Schiff Screening of Relativistic Nucleon EDMs

J. Engel

University of North Carolina

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Prologue: What Do EDM's Have to Do With T?

Consider nondegenerate ground state $|\mathrm{g.s.} : J, M\rangle$. Symmetry under rotations $R_y(\pi) \Rightarrow$ for a vector operator like $\boxed{\vec{d} \equiv \sum_i e_i \vec{r}_i}$,

$$\langle \mathrm{g.s.} : J, M|\, \vec{d}\, |\mathrm{g.s.} : J, M\rangle = -\, \langle \mathrm{g.s.} : J, -M|\, \vec{d}\, |\mathrm{g.s.} : J, -M\rangle \ .$$

T takes M to $-M$, like $R_y(\pi)$. But $\vec{d}$ is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

$$\langle \mathrm{g.s.} : J, M|\, \vec{d}\, |\mathrm{g.s.} : J, M\rangle = +\, \langle \mathrm{g.s.} : J, -M|\, \vec{d}\, |\mathrm{g.s.} : J, -M\rangle \ .$$

Together with the first equation, this implies

$$\boxed{\langle \vec{d} \rangle = 0 \ .}$$

If T is violated, argument fails because T can take $|\mathrm{g.s.} : JM\rangle$ to $|\mathrm{ex.} : J, -M\rangle$, a state in a *different* multiplet.

Interesting Paper

Nuclear electric dipole moment with relativistic effects in Xe and Hg atom

(Phys. Rev. C75, 035501 (2007))

Sachiko Oshima, Takehisa Fujita, and Tomoko Asaga

The atomic electric dipole moment (EDM) is evaluated by considering the relativistic effects as well as nuclear finite size effects in Xe and Hg atomic systems. . . . As the results, the finite contribution to the atomic EDM comes from the first order perturbation energy of relativistic effects, and it amounts to around 0.3 and 0.4 percents of the neutron EDM d_n for Xe and Hg, respectively, though the calculations are carried out with a simplified single-particle nuclear model. From this relation in Hg atomic system, we can extract the neutron EDM which is found to be just comparable with the direct neutron EDM measurement.

Relativitistic corrections to dipole operator unshielded for electrons

Claim: For nucleons, the corrections suppressed by $v^2/c^2 \approx .01$

Usual screening suppression on order of $10Z^2 R_N^2/R_A^2 \approx .001$

Atomic EDMs about 10 times more sensitive than peviously thought!

Really Correct?

Relativistic dipole operator:

$$\boldsymbol{d} = \beta \boldsymbol{\Sigma} = \quad \beta \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

Perturbing Hamiltonian:

$$H_1 = -\sum_{j=1}^{A} d_N^j \beta \Sigma_j \cdot \boldsymbol{E} \; + \; e \sum_{i=1}^{Z} \boldsymbol{r}_i \cdot \boldsymbol{E} \quad \left(-e \sum_{k=1}^{Z} \boldsymbol{R}_k \cdot \boldsymbol{E} \right)$$

Nucleon EDMs Electron coords. Proton coords.

Unperturbed Hamiltonian:

$$H_0 = H_e^{\mathrm{int}} + H_{\mathrm{nuc}}^{\mathrm{int}} + H_{e\text{–}\mathrm{nuc}} \qquad\qquad H_{e\text{–}\mathrm{nuc}} = \sum_{i=1}^{Z} \frac{Z\, e^2}{r_i} + \dots$$

First order energy shift in external field:

$$\Delta E^{(1)} = -\langle \mathrm{g.s.}| \sum_{j=1}^{A} d_N^j \, \beta \, \boldsymbol{\Sigma}_j \, |\mathrm{g.s.}\rangle \cdot \boldsymbol{E}_{\mathrm{ext}}$$

But must consider atomic polarization:

$$\Delta E^{(2)} = -\sum_n \frac{1}{E_{\mathrm{g.s.}} - E_n} \langle \mathrm{g.s.}| \left(\sum_{j=1}^{A} d_N^j \beta \boldsymbol{\Sigma}_j \right) \cdot \left(\sum_{i=1}^{Z} \boldsymbol{\nabla}_i A_0 \right) |n\rangle$$

$$\times \langle n| e \sum_{k=1}^{Z} \boldsymbol{r}_k \cdot \boldsymbol{E}_{\mathrm{ext}} |\mathrm{g.s.}\rangle + \mathrm{h.c.}$$

