T-Odd Observables in Light-Front QCD

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Based on: Stan Brodsky (SLAC), SG, & Dae Sung Hwang (Sejong), hep-ph/0601037, Phys. Rev. D 73, 036007 (2006);
Stan Brodsky & SG, hep-ph/0608219, Phys. Lett. B643, 22 (2006).

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The Electric Dipole Moment - A Prequel

The electric dipole moment d of a particle with spin S is defined via $\mathcal{H} = -d\frac{\mathbf{S}}{S} \cdot \mathbf{E}$

$d \neq 0$ violates both T and P.

E. M. Purcell and N. F. Ramsey, "On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei," Phys. Rev. 78, 807 (1950):

> The argument against electric dipoles, in another form, raises the question of parity.... But there is no compelling reason for excluding this possibility....

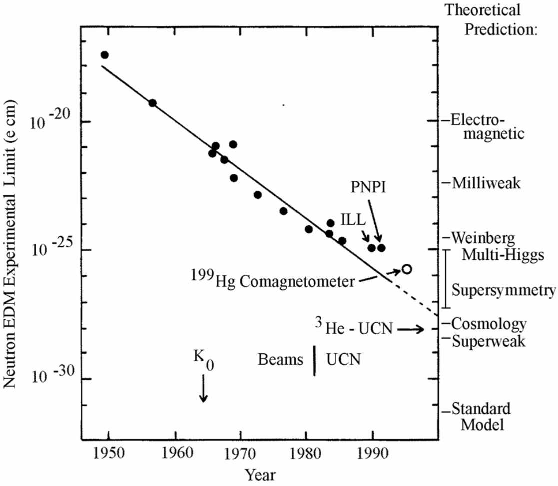
Context: Dirac (1949) – A magnetic monopole violates P, T.

A experimental strategy for d:

$\mathcal{H} = -\mu\frac{\mathbf{S}}{S} \cdot \mathbf{B} - d\frac{\mathbf{S}}{S} \cdot \mathbf{E}$

Limit set by neutron density, observation time, and the strength of the applied electric field.

Neutron EDM Timeline – The "Model Killer"

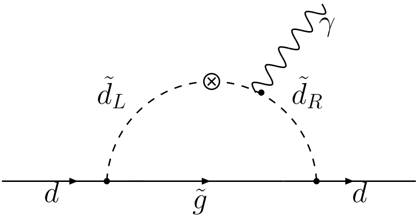


[S. Lamoreaux (Yale)]

Electric Dipole Moments in Supersymmetry

Supersymmetric models naturally "overproduce" electric dipole moments – picking parameters at random invariably lead to EDMs at odds with experiment, the "SUSY CP problem".

[Pospelov and Ritz, 2005]



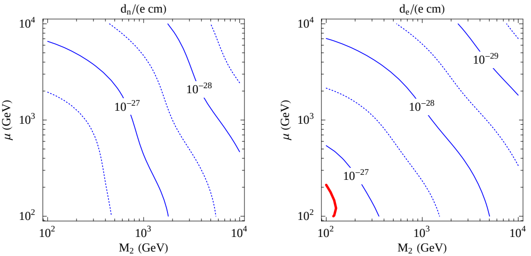
Can resolve by making scalars heavy or CP-violating parameters small.

Enter split supersymmetry.... [Arkani-Hamed, Dimopoulos (2004); Giudice, Romanino (2004)]

Electric Dipole Moments in Split Supersymmetry

Models with “split” supersymmetry (heavy scalars!) can still produce significant EDMs at two-loop order:

[Chang, Chang, Keung, 2005; Giudice and Romanino, 2006]



Both d_e and d_n are expected to improve.

$|d_n| \leq 3 \cdot 10^{-26}$ e-cm [Baker et al.,ILL, hep-ex/0602020] to 10^{-28} e-cm [LANL/SNS EDM expt]

$|d_e| \leq 1.6 \cdot 10^{-27}$ e-cm [Regan et al., PRL 88, 071805 (2002)] to $10^{-29\,(31)}$ e-cm [DeMille et al., 2002.]

If we observe an EDM, how sharply can we constrain a model’s parameters?

The Nucleon Electric Dipole Moments in QCD

Estimates of hadronic electric dipole moments depend on the hadron's non-perturbative structure.

