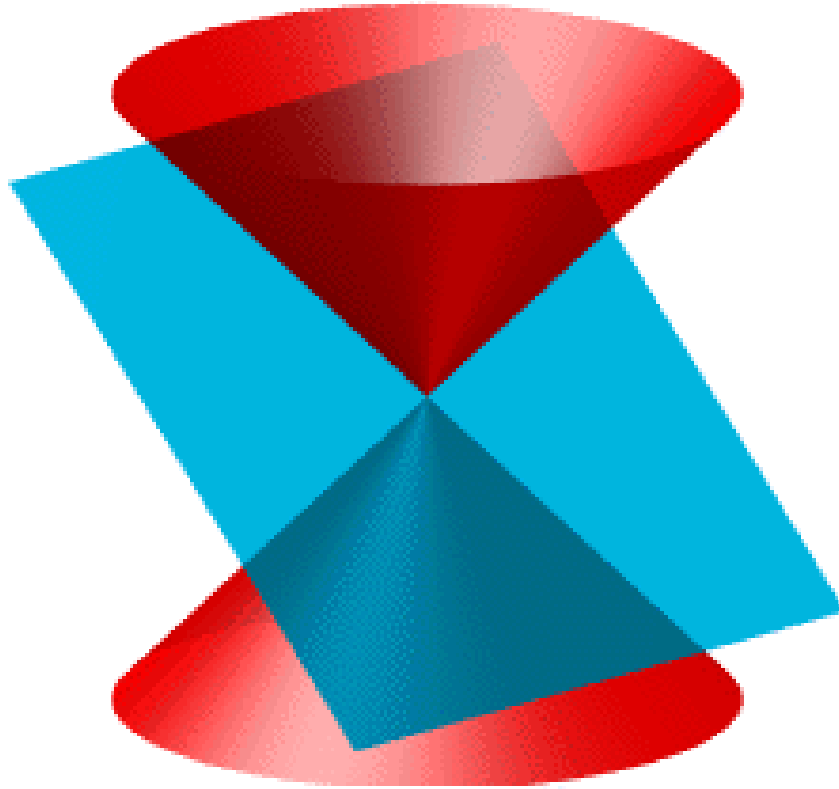


# Semileptonic Decay Processes in Light-Front Dynamics



INT Seattle, May 4, 2007

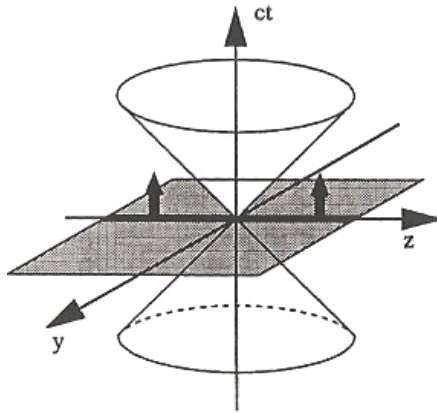
# Motivation

- Precision test of standard model seems promising.
    - Unitarity of CKM mixing matrix (CP-violation)
    - B Physics (Babar,Belle,BTeV,LHCB,...)
    - UCN Collaborations (... ,NC State,...)
    - Demand on finding hadron wavefunction (QCD)
  - LFD has progressed for last several years.
    - Distinguished Features (Vacuum,Symmetry)
    - Treacherous Points (Zero-Mode,Arc-Contribution)
    - Applications to Phenomenology (JLab,RHIC,...)
- Time to review progress and scrutinize  
phenomenological model building based on QCD...

# Outline

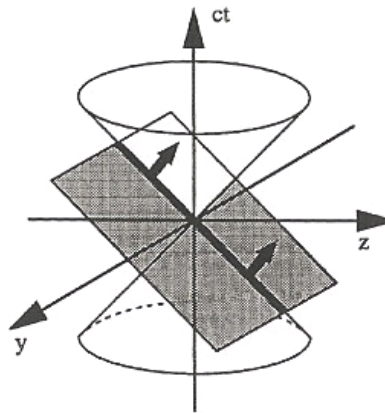
- Why LFD?
  - Distinguished Features in LFD
  - Application to Hadron Phenomenology
- Treacherous Points in Semileptonic Processes
  - Non-Valence Contribution
  - Zero-Modes
- Power Counting Method
  - Correct Assessment of Zero-Modes
  - Scrutinization of LFQM
- Conclusions

# Distinguished Features in LFD



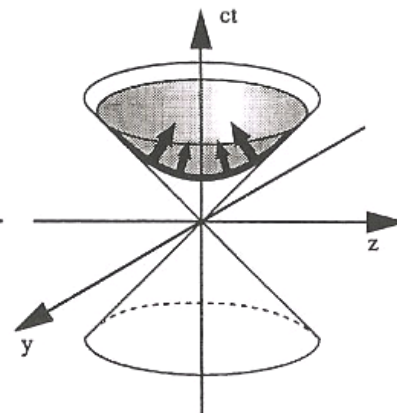
The instant form

$$\begin{aligned}\bar{x}^0 &= ct \\ \bar{x}^1 &= x \\ \bar{x}^2 &= y \\ \bar{x}^3 &= z\end{aligned}$$



The front form

$$\begin{aligned}\bar{x}^0 &= ct+z \\ \bar{x}^1 &= x \\ \bar{x}^2 &= y \\ \bar{x}^3 &= ct-z\end{aligned}$$



The point form

$$\begin{aligned}\bar{x}^0 &= \tau, & ct &= \tau \cosh \omega \\ \bar{x}^1 &= \omega, & x &= \tau \sinh \omega \sin \theta \cos \phi \\ \bar{x}^2 &= \theta, & y &= \tau \sinh \omega \sin \theta \sin \phi \\ \bar{x}^3 &= \phi, & z &= \tau \sinh \omega \cos \theta\end{aligned}$$

LFD is like sweeping dirt to a corner:

Simple Vacuum except Zero-Modes, Maximum Number of Kinematic Generators



*Equal t*

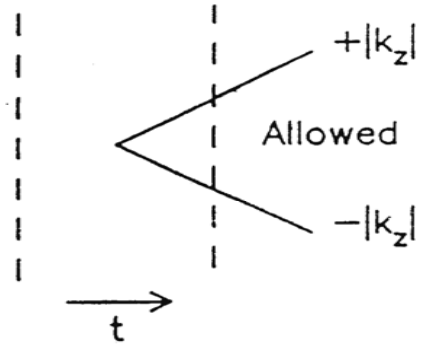
*Equal  $\tau$*

$$\begin{aligned} p^0 &\leftrightarrow p^- = p^0 - p^3 \\ (p^1, p^2) &\leftrightarrow \vec{p}_\perp \\ p^3 &\leftrightarrow p^+ = p^0 + p^3 \end{aligned}$$

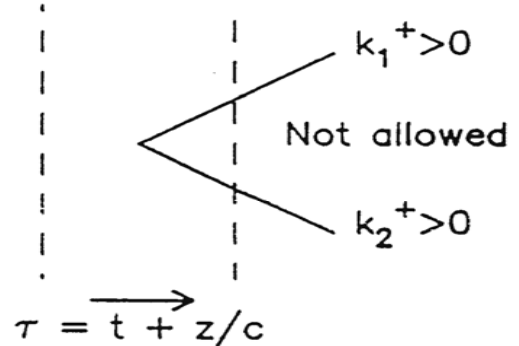
### Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^2 + m^2}$$

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$$



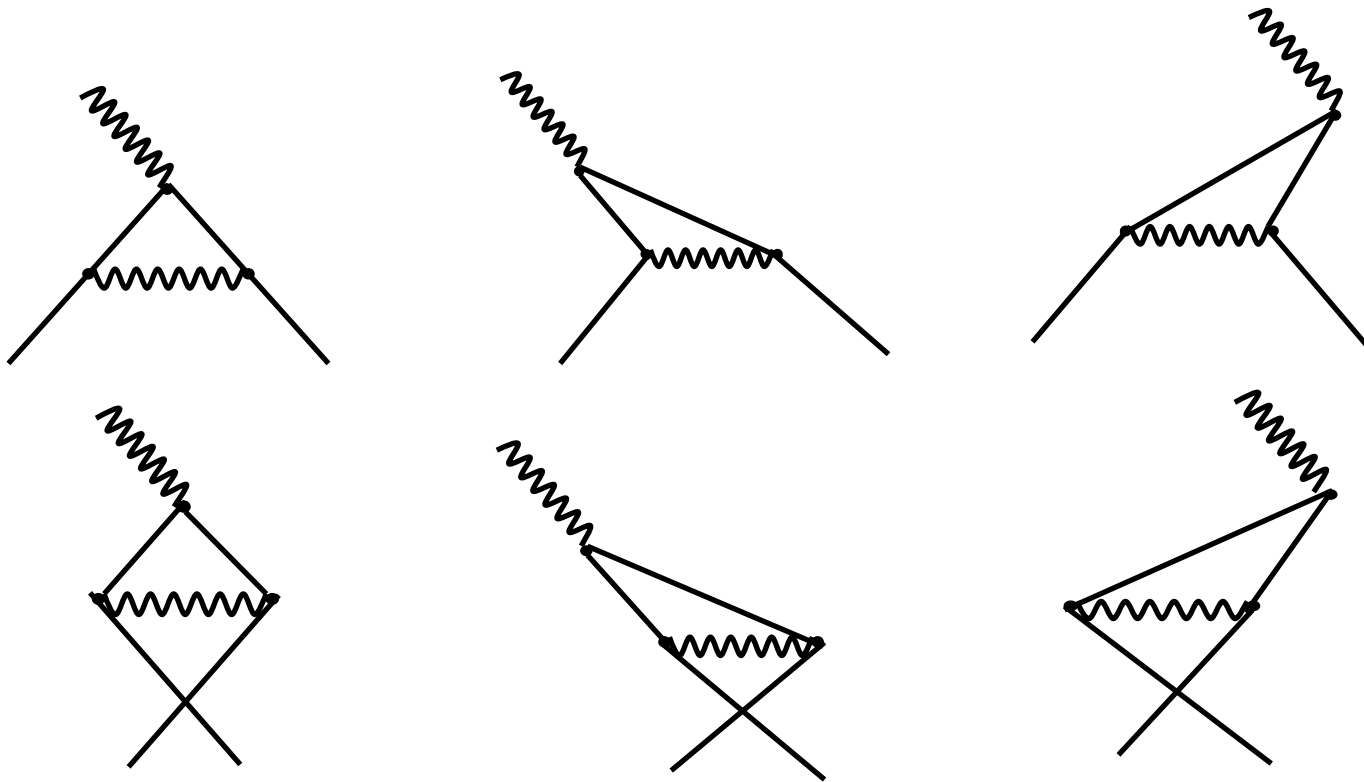
(a)



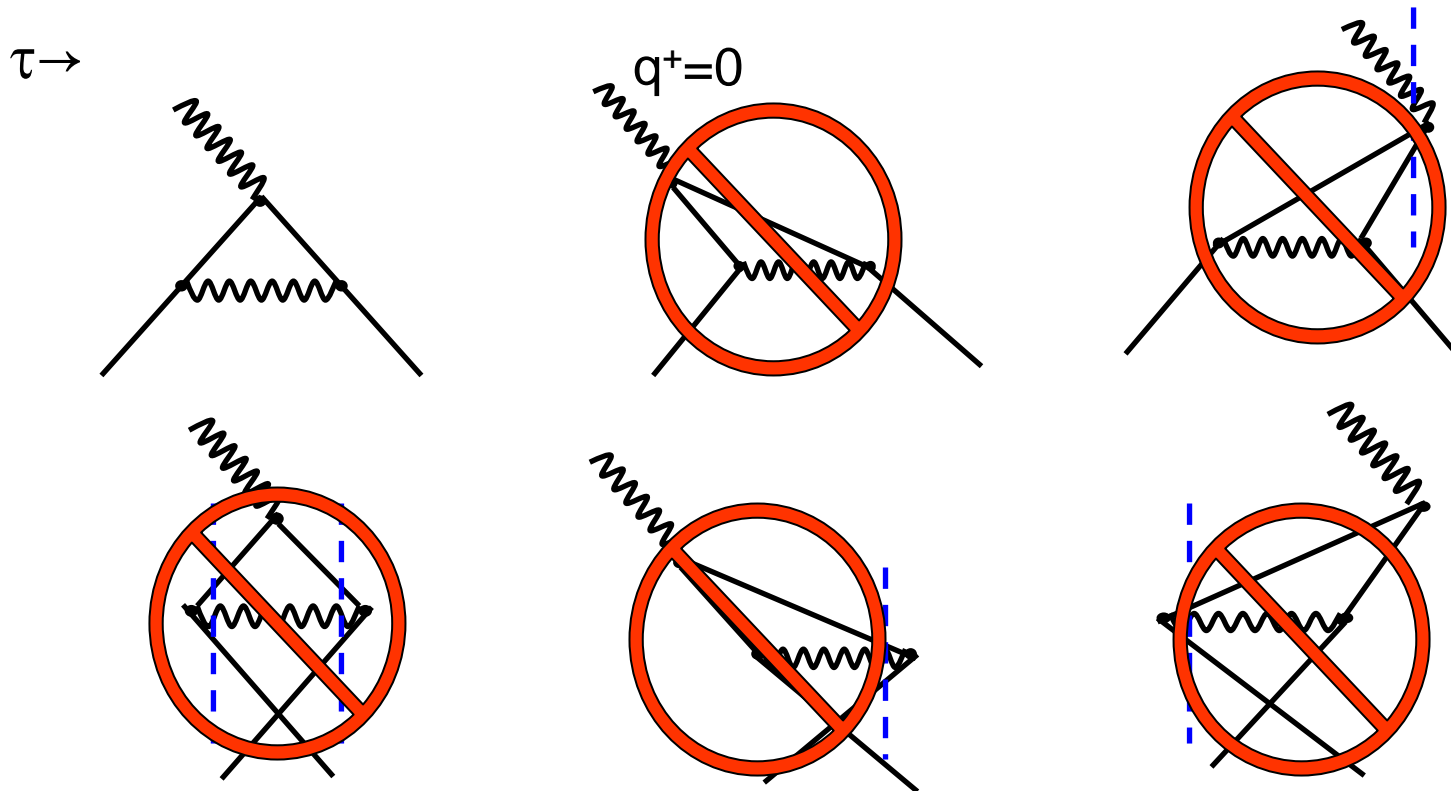
(b)

