TWO-PION EXCHANGE

PARITY-VIOLATING NN INTERACTION

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Some work with Chang Ho Hyun and Shung-ichi Ando (Suwon)

MOTIVATIONS

New work in the EFT formalism (Zhu et al.)

- two components retained
- effects up to 30% of the one-pion exchange when a comparison is possible!

Earlier work in the 70's

- many more components for the isovector part (6)
- no large contribution expected

Present talk: comparison of the different approaches

- role of LEC's
- "velocity" dependent (non-local) terms
- range

(isoscalar and isotensor components ignored)

OUTLINE

- Dispersion relation approach
- EFT approach
- Relation to the contribution of time-ordered diagrams
- Similarities and differences
- Numerical comparisons

DISPERSION-RELATION FORMALISM

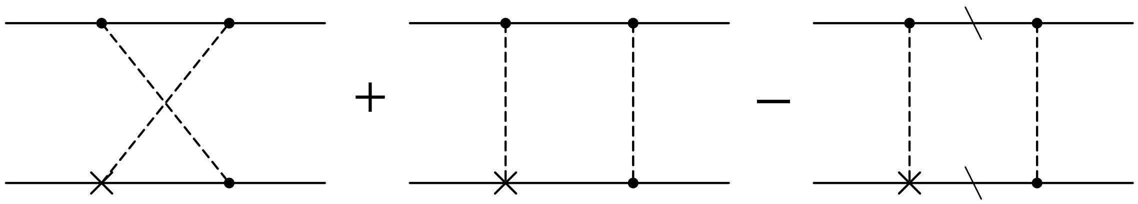
A powerful method

Schematically:

$$A(NN \to NN)(t) \propto \int \frac{dt'}{t'-t} A(NN \to 2\pi)(t') \times A^*(NN \to 2\pi)(t')$$

Some model:

approximation of the $A(NN \to 2\pi)(t')$ amplitude
by the contribution of the nucleon and its excitations
($\Delta(1232)$ resonance in particular)



PV NN INTERACTION IN MOMENTUM SPACE ($\Delta T = 1$ part)

General expression (in notations of M. Chemtob and B.D.):

$$\begin{aligned}
V(\vec{p}\,', \vec{p}) &= V_{44} + V_{34} + V_{56} + V_{75} + V_{66} + V_{85} \\
&= i\,(\tau_1 + \tau_2)^z \;\; (\vec{\sigma}_1 \times \vec{\sigma}_2)\cdot(\vec{p}\,' - \vec{p})\; v_{44}(q, \cdots) \\
&\quad + (\tau_1 + \tau_2)^z \;\; (\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{p}\,' + \vec{p})\; v_{34}(q, \cdots) \\
&\quad + i\,(\tau_1 \times \tau_2)^z \;\; (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{p}\,' - \vec{p})\; v_{56}(q, \cdots) \\
&\quad + (\tau_1 - \tau_2)^z \;\; (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{p}\,' + \vec{p})\; v_{75}(q, \cdots) \\
&\quad + (\tau_1 \times \tau_2)^z \;\; (\vec{\sigma}_1\cdot\vec{q}\; \vec{\sigma}_2\cdot(\vec{p}\,' + \vec{p})\times\vec{q} + (\vec{\sigma}_1 \leftrightarrow \vec{\sigma}_2))\; v_{66}(q, \cdots) \\
&\quad - i\,(\tau_1 - \tau_2)^z \;\; (\vec{\sigma}_1\cdot(\vec{p}\,' + \vec{p})\; \vec{\sigma}_2\cdot(\vec{p}\,' + \vec{p})\times\vec{q} + (\vec{\sigma}_1 \leftrightarrow \vec{\sigma}_2))\; v_{85}(q, \cdots)\,.
\end{aligned}$$

($\vec{p}$ and $\vec{p}\,'$: c.m. nucleon momenta in initial and final states,
$\vec{q}$: momentum transfer carried by pions)

Four terms at order M^{-1} and two at order M^{-3} (generally discarded)

Functions $v_{ij}(q, \cdots) \rightarrow$ assume a dispersion relation form:

$$v_{ij}^{\rm COV}(q, \cdots) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dt'\; \frac{g_{ij}(t', \cdots)}{\sqrt{t'}\,(t' + q^2)}\,,$$

PV NN INTERACTION IN CONFIGURATION SPACE ($\Delta T = 1$ part)