For example, in the SM (CKM mechanism of CP violation), long-distance effects (π-loop) give for the neutron

$d_n^{\rm KM} \simeq 10^{-32}$e-cm [Gavela et al., PLB 1982; Khriplovich & Zhitnitsky, PLB 1982]

whereas a LL computation in three-loops yields

$d_d^{\rm KM} \simeq 10^{-34}$e-cm. [Czarnecki & Krause, PRL 1997]

cf. QCD sum rules w/ dim $\leq$ 5 CP-violating ops. give d_n to $\sim$ 50%

[Pospelov & Ritz, PRL 1999]

Evaluating d_n and d_p is also important to interpreting the ^2H EDM.

[Lebedev et al., PRD 2004]

Here we analyze the nucleon electric dipole moment in the light-front formalism of QCD.

Overview

A variety of observables are controlled by nucleon spin-flip matrix elements — we use light-front QCD to study the connections.

Central to us is

the anomalous magnetic moment κ_N

as well as to...

the electric dipole moment d_N.

and its connection to...

the Sivers single-spin asymmetry $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$

The Sivers asymmetry is a $\mathbf{S} \cdot (\mathbf{p} \times \mathbf{q})$ correlation.

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Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S_z' | J^\mu(0) | P, S_z \rangle =$$
$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

We ignore the anapole form factor and define

$$\kappa = \frac{e}{2M}\left[F_2(0)\right] , \qquad d = \frac{e}{M}\left[F_3(0)\right]$$

We will find a close connection between κ and d, as long anticipated. [Bigi, Uralstev, NPB 1991]

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame, and assumed simple vacuum imply ($q^{R/L} \equiv q^1 \pm iq^2$):

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Discrete Symmetries on the Light Front

Particles remain on their on-mass-shell as in the equal-time formalism.
Consider transformations on $\mathbf{k}_\perp$; thus $|\mathbf{k}_\perp|^2$, k^+, k^- are unchanged.

Parity $\mathcal{P}_\perp$:

A vector d^μ transforms as $d^R \to -d^L$, $d^L \to -d^R$, $d^\pm \to d^\pm$.
Note $d^{R,L} \equiv d^1 \pm i d^2$ so that $d^1 \to -d^1, d^2 \to d^2$.

$\mathcal{P}_\perp$ is unitarity; it flips the spin as well:

$$\begin{aligned} \mathcal{P}_\perp a^\lambda_{p^L,p^R} \mathcal{P}_\perp^\dagger &= \eta_a a^{-\lambda}_{-p^R,-p^L} , \\ \mathcal{P}_\perp b^\lambda_{p^L,p^R} \mathcal{P}_\perp^\dagger &= \eta_b b^{-\lambda}_{-p^R,-p^L} , \end{aligned}$$

$$\mathcal{P}_\perp \psi^\dagger(x) \mathcal{P}_\perp^\dagger = \eta_a^* \gamma^1 \gamma_5 \psi^\dagger(x^+, x^-, -x^R, -x^L) ,$$

so that, e.g.,

$$\begin{aligned} \mathcal{P}_\perp \bar\psi\psi(x) \mathcal{P}_\perp^\dagger &= \bar\psi\psi(x^+, x^-, -x^R, -x^L) \\ \mathcal{P}_\perp i\bar\psi\gamma_5\psi(x) \mathcal{P}_\perp^\dagger &= -i\bar\psi\gamma_5\psi(x^+, x^-, -x^R, -x^L) \end{aligned}$$

$F_1(q^2)$, $F_2(q^2)$ are even and $F_3(q^2)$ is odd under $\mathcal{P}_\perp$.

Discrete Symmetries on the Light Front

Time Reversal $\mathcal{T}_\perp$:

Momentum q^μ transforms as $q^R \to -q^L$, $q^L \to -q^R$, $q^\pm \to q^\pm$.
Thus $x^\mu = (x^+, x^-, x^L, x^R) \to (-x^+, -x^-, x^R, -x^L)$.

