# Charge Density of the Neutron

Gerald A. Miller
University of Washington

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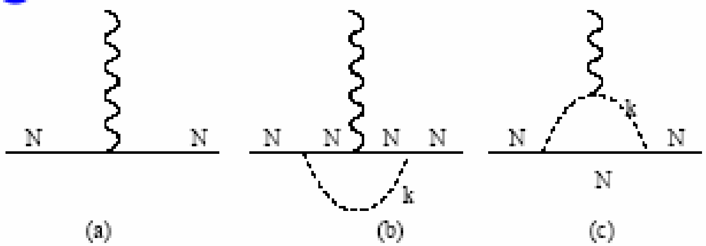
# What is charge density at the center of the neutron?

- Neutron has no charge, but charge density need not vanish
- Is central density positive or negative?

Neutron: Need $\pi$ cloud effect at low $Q^2$

Cloudy Bag Model 1980




Relativistic treatment needed Feynman graphs

Light front cloudy bag model LFCBM 2002

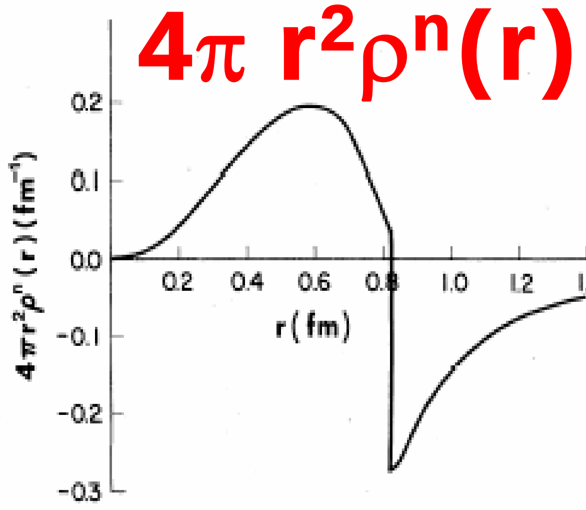



FIG. 11. Neutron charge density.

# One gluon exchange also gives positive central charge density

- Friar, Ellis et al, Isgur & Karl
- dd one gluon interaction is repulsive

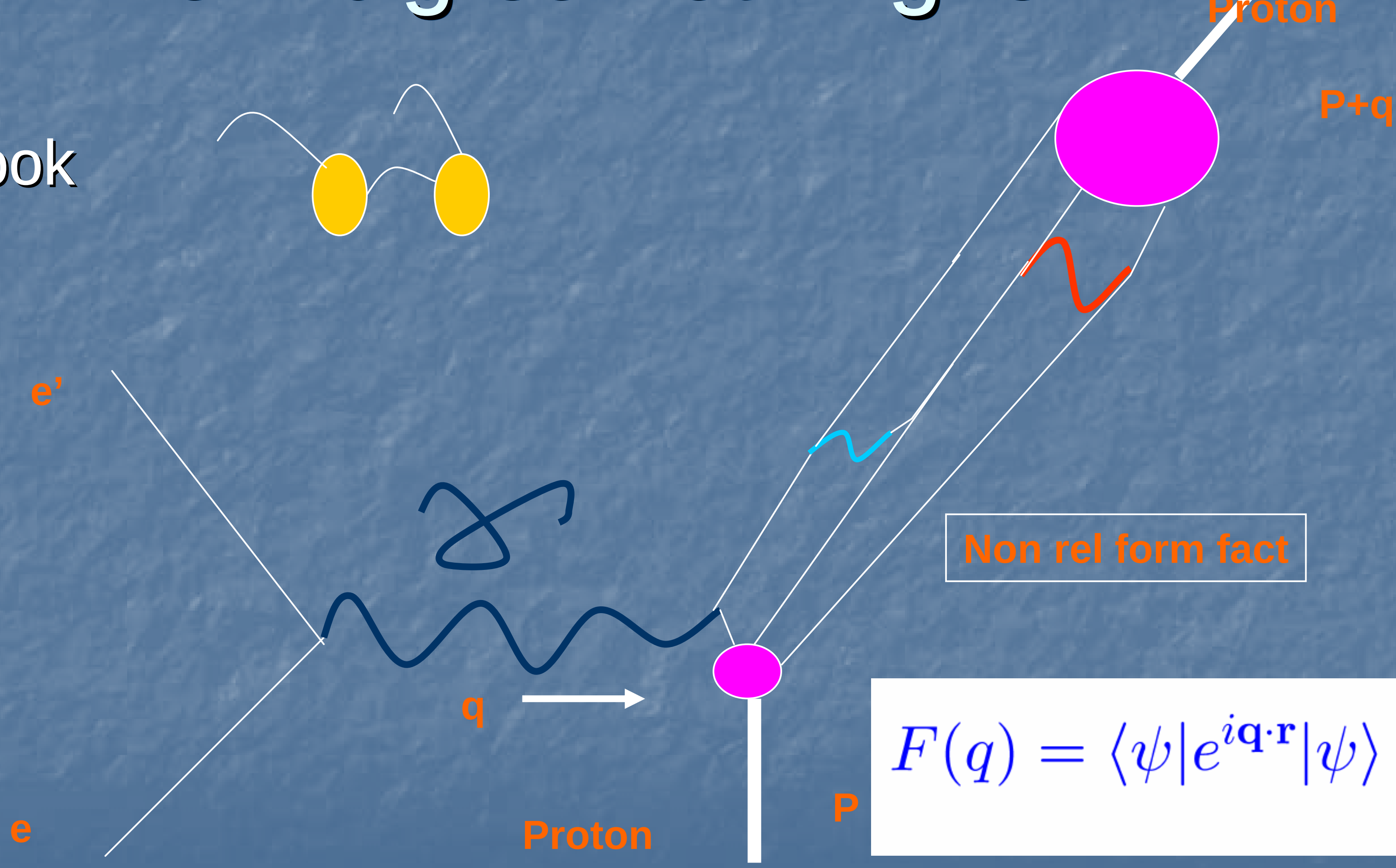
Main question today: obtain model independent information

