False EDM from Geometric phase

- Commins Am J Phys **59**, 1077 (91)

- Pendlebury et al PRA **70** 032102 (04)
  - Classical spin trajectory in rotating frame
    - recall Gamblin and Carver PR **138**, A946 (65)
- Lamoreaux and Golub PRA **71** 032104 (05)
  - Perturbation theory approach (density function)
    - Recall Colegrove & Walters PR **139**, A1398 (65)

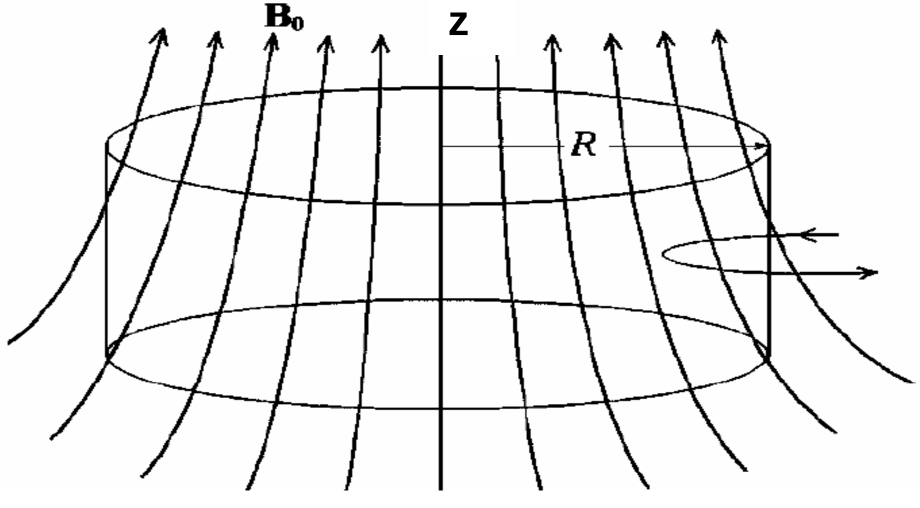
Caveats: - assembled without notes in last 24 hours
- has a strong neutron & ^{3}He focus

Geometric phase contributions

- Recall EDM measurements rely on E-field dependent frequency shift wrt Larmor precession frequency $\omega_0 = \gamma_n B_0$
- **B**-Gradients combined with **v** x **E** field can shift ω
  - Precession phase +/- Geo. phase
  - Can lead to false EDM: phase shift $\propto$ E-field direction
- Impacts both neutron and magnetometers
  - Magnetometers (e.g. ^{199}Hg & ^{3}He) pick up phase at different rate due to higher velocities

