The Berry phase
Its derivation and its role in UCN EDM experiments

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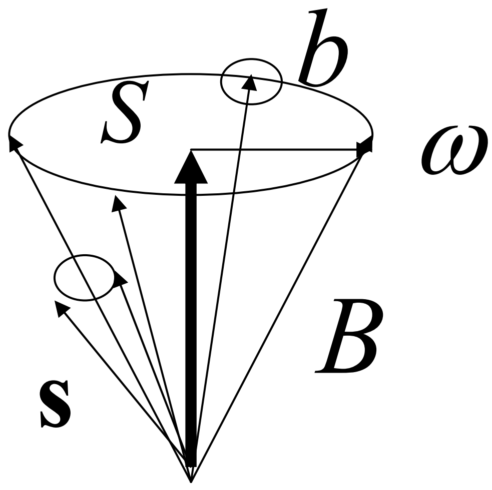
INT-1.06 2007

What is it

$$\varepsilon = \frac{\omega}{\gamma B} << 1$$

$$\varphi(t) = \varphi_d(t) + \varphi_g(t)$$



$$\varphi_d(t) = \gamma B t / 2$$

$$\gamma = \frac{2\mu}{\hbar}$$

$$\varphi_g(t) = \frac{1}{2}\frac{b^2}{2B^2}\omega t$$

$$T = \frac{2\pi}{\omega}$$

$$\theta_B = \varphi_g(T) = \frac{1}{2}\frac{S}{B^2}$$

$$S = \pi b^2$$

In fact

$$\varphi(t,\varepsilon) = \varphi(t,0) + \varepsilon \frac{d}{d\varepsilon}\varphi(t,\varepsilon)\big|_{\varepsilon=0}$$

$$\varphi_d(t) = \varphi(t,0)$$

$$\varphi_g(t) = \varepsilon \frac{d}{d\varepsilon}\varphi(t,\varepsilon)\big|_{\varepsilon=0}$$

An analytically solvable example

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}-\boldsymbol{\mu}\left(\mathbf{B}_0+\mathbf{B}_{rf}(t)\right)\right]\Psi(x,t)$$

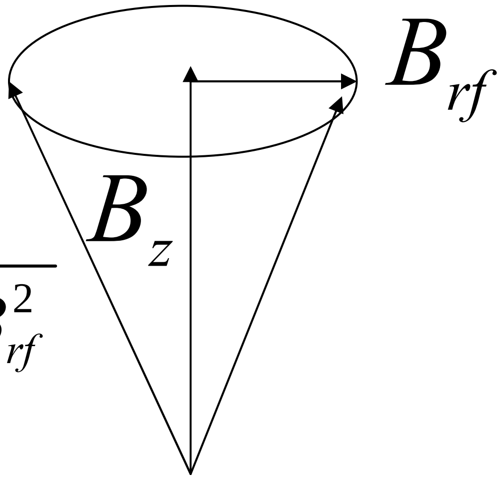
$$\boldsymbol{\mu}=-\mu_n\boldsymbol{\sigma} \qquad \boldsymbol{\sigma}=(\sigma_x.\sigma_y.\sigma_z)$$

$$\Psi(x,t)=\exp\left(ikx-i\frac{\hbar k^2}{2m}t\right)\Phi(t)$$

$$i\frac{\partial}{\partial t}\Phi(t)=\frac{\gamma}{2}\boldsymbol{\sigma}\left(\mathbf{B}_0+\mathbf{B}_{rf}(t)\right)\Phi(t)$$

$$\mathbf{B}_0=(0,0,B_z) \qquad \mathbf{B}_{rf}(t)=B_{rf}\left(\cos(\omega t),\sin(\omega t),0\right)$$

What can we expect?



$$S = \pi B_{rf}^2$$

$$\theta_B = \frac{1}{2}\frac{\pi B_{rf}^2}{B^2}$$

$$B = \sqrt{B_z^2 + B_{rf}^2}$$

$$\varphi_g(t) = \frac{1}{2}\frac{B_{rf}^2}{2B^2}\omega t$$

$$i\frac{\partial}{\partial t}\Phi(t) = \frac{\gamma}{2}\left(\sigma_z B_z + \boldsymbol{\sigma}\mathbf{B}_{rf}(t)\right)\Phi(t)$$