# Outline

- Electromagnetic form factors
- Light cone coordinates, kinematic subgroup
- Generalized parton distribution functions
- Bit of math
- Two dimensional Fourier transform of $F_1$, gives $\rho(b)$ Soper '77
- Data analysis

# What is a form factor? How to tell how big something is?

- Look




$$F(q) = \langle \psi | e^{i\mathbf{q}\cdot\mathbf{r}} | \psi \rangle$$

Structure factor

# Definitions

$$\langle N, \lambda' p' | J^\mu | N, \lambda p \rangle = \bar{u}_{\lambda'}(p')[F_1(Q^2)\gamma^\mu + F_2(Q^2)\sigma^{\mu\nu} \frac{(p'-p)_\nu}{2\, M_p}] u_\lambda(p)$$

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2 \,, \qquad G_M = F_1 + F_2$$

Interpretation- Breit frame $\vec{p}\,' = -\vec{p}$

$G_E$ is helicity flip matrix element of $J^0$

$G_M$ is helicity conserving matrix element of $J^i$

# Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

Correct non-relativisticaly

Non-relativistic two particle :

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = e^{i\mathbf{P}\cdot\mathbf{R} - i(\frac{P^2}{2M} - \epsilon)t}\phi(\mathbf{r})$$

**$\phi$ invariant under Galilean transformation**

Relativity: $(\mathbf{r}_1, t_1), (\mathbf{r}_2, t_2)$ $t_1 \neq t_2$

$e^{i\,H(t_1 - t_2)}$ Interactions!

$\phi$ is frame dependent,

interpretation of Sachs wrong

# Why relativity if $Q^2 \ll M^2$

QCD- photon hits $\approx$ massless quarks

No matter how small $Q^2$ is, there is a boost correction that is $\propto Q^2$

$F_1 \sim Q^2(|\psi|^2 + C/(m_q R_N)^2)$

# Light cone coordinates

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2}$$

$$\text{“Evolution”}\quad p^- = (p^0-p^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (ct-z)/\sqrt{2} = (x^0-x^3)/\sqrt{2},\ \text{If}\ x^+=0,\ \ x^- = -\sqrt{2}z$$

$$\text{“Momentum”}\quad p^+ = (p^0+p^3)/\sqrt{2}$$

$$\text{Transverse :}\ \ \text{“Position” b “Momentum” p}$$

# Relativistic formalism-kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

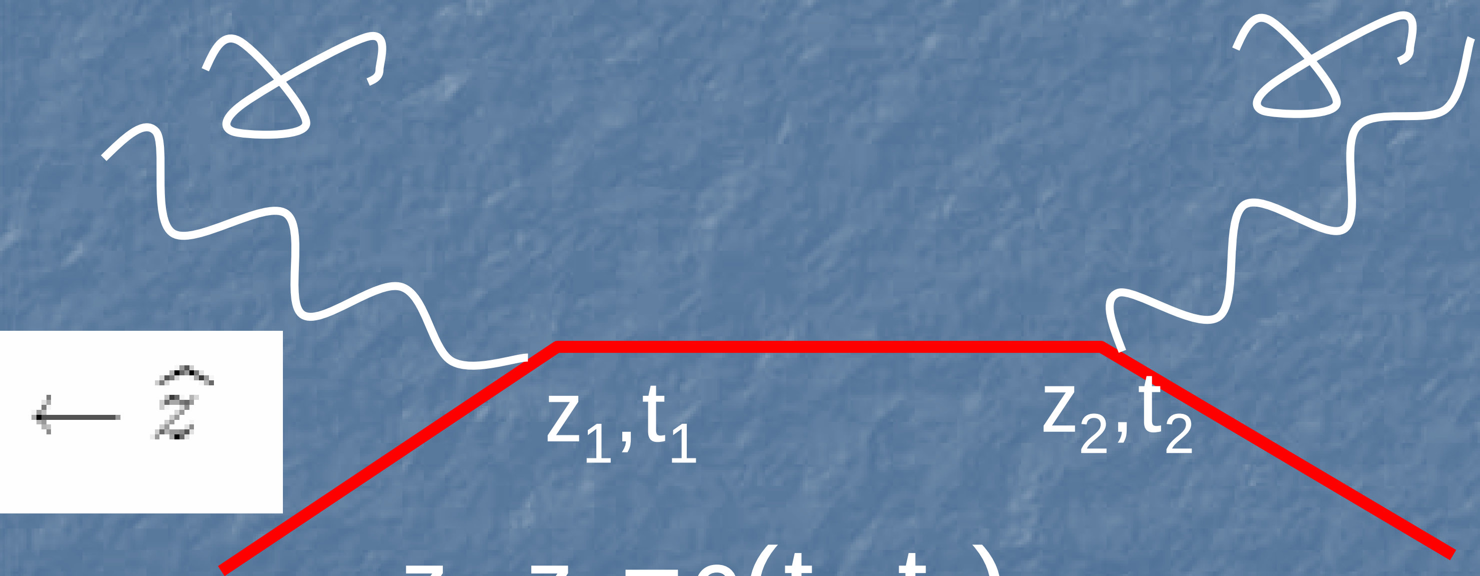
$$k^+ \longrightarrow k^+, \;\; \mathbf{k} \longrightarrow \mathbf{k} - k^+\mathbf{v}$$