General expression:

$$
\begin{aligned}
V(r,\vec{p}\,',\vec{p}) = {} & i\,(\tau_1+\tau_2)^z\;\;(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot[\vec{p},\;v_{44}(r,\cdots)] \\
& +(\tau_1+\tau_2)^z\;\;(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\{\vec{p},\;v_{34}(r,\cdots)\} \\
& +i\,(\tau_1\times\tau_2)^z\;\;(\vec{\sigma}_1+\vec{\sigma}_2)\cdot[\vec{p},\;v_{56}(r,\cdots)] \\
& +(\tau_1-\tau_2)^z\;\;(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\{\vec{p},\;v_{75}(r,\cdots)\} \\
& +2i\,(\tau_1\times\tau_2)^z\;\;(\vec{\sigma}_1\cdot[\vec{p},\;\vec{\sigma}_2\cdot\vec{l}\,\frac{1}{r}\frac{d}{dr}v_{66}(r,\cdots)]+(\vec{\sigma}_1\leftrightarrow\vec{\sigma}_2)) \\
& -2\,(\tau_1-\tau_2)^z\;\;(\vec{\sigma}_1\cdot\{\vec{p},\;\vec{\sigma}_2\cdot\vec{l}\,\frac{1}{r}\frac{d}{dr}v_{85}(r,\cdots)\}+(\vec{\sigma}_1\leftrightarrow\vec{\sigma}_2))\;\;,
\end{aligned}
$$

where:

$$
v_{ij}^{\rm COV}(r,\cdots) = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} dt'\;g_{ij}(t',\cdots)\;\frac{e^{-r\sqrt{t'}}}{r\;\sqrt{t'}}\;.
$$

Notice the appearance of the orbital angular momentum operator in the last two terms

A FEW DETAILS

Convergence properties:

depend on the way the iterated one-pion exchange is removed

(choice of the Green's function, $(2E_p - 2E_0)^{-1}$ or $E_0(E_p^2 - E_0^2)^{-1}$)

Below $\to$ quadratic mass operator

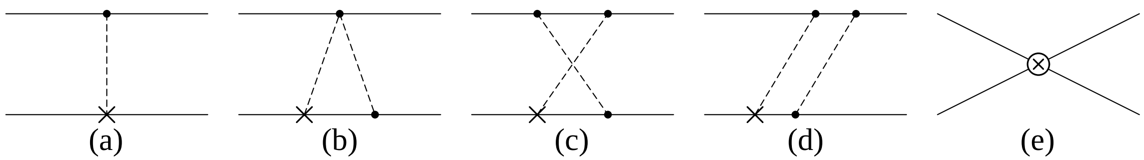
$$
\begin{aligned}
g_{44}(t')_{t' \to \infty} &= \tilde{K} \frac{2}{M\sqrt{t'}} \Big(\log(\frac{t'}{M^2}) - 1\Big)\,, \\
g_{34}(t')_{t' \to \infty} &= \tilde{K} \frac{2}{M\sqrt{t'}} \Big(\log(\frac{t'}{M^2}) - 1\Big)\,, \\
g_{56}(t')_{t' \to \infty} &= -\tilde{K} \frac{1}{M\sqrt{t'}} \Big(\frac{5}{8}\log(\frac{t'}{M^2}) + \frac{15}{8} - \frac{3}{2}\log(2)\Big)\,, \\
g_{75}(t')_{t' \to \infty} &= \tilde{K} \frac{1}{M\sqrt{t'}} \Big(\frac{9}{4}\log(\frac{t'}{M^2}) - \frac{1}{4} + \log(2)\Big)\,.
\end{aligned}
$$

$(t')^{-1/2}$ behavior $\to$ convergence of dispersion integrals ensured, though $\cdots$

can provide some benchmark result

TWO-PION EXCHANGE IN THE EFT FORMALISM (ZHU et al.)

Graphical representation:



$$v_{44}^{\rm EFT}(q) = -4\sqrt{2}\,\pi\frac{h_\pi^1}{\Lambda_\chi^3}(g_A^3\,L(q) - LEC_2)\,,$$

$$v_{56}^{\rm EFT}(q) = -\sqrt{2}\,\pi\frac{h_\pi^1}{\Lambda_\chi^3}(g_A\,L(q) - g_A^3\,(3-\frac{4\,m_\pi^2}{4\,m_\pi^2+q^2})\,L(q) + LEC_6)\,.$$