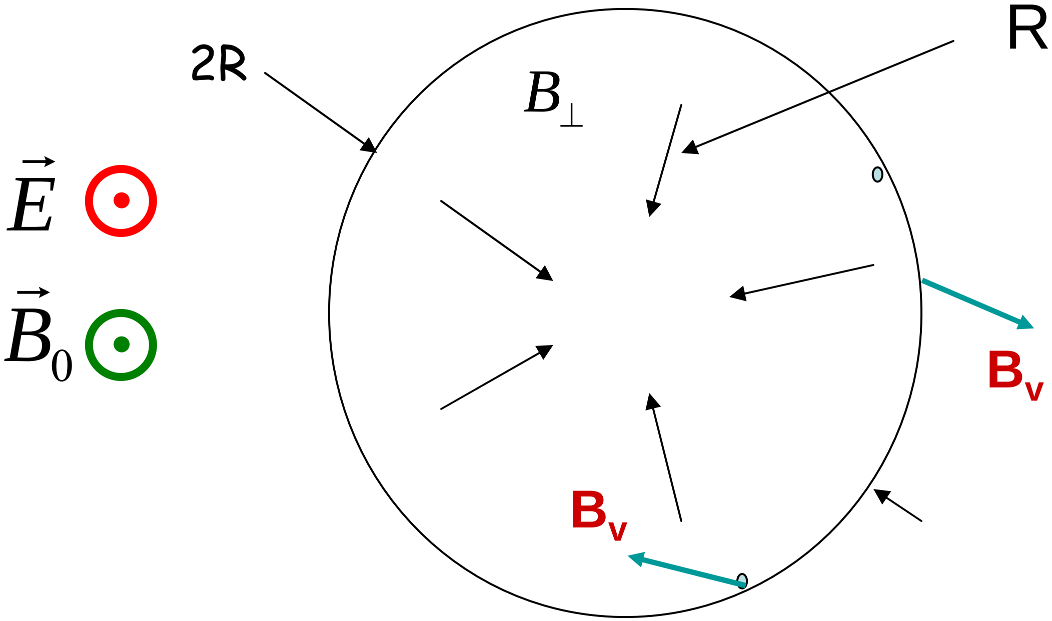
False EDM from Geometric phase

- Geometric phase contribution to false EDM's depends on radial fields perpendicular to B_0 that rotate as one circulates about B_0
  - These can result from dB_z/dz in the diagram below
  - Will add to the motional **v** x **E** field seen by moving particle



False EDMs from $B_v = v \times E$ field

Radial B-field due to gradient



- Motion in B – field shifts the precession frequency - ω_0 :

$$\Delta\omega \cong \frac{\gamma_n^2 \left(B_\perp \mp |\vec{v}_n \times \vec{E}|/c^2\right)^2}{4\left(\omega_0 \mp v_n/R\right)}$$

- $\mp$ due to different trajectories
- Does NOT average to 0
- Gives $\Delta\omega$ that depends on direction of $\vec{E}$

v x **E** field changes sign with neutron direction

Adiabatic vs Non-Adiabatic Cases

- UCN usually adiabatic, magnetometers usually non-adiabatic

- Consider cylindrical box of radius R
- Time to circulate around box $\leq 2\pi R/v$
  - This is a frequency $\omega_R \sim v/R$

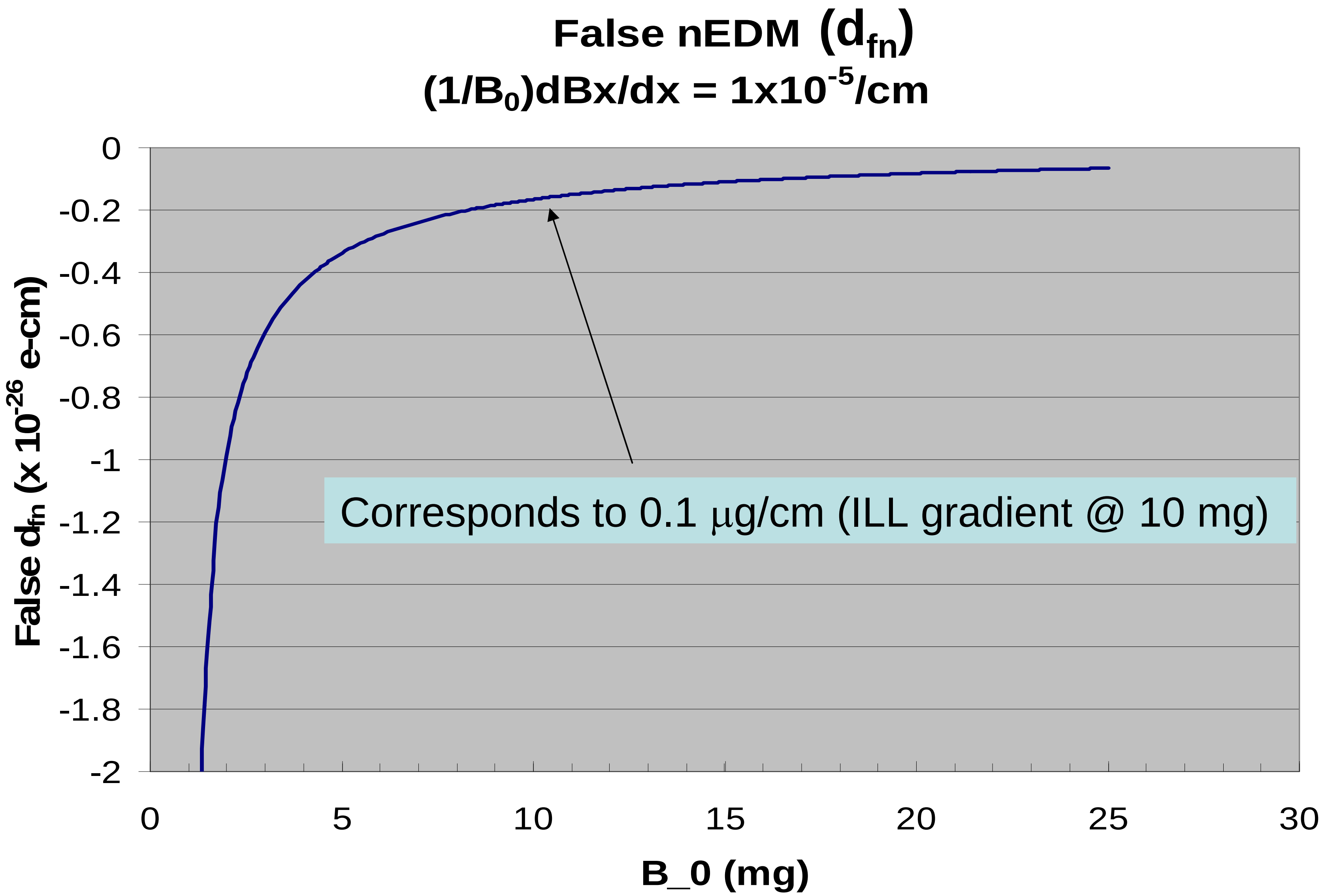
- Geo. Phase depends on size of ω_R
  - For slow velocities $|\omega_R| < |\omega_0|$ → adiabatic
  - For high velocities $|\omega_R| > |\omega_0|$ → non-adiabatic