$k^-$ chosen so $k^2$ not changed

**Just like non-relativistic**

# Why light cone coordinates?

Deep inelastic scattering:




$$z_1 - z_2 = c(t_2 - t_1)$$

$$z_1 + ct_1 = z_2 + ct_2 \qquad x^+ = \text{fixed}$$

# Generalized Parton Distribution

$$H_q(x,t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

$$H_q(x, \xi = 0, t) \equiv H_q(x,t)$$

$A^+=0$, $t=(p-p')^2 = -Q^2 = -(\mathbf{p}' - \mathbf{p})^2$

# Generalized Parton Distribution

$$H_q(x,t)=\int\frac{dx^-}{4\pi}\langle p^+,\mathrm{p}',\lambda|\bar{q}(-\frac{x^-}{2},0)\gamma^+q(\frac{x^-}{2},0)|p^+,\mathrm{p},\lambda\rangle e^{ixp^+x^-}$$

$H_q(x,0)=q(x)$ (PDF)

$$F_1(t)=\sum_q e_q\int dx H_q(x,t).$$

$$H_q(x,t) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

transverse center of mass **R**

$$\left| p^+, \mathbf{R} = \mathbf{0}, \lambda \right\rangle \equiv \mathcal{N} \int \frac{d^2\mathbf{p}}{(2\pi)^2} \left| p^+, \mathbf{p}, \lambda \right\rangle$$

$$\hat{O}_q(x, \mathrm{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger \left( -\frac{x^-}{2}, \mathrm{b} \right) q_+ \left( \frac{x^-}{2}, \mathrm{b} \right) e^{ixp^+x^-}$$

# Integrate over x

$$\hat{O}_q(x,\mathrm{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger\left(-\frac{x^-}{2},\mathrm{b}\right) q_+\left(\frac{x^-}{2},\mathrm{b}\right) e^{ixp^+x^-}$$

$$\int dx \hat{O}_q(x,\mathbf{b}) = \frac{1}{2} q_+^\dagger(0,\mathbf{b}) q_+(0,\mathbf{b})$$

**Density and matrix element of $\gamma^+$**

$$H_q(x,t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

$$\hat{O}_q(x, \mathbf{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger \left(-\frac{x^-}{2}, \mathbf{b}\right) q_+ \left(\frac{x^-}{2}, \mathbf{b}\right) e^{ixp^+x^-}$$

$$q(x, \mathbf{b}) \equiv \left\langle p^+, \mathbf{R} = 0, \lambda \middle| \hat{O}_q(x, \mathbf{b}) \middle| p^+, \mathbf{R} = 0, \lambda \right\rangle.$$

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\,\mathbf{q}\cdot\mathbf{b}} H_q(x, t = -\mathbf{q}^2),$$

Burkardt

$$\hat{O}_q(x,\mathrm{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger\left(-\frac{x^-}{2},\mathrm{b}\right) q_+\left(\frac{x^-}{2},\mathrm{b}\right) e^{ixp^+x^-}$$

$$q(x,\mathrm{b}) \equiv \left\langle p^+,\mathrm{R}=0,\lambda \middle| \hat{O}_q(x,\mathrm{b}) \middle| p^+,\mathrm{R}=0,\lambda \right\rangle .$$

$$\rho(b) \equiv \sum_q e_q \int dx \; q(x,\mathbf{b}) = \int d^2 q F_1(Q^2 = \mathbf{q}^2) e^{i\,\mathbf{q}\cdot\mathbf{b}} .$$

**Soper '77**

# Main result

$$\rho(b) \equiv \sum_q e_q \int dx\, q(x, \mathrm{b}) = \int d^2q F_1(Q^2 = \mathrm{q}^2) e^{i\,\mathrm{q}\cdot\mathrm{b}}$$

Soper '77

$$\rho(b) = \int_0^\infty \frac{dQ\, Q}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}$$

