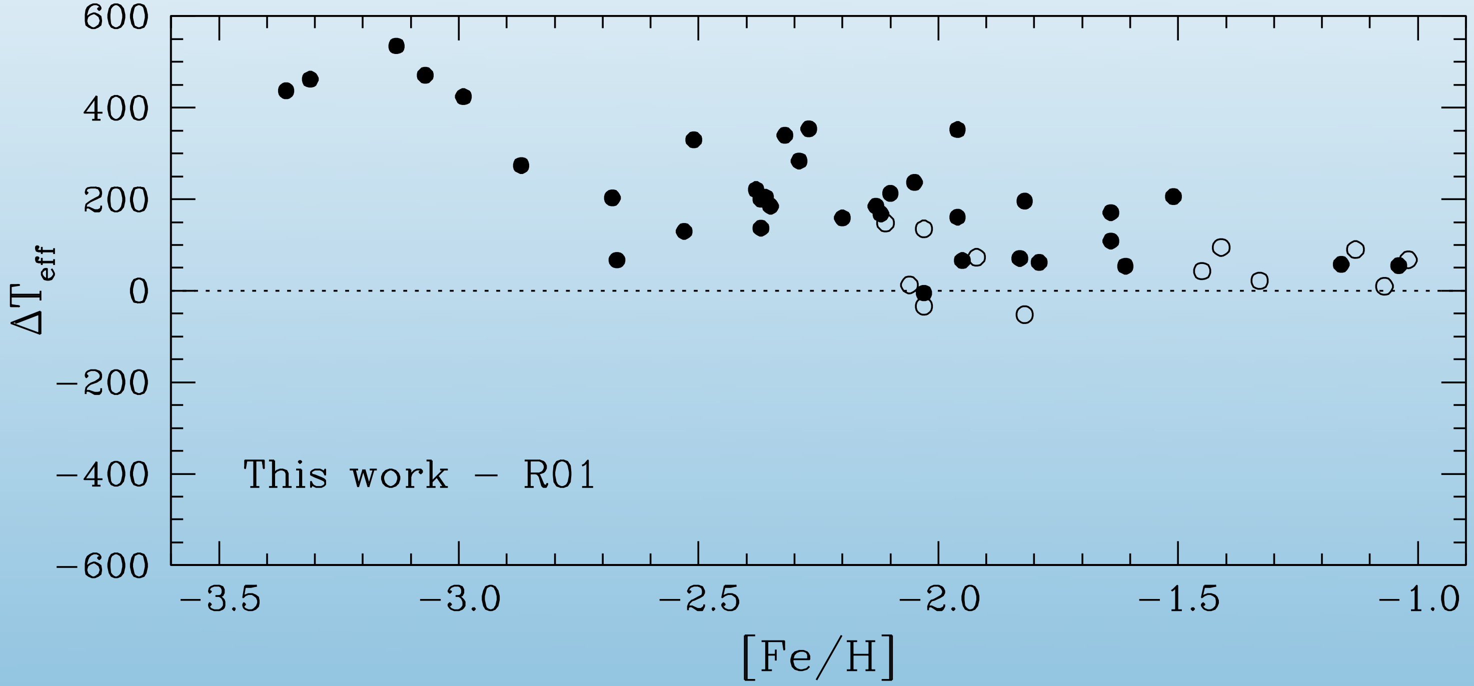
Reappraising the Spite Lithium Plateau: Extremely Thin and Marginally Consistent with WMAP

Jorge Meléndez[1] and Iván Ramírez[2]

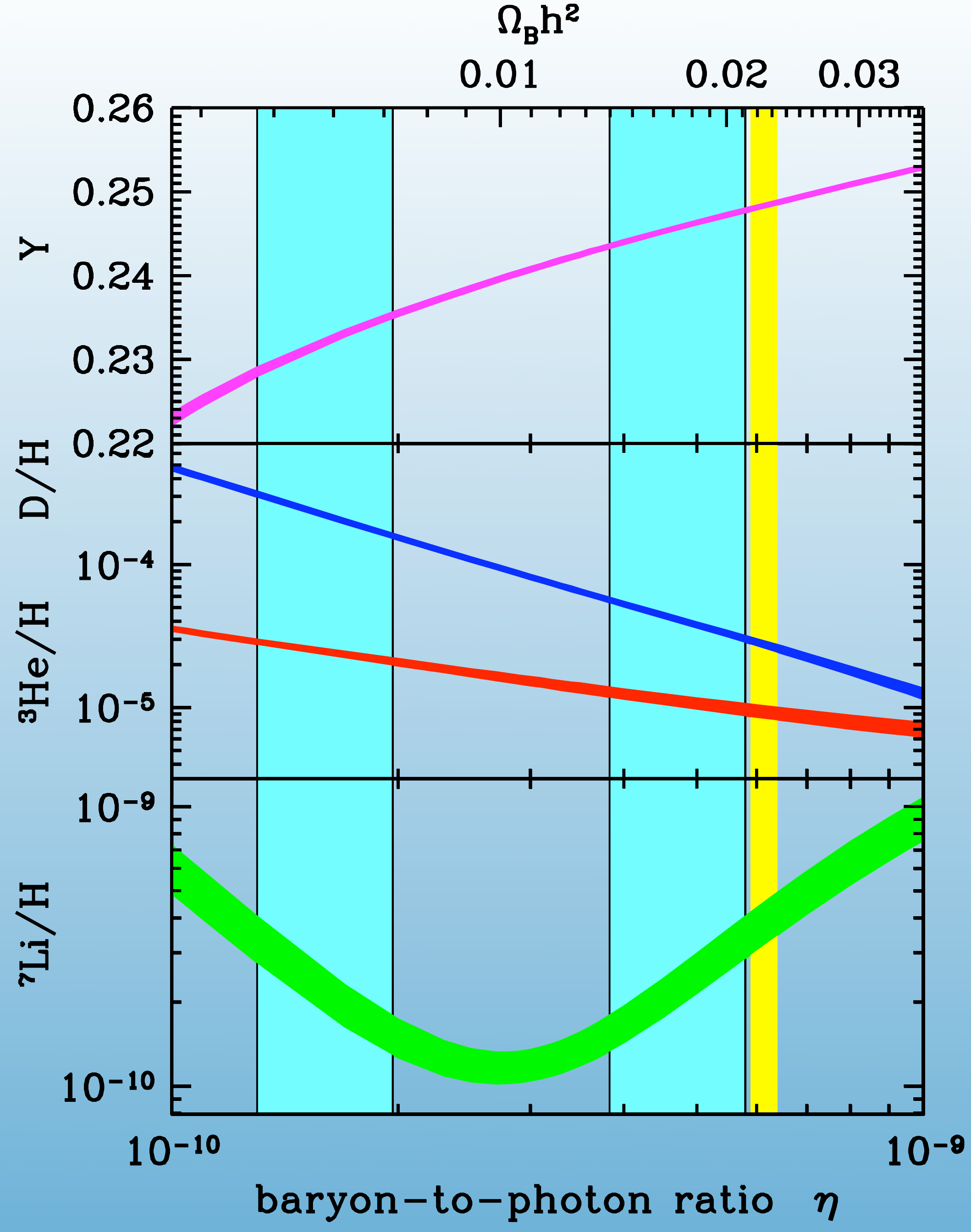
New evaluation of surface temperatures in 41 halo stars with systematically higher temperatures (100-300 K)

$$[Li] = 2.37 \pm 0.1$$

$$Li/H = 2.34 \pm 0.54 \times 10^{-10}$$



600
400
200
0
−200
−400
−600
ΔTeff
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
[Fe/H]
This work − R01



Ω_B h²
0.01
0.02
0.03
0.26
0.25
0.24
0.23
0.22
Y
D/H
10⁻⁴
³He/H
10⁻⁵
10⁻⁹
⁷Li/H
10⁻¹⁰
10⁻¹⁰
10⁻⁹
baryon−to−photon ratio η

^{6}LiBeB

For $\eta_{10} \approx 6$

$^{6}Li/H \approx 10^{-14}$

$^{9}Be/H \approx 0.5 - 5 \times 10^{-19}$

$^{10}B/H \approx 2 \times 10^{-20}$

$^{11}B/H \approx 3 \times 10^{-16}$

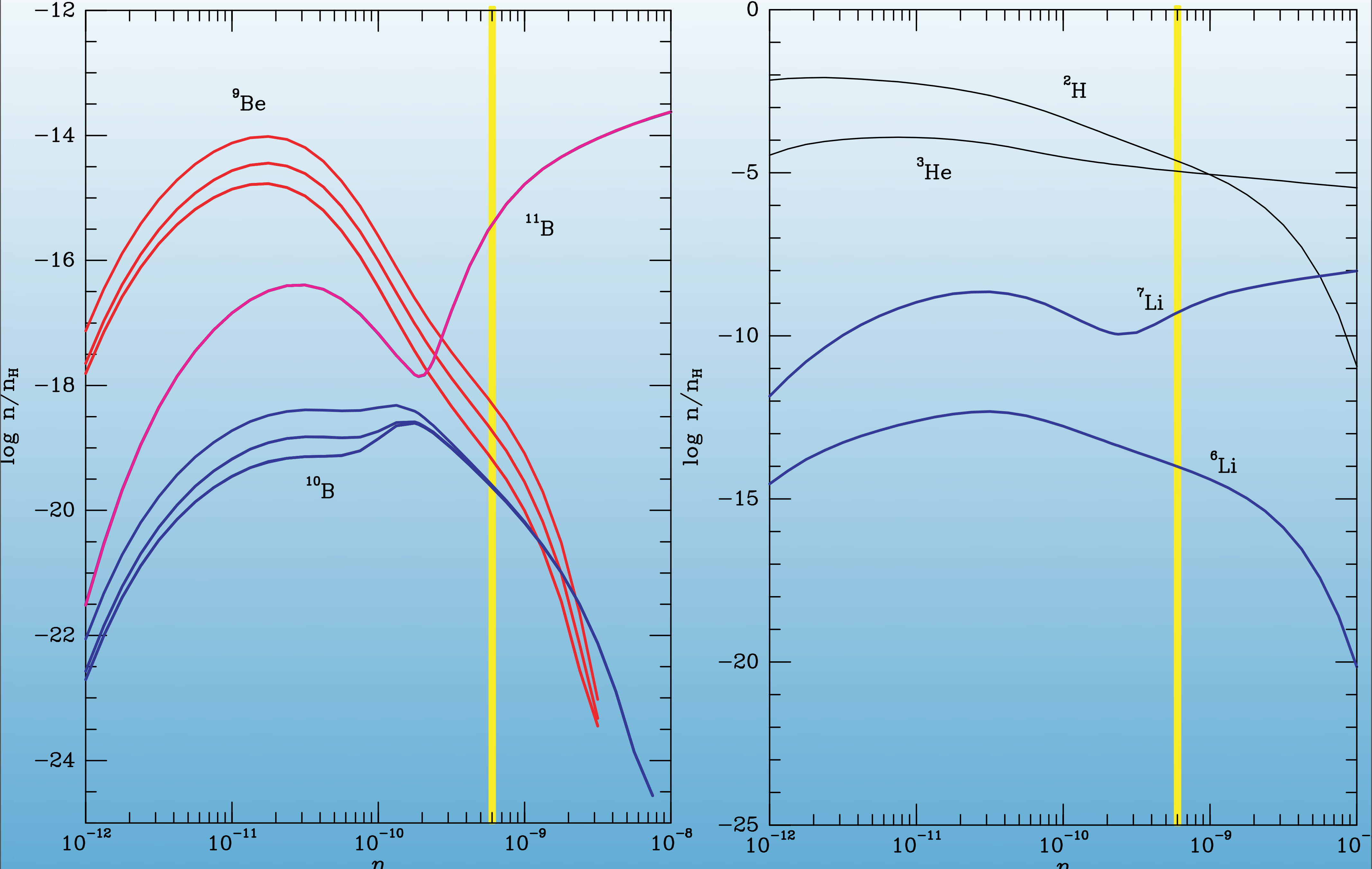
Far Below the observed values in Pop II stars

$^{6}Li/H \approx$ few $\times 10^{-12}$

$^{9}Be/H \sim 1 - 10 \times 10^{-13}$ B/H $\sim 1 - 10 \times 10^{-12}$

These are not BBN produced.

