Where is Two-photon-exchange effects and Why should you be bothered by them……

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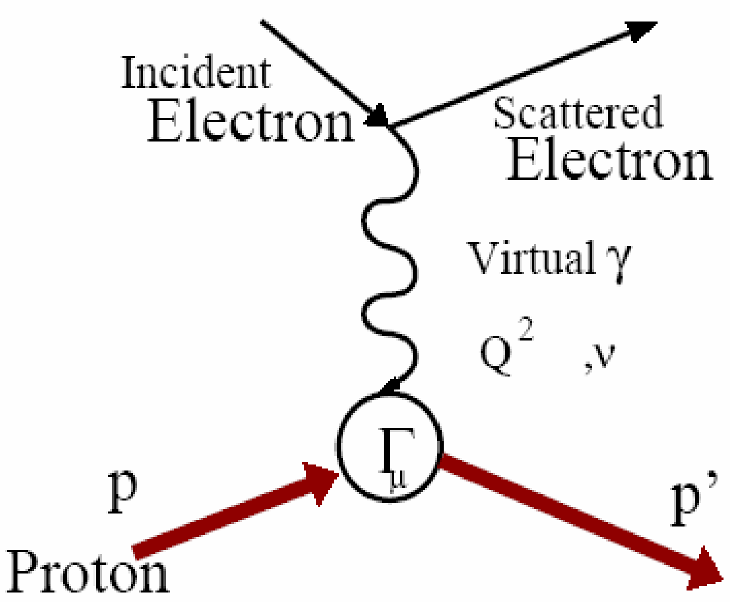
Nucleon Form Factors

- **Hofstadter determined the precise size of the proton and neutron by measuring their form factor.**



$\tau \equiv Q^2/4M^2$

Nucleon vertex:

$$\Gamma_\mu(p',p) = \underbrace{F_1(Q^2)}_{Dirac}\gamma_\mu + \frac{i\kappa_p}{2M_p}\underbrace{F_2(Q^2)}_{Pauli}\sigma_{\mu\nu}q^\nu$$

$$G_E(Q^2) = F_1(Q^2) - \kappa_N \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + \kappa_N F_2(Q^2$$

At $Q^2 = 0$ $\quad G_{Mp} = 2.79 \quad G_{Mn} = -1.91$

$G_{Ep} = 1 \quad G_{En} = 0$

Rosenbluth Separation Method

Within one-photon-exchange framework:

$$\sigma_R(Q^2,\varepsilon) \equiv \frac{d\sigma}{d\Omega_{Lab}} \frac{\varepsilon(1+\tau)}{\tau\sigma_{Mott}} = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

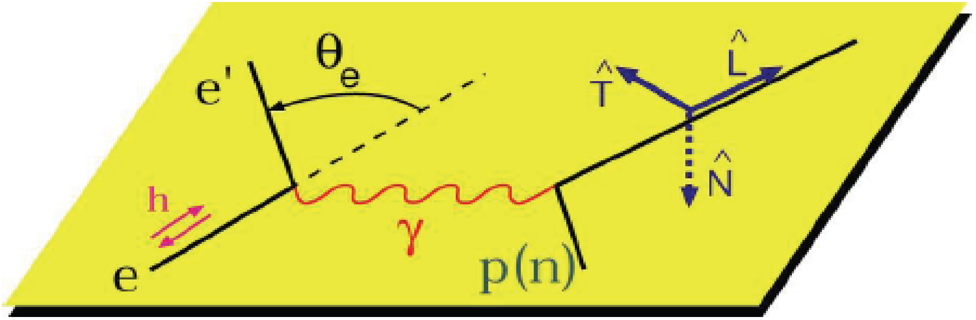
$$1/\varepsilon \equiv 1 + 2(1+\tau)\tan^2\theta_{Lab}/2$$

$$\sigma_{\rm Mott} = \frac{\alpha^2 E_3 \cos^2\frac{\theta}{2}}{4E_1^3 \sin^4\frac{\theta}{2}}$$

$$0 \leq \varepsilon \leq 1 \qquad \tau_N = Q^2/4M_N^2$$

the slope of $\sigma_R(\varepsilon)$ is directly related to G_E and the intercept to G_M

Polarization Transfer Method



$$R = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \cdot \frac{P_t}{P_l},$$

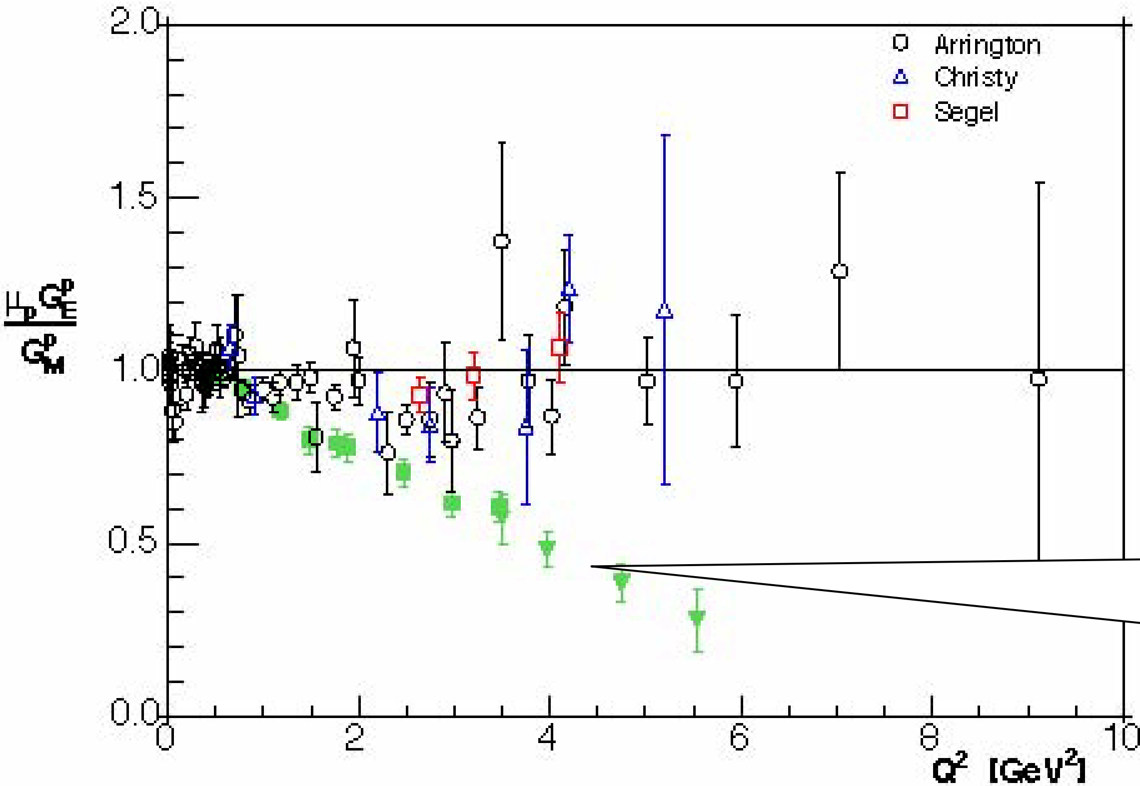
$$R = G_E/G_M$$

P_l is the polarization parallel to its momentum

P_t is the polarization perpendicular to its momentum

Polarization transfer cannot determine the values of G_E and G_M but can determine their ratio R.

Two methods, Two Results!



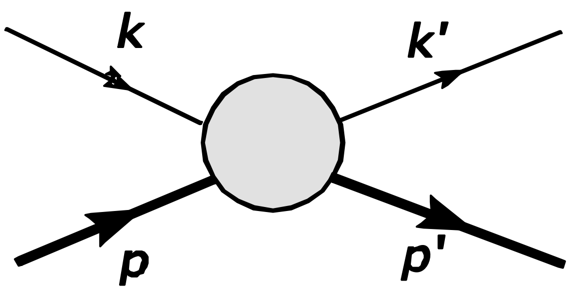
SLAC, JLab
Rosenbluth data

JLab/HallA
Polarization data
Jones et al. (2000)
Gayou et al (2002)

$$\mu_p R = \mu_p G_E^p / G_M^p = 1 - 0.13(Q^2\ [\mathrm{GeV}^2] - 0.04)$$

How to explain it?