# g-2 calculation

t →



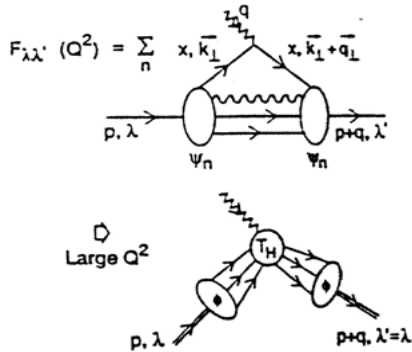
# g-2 calculation



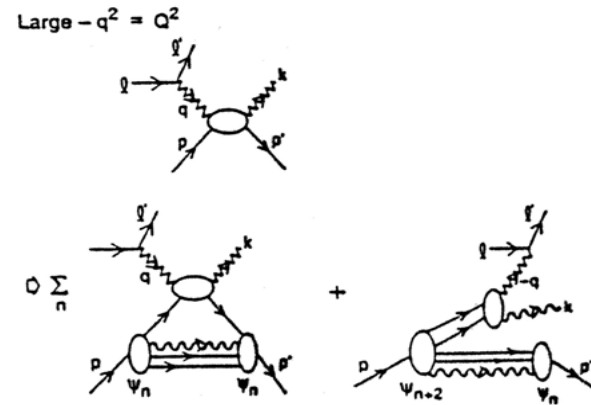
- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

# Applications to Hadron Phenomenology

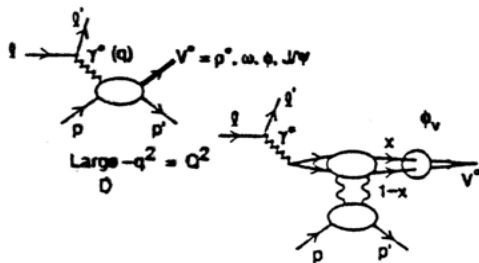
Form Factors  $|p \rightarrow l' p'$   
 $\langle p' \lambda' | J^+(0) | p \lambda \rangle$



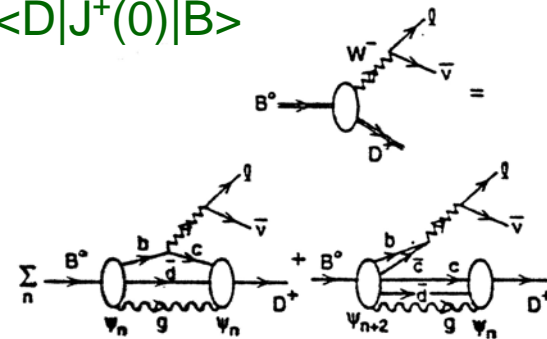
Virtual Compton  $\gamma^* p \rightarrow \gamma' p'$   
 $\langle p' \lambda' | J^\mu(z) J^\nu(0) | p \lambda \rangle$



Vector Meson Leptoproduction  $\gamma^* p \rightarrow V^* p'$

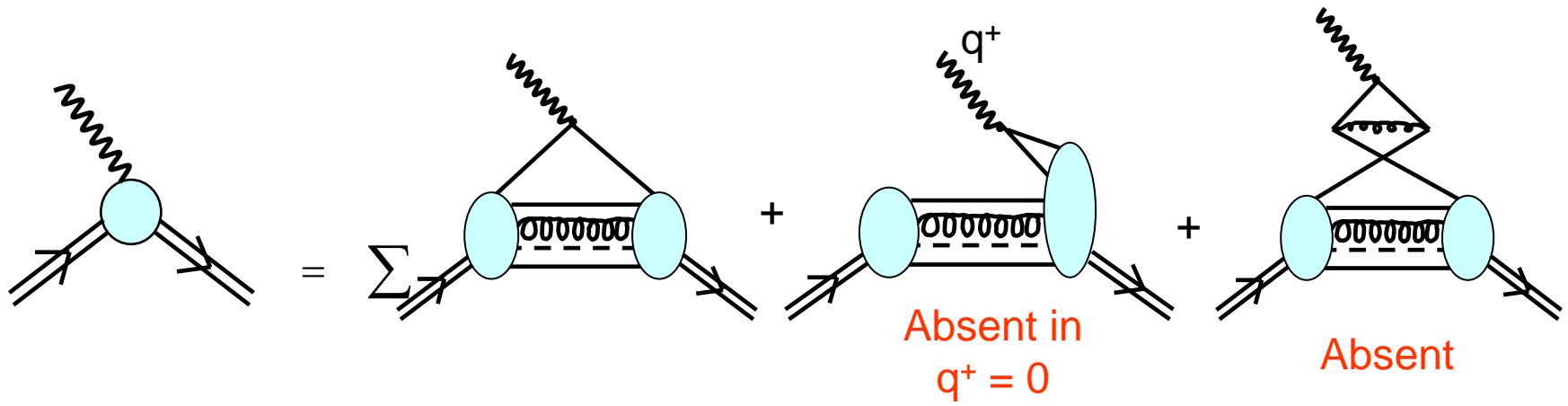


Weak Decay  
 $\langle D | J^+(0) | B \rangle$

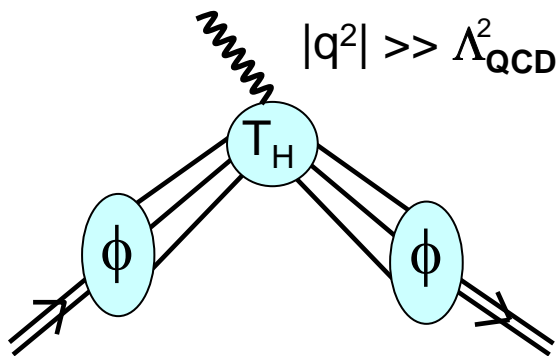
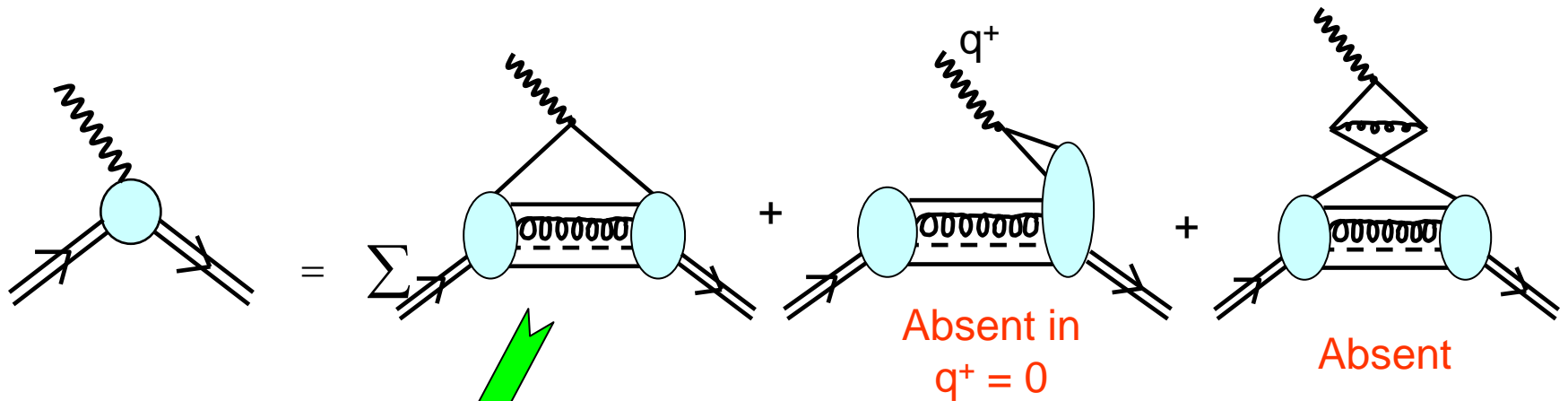




# LFD in Exclusive Processes



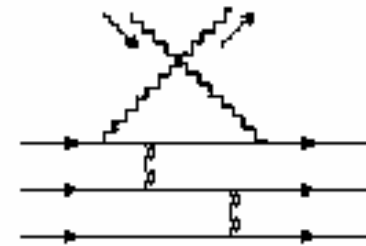
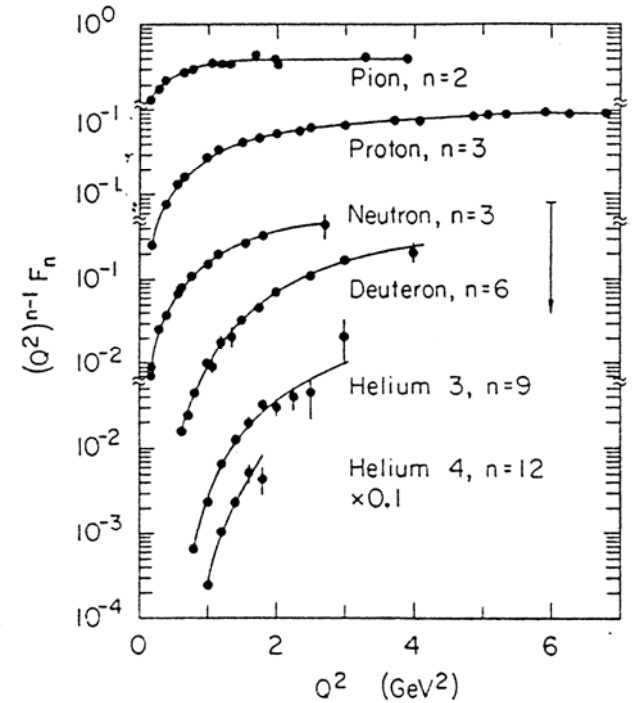
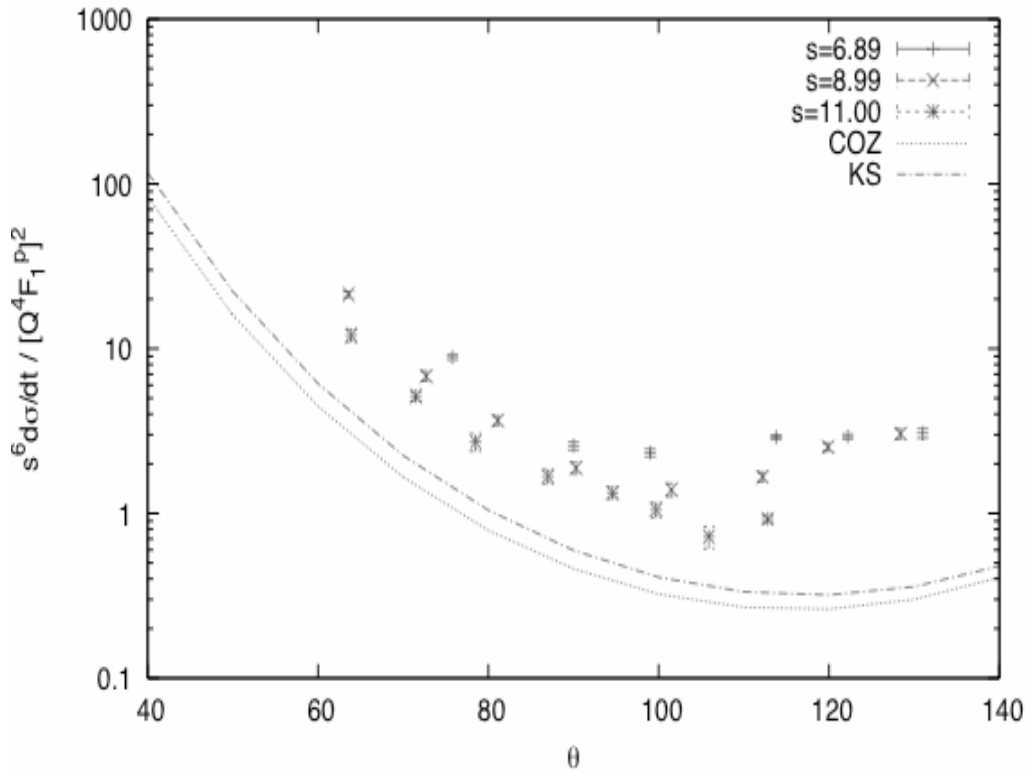
# LFD in Exclusive Processes



$$T_H = \sum \left[ \begin{array}{c} x_1 \text{---} \text{wavy} \text{---} y_1 \\ x_2 \text{---} \text{gluon} \text{---} y_2 \\ x_3 \text{---} \text{gluon} \text{---} y_3 \end{array} + \dots \right]$$