GCR Nucleosynthesis



-12
-14
-16
-18
-20
-22
-24
log n/n_H
^{9}Be
^{11}B
^{10}B
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
η
0
-5
-10
-15
-20
-25
^{2}H
^{3}He
^{7}Li
^{6}Li

^{6}Li

In the happy but not too distant past:

^{6}Li (@ [Fe/H] ~ -2.3):

HD 84937: ^{6}Li/Li $= 0.054 \pm 0.011$

BD 26^o3578: ^{6}Li/Li $= 0.05 \pm 0.03$

SLN

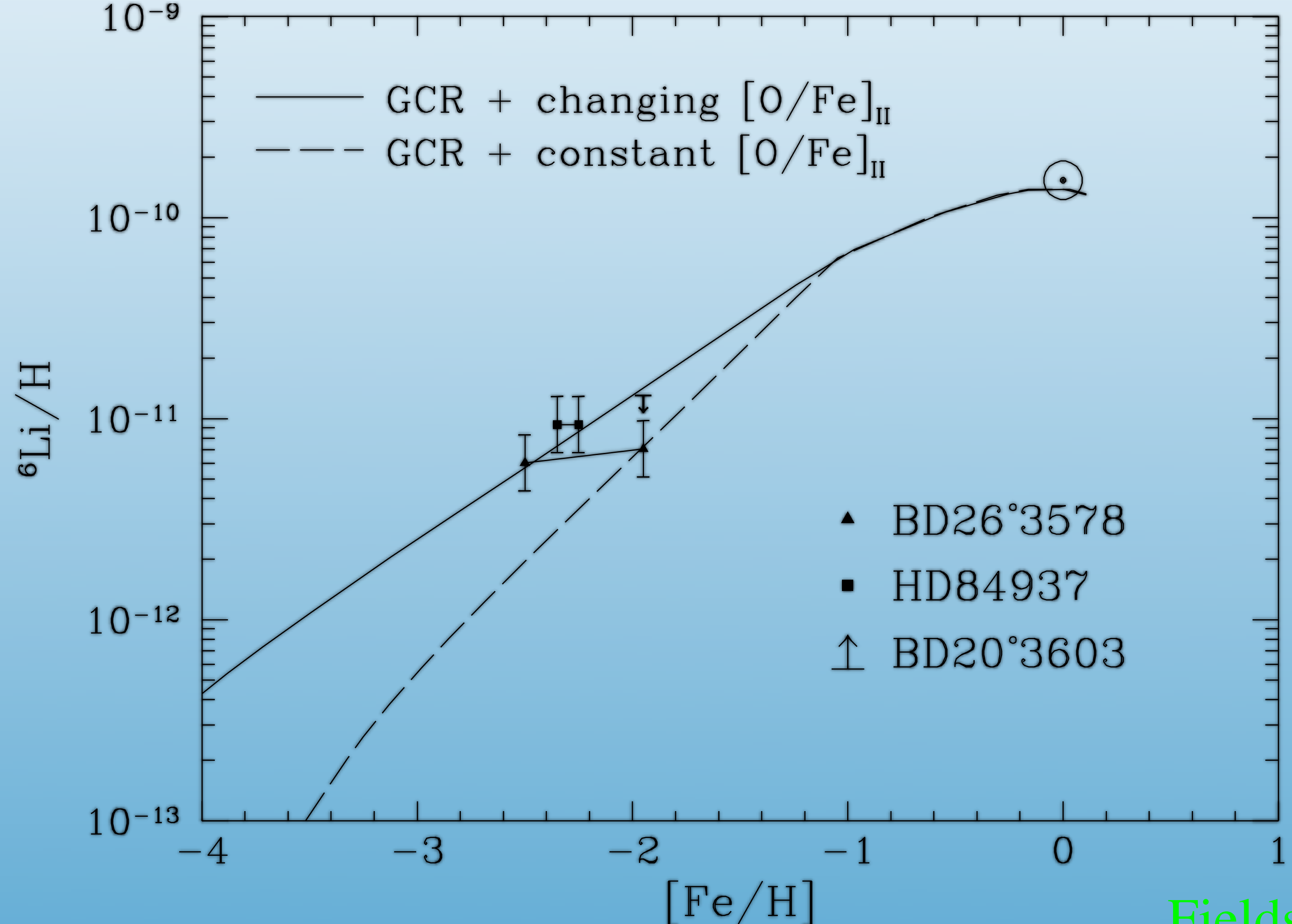
Hobbs & Thorburn

Cayrel etal

cf. BBN abundance of about ^{6}Li/H = 10^{-14}

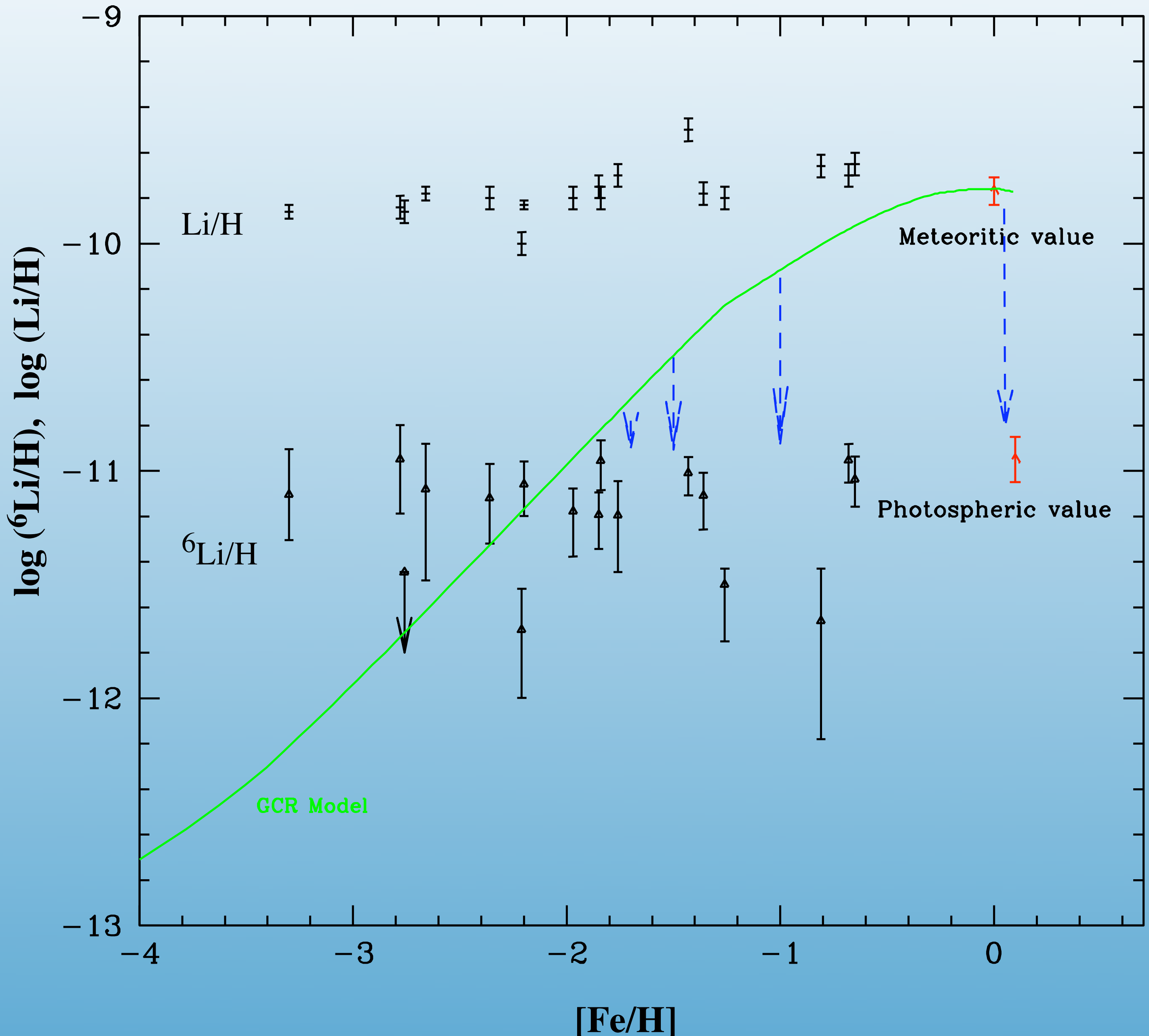
or ^{6}Li/Li $< 10^{-4}$

These data nicely accounted for by Galactic Cosmic Ray Nucleosynthesis

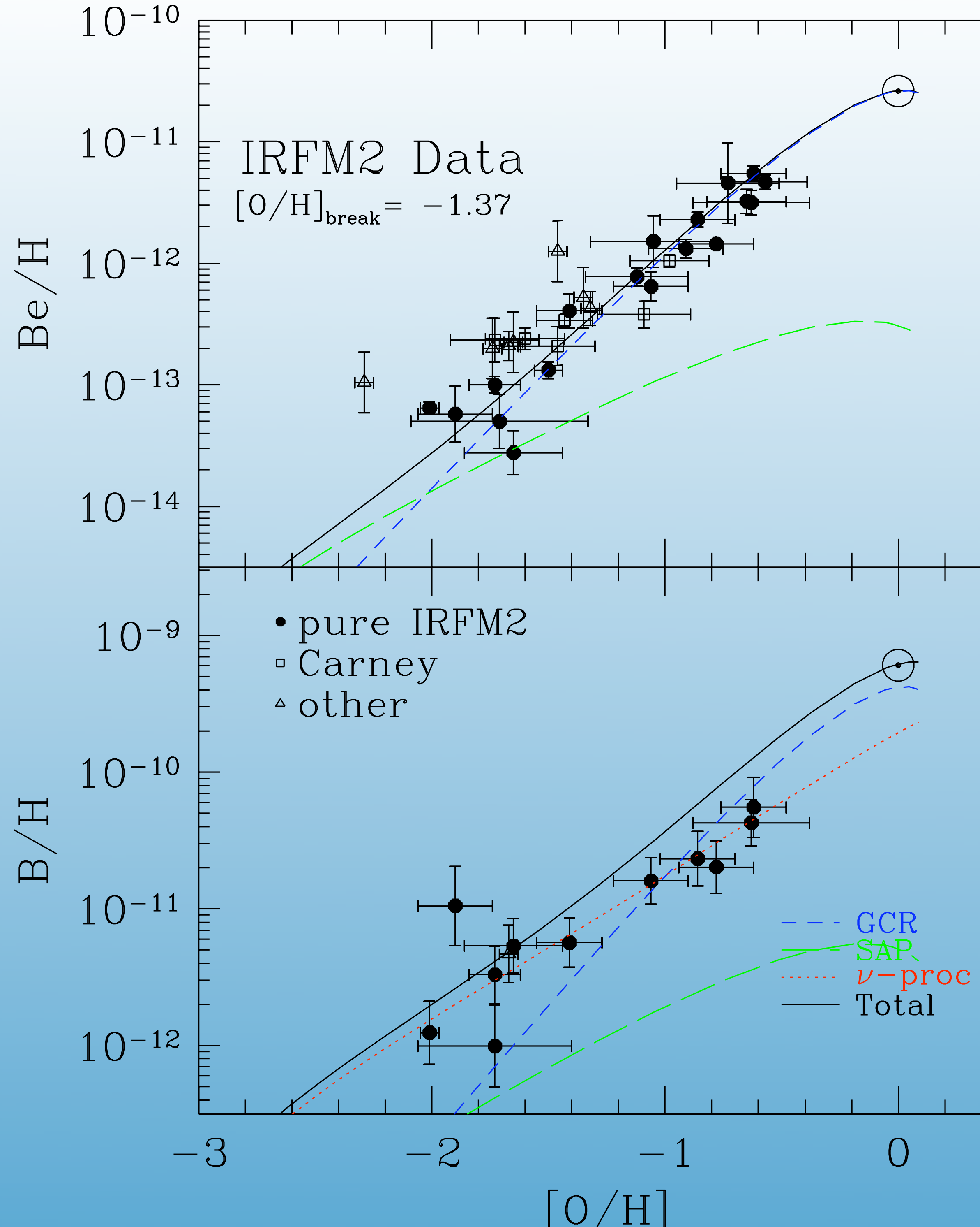


Fields and Olive
Vangioni et al.

Problem 2: There appears to be a ^{6}Li plateau



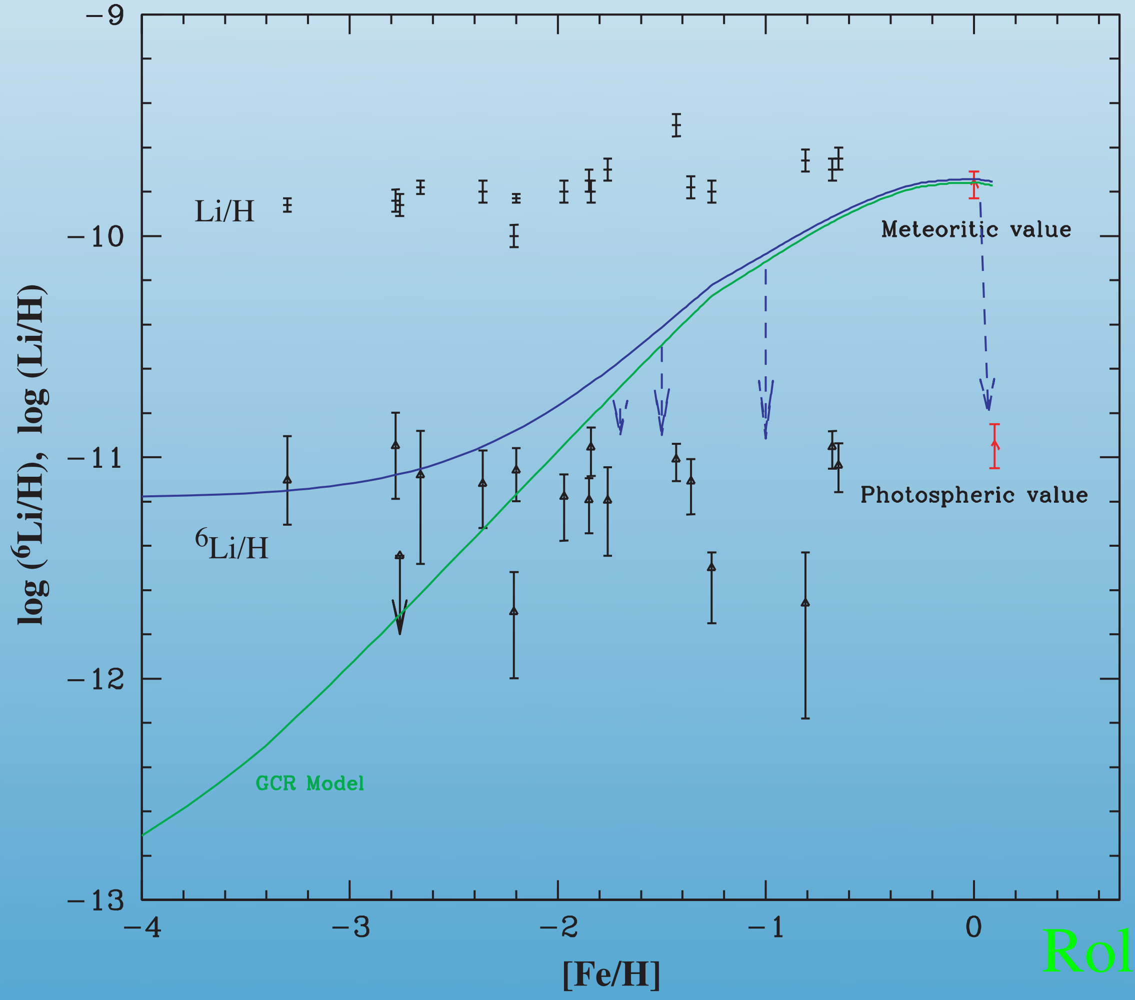
Data from Asplund et al and Inoue



GCRN production of Be and B including primary and secondary sources

Possible Solution: Cosmological Cosmic Rays (to problem two only)

- Cosmic Chemical Evolution
- Early Reionization and Massive Stars
- Cosmic Ray Production and Propagation in an expanding Universe



Rollinde. Vangioni, Olive

Summary

- D, He are ok -- issues to be resolved

- Li: 2 Problems
  - BBN ^{7}Li high compared to observations