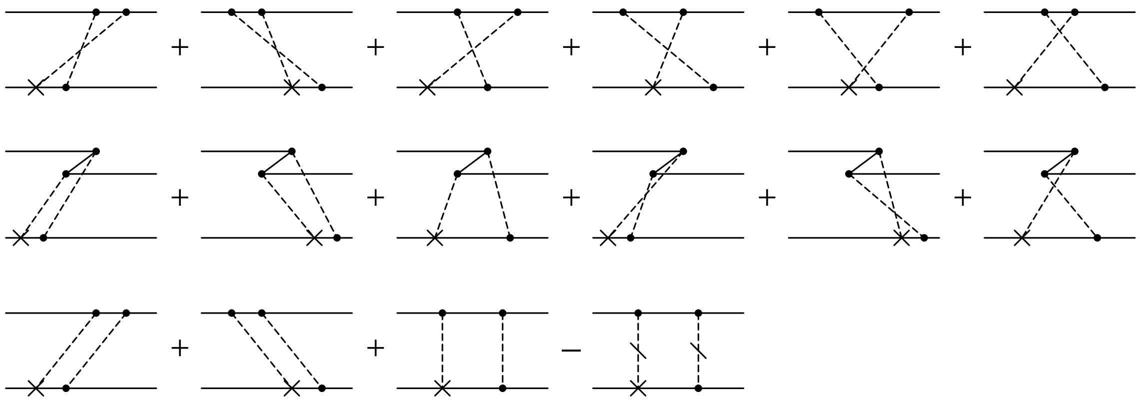
Scale Λ_χ: roughly given by $\Lambda_\chi = 4\pi f_\pi = 4\pi\, g_A\, M/g_{\pi NN} \simeq 1$ GeV.

Function $L(q) \rightarrow$ defined as:

$$L(q) = \frac{\sqrt{q^2+4m_\pi^2}}{q}\log\left(\frac{\sqrt{q^2+4m_\pi^2}+q}{2m_\pi}\right) = \frac{\sqrt{q^2+4m_\pi^2}}{2\,q}\log\left(\frac{\sqrt{q^2+4m_\pi^2}+q}{\sqrt{q^2+4m_\pi^2}-q}\right)\,.$$

TWO-PION EXCHANGE IN THE TIME-ORDERED DIAGRAM APPROACH

Graphical representation:



$$v^{\rm TO}_{44}(q) = \frac{g^3_{\pi NN}\, h^1_\pi}{4\, M^3 \sqrt{2}}$$

$$\times \int \frac{d\vec{k}}{(2\pi)^3} \frac{k^2 - (\vec{k}\cdot\hat{q})^2}{\omega_i\ \omega_j} \left(\frac{1}{\omega_i^2\,(\omega_i + \omega_j)} + \frac{1}{(\omega_i + \omega_j)\,\omega_j^2} + \frac{1}{\omega_i\,(\omega_i + \omega_j)\,\omega_j}\right),$$

$$v^{\rm TO}_{56}(q) = \frac{g^3_{\pi NN}\, h^1_\pi}{4\, M^3 \sqrt{2}} \left(\frac{1}{2} \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{\omega_i\,\omega_j\,(\omega_i + \omega_j)}\right.$$

$$\left. - \frac{1}{2} \int \frac{d\vec{k}}{(2\pi)^3}\, \frac{k^2 - \frac{q^2}{4}}{\omega_i\ \omega_j} \left(\frac{1}{\omega_i^2\,(\omega_i + \omega_j)} + \frac{1}{(\omega_i + \omega_j)\,\omega_j^2} + \frac{1}{\omega_i\,(\omega_i + \omega_j)\,\omega_j}\right)\right),$$

TIME-ORDERED DIAGRAM APPROACH → DISPERSION FORM

Supposes a complete set of topologically equivalent diagrams

$$\int d\vec{k}\,\frac{k^2-(\vec{k}\cdot\hat{q})^2}{\omega_i\,\omega_j}\,\Big(\frac{1}{\omega_i^2\,(\omega_i+\omega_j)}+\frac{1}{(\omega_i+\omega_j)\,\omega_j^2}+\frac{1}{\omega_i\,(\omega_i+\omega_j)\,\omega_j}\Big)$$

$$= 4\pi(1-L(q))+\int d\vec{k}\,\frac{k^2}{\omega_k^5} = \pi\int_{4m_\pi^2}^{\infty} dt'\,\frac{2\sqrt{t'-4\,m_\pi^2}}{\sqrt{t'}\,(t'+q^2)}\,,$$

$$\int d\vec{k}\,\frac{1}{\omega_i\,\omega_j\,(\omega_i+\omega_j)} = 2\pi(1-L(q))+\frac{1}{2}\int d\vec{k}\,\frac{1}{\omega_k^3} = \pi\int_{4m_\pi^2}^{\infty} dt'\,\frac{\sqrt{t'-4\,m_\pi^2}}{\sqrt{t'}\,(t'+q^2)}\,,$$

$$\int d\vec{k}\,\frac{k^2-\frac{q^2}{4}}{\omega_i\,\omega_j}\,\Big(\frac{1}{\omega_i^2\,(\omega_i+\omega_j)}+\frac{1}{(\omega_i+\omega_j)\,\omega_j^2}+\frac{1}{\omega_i\,(\omega_i+\omega_j)\,\omega_j}\Big)\Big)\,,$$

$$= 2\pi\Big(3(1-L(q))+\frac{4\,m_\pi^2}{4\,m_\pi^2+q^2}L(q)\Big)+\frac{3}{2}\int d\vec{k}\,\frac{k^2}{\omega_k^5}$$

$$= \pi\int_{4m_\pi^2}^{\infty} dt'\;\;\frac{3(t'-4m_\pi^2)+4m_\pi^2}{\sqrt{t'}\,\sqrt{t'-4m_\pi^2}\,(t'+q^2)}\,.$$