$\mathcal{T}_\perp$ is antiunitarity, but it does not flip the spin.

$$\begin{aligned} \mathcal{T}_\perp a^\lambda_{p^L,p^R} \mathcal{T}_\perp^\dagger &= \tilde{\eta}_a a^\lambda_{-p^R,-p^L} \,, \\ \mathcal{T}_\perp b^\lambda_{p^L,p^R} \mathcal{T}_\perp^\dagger &= \tilde{\eta}_b b^\lambda_{-p^R,-p^L} \,, \end{aligned}$$

$$\mathcal{T}_\perp \psi^\dagger(x) \mathcal{T}_\perp^\dagger = \tilde{\eta}_a^* \sigma^{12} \psi^\dagger(-x^+, -x^-, x^R, x^L)$$

so that, e.g.,

$$\begin{aligned} \mathcal{T}_\perp \bar{\psi}\psi(x) \mathcal{T}_\perp^\dagger &= \bar{\psi}\psi(-x^+, -x^-, x^R, x^L) \\ \mathcal{T}_\perp i\bar{\psi}\gamma_5\psi(x) \mathcal{T}_\perp^\dagger &= -i\bar{\psi}\gamma_5\psi(-x^+, -x^-, x^R, x^L) \end{aligned}$$

With $\mathcal{C}$ as usual all scalar fermion bilinears are invariant under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$.
Re(F_1), Re(F_2), and Im(F_3) are even and Re(F_3), Im(F_1), and Im(F_2) are odd under $\mathcal{T}_\perp$.

A Universal Relation for $F_2(q^2)$ and $F_3(q^2)$

β_a violates $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$.

$$\psi_a^\uparrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^\uparrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{+i\beta_a/2} \,,$$
$$\psi_a^\downarrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^\downarrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{-i\beta_a/2}$$

$$\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a)\Xi_a \quad ; \quad \frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a)\Xi_a \,,$$

$$\Xi_a = \int \frac{\left[\mathrm{d}^2\vec{k}_\perp \mathrm{d}x\right]}{16\pi^3} \sum_j e_j \frac{1}{-q^1 + iq^2} \left[\phi_a^{\uparrow *}(x_i, \vec{k}'_{\perp\, i}, \lambda_i)\, \phi_a^\downarrow(x_i, \vec{k}_{\perp\, i}, \lambda_i)\right].$$

For Fock component a:

$$[F_3(q^2)]_a = (\tan\beta_a)[F_2(q^2)]_a$$
$$d_a = (\tan\beta_a)2\kappa_a \quad \text{or} \quad d_a = 2\kappa_a\beta_a \quad \text{as} \quad q^2 \to 0$$

cf. μ $g-2$ and EDM [Feng, Matchev, and Shadmi, 2001]

Both the EDM and anomalous magnetic moment should be calculated within a given method, to test for consistency.

The Anomalous Magnetic Moment in Light-Front QCD

Each Fock state of the light-front wave function for a nucleon of spin J^z obeys

$$J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z$$

There are n-1 orbital angular momenta in a Fock state of n constituents.

Recall [Brodsky, Drell,1980]

$$\kappa = -\sum_a \sum_j e_j \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\psi_a^*(x_i, \mathbf{k}_{\perp i}, \lambda_i)\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j}\psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)\,,$$

with $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j} \equiv (S_+ L_-^{q_j} + S_- L_+^{q_j})/2$

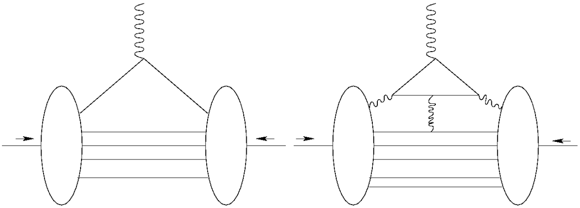
where $S_\pm = S_1 \pm i S_2$ and $L_\pm^{q_j} = \sum_{i\neq j} x_i(\partial/\partial k_{1i} \mp i\partial/\partial k_{2i})$

Empirically, $\kappa_n = -1.91\mu_N$ and $\kappa_p = 1.79\mu_N$.

- The $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j}$ matrix element is large!
- $\kappa_p + \kappa_n \ll \kappa_p - \kappa_n$
  $\Longrightarrow$ The isoscalar anomalous magnetic moment is *very* small.

The Anomalous Magnetic Moment in Light-Front QCD

Both intrinsic quarks and gluons can contribute to the anomalous magnetic moment.