  - BBN ^{6}Li low compared to observations
    ^{6}Li plateau?
- Important to consider:
  - Depletion
  - Li Systematics - T scale
  - Particle Decays?
  - PreGalactic production of ^{6}Li
  - Tie in to Be and B production

How does a Fundamental Constant Change?

$$\mathcal{L} \sim \phi R \qquad \langle\phi\rangle = \frac{1}{16\pi G_N} = \frac{M_P^2}{16\pi}$$

$$\mathcal{L} \sim \phi F^2 \qquad \langle\phi\rangle = \frac{1}{4e^2} = \frac{1}{16\pi\alpha}$$

Does this ever happen?

e.g. JBD Theory

$$S = \int d^4x\sqrt{g}\left[\phi R - \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi + \mathcal{L}_m\right]$$

$$\mathcal{L}_m = -\frac{1}{4e^2}F^2 - \frac{1}{2}\partial_\mu y\partial^\mu y - V(y)$$

$$-\overline{\Psi}\,\not{D}\Psi - m\overline{\Psi}\Psi + \Lambda$$

with a conformal rescaling,

$$S = \int d^4x \sqrt{\overline{g}} \Big[\overline{R} - (\omega + \tfrac{3}{2}) \frac{(\partial_\mu \phi)^2}{\phi^2}$$

$$- \frac{1}{2} \frac{(\partial_\mu y)^2}{\phi} - \frac{V(y)}{\phi^2} - \frac{\overline{\Psi} \not{D} \Psi}{\phi^{3/2}}$$

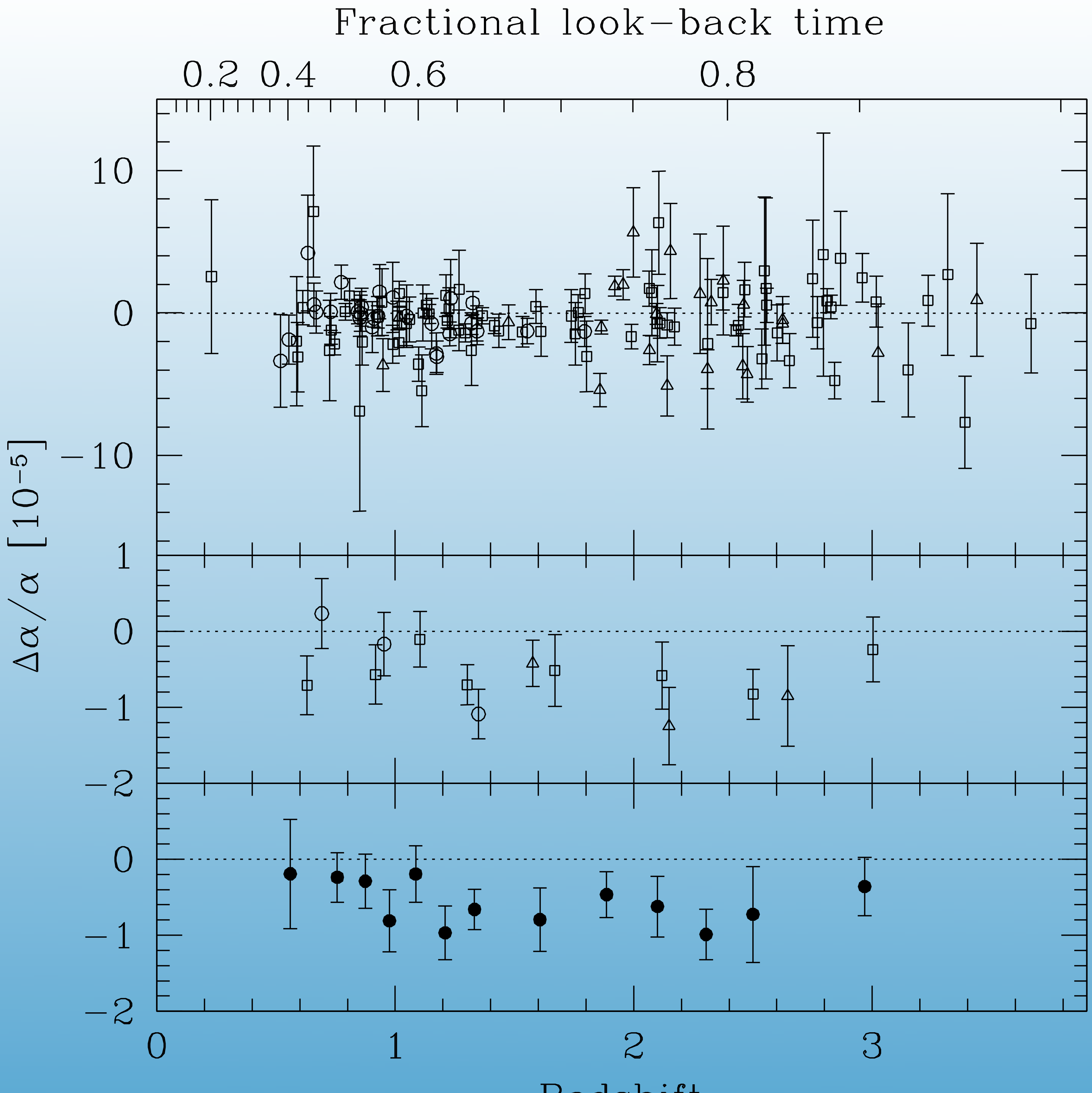
$$- \frac{m \overline{\Psi} \Psi}{\phi^2} - \frac{1}{4e^2} F^2 + \frac{\Lambda}{\phi^2} \Big]$$