Notice: the $L(q)$ part and the total result have opposite signs

SIMILARITIES AND DIFFERENCES

Spectral functions $g(t')$: to be noticed

$\rightarrow$ large M limit (large distances) $\neq$ large t' limit (small distances)

$$
\begin{aligned}
g_{44}(t')_{M\rightarrow\infty} &= \tilde{K}\,\frac{4q_\pi}{M^3} = \tilde{K}\,\frac{2\sqrt{t'-4m_\pi^2}}{M^3}\,,\\
g_{34}(t')_{M\rightarrow\infty} &= \tilde{K}\,\frac{\pi}{M^4}\,\frac{x}{2} = \tilde{K}\,\frac{\pi}{M^4}\frac{t'-2m_\pi^2}{4}\,,\\
g_{56}(t')_{M\rightarrow\infty} &= -\tilde{K}\,\frac{x}{q_\pi\,M^3} = -\tilde{K}\,\frac{(t'-2m_\pi^2)}{M^3\,\sqrt{t'-4m_\pi^2}}\,,\\
g_{75}(t')_{M\rightarrow\infty} &= \tilde{K}\,\frac{\pi}{M^4}\,(\frac{t'-4m_\pi^2}{16}+\frac{3}{2}\,\frac{t'-2m_\pi^2}{4})\,.
\end{aligned}
$$

Large M limit: $g_{44}(t')$ and $g_{56}(t')$ dominate

$\rightarrow$ in agreement with EFT and TO approaches

SIMILARITIES AND DIFFERENCES

Potentials in momentum space

$$v_{44}^{\rm LM}(q) = v_{44}(q)_{M\to\infty} = \frac{g_{\pi NN}^3\; h_\pi^1}{16\,\pi^2\; M^3\sqrt{2}} \;\; \int_{4m_\pi^2}^{\infty} dt' \; \frac{\sqrt{t' - 4\,m_\pi^2}}{\sqrt{t'}\;(t'+q^2)}$$

$$= -4\sqrt{2}\,\pi \frac{g_{\pi NN}^3\;\; h_\pi^1}{(4\,\pi g_A M)^3} \;\; (g_A^3\; L(q) - LEC_2)\,,$$

$$v_{56}^{\rm LM}(q) = v_{56}(q)_{M\to\infty} = -\; \frac{g_{\pi NN}^3\; h_\pi^1}{32\,\pi^2\; M^3\sqrt{2}} \; \int_{4m_\pi^2}^{\infty} dt' \; \frac{(t' - 2m_\pi^2)}{\sqrt{t'}\;\sqrt{t' - 4m_\pi^2}\;(t'+q^2)}$$

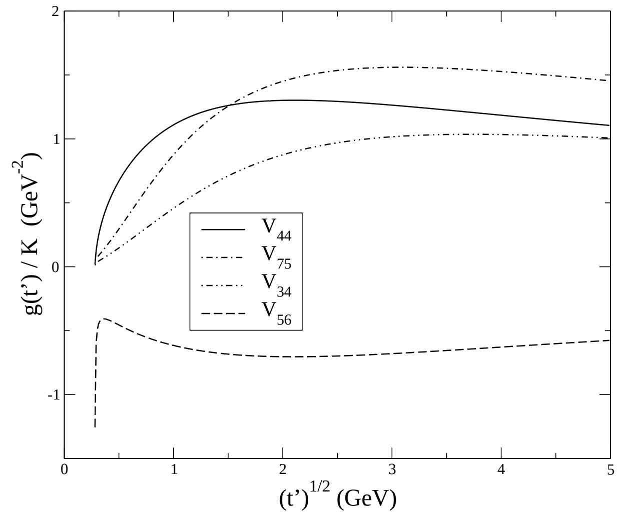
$$= -\sqrt{2}\,\pi \frac{g_{\pi NN}^3\; h_\pi^1}{(4\,\pi g_A M)^3}\; (g_A^3\; L(q) - g_A^3\;(3 - \frac{4\,m_\pi^2}{4\,m_\pi^2 + q^2})\; L(q) + LEC_6)\,.$$