Go beyond One-Photon Exchange….



$$K \equiv \frac{1}{2}(k+k')$$

$$P \equiv \frac{1}{2}(p+p')$$

$$q \equiv k - k' = p' - p$$

New Structure

$$T^{non-flip}_{h'\lambda'_N, h\lambda_N} = \frac{e^2}{Q^2}\bar{u}(k',h')\gamma_\mu u(k,h)$$

$$\times \quad \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \boxed{\tilde{F}_3\frac{\gamma.K P^\mu}{M^2}}\right)u(p,\lambda_N)$$

$$\tilde{G}_M(\nu,Q^2) = G_M(Q^2) + \delta\tilde{G}_M$$

$$\tilde{F}_2(\nu,Q^2) = F_2(Q^2) + \delta\tilde{F}_2$$

$$\tilde{F}_3(\nu,Q^2) = 0 + \delta\tilde{F}_3$$

$$\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\,\tilde{F}_2$$

$$\tilde{G}_E(\nu,Q^2) = G_E(Q^2) + \delta\tilde{G}_E$$

Two-Photon-Exchange Effects

on two techniques

$$\sigma_R = G_M^2 \left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \varepsilon \left\{ \frac{1}{\tau} G_E^2 \left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_E}\right) + 2G_M^2 \left(1 + \frac{1}{\tau}\frac{G_E}{G_M}\right) \frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M} \right\} + \mathcal{O}(e^4)$$

large

small

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \left\{ \frac{G_E}{G_M}\left(1 - \frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_M} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M} \right\} + \mathcal{O}(e^4)$$

Possible explanation

- 2-photon-exchange effect can be large on Rosenbluth method when Q^2 is large.
- 2-Photon-exchange effect is much smaller on polarization transfer method.
- Therefore 2-photon-exchange may explain the difference between two results.

Guichon, Vanderhaeghen, PRL 91 (2003)

One way or another…….

- There are two ways to estimate the TPE effect:

Use models to calculate Two-Photon-Exchange diagrams: Like parton model, hadronic model and so on…..

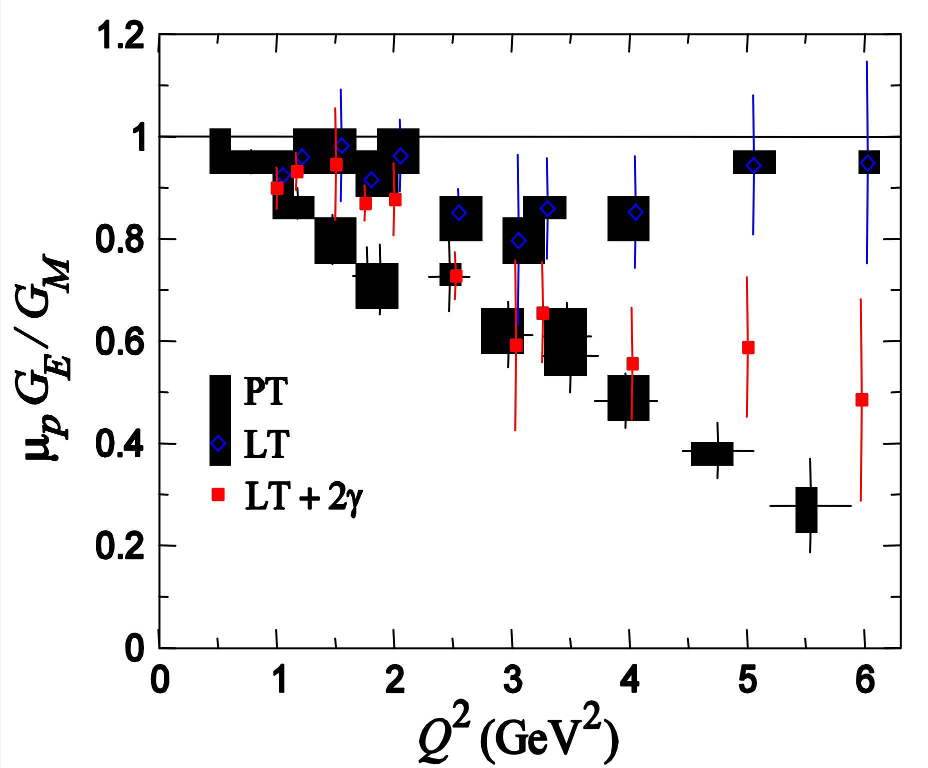
Direct analyze the cross section data by including the TPE effects:

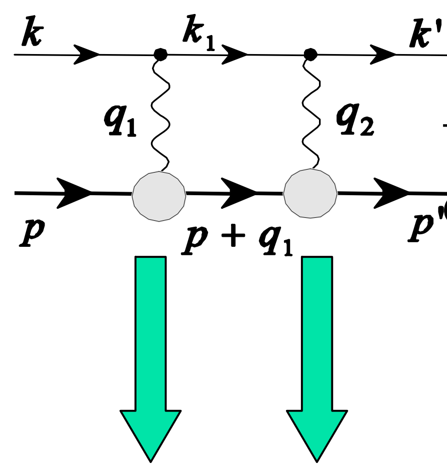
$$\sigma_R(Q^2,\varepsilon) = \boxed{G_M^2(Q^2) + \frac{\varepsilon}{\tau}G_E^2(Q^2)} + \boxed{F(Q^2,\varepsilon)} = \sigma_R^{1\gamma}(1+\delta_{2\gamma})$$

One-Photon-exchange

Two-photon-exchange

Hadronic Model Result





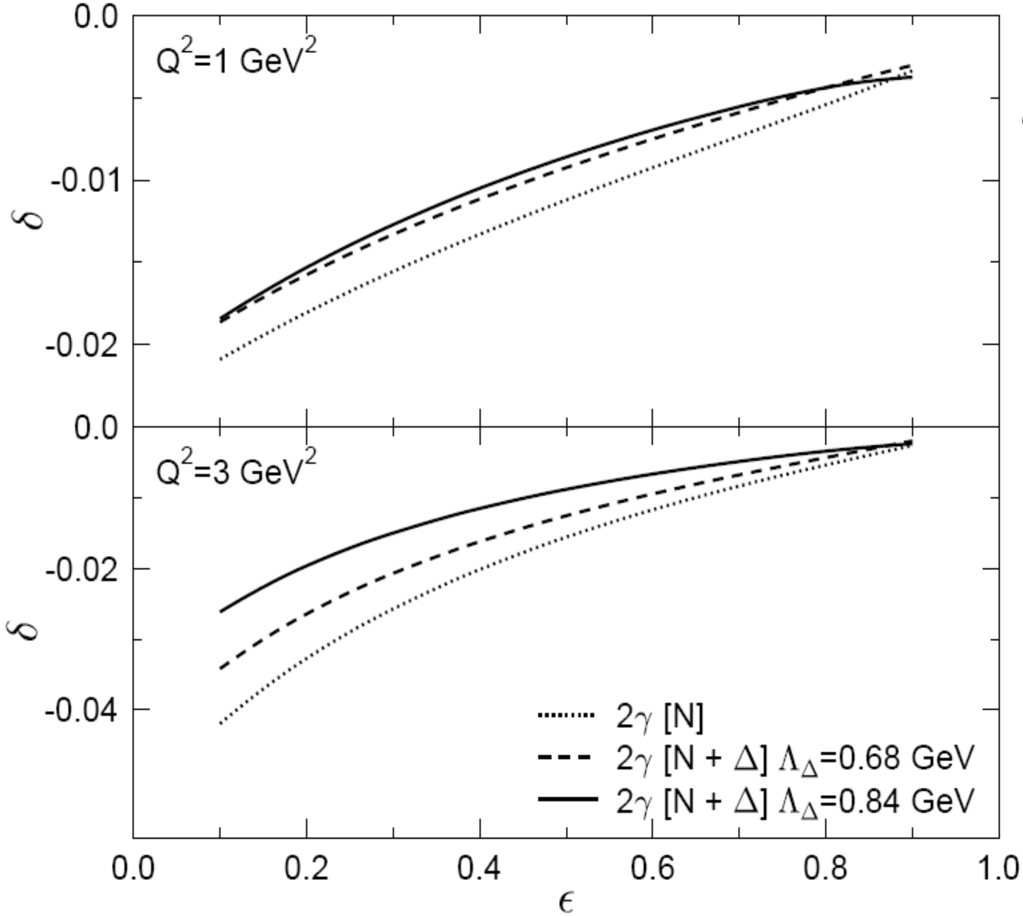
+ Cross diagram

Insert on-shell form factors

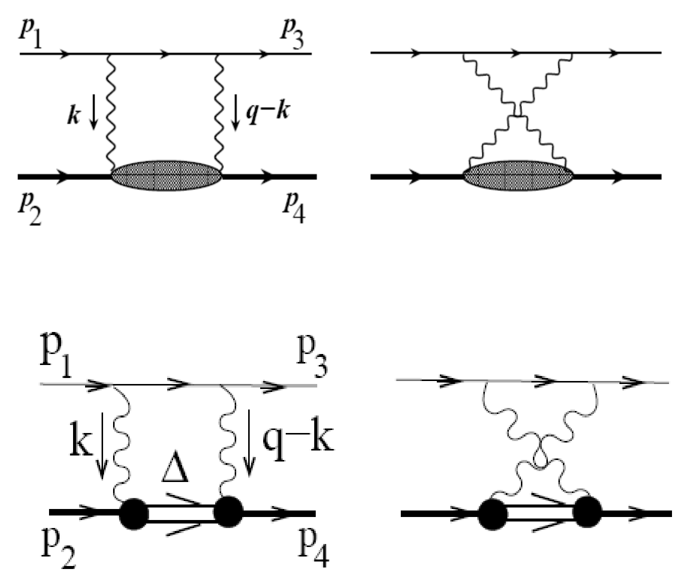
$$1 + \delta = \frac{|\mathcal{M}_0 + \mathcal{M}_1|^2}{|\mathcal{M}_0|^2}$$