The gluon mechanism generates isoscalar ($I = 0$) contributions exclusively.
The empirical anomalous magnetic moments show that the gluon mechanism is much smaller.

Implications for Models of CP Violation

Evaluating d_n and d_p is also important to interpreting the ${}^2H$ EDM.

[Lebedev et al., PRD 2004]

CP violation via a QCD $\bar{\theta}$-term.

In a $q(qq)_0$ model of the nucleon

$$d^n \approx e\beta^n\kappa^n(2\cdot 10^{-14}\,\mathrm{cm})\,,\quad d^p \approx e\beta^p\kappa^p(2\cdot 10^{-14}\,\mathrm{cm})\,,$$

Since δL_{CP} is isoscalar, $\beta^n = \beta^p$ and
$(d^n + d^p)/(d^p - d^n) = (\kappa^n + \kappa^p)/(\kappa^p - \kappa^n) \approx -0.12/3.70 \approx -0.03$

Much smaller than leading-order QCD sum rule estimate. [Pospelov and Ritz, PRL 1999]

Note context: $d_D(\bar{\theta}) = -e[(3.5 \pm 1.4) + (1.4 \pm 0.4)] \times 10^{-3}\bar{\theta}\,(\mathrm{GeV}^{-1})$

[Lebedev et al., PRD 2004]

The first term is controlled by $d_n + d_p$.

Decreasing its size weakens the overall constraint on $\bar{\theta}$.

The "classic" QCD sum rule calculation of the anomalous magnetic moments yields $\kappa_{p,n} = \pm 2$, in accord with experiment. The isoscalar magnetic moment is predicted to vanish, however; empirically, it is non-zero but small.

[Ioffe and Smilga, 1984]

Outlook

Supersymmetric models which evade existing empirical constraints predict EDMs of a scale to which upcoming experiments are sensitive, and we possess sufficient theoretical knowledge to be able to interpret the experimental results.

The Single-Spin Asymmetry in Semi-Inclusive DIS

We define the single-spin asymmetry in $lp^{\uparrow} \to l'\pi^{\pm}X$ via

$$A_{UT}^{\pi^{\pm}}(\phi,\phi_s) \equiv \frac{1}{|\langle S_p\rangle|}\left(\frac{N^{\uparrow}_{\pi^{\pm}}(\phi,\phi_s) - N^{\downarrow}_{\pi^{\pm}}(\phi,\phi_s)}{N^{\uparrow}_{\pi^{\pm}}(\phi,\phi_s) + N^{\downarrow}_{\pi^{\pm}}(\phi,\phi_s)}\right),$$

$$\equiv A_{UT}^{C}\sin(\phi+\phi_S) + A_{UT}^{S}\sin(\phi-\phi_S) + \ldots.,$$

We consider the Sivers effect A_{UT}^S exclusively.

Physically [Brodsky, Hwang, Schmidt, 2002]

- $A_{UT}^S \propto \mathrm{Im}(\mathcal{M}[J_p^y = +1/2]^*\mathcal{M}[J_p^y = -1/2])$.
- Fixed quark helicity $\Longrightarrow |\Delta L^y| = 1$.
- Final-state interactions are necessary!
- A_{UT}^S is of leading twist.

Formally A_{UT}^S is ascribed to a “T-odd” distribution function $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)$

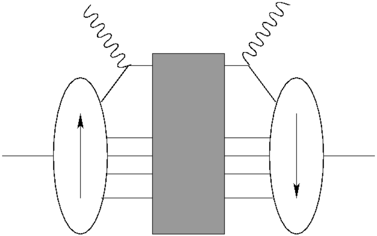
$$f_{q/p\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{\epsilon^{\mu\nu\rho\sigma}P_\mu k_\nu S_\rho n_\sigma}{M(P\cdot n)}$$

The Single-Spin Asymmetry in Light-Front QCD

In the light-front formalism in $A^+ = 0$ gauge, with appropriate boundary conditions, the FSI are captured with a phase:

$$\tilde{\psi}_a^{S_y} = \psi_a^{S_y} \exp(i\phi_a^{S_y})$$

The needed $|\Delta L^y|$ can be provided by either quark or gluon degrees of freedom. Note the quark mechanism:



The gluon mechanism generates isoscalar ($I = 0$) contributions exclusively.