$\tau = Q^2 / 4M^2$

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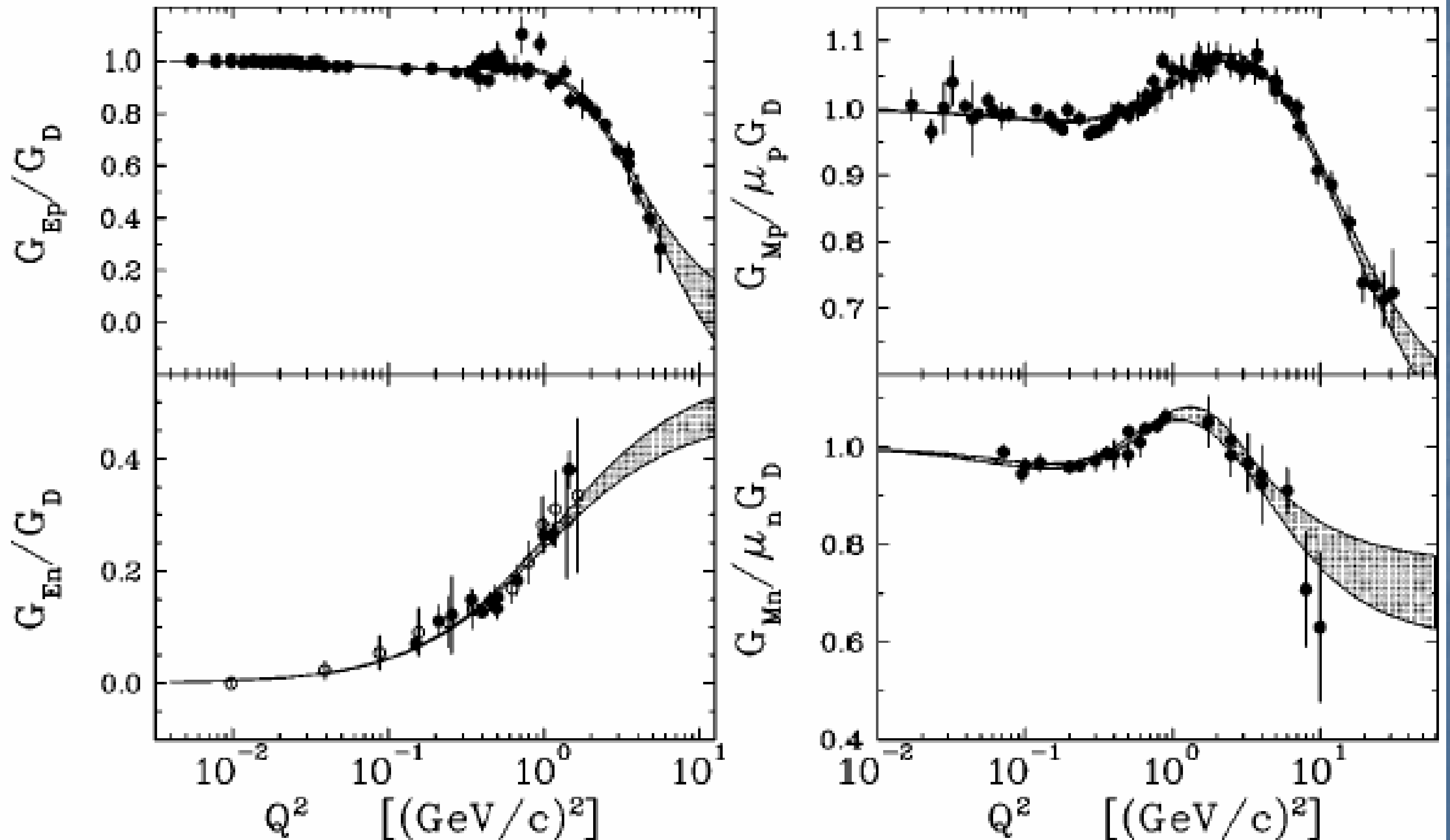
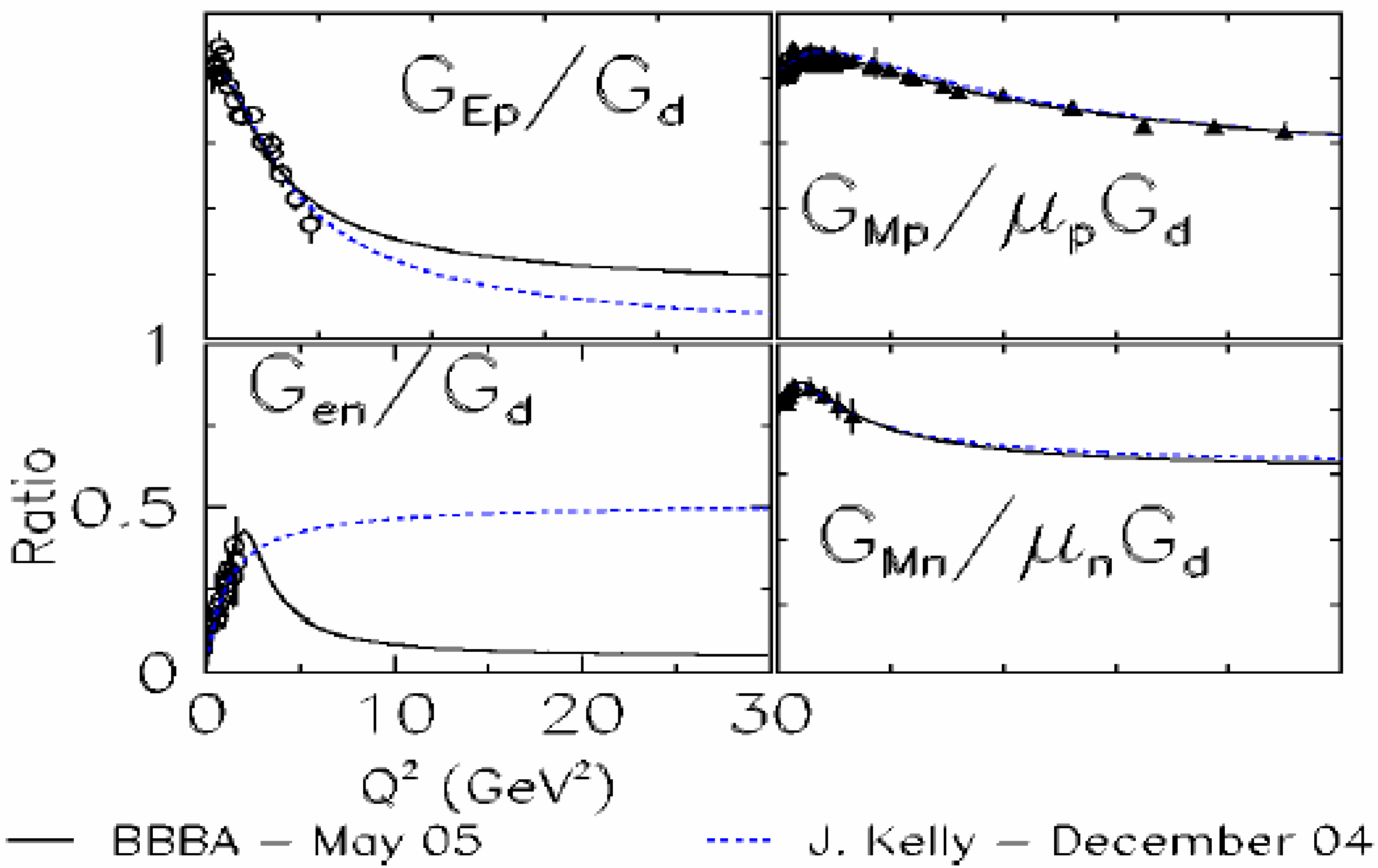
# Simple parametrization of nucleon form factors