now, $M_p(G_N)$, and α are fixed but particle masses scale with ϕ,

$$m \sim 1/\phi^{1/2}$$

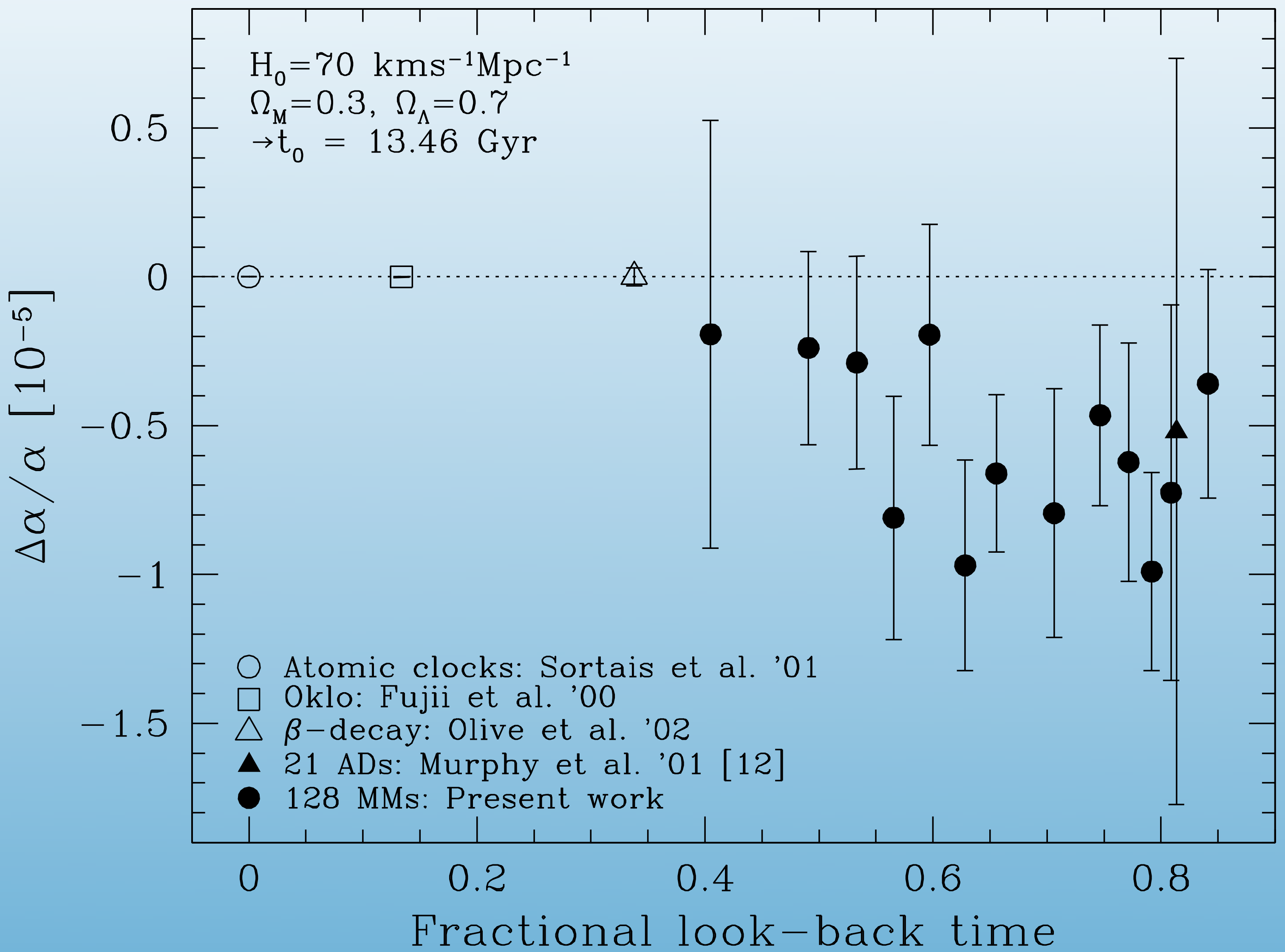
the same is true for the Higgs expectation value,

$$G_F \sim \frac{1}{v^2} \sim 1/\phi$$



Fractional look−back time
0.2
0.4
0.6
0.8
10
0
−10
1
0
−1
−2
0
−1
−2
Δα/α [10⁻⁵]
0
1
2
3
Redshift

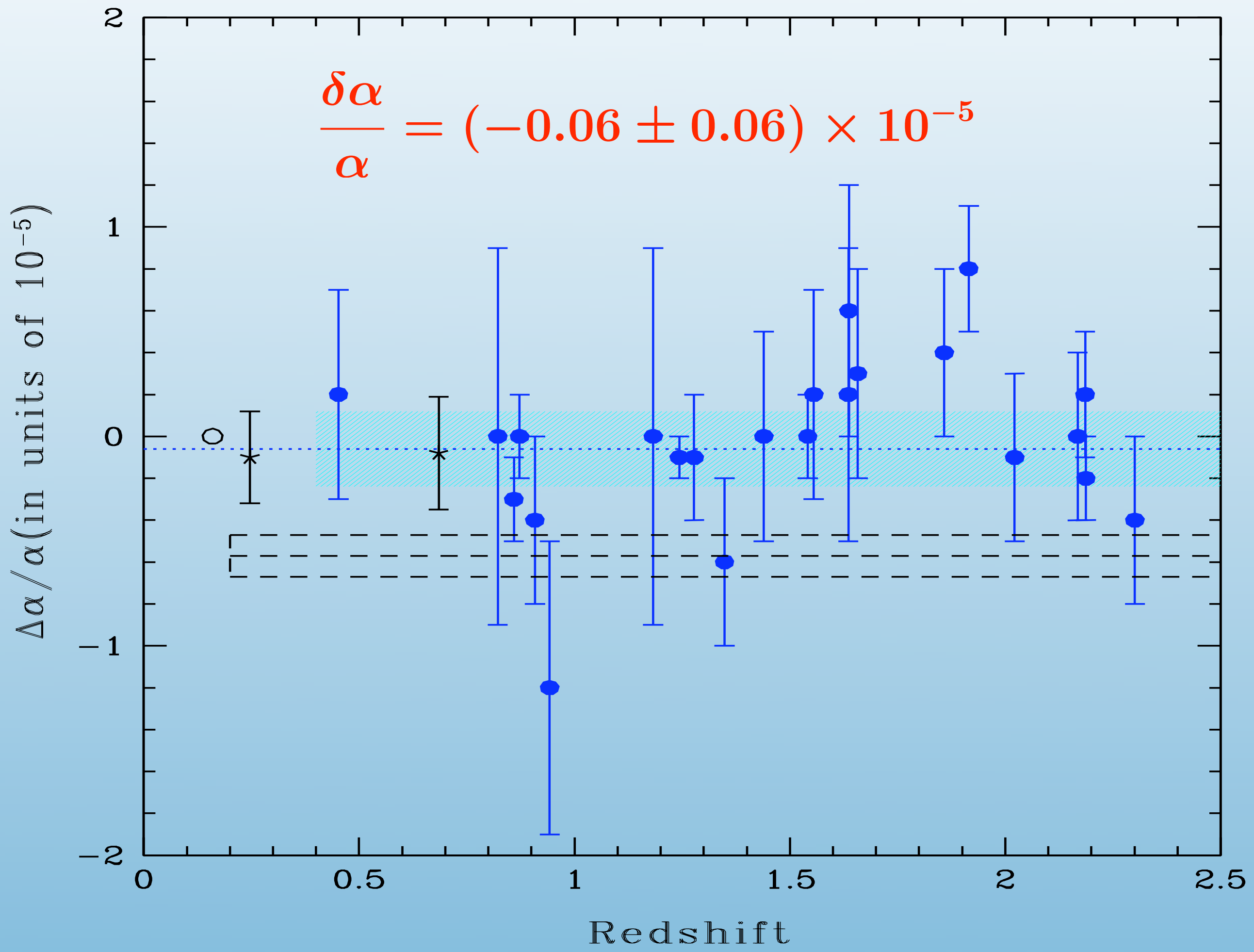
Keck/HIRES data



Murphy et al.

Newer Data* VLT/UVES

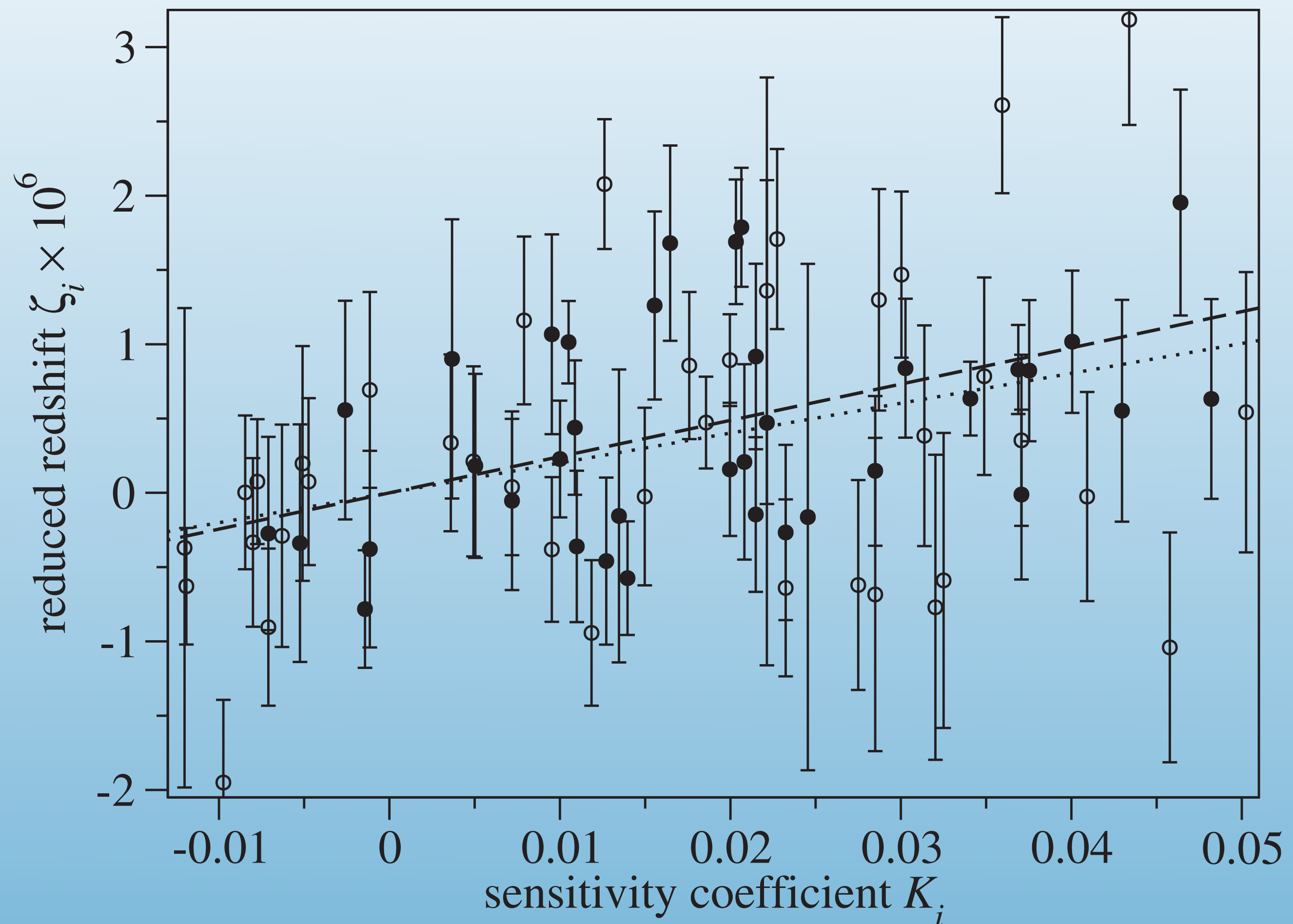
Chand et al.