Blunden, Tjon, Melnitchouk (2003, 2005)

Results of hadronic model

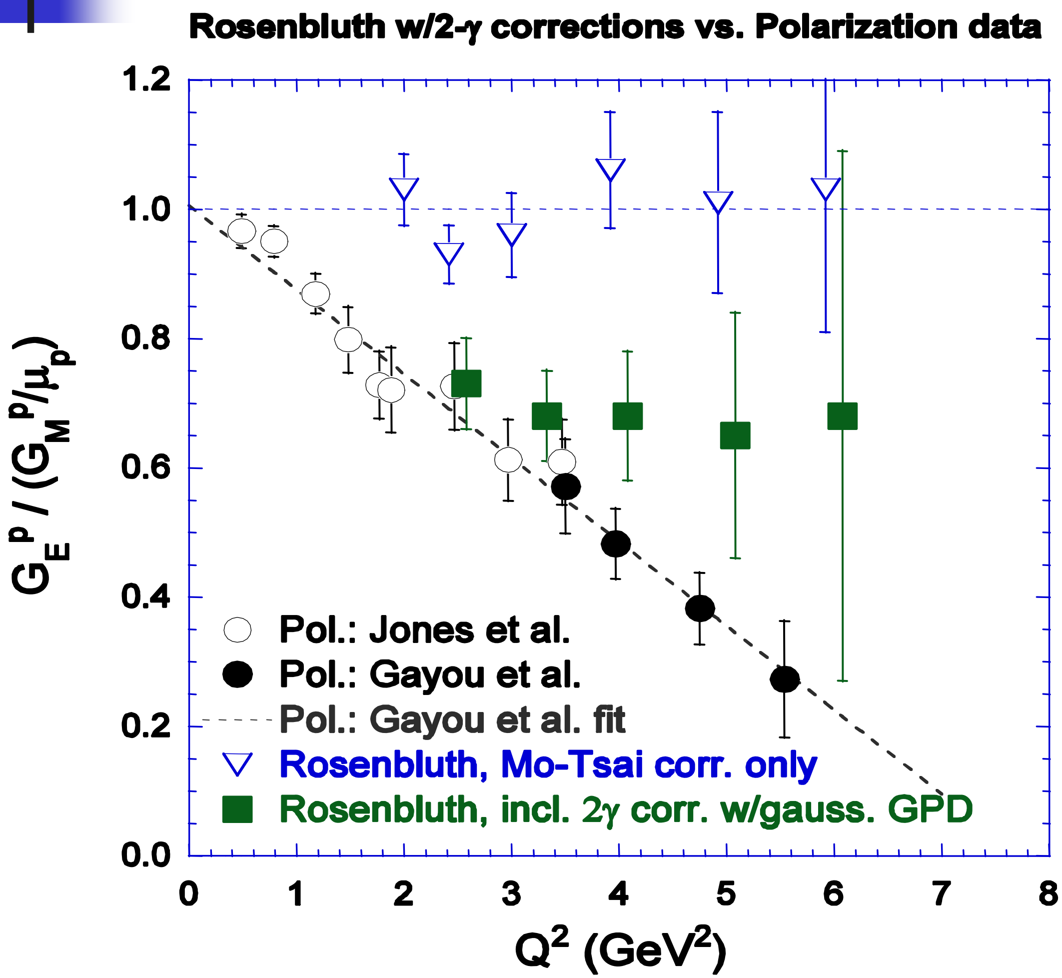


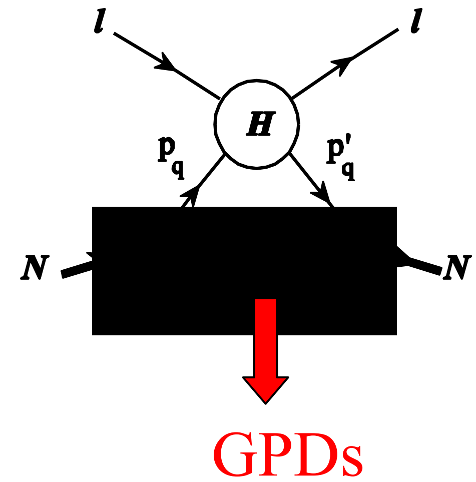
$$d\sigma_R = \left[G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2) \right] (1 + \delta_N + \delta_\Delta)$$



Blunden, Tjon, Melnitchouk (2003, 2005)

Partonic Model Calculation





Y.C.Chen, Afanasev,Brodsky, Carlson, Vanderhaeghen (2004)

Model-independent analysis

$R = G_E/G_M$ Determined from polarization transfer data

$$\sigma_R = G_M^2(Q^2)\left(1 + \frac{\varepsilon}{\tau}R^2\right) + F(Q^2,\varepsilon)$$

Inputs

TPE effects

From crossing symmetry and charge conjugation:

$$F(Q^2, y) = -F(Q^2, -y) \qquad y = \sqrt{\frac{1-\varepsilon}{1+\varepsilon}}$$

Our Choice of $F(Q^2, \varepsilon)$

$\varepsilon \to 1, y \to 0, F \to 0$

$\varepsilon \to 0, y \to 1, F \neq 0$

Fit (A)

$$\sigma_R = G_M^2(Q^2)\left(1 + \frac{\varepsilon}{\tau}R^2\right) + A(Q^2)y + B(Q^2)y^3$$

Fit (B)

$$\sigma_R = G_M^2(Q^2)\left(1 + \frac{\varepsilon}{\tau}R^2\right) + \hat{A}(Q^2)y + \hat{B}(Q^2)y(\ln|y|)^2$$

Fit (A) :

$$A(Q^2) = \alpha G_D^2(Q^2), \quad B(Q^2) = \beta G_D^2(Q^2), \quad G_D = \frac{1}{(1 + Q^2/0.71)^2},$$

$$\alpha = -0.221 \qquad \beta = -0.28$$

Fit (B):

$$\hat{A}(Q^2) = \hat{\alpha} G_D^2(Q^2), \quad \hat{B}(Q^2) = \hat{\beta} G_D^2(Q^2), \quad G_D = \frac{1}{(1 + Q^2/0.71)^2},$$

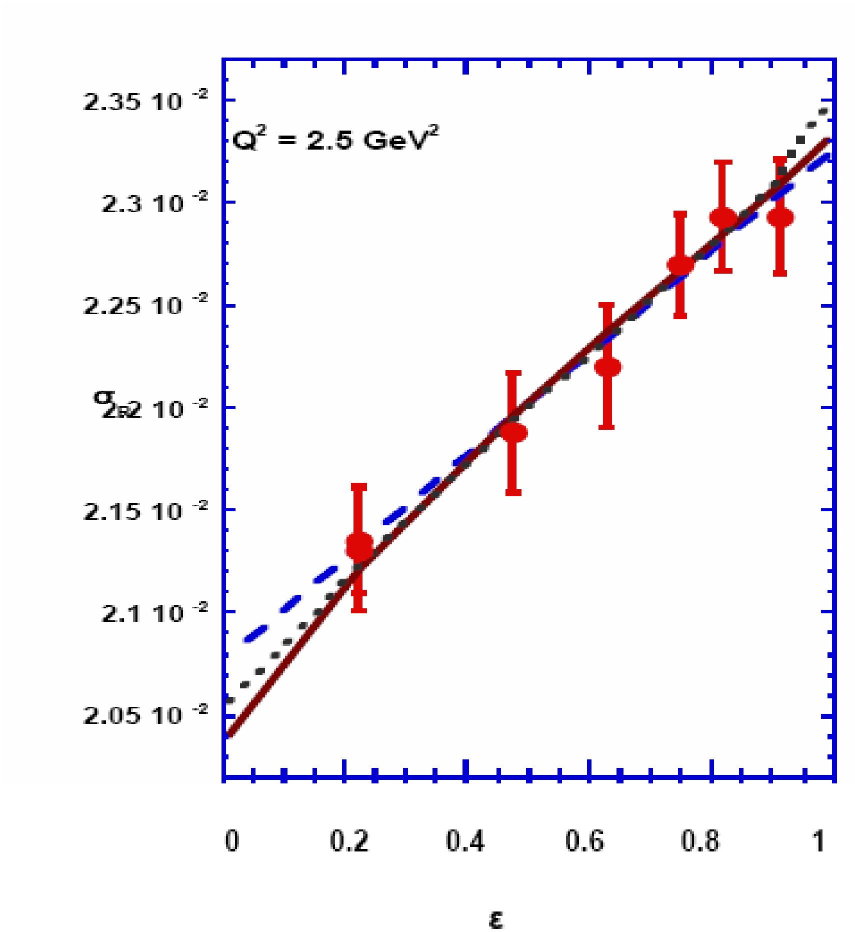
$$\hat{\alpha} = -0.614 \qquad \hat{\beta} = -0.205$$