Adiabatic false EDM d_f

- Pendlebury et al formula (for simple orbit at v)

$$d_f \cong -\frac{J\hbar}{2}\left(\frac{\partial B_z/dz}{B_z^2}\right)\left(\frac{v_{xy}^2}{c^2}\right)\left(\frac{1}{1-\frac{\omega_r^2}{\omega_0^2}}\right) \; ; \quad (SI)$$

  - After averaging over orbits d_f depends on $\left\langle \frac{\partial B_z}{\partial z} \right\rangle_{Volume}$
  - Since magnetic coils give $\left(\frac{\partial B_z/dz}{B_z}\right) \approx \text{constant}$ → $d_f \sim 1/B_z$
  - d_f has a resonant term $\frac{1}{1-\frac{\omega_r^2}{\omega_0^2}}$; $\omega_r \approx \frac{v}{R} \propto 1/\text{time to circulate cell}$
    - Somewhat washed out by averaging
  - Can be related directly to Berry's phase



False nEDM (d_{fn})
$(1/B_0)dBx/dx = 1x10^{-5}$/cm
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
False d_{fn} (x 10^{-26} e-cm)
Corresponds to 0.1 μg/cm (ILL gradient @ 10 mg)
0
5
10
15
20
25
30
B_0 (mg)

Non-adiabatic false EDM

- Simple orbit at velocity v gives

$$d_f \cong \frac{J\hbar}{2}\left(\frac{\partial B_z}{\partial z}\right)\left(\frac{\gamma^2 R^2}{c^2}\right)\left(\frac{1}{1-\frac{\omega_0^2}{\omega_r^2}}\right) \; ; \quad \text{(SI)}$$

