Order-α radiative correction calculations for neutron and nuclear beta decays

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Particle physics information from low energy beta decays; advantages compared to high energy beta decays

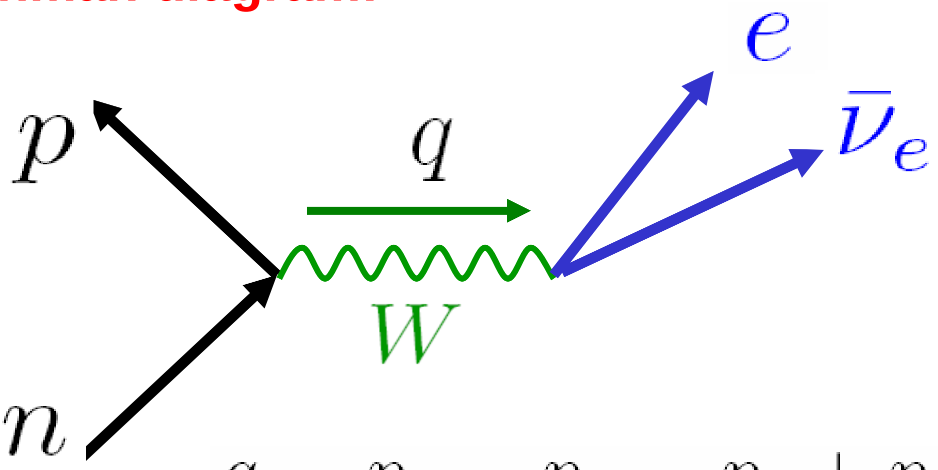
Photon bremsstrahlung

Virtual part of radiative correction

Model independent radiative correction results

Model dependent corrections

neutron decay Feynman diagram:



$$q = p_n - p_p = p_e + p_\nu$$

SM decay amplitude:

$$\mathcal{M}_0 = \frac{G_V}{\sqrt{2}} h_\mu l^\mu$$

$$l^\mu = \bar{u}_e \gamma^\mu (1 - \gamma_5) v_\nu, \quad h_\mu = \bar{u}_p H_\mu u_n$$

General spin-1/2 → spin-1/2 semileptonic decay:

$$H_\mu = f_1(q^2)\gamma_\mu + f_2(q^2)\sigma_{\mu\alpha}q^\alpha/m_n + f_3(q^2)q^\mu/m_n + \\ + \left(g_1(q^2)\gamma_\mu + g_2(q^2)\sigma_{\mu\alpha}q^\alpha/m_n + g_3(q^2)q^\mu/m_n\right)\gamma_5$$

6 complex form factor functions !

Dipole approximation for small q :

$$f_i(q^2) \approx f_i(0) + C_{fi}\frac{q^2}{m_n^2}, \quad g_i(q^2) \approx g_i(0) + C_{gi}\frac{q^2}{m_n^2}$$

$$q \approx MeV, m_n \approx GeV$$

(C_{fi}, C_{gi} : order-1 numbers)

→ **q^2 dependence of form factors in neutron and in small decay energy nuclear beta decays is negligible (order-10^{-6} effect)**

Absence of second class currents → $f_3 = g_2 = 0$

Goldberger-Treiman relation → g_3 **negligible**

CVC (conserved vector current), SU(2) isospin symmetry:

$$f_1(0) = 1 \qquad f_2(0) = f_2 = \frac{\mu_p - \mu_n}{2}$$

Behrends-Sirlin-Ademollo-Gatto theorem: isospin symmetry breaking for vector form factors is quadratic (proportional to $(m_n - m_p)^2/m_n^2$)

→ CVC prediction is precise for the vector form factors f_1 and f_2 !

Time reversal invariance → form factors are real

Only 2 free parameters within the Standard Model:

G_V and $\lambda = G_A/G_V = g_1/f_1$

Vector coupling constant connected to muon decay coupling (lepton-quark universality with quark mixing):

$$G_V = G_\mu V_{ud}$$

V_{ud} : up-down element of CKM quark-mixing matrix

Neutron decay rate: $\tau^{-1} \sim G_V^2(1+3\lambda^2)$

(τ: lifetime)

λ can be determined from angular correlation experiments (electron and proton asymmetry, electron-neutrino correlation)

$\tau,\ \lambda \Longrightarrow G_V$ (vector coupling determination from neutron decay)

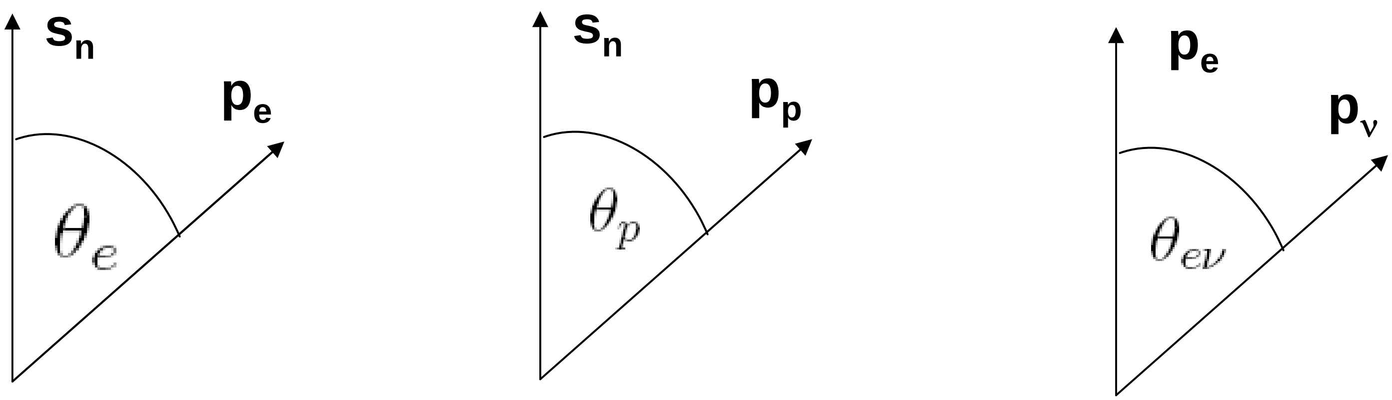
Comparison with $0^+ \rightarrow 0^+$ and π_{l3} lifetimes ⟹ CVC test

CKM unitarity test: $V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$

$V_{us}^2 \sim 10^{-2}, \quad V_{ub}^2 \sim 10^{-5}$ ⟹ main experimental and theoretical errors could come from V_{ud}

- **neutron lifetime important for big bang cosmology (H/He ratio, light element abundances)**
- **λ important for neutrino cross section calculations, quark models, pp fusion in Sun (pp→ $de^+\nu$: inverse neutron decay)**

Measurement of λ possible form various experiments: electron asymmetry, proton asymmetry, electron-neutrino correlation



Comparison of various λ values:

- test of experimental systematic effects (within framework of SM)

- test of non-standard weak couplings

Non-standard weak couplings

Standard Model:

zero neutrino mass, only left-handed coupling

Parametrization of phenomenological weak couplings by Lee, Yang (1956):

$$\mathcal{M}_\beta = \sum_j (\bar{u}_p \Gamma_j u_n)[\bar{u}_e \Gamma_j (C_j - C'_j \gamma_5) v_\nu]$$

$j = S, V, A, T$ (scalar, vector, axialvector, tensor)

Better to use linear combinations L_j and R_j :

$$C_j = \frac{G_W}{\sqrt{2}}(L_j + R_j) \qquad C'_j = \frac{G_W}{\sqrt{2}}(L_j - R_j)$$

(for advantages see: F. Glück et al., Nucl. Phys. A 593, 125 (1995))

Interference theorem

Beta decay amplitude is sum of left-handed and right-handed amplitudes:

$$\mathcal{M}_\beta = \mathcal{M}_L + \mathcal{M}_R$$

$$\mathcal{M}_L = \frac{G_W}{\sqrt{2}} \sum_j L_j (\bar{u}_p \Gamma_j u_n)[\bar{u}_e \Gamma_j (1 - \gamma_5) v_\nu],$$

$$\mathcal{M}_R = \frac{G_W}{\sqrt{2}} \sum_j R_j (\bar{u}_p \Gamma_j u_n)[\bar{u}_e \Gamma_j (1 + \gamma_5) v_\nu].$$

Zero neutrino mass:

no interference between left-handed and right-handed couplings

(see: F. Glück et al., Nucl. Phys. A 593, 125 (1995))

Standard (V-A) Model: L_V and L_A are non-zero

Beta decay observables (with zero neutrino mass) are quadratic in the small right-handed couplings