$$\mathbf{B}_{rf}(t) = B_{rf}\left(\cos(\omega t), \sin(\omega t), 0\right)$$

Solution

$$i\frac{\partial}{\partial t}\Phi(t)=\frac{\gamma}{2}\left(\sigma_z B_z+\boldsymbol{\sigma}\mathbf{B}_{rf}(t)\right)\Phi(t)$$

$$\boldsymbol{\sigma}\mathbf{B}_{rf}(t)=B_{rf}\left(\sigma_x\cos(\omega t)+\sigma_y\sin(\omega t)\right)=B_{rf}e^{-i\omega\sigma_z t/2}\sigma_x e^{i\omega\sigma_z t/2}$$

$$\Phi(t)=e^{-i\omega\sigma_z t/2}\xi(t) \qquad \Phi(0)=\xi(0)$$

$$i\frac{\partial}{\partial t}\xi(t)=\frac{1}{2}\left(\sigma_x\gamma B_{rf}+\sigma_z(\gamma B_z-\omega)\right)\xi(t)\equiv\frac{\boldsymbol{\Omega\sigma}}{2}\xi(t)$$

$$\boldsymbol{\Omega}\equiv\boldsymbol{\Omega}(0)=(\gamma B_{rf},0,\gamma B_z-\omega)$$

$$\xi(t)=\exp\left(-i\boldsymbol{\Omega\sigma}t/2\right)\xi(0)$$

$$\Phi(t)=e^{-i\omega\sigma_z t/2}e^{-i\boldsymbol{\Omega\sigma}t/2}\Phi(0)$$

Solution

$$\Phi(t) = e^{-i\omega\sigma_z t/2} e^{-i\boldsymbol{\Omega\sigma} t/2} \Phi(0)$$

$$\Phi(0) \equiv \Phi_{\boldsymbol{\Omega}(0)} = \frac{I + \mathbf{o}(0)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}} \xi_u \qquad \mathbf{o}(0) = \frac{\boldsymbol{\Omega}(0)}{\Omega} \qquad \xi_u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Phi(t) = e^{-i\Omega t/2} e^{-i\omega\sigma_z t/2} \frac{I + \mathbf{o}(0)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}} e^{i\omega\sigma_z t/2} e^{-i\omega\sigma_z t/2} \xi_u =$$

$$= e^{-i\Omega t/2} e^{-i\omega t/2} \frac{I + \mathbf{o}(t)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}} \xi_u = e^{-i(\Omega+\omega)t/2} \Phi_{\boldsymbol{\Omega}(t)} = e^{-i\chi t} \Phi_{\boldsymbol{\Omega}(t)}$$

$$\mathbf{o}(t)\boldsymbol{\sigma} = e^{-i\omega\sigma_z t/2} \mathbf{o}(0)\boldsymbol{\sigma} e^{i\omega\sigma_z t/2} = \frac{\boldsymbol{\Omega}(t)\boldsymbol{\sigma}}{\Omega}$$

$$\boldsymbol{\Omega}(t) = \left(\gamma B_{rf} \cos(\omega t),\ \gamma B_{rf} \sin(\omega t),\ \gamma B_z - \omega\right)$$

Final representation

$$\Phi(t) = e^{-i\chi t}\,\frac{I+\mathbf{o}(t)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}}\,\xi_u \qquad \chi = \frac{\Omega + \omega}{2}$$

$$|\,\boldsymbol{\Omega}(t)\,| = \Omega(0) = \sqrt{\gamma^2 B_{rf}^2 + (\gamma B_z - \omega)^2}$$

Expansion over adiabatic parameter

$$\varepsilon = \frac{\omega}{\gamma B} << 1 \qquad \Omega = \sqrt{\gamma^2 B_{rf}^2 + (\gamma B_z - \omega)^2} \approx \gamma B - \frac{\omega B_z}{B}$$

$$\chi = \frac{\Omega + \omega}{2} \approx \frac{\gamma B}{2} + \frac{B - B_z}{B}\frac{\omega}{2} \approx \frac{\gamma B}{2} + \frac{B_{rf}^2}{2B^2}\frac{\omega}{2}$$

$$B = \sqrt{B_{rf}^2 + B_z^2} \qquad B_z = \sqrt{B^2 - B_{rf}^2}$$

The opposite limit

$$\varepsilon = \frac{\omega}{B_z} >> 1 \qquad \frac{\gamma^2 B_{rf}^2}{\omega^2} << 1$$