J. J. Kelly

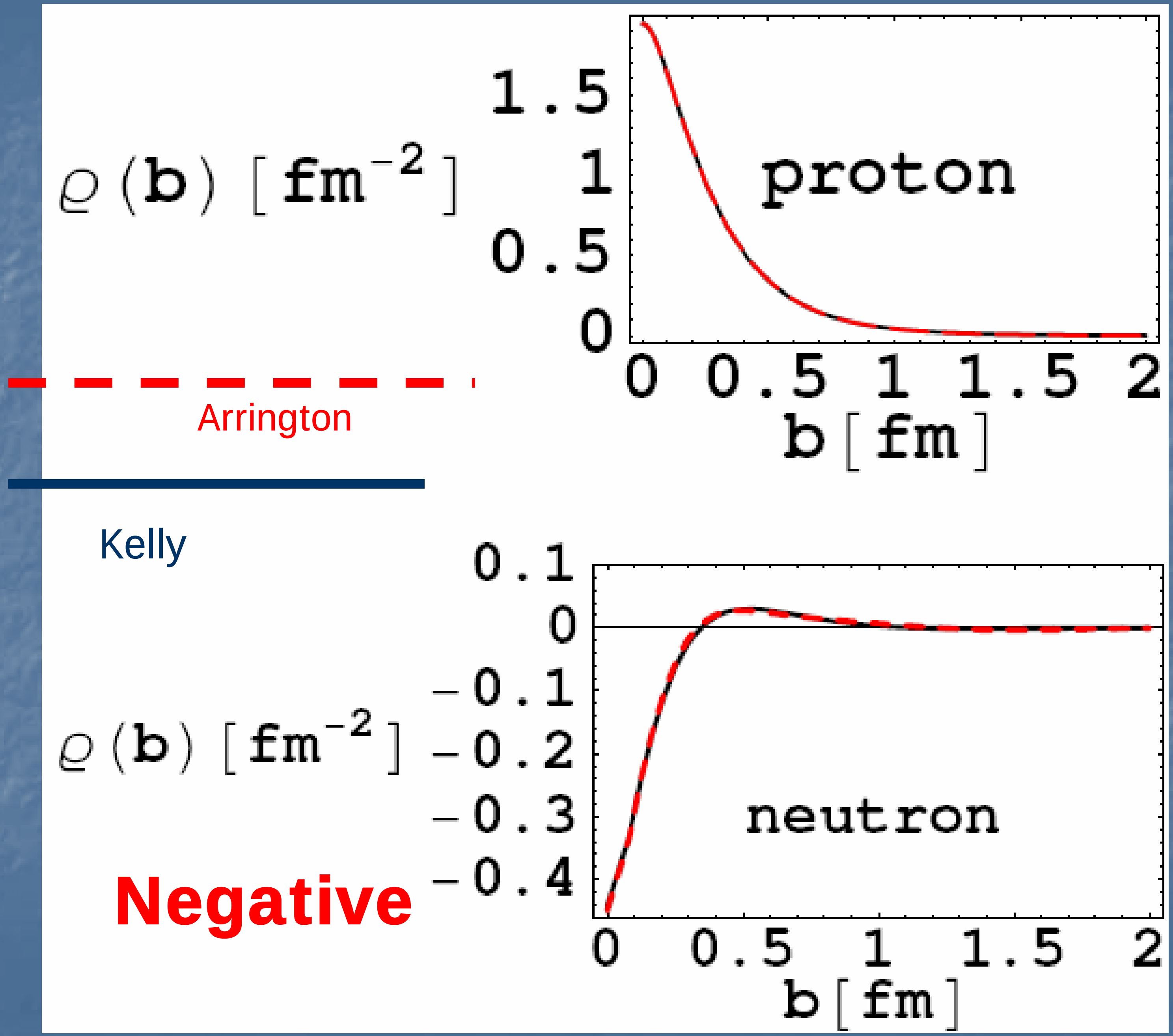


# A New Parameterization of the Nucleon Elastic Form Factors

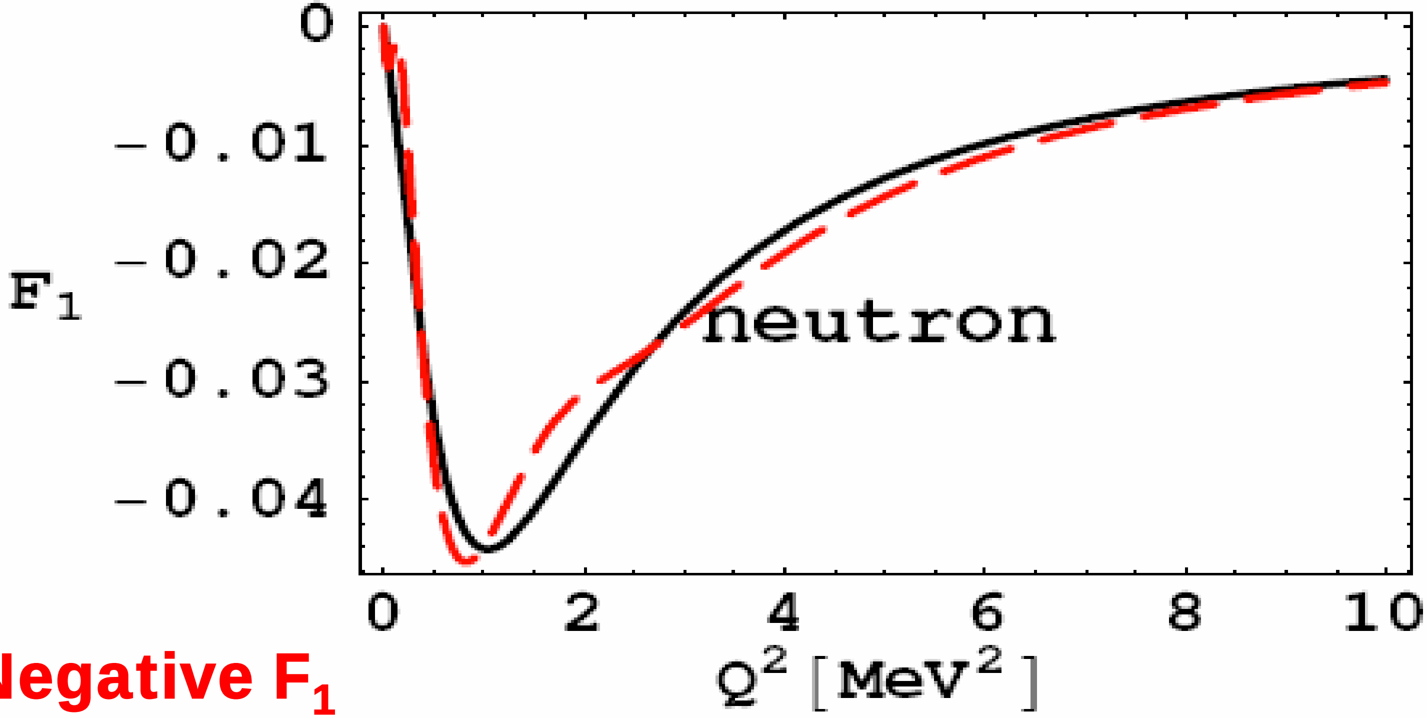
R. Bradford,[a] A. Bodek,[a] H. Budd,[a] and J. Arrington[b]