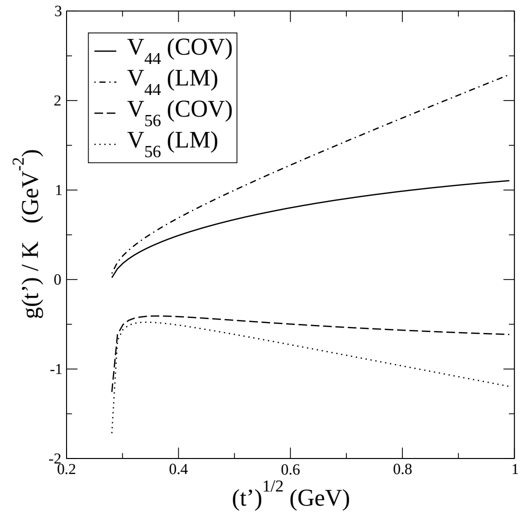
- v_{44}: EFT, TO and LM coincide (up tp LEC's and choice of Λ_χ)
- v_{56}: EFT differs for the triangle diagram: g_A instead of g_A^3
  $\to$ role of the $\Delta(1232)$ in the dispersion- relation approach

FUNCTIONS $g(t')$

Left panel:

- $g_{44}(t')$, $g_{34}(t')$, $g_{75}(t')$, $g_{56}(t')$: comparable at large t'
- $g_{44}(t')$, $g_{56}(t')$: dominate at low t'



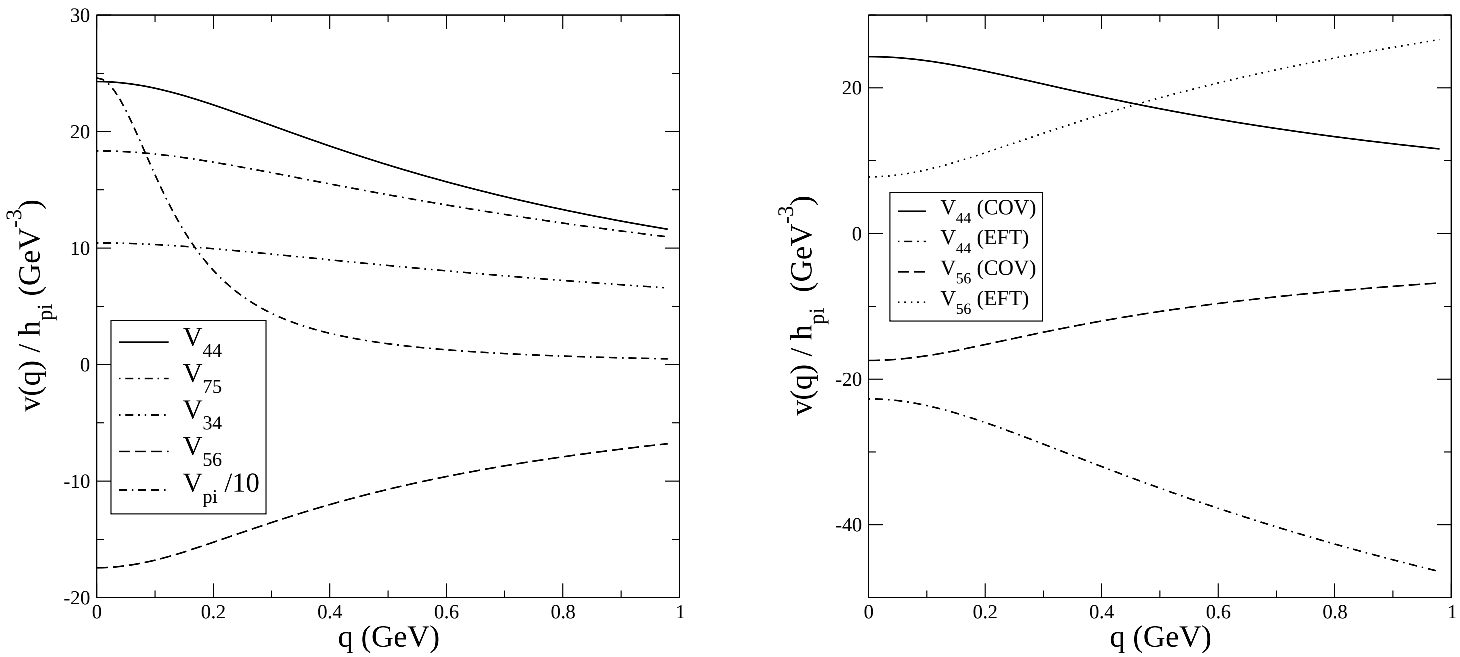


Right panel:

- large M limit for $g_{44}(t')$ and $g_{56}(t') \rightarrow$ overestimates the full result

POTENTIALS $v(q)$

Left panel: all potentials are comparable (apart V_{pi}, expected)

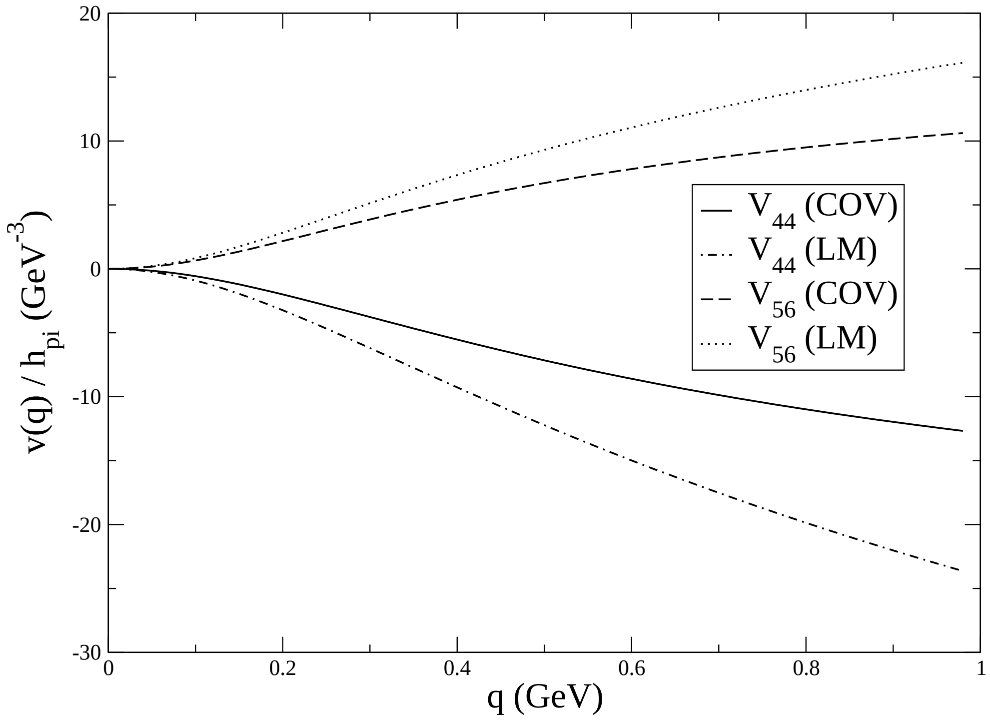


Right panel:

- v^{COV} and v^{EFT} have opposite signs for both components, v_{44} and v_{56}
- similar q dependence however $\rightarrow$ role of LEC's