$$\Omega = \sqrt{\gamma^2 B_{rf}^2 + (\gamma B_z - \omega)^2} \approx \sqrt{\gamma^2 B_{rf}^2 + \omega^2} - \frac{\gamma B_z \omega}{\sqrt{\gamma^2 B_{rf}^2 + \omega^2}}$$

$$\chi = \frac{\Omega + \omega}{2} \approx \omega - \frac{\gamma B_z}{2} + \frac{\gamma^2 B_{rf}^2}{4\omega^2}\omega \qquad \varepsilon >> 1$$

$$\chi = \frac{\Omega + \omega}{2} \approx \frac{\gamma B}{2} + \frac{B_{rf}^2}{2B^2}\frac{\omega}{2} \qquad \varepsilon << 1$$

Arbitrary polarization

$$\Phi(0)=\alpha_{\Omega}\Phi_{\Omega(0)}+\alpha_{-\Omega}\Phi_{-\Omega(0)}$$

$$\alpha_{\pm\Omega}=\Phi^{+}_{\pm\Omega(0)}\Phi(0)$$

$$\Phi_{\Omega(0)}=\frac{I+\mathbf{o}(0)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}}\xi_u$$

$$\Phi_{-\Omega(0)}=\frac{I-\mathbf{o}(0)\boldsymbol{\sigma}}{\sqrt{2(1+o_z)}}\xi_d$$

$$\Phi(t)=\alpha_{\Omega}e^{-i\chi t}\Phi_{\Omega(t)}+\alpha_{-\Omega}e^{i\chi t}\Phi_{-\Omega(t)}$$

$$\mathbf{s}(t)=\Phi^{+}(t)\boldsymbol{\sigma}\Phi(t)$$

$$s_{x,y}(t)=2\,|\,\alpha^{*}_{-\Omega}\alpha_{\Omega}\,|\left(\cos\eta-o_{x,y}(t)\frac{B_{rf}\cos(\omega t-\eta)}{\Omega+\Omega_z}\right)+\left(|\,\alpha_{\Omega}\,|^{2}-|\,\alpha_{-\Omega}\,|^{2}\right)o_{x,y}(t)$$

$$s_z(t)=2\,|\,\alpha^{*}_{-\Omega}\alpha_{\Omega}\,|\frac{B_{rf}\cos(\omega t-\eta)}{\Omega}+\left(|\,\alpha_{\Omega}\,|^{2}-|\,\alpha_{-\Omega}\,|^{2}\right)o_z$$

$$\eta=(\omega+\Omega)t+\varphi$$

$$\varphi=\arg\left(\alpha^{*}_{-\Omega}\alpha_{\Omega}\right)$$

Application to EDM experiments

$$\hbar\omega = 2\mu_n \boldsymbol{\sigma}(\mathbf{B} + d_n \mathbf{E}) \qquad \mu_n = |\mu_n| \qquad d_n = \frac{eD}{|\mu_n|}$$

$$D_n = 2\cdot 10^{-26}\,\mathrm{cm} \Rightarrow \mathrm{d_n} = 10^{-12}$$

$$\omega_{\uparrow\uparrow} = \gamma(B + dE) \qquad \omega_{\uparrow\downarrow} = \gamma(B - dE) \qquad \gamma = \frac{2\mu}{\hbar}$$

