
QUANTIZED BLACK HOLES, THEIR SPECTRUM AND RADIATION

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- Introduction: entropy, temperature, radiation of black holes
 - Holographic bound
 - Count of states
 - Quantization of area
 - Radiation spectrum of quantized black holes
 - Is radiation of quantized black holes observable?
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Introduction

Wheeler's paradox. Entropy of black hole. Bekenstein.

Then temperature of black hole. Radiation! Gibov, Hawking.

Propagation of semiclassical wave packet from horizon vicinity.

Independently of its initial spectral content, at infinity it is

$|f(\omega)|^2 \sim \exp(-8\pi k M \omega)$. Boltzmann distribution with

$$T = (8\pi k M)^{-1}$$

Now, with $dE(= dM) = T dS = (8\pi k M)^{-1} dS$,

we obtain $S = 4\pi k M^2$. Since the horizon area is

$$A = 4\pi r_g^2 = 16\pi k^2 M^2,$$

$$S = A/4l_p^2; \quad l_p^2 = k. \quad \text{Hawking.}$$

The area of a black hole horizon behaves as an adiabatic invariant
(Christodoulou, Ruffini, Bekenstein).

Quantization of horizon area

Bekenstein, later Mukhanov, Kogan (strings).

Simple-minded arguments: $A = 8\pi\gamma l_p^2 N$; $l_p^2 = \hbar k / c^3$,

N generalized quantum number (the same power as \hbar).

No sound arguments in favor of integer N , or equidistant spectrum!

$$A = 8\pi\gamma l_p^2 \sum_{jm} a(j)\nu_{jm}$$

$$S = A/4l_p^2 = 2\pi\gamma \sum_{jm} a(j)\nu_{jm} = 2\pi\gamma \sum_j a(j)\nu_j$$

Holographic bound

(Bekenstein, 't Hooft, Susskind)

Entropy S of any spherical nonrotating body confined inside a sphere of area A is bounded as follows:

$$S \leq A/4l_p^2,$$

with the equality attained only for a body that is a black hole.

To prove it, let this body collapse into a black hole. Then, we have

$$S \leq S_{bh} = A_{bh}/4l_p^2 \leq A/4l_p^2.$$

Quite unexpected,

but for common objects too mild to contradict the common experience.

Alternative formulation:

Among the spherical surfaces of a given area, it is the surface of a black hole horizon that has the maximum entropy.

Number of quantum states

$$S = \ln K ,$$

K is the total number of quantum states, which depends essentially on the assumptions related to the distinguishability of the sites.

1 Complete distinguishability

$$K = \nu! , \quad \nu = \sum_{jm} \nu_{jm} = \sum_j \nu_j .$$

$S = \ln \nu! = \nu \ln \nu$ reaches maximum for fixed A when all j correspond to the smallest possible $a(j) = a_{\min}$. Then $A \sim \nu$ and

$$S \sim A \ln A .$$

Wrong!

2 Complete indistinguishability

Number of states of given j is

$$K(j) = (\nu_j + g(j) - 1)! / \nu_j! (g(j) - 1)!,$$

$g(j)$ is the number of possible values of m for a given j .

$$S(j) = \ln K(j) = g(j) \ln j,$$

under natural assumption $\nu_j \gg g(j)$.

$$A(j) \sim a(j) \nu_j \gg g(j) \ln \nu_j.$$

Wrong!

3 (and quite popular)

Total number of states is $K = \sum_j g(j)^{\nu_j}$.

It corresponds to the assumptions:

same j ,	different m	→	distinguishable,
different j ,	any m	→	indistinguishable,
same j ,	same m	→	indistinguishable.

Obviously, the first two of them, taken together, look unnatural!

except the case when only a single value j is allowed.

The reasonable assumption

Only sites with the same jm are indistinguishable. Then

$$S = \ln \left[\nu! \prod_{jm} 1 / (\nu_{jm}!) \right].$$

But what is in common with

$$A \sim N = \sum_{jm} a(j) \nu_{jm} ?$$

We need maximum S for given N (“microcanonical” entropy), or with logarithmic accuracy maximum of

$$\nu \ln \nu - \sum_{jm} \nu_{jm} \ln \nu_{jm} - \mu \sum_{jm} a(j) \nu_{jm},$$

where μ is a Lagrangian multiplier.

Variation in ν_{jm} gives (with logarithmic accuracy)

1. $\ln \nu - \ln \nu_{jm} - \mu a(j) = 0$, or

2. $\nu_{jm} = \nu e^{-\mu a(j)}$. Sum 2. over jm and divide by ν :

$$\sum_{jm} e^{-\mu a(j)} = \sum_j g(j) e^{-\mu a(j)} = 1$$

(a secular equation for the Lagrange multiplier μ).