SUBSTRACTED POTENTIALS $v(q)$

Potentials v_{44} and v_{56} substracted so that to vanish at $q = 0$

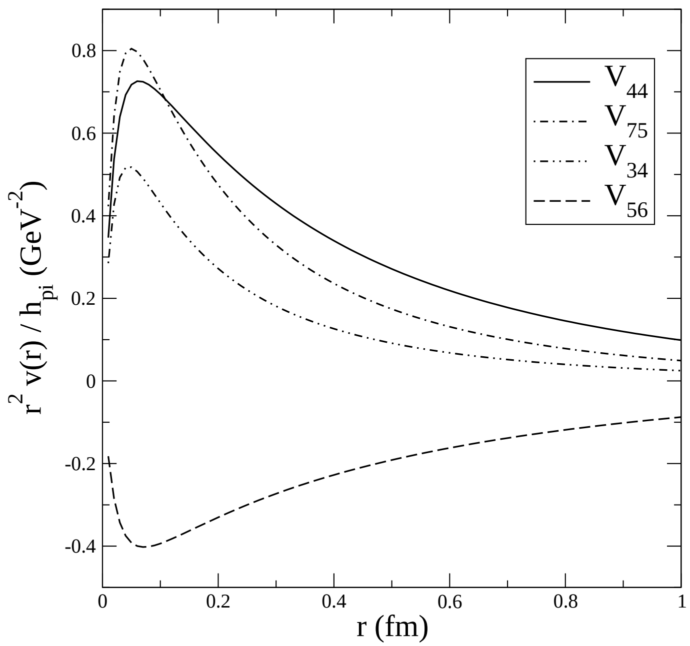


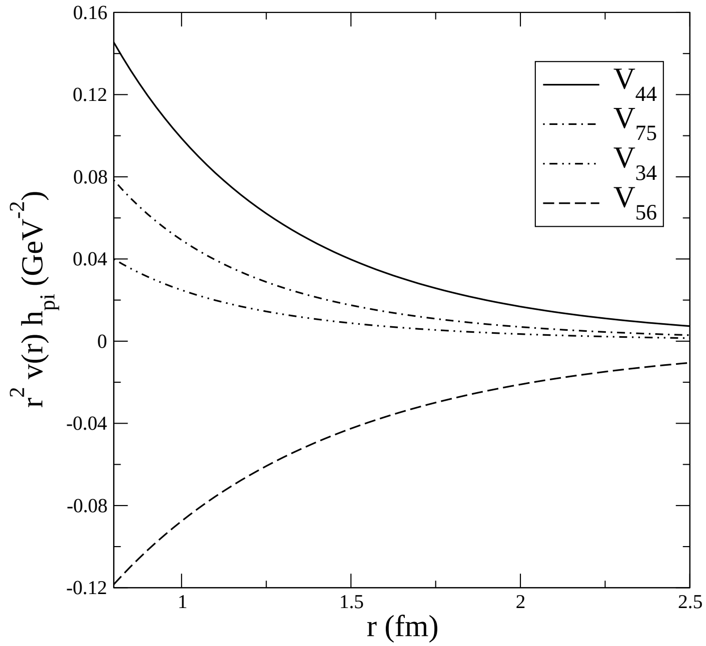
The large M limit tends to overestimate the “covariant” result

$\rightarrow$ cutoff role due to a relativistic treatment

POTENTIALS $v(r)$

- All potentials compare at (very) small distances
- The potentials v_{44} and v_{56} dominate at medium and large distances but contribution in these ranges relatively small



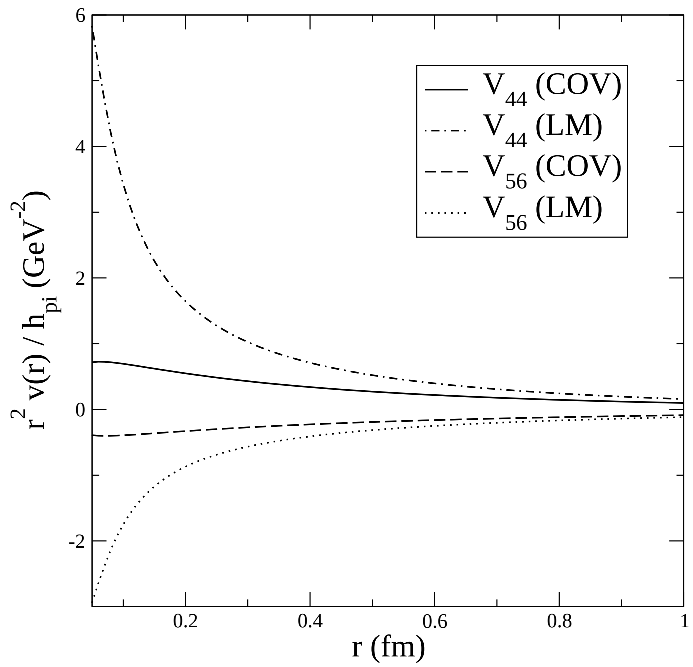


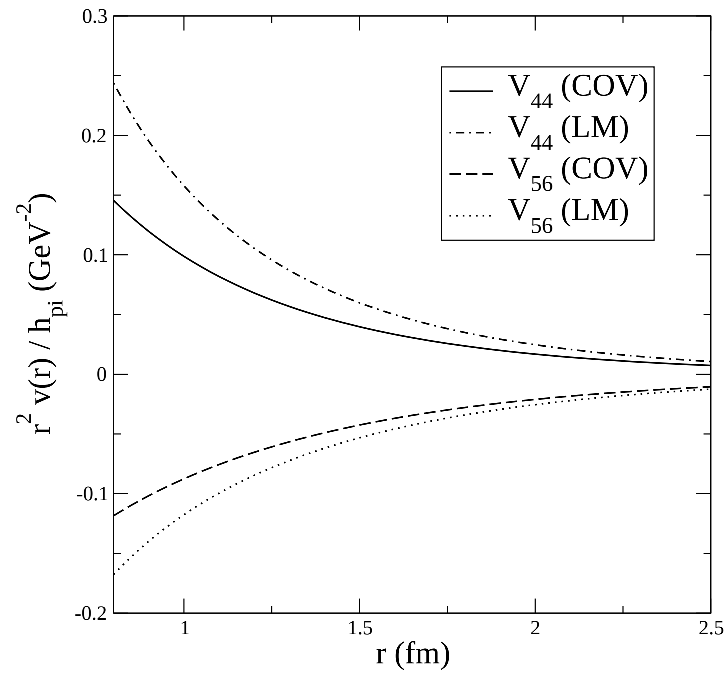
Notice ratios: $v_{34}/v_{44} \rightarrow \rho$-exchange like around 1fm for ${}^1S_0 \leftrightarrow {}^3P_0$
$v_{75}/v_{56} \rightarrow$ destructive interference for ${}^3S_1 \leftrightarrow {}^3P_1$