Good constraints for L_S, L_T, poor constraints for R_V, R_A, R_S, R_T

Particle physics information from beta decays

i, weak interaction

- parity violation (right-handed couplings?)
- CP and time reversal symmetry violation
- Lorentz structure (V, A, S, T)
- CKM unitarity, matrix elements
- neutrino properties (masses, lepton mixing)

ii, strong interaction

- form factors:
  meson β decay: $f_+(q^2)$, $f_-(q^2)$
  baryon β decay: $f_i(q^2)$, $g_i(q^2)$ (i=1,2,3)
  comparisons with quark models, Cabibbo model, CVC,
  Behrend-Sirlin-Ademollo-Gatto theorem
- quark masses

Advantages of low energy beta decays

- q^2 dependence of form factors: small or negligible
- recoil-order form factors: small; reliable theoretical predictions
- (CVC, absence of second class currents, Goldberger-Treiman rel.)
- SU(2) symmetry breaking effects: small
  - neutron decay: $f_1(0)=1$
  - $0^+ \rightarrow 0^+$ Fermi decays: symm. breaking corr. can be calculated
- radiative correction calc.: reliable
- (f.e.: no strong interaction uncertainties in photon bremsstrahlung)

High energy beta decays:

- sensitive to strong interaction models
- precise V-A test is difficult

Low energy beta decays:

- not so sensitive to strong interaction models
- precise V-A test is easier

Photon bremsstrahlung

Bloch-Nordsieck theorem (1937)

Charged particle processes: bremsstrahlung (BR) photons are always present; probability(no BR photons)=0

Finite energy resolution: only $K>K_{min}$ BR events can be distinguished from processes without any photons

neutron decay: K_{max}=780 keV

with K_{min}=1 keV:

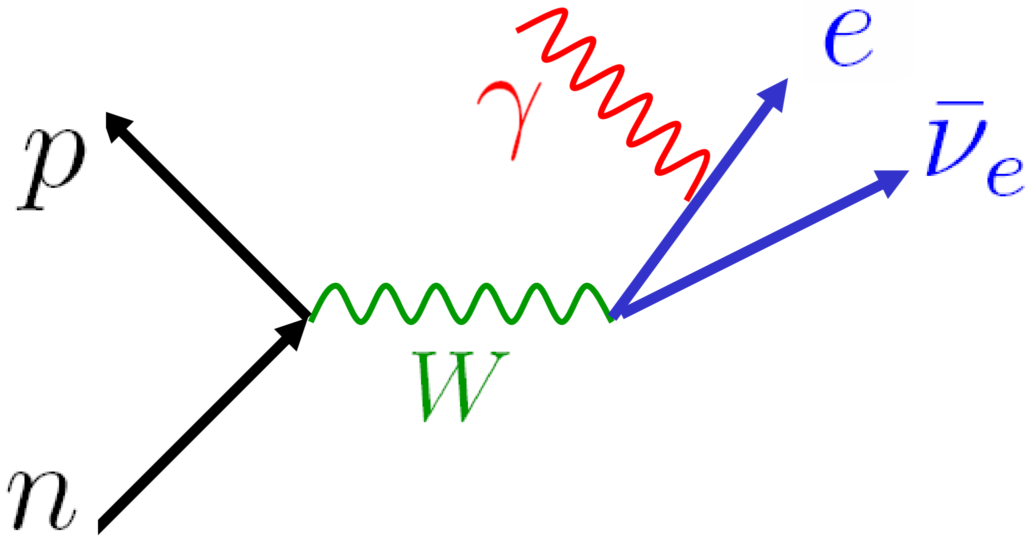
$P(1\ \gamma)=0.5\ \%$

$P(2\ \gamma)=0.001\ \%$

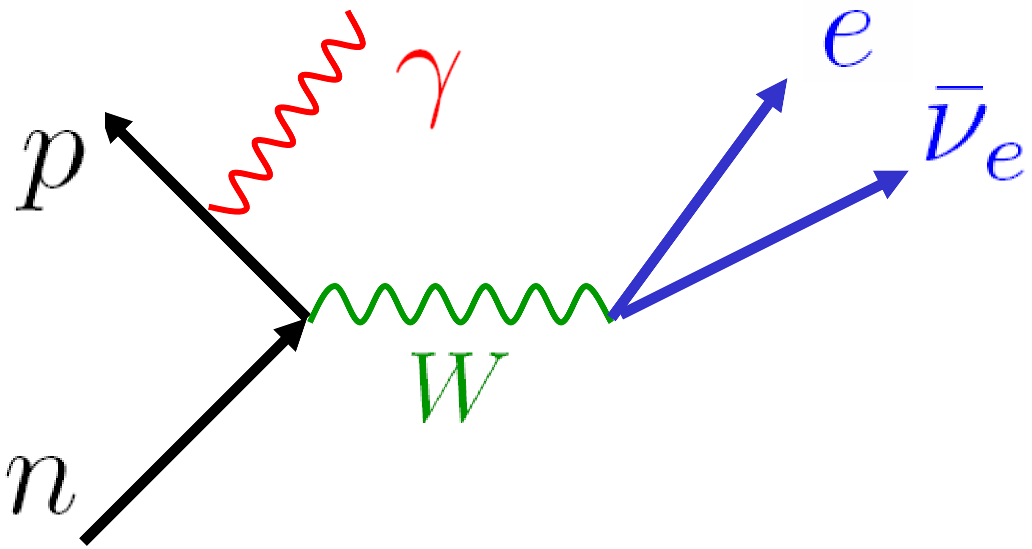
internal photon bremsstrahlung in neutron decay:

$$n \longrightarrow p e \bar{\nu}_e \gamma$$

$\mathcal{M}_l$

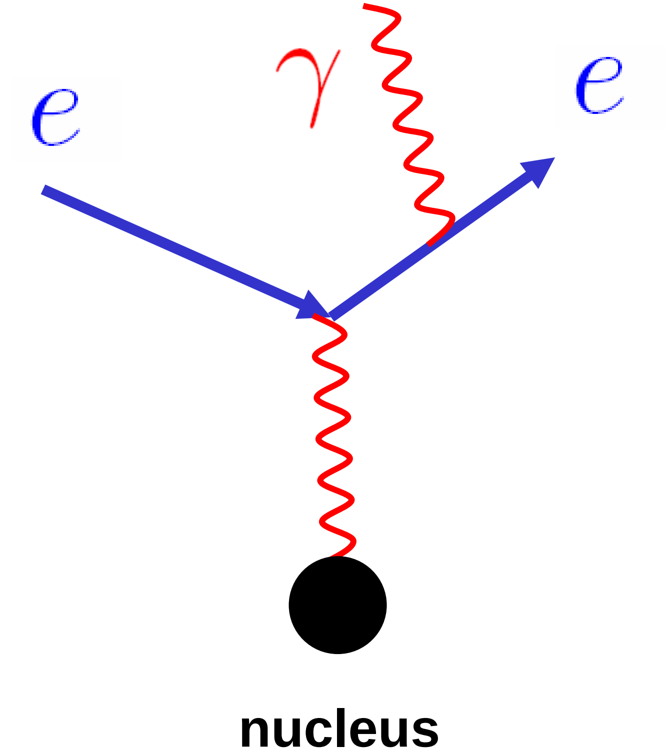


$\mathcal{M}_h$



Internal (inner) bremsstrahlung completely different from external BR. External BR is independent of the decay, internal BR occurs during the decay.

external BR of electron:



Possible confusion: inner and outer radiative correction.
The inner radiative correction (completely virtual process) has nothing to do with the inner bremsstrahlung !

Photon bremsstrahlung amplitude (gauge invariant):

$$\mathcal{M}_{BR} = \mathcal{M}_l + \mathcal{M}_h$$

$\mathcal{M}_l \longrightarrow$ QED (accurate,reliable calc.)

$\mathcal{M}_h \longrightarrow$ generally model (strong int.) dependent

BUT !