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i)$$

# The Quark Counting Rule and PQCD Predictions of Exclusive Processes



$$6 \times 7 \times 8 = 336$$

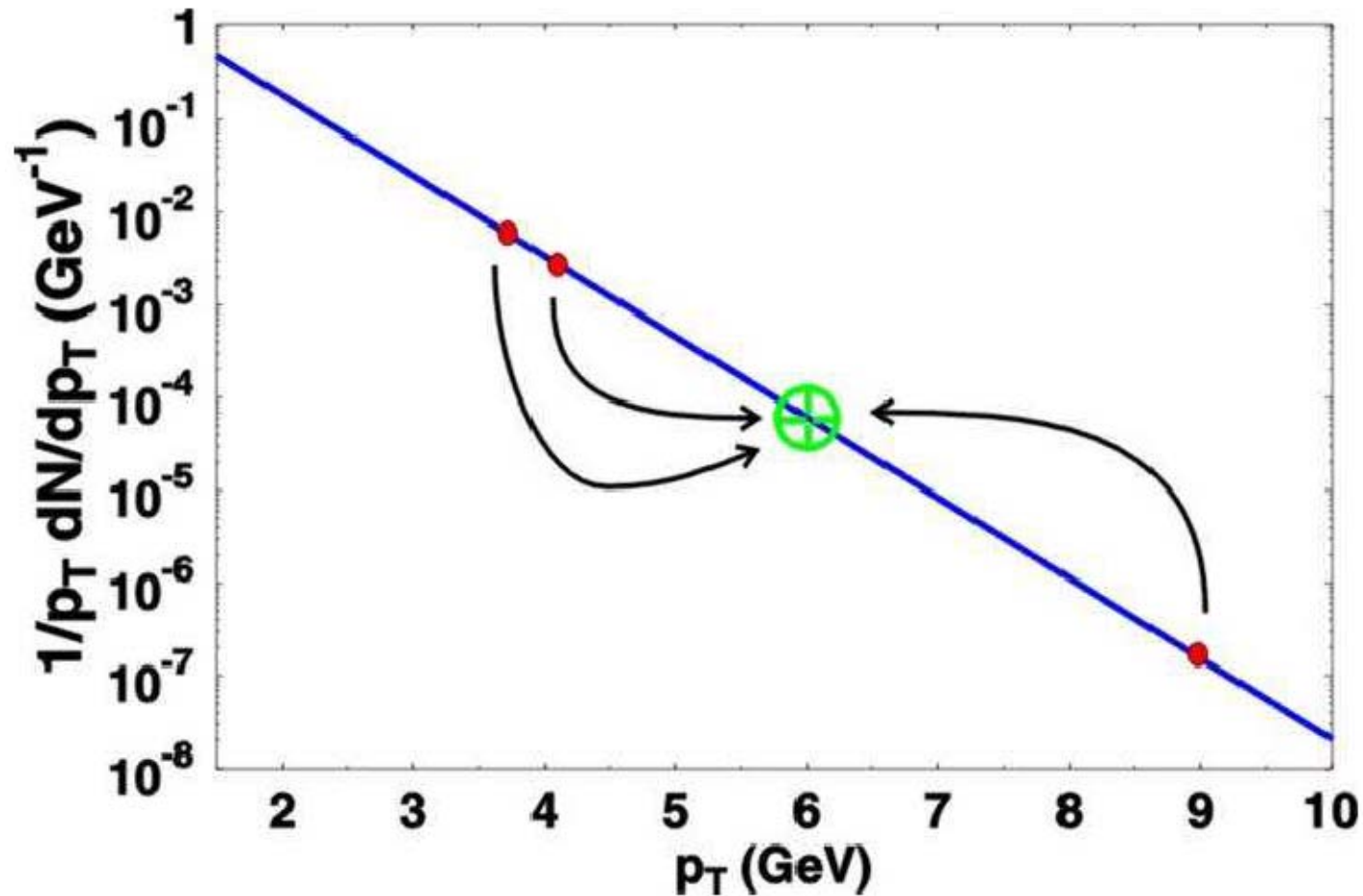
Pang & Ji, J.Comp.Phys.115,267(94)

R. Thomson, A. Pang and C.Ji, PRD73,054023(2006)  
JLab Hall A Collaboration, PRL98, 152001(2007)

QuickTime™ and a  
YUV420 codec decompressor  
are needed to see this picture.

# Hadronization Mechanisms

R.J. Fries, nucl-th/0403036, PRC 68, 044902 (2003)

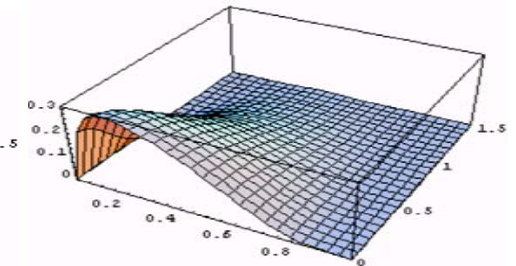
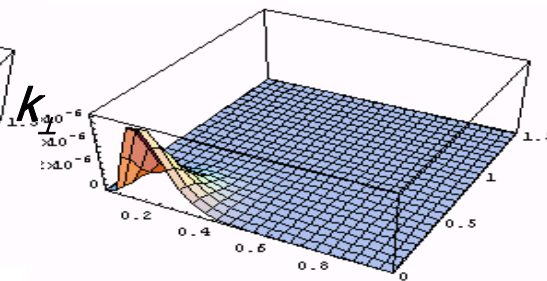
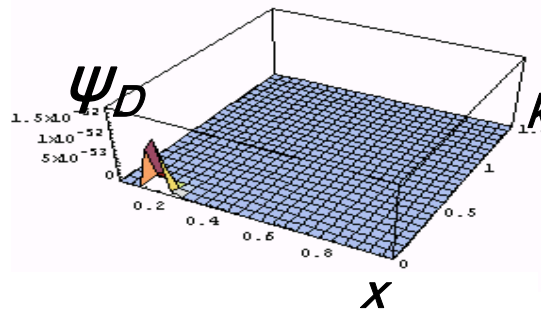
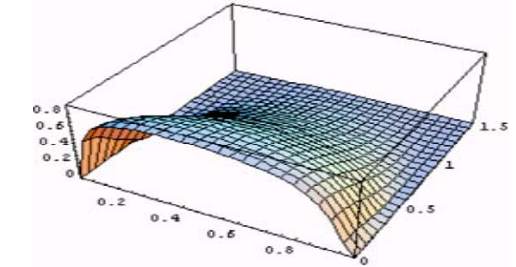
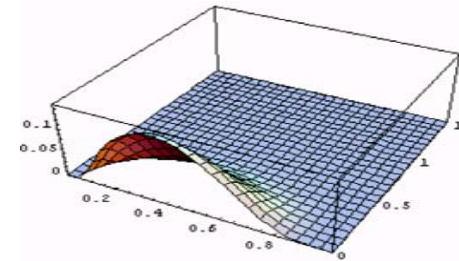
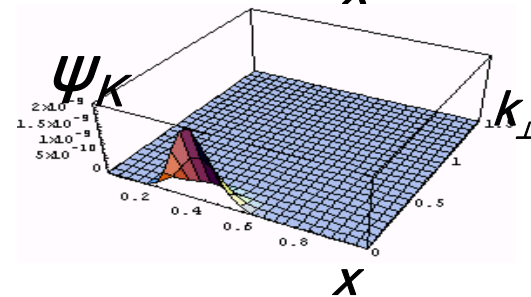
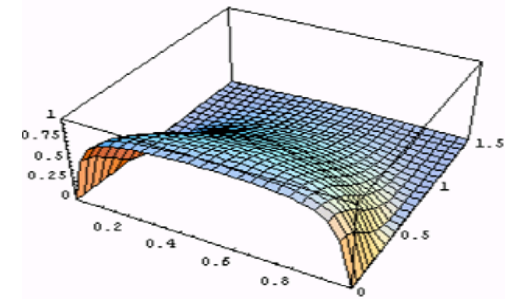
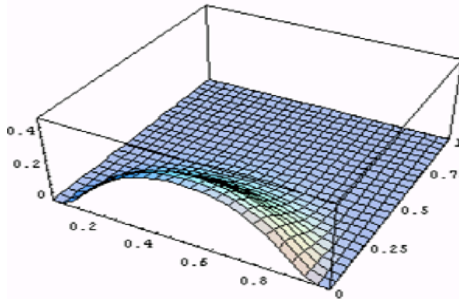
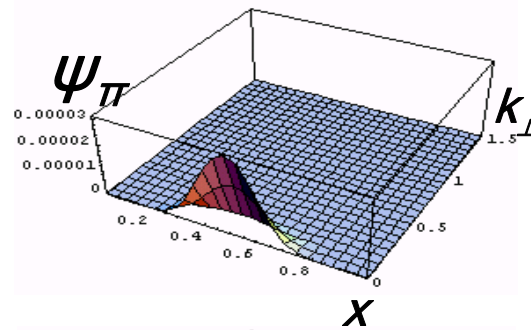


# Light-Front Wavefunctions

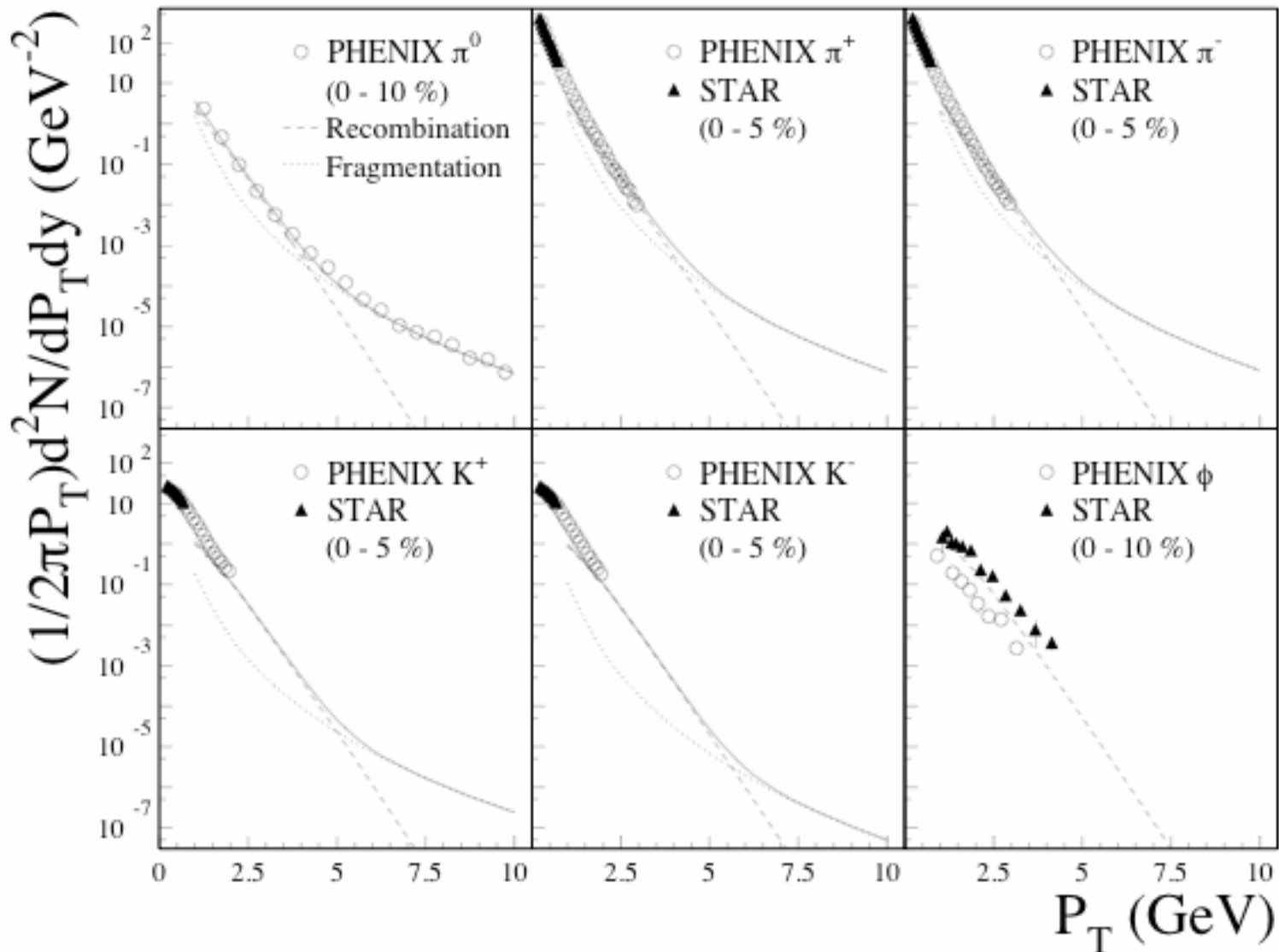
$\beta^2 (\text{GeV}^2) = 0.026$

0.26

2.6



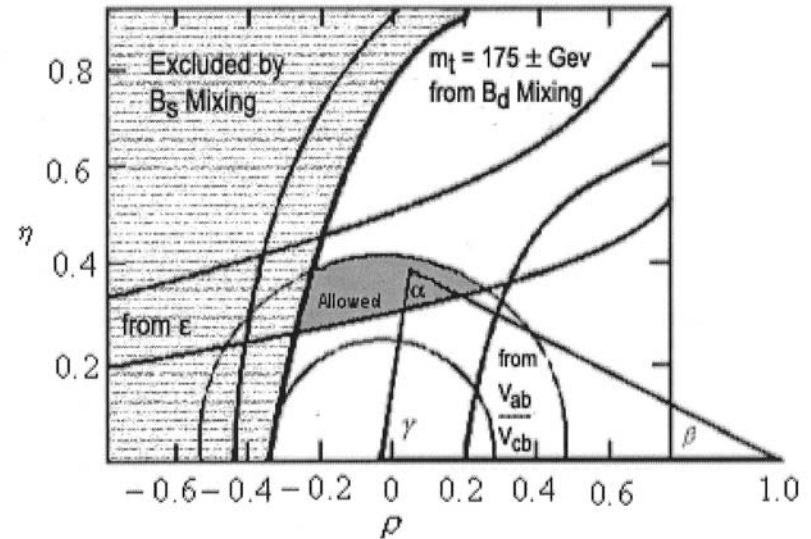
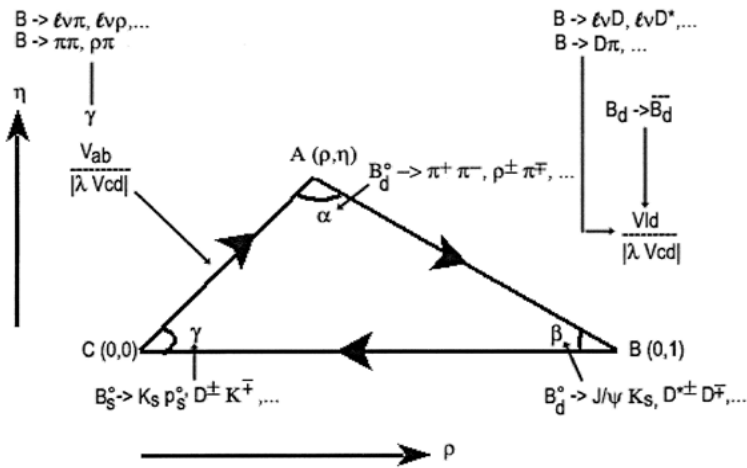
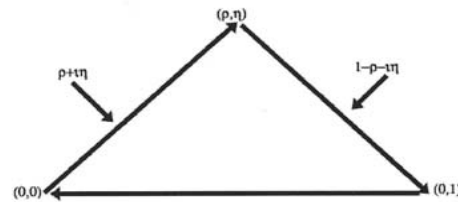
# Single Spectra of Mesons