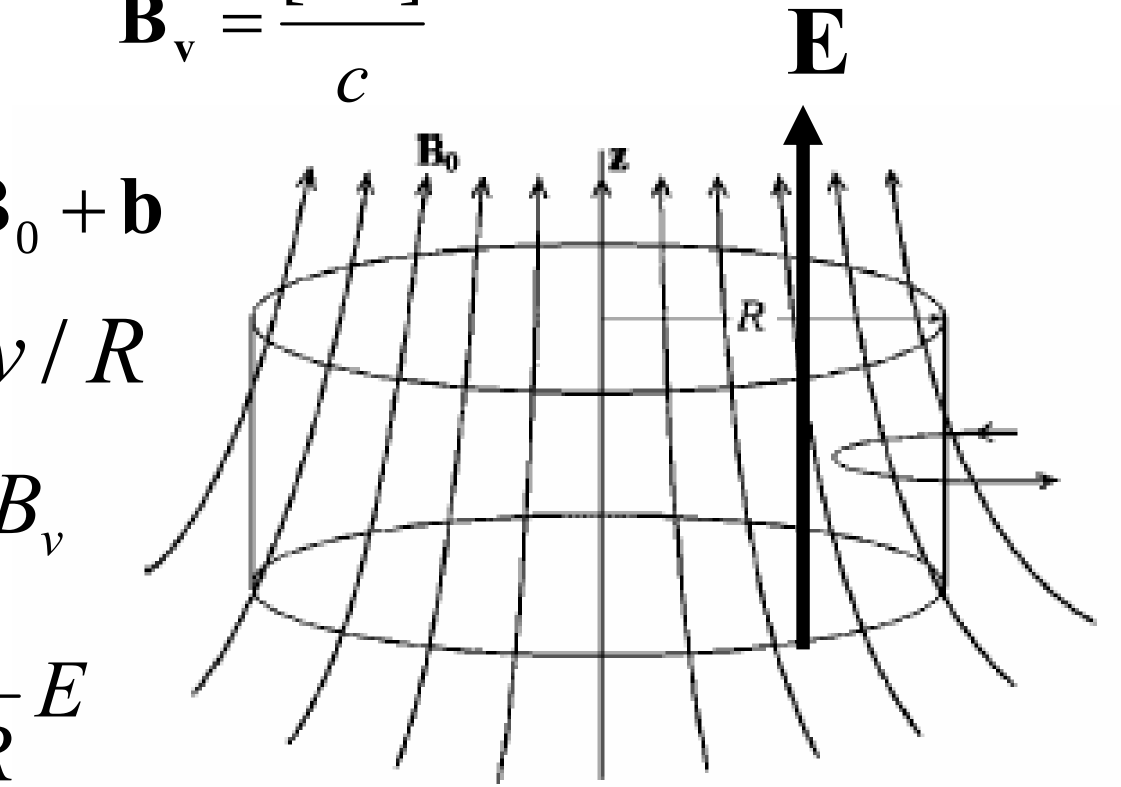
$$\Delta\omega = \omega_{\uparrow\uparrow} - \omega_{\uparrow\downarrow} = 2\gamma dE \qquad d = \frac{\Delta\omega}{2\gamma E}$$

A false effect $\qquad \mathbf{B}_v = \frac{[\mathbf{vE}]}{c}$

$$\varphi_g(t) = \frac{B_v^2}{2B_z^2}\frac{\omega_v t}{2} \propto E^2 \qquad \omega_v \propto \frac{v}{R}$$

Application to EDM experiments

$$\mathbf{B_r} = -\frac{\partial B_z}{\partial z}\frac{\mathbf{r}}{2} = -B'\frac{\mathbf{r}}{2} \qquad \mathbf{B_v} = \frac{[\mathbf{vE}]}{c}$$

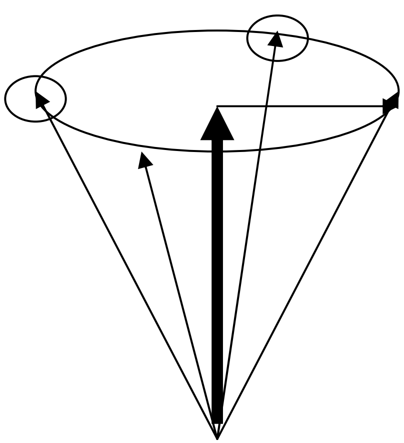


$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B_r} + \mathbf{B_v} = \mathbf{B}_0 + \mathbf{b}$$