BR photon energy in neutron decay) < 0.78 MeV

$\longrightarrow$ BR photon wavelength >> 1 fm

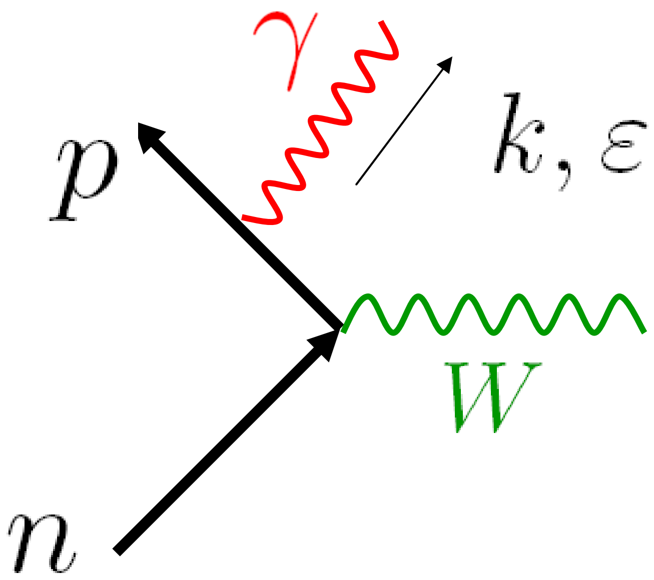
BR photons in neutron decay can see only the proton charge (and slightly the nucleon magnetic moment), but not the inner structure of the nucleons !

Order-K^{-1} part of the hadronic BR amplitude:

$$\mathcal{M}_h[K^{-1}] = e\frac{(p\varepsilon)}{(pk)}\mathcal{M}_0$$

$$k = (K, \mathbf{k}), \quad K = |\mathbf{k}|$$

$\mathcal{M}_0$ **: zeroth-order amplitude (without radiative corr.)**



→ 1/K behaviour of low energy BR photon spectrum

Low theorem **(F. E. Low, Phys. Rev. 110 (1958) 974)**

From EM current conservation (gauge invariance) the order-K^0 part (next order, subleading) of the hadronic BR amplitude can also be reliably (model independently) computed

(depends on magnetic moments of the nucleons)

Many experimental tests of Low theorem in high energy decay and scattering processes

From Low theorem: only the order-K part of the BR photon amplitude is model dependent

$$O(K^0) \sim \frac{K}{m_n} O(K^{-1}) \sim 10^{-3} \cdot O(K^{-1})$$ **(K=1 MeV)**

$$O(K) \sim \left(\frac{K}{m_n}\right)^2 O(K^{-1}) \sim 10^{-6} \cdot O(K^{-1})$$ **(K=1 MeV)**

→ 10^{-6} accuracy of photon BR calc. in neutron decay (for K=100 keV: 10^{-8} accuracy)

No information about strong interaction dynamics from photon bremsstrahlung in neutron decay !

Photon BR measurement in neutron decay: test of QED and Low theorem in a low energy weak decay process

Photon bremsstrahlung: **part of radiative correction, calc. in neutron and nuclear beta decays is accurate and reliable**

BR calculation: **- theoretically simple**
- technically complicated

Integration in many dimensional phase space:

$$\int \frac{d^3\mathbf{p}_p}{E_p}\frac{d^3\mathbf{p}_e}{E_e}\frac{d^3\mathbf{p}_\nu}{E_\nu}\frac{d^3\mathbf{k}}{K}\delta^4(p_n - p_p - p_e - p_\nu - k)\cdot\sum_{spin}|\mathcal{M}_{BR}|^2$$

$\sum_{spin}|\mathcal{M}_{BR}|^2$ **: computation by symbolic algebra program (Reduce)**

(Dirac matrix algebra, Lorentz-indices)

Phase space integration

i, analytical, semianalytical:

F. Glück, T. Toth, Phys. Rev. D 41, (1990) 2160,
Phys. Rev. 46 (1992) 2090;
F. Glück, Phys. Rev. D 47 (1993) 2840.

ii, Monte Carlo:

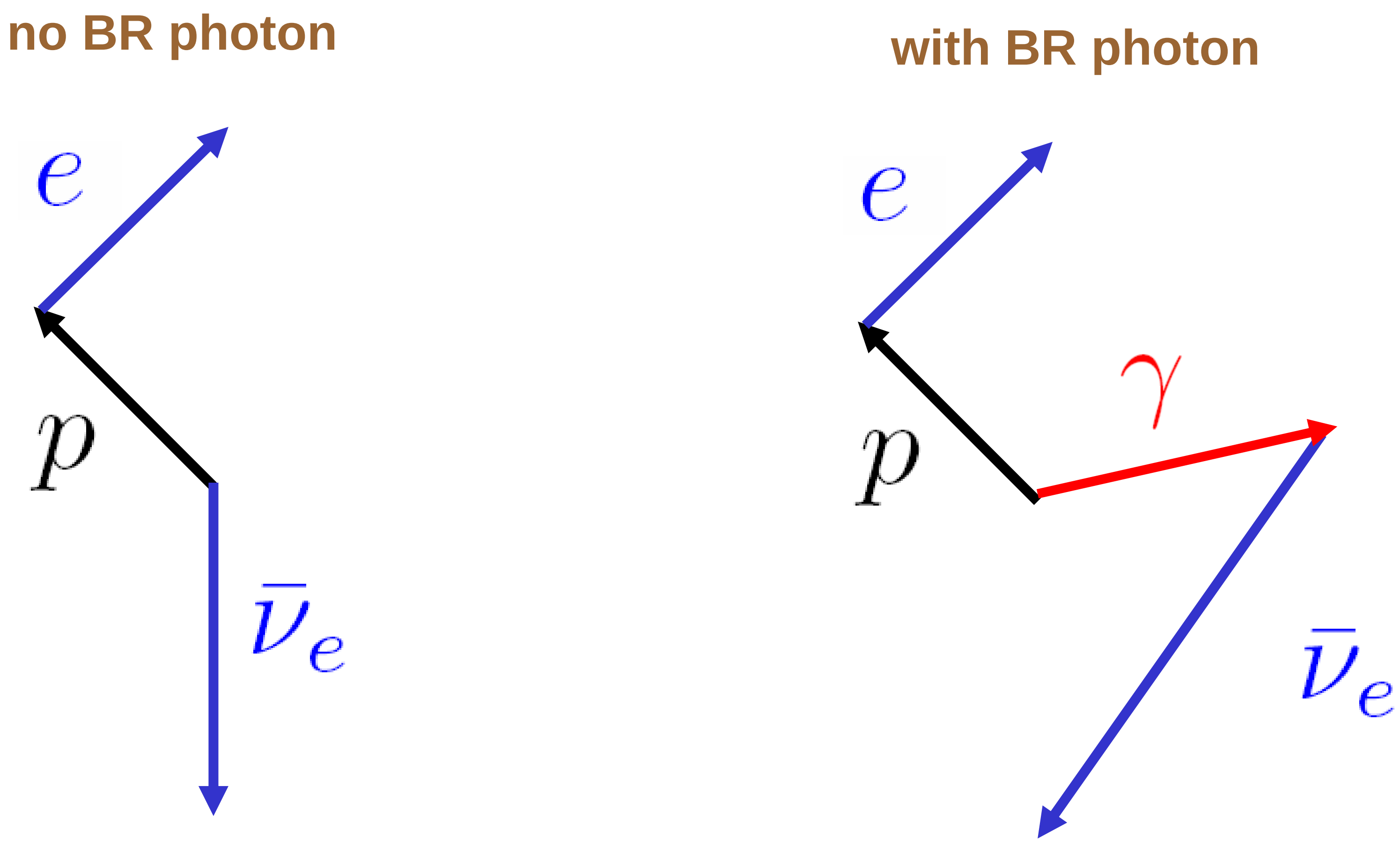
F. Glück, Comp. Phys. Comm. 101 (1997) 223.

Advantages of MC : easier, flexible for experimental details, any kind of quantity can be computed; few hundred million events can be generated within 1 hour computation time (Poisson error<0.1 %)

Many comparisons among various computation methods.
Good agreement between semianalytical and MC results.

MC generator FORTRAN codes for unpolarized nuclear beta decay and for polarized neutron decay are available. Used in analysis of $^{38}K_m$ Fermi-type beta decay electron-neutrino correlation experiment (A. Gorelov et al., Phys. Rev. Lett. 94 (2005) 142501)

BR photon changes the decay kinematics !



Kinematics important for experimental details !!!

Radiative correction calculations to electron-neutrino correlation of Y. Yokoo and M. Morita (1976), K. Fujikawa and M.Igarashi (1976) Augusto Garcia and M. Maya (1978), Augusto Garcia (1982):

p_e, p_ν fixed, integration over photon momentum *k*:,

analytical integration possible

Problem: proton momentum changes with photon momentum *k* (momentum conservation), and no information about neutrino momentum (neutrino is usually not detected)

→ these radiative correction calculations to electron-neutrino correlation are not suitable for experimental analyses

(K. Toth, KFKI-1984-52, K. Toth et al., Phys. Rev. D33 (1986) 3306, Phys. Rev. D 40 (1989) 119)

Experiments: electron (positron) and proton (recoil nucleus) is detected, usually no information about neutrino and BR photon.