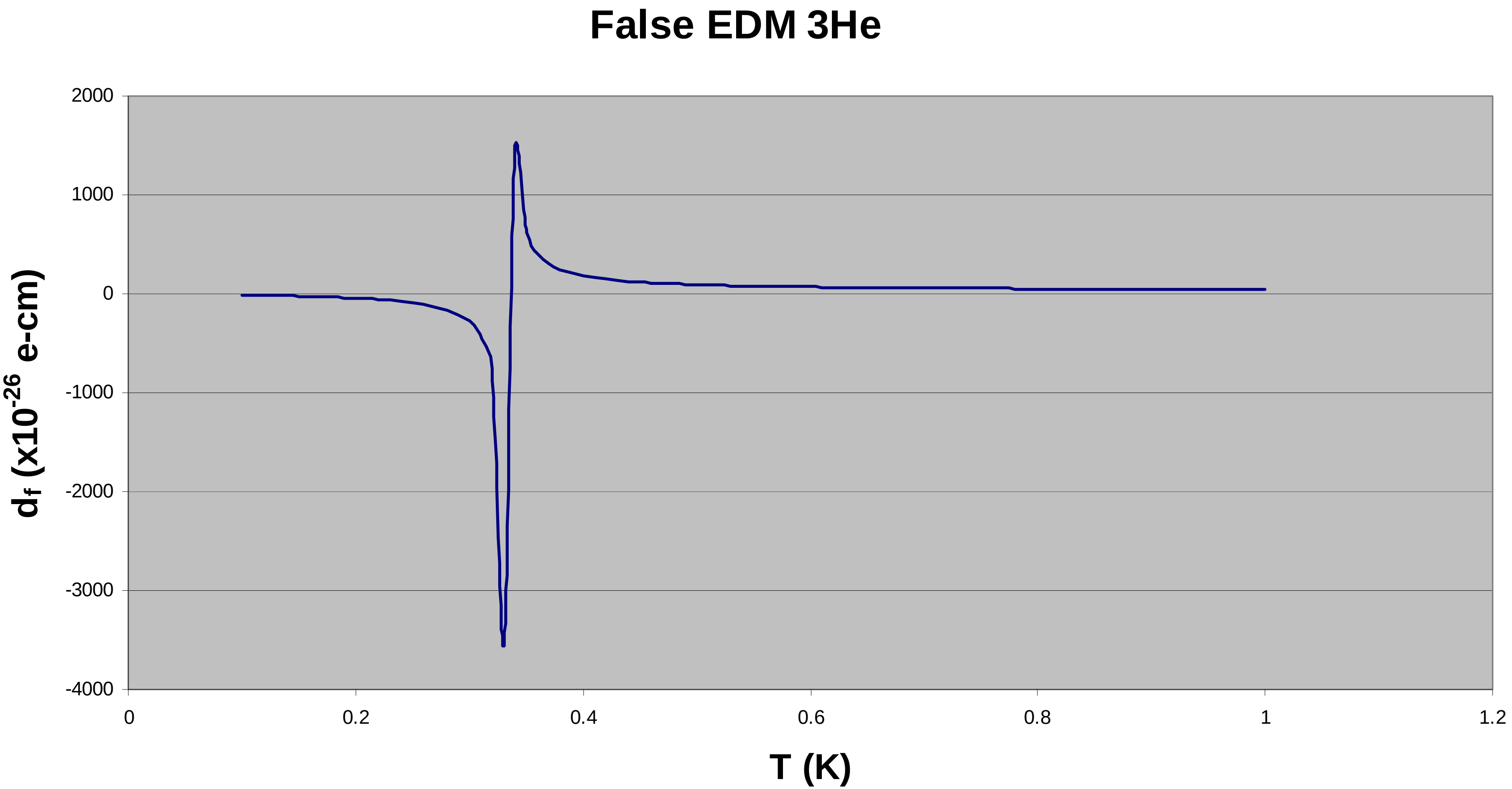
  – d_f again has a resonant term

  – After averaging over orbits d_f again depends on $\left\langle \frac{\partial B_z}{\partial z} \right\rangle$

    - NOTE: Above formula underestimates d_f if complex gradients exist

      (factor of 1.5 – 7 for OILL experiment)

For ^{3}He can see resonance vs. T (since $\omega_r \propto \mathbf{v} \propto \sqrt{T}$)

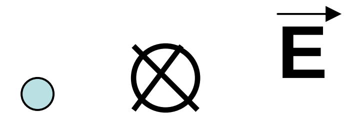


Note: $\gamma_{3He} \sim 4\gamma_{Hg}$

B_0=10mg, dB/dx=0.1μg/cm

Effects of Collisions

- Collisions can produce *very* fast rotation of the **v** x **E** B-field



- Calculation of spin re-orientation (Pendlebury et al)