B.Hong, C.Ji, D.-P.Min, PRC73, 054901 (2006)

# Unitarity Triangle

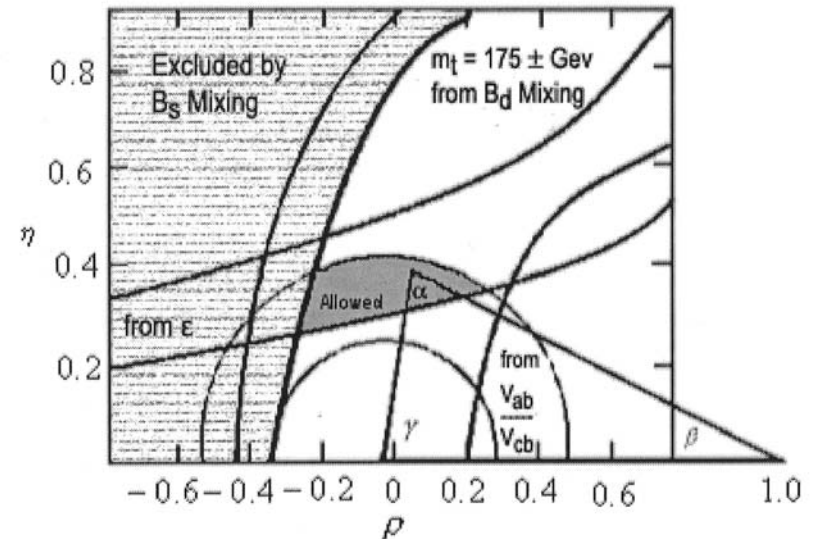
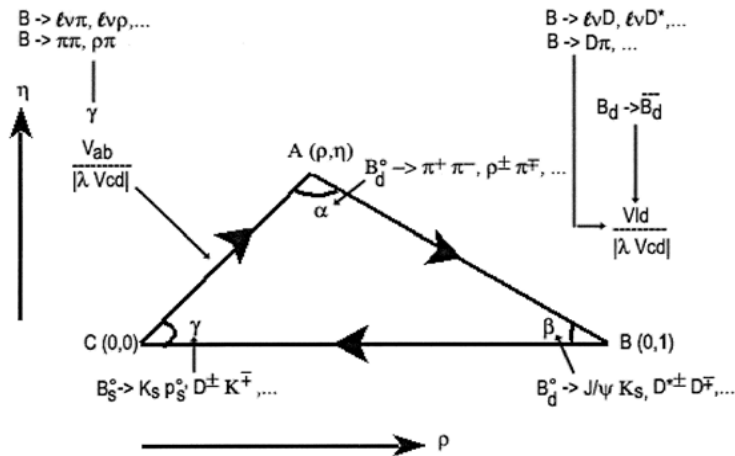
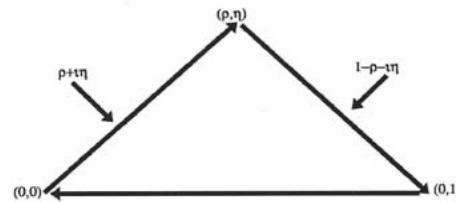
## Triangle in Wolfenstein parametrization





# Unitarity Triangle

Triangle in Wolfenstein parametrization



C.Ji and H.-M.Choi, NPB (Proc. Suppl.) 90 (2000) 93-99  
 B-physics phenomenology with emphasis on the light-cone

# Mixing Matrix Elements $V_{ij}$

$$|V_{ud}|=0.9740\pm 0.0005$$

Superallowed  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decay  
nucleon  $\beta$  decay, pion  $\beta$  decay, ...

$$|V_{us}|=0.2196\pm 0.0023$$

$$K \rightarrow \pi l \bar{\nu}_l (K_{l3}), \Lambda \rightarrow p e \bar{\nu}_e, \Sigma^- \rightarrow n e \bar{\nu}_e, \dots$$

$$|V_{cd}|=0.224\pm 0.016$$

$$D^0 \rightarrow \pi^- e^+ \nu_e, D^+ \rightarrow \pi^0 e^+ \nu_e, D^+ \rightarrow \mu^+ \nu_\mu, \dots$$

$$|V_{cs}|=1.04\pm 0.16$$

$$D^0 \rightarrow K^- e^+ \nu_e, D^+ \rightarrow \bar{K}^0 e^+ \nu_e, \dots$$

$$|V_{cb}|=0.0395\pm 0.0017$$

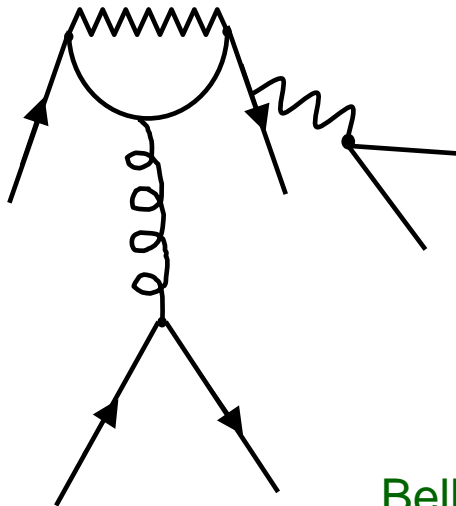
$$B^+ \rightarrow \bar{D}^0 l^+ \nu_l, B^+ \rightarrow \bar{D}^0 l^+ \nu_l, \dots (HQET)$$

$$|V_{ub}|=(0.08\pm 0.02)|V_{cb}|=(3.3\pm 0.4\pm 0.7)\times 10^{-3}$$

$$B \rightarrow \pi l \nu_l, \rho l \nu_l, \omega l \nu_l, B^+ \rightarrow \mu^+ \nu_\mu, \tau \nu_\tau, \dots$$

$$|V_{tb}| \approx 1, |V_{ts}| \approx 0.04, |V_{td}| \approx 0.005 \sim 0.013$$

# Penguin Process



$$BR(B \rightarrow K l^+ l^-) = 4.96 \times 10^{-7} \left| \frac{V_{ts}}{V_{cb}} \right|^2$$

$$BR^{Expt}(B \rightarrow K e^+ e^-) < 1.2 \times 10^{-6}$$

$$BR^{Expt}(B \rightarrow K \mu^+ \mu^-) < \left( 0.99_{-0.32-0.15}^{+0.39+0.13} \right) \times 10^{-6}$$

Belle Collaboration, Phys.Rev.Lett.88,021801(2002)

H.-M.Choi, C.Ji and L.S.Kisslinger, Phys.Rev.D65,074032(2002)

$t \rightarrow (b, s, d) l^+ \nu$  (CDF/D0)

$$M_t = 173.8 \pm 5.2 \text{ GeV}$$

$\tau_t \ll$  strong interaction time scale

# Semileptonic Decay Processes in Light-Front Quark Model

W.Jaus, PRD63, 053009(2001)

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

$$\begin{aligned} & \langle P'';00 | J_{V-A}^{\mu} | P';00 \rangle \\ & = f_+(q^2) (P' + P'')^{\mu} + f_-(q^2) q^{\mu} \end{aligned}$$

where

$$q = P' - P''$$

# Semileptonic Decay Processes in Light-Front Quark Model

W.Jaus, PRD63, 053009(2001)

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TIFF (LZW) decompressor  
are needed to see this picture.

# $\beta$ Decay up to $O(\alpha)$

*Total Decay Rate* :  $1/\tau = 1/\tau_0(1 + \delta)$

where

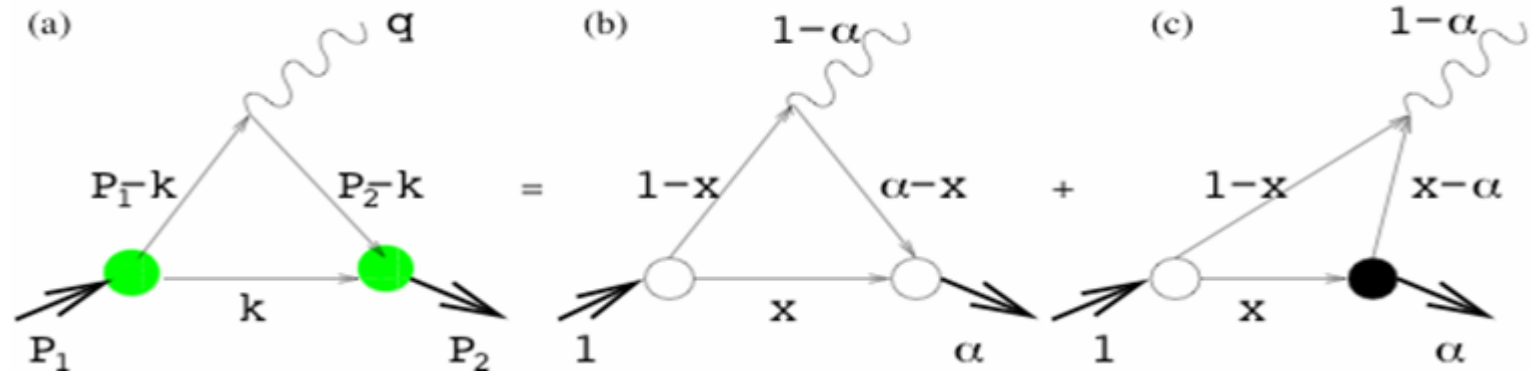
$$\delta = \frac{\alpha}{2\pi} \left[ g(E_0) + 3 \ln \frac{M_Z}{M_p} + A_g \right] + \frac{\alpha}{2\pi} \left[ 3(Q_u + Q_d) \ln \frac{M_Z}{M_A} + 2C \right]$$

A.Sirlin, RMP50, 573 (1978); PR164, 1767(1967)

W.J.Marciano and A.Sirlin, PRL56, 22 (1986)

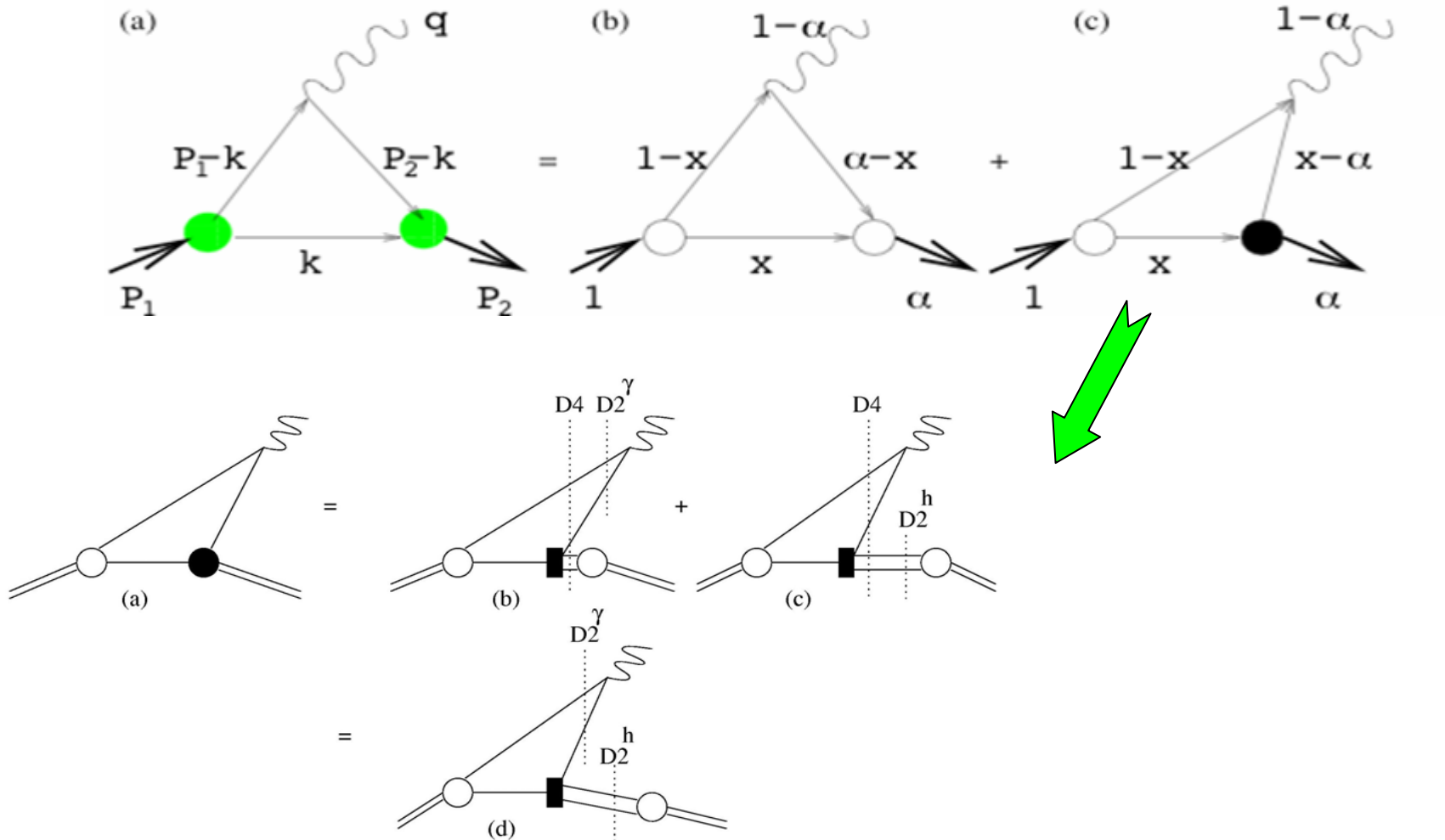
# Semileptonic Decay Processes

are timelike :  $q^2 = q^+q^- - q_{\perp}^2 > 0$  or  $q^+ \neq 0$ .