Radiative correction calculations should integrate over the BR photon with fixed charged lepton and recoil particle momenta.

F.e.: proton spectrum in neutron decay → integration over electron and BR photon with fixed proton energy

Analytical integration in this case is difficult, but no extra problem with Monte Carlo method!

Experimental details (particle kinematics, cuts, energy resolution, etc.) could be important for the radiative correction calculation results !!!

Infrared divergence

Photon bremsstrahlung experiment: low energy photons are not detected

Observable: $$\int_{K_{min}}^{K_{max}} \frac{dK}{K} \longrightarrow$$ **finite**

Radiative correction calculation: low energy photons cannot be excluded, they should be taken into account

$$\longrightarrow \int_{0}^{K_{max}} \frac{dK}{K} \longrightarrow$$

photon bremsstrahlung part of radiative correction has infrared divergence

Solution of infrared divergence problem

i, photon mass regularization:

$$k = (K_0, \mathbf{k}), \quad K_0 = \sqrt{\mathbf{k}^2 + m_\gamma^2}$$

$$\int_0^{K_{max}} \frac{dK}{\sqrt{\mathbf{k}^2 + m_\gamma^2}} \sim C \ln\left(\frac{K_{max}}{m_\gamma}\right)$$

ii,same mass regularization in virtual correction integrals:

order-α virtual correction $\sim C \ln m_\gamma$

Radiative correction = bremsstrahlung + virtual

$\longrightarrow$ **no infrared divergence in the sum !**

For correct radiative correction result it is important to treat in the integrals the BR photon as massive, with all 4 polarization states (massles photon has only 2 transverse polarization states)

Early radiative correction results for muon decay (1956-57) were wrong, because of incorrect IR regularization in the BR integral.

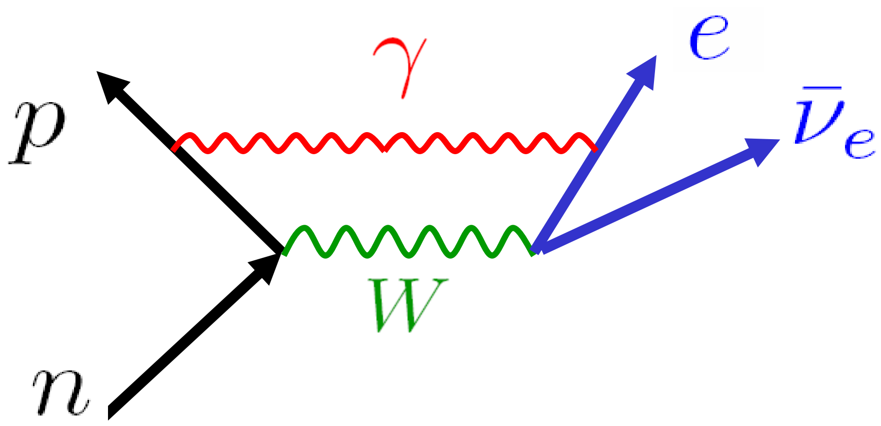
See the historical paper:

T. Kinoshita: Everyone makes mistakes – including Feynman, J. Phys. G 29 (2003) 9.

Another IR regularization method: dimensional regularization (dimension of phase space not integer); invented in 1972-73, useful in electroweak correction calculations

Virtual corrections

Photon exchange between charged particles:



BR photon is on-shell:

$$k_{BR} = (K, \mathbf{k}), \quad k_{BR}^2 = K^2 - \mathbf{k}^2 = 0$$

Virtual photon is off-shell: $k_{VIRT}^2 \neq 0$

Energy (K) and momentum (k) of virtual photon are independent !

Order-α virtual amplitude by 4-dimensional integral:

$$\mathcal{M}_{VIRT} \sim \int d^4k \left\{ \begin{array}{l} \text{wave} \quad \text{functions} \\ \text{propagators} \\ \text{vertices} \end{array} \right\}$$

Interference between zeroth-order amplitude $\mathcal{M}_0$ and virtual correction amplitude $\mathcal{M}_{VIRT}$

(virtual process indistinguishable from zeroth-order process)

Photon bremsstrahlung: no interference with zeroth-order amplitude (BR photon is in principle detectable)

Order-a radiative correction calculation of observable quantities:

$$\int |\mathcal{M}_0 + \mathcal{M}_{VIRT}|^2 + \int |\mathcal{M}_{BR}|^2$$

Order-α terms:

$$\int |\mathcal{M}_{BR}|^2 = -C_1 \ln m_\gamma + C_2$$

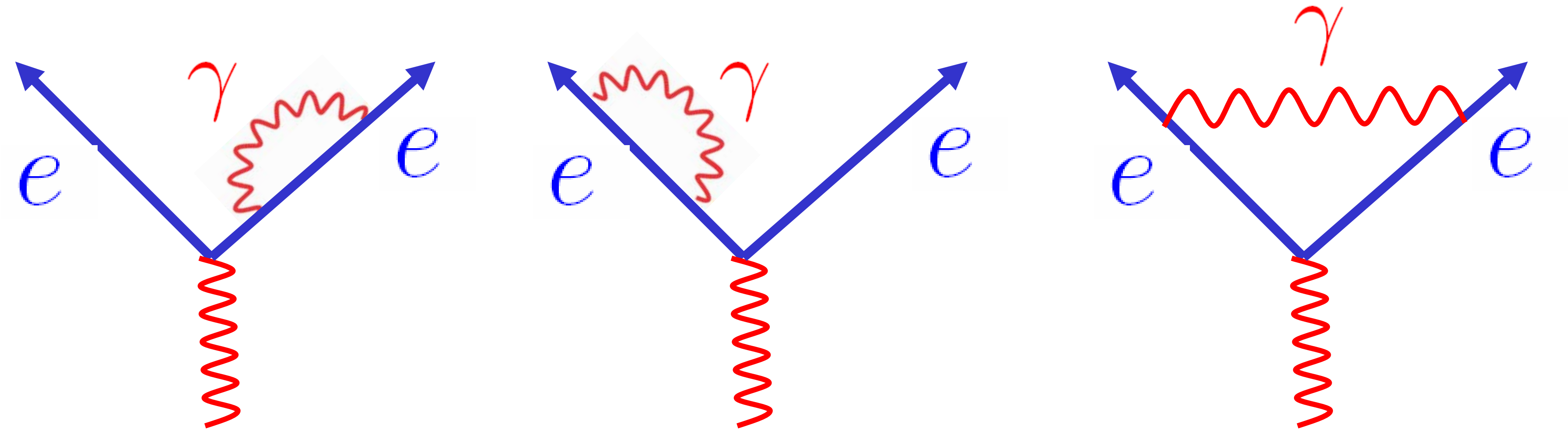
$$2 \int \Re(\mathcal{M}_0 \mathcal{M}^*_{VIRT}) = C_1 \ln m_\gamma + C_3$$

Infrared divergent terms cancel in the VIRTUAL+BR sum

Since both the virtual and the bremsstrahlung correction is IR divergent: it is not meaningful to give quantitative results only for the virtual, or only for the BR correction: only their sum is quantitatively meaningful

UV divergence of virtual correction

QED:



self-energy diagrams **vertex diagram**

UV divergence in each graph, but with mass and charge renormalization:

sum of virtual amplitudes is finite

Similar cancelation of order-α UV divergent terms in muon decay with V-A (4-fermion) theory

Neutron decay

UV divergence is present in 4-fermion and in intermediate vector boson theories

Conjecture in 60´s: perhaps strong interaction can help to solve the UV divergence problem?

$$(F_{EM}(q^2) \to 0 \quad \text{with} \quad q^2 \to \infty)$$

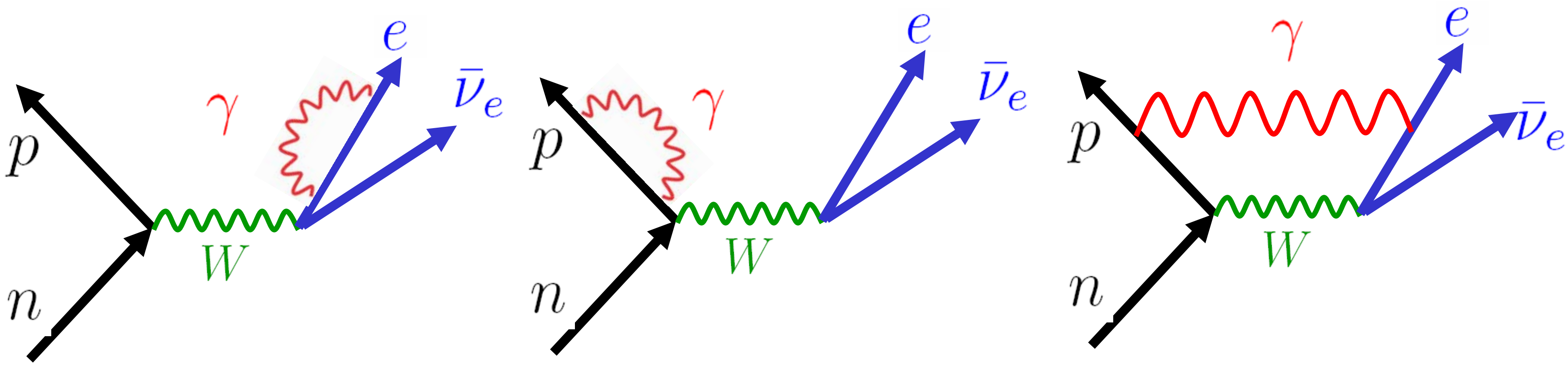
(Feynman, Källen, Berman, Sirlin)

Current algebra (middle 60´s): the strong interaction cannot solve the UV-divergence problem !