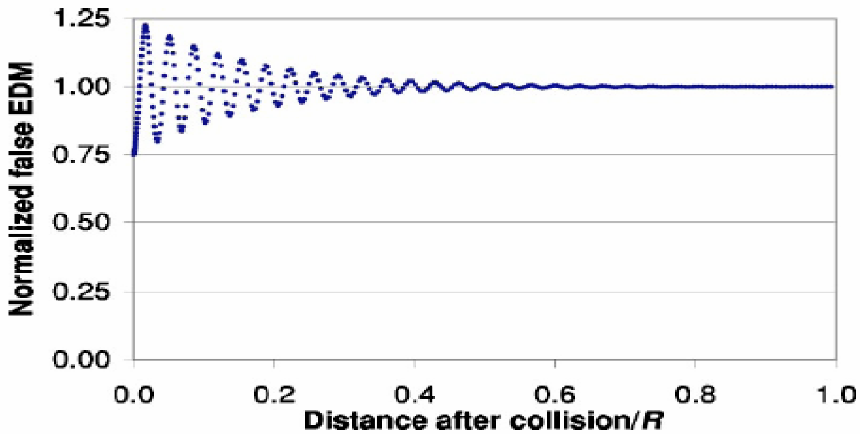


FIG. 6. (Color online) A transient oscillation following an instantaneous rotation of $\mathbf{B}_{xy}$ at a collision. The resulting false EDM settles down to the same value as it would have had if the head of the $\mathbf{B}_{xy}$ vector had followed a straight line slowly to the new position.

If spin has **enough** time it settles down to value it would have had if vxE had rotated slowly. **But** needs a few Larmor periods to *settle* down!

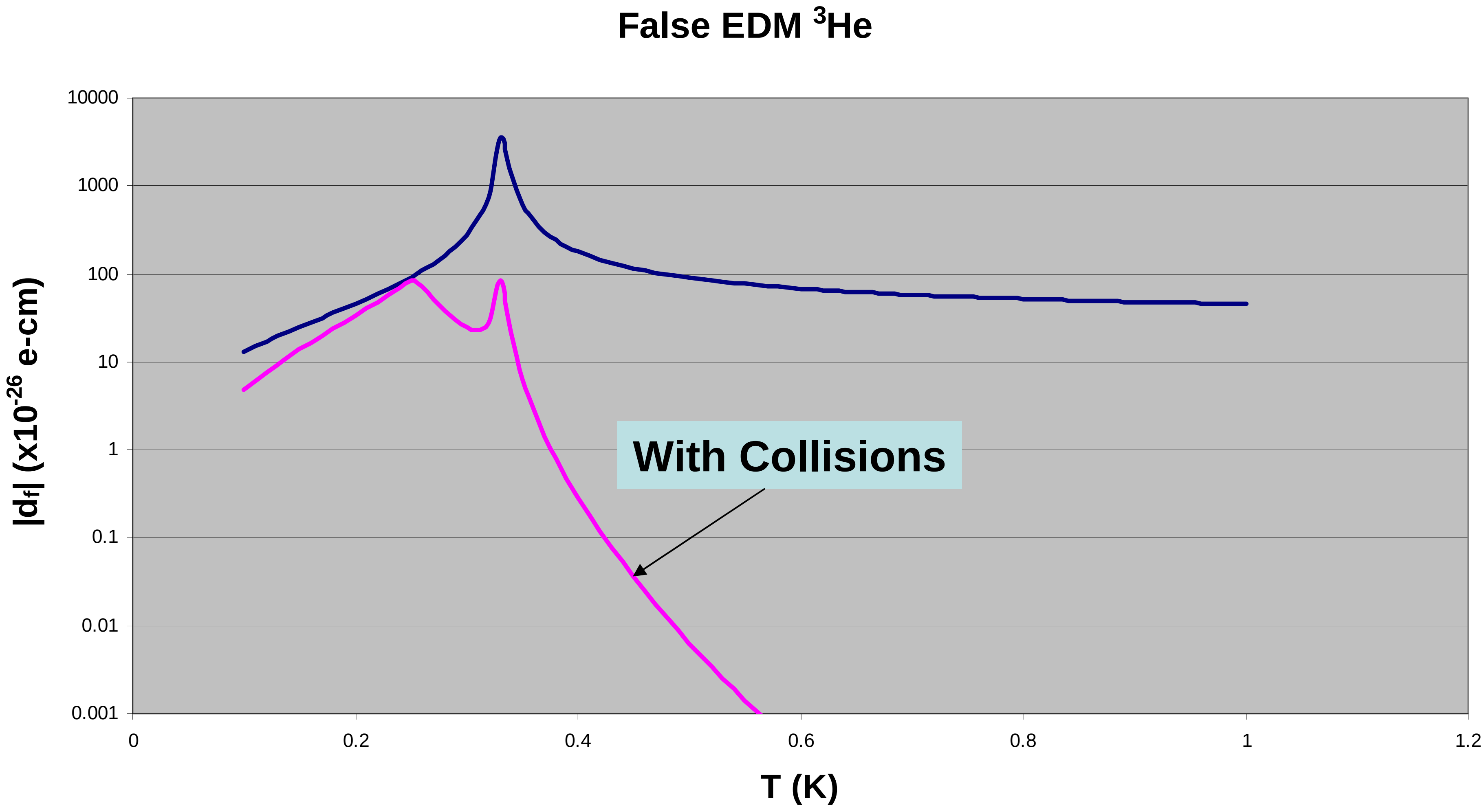
But if collisions are fast, less phase is accumulated

