Electric Dipole Moment in (S)SUSY models

We-Fu Chang

National Tsing-Hua University, Taiwan

hep-ph/0503055

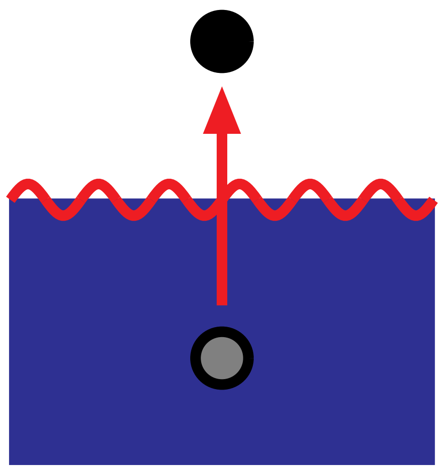
(with Darwin Chang, Wai-Yee Keung)

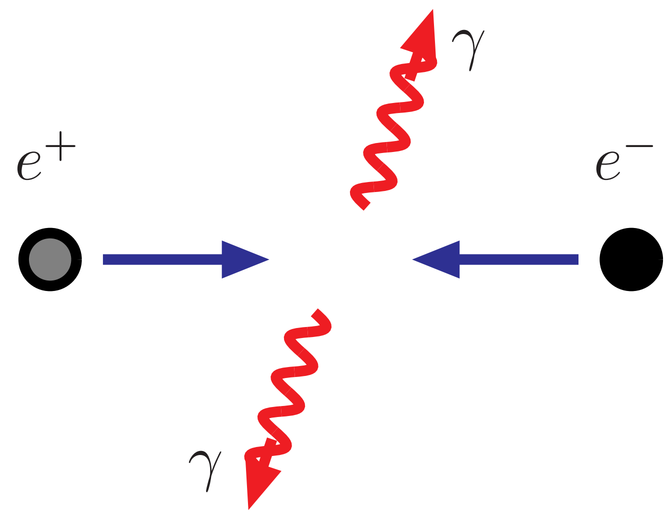
INT EDM Workshop

March 21, 2007

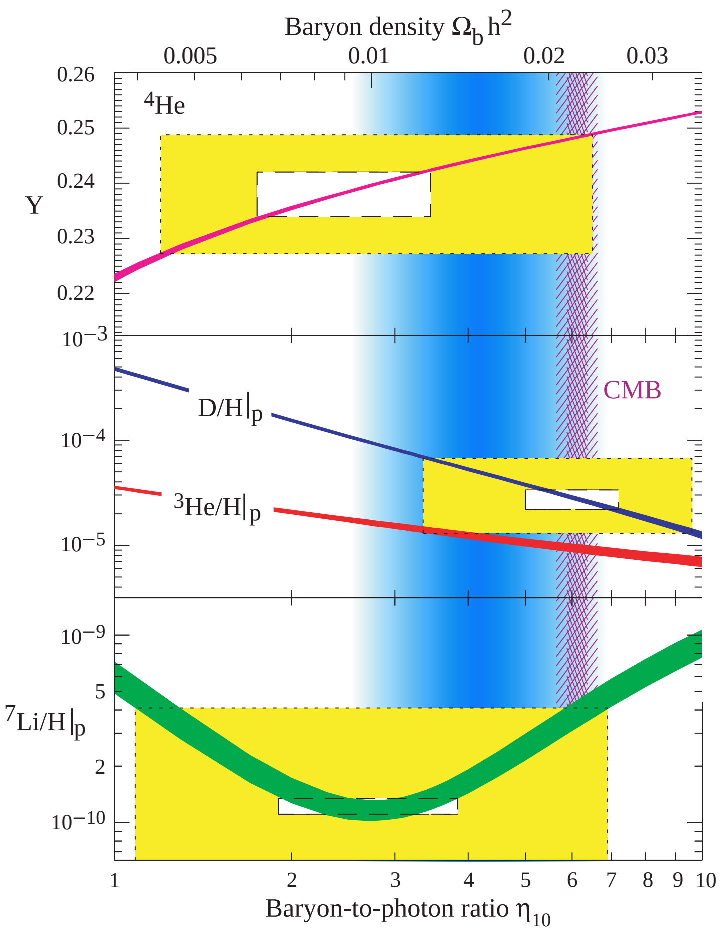
An everyday mystery

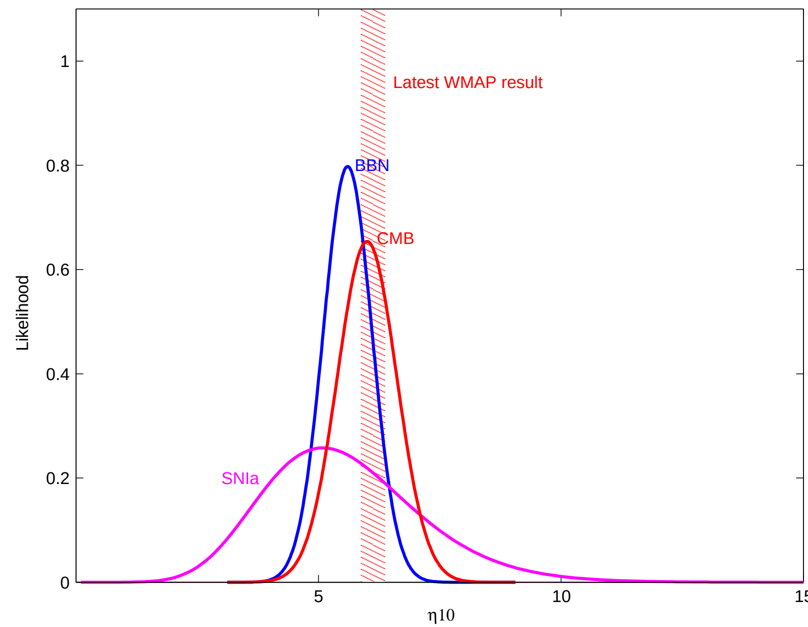
- Every single second, we witness one of Nature's great mysteries.
- How can we be here sound (and sleeping?)