Solution by the $SU(3)_c$ x $SU(2)_L$ x $U(1)$ non-Abelian gauge theory (Standard Model)

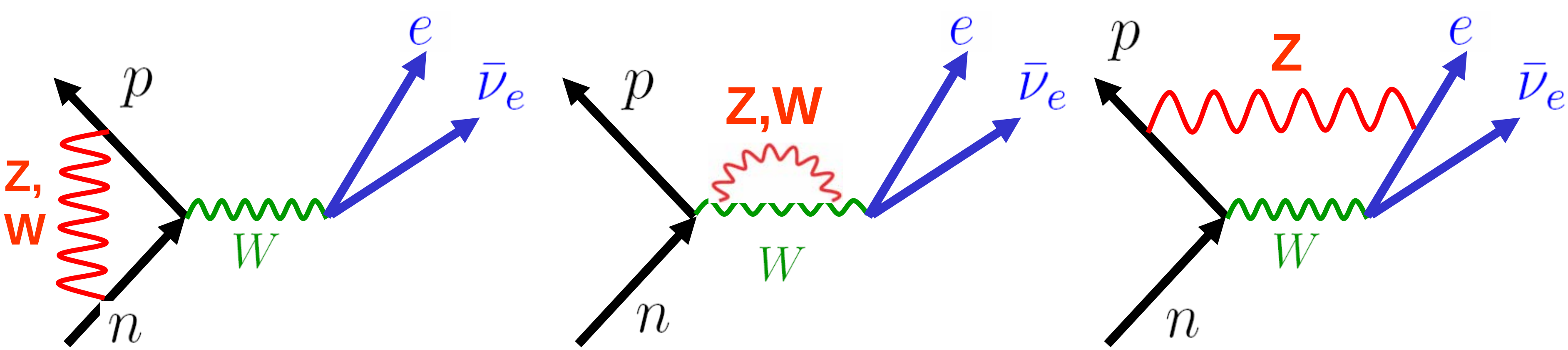
Sirlin (1974,1978)

Photonic diagrams (+3 WWγ graphs):



self-energy e **self-energy p** **box**

Non-photonic diagrams (examples):



Duplication of photonic self-energy integrals by photon propagator decomposition:

$$\frac{1}{k^2} = \underbrace{\frac{1}{k^2 - M_W^2}}_{\text{1. part}} - \underbrace{\frac{M_W^2}{k^2 - M_W^2}\frac{1}{k^2}}_{\text{2. part}}$$

weak correction: all non-photonic + WWγ graphs + 1. part ph. self-energy

photonic correction: photonic box + 2. part photonic self-energy

photonic corrections are UV finite

weak correction: asymptotoc freedom of QCD and electroweak renormalization → cancelation of UV divergences, finite rad. corr.; also IR finite

Weak correction to total beta decay rate:

r_{WEAK}**=0.02 %** **(A. Sirlin, Rev. Mod. Phys. 50 (1978) 573)**

Alberto Sirlin

The model independent (MI, outer) correction

Photonic virtual correction: - IR divergent
- strong interaction dependent

(1 GeV photons disturb the nucleon inner structure)

Radiative correction contribution with small photon energy (BR + virtual): IR divergent, no strong interaction dependence, depends on particle momenta (changes the spectrum shapes)

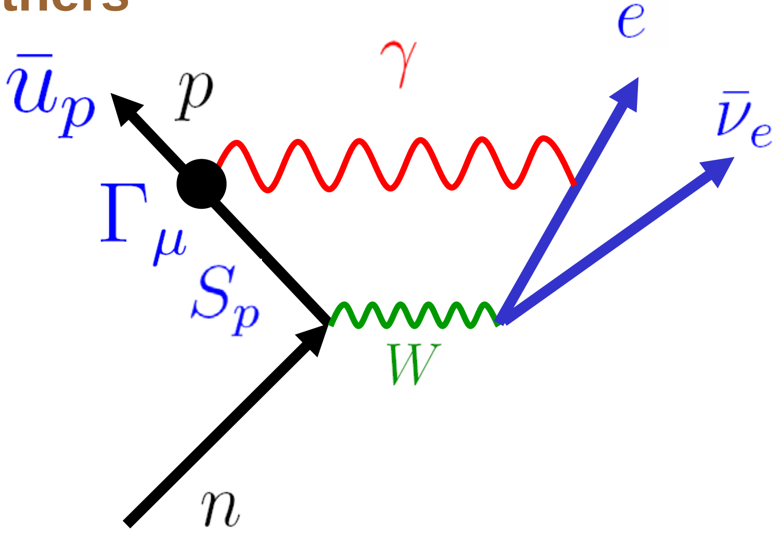
→ **should be separated from the others**

Sirlin, 1967:

$$Z_\mu = \bar{u}_p \Gamma_\mu S_p$$

Point-like hadron model:

$$Z_\mu = \bar{u}_p \gamma_\mu \frac{\not{p} + \not{k} + m}{(p+k)^2 - m^2}$$



Convective term – spin term separation
(Yennie, Frautschi, Suura,1961; Meister, Yennie,1962):

$$\bar{u}_p \gamma_\mu (\not{p} + \not{k} + m) = \bar{u}_p \left\{ \underbrace{2p_\mu + k_\mu}_{\text{convective term}} + \underbrace{\frac{1}{2}[\gamma_\mu, \not{k}]}_{\text{spin term}} \right\}$$

Model independent (MI) virtual correction:

photonic virtual integrals with convective term

$$Z_\mu = \bar{u}_p \Gamma_\mu S_p = Z_\mu^{\mathrm{MI}} + Z_\mu^{\mathrm{MD}}$$

$$Z_\mu^{\mathrm{MI}} = \bar{u}_p \frac{2p_\mu + k_\mu}{(p+k)^2 - m^2}$$

Z_μ^{MD} **: precise calculation difficult, but its general properties are similar to spin term**
(see later)

MI radiative correction= MI virtual + bremsstrahlung

Properties of model independent correction:

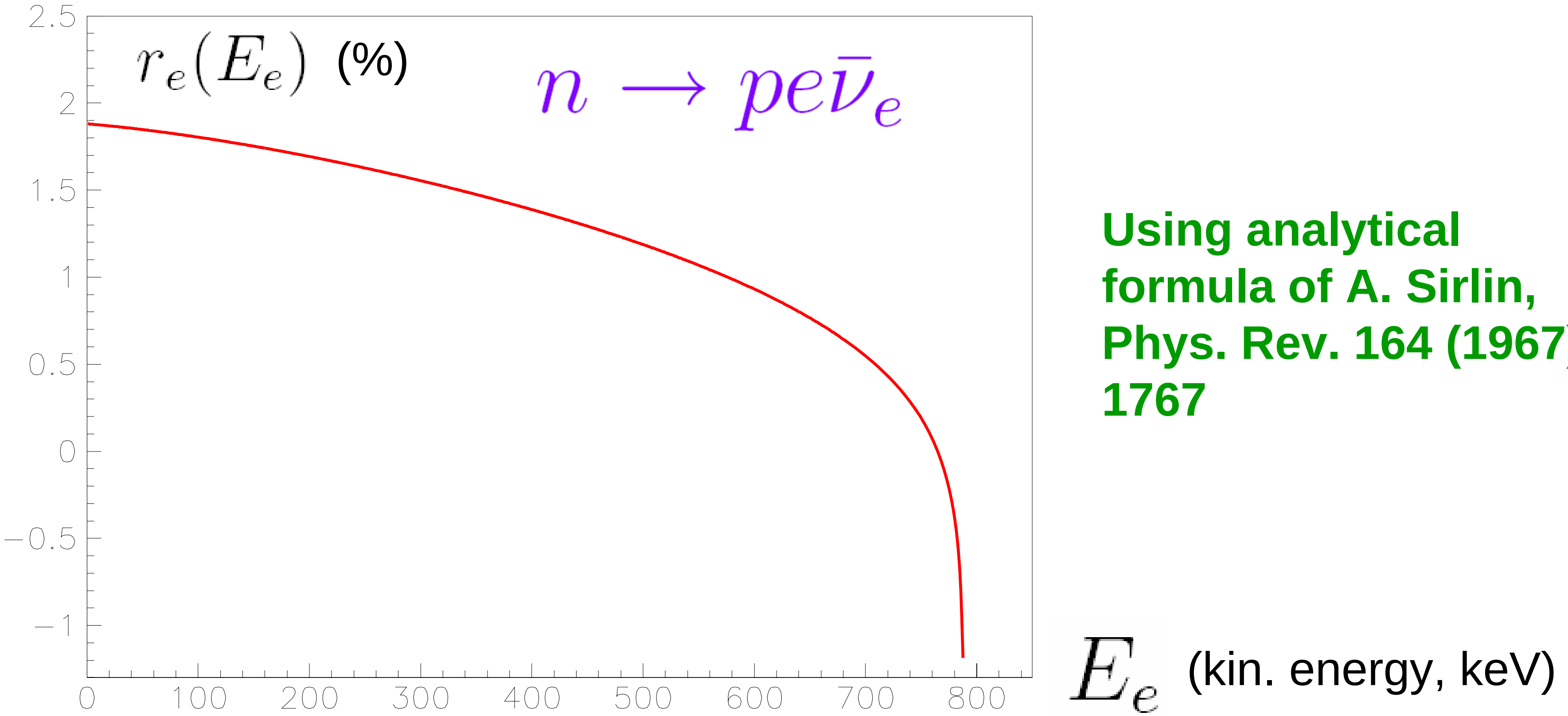
i, no strong interaction dependence, reliable

ii, sensitive to experimental details (f.e.: photon bremsstrahlung changes the kinematics)

iii, changes the spectrum shapes and asymmetries

Model independent radiative correction is important in the experimental analyses !

MI radiative correction to electron energy spectrum in neutron decay



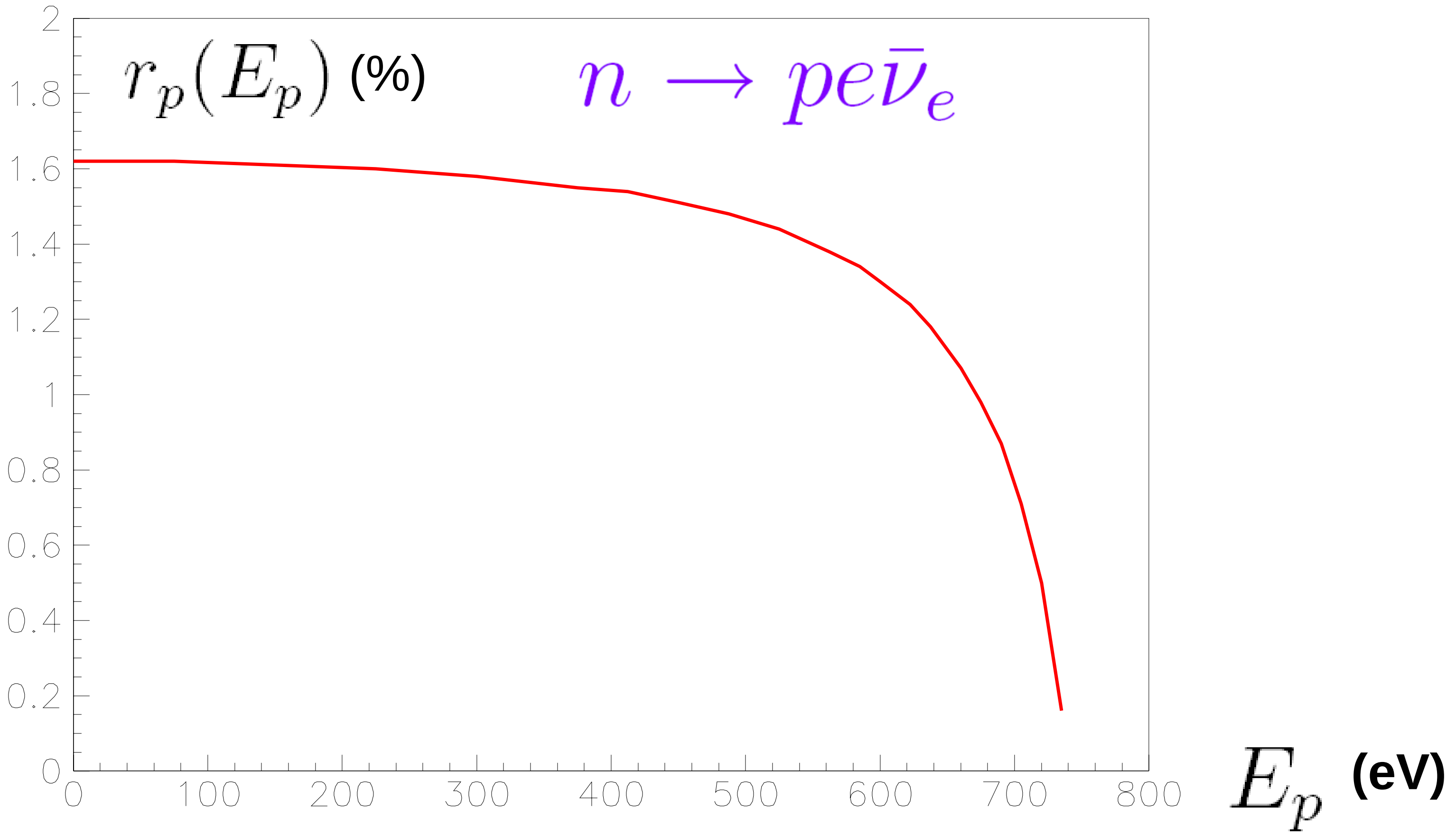
Using analytical formula of A. Sirlin, Phys. Rev. 164 (1967) 1767

$$E_e \longrightarrow E_{emax} \quad : \quad r_e \sim \ln(E_{emax} - E_e)$$

(electron energy goes to maximum → BR phase space decreases → IR divergence of virtual correction starts to appear)