*Recently revised by Murphy etal to $\frac{\delta\alpha}{\alpha} = (-0.44 \pm 0.16) \times 10^{-5}$

Also from quasar absorption systems:

Using molecular rotation lines (which depend on $\mu = m_p/m_e$)



$$\frac{\Delta\mu}{\mu} = (2.4 \pm 0.6) \times 10^{-5}$$

New high resolution measurements

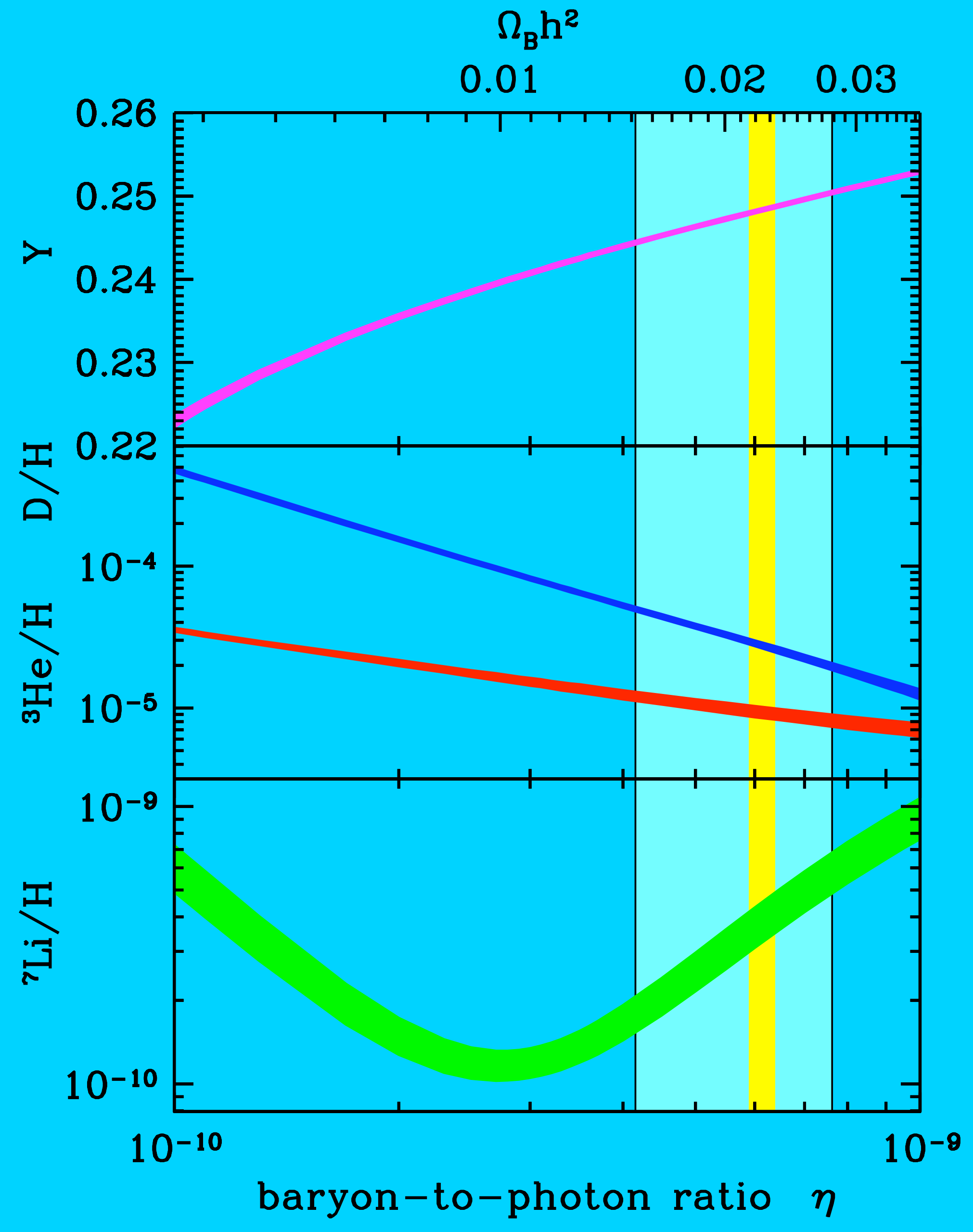
Reinhold etal.

Limits on the variations of α

- Cosmology
  - BBN
  - CMB
- The Oklo Reactor
- Meteoritic abundances
- Atomic clocks

BBN Concordance

- Concordance rests on balance between interaction rates and expansion rate.
- Allows one to set constraints on:
  - Particle Types
  - Particle Interactions
  - Particle Masses
  - Fundamental Parameters



Cyburt, Fields, KAO

How could varying α affect BBN?

$$G_F^2 T^5 \sim \Gamma(T_f) \sim H(T_f) \sim \sqrt{G_N N} T_f^2$$

Recall in equilibrium,

$$\frac{n}{p} \sim e^{-\Delta m/T}$$ **fixed at freezeout**

Helium abundance,

$$Y \sim \frac{2(n/p)}{1+(n/p)}$$

If T_f is higher, (n/p) is higher, and Y is higher

Contributions to ΔY:

Kolb, Perry, and Walker
Campbell and Olive
Bergstrom, Iguri, and Rubenstein

$$\frac{\Delta Y}{Y} \simeq \frac{1}{1+n/p}\frac{\Delta(n/p)}{(n/p)}$$

$$\frac{\Delta(n/p)}{(n/p)} \simeq \frac{\Delta m_N}{T_f}\left(\frac{\Delta T_f}{T_f} - \frac{\Delta^2 m_N}{\Delta m_N}\right)$$

Contributions to Δm_N:

$$\Delta m_N \sim a\alpha_{em}\Lambda_{QCD} + bv$$

electromagnetic **weak**

-0.8 MeV **2.1 MeV**

Changes in α, Λ_{QCD}, and/or v
all induce changes in Δm_N and hence Y

Limits:

Campbell & Olive
see also Ichikawa & Kawaski
Nollett & Lopez

$$\frac{\Delta Y}{Y} \lesssim \frac{\pm 0.012}{0.24} = \pm 0.05$$

$$\frac{\Delta(n/p)}{(n/p)} \simeq \frac{\Delta m_N}{T_f}\left(\frac{\Delta T_f}{T_f} - \frac{\Delta^2 m_N}{\Delta m_N}\right)$$

If the dominant contribution from $\Delta\alpha$ is in Δm_N then:

$$\frac{\Delta Y}{Y} \simeq \frac{\Delta^2 m_N}{\Delta m_N} \sim \frac{\Delta\alpha}{\alpha} < 0.05$$

If $\Delta\alpha$ arises in a more complete theory the effect may be greatly enhanced:

$$\frac{\Delta Y}{Y} \simeq O(100)\frac{\Delta\alpha}{\alpha} \textbf{ and } \frac{\Delta\alpha}{\alpha} < \textbf{few } \times 10^{-4}$$

Approach:

Consider possible variation of Yukawa, h,
or fine-structure constant, α

Include dependence of Λ on α; of v on h, etc.

Consider effects on: $Q = \Delta m_N$, τ_N, B_D

Coc, Nunes, Olive, Uzan, Vangioni

Quantities of importance for BBN

- $Q = \Delta m_N$ nucleon mass difference

$$Q \equiv m_n - m_p = a\,\alpha\,\Lambda + (h_d - h_u)\,v\,,$$

$$\frac{\Delta Q}{Q} = -0.6\left[\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right] + 1.6\left[\frac{\Delta(h_d - y_u)}{h_d - h_u} + \frac{\Delta v}{v}\right]$$

- τ_n neutron lifetime

$$\tau_n^{-1} = \frac{1}{60}\frac{1 + 3\,g_A^2}{2\pi^3}\,G_F^2\,m_e^5\left[\sqrt{q^2 - 1}(2q^4 - 9q^2 - 8) + 15\ln(q + \sqrt{q^2 - 1})\right], \qquad ($$