Interpreting the Sivers Effect

We define

$$\kappa \equiv \sum_q e_q a_q \quad ; \quad \frac{f_{1T}^{\perp\, q}(\eta, \mathbf{l}_\perp^2)}{M} \equiv -\tilde{a}_q(\eta, \mathbf{l}_\perp^2)\,.$$

Assuming isospin symmetry and neglecting anti-quark contributions:

$$\kappa_p = 1.79 = (+2/3)a_u^p + (-1/3)a_d^p$$
$$\kappa_n = -1.91 = (-1/3)a_d^n + (+2/3)a_u^n = (+2/3)a_d^p + (-1/3)a_u^p\,,$$

yields $a_d^p = a_u^n = -2.03$ and $a_u^p = a_d^n = 1.67$.
Assuming the isospin structure of the empirical anomalous magnetic moments is that of the Sivers SSA predicts the observed sign of the SSA from HERMES for $\pi^\pm$-mesons produced from the proton at large z.

[Burkardt, 2004]

Fits of the HERMES data, however, yield: $\tilde{a}_d^p = -1.86 \pm 0.28$ and $\tilde{a}_u^p = +0.81 \pm 0.07$. [Vogelsang and Yuan, 2005]
Nevertheless, for the deuteron data, we note $a_u^p + a_u^n = a_d^p + a_d^n = -0.360$, so that the deuteron SSA should be significantly suppressed.

[Anselmino, et al., 2005; Vogelsang and Yuan, 2005]

Evidence for Absence of Gluon Orbital Angular Momentum

What does the observed cancellation *mean*?

The gluon mechanism is $l = 0$ and cannot cancel in the SSA for ^{2}H.

The kinematic region in x and z studied by COMPASS should permit the gluon mechanism to contribute.

Thus the ^{2}H data allows us to conclude that gluon mechanism is small compared to the quark contributions.

We have evidence for the absence of gluon orbital angular momentum!

How small is the gluon mechanism?

We assume the phases scale, gluon to quark, as 2.25, as per a leading-order QCD analysis of the ratio of the rapidity plateaus in gluon versus quark jets.

[Brodsky, Gunion, 1976; Konishi et al., 1978; Capella et al., 2000]

Thus the ratio of the SSA from positively charged, leading hadrons on a ^{2}H target to that from leading π^+ production on a proton target makes the gluon mechanism is smaller than the quark mechanism by a factor of 0.2.

Can reduce this by a factor of 2.5 with new COMPASS data! [E.S. Ageev et al., COMPASS, 2007]

The nucleon's orbital angular momentum appears to be largely carried by its quarks.

Summary and Outlook

In the light-front formalism of QCD, the nucleon anomalous magnetic moment, the nucleon electron dipole moment, the single-spin asymmetry in semi-inclusive deeply inelastic scattering, and the generalized parton distribution function E are all spin-flip matrix elements. – and they are interconnected.

Our analysis reveals that the orbital angular momentum carried by gluons is small, which appears to be so, and provides an independent constraint on the electric dipole moment matrix element, crucial to constraining physics beyond the Standard Model.

Backup Slides

such that

$$A_{UT}^{S} = -\frac{2}{M} \frac{\langle \sum_q |\mathbf{k}_\perp| e_q^2 f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\mathcal{D}(z,\mathbf{p}_\pi,\mathbf{k}_\perp)\sin^2(\phi-\phi_S)\rangle}{\langle \sum_q e_q^2 f_1^q(x,\mathbf{k}_\perp^2)\mathcal{D}(z,\mathbf{p}_\pi,\mathbf{k}_\perp)\rangle} \,.$$

We can encode the needed final-state interactions (FSI) in a gauge-invariant way using the "gauge-link" formalism: [Belitsky, Ji, Yuan 2003]

$$f_{q/p\uparrow}(x,\mathbf{k}_\perp) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{-ik^+\xi^- + i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp}$$

$$\times \langle P|\bar\psi(\xi_i,\boldsymbol{\xi})[\infty,\infty;\xi^-,\boldsymbol{\xi}_\perp]_C^\dagger \gamma^+ [\infty,\infty;0^-,\mathbf{0}_\perp]_C \psi(0,\mathbf{0}_\perp)|P\rangle \,,$$