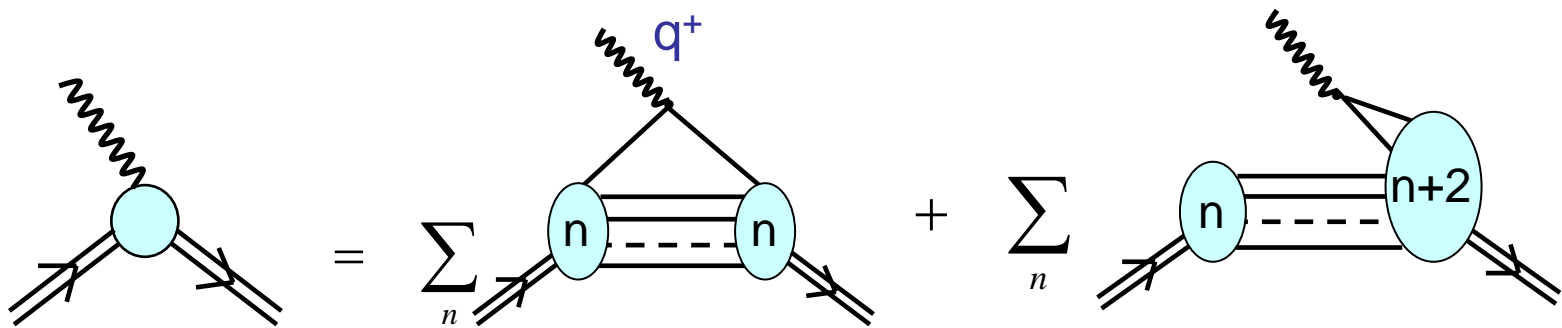
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are timelike :  $q^2 = q^+ q^- - q_{\perp}^2 > 0$  or  $q^+ \neq 0$ .





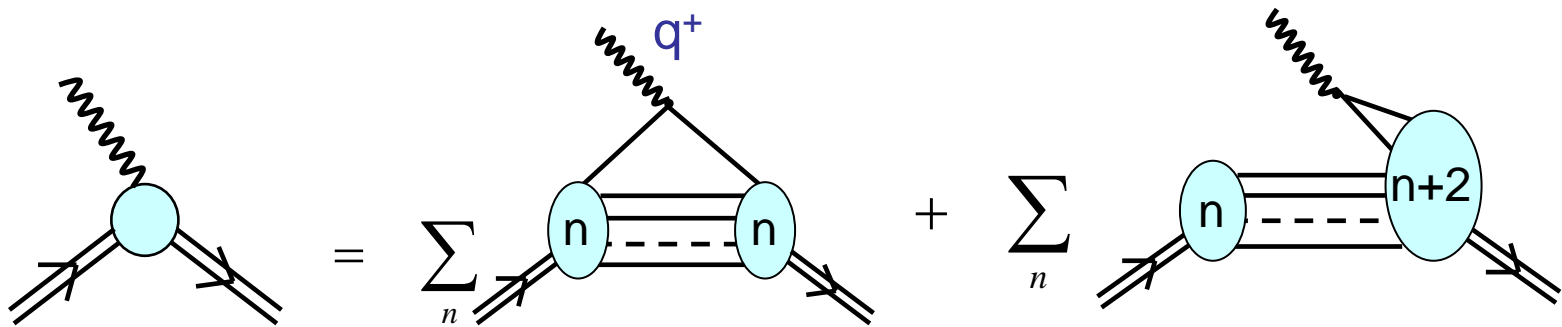
# Zero-Mode Issue in LFD



- Even if  $q^+ \rightarrow 0$ , the off-diagonal elements do not go away in some cases.

$$\lim_{q^+ \rightarrow 0} \int_{p^+}^{p^+ + q^+} dk^+ (\dots) \neq 0$$

# Zero-Mode Issue in LFD



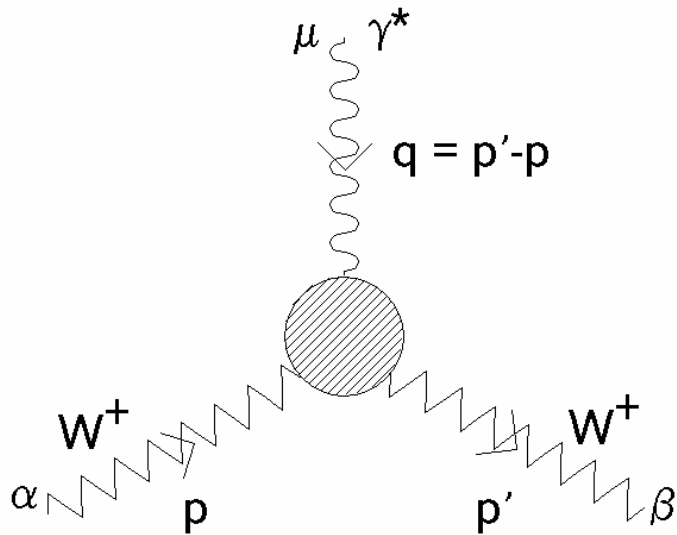
- Even if  $q^+ \rightarrow 0$ , the off-diagonal elements do not go away in some cases.

$$\lim_{q^+ \rightarrow 0} \int_{p^+}^{p^+ + q^+} dk^+ (\dots) \neq 0$$

Vector Anomaly in SM (Anomalous Magnetic Moment of  $W^\pm$ )  
 B.Bakker and C.Ji, PRD71,053005(2005)

# CP-Even Electromagnetic Form Factors of $W^\pm$ Gauge Bosons

$$\Gamma_{\alpha\beta}^\mu = ie \left\{ A[(p+p')^\mu g_{\alpha\beta} + 2(q_\beta g_\alpha^\mu - q_\alpha g_\beta^\mu)] + (\Delta\kappa)(g_\alpha^\mu q_\beta - g_\beta^\mu q_\alpha) + \frac{(\Delta Q)}{2M_W^2} (p+p')^\mu q_\alpha q_\beta \right\}$$



At tree level, for any  $q^2$ ,

$$A = 1, \quad \Delta\kappa = 0, \quad \Delta Q = 0$$

Beyond tree level,

$$A = F_1(q^2),$$

$$-(\Delta\kappa) = F_2(q^2) + 2F_1(q^2),$$

$$-(\Delta Q) = F_3(q^2),$$

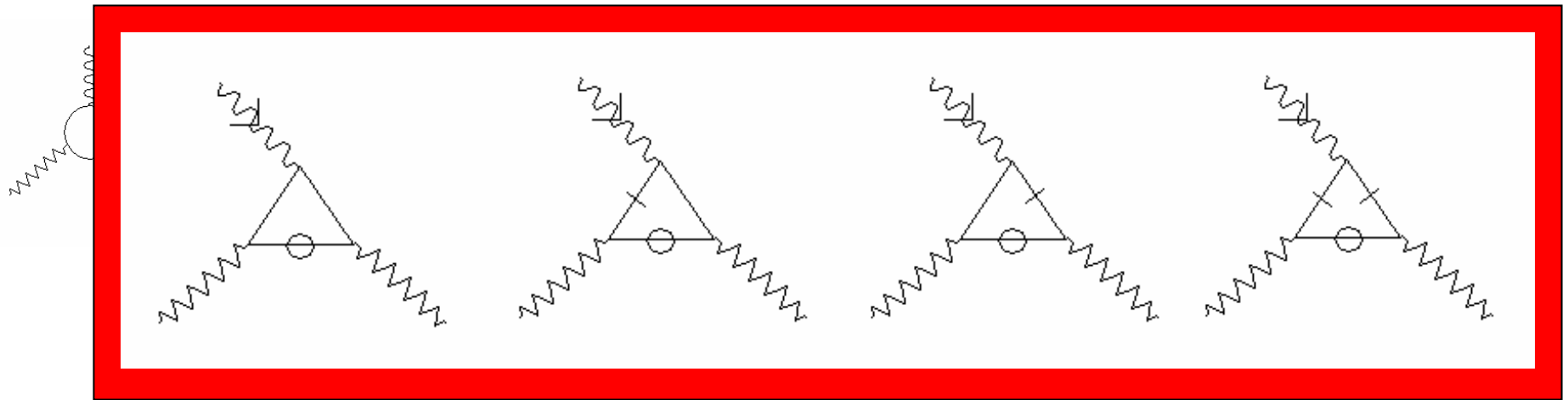
$$\Gamma_{\alpha\beta}^\mu = -ie J_{\alpha\beta}^\mu$$

$$J_{\alpha\beta}^\mu = \left\{ -(p+p')^\mu g_{\alpha\beta} F_1(q^2) + (g_\alpha^\mu q_\beta - g_\beta^\mu q_\alpha) F_2(q^2) + \frac{q_\alpha q_\beta}{2M_W^2} (p+p')^\mu F_3(q^2) \right\}$$

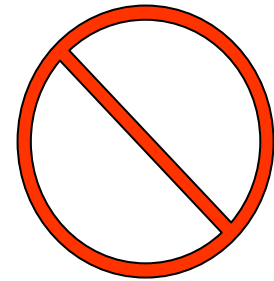
# LFD Results

$$G_{hh}^+ = \langle h', p' | J^+ | h, p \rangle \text{ in } q^+ = 0 \text{ frame with } \eta = Q^2 / 4M_W^2 \quad (Q^2 = -q^2),$$

$$G_{++}^+ = 2p^+ (F_1 + \eta F_3), G_{+0}^+ = p^+ \sqrt{2\eta} (2F_1 + F_2 + 2\eta F_3), G_{+-}^+ = -2p^+ \eta F_3, G_{00}^+ = 2p^+ (F_1 - 2\eta F_2 - 2\eta^2 F_3)$$



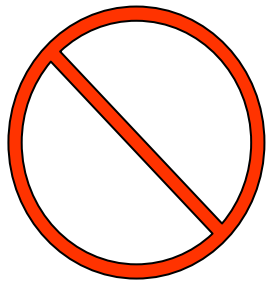
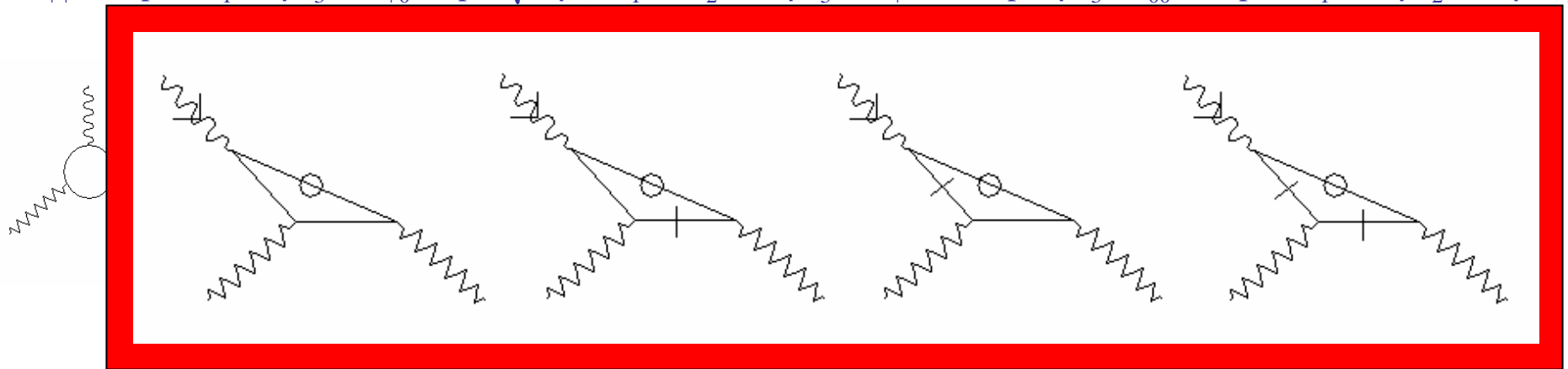
$J^+$



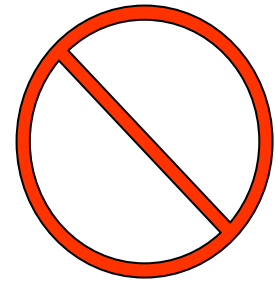
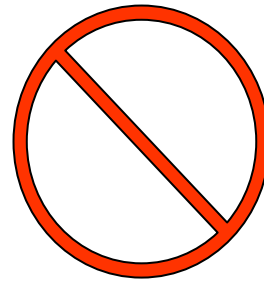
# LFD Results

$G_{hh}^+ = \langle h', p' | J^+ | h, p \rangle$  in  $q^+ = 0$  frame with  $\eta = Q^2 / 4M_W^2$  ( $Q^2 = -q^2$ ),

$G_{++}^+ = 2p^+(F_1 + \eta F_3), G_{+0}^+ = p^+ \sqrt{2\eta}(2F_1 + F_2 + 2\eta F_3), G_{+-}^+ = -2p^+ \eta F_3, G_{00}^+ = 2p^+(F_1 - 2\eta F_2 - 2\eta^2 F_3)$



$J^+$   
 $q^+ = 0$



# Effective Constituent Quark Model for Low $Q^2$

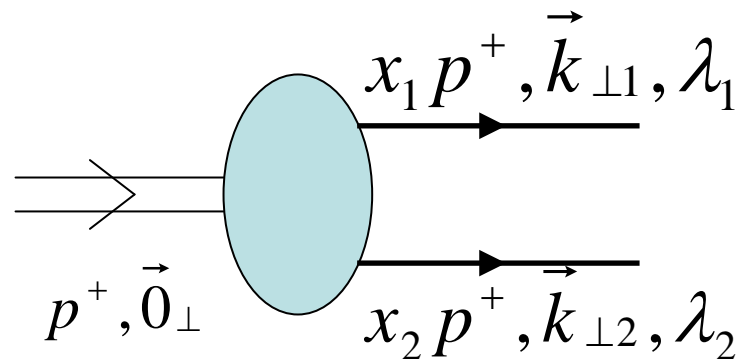
$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \dots$$

$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$

where

$$|Q\rangle = \psi_q^Q|q\rangle + \psi_{qg}^Q|qg\rangle + \dots$$

$$|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}}|\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}}|\bar{q}g\rangle + \dots$$



$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i}) \chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial

(Dependent on the model potential)

$$H = T + V$$

$V$  includes Coulomb, Confinement,  
Spin-Spin, Spin-Orbit interactions.