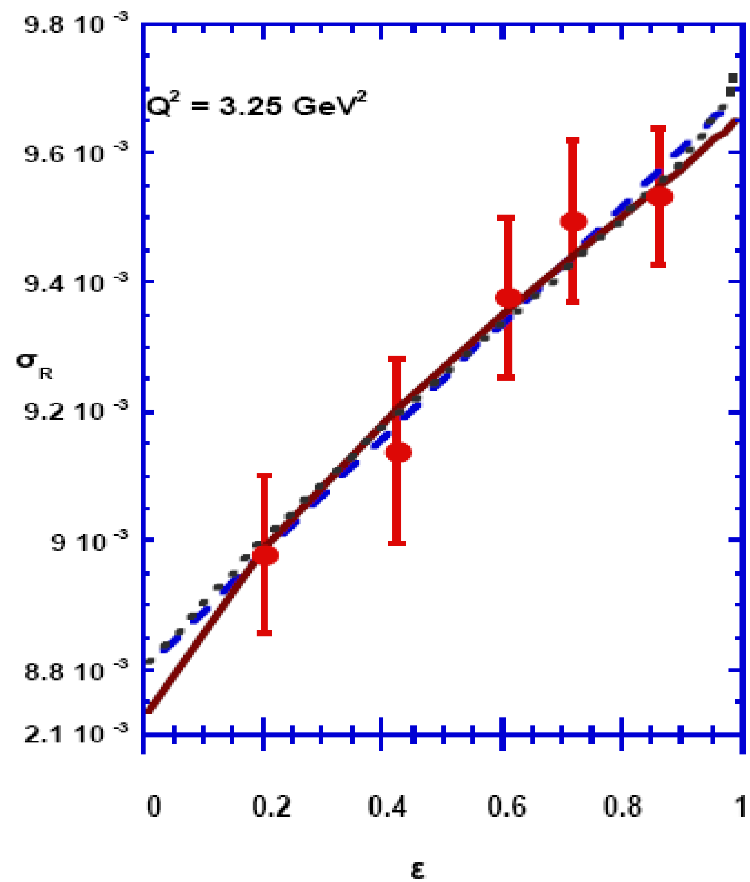
Result of fits

Dashed Line: Rosenbluth

Solid line: Fit (A)

Dotted line: Fit (B)





Result of fits

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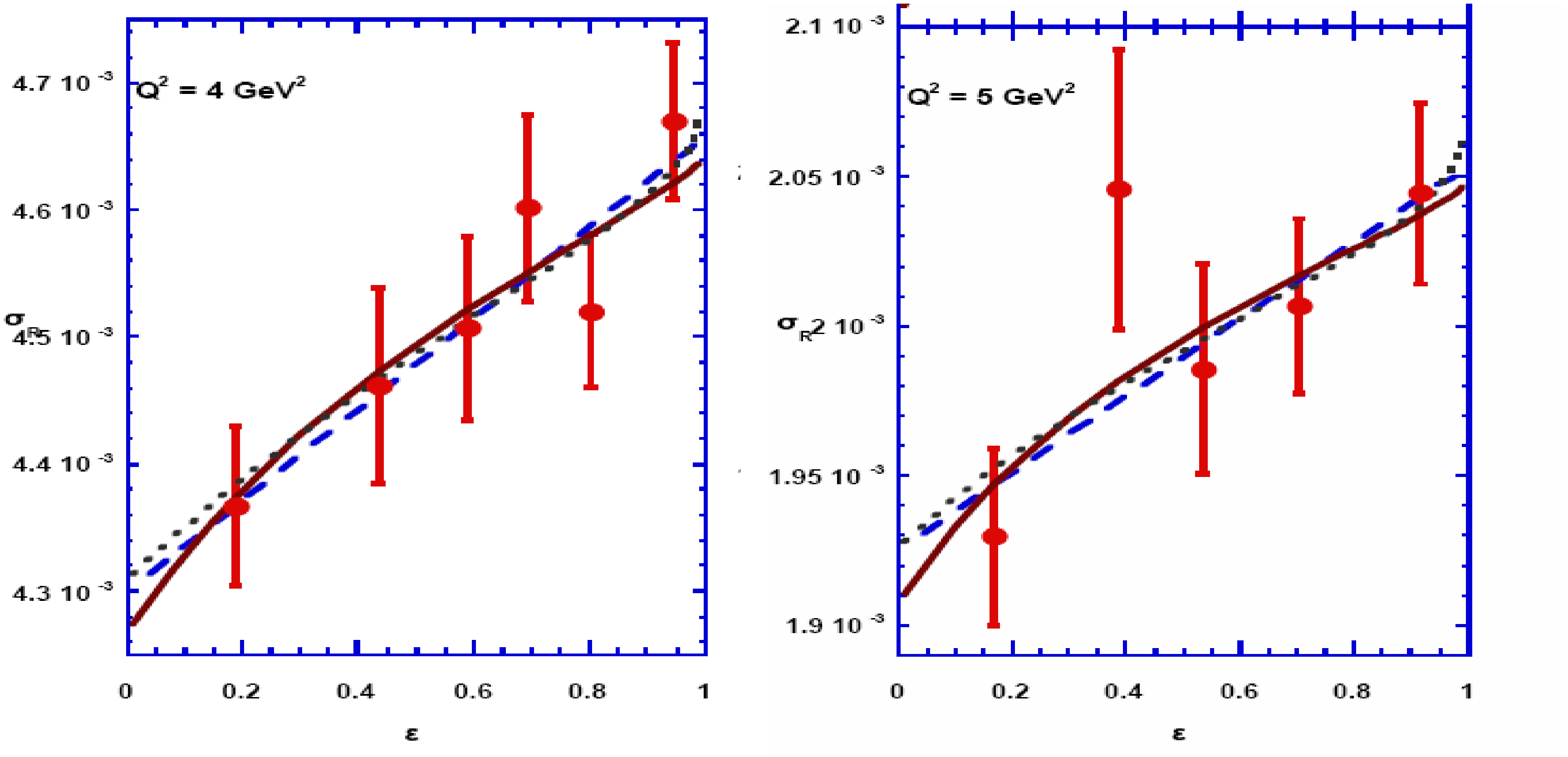
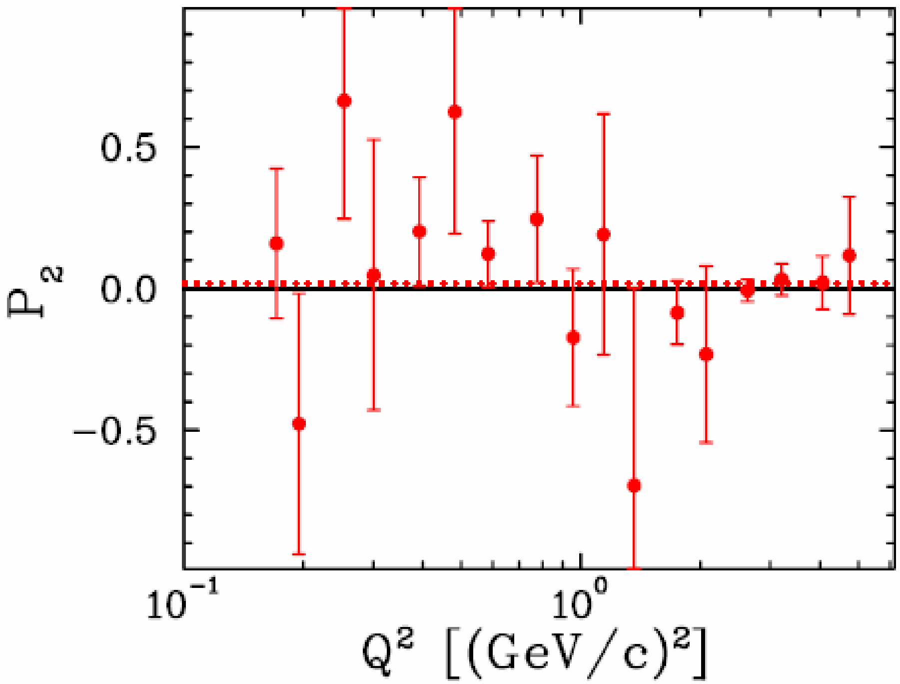