  Where goes the antimatter?





Baryon Asymmetry of the Universe



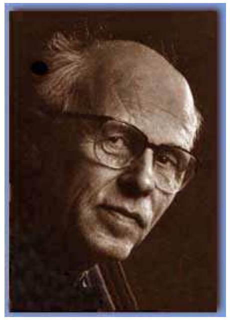


It's now well established that:

$$\eta_B \sim 5.6 \times 10^{-10}$$

$$\left(Y_B \sim \frac{\eta}{7} \right)$$

Sakharov's 3 conditions



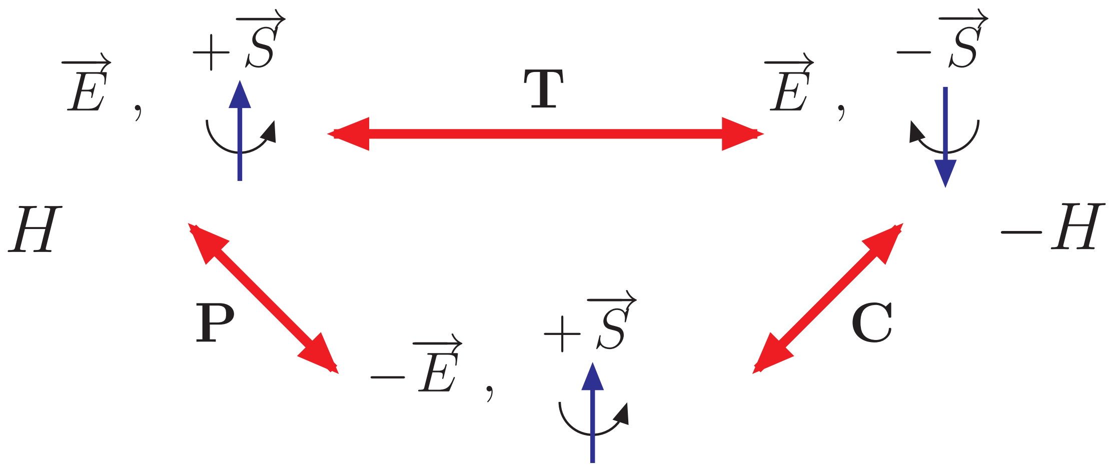
It was first realized by A. Sakharov in 1967 that to generate the matter anti-matter asymmetry from the initially symmetrical phase, the following three necessary conditions must be satisfied.