Spin-Orbit

(Interaction independent Melosh transformation)

$$J^{PC} = 0^{++}(f_0, a_0, \dots)$$

$$0^{-+}(\pi, K, \eta, \eta', \dots)$$

$$1^{-}(\rho, K^*, \omega, \phi, \dots)$$

...

# Mass Spectra of $0^{-+}$ and $1^{-}$ Mesons

H.M.Choi and C.Ji, Phys.Rev.D59,074015(99)

$^1S_0$	Expt. [1]	Prediction	$^3S_1$	Expt. [1]	Prediction
$\pi$	$135 \pm 0.0006$	<u>135</u>	$\rho$	$770 \pm 0.8$	<u>770</u>
$K$	$498 \pm 0.016$	478	$K^*$	$892 \pm 0.26$	850
$\eta$	$547 \pm 0.12$	<u>547</u>	$\omega$	$782 \pm 0.12$	<u>782</u>
$\eta'$	$958 \pm 0.14$	<u>958</u>	$\phi$	$1020 \pm 0.008$	<u>1020</u>
$D$	$1865 \pm 0.5$	1836	$D^*$	$2007 \pm 0.5$	1998
$D_s$	$1969 \pm 0.6$	2011	$D_s^*$	$2112 \pm 0.7$	2109
$\eta_c$	$2980 \pm 2.1$	3171	$J/\psi$	$3097 \pm 0.04$	3225
$B$	$5279 \pm 1.8$	5235	$B^*$	$5325 \pm 1.8$	5315
$B_s$	$5369 \pm 2.0$	5375	$(b\bar{s})$	–	5424
$(b\bar{b})$	–	9657	$\Upsilon$	$9460 \pm 0.21$	9691

# Decay Constants and Charge Radii

Observables	$\delta_V = -3.3^\circ \pm 1^\circ$		$\delta_V = +3.3^\circ \pm 1^\circ$		Experiment
	HO	Linear	HO	Linear	
$f_\pi$ [MeV]	92.4	91.8	92.4	91.8	$92.4 \pm 0.25$
$f_K$ [MeV]	109.3	114.1	109.3	114.1	$113.4 \pm 1.1$
$f_\rho$ [MeV]	151.9	173.9	151.9	173.9	$152.8 \pm 3.6$
$f_{K^*}$ [MeV]	157.6	180.8	157.6	180.8	
$f_\omega$ [MeV]	$45.9 \pm 1.4$	$52.6 \pm 1.6$	$55.1 \pm 1.3$	$63.1 \pm 1.5$	$45.9 \pm 0.7$
$f_\phi$ [MeV]	$82.6 \mp 0.8$	$94.3 \mp 0.9$	$76.7 \mp 1.0$	$87.6 \mp 1.1$	$79.1 \pm 1.3$
$r_\pi^2$ [fm <sup>2</sup> ]	0.449	0.425	0.449	0.425	$0.432 \pm 0.016$ [32]
$r_{K^+}^2$ [fm <sup>2</sup> ]	0.384	0.354	0.384	0.354	$0.34 \pm 0.05$ [32]
$r_{K^0}^2$ [fm <sup>2</sup> ]	-0.091	-0.082	-0.091	-0.082	$-0.054 \pm 0.101$ [32]



# Radiative Decay Processes

Widths [keV]	$\delta_V = -3.3^\circ \pm 1^\circ$		$\delta_V = +3.3^\circ \pm 1^\circ$		Experiment
	HO	Linear	HO	Linear	
$\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma)$	76	69	76	69	$68 \pm 8$
$\Gamma(\omega \rightarrow \pi \gamma)$	$730 \pm 1.3$	$667 \pm 1.3$	$730 \mp 1.3$	$667 \mp 1.3$	$717 \pm 51$
$\Gamma(\phi \rightarrow \pi \gamma)$	$5.6_{+3.9}^{-2.9}$	$5.1_{+3.6}^{-2.6}$	$5.6_{-2.9}^{+3.9}$	$5.1_{-2.6}^{+3.6}$	$5.8 \pm 0.6$
$\Gamma(\rho \rightarrow \eta \gamma)$	59	54	59	54	$58 \pm 10$
$\Gamma(\omega \rightarrow \eta \gamma)$	$8.7 \mp 0.3$	$7.9 \mp 0.3$	$6.9 \mp 0.3$	$6.3 \mp 0.3$	$7.0 \pm 1.8$
$\Gamma(\phi \rightarrow \eta \gamma)$	$38.7 \pm 1.6$	$37.8 \pm 1.5$	$49.2 \pm 1.6$	$47.6 \pm 1.5$	$55.8 \pm 3.3$
$\Gamma(\eta' \rightarrow \rho \gamma)$	68	62	68	62	$61 \pm 8$
$\Gamma(\eta' \rightarrow \omega \gamma)$	$4.9 \pm 0.4$	$4.5 \pm 0.4$	$7.6 \pm 0.4$	$7.0 \pm 0.4$	$6.1 \pm 1.1$
$\Gamma(\phi \rightarrow \eta' \gamma)$	$0.41 \mp 0.01$	$0.39 \mp 0.01$	$0.36 \mp 0.01$	$0.34 \mp 0.01$	$< 1.8$
$\Gamma(K^{*0} \rightarrow K^0 \gamma)$	124.5	116.6	124.5	116.6	$117 \pm 10$
$\Gamma(K^{*+} \rightarrow K^+ \gamma)$	79.5	71.4	79.5	71.4	$50 \pm 5$

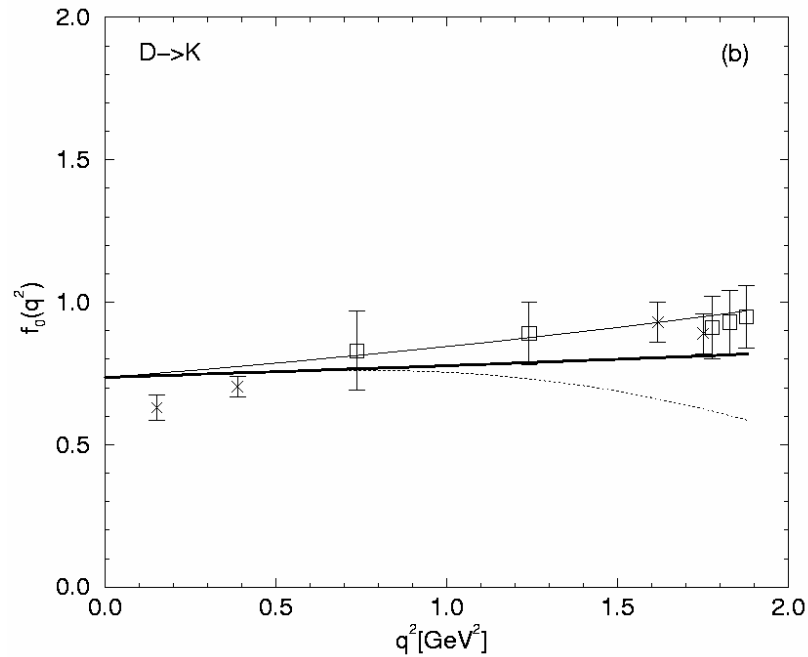
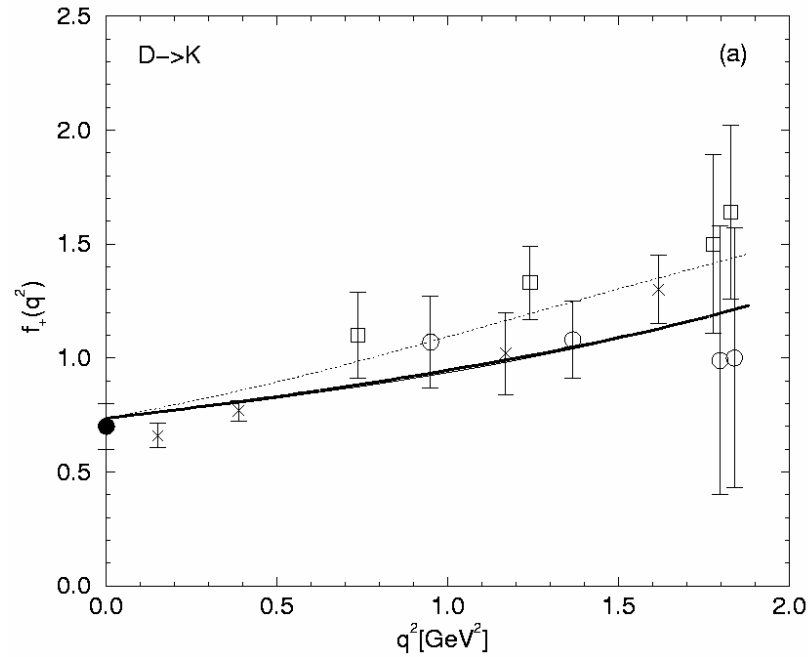
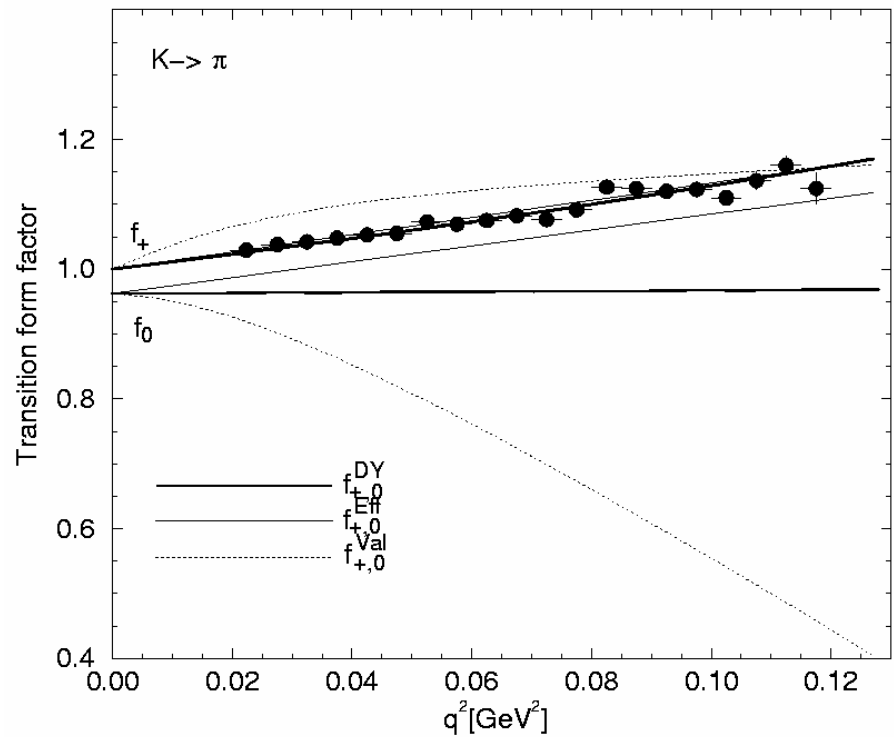
# Semi-leptonic Decay: $0^- \rightarrow 0^-$

$$f_0(q^2) = f_+(q^2) + q^2 f_-(q^2) / (M_1^2 - M_2^2)$$

## Slope at $q^2=0$

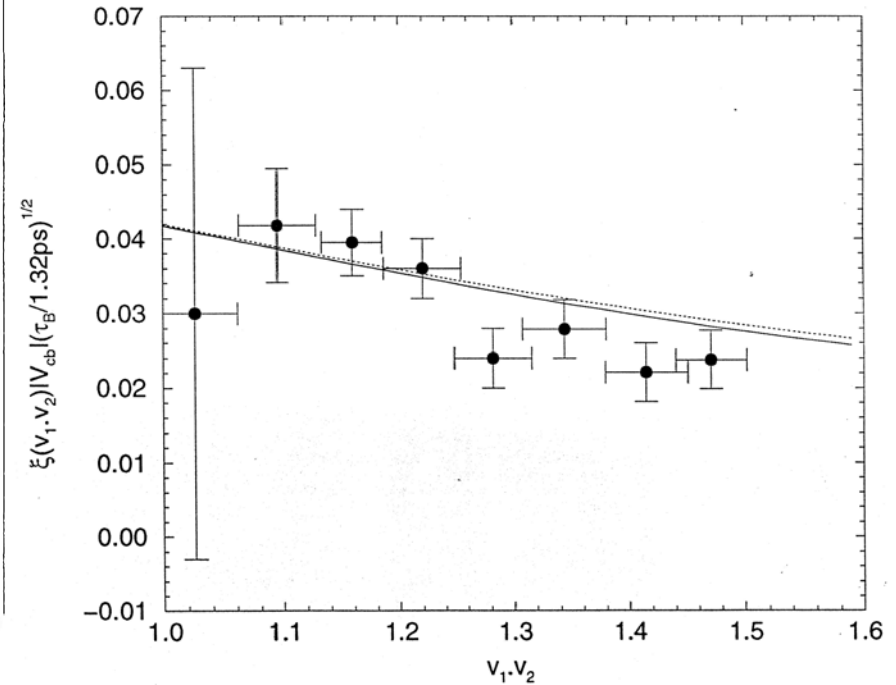
$$\lambda_+ = 0.026 [0.083] \quad \text{Expt}(K_{e3}^0) : 0.0288 \pm 0.0015$$