Including higher orders: logarithmic singularity disappears

MI radiative correction to proton energy spectrum in neutron decay

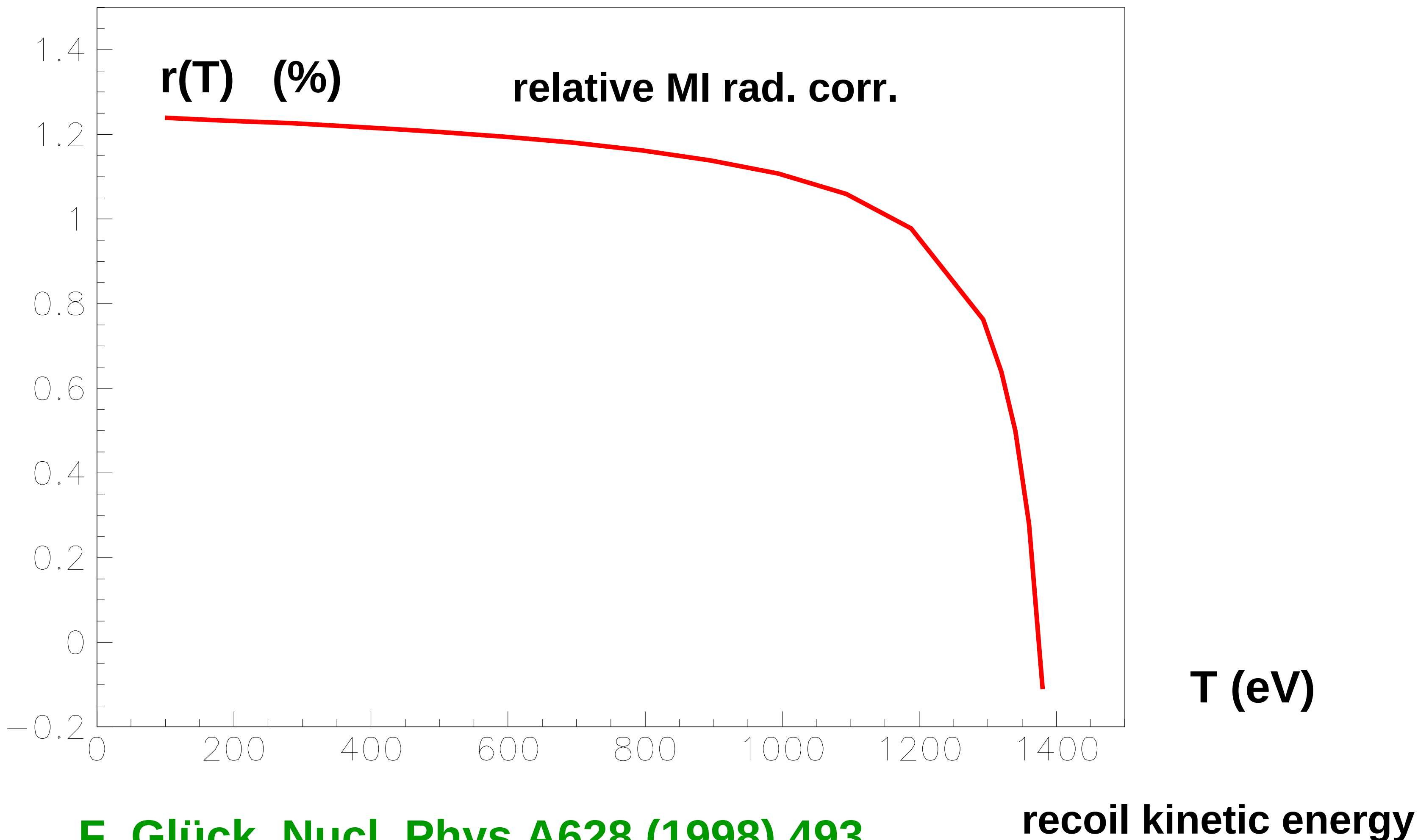


R. Christian, H. Kühnelt, Acta Phys. Austriaca, 49 (1978) 229;
F. Glück, Phys. Rev. D47 (1993) 2840

Change of fitted axialvector-to-vector coupling constant ratio:

$$\delta\lambda \approx 0.01$$

MI radiative correction to recoil energy spectrum in ^{6}He decay



F. Glück, Nucl. Phys A628 (1998) 493

Allowed nuclear beta decay similar to neutron decay
(see f.e.: B. R. Holstein and S. B. Treiman, Phys. Rev. C3 (1971) 1921)

Experimental electron-neutrino correlation result of C. Johnson et al., Phys. Rev. 132 (1963) 1149:

$$a_{e\nu}^{0}[{}^{6}\mathrm{He},\ \mathrm{Johnson}\ 1963] = -0.3343 \pm 0.0030$$

With radiative correction:

$$a_{e\nu}^{\gamma}[{}^{6}\mathrm{He},\ \mathrm{Glück}\ 1998] = -0.3308 \pm 0.0030$$

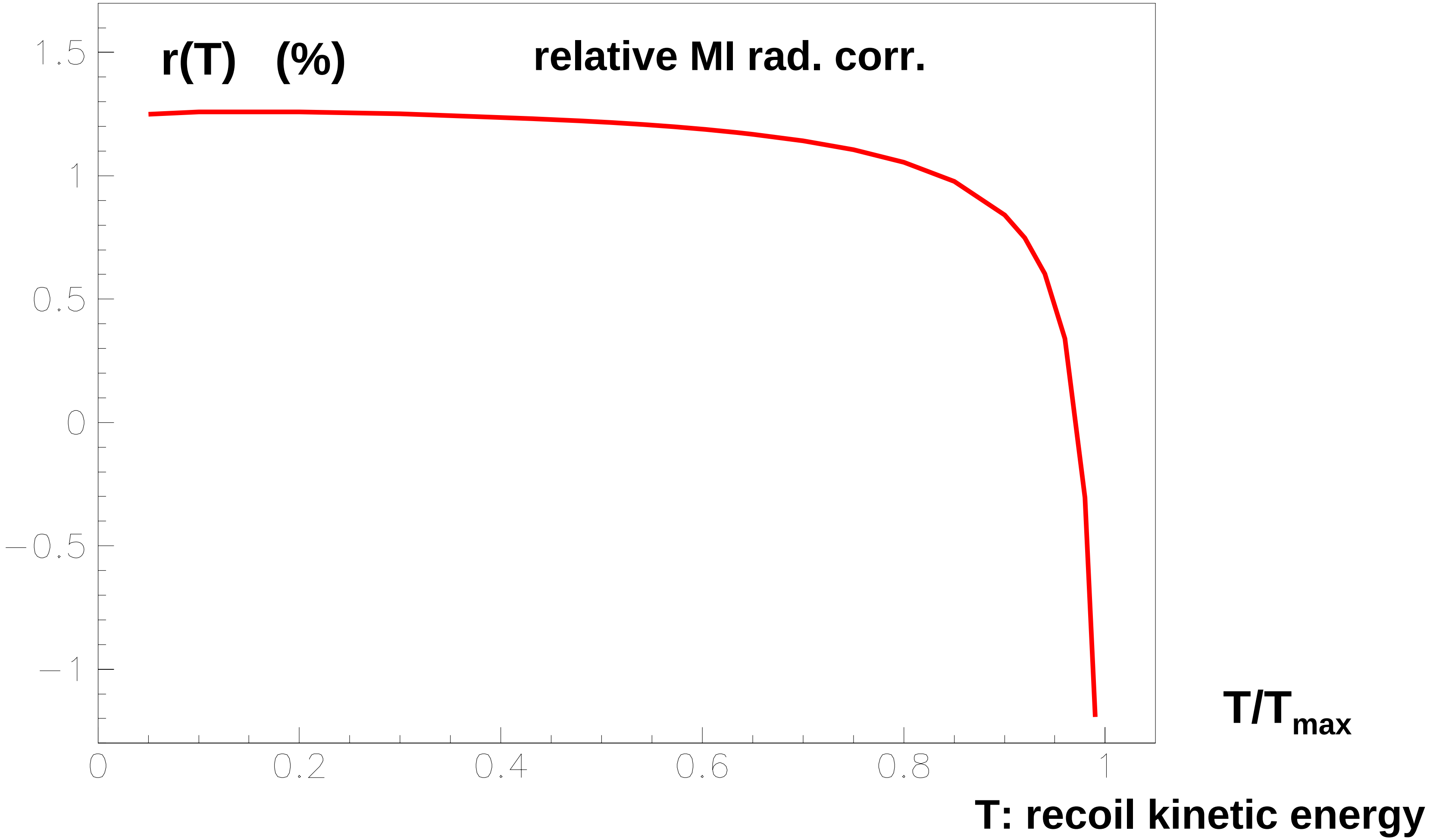
0.0035 (>1 σ) shift due to radiative correction !

Using the rad. corr. result of Y. Yokoo and M. Morita, Suppl. Prog. Theor. Phys. 60 (1976) 37, discussed in W. Kleppinger et al. , Nucl. Phys. A293 (1977) 46:

shift is only 0.0015

The kinematical change of the decay due to BR photons was not taken into account in the calculation of Yokoo and Morita !

MI radiative correction to recoil energy spectrum in ^{32}Ar decay



F. Glück, Nucl. Phys A628 (1998) 493

Estimated increase of electron-neutrino correlation parameter due to radiative correction:

$$(a_{e\nu}^{\gamma} - a_{e\nu}^{0})[^{32}\mathrm{Ar}] = 0.0036$$

Experimental result of E. Adelberger et al., Phys. Rev. Lett. 83 (1999) 1299 (beta delayed proton spectrum shape):

$$a_{e\nu}[^{32}\mathrm{Ar}] = 0.9989 \pm 0.0052 \pm 0.0036$$

(above radiative correction to recoil spectrum was taken into account)

$$\Lambda \to pe\bar{\nu}$$

Most precise electron-neutrino correlation measurement,
J. Dworkin et al., Phys. Rev. D41 (1990) 780:

$$\lambda[\Lambda \to pe\bar{\nu}] = g_1/f_1[\Lambda \to pe\bar{\nu}] = 0.731 \pm 0.016$$

Using wrong radiative correction results of A. Garcia (kinematics effect of BR photons was not taken into account)

With suitable radiative correction calculation, taking into account some important experimental details
(F. Glück et al, Phys. Lett. B340 (1994) 240):

$$\lambda[\Lambda \to pe\bar{\nu}] = 0.766 \pm 0.016$$ **(2 σ shift)**

$(F+D)_{\mathrm{hyp}}$: **changes from 1.261 to 1.313**

(*F, D*: Cabibbo fit parameters; in Cabibbo model *F+D*=λ[n→peν])

$V_{us}[\Lambda \to pe\bar{\nu}]$ **: smaller by few %**

Model independent radiative correction results for polarization asymmetries in polarized neutron decay:

R. T. Shann, Nuovo Cimento 5A (1971) 591,
F. Glück, K. Toth, Phys. Rev. D46 (1992) 2090,
F. Glück, Phys. Lett. B376 (1996) 25,
F. Glück, Phys. Lett. B436 (1998) 25.