- Baryon (or Lepton) number violation
  - Because at the very beginning, $n_B - n_{\bar{B}} = 0$.
- C and CP violation
  - C violation is for distinguishing baryon from anti baryon.
  - CP violation is to mark a special reaction rate direction in the thermal soup.
- Out of equilibrium
  - Since CPT predicts $m_P = m_{\bar{P}}$, if it is in thermal equilibrium,

$$n_P = \int \frac{d^3k}{e^{-\beta\sqrt{k^2+m_P^2}}+1} = n_{\bar{P}}$$

EDM of a fundamental particle

- The spin of en elementary particle provides a vector, an intrinsic direction, to be associated with a possible permanent EDM.
- EDM violates CP symmetry,



- in QFT,

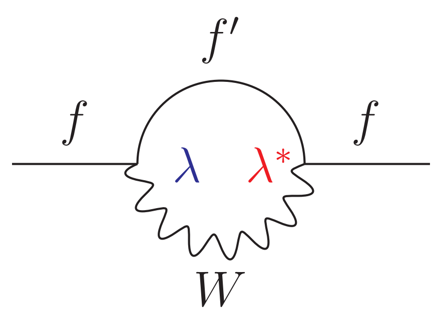
$$CP violation \Leftrightarrow Physical\ complex\ couplings$$

EDM in QFT

- In QFT, EDM corresponds to a dim-5 operator after EWSB(dim-6 if SM symmetry),

$$\mathcal{L}_{EDM} = -i\frac{d_f}{2}\bar{f}_{L/R}\sigma^{\mu\nu}\gamma_5 f_{R/L}F_{\mu\nu} \rightarrow d_f\,\overrightarrow{s}\cdot\overrightarrow{E}\;(NR\;limit)$$

  which does not appear at the tree-level.
- In a renormalizable QFT, EDM comes from quantum corrections. No counter term in the Lagrangian to cancel the div. Therefore, EDM is finite.
- In SM the CP violation is in the CKM, charged currents. It is very hard to make the result complex. The complex coupling tend to appear in conjugated pair.

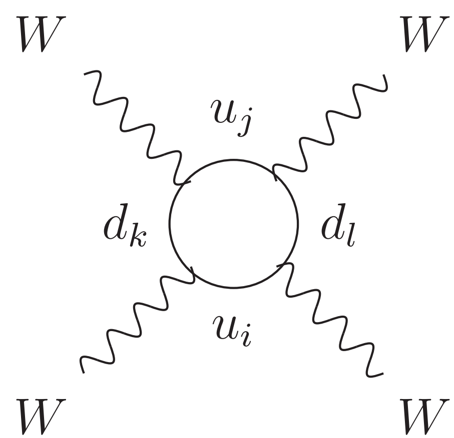


EDM in SM-1

- In SM, a basis independent invariance, Jarlskog, is used to quantify how large the CP violation is. It is defined as:

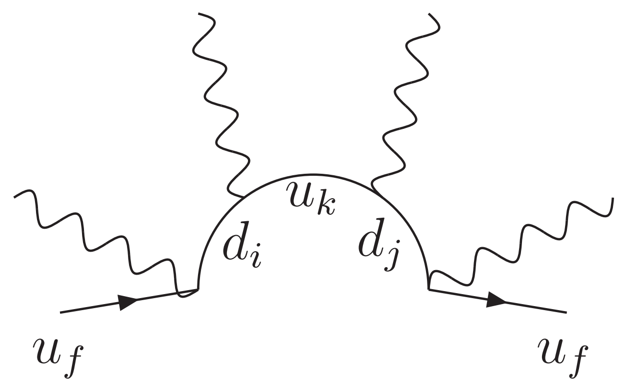
$$\sum \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}J = Im[V^*_{\beta i}V_{\beta j}V^*_{\alpha j}V_{\alpha i}]$$

  and Nature picks a small value for $J \sim 10^{-5}$.
- Therefore, it's easy to see that the minimum possible EDM diagram must involve FOUR W bosons vertices as shown