$$\lambda_0 = 0.025 [-0.017] \quad \text{Expt}(K_{\mu 3}^0) : 0.025 \pm 0.006$$



# Branching Ratios and Isgur-Wise Function

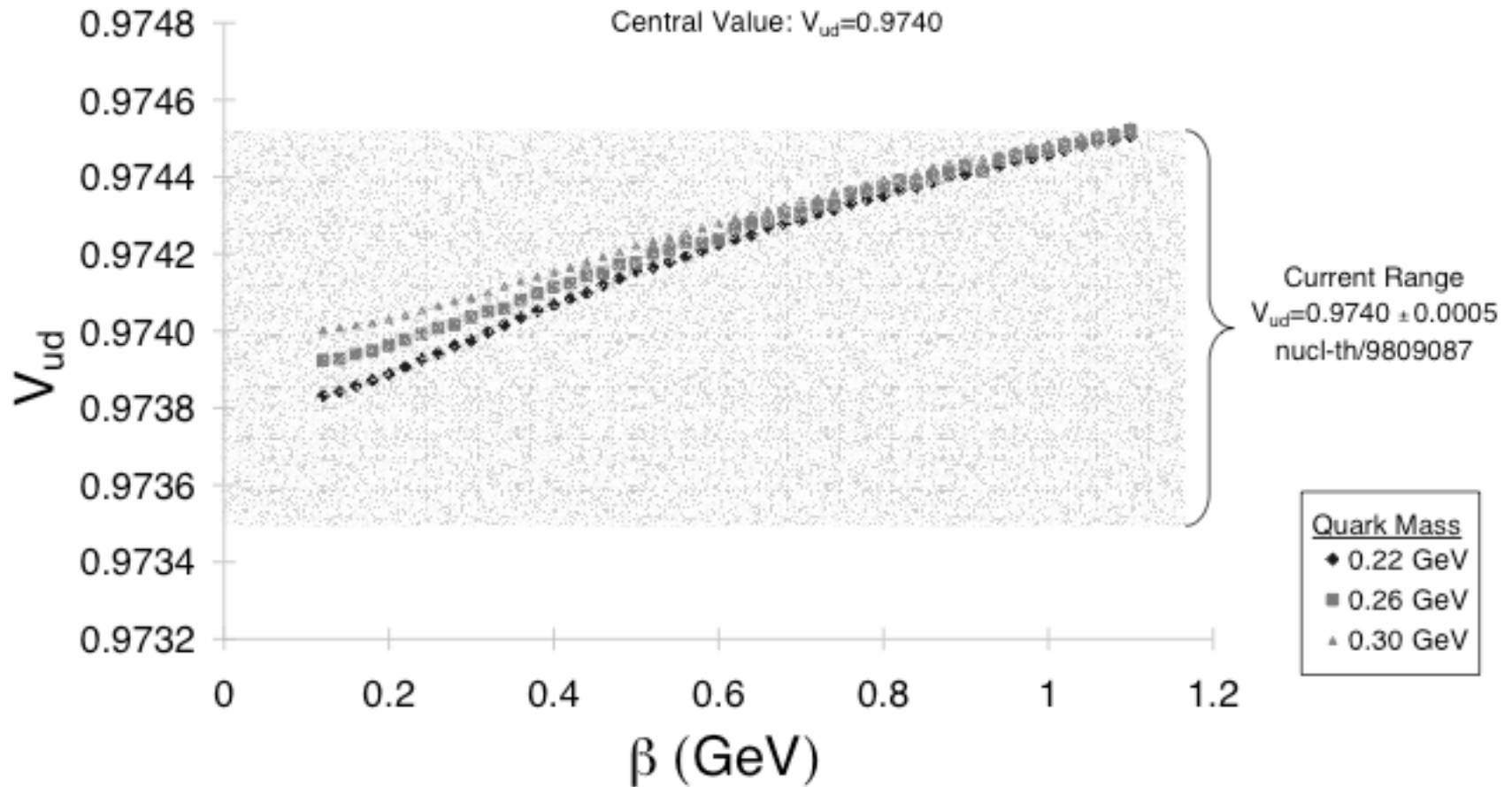
Processes		$f_+(0)$	$f_+(q_{\max}^2)$	$B_r^{\text{Th.}}$	$B_r^{\text{exp.}}$
$B \rightarrow \pi$	H.O.	0.234	2.77	$(1.20 \pm 0.14^{+0.24+0.63}_{-0.25-0.36}) \times 10^{-4}$	$(1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$
	Linear	0.273	2.43	$(1.33 \pm 0.16^{+0.27+0.71}_{-0.28-0.40}) \times 10^{-4}$	
$D \rightarrow K$	H.O.	0.724	1.23	$(3.43 \pm 1.33)\%$	$(3.64 \pm 0.20)\%$
	Linear	0.736	1.21	$(3.46 \pm 1.34)\%$	
$D \rightarrow \pi$	H.O.	0.593	1.72	$(2.24 \pm 0.33) \times 10^{-3}$	$(3.8^{+1.2}_{-1.0}) \times 10^{-3}$
	Linear	0.618	1.56	$(2.28 \pm 0.34) \times 10^{-3}$	
$B \rightarrow D$	H.O.	0.686	1.14	$(2.36 \pm 0.36)\%$	$(2.35 \pm 0.2 \pm 0.44)\%$
	Linear	0.709	1.14	$(2.47 \pm 0.37)\%$	



# $V_{ud}$ vs. $\beta$

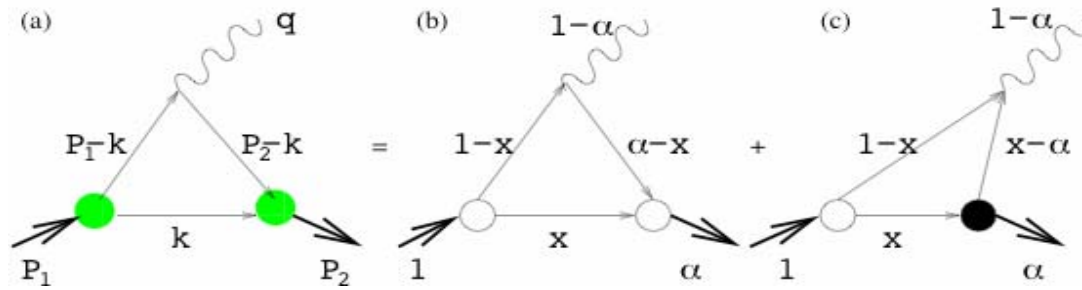
Used Jaus' values of  $M_A$  from pion radiative corrections to compute  $V_{ud}$  from superallowed data.

Central Value:  $V_{ud}=0.9740$



# Pinning Down Which Form Factors

- Jaus's  $\omega$ -dependent formulation yields zero-mode contributions both in  $G_{00}$  and  $G_{01}$ .  
W.Jaus, PRD60,054026(1999);PRD67,094010(2003)
- However, we find only  $G_{00}$  gets zm-contribution.  
B.Bakker,H.Choi and C.Ji,PRD67,113007(2003)  
H.Choi and C.Ji,PRD70, 053015(2004)
- Also,discrepancy exists in weak transition form factor  $A_1(q^2)=f(q^2)/(M_P+M_V)$ .  
**Power Counting Method**  
H.Choi and C.Ji, PRD72, 013004(2005)



## Electroweak Transition Form Factors

$$\begin{aligned}
 \langle P_2; 1h | J_{V-A}^\mu | P_1; 00 \rangle &= ig(q^2) \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* P_\alpha q_\beta \\
 &- f(q^2) \varepsilon^{*\mu} - a_+(q^2) (\varepsilon^* \cdot P) P^\mu - a_-(q^2) (\varepsilon^* \cdot P) q^\mu
 \end{aligned}$$

where

$$P = P_1 + P_2, \quad q = P_1 - P_2$$

$$\langle J_{V-A}^\mu \rangle_h = i \int \frac{d^4 k}{(2\pi)^4} \frac{S_{\Lambda_1}(P_1 - k) S_h^\mu S_{\Lambda_2}(P_2 - k)}{D_{m_1} D_m D_{m_2}}$$

where

$$D_m = k^2 - m^2 + i\varepsilon,$$

$$S_{\Lambda_i}(P_i) = \Lambda_i^2 / (P_i^2 - \Lambda_i^2 + i\varepsilon),$$

$$S_h^\mu = \text{Tr}[(\not{p}_2 + m_2)\gamma^\mu(1 - \gamma_5)(\not{p}_1 + m_1)\gamma_5(-\not{k} + m)\varepsilon^* \cdot \Gamma],$$

$$\Gamma^\mu = \gamma^\mu - \frac{(P_2 - 2k)^\mu}{D},$$

and

$$(1) \quad D_{\text{cov}}(M_V) = M_V + m_2 + m,$$

$$(2) \quad D_{\text{cov}}(k \cdot P_2) = [2k \cdot P_2 + M_V(m_2 + m) - i\varepsilon] / M_V,$$

$$(3) \quad D_{LF}(M_0) = M_0 + m_2 + m.$$

# Power Counting Method

$$\begin{aligned} \langle J_A^+ \rangle_{z.m.}^h &\propto \lim_{\alpha \rightarrow 1} \int_{\alpha}^1 dx \frac{(1-x)^2}{(1-\alpha)^2} S_h^+(k_{m_1}^-) [\dots] \\ &= \lim_{\alpha \rightarrow 1} (1-\alpha) \int_0^1 dz (1-z)^2 S_h^+(k_{m_1}^-) [\dots], \end{aligned}$$

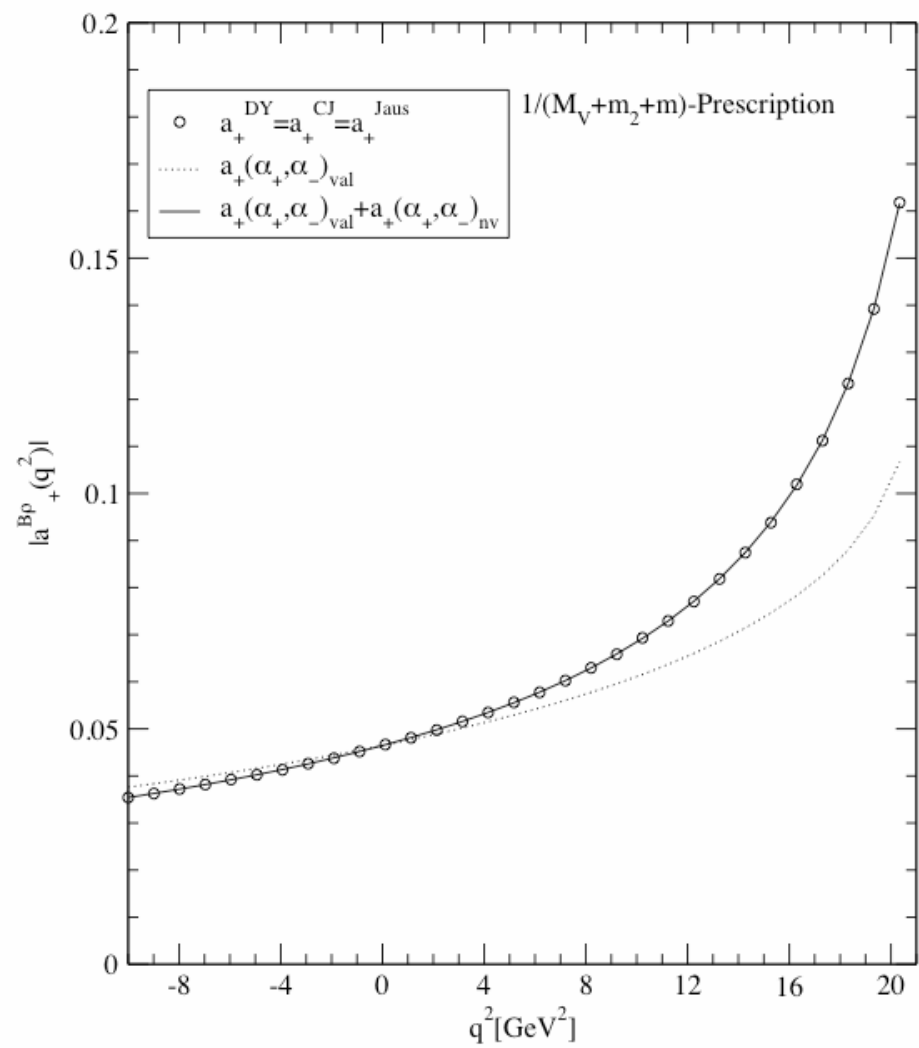
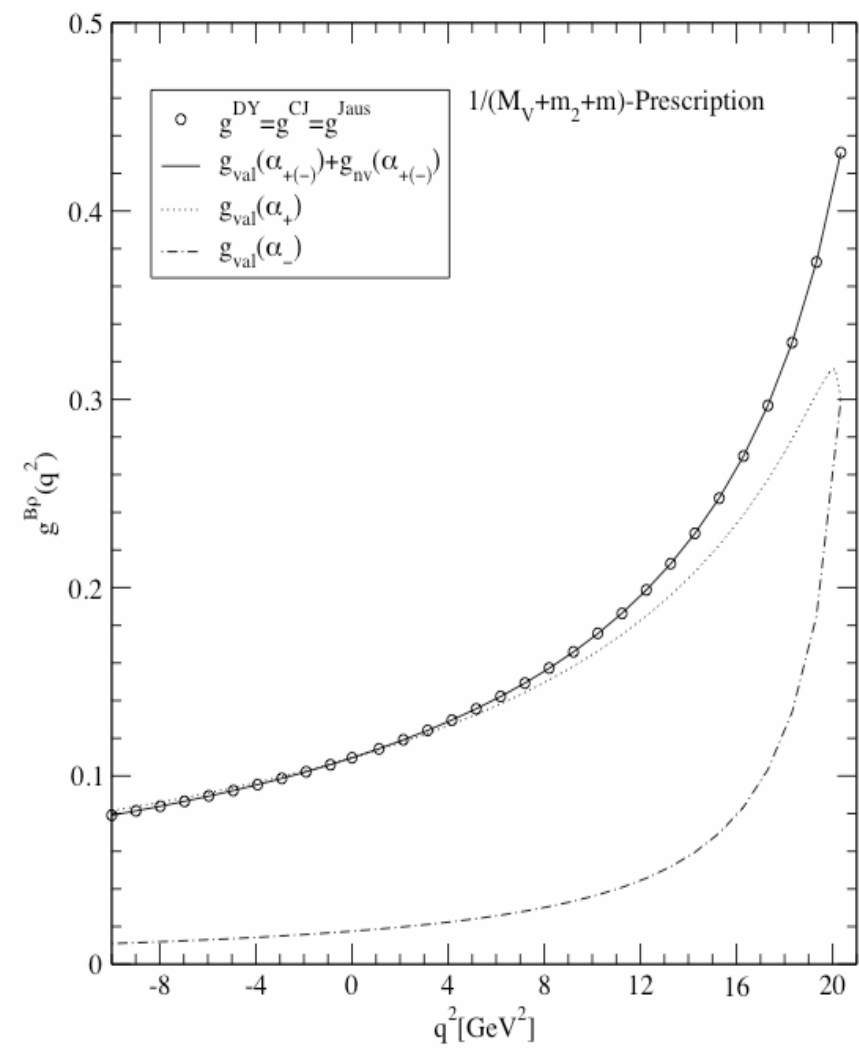
where