Puzzle about nonlinearity



V.Tvaskis et al, PRC 73, 2005

$$\Delta_{max} = \frac{(\sigma - \sigma_{fit})_{max}}{\sigma} \approx P_2 \cdot (\Delta\varepsilon)^2/8$$

	$\langle P_2 \rangle$	$\mid P_2 \mid_{max}$ 95% C.L.	Δ_{max} 95% C.L.
Elastic	0.019(27)	0.064	$0.8\% \cdot (\Delta\varepsilon)^2$
Resonance	-0.060(42)	0.086	$1.1\% \cdot (\Delta\varepsilon)^2$
DIS	-0.012(71)	0.146	$1.8\% \cdot (\Delta\varepsilon)^2$

Purely due to TPE

$$\sigma_R = P_0 \cdot \left[1 + P_1(\varepsilon - \frac{1}{2}) + P_2(\varepsilon - \frac{1}{2})^2\right]$$

TPE vs OPE

$$\sigma_R(\varepsilon) = \left(G_M^2 + \frac{G_E^2}{2\tau} + C_0\right) + \left(\frac{G_E^2}{\tau} + C_1\right)\left(\varepsilon - \frac{1}{2}\right) + C_2\left(\varepsilon - \frac{1}{2}\right)^2 + \mathcal{O}((\varepsilon - \frac{1}{2})^3)$$

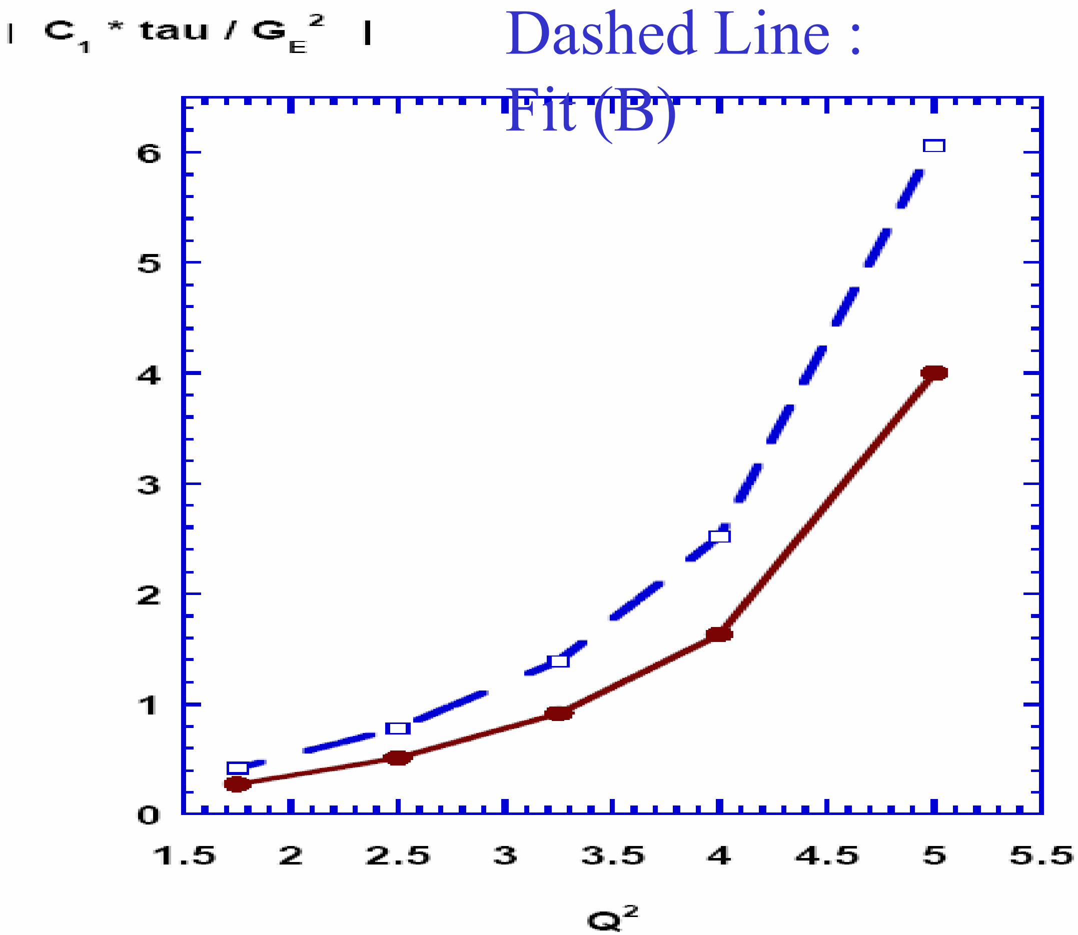
Fit (A) :

$$C_0 = \frac{A + 3B}{\sqrt{3}}, \quad C_1 = \frac{4(A+B)}{3\sqrt{3}}, \quad C_2 = \frac{16B}{9\sqrt{3}},$$

Fit (B):

$$C_0 = \frac{4\hat{A} + \ln^2 3\hat{B}}{4\sqrt{3}}, \quad C_1 = \frac{4\hat{A} + (4\ln 3 - \ln^2 3)\hat{B}}{3\sqrt{3}}, \quad C_2 = \frac{(16 - 8\ln 3)\hat{B}}{9\sqrt{3}}$$

TPE contribution to slope



Dashed Line : Fit (B)

Solid line: Fit (A)

SLOPE(TPE)/SLOPE(OPE) $=C_1/(G_E^2/\tau)$

$$\sigma_R = P_0 \cdot \left[1 + P_1(\varepsilon - \frac{1}{2}) + P_2(\varepsilon - \frac{1}{2})^2\right]$$

Fit (A)

$Q^2[\mathrm{GeV}^2]$	$G_M^2(10^{-3})$	$\frac{1}{\tau}G_E^2(10^{-3})$	$A(10^{-3})$	$B(10^{-3})$	$P_2(\%)$	χ^2	N_{points}
1.75	62.68	9.75	−1.533	-1.943	-3.15	0.110	4
2.50	21.55	1.80	-0.529	−0.670	-3.28	0.177	7
3.25	9.236	0.436	-0.228	-0.289	-3.36	0.095	5
4.00	4.526	0.120	-0.114	−0.145	-3.44	0.366	6
5.00	2.027	0.023	-0.053	-0.068	-3.69	0.582	5

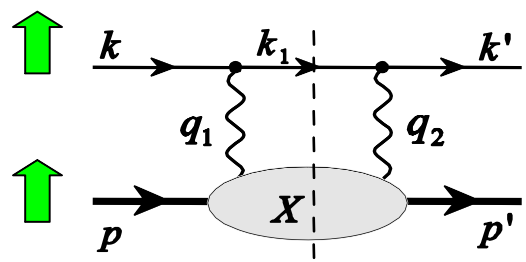
Fit (B)

$Q^2[\mathrm{GeV}^2]$	$G_M^2\ (10^{-3})$	$\frac{1}{\tau}G_E^2(10^{-3})$	$\hat{A}(10^{-3})$	$\hat{B}(10^{-3})$	$P_2(\%)$	χ^2	N_{points}
1.75	63.88	9.982	-4.261	-1.422	-0.99	0.191	4
2.50	22.00	1.841	-1.470	-0.491	-1.03	0.162	7
3.25	9.436	0.446	-0.635	-0.212	-1.06	0.122	5
4.00	4.625	0.123	-0.317	-0.106	-1.09	0.329	6
5.00	2.073	0.024	-0.147	-0.049	-1.14	0.629	5

Common features of Two fits

- G_M increase few percents compared with Rosenbluth results
- G_E are much smaller than Rosenbluth
- Result at high Q^2
- OPE-TPE interference effects are always destructive
- TPE play important role in the slope
- TPE give very small curvature

Any other places for TPE?