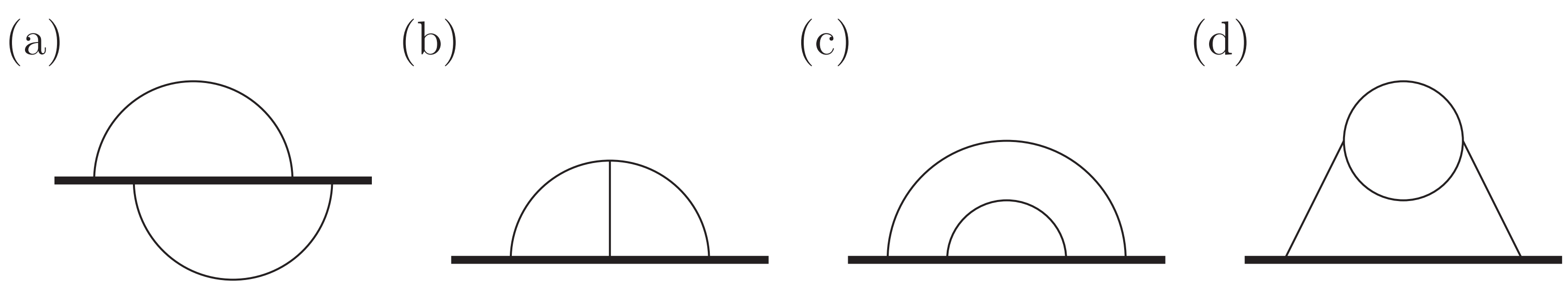


EDM in SM-2

- So, let's try to make a 2-loop quark EDM from the previous diagram.
- The most economic way is cutting one of the quark lines and make the 2 ends external as shown (external up quark as example)



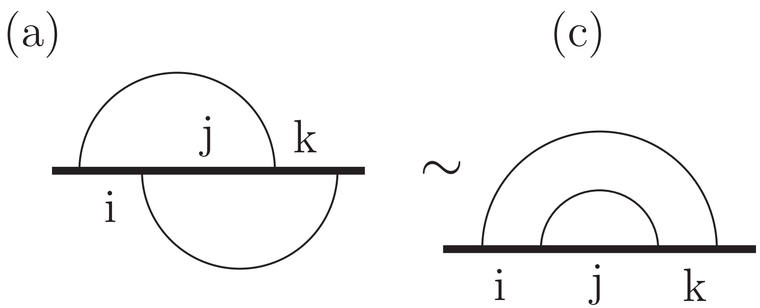
- and then try to hide the open W lines to form any one of the following four 2-loop topology:



It's clear that only type -(a) and (c) are possible.

EDM in SM-3

- First, if all quark masses are degenerated, the unitarity of CKM matrix guarantees that CP violation vanishes.



$$\sim \sum_{ijk} V_{iu}(V^\dagger)_{ji} V_{kj} (V^\dagger)_{uk} = 1$$

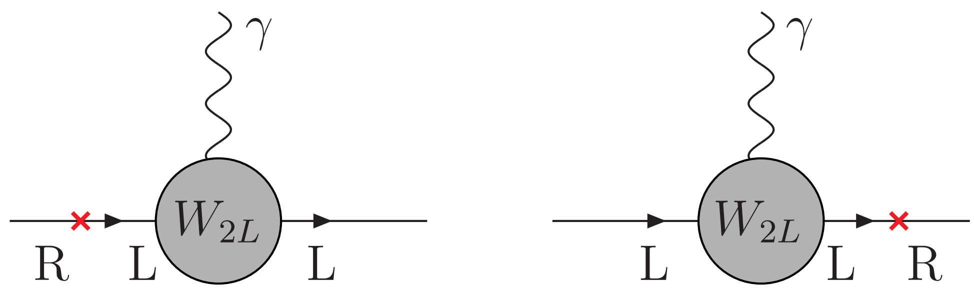
- How about putting in the quark masses?
By dimensional analysis, we can guess the masses splitting effects must be proportional to the following factor

$$(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_s^2)(m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2) M_W^{-12} \sim 10^{-20}$$

- It's amazingly small. But is it in principle non-zero?

EDM in SM-4

- To answer it, I want to remind you that the SM Charged Current interaction is purely left-handed!
- Also we know the EDM operator must flips chirality of the external fermion!
- Which means, no matter how you play with the mass insertion game on the internal quark lines, eventually we need a mass insertion at one of the external fermions to make the chirality right. And we have tow ways to do it:



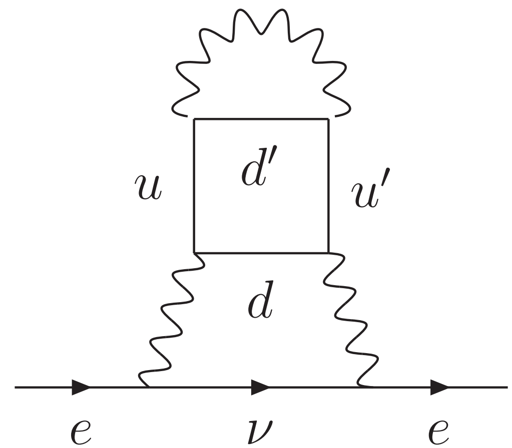
- Although each one is complex, their EDM parts happen to cancel!

$$c\bar{f}\sigma^{\mu\nu}(1+\gamma_5)fF_{\mu\nu}+c\bar{f}\sigma^{\mu\nu}(1-\gamma_5)fF_{\mu\nu}$$

- This shows you another tricky part of doing EDM calculation. You always need to worry about the conjugated diagram!