Electric field at nucleus from i^{th} electron

Atomic excitations

$$A_0 = \sum_{i=1}^{Z} \frac{e}{r_i} = -\frac{1}{Z e} H_{e\text{–nuc}}$$

$$\left(\sum_{j=1}^{A} d_N^j \beta \boldsymbol{\Sigma}_j \right) \cdot \left(\sum_{i=1}^{Z} \boldsymbol{\nabla}_i A_0 \right) = -\frac{1}{Z e} \left[\sum_{j=1}^{A} \sum_{i=1}^{Z} d_N^j \beta \boldsymbol{\Sigma}_j \cdot \boldsymbol{\nabla}_i \,,\, H_{e\text{–nuc}} \right]$$

$$= -\frac{1}{Z e} \left[\sum_{j=1}^{A} \sum_{i=1}^{Z} d_N^j \beta \boldsymbol{\Sigma}_j \cdot \boldsymbol{\nabla}_i \,,\, H_0 - H_e^{\mathrm{int}} - H_{\mathrm{nuc}}^{\mathrm{int}} \right]$$

Use:

$$\sum_{i=1}^{Z}[d_N^j\,\beta\,\boldsymbol{\Sigma}_j\cdot\boldsymbol{\nabla}_i\,,\,H_e^{\rm int}] \quad = \sum_{i=1}^{Z} d_N^j\,\beta\,\boldsymbol{\Sigma}_j\cdot[\boldsymbol{\nabla}_i\,,\,H_e^{\rm int}] = 0$$

and

$$\begin{aligned}{}_{\rm nuc}\langle{\rm g.s.}|[d_N^j\,\beta\,\boldsymbol{\Sigma}_j\cdot\boldsymbol{\nabla}_i\,,\,H_{\rm nuc}^{\rm int}]|{\rm g.s.}\rangle_{\rm nuc} &= {}_{\rm nuc}\langle{\rm g.s.}|[d_N^j\,\beta\,\boldsymbol{\Sigma}_j\,,\,H_{\rm nuc}^{\rm int}]|{\rm g.s.}\rangle_{\rm nuc}\cdot\boldsymbol{\nabla}_i \\ &= 0\end{aligned}$$

and note that commutator with H_0 adds a factor $E_{\rm g.s.} - E_n$ so that you can use closure to sum over excited states. Get

$$\begin{aligned}\Delta E^{(2)} =& \frac{1}{Z}\,\langle{\rm g.s}|\left[\left(\sum_{j=1}^{A}\sum_{i=1}^{Z} d_N^j\,\beta\,\boldsymbol{\Sigma}_j\cdot\boldsymbol{\nabla}_i\right),\sum_{k=1}^{Z}\boldsymbol{r}_k\cdot\boldsymbol{E}_{ext}\right]|{\rm g.s.}\rangle \\ =& \langle{\rm g.s.}|\sum_{j=1}^{A} d_N^j\,\beta\,\boldsymbol{\Sigma}_j\,|{\rm g.s.}\rangle\cdot\boldsymbol{E}_{\rm ext} \\ =& -\Delta E^{(1)}\end{aligned}$$

Why Aren't Electron EDMs Shielded?

$$_{\rm nuc}\langle {\rm g.s.}|[d_N^j\,\beta\,\mathbf{\Sigma}_j\cdot\mathbf{\nabla}_i\,,\;H^{\rm int}_{\rm nuc}]|{\rm g.s.}\rangle_{\rm nuc}=0$$

but if j labels electrons instead of nucleons

$$\langle n|[d_e^j\,\beta\,\mathbf{\Sigma}_j\cdot\mathbf{\nabla}_i\,,\;H^{\rm int}_{\rm e}]|{\rm g.s.}\rangle\neq 0\;.$$

Why?

Because $\beta\Sigma$ does not commute with $\boldsymbol{\alpha}\cdot\boldsymbol{p}$, which is in free Dirac Hamiltonian. Doesn't matter for nucleons because expectation value is all that occurs, and it is killed by commutator with Hamiltonian.

Reason only expectation value occurs for nucleons: Gradient acts on electrons, leaving positive-parity nuclear operator $\beta\mathbf{\Sigma}$, which cannot excite states of opposite parity. But those are needed because deexcitation is by negative-parity operator $\sum_k \boldsymbol{R}_k\cdot\boldsymbol{E}$.

Differences are due to monopole approximation for $H_{e-{\rm nuc}}$, i.e. point-like approximation for nucleus.

THE END