Normal spin asymmetries in elastic eN scattering

directly proportional to the imaginary part of 2-photon exchange amplitudes

spin of beam OR target NORMAL to scattering plane

Comparison of e^-p/e^+p :

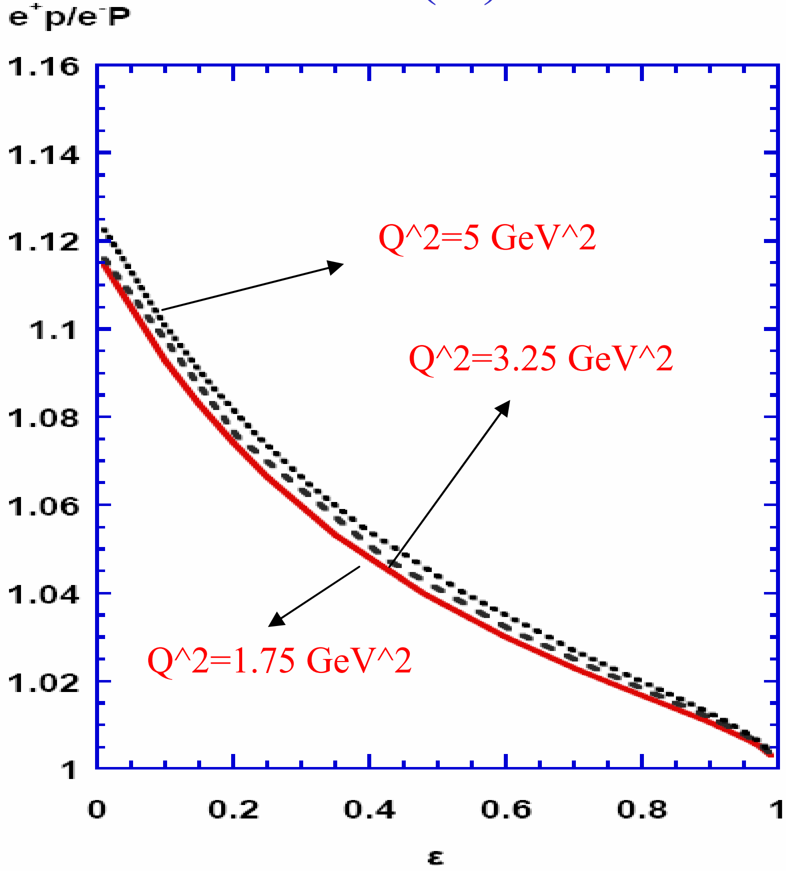
$Amp(e^-p)=Amp(1\gamma)+Amp(2\gamma)$

$Amp(e^+p)=Amp(1\gamma)-Amp(2\gamma)$

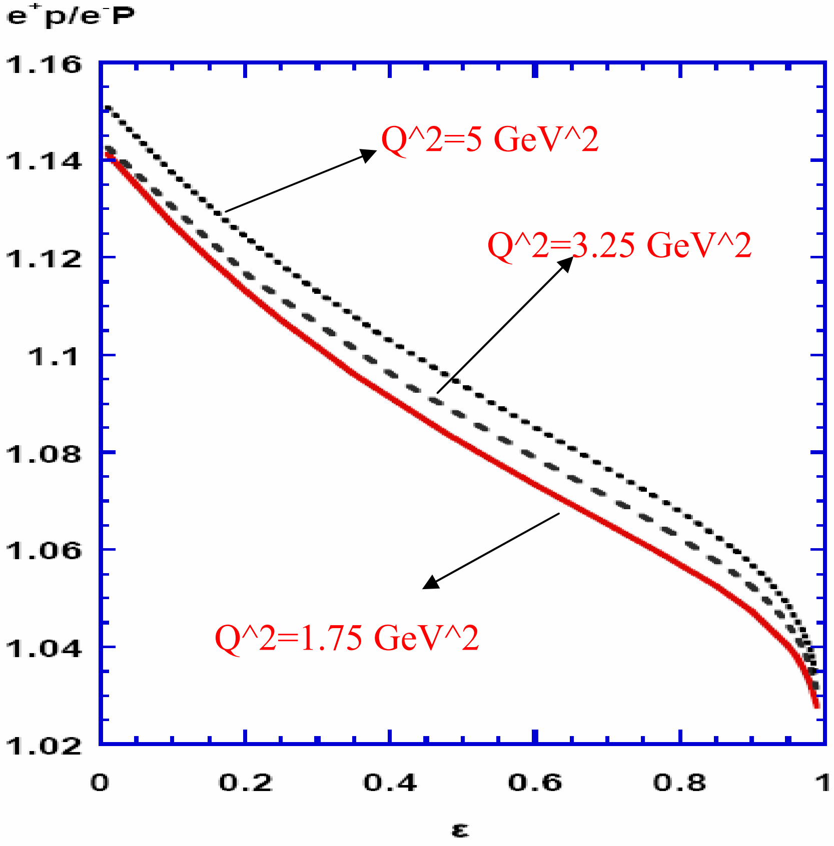
} Due to Charge conjugation

$R=\sigma(e^+p) / \sigma(e^-p)$

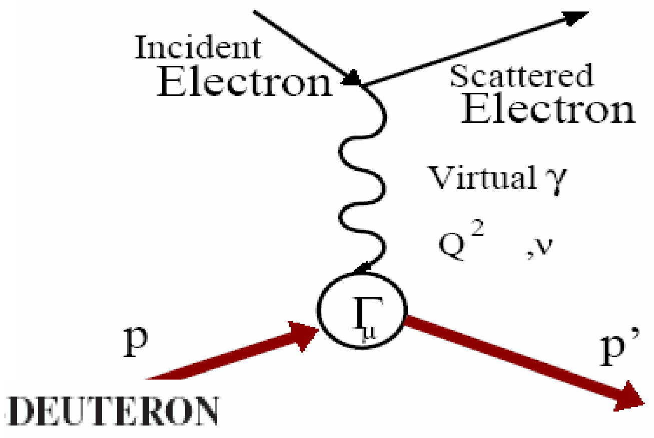
Fit (A)



Fit (B)



Electron-Deuteron elastic scattering



$$K \equiv \frac{1}{2}(k + k')$$
$$P \equiv \frac{1}{2}(p + p')$$
$$q \equiv k - k' = p' - p$$

$$G_M = G_2, \quad G_Q = G_1 - G_2 + (1+\tau)G_3,$$
$$G_C = G_1 + \tfrac{2}{3}\tau G_Q.$$

$$\Gamma_\mu = -e_D \left\{ \left[G_1(Q^2)\xi'^*(\lambda') \cdot \xi(\lambda) - G_3(Q^2)\frac{(\xi'^*(\lambda') \cdot q)(\xi(\lambda) \cdot q)}{2M_D^2} \right] \cdot P_\mu + G_2(Q^2)\left[\xi_\mu(\lambda)(\xi'^*(\lambda') \cdot q) - \xi'^*_\mu(\lambda')(\xi(\lambda) \cdot q)\right] \right\}$$

Electron-Deuteron elastic scattering

Within One-Photon-Exchange Framework:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}|_{Mott} I_0(OPE) \qquad I_0(OPE) = [A + B tan^2\theta/2]$$

$$A = G_c^2 + \frac{2}{3}\tau G_M^2 + \frac{8}{9}\tau^2 G_Q^2 \qquad B = \frac{4}{3}\tau(1+\tau)G_M^2$$

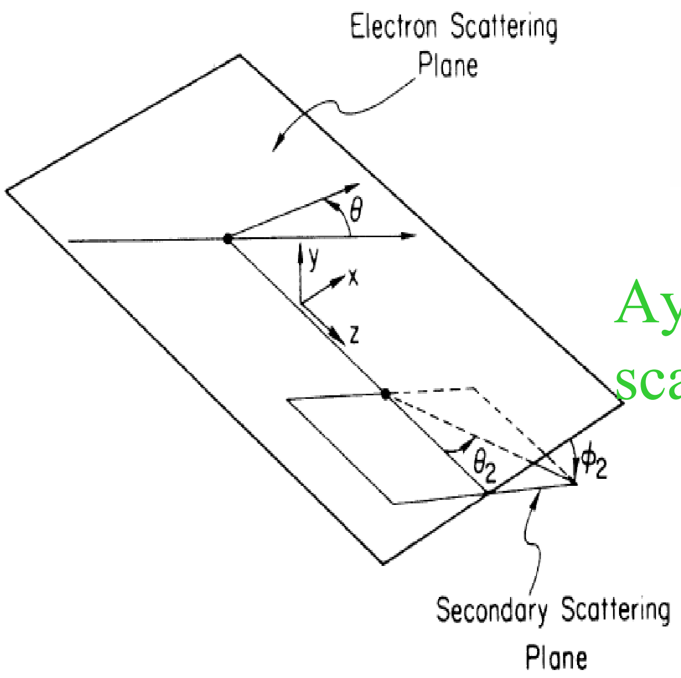
$$G_c(0) = 1, \qquad G_Q(0) = M^2 Q_d = 25.83, \qquad G_M(0) = 1.714.$$

To extract three form factors, one needs data of other observables.