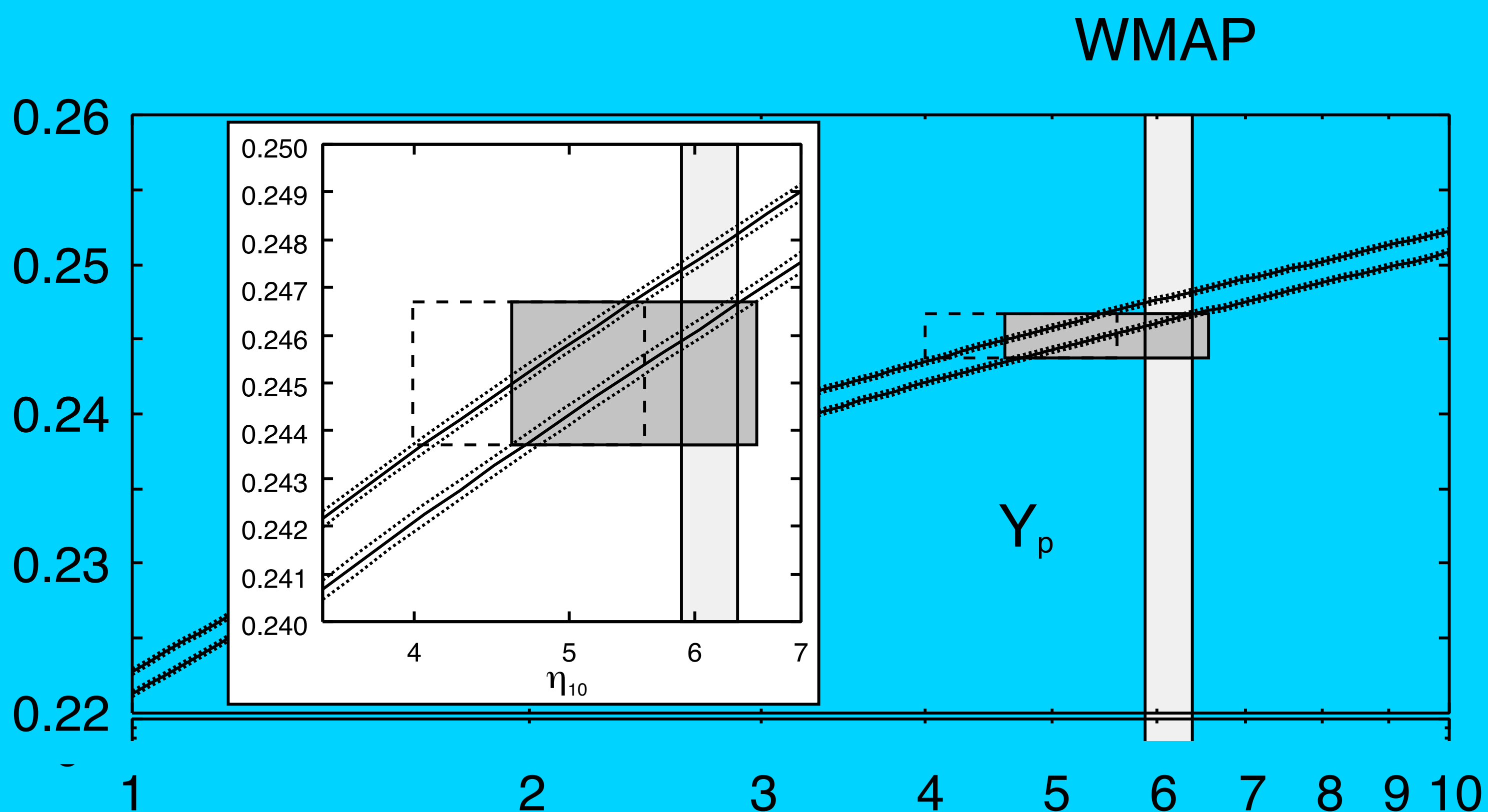
$$\frac{\Delta\tau_n}{\tau_n} = -4.8\,\frac{\Delta v}{v} + 1.5\,\frac{\Delta h_e}{h_e} - 10.4\,\frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right).$$

Neutron Lifetime Measurement

Used: $\tau_n = 885.7 \pm 0.8$ (RPP world average)

There was a new determination $878.5 \pm 0.7 \pm 0.3$ (Serebrov et al.)

$\Rightarrow$ lower ^{4}He



Mathews et al.

- B_D binding energy of deuterium

Using a potential model, Dimitriev & Flambaum

$$\frac{\Delta B_D}{B_D} = -48\,\frac{\Delta m_\sigma}{m_\sigma} + 50\,\frac{\Delta m_\omega}{m_\omega} + 6\,\frac{\Delta m_N}{m_N}\,.$$

and the dependence on Λ, by dimensional grounds,

$$\Delta B_D / B_D = 8\,\Delta\Lambda/\Lambda.$$

But there is also a dependence on quark masses.

Spin-independent Neutralino-p cross section

The scalar cross section

$$\sigma_3 = \frac{4m_r^2}{\pi}\left[Zf_p + (A-Z)f_n\right]^2$$

where

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} f_{Tq}^{(p)}\frac{\alpha_{3q}}{m_q} + \frac{2}{27}f_{TG}^{(p)}\sum_{c,b,t}\frac{\alpha_{3q}}{m_q}$$

and

$$m_p f_{Tq}^{(p)} \equiv \langle p|m_q\bar{q}q|p\rangle \equiv m_q B_q$$

determined by

$$\sigma_{\pi N} \equiv \Sigma = \frac{1}{2}(m_u+m_d)(B_u+B_d)$$

will take:

Σ = 45 GeV or 64 GeV

Strangeness contribution $y = 2B_s/(B_u + B_d)$

with $\Sigma(1-y) = 36 \pm 7$ MeV

and $$z \equiv \frac{B_u - B_s}{B_d - B_s} = \frac{m_{\Xi^0} + m_{\Xi^-} - m_p - m_n}{m_{\Sigma^+} + m_{\Sigma^-} - m_p - m_n} = 1.49$$

giving $$\frac{\Delta m_N}{m_N} = \left(\frac{m_s B_s}{m_N}\right) \frac{\Delta m_s}{m_s} \simeq 0.19 \frac{\Delta m_s}{m_s}.$$

and $$\frac{\Delta m_N}{m_N} \simeq 0.052 \frac{\Delta m_q}{m_q}.$$

This implies that

$$\frac{\Delta m_p}{m_p} \simeq \frac{\Delta \Lambda}{\Lambda} + 0.24 \left(\frac{\Delta h}{h} + \frac{\Delta v}{v}\right).$$

Repeat calculation for contribution of quark masses to σ and ω

Dimitriev & Flambaum

$$\frac{\Delta B_D}{B_D} = 8\,\frac{\Delta\Lambda}{\Lambda} - 17\left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s}\right)$$

contributions from u and d are negliglible

Alternative:

Use dependence from pion mass

Beane & Savage
Yoo & Scherrer

$$\frac{\Delta B_D}{B_D} = -r\,\frac{\Delta m_\pi}{m_\pi}$$

r = 6-10

$$\frac{\Delta B_D}{B_D} = \frac{-r}{2}\left(\frac{\Delta\Lambda}{\Lambda} + \frac{\Delta v}{v} + \frac{\Delta h}{h}\right)$$

Coupled Variations

Campbell and Olive

Langacker, Segre, and Strassler

Dent and Fairbairn

Calmet and Fritzsch

Damour, Piazza, and Veneziano

Recall,

$$\alpha_s(M_{UV}^2) \equiv \frac{g_s^2(M_{UV}^2)}{4\pi} = \frac{4\pi}{b_3 \ln(M_{UV}^2/\Lambda^2)}$$

$$\Lambda = \mu \left(\frac{m_c\, m_b\, m_t}{\mu^3} \right)^{2/27} \exp\left(-\frac{2\pi}{9\alpha_s(\mu)} \right)$$

$$\frac{\Delta\Lambda}{\Lambda} = R\,\frac{\Delta\alpha}{\alpha} + \frac{2}{27}\left(3\,\frac{\Delta v}{v} + \frac{\Delta h_c}{h_c} + \frac{\Delta h_b}{h_b} + \frac{\Delta h_t}{h_t} \right)$$

R ~ 30, but very model dependent

Dine et al.

Net sensitivities due to Λ

$$\begin{aligned}
\frac{\Delta B_D}{B_D} &= -15\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) + 8R\,\frac{\Delta\alpha}{\alpha}\,, \\
\frac{\Delta Q}{Q} &= 1.5\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) - 0.6(1+R)\,\frac{\Delta\alpha}{\alpha}\,, \\
\frac{\Delta\tau_n}{\tau_n} &= -4\,\frac{\Delta v}{v} - 8\,\frac{\Delta h}{h} + 3.8(1+R)\,\frac{\Delta\alpha}{\alpha}\,.
\end{aligned}$$

Fermion Masses:

$$\boldsymbol{m_f \propto h_f v \qquad G_F \propto 1/v^2}$$

Also expect variations in Yukawas,

$$\frac{\Delta h}{h} = \frac{1}{2}\frac{\Delta \alpha_U}{\alpha_U}$$

But in theories with radiative electroweak symmetry breaking

$$v \sim M_P \exp(-2\pi c/\alpha_t)$$

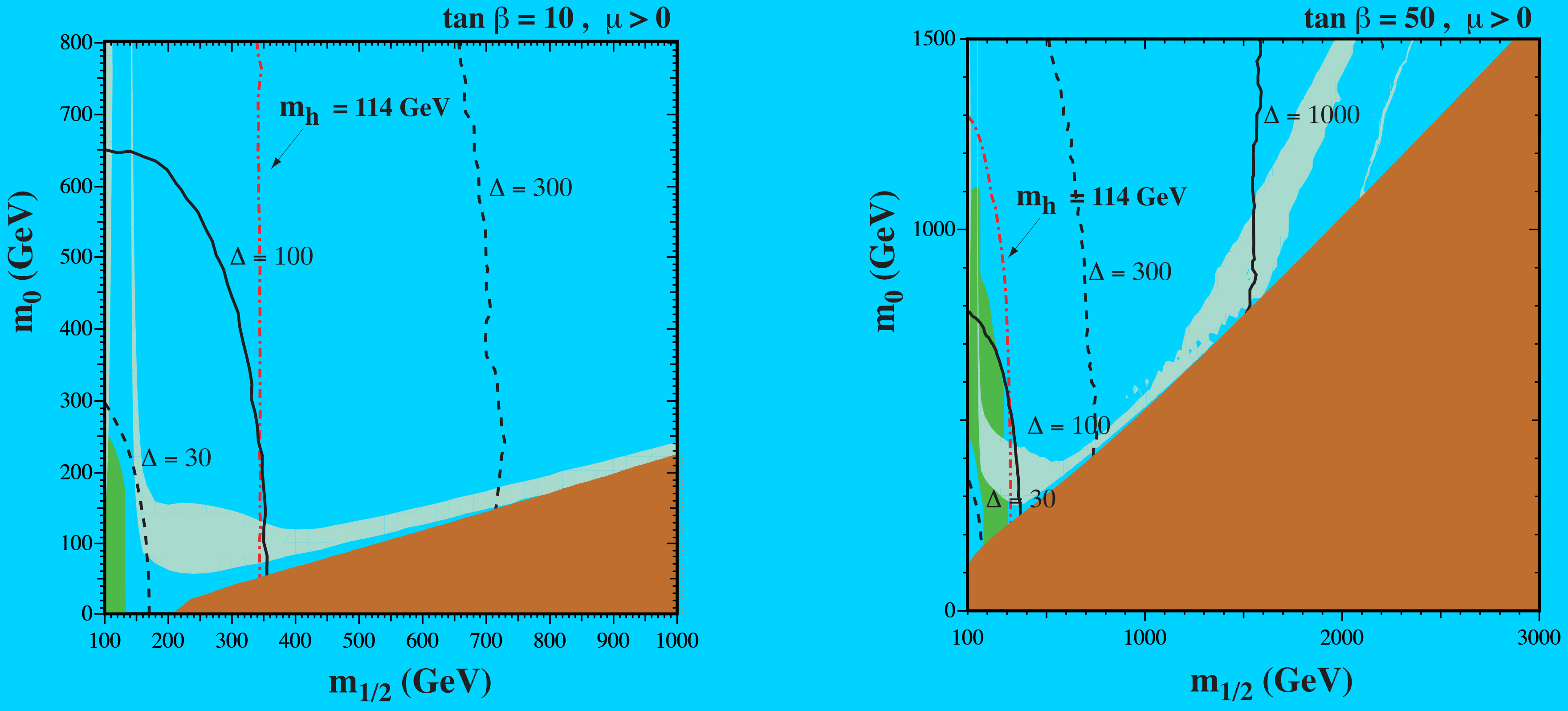
Thus small changes in h_t
will induce large changes in v

$$\frac{\Delta v}{v} \sim 80\frac{\Delta \alpha_U}{\alpha_U}$$

Define another sensitivity parameter $\quad \dfrac{\Delta v}{v} \equiv S \, \dfrac{\Delta h}{h} \, ,$

related SUSY finetuning parameters

$$\Delta = \sqrt{\sum_i \ \Delta_i{}^2} \, , \quad \Delta_i \equiv \frac{\partial \ln m_W}{\partial \ln a_i}$$