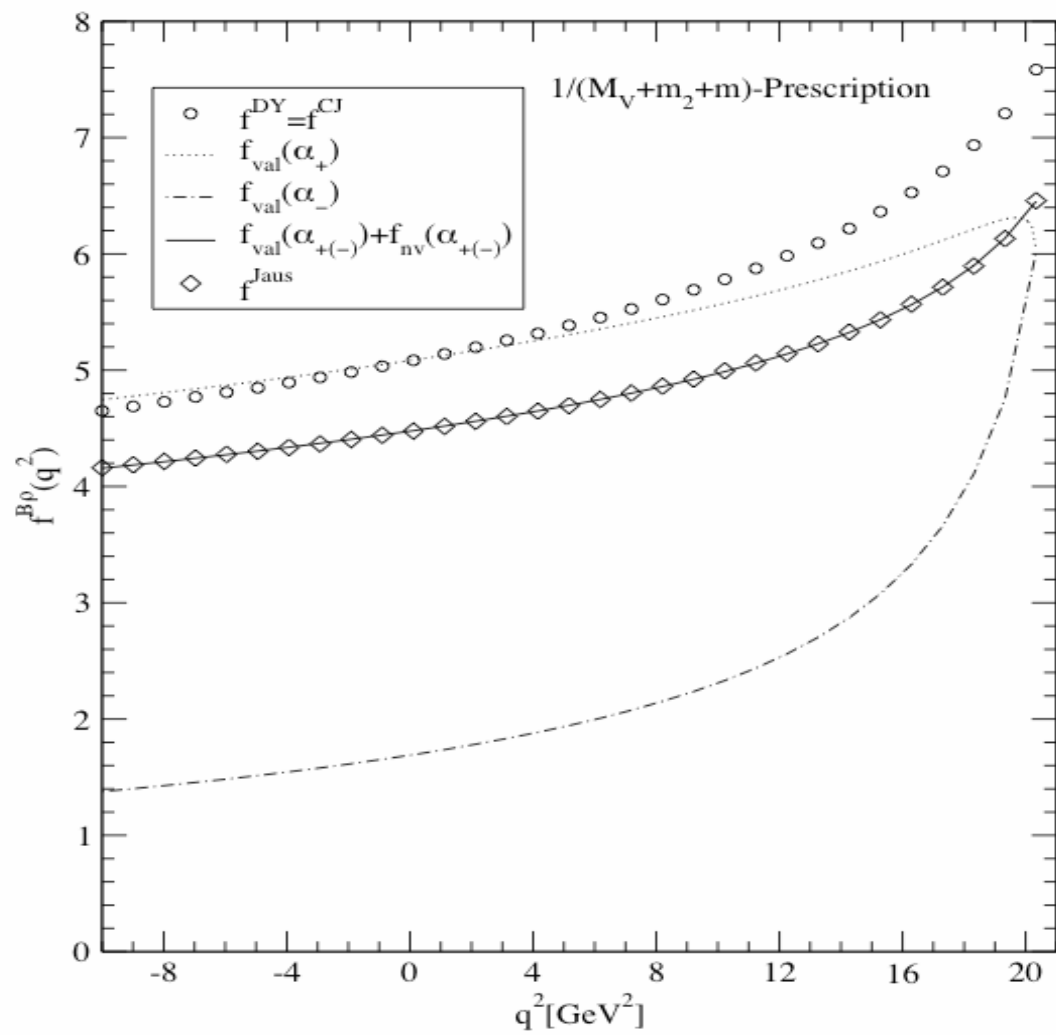
$$x = \alpha + (1-\alpha)z \quad \text{and} \quad [\dots] \text{ is regular as } \alpha \rightarrow 1.$$

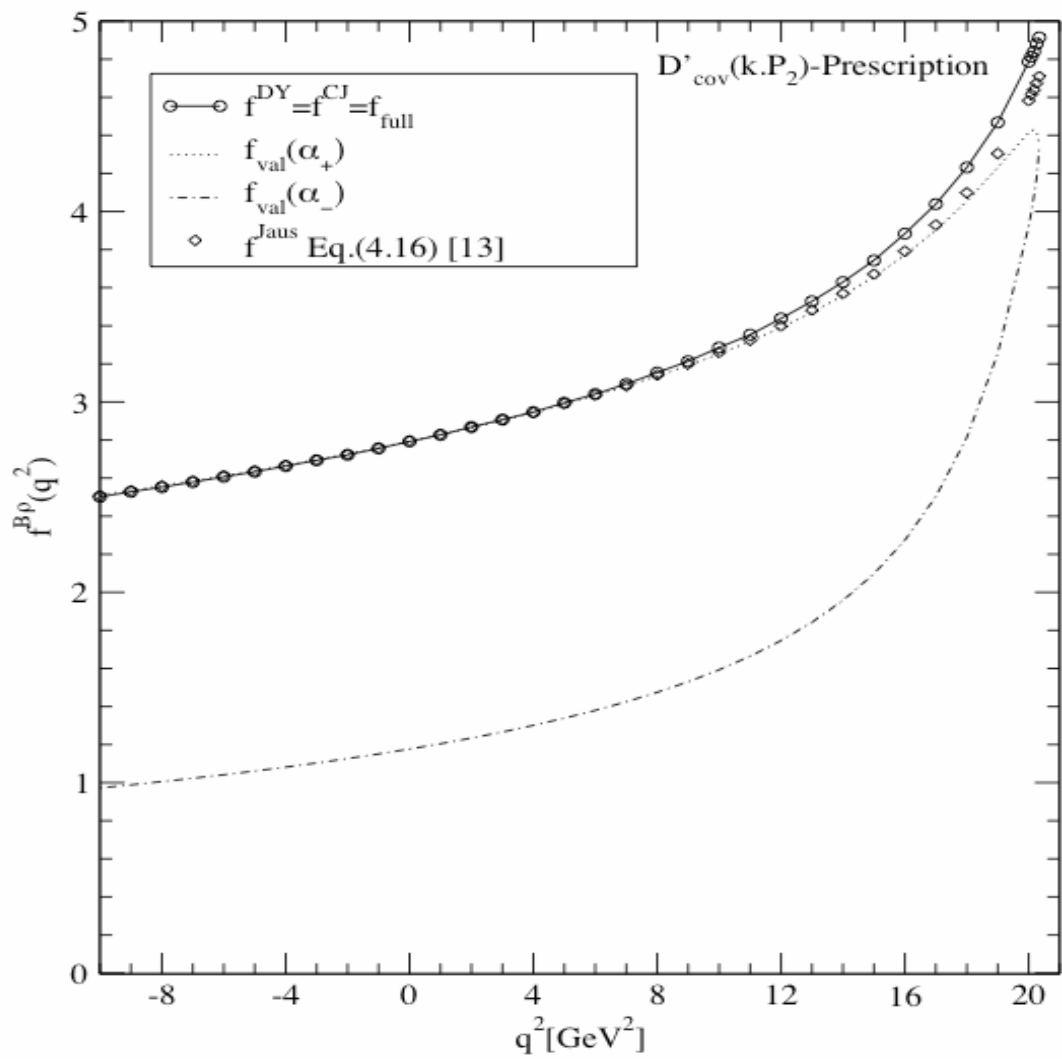
$S_{h=0}^+$  Power Counting:

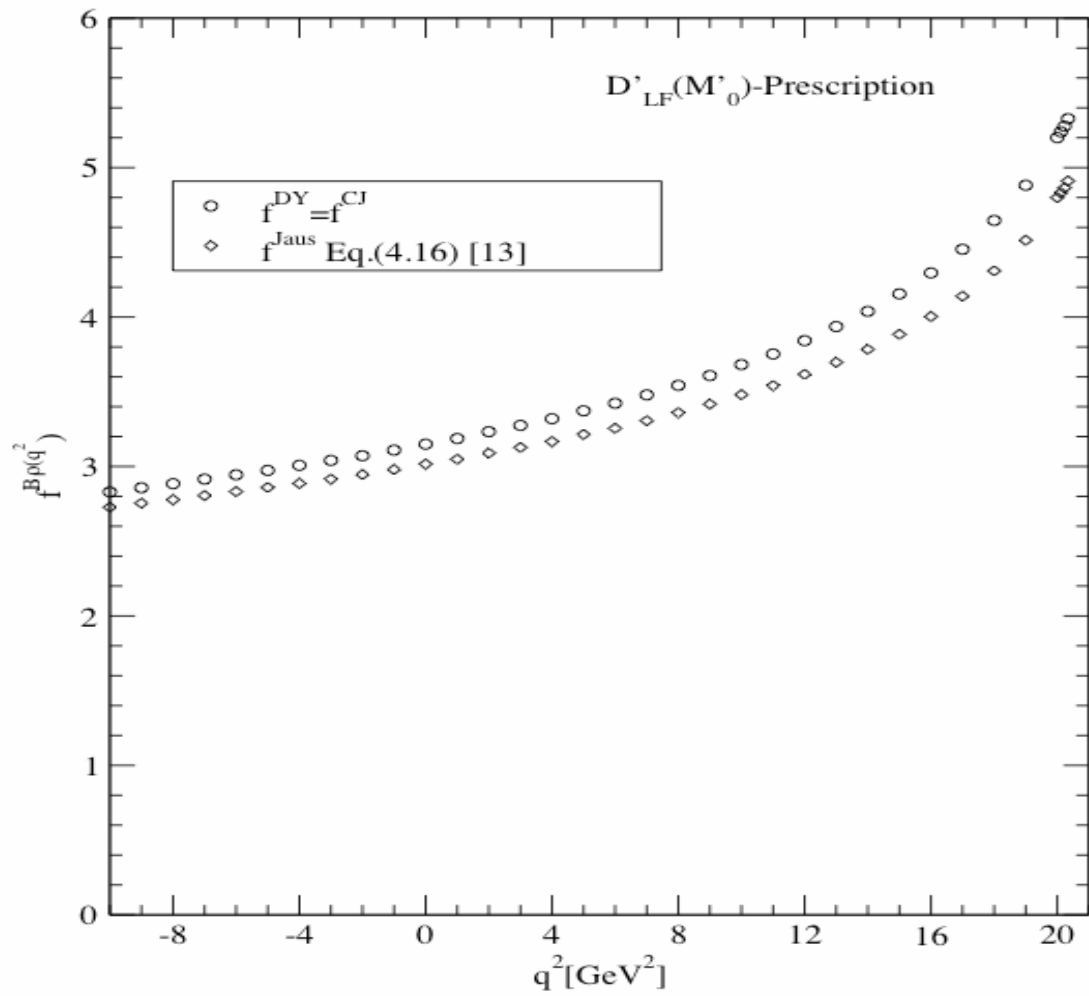
- (1)  $(1-x)^{-1} = [(1-\alpha)(1-z)]^{-1}$  for  $D_{\text{cov}}(M_V)$ ,
- (2)  $(1-x)^0$  for  $D_{\text{cov}}(k \cdot P_2)$ ,
- (3)  $(1-x)^{-1/2} = [(1-\alpha)(1-z)]^{-1/2}$  for  $D_{\text{LF}}(M_0)$ .











# Conclusions

- LFD provides a unified framework to analyze various hadron phenomenologies in Jlab, RHIC, B-factories, etc.
- LFQM progressed in calculations of meson spectra and wavefunctions making some basis for the extension to the study of baryons.
- New precision data can scrutinize the model parameters.
- For the good phenomenology, it is significant to correctly pin down the zero-mode contribution and the power counting method offers a good way to do this.