Polarization transfer

a=±1/2, polarization of incoming elect

$$\frac{d^2\sigma}{d\Omega d\Omega_2} = \frac{d^2\sigma}{d\Omega d\Omega_2}\bigg|_0 \left\{1 + \tfrac{3}{2} a p_x A_y \sin\phi_2 + \tfrac{1}{2} p_{zz} A_{zz} + \tfrac{2}{3} p_{xz} A_{xz} \cos\phi_2 + \tfrac{1}{6}(p_{xx} - p_{yy})(A_{xx} - A_{yy}) \cos 2\phi_2 \right\}$$



Ay: Vector analyzing power of the secondary scattering

Azz, Axz, Axx-Ayy: Tensor polarization of the second scattering

Px, Pzz, Pxz and Pxx-Pyy are functions of form factors Gc,Gm and Gq.

Polarized deuteron target

$$\frac{\sigma}{\sigma_0} = 1 + (P_{zz}/\sqrt{2})\left(\frac{3\cos^2\theta^* - 1}{2}T_{20} - \sqrt{\frac{3}{2}}\sin 2\theta^* \cos\phi^* T_{21} + \sqrt{\frac{3}{2}}\sin^2\theta^* \cos 2\phi^* T_{22}\right)$$

$$P_{zz} = \sqrt{2}T_{20}, \quad P_{xz} = -\sqrt{3}Re(T_{21}), \quad (P_{xx} - P_{yy}) = 2\sqrt{3}Re(T_{22}),$$

$$P_z = -\sqrt{\tfrac{2}{3}}T_{10}, \quad P_y = -\frac{2\sqrt{3}}{3}Im(T_{11}), \quad P_x = -\frac{2\sqrt{3}}{3}Re(T_{11}).$$

Py=0 within One-Photon-Exchange framework.

$P_{zz}=n_{+}+n_{-}-2n_{0}$: degree of polarization of the target deuteron

Polarization observables of e-D scattering

Within One-Photon-Exchange framework

$$-I_0 P_{zz} = \frac{8}{3}\tau(G_C G_Q) + \frac{8}{9}\tau^2 G_Q^2 + \frac{1}{3}\tau\left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right] G_M^2$$

$$I_0 P_{xz} = -\tau \frac{K_0}{M_D} \tan\frac{\theta}{2} G_M G_Q$$

$$I_0 (P_{xx} - P_{yy}) = -\tau G_M^2$$

$$I_0 P_z = \frac{1}{3}\frac{K_0}{M_D}\sqrt{\tau(\tau+1)}\tan^2\frac{\theta}{2} G_M^2$$

$$K_0^2 = 4M_D^2\tau\left[(1+\tau) + \cot^2\frac{\theta}{2}\right]$$

$$I_0 P_x = -\frac{4}{3}\sqrt{\tau(\tau+1)}\tan\frac{\theta}{2} G_M \left(G_c + \frac{1}{3}\tau G_Q\right)$$

Constraints between observables

$$\mathcal{C}_1 = I_0(1+2P_{zz}) = G_C^2 - \frac{16}{3}\tau G_C G_Q - \frac{8}{9}\tau^2 G_Q^2,$$

$$\mathcal{C}_2 = \frac{(I_0 P_{xz})(I_0 P_x)}{I_0 P_z} = 4\tau G_Q \left(G_C + \frac{\tau}{3} G_Q\right).$$

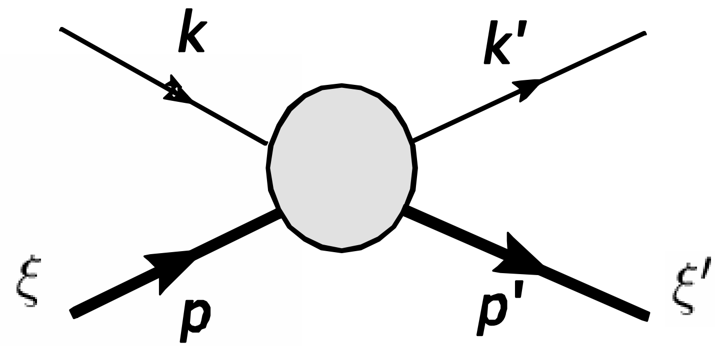
$$\mathcal{C}_1 + \tfrac{4}{3}\mathcal{C}_2 = G_c^2 + \tfrac{8}{9}\tau^2 G_Q^2 > 0.$$

Those combinations are independent of θ when all observables are *Θ*-dependent.
It is easy to use the above combinations to test under which kinematic conditions TPE become important.

Amplitudes of e-D scattering (beyond OPE)

$$\mathcal{M}^{eD} = -e^2 \bar{u}(p_3, s_3)\gamma_\mu u(p_1, s_1)\frac{1}{q^2}\sum_{i=1}^{6} \tilde{G}_i M_i^\mu$$

$$\begin{aligned}
M_1^\mu &= (\xi'^* \cdot \xi) P^\mu, \\
M_2^\mu &= \left[\xi^\mu (\xi'^* \cdot q) - (\xi \cdot q)\xi'^{*\mu}\right], \\
M_3^\mu &= -\frac{1}{2M_D^2}(\xi \cdot q)(\xi'^* \cdot q) P^\mu, \\
M_4^\mu &= \frac{1}{2M_D^2}(\xi \cdot K)(\xi'^* \cdot K) P^\mu, \\
M_5^\mu &= \left[\xi^\mu (\xi'^* \cdot K) + (\xi \cdot K)\xi'^{*\mu}\right], \\
M_6^\mu &= \frac{1}{2M_D^2}\left[(\xi \cdot q)(\xi'^* \cdot K) - (\xi \cdot K)(\xi'^* \cdot q)\right] P^\mu
\end{aligned}$$



$$K \equiv \frac{1}{2}(k + k')$$

$$P \equiv \frac{1}{2}(p + p')$$

$$q \equiv k - k' = p' - p$$

Cross section in term of $G^{(2)}_{1\text{-}6}$

$$\frac{d\sigma}{d\Omega} = \sigma_0\left\{\left[(A+\Delta A)\cot^2\frac{\theta}{2} + (B+\Delta B)\right] + \Delta\sigma(\theta,Q^2)\cot^2\frac{\theta}{2}\right\}$$

$$\begin{aligned}\Delta A &= 2\Big[G_c Re(G_C^{(2)*}) + \frac{2}{3}\tau G_M Re(G_M^{(2)*}) + \frac{8}{9}\tau^2 G_Q Re(G_Q^{(2)*})\Big] \\ &\quad + \frac{4\tau^2}{3}\Big[(2\tau+1)G_1 - 2(\tau+1)G_2 + 2\tau(\tau+1)G_3\Big]Re(G_4^{(2)*})\end{aligned}$$

$$\Delta B = \frac{8}{3}\tau(1+\tau)G_M Re(G_M^{(2)*})$$

$$\begin{aligned}\Delta\sigma(\theta,Q^2) &= \frac{2}{3}\Big\{2\tau\cot^2\frac{\theta}{2}\Big[(2\tau-1)G_1 - 2\tau G_2 + 2\tau^2 G_3\Big]Re(G_4^{(2)*}) \\ &\quad + \frac{K_0}{M_D}\Big[\Big((2\tau-1)G_1 - 2\tau G_2 + 2\tau^2 G_3 - 2\tau\tan^2\frac{\theta}{2}G_2\Big)Re(G_5^{(2)*}) \\ &\quad + 2\tau\Big((2\tau+1)G_1 - (2\tau+1)G_2 + 2\tau(\tau+1)G_3\Big)Re(G_6^{(2)*})\Big]\Big\},\end{aligned}$$

Y-B. Dong, C. -W. Kao, S.-N. Yang, Y.-C. Chen, 2006

Polarization observables in term of G $^{(2)}{}_{1\text{-}6}$

$$-I_0 P_{zz} = \frac{8}{3}\tau(G_C G_Q) + \frac{8}{9}\tau^2 G_Q^2 + \frac{1}{3}\tau\left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]G_M^2 + \Delta P_{zz}$$

$$\Delta P_{zz} = \delta P_{zz} + \delta_0 P_{zz}$$

$$\begin{aligned}
\delta P_{zz} = & \frac{4\tau^2}{3}\left[2(2\tau+1)G_1 - (4\tau+1)G_2 + 4\tau(\tau+1)G_3\right]Re(G_4^{(2)*}) \\
& + \frac{4}{3}\tau\cot^2\frac{\theta}{2}\left[\frac{4\tau^2+2\tau+1}{\tau+1}G_1 - \frac{\tau(4\tau+1)}{\tau+1}G_2 + 4\tau^2 G_3\right]Re(G_4^{(2)*}) \\
& + \frac{2K_0}{3M}\left[\left(\frac{4\tau^2+2\tau+1}{\tau+1}G_1 - \frac{3\tau(2\tau+1)}{2(\tau+1)}G_2 + 4\tau^2 G_3 + 2\tau^2\tan^2\frac{\theta}{2}G_2\right)Re(G_5^{(2)*})\right. \\
& \left. + \tau\left(4(2\tau+1)G_1 - (8\tau+1)G_2 + 8\tau(\tau+1)G_3\right)Re(G_6^{(2)*})\right]
\end{aligned}$$

$$\begin{aligned}
\delta_0 P_{zz} = & \frac{8}{3}\tau\left[G_C Re(G_Q^{(2)*}) + G_Q Re(G_C^{(2)*})\right] \\
& + \frac{16}{9}\tau^2 G_Q Re(G_Q^{(2)*}) + \frac{2}{3}\tau\left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]G_M Re(G_M^{(2)*})
\end{aligned}$$