- Collisional reduction of d_f for adiabatic case
  - Numerical calculations give $d_{fn} \propto \frac{1}{1+\left(\frac{v}{\omega_0 \lambda}\right)^2}$ for mean free path = λ

    $\frac{v}{\omega_0 \lambda} \approx \frac{\tau_{Larmor}}{\tau_{Collision}}$
  - But can't use collisions for neutrons!
- Collisional reduction for non-adiabatic case

$$d_{f\,^3He} \propto \frac{1}{1+\left(\frac{3\pi\ D}{2R^2\omega_0}\right)^2} \quad ;\text{with } D \propto 1/T^7$$

Effect of collisions on ^3He $|d_f|$



B_0=10mg, dB/dx=0.1μg/cm

OILL Geometric Phase

GEOMETRIC-PHASE-INDUCED FALSE ELECTRIC…

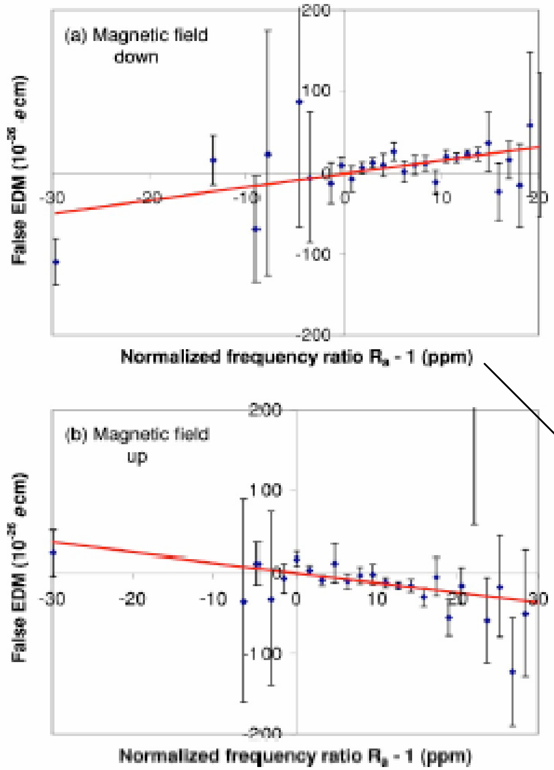


FIG. 13. (Color online) A subset of data from the neutron EDM experiment at the ILL, showing the measured false EDM as a function of the measured neutron to mercury frequency ratio. We expect this frequency ratio to be proportional to the magnetic field gradient. (Small, constant vertical offsets have been applied to the data in each plot.)

OILL uses frequency ratio measurements to characterize systematic uncertainties due to geometric phase.

$$R_a \equiv \frac{\left|\frac{\omega_{Ln}}{\omega_{LHg}}\right|}{\left|\frac{\gamma_n}{\gamma_{Hg}}\right|}$$

Summary

- False EDMs from geo. phase depend on velocity, B_0, B-gradients, collisions, ...
  - Real EDMs don't !!
  - Several handles to check systematics
  - Can make effects larger/smaller
- To achieve d_n~ $1x10^{-28}$ e-cm requires

$$\frac{1}{B_z}\left\langle\frac{\partial B_z}{\partial z}\right\rangle \approx 1 \bullet 10^{-6}\,/\,cm \quad \text{at}\, 10\, \text{mg}$$

- Interesting questions for next generation experiments