POTENTIALS $v(r)$ AND LARGE M LIMIT

Results in qualitative agreement with those in momentum space:

large M limit $\rightarrow$ overestimates the "covariant" result,
especially at small r's

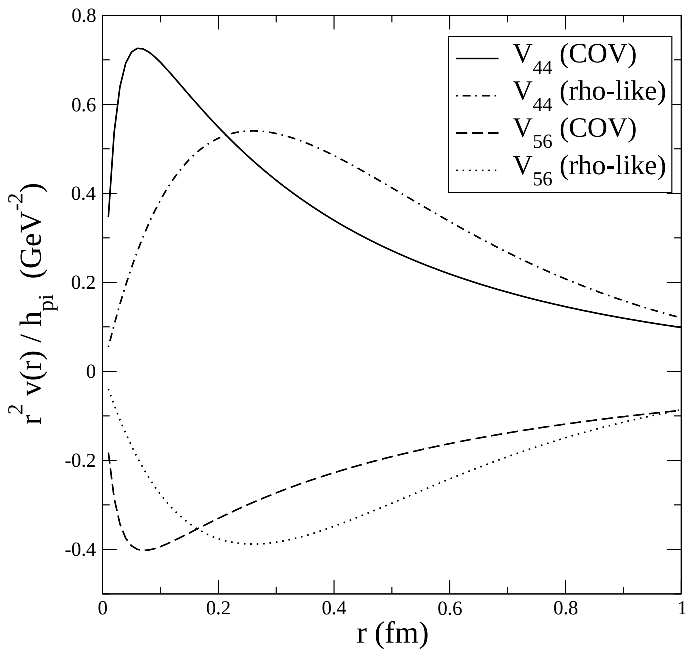


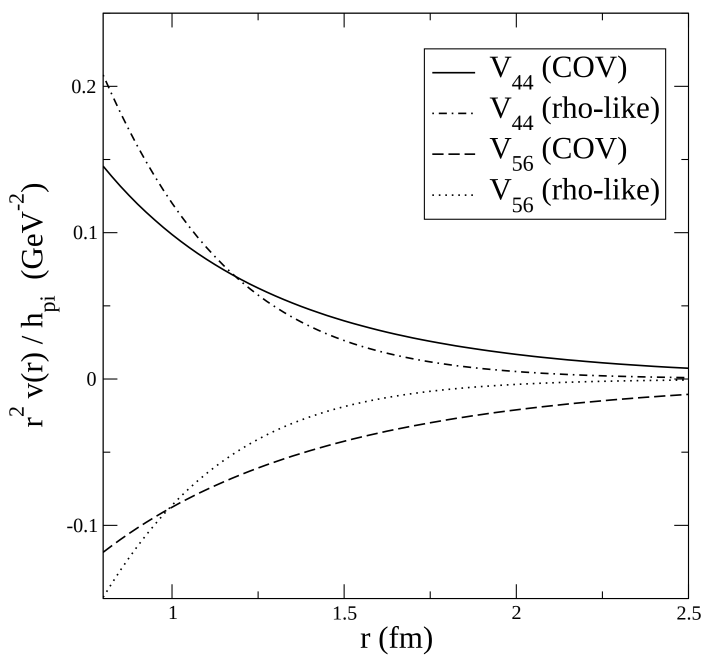


Important notice: large M limit $\rightarrow$ divergent r^{-1} behavior for $r \rightarrow 0$,
compensated by a $\delta(r)$ function of opposite sign in the EFT approach (LEC's!)

POTENTIALS $v(r)$ AT LARGE AND SMALL DISTANCES

Two-pion exchange: a medium-range interaction? $\rightarrow$ compare to ρ exchange





Difference with the strong-interaction case:

no S-state two-pion exchange, only in a P-state

CONCLUSION AND OUTLOOK

Comparison $\rightarrow$ no fundamental discrepancy

Suggests however many corrections to the lowest-order EFT approach

- higher M^{-1} order terms, not negligeable
- smaller long-range contribution

Role of LEC's

$\rightarrow$ a different substraction scheme is to be preferred

$$L(q) \rightarrow L(q) - 1 - \log(\frac{\mu}{m_\pi}) \text{ with } \mu \simeq M$$

Range of two-pion exchange in comparison with the ρ-exchange one:

- longer range (two-pion tail),
- but also shorter range (extra log (r) dependence)