With, $\Delta \sim 100$ - 400 (1000), $\Delta_t \sim 80$ - 250 (500)

Putting both relations together:

$$\frac{\Delta B_D}{B_D} = -15(1+S)\frac{\Delta h}{h} + 8R\frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta Q}{Q} = 1.5(1+S)\frac{\Delta h}{h} - 0.6(1+R)\frac{\Delta\alpha}{\alpha},$$

$$\frac{\Delta\tau_n}{\tau_n} = -(8+4S)\frac{\Delta h}{h} + 3.8(1+R)\frac{\Delta\alpha}{\alpha}.$$

$$\frac{\Delta B_D}{B_D} = -0.6r(1+S)\frac{\Delta h}{h} - 0.5rR\frac{\Delta\alpha}{\alpha}$$ from m_π

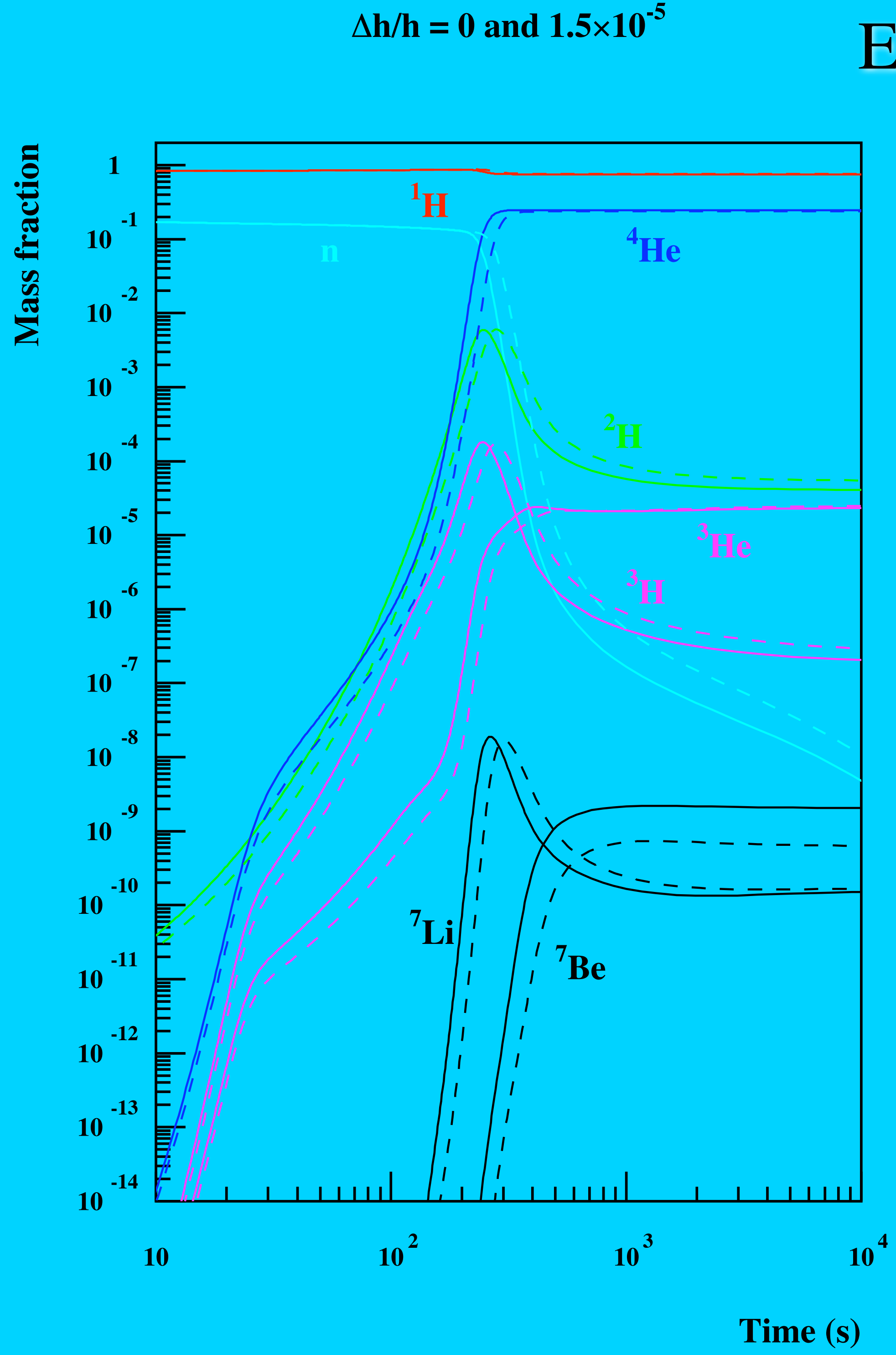
and with $\frac{\Delta h}{h} = \frac{1}{2}\frac{\Delta\alpha_U}{\alpha_U}$

$$\frac{\Delta B_D}{B_D} = -[7.6(1+S) - 8R]\frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta Q}{Q} = (0.1 + 0.7S - 0.6R)\frac{\Delta\alpha}{\alpha}$$

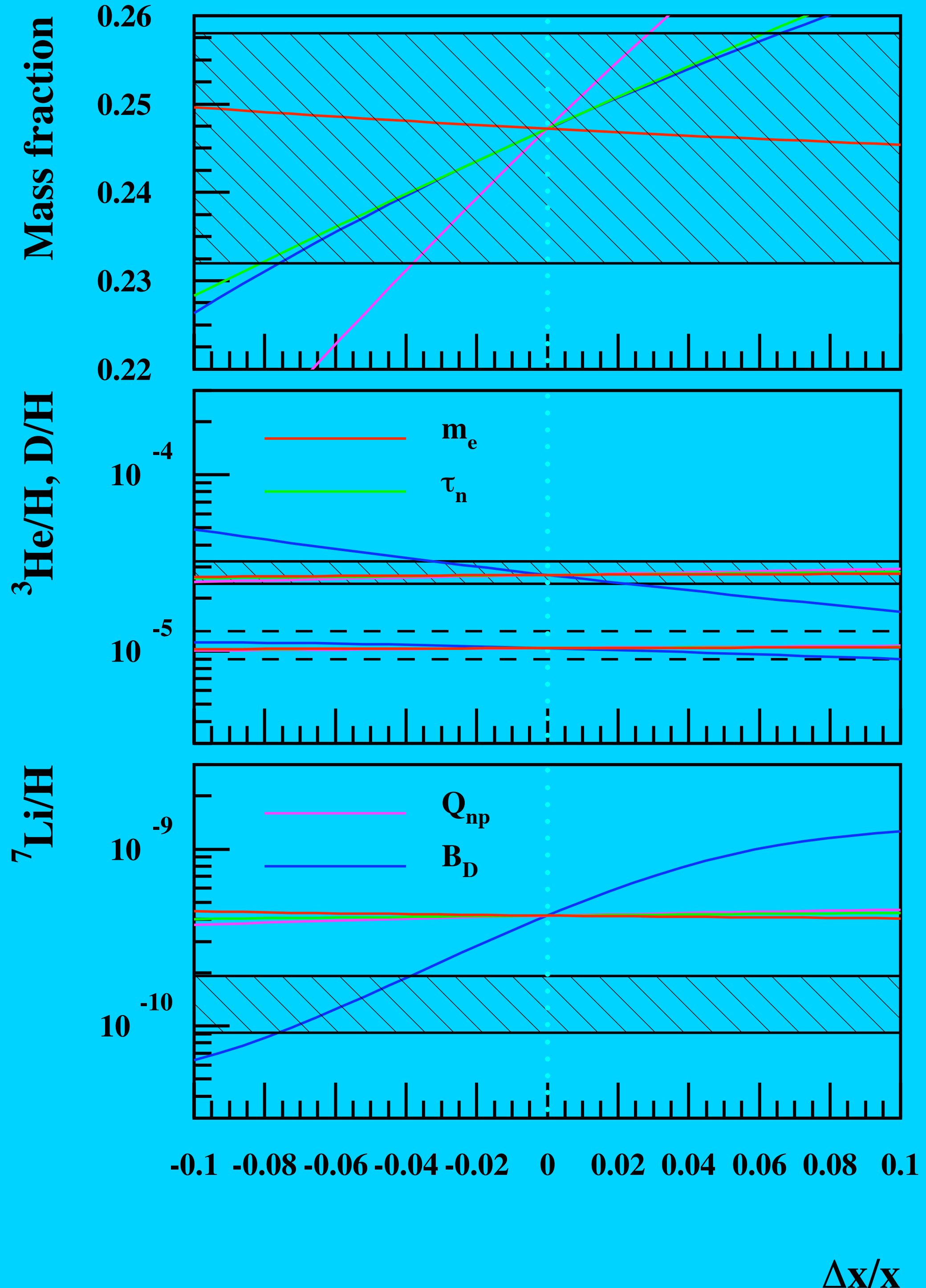
$$\frac{\Delta\tau_n}{\tau_n} = -[0.2 + 2S - 3.8R]\frac{\Delta\alpha}{\alpha},$$

Effect of variations of h (S = 160)