Relative MI radiative corrections:

electron asymmetry: $\delta\alpha_e \approx -0.01\%$

proton asymmetry: $\delta\alpha_p \approx 0.04\%$

electron-proton asymmetry (PERKEO experiment): $\delta\alpha_{ep} \approx -0.05\%$

Model independent radiative correction to neutron decay rate:

$$\delta_{\mathrm{MI}} = 1.5\ \%$$

$$\delta_{\mathrm{MI}} = \delta_{\mathrm{BR}} + \delta_{\mathrm{MI,\,virt}}$$

Due to IR divergence:

$$\delta_{\mathrm{BR}} = +\infty$$

$$\delta_{\mathrm{MI,\,virt}} = -\infty$$

Gaponov, Khafizov, Nucl. Inst. Meth. A440 (2000) 557:

incorrect understanding of model independent correction;
BR measurement down to light domain provides no quantitative information about the MI radiative correction!

Model dependent (MD, inner) correction

Radiative corr.= BR + virtual = MI + MD

MD = weak + MD part of photonic virtual corr.

→ MD correction is pure virtual (no IR divergence)

MI virtual: main contribution from small energy virtual photons (small energy = much smaller than nucleon mass)

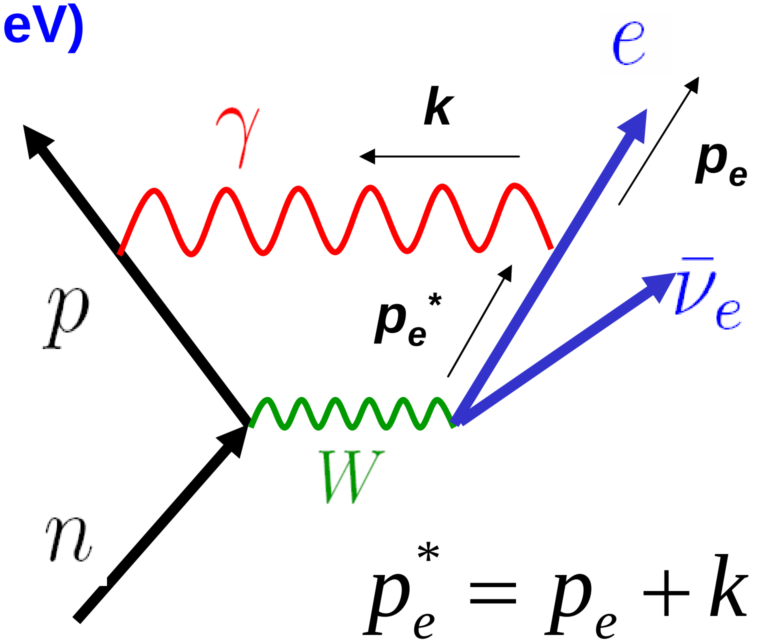
MD: main contribution from intermediate and high energy virtual photons (intermediate energy: not far from 1 GeV; high energy: much larger than 1 GeV)

Small photon energy (momentum):

Propagator momenta are sensitive to external momenta

Large photon energy (momentum):

Propagator momenta depend mainly on virtual photon momentum, they are not sensitive to the external momenta



→ no change of spectrum shapes and angular distributions due to the model dependent correction

A. Sirlin, Phys. Rev. 164 (1967) 1767

Neglecting terms of order $\sim \alpha \frac{E_e}{m_n} \ln\left(\frac{m_n}{E_e}\right) \sim 10^{-5}$,

the MD correction can be absorbed into the dominant form factors f_1 and g_1

Effective form factors:

$$f_1' := f_1\left(1 + \frac{\alpha}{2\pi}c\right), \quad g_1' := g_1\left(1 + \frac{\alpha}{2\pi}d\right)$$

MD corr.: 2 numbers (c, d)

Redefinition of G_V and λ :

$$G_V := G_\mu V_{ud} f_1', \quad \lambda := g_1'/f_1'$$

All measureable quantities in neutron decay depend on these effective parameters (c and d are the same for all quantities)

SM tests by comparison of λ from different types of experiments (like electron asymmetry and electron-neutrino correlation) are independent of the MD correction !

Model dependent correction to the vector coupling constant is important for V_{ud} determination and for CKM unitarity test !

Model dependent correction of the decay rate:

$$\rho_0 \rightarrow \rho_0(1 + \delta_{MD})$$

$$\delta_{\text{MD}} = \delta_{\text{as}} + \delta_{\text{QCD}} + \delta_{\text{med}}$$

δ_{as} : high energy (>>1 GeV) virtual photons

Asymptotic part: **reliable calculation is possible due to the non-Abelian feature of QCD (asymptotic freedom)**

Sirlin, Rev. Mod. Phys. 50 (1978) 573:

$$\delta_{\rm as} = \frac{3\alpha}{2\pi}(1+2\bar{Q})\ln\left(\frac{M_Z}{m_p}\right) \qquad \bar{Q} = \frac{1}{2}(Q_u+Q_d) = \frac{1}{6}$$

$$\delta_{\rm as} = \frac{2\alpha}{\pi}\ln\left(\frac{M_Z}{m_p}\right) = 2.13\ \%$$

$\delta_{\rm QCD}$ **: perturbative QCD correction to the asymptotic part**

$$\delta_{\rm QCD} = -0.04\ \%$$

$\delta_{\rm med}$ **: intermediate energy (near 1 GeV) virtual photons**

Reliable, precise calculation of the intermediate correction is difficult !

W. Marciano, A. Sirlin, Phys. Rev. Lett. 56 (1986) 22 :

$$\delta_{\text{med}} = 0.12\ \% \pm 0.20\ \%$$

W. Marciano, A. Sirlin, Phys. Rev. Lett. 96 (2006) 032002 :

$$\delta_{\text{med}} = 0.10\ \% \pm 0.10\ \%$$

Total decay rate of neutron decay with order-α rad. corr. :

$$\tau^{-1} \sim G_\mu^2 V_{ud}^2 (1 + 3\lambda^2)(1 + \delta_{\text{CB}} + \delta_{\text{MI}} + \delta_{\text{MD}})$$

$$\left.\begin{array}{l} \delta_{\text{CB}} = 3.5\ \% \\ \delta_{\text{MI}} = 1.5\ \% \\ \delta_{\text{MD}} = 2.2\ \% \end{array}\right\} \Longrightarrow \begin{array}{l} \delta\tau = -70\ \text{ s} \\ (\tau \approx 885\ \text{ s}) \end{array}$$

Historical remarks

Dominant term of radiative correction with UV-cutoff Λ in V-A theory:

$$\delta_{\rm dom} = \frac{3\alpha}{2\pi} \ln\left(\frac{\Lambda}{2E_{emax}}\right)$$

In intermediate vector boson theory: $\Lambda \longrightarrow M_W$

In Standard Model: $\Lambda \longrightarrow M_Z$

Beginning of 60`s: UV-cutoff or W mass cannot be smaller than 1 GeV → order +2 % correction to neutron (nuclear) beta decay rate → small discrepancy of beta decay universality → Cabibbo model !

Without radiative correction:

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 > 1$$

→ radiative correction important for precise test of the quark mixing theory

$$\delta_{\mathrm{dom}} = \frac{3\alpha}{2\pi} \ln\left(\frac{M_W}{2E_{emax}}\right) \longrightarrow$$

⟶ Blin-Stoyle, Freeman, 1971: radiative correction requires large W boson mass (M_W>>1 GeV) !

$$\delta_{\mathrm{as}} = \frac{3\alpha}{2\pi}(1+2\bar{Q}) \ln\left(\frac{M_Z}{m_p}\right)$$

Wilkinson, 1975: with $0^+ \rightarrow 0^+$ ft data and M_W=80 GeV, agreement between theory and experiments using the fractional charged quark model !

Summary

Neutron and nuclear beta decay experiments provide important information about the weak and strong interactions. Theoretical analysis of neutron decay is more simple than that of high energy beta decays.

Bremsstrahlung photons change the kinematics of the decays.

The model independent radiative corrections are important for the precise analyses of spectrum shape and angular correlation experiments.

Experimental details (particle kinematics, cuts, energy resolution, etc.) could be essential for the radiative correction calculation results !!!

The model dependent correction is important for the CKM unitarity test.