Small θ Limit

$$\Delta\sigma = \frac{1}{3}\Big\{4\tau\cot^2\frac{\theta}{2}\Big[(2\tau-1)G_1 - 2\tau G_2 + 2\tau^2 G_3\Big] Re(G_4^{(2)*})$$

$$+4\sqrt{\tau}\cot\frac{\theta}{2}\Big[\Big((2\tau-1)G_1 - 2\tau G_2 + 2\tau^2 G_3\Big) Re(G_5^{(2)*})$$

$$+2\tau\Big((2\tau+1)G_1 - (2\tau+1)G_2 + 2\tau(\tau+1)G_3\Big) Re(G_6^{(2)*})\Big]\Big\}$$

$$\Delta P_{zz} \sim \frac{4}{3}\tau\cot^2\frac{\theta}{2}\Big[\frac{4\tau^2+2\tau+1}{\tau+1}G_1 - \frac{\tau(4\tau+1)}{\tau+1}G_2 + 4\tau^2 G_3\Big] Re(G_4^{(2)*})$$

$$+\frac{4}{3}\sqrt{\tau}\cot\frac{\theta}{2}\Big[\Big(\frac{4\tau^2+2\tau+1}{\tau+1}G_1 - \frac{3\tau(2\tau+1)}{2(\tau+1)}G_2 + 4\tau^2 G_3\Big) Re(G_5^{(2)*})$$

$$+\tau\Big(4(2\tau+1)G_1 - (8\tau+1)G_2 + 8\tau(\tau+1)G_3\Big) Re(G_6^{(2)*})\Big]$$

$$\Delta P_{xz} \sim \tau\Big\{\Big[2\sqrt{\tau}\cot^2\frac{\theta}{2}\Big(\frac{2\tau}{\tau+1}G_1 - \frac{3\tau+1}{\tau+1}G_2 + 2\tau G_3\Big)\Big] Re(G_4^{(2)*})$$

$$+2\tau\cot\frac{\theta}{2}\Big[\Big(\frac{1}{\tau+1}G_1 + 2G_3\Big) Re(G_5^{(2)*})$$

$$+\Big(G_1 - 4G_2 + 2(\tau+1)G_3\Big) Re(G_6^{(2)*})\Big]\Big\},$$

Small θ Limit (continued)

$$\Delta(P_{xx} - P_{yy}) \sim \frac{4\tau}{\tau+1}\cot^2\frac{\theta}{2}\,(G_1 + \tau G_2)\,Re(G_4^{(2)*}) + \frac{4\sqrt{\tau}}{\tau+1}\cot\frac{\theta}{2}\Big[(G_1+\tau G_2)\,Re(G_5^{(2)*}) + \tau(\tau+1)G_2 Re(G_6^{(2)*})\Big]$$

$$\Delta P_z \sim -\frac{2\tau}{3}\sqrt{\frac{\tau}{\tau+1}}\Big[2\sqrt{\tau}\cot\frac{\theta}{2}Re(G_4^{(2)*}) + 3Re(G_5^{(2)*}) + 2(\tau+1)Re(G_6^{(2)*})\Big]G_2.$$

$$\Delta P_x \sim \frac{2\tau\sqrt{\tau+1}}{3}\Big[\Big(2G_1 - \frac{4\tau+1}{\tau+1}G_2 + 2\tau G_3\Big)Re(G_5^{(2)*}) - 4\tau G_2 Re(G_6^{(2)*})\Big] - \frac{4}{3}\frac{\tau^2\sqrt{\tau}}{\sqrt{\tau+1}}\cot\frac{\theta}{2}G_2 Re(G_4^{(2)*}).$$

$$Re\,G_4^{(2)}(\theta,Q^2) \le \theta^2,\quad Re\,G_5^{(2)}(\theta,Q^2) \le \theta,\quad Re\,G_6^{(2)}(\theta,Q^2) \le \theta \text{ when } \theta \to 0.$$

TPE on Px and Pz vanish at small angels but
TPE on other obsevables survive.

Large θ Limit

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta\to\pi} \sim \sigma_0(B + \Delta B')$$

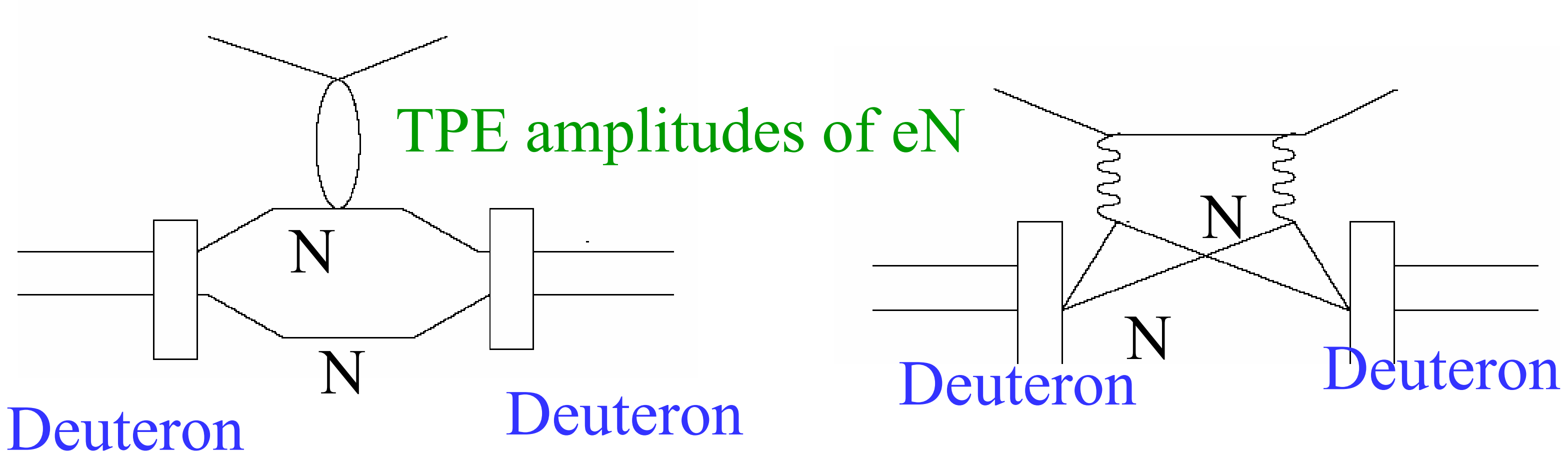
$$\Delta B' = -\frac{8}{3}\tau\sqrt{\tau(1+\tau)}G_2(Q^2)Re(G_5^{(2)*}) + \frac{8}{3}\tau(1+\tau)G_2Re(G_2^{(2)*})$$

$$\Delta P_{zz} \sim \frac{8}{3}\tau^2\sqrt{\tau(\tau+1)}\tan^2\frac{\theta}{2}G_2Re(G_5^{(2)*}) + \frac{4}{3}\tau(\tau+1)\tan^2\frac{\theta}{2}G_2Re(G_2^{(2)*}),$$

$$\Delta P_z \sim \frac{4}{3}\sqrt{\tau(\tau+1)}\tan^2\frac{\theta}{2}\left[\sqrt{\tau(\tau+1)}Re(G_M^{(2)*}) - Re(G_5^{(2)*})\right]G_2.$$

TPE effects on other observables vanish when **Θ** approaches π/2

Calculation of TPE amplitude



In progress,
collaboration with H-Q Zhou, S.N Yang (NTU),
and Y.B. Dong (CAS)

Summary and Outlook

- The different results of Rosenbluth separation and Polarization transfer method stimulate a lot of theoretical and experimental research of TPE.
- Calculation based on models show TPE is important for extraction of form factors.
- Our model-independent analysis shows that the more precise data at lower ε is crucial for the extraction of TPE and nucleon form factors.
- TPE effects of e-D scattering is under investigation.
- TPE effects of e-D is more complicated.
- More TPE-related research is going: $N \rightarrow \Delta$ transition form factor, normal beam asymmetry and so on…