The gauge links $[\ldots]_C$ are stretched in both light-like and transverse directions:

$$[\infty,\infty;\xi_-,\boldsymbol{\xi}_\perp]_C = [\infty,\infty;\infty,\boldsymbol{\xi}_\perp][\infty,\boldsymbol{\xi}_\perp;\xi_-,\boldsymbol{\xi}_\perp]$$

with

$$[\xi_-,\boldsymbol{\xi}_\perp;0,\boldsymbol{\xi}_\perp] \equiv P\exp\left(-ig\int_0^{\xi_-} d\xi_-\, A_+(\xi_-,\boldsymbol{\xi}_\perp)\right) \,.$$

The Single-Spin Asymmetry in Light-Front QCD

Working in the $q^+ = 0$ frame we thus identify

$$f_{1T}^{\perp\,q}(\eta, \mathbf{l}_\perp^2)\frac{l_1}{M} = -\frac{i}{2}\sum_a \sum_{j=1}^{n} \delta_{qq_j} \int [dx][d^2\mathbf{k}_\perp] \Big\{ \psi_a^{\uparrow\,*}(x_i', \mathbf{k}_{\perp i}', \lambda_i)\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$$

$$\times\, \mathrm{Im}\,\exp\left(i(\phi^\downarrow - \phi^\uparrow)\right) + \psi_a^{\downarrow\,*}(x_i', \mathbf{k}_{\perp i}', \lambda_i)\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)\mathrm{Im}\,\exp\left(-i(\phi^\downarrow - \phi^\uparrow)\right) \Big\},$$

with $\mathbf{k}_{\perp j}' = \mathbf{k}_{\perp j} + \mathbf{l}_\perp$ and $x_j' = x_j + \eta$ for the struck constituent j and $\mathbf{k}_{\perp i}' = \mathbf{k}_{\perp i} - x_i\mathbf{l}_\perp/(1-x_j)$ and $x_i' = x_i[1-\eta/(1-x_j)]$ for each spectator i.
Retaining the leading terms as $\mathbf{l}_\perp \to 0$:

$$\frac{f_{1T}^{\perp\,q}(\eta, 0)}{M} = \sum_a \sum_j \delta_{qq_j} \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \frac{1}{2i(1-x_j)} \Big[\tilde{\psi}_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i)\mathbf{S}_{\perp T}\cdot\mathbf{L}_{\perp T}^{q_j}\tilde{\psi}_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)$$

$$-\bar{\psi}_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i)\mathbf{S}_{\perp T}\cdot\mathbf{L}_{\perp T}^{q_j}\bar{\psi}_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big],$$

Here $\bar{\psi}_a^{S_y} \equiv \psi_a^{S_y}\exp(-i\phi_a^{S_y})$, $\mathbf{S}_{\perp T}\cdot\mathbf{L}_{\perp T}^{q_j} \equiv (S_{+T}L_{-T}^{q_j} - S_{-T}L_{+T}^{q_j})/2$, $S_{\pm T} = S_3 \pm iS_1$, and $L_{\pm T}^{q_j} = \sum_{i\neq j} x_i(\partial/\partial k_{3i} \mp i\partial/\partial k_{1i})$.

Outlook

- The anomalous magnetic moment, the Sivers function, and the generalized parton distribution E can all be connected to matrix elements involving the orbital angular momentum of the nucleon's constituents.
- The SSA can be generated by either a quark or gluon mechanism, and the isospin structure of the two mechanisms is distinct. The approximate cancellation of the SSA measured on a deuterium target suggests that the gluon mechanism, and thus the orbital angular momentum carried by gluons in the nucleon, is small.
- Studies of the SSA in ϕ or K^+K^- production, via $\gamma^* g \to s\bar{s} \to \phi + X$ or $\gamma^* g \to s\bar{s} \to K^+K^- + X$ should provide additional constraints on the gluon mechanism.

The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \boldsymbol{\Delta}_\perp, (t + \boldsymbol{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]
We find, under $\boldsymbol{q}_\perp \to \boldsymbol{\Delta}_\perp$, for $\zeta \le x \le 1$,

$$\begin{aligned} \frac{E(x,\zeta,0)}{2M} &= \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x - x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \\ &\quad \times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \, , \end{aligned}$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i.
The E distribution function is related to a $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}}$ matrix element at finite ζ as well.