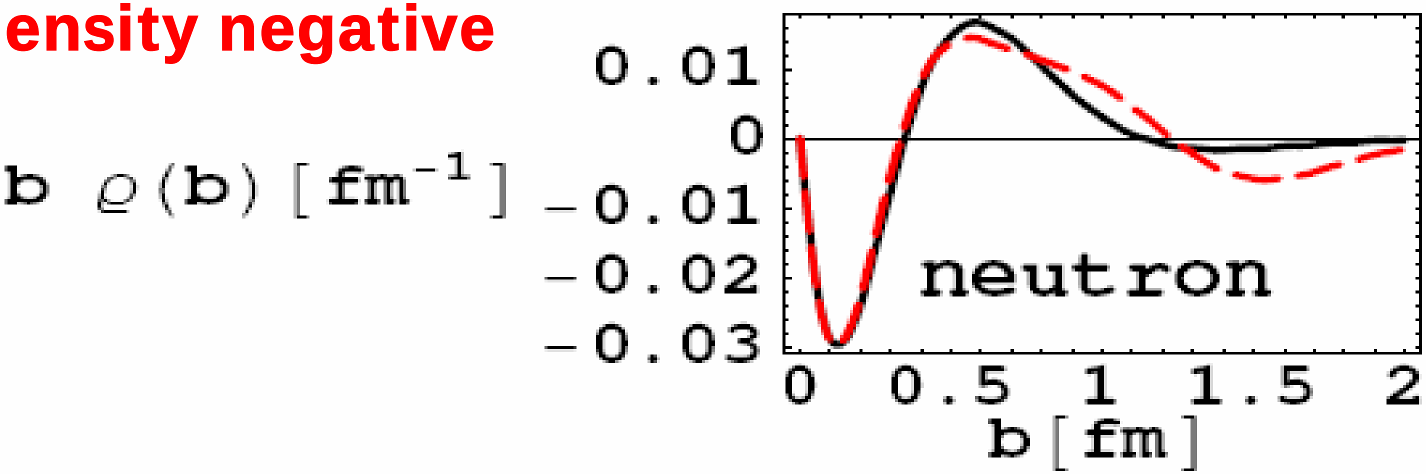
# Results



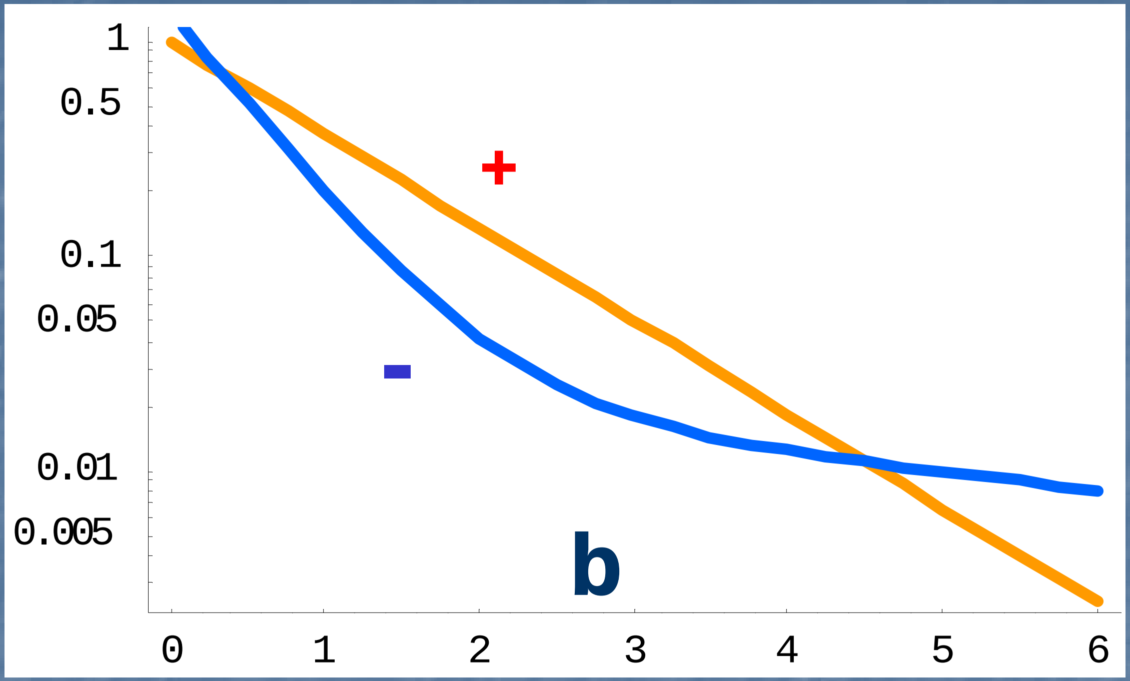
ρ(b) [fm⁻²]
1.5
1
0.5
0
proton
0 0.5 1 1.5 2
b [fm]
Arrington
Kelly
0.1
0
-0.1
-0.2
-0.3
-0.4
ρ(b) [fm⁻²]
neutron
Negative
0 0.5 1 1.5 2
b [fm]