$$\mathbf{b} = \mathbf{B_r} + \mathbf{B_v} \qquad \omega_v \approx v / R$$

$$b^2 = B_v^2 + B_r^2 + 2B_r B_v$$

$$B_v \omega_v \approx \frac{v^2}{cR} E$$

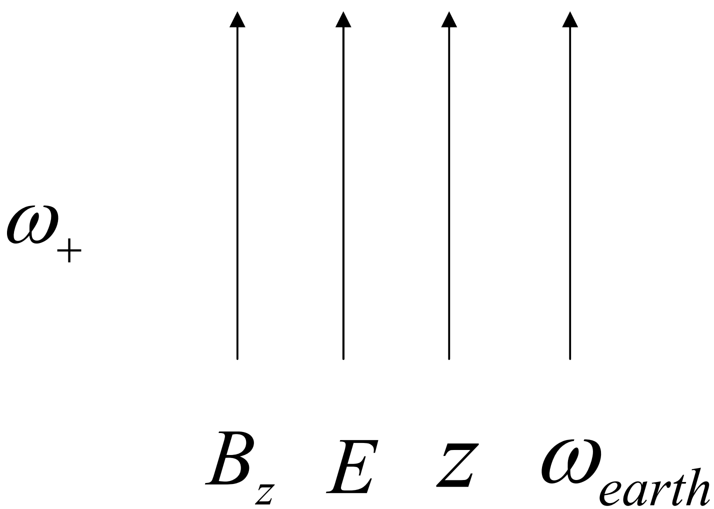


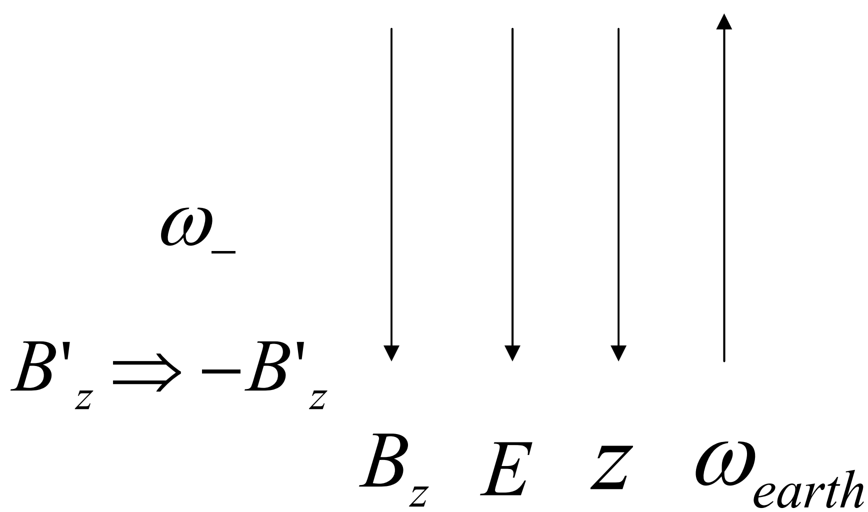
$$\chi t \approx \gamma B_z t + \frac{B_r B_v}{B_z^2}\frac{\omega_v t}{2} \approx$$

$$\approx \gamma B_z t - B'_z \frac{v^2}{cB_z^2} Et$$

All the fields for precession

$$\hbar\omega = 2\mu\left[\int dz\rho(z)B_z(z) + \left(d + B'_z\frac{\alpha v^2}{c\gamma B_z^2}\right)E - \frac{\omega_{earth}}{\gamma}\right]$$



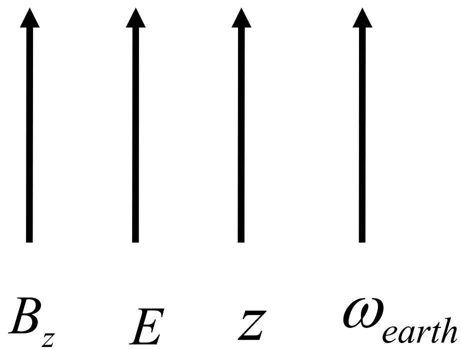


$$\omega_+ + \omega_- = 2\gamma\left[\int dz\rho(z)B_z(z) + dE\right]$$

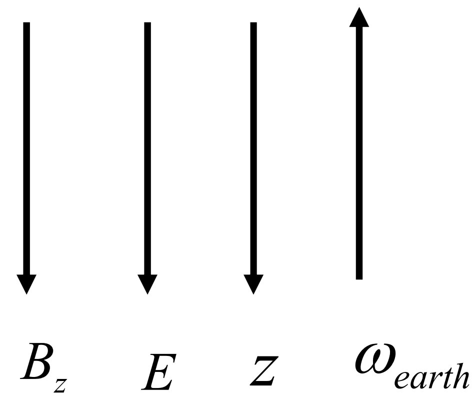
$$\frac{\omega_{earth}}{\gamma B_z} \Rightarrow -\frac{\omega_{earth}}{\gamma B_z}$$

$$\overline{\omega}(E) = \frac{\omega_+ + \omega_-}{2} = \gamma\left[\int dz\,\rho(z) B_z(z) + dE\right]$$