Multiply 1. by ν_{jm} and sum over jm :

$$S_{\max} = \mu N = \frac{\mu}{8\pi\gamma l_p^2} A. \quad \gamma = \mu/2\pi.$$

$$A = 8\pi\gamma l_p^2 \nu \sum_j e^{-\mu a(j)} g(j) a(j).$$

Example

In Loop Quantum Gravity (LQG) $a(j) = \sqrt{j(j+1)}$; $g(j) = 2j + 1$.

The secular equation is $\sum_{j=1/2}^{\infty} (2j + 1) e^{-\mu\sqrt{j(j+1)}} = 1$,

with the solution $\mu = 1.722$,

and Barbero – Immirzi parameter of LQG $\gamma = \mu/(2\pi) = 0.274$.

(Khriplovich, Korkin (2001); Ghosh, Mitra (2004); Corichi et al (2006))

Radiation spectrum of quantized black holes

If, due to quantum effects, the minimum change of the horizon area under adiabatic process is (Bekenstein)

$$(\Delta A)_{\min} = \xi l_p^2,$$

then there are no “combinatorial” frequencies, i.e., the radiation occurs when a site of given j disappears:

$$\Delta N = a(j), \quad \omega_j = \mu T a(j).$$

Discrete spectrum! $\omega_{\min} = \mu T a(j_{\min})$.

Even finite number of lines: since $\nu_{jm} \geq 1$,
 $a(j_{\max}) = \ln \nu / \mu$. $\omega_{\max} = T \ln(A/l_p^2)$.

However, anyway, exponential decrease.

Natural assumption: $\Gamma_j \sim \nu_j = \nu g(j)e^{-\omega_j/T}$. Then

$$I_j \sim \omega_j \nu_j = \nu \omega_j g(j) e^{-\omega_j/T}$$

Exponential Wien profile for $\omega_j \gg T$ (almost free)!

As distinct from the common thermal radiation, here the typical wave length is roughly on the same order of magnitude as the radius of the black body. Therefore, due to the centrifugal barrier, this radiation is strongly dominated by the lowest possible partial wave, $J=1$ for photon.

Still, according to calculations by [Page](#), the total intensity of γ radiation is about the same as that given by the naïve Planck formula.

Using the Page profile as the envelope for the discrete spectrum, one can demonstrate that

the ratio of the line width to the line separation is $\lesssim 0.04$.

Spectrum is really discrete!

The result refers to photons with 2 polarizations.

Similar formulae hold for other particles,

but one should change, in particular, the statistical weight.

Is Radiation of Quantized Black Holes Observable?

Dark matter of density $\rho(r)$ in Solar system (interacting with the usual one only gravitationally) results in additional perihelion rotation

$$\frac{\delta\phi}{2\pi} = -\frac{2\pi\rho(r)r^3}{M}.$$

We assume that $\rho(r)$ is spherically-symmetric and centered on the Sun, and that the orbit eccentricity is small; M is mass of the Sun.

Analysis of observational data for secular perihelion precession of Earth and Mars results in the upper limit (Khriplovich, Pitjeva):

$$\rho(r) < 3 \times 10^{-19} \text{ g/cm}^3.$$

Estimates for the expected signal from PBHs (Khriplovich, Produit) are performed under the (optimistic) assumption that their density is $\rho \simeq 10^{-19} \text{ g/cm}^3$.

PBH with initial mass $m \lesssim m_0 = 5 \times 10^{14} \text{ g}$ cannot survive till our time due to their radiation. For masses larger than 10^{17} g , the signal gets hopelessly small. The estimates for reasonable masses are as follows:

$m, \text{ g}$	$n, \text{ cm}^{-3}$	$\bar{r}, \text{ cm}$	$T, \text{ MeV}$	$N, \text{ ph s}^{-1}$	$\nu, \text{ ph cm}^{-2} \text{ s}^{-1}$
5×10^{14}	2×10^{-34}	1.7×10^{11}	20	6×10^{19}	1.6×10^{-4}
2×10^{15}	5×10^{-35}	2.7×10^{11}	5	1.5×10^{19}	1.6×10^{-5}
10^{16}	10^{-35}	0.5×10^{12}	1	3×10^{18}	10^{-6}
10^{17}	10^{-36}	10^{12}	0.1	3×10^{17}	2×10^{-8}

Table 1: Predictions for radiation of primordial black holes in Solar system

The typical signature of radiating PBH would be 2 — 3 relatively strong lines in the spectrum, for instance, with energies about 5, 10, 15 MeV for PBH with mass 2×10^{15} g.