**Negative $F_1$ means central density negative**



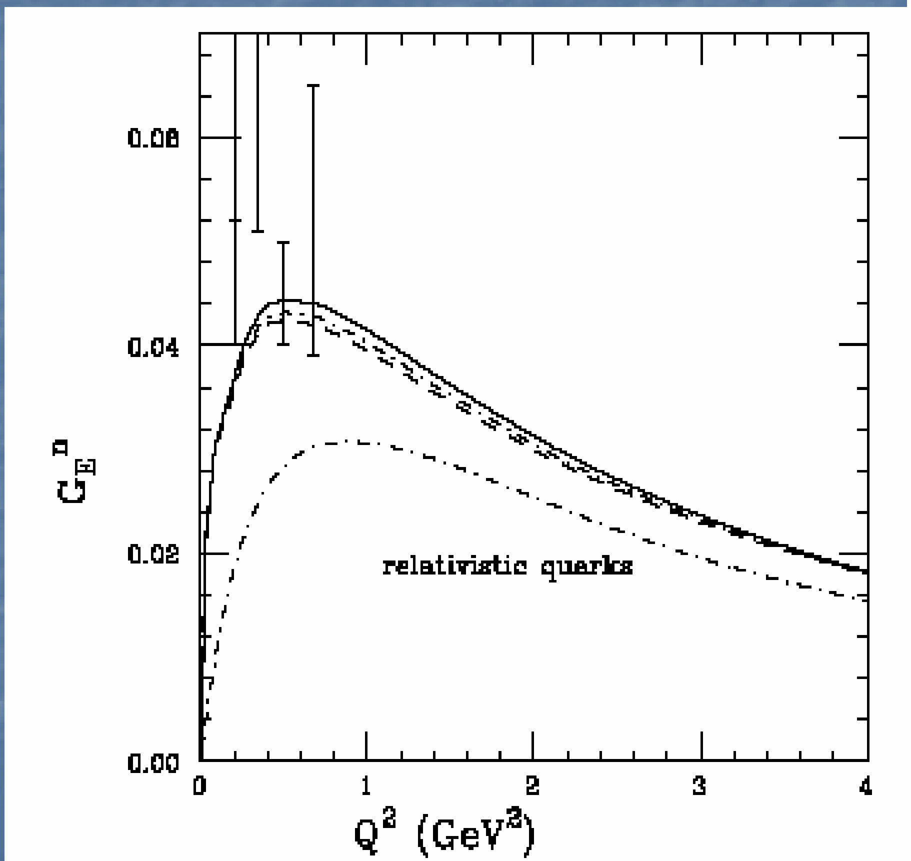
# Neutron Interpretation

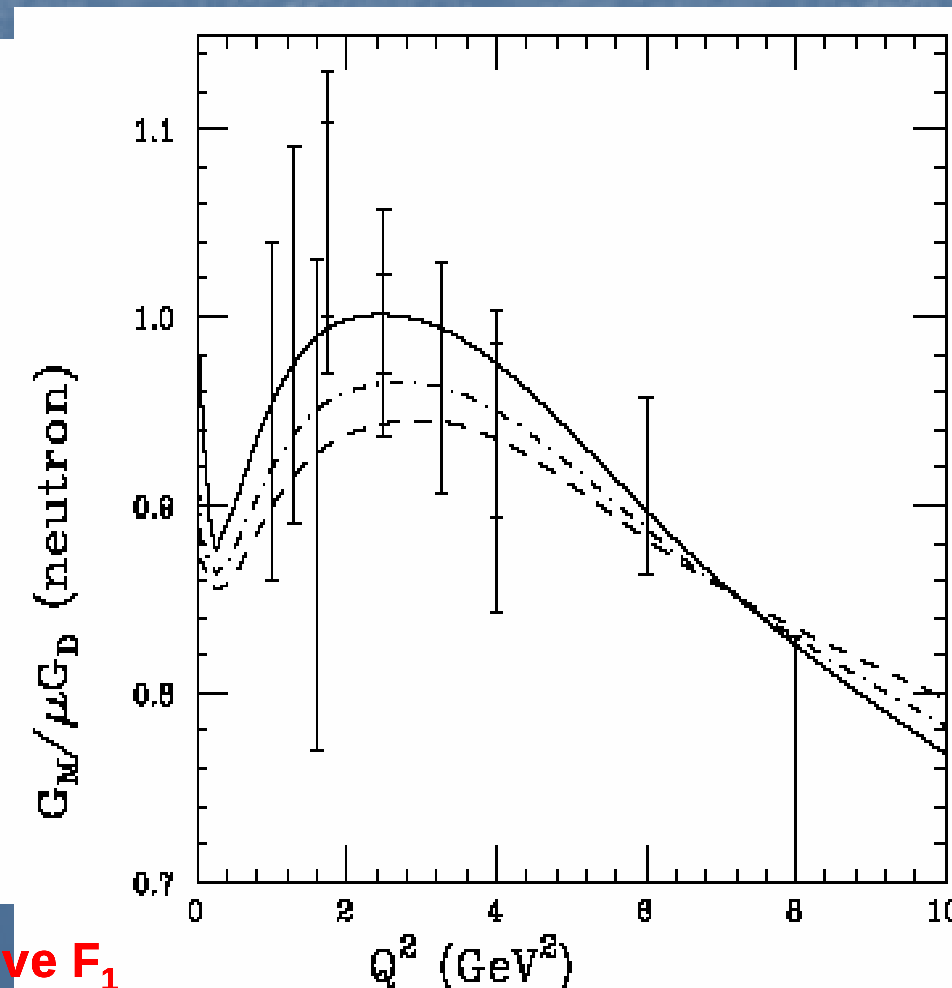



? $\pi^-$ at short distance ?

# Neutron Form Factors in LFCBM

## Miller 2002

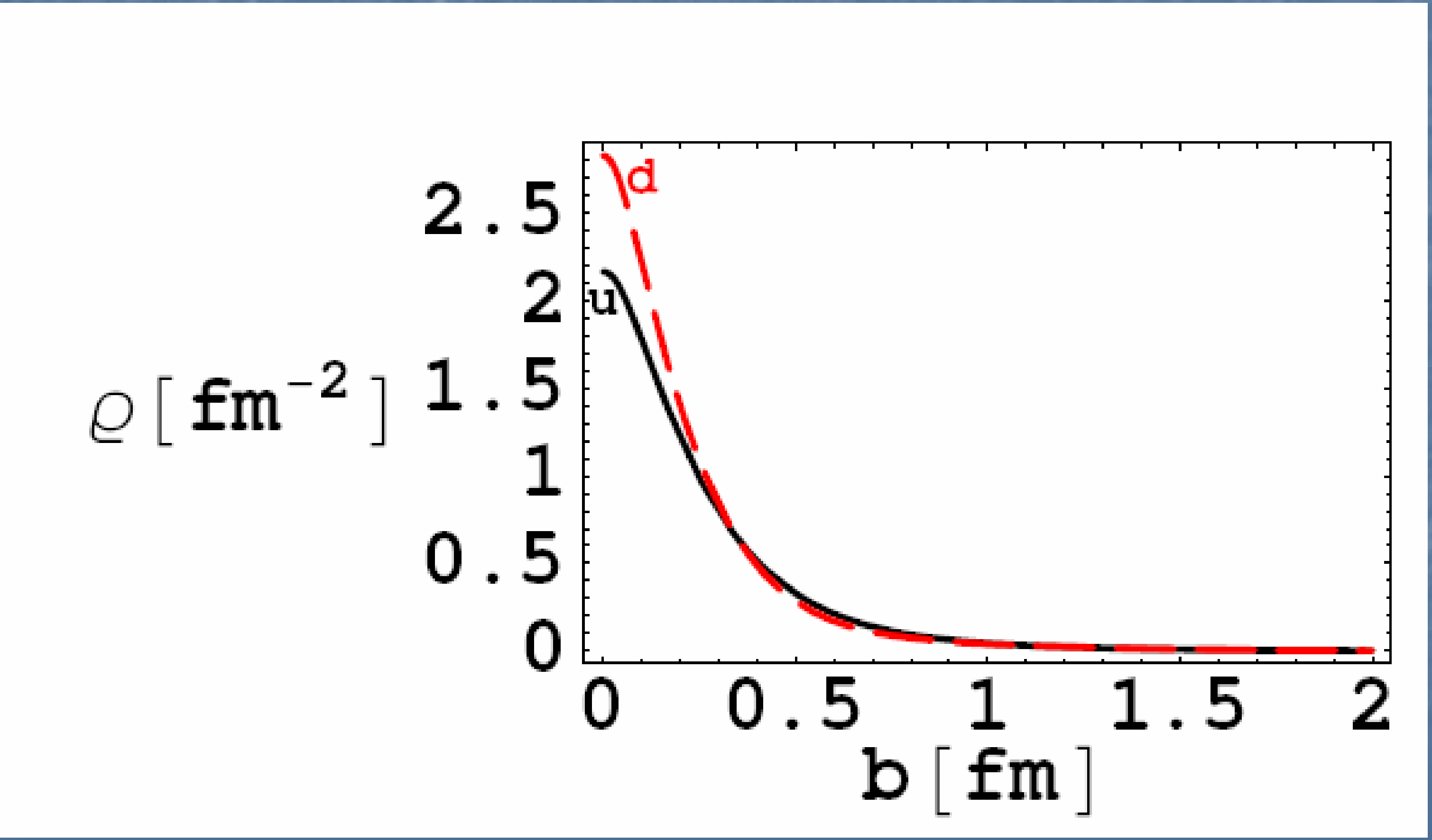



**This gives negative $F_1$**

# Charge symmetry

$\rho_u = \rho_p - \rho_n/2 \quad \rho_d = \rho_p - 2\rho_n$



# ?Quark interpretation?

- b=0, high transverse momentum, low Bjorken x
- low x, sea
- u bar u is suppressed by Pauli principal, Signal & Thomas

# Summary

- Model independent information on charge density

$$\rho(b) \equiv \sum_q e_q \int dx\, q(x, \mathbf{b}) = \int d^2q F_1(Q^2 = \mathbf{q}^2) e^{i\,\mathbf{q}\cdot\mathbf{b}}$$

- Central charge density of neutron is negative

- Evidence for pion cloud at large b

# What do electromagnetic form factors measure?