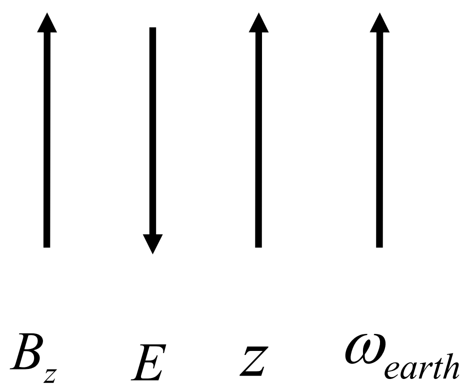
$\omega_+(E)$



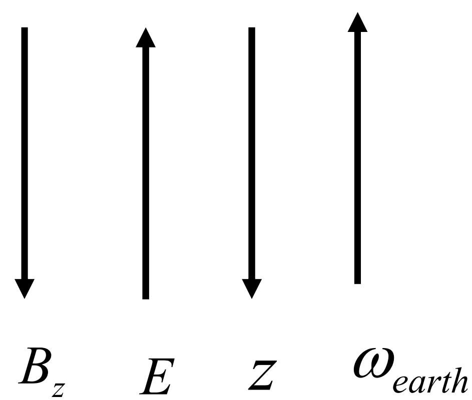
$\omega_-(E)$



$\omega_+(-E)$



$\omega_-(-E)$



$$\overline{\omega}(E) - \overline{\omega}(-E) = 2\gamma dE$$

Usually measurements are with Hg

$$R_a(\uparrow,\uparrow) \equiv \frac{\omega_n \mu_{Hg}}{\omega_{Hg}\mu_n} = \frac{\int dz \rho_n(z) B_z(z) + \left(d_n + B'_z \frac{\alpha_n v_n^2}{c\gamma_n B_z^2} \right) E - \frac{\omega_{earth}}{\gamma_n}}{\int dz \rho_{Hg}(z) B_z(z) + \left(d_{Hg} + B'_z \frac{\alpha_{Hg}\gamma_{Hg} R^2}{c} \right) E + \frac{\omega_{earth}}{\gamma_{Hg}}}$$

$$T_a(\uparrow_B) \equiv R_a(\uparrow_B, \uparrow_E) - 1 \approx -\frac{\Delta h B'_z}{B_z} +$$

$$+ d_n \frac{E}{B_z} - \left(\frac{1}{\gamma_n} + \frac{1}{\gamma_{Hg}} \right) \frac{\omega_{earth}}{B_z} + B'_z \left(\frac{\alpha_n v_n^2}{c\gamma_n B_z^2} - \frac{\alpha_{Hg}\gamma_{Hg} R^2}{c} \right) \frac{E}{B_z}$$

$$d_{Hg} \approx 0$$

In the experiment B'_z is varied with trimming coils

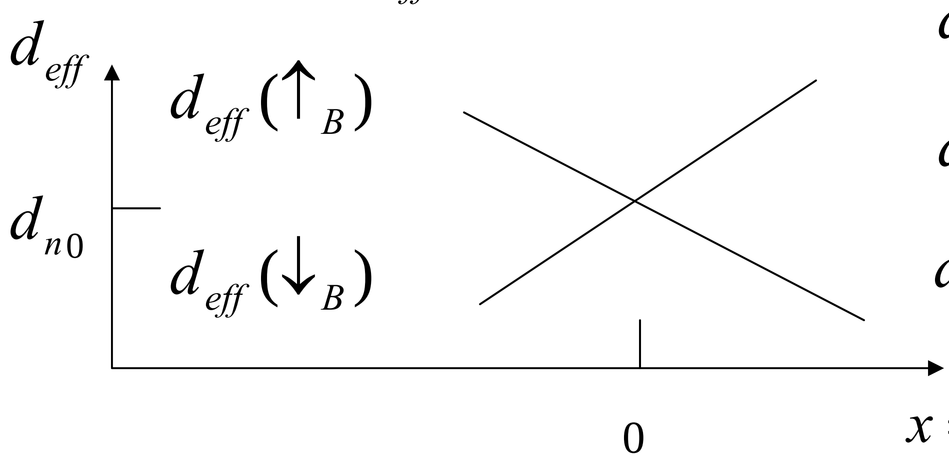
If you do not account for earth rotation

$$d_{eff}(\uparrow_B) = \frac{1}{2}\left(R_a(\uparrow_B, \uparrow_E) - R_a(\uparrow_B, \downarrow_E)\right) \approx (d_n - QB'_z)\frac{E}{B_z}$$

$$d_{eff}(\downarrow_B) = \frac{B_z}{2E}\left(R_a(\downarrow_B, \downarrow_E) - R_a(\downarrow_B, \uparrow_E)\right) \qquad Q = \frac{\alpha_n v_n^{\ 2}}{c\gamma_n B_z^2} - \frac{\alpha_{Hg}\gamma_{Hg} R^2}{c}$$