In particular, the plot below demonstrates that the sensitivity of SPI spectrometer aboard the INTEGRAL satellite is in principle sufficient to observe the line around 5 MeV.

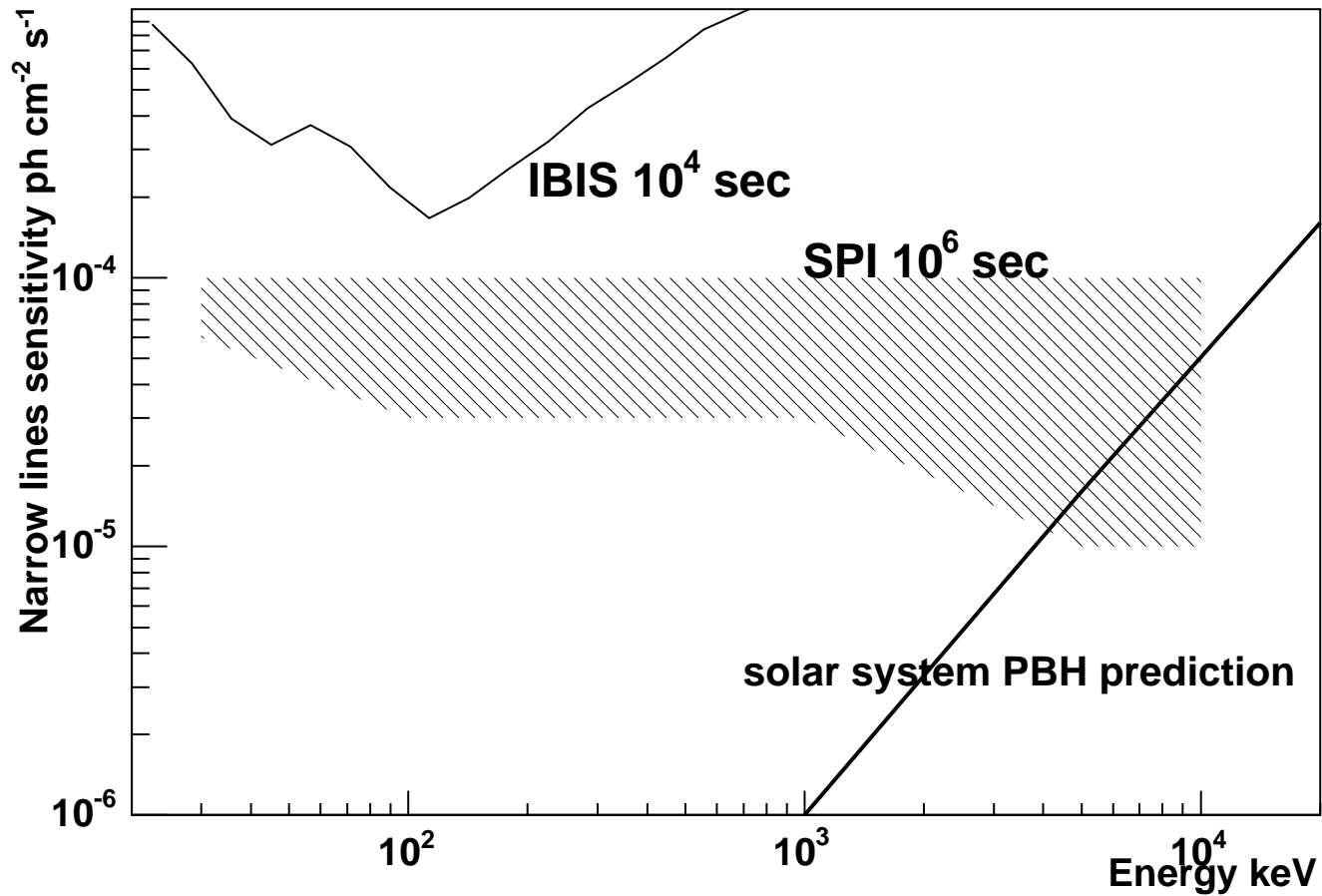


Figure 1: Sensitivity of IBIS imager for narrow line of point-like source exposed for 10⁴ s and of the SPI spectrometer for point-like source exposed for 10⁶ s

It can be demonstrated that our assumption on the density of PBHs in the Solar system does not contradict the best observational upper limits on it.

These limits do not preclude the searches for quantized PBHs in the Solar system.

One should try! (Telegdi)