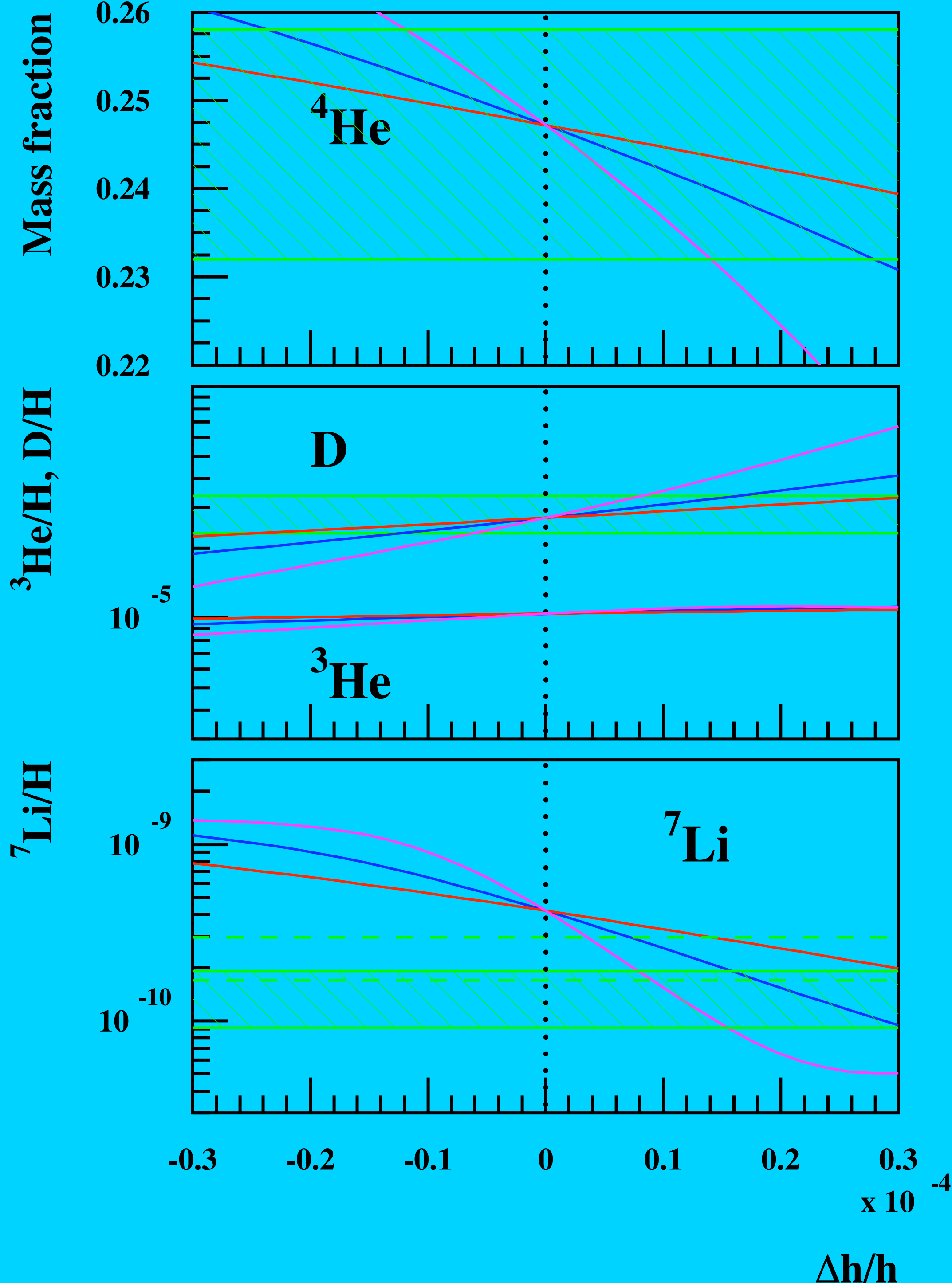


Notice effect on ^{7}Li

m_e, B_D, Q_{np} and τ_n variations

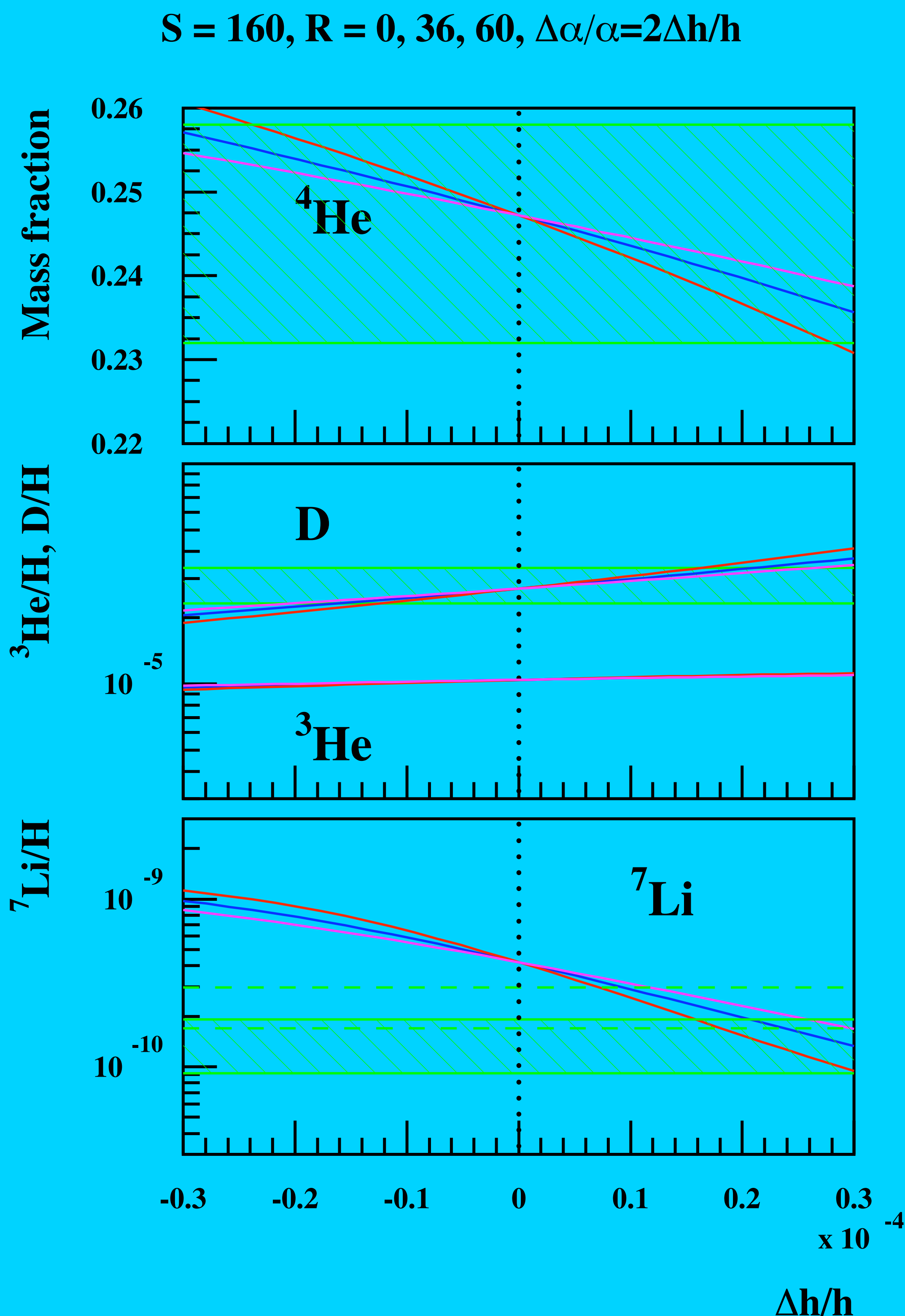


S = 80, 160, 320, $\Delta\alpha/\alpha=0$



For S = 160,

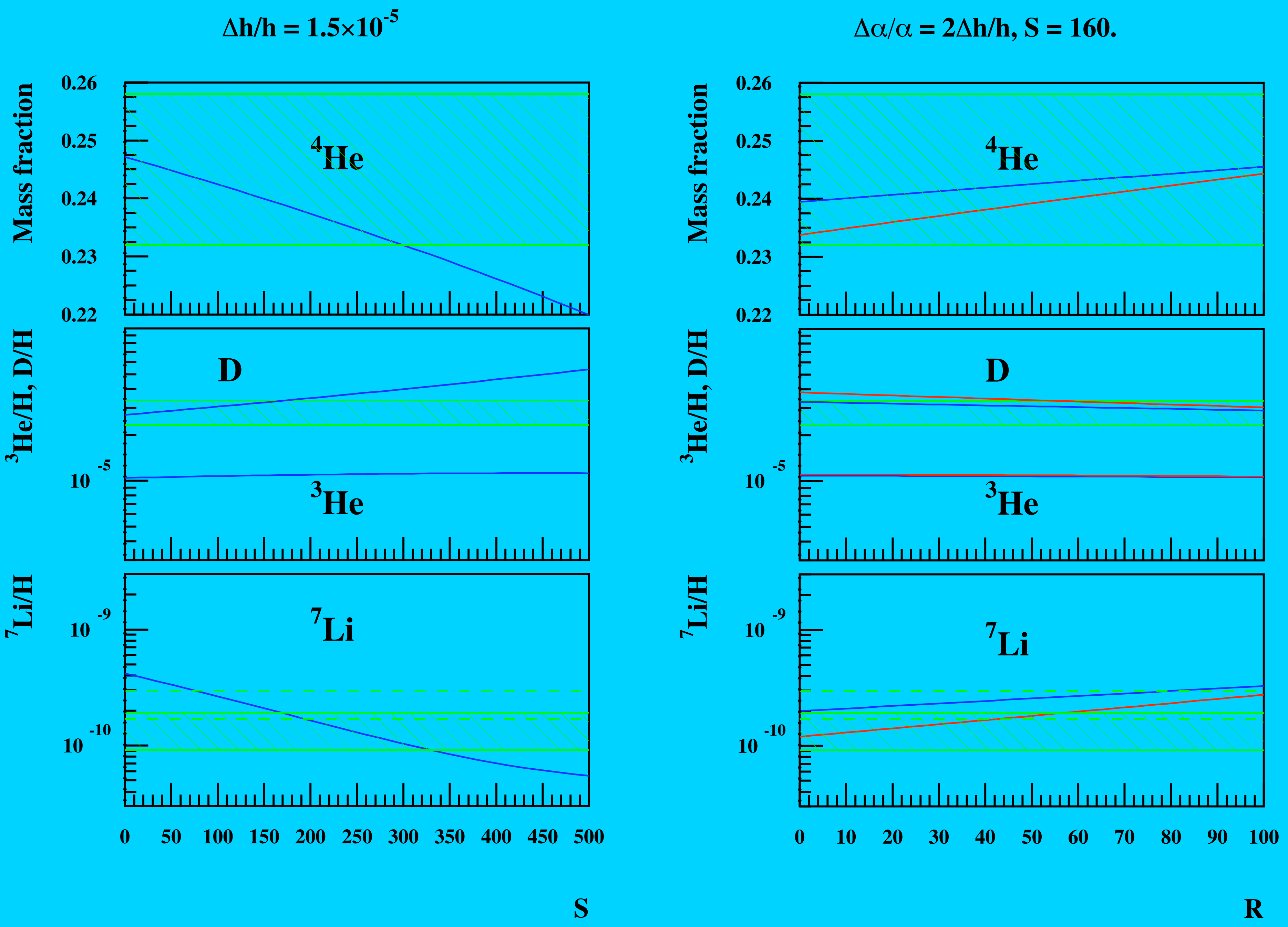
$$-1.2\times10^{-3} < \frac{\Delta h}{h} < 1.6\times10^{-5}.$$



For S = 160, R = 36,

$$-1.8 \times 10^{-5} < \frac{\Delta h}{h} < 2.1 \times 10^{-5},$$

Finally,



$\Delta h/h = 1.5\times10^{-5}$
$\Delta\alpha/\alpha = 2\Delta h/h$, S = 160.
Mass fraction
0.26
0.25
0.24
0.23
0.22
^{4}He
^{3}He/H, D/H
D
10^{-5}
^{3}He
^{7}Li/H
10^{-9}
10^{-10}
^{7}Li
0 50 100 150 200 250 300 350 400 450 500
S
0 10 20 30 40 50 60 70 80 90 100
R

Summary

- While possible, there are many constraints on the variations of α
- BBN constraints (when coupled variations are considered) are of order 10^{-5}
- Solution to ^{7}Li problem?