$$T_a(\uparrow_B) \approx R_a(\uparrow_B, \uparrow_E) - 1 \approx -\frac{\Delta h B'_z}{B_z} \qquad T_a(\downarrow_B) \approx R_a(\downarrow_B, \downarrow_E) - 1 \approx -\frac{\Delta h B'_z}{B_z}$$

$$d_{eff} = d_n + QB'_z$$



$$d_{eff}(\downarrow_B, x) = b + \alpha x$$

$$d_{eff}(\uparrow_B, x) = a - \alpha x$$

$$d_n - d_{Hg} = (a+b)/2$$

$$x = -\Delta h B'_z / E$$

If you do account for earth rotation

$$T_a(\uparrow_B) \approx -\frac{\Delta h B'_z}{B_z} - \frac{\omega_{earth}}{\gamma B_z}$$

$$T_a(\downarrow_B) \approx -\frac{\Delta h B'_z}{B_z} + \frac{\omega_{earth}}{\gamma B_z}$$

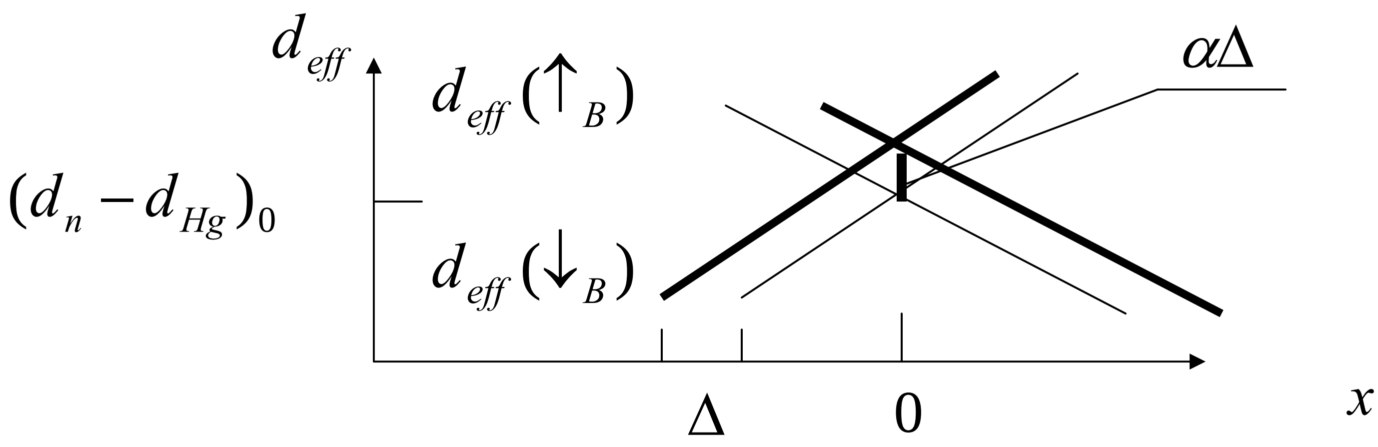
$$-hB'_z / B_z = T_a(\uparrow_B) + \omega_{earth} / \gamma B_z$$

$$-hB'_z / B_z = T_a(\downarrow_B) - \omega_{earth} / \gamma B_z$$

$$d_{eff}(\uparrow_B, x) = a - \alpha(x - \Delta)$$

$$d_{eff}(\downarrow_B, x) = b + \alpha(x + \Delta)$$

$$d_n = \frac{a+b}{2} + \alpha\Delta \approx d_{n0} + \alpha\Delta$$



$$\Delta = \omega_{earth} / \gamma E$$

You do not need to calculate the geometric phase,
Because your goal is to exclude it

However, if you want to measure it, It is possible
to calculate it with any required precision

I think that such an experiment will be
even more interesting than the EDM search!

References

- C. A. Baker et al., Phys. Rev. Lett. 97 (2006) 131801.
- J. Pendlebury et al., Phys. Rev. A 70 (2004) 032102.
- S.K. Lamoreaux and R. Golub, Phys. Rev. Lett. 98 (2007) 149101.
- Baker *et al.* Reply

  Phys. Rev. Lett. **98**, 149102 (2007)

The end,

Thank you

Special thanks to Chris Crawford for consultations