EDM in SM-5

- In the last 4 pages, I have shown you that in SM the quark EDM don't have any 2-loop contribution!!
- Same method can be applied to the charged lepton EDM. The minimal possible diagram is 3-loop. Because two of the W boson lines must end at the charged lepton. It looks like:



- Again, the pure left-handed CC interaction and the conjugated diagrams make this 3-loop EDM vanish!
- Assume that the electron(quark) EDM starts at 4-loop(3-loop), in SM the values are extremely tiny:

$$|d_n| < 10^{-30}\text{e-cm} \, , \, |d_e| < 10^{-38}\text{e-cm}$$

Therefore, EDM will be a very clear signal beyond SM.

Small Recap

Ok, let me summarize a little bit here:

- Even from the daily experience, we know there must be new CP violating source(s) beyond SM.

- EDM will be a very clean probe and constraint for CP violation beyond SM.

2-loop EDM for any new physics

- Why do we care about 2-loop contribution rather than 1-loop ones?
- The main reason is the current limits on EDM are already very stringent.

$$d_e < 1.7 \times 10^{-27} e\, cm\,,\; d_n < 6.3 \times 10^{-26} e\, cm$$

- From a simple estimation, the 1-loop induced electron EDM has a typical value:

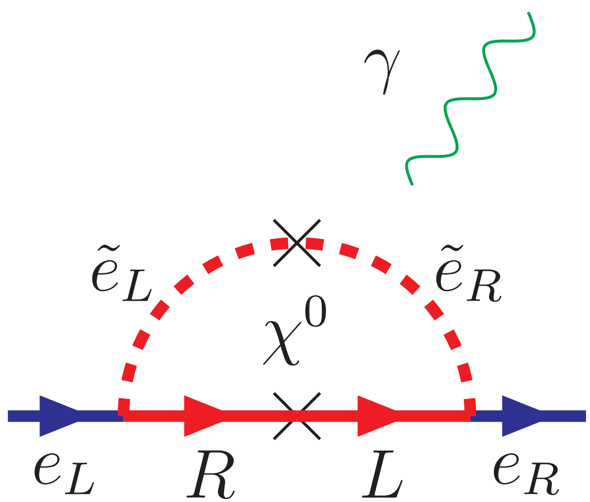
$$\sim \frac{m_e g^2}{16\pi^2} \frac{e \sin\phi_{CP}}{M^2} \ln \frac{M}{M_e} \sim 10^{-23} g^2 \sin\phi_{CP} \left(\frac{100\text{GeV}}{M}\right)^2 \text{e-cm}$$

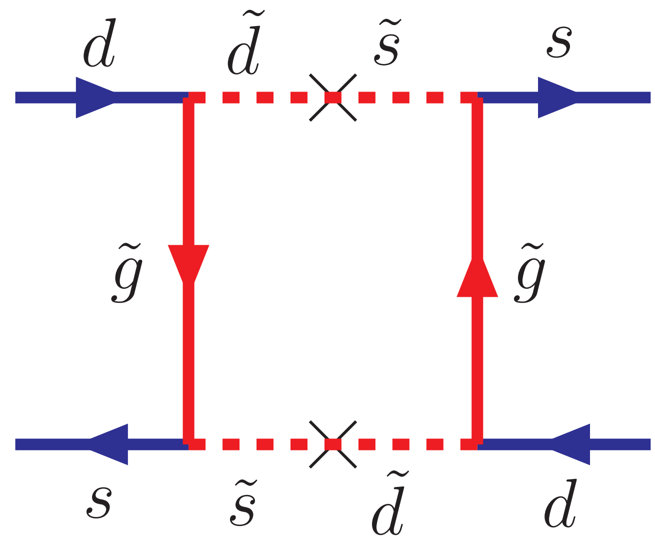
Assume $g \sim \sin\phi_{CP} \sim \mathcal{O}(1)$, this 1-loop EDM is way too large.

- Anyone who wants to build a realistic model beyond SM must find a way (natural or not) to suppress the EDM generated at 1-loop level.

Typical diagrams for FCNC and CP violation in MSSM

- The typical 1-loop diagrams lead to FCNC and EDM and K- mixing.





1-loop EDM in MSSM

To suppress the 1-loop EDM in MSSM, one needs either

- Small SUSY CP-phase $(\leq 10^{-2})$,
- OR Heavy SUSY scalars, $m_{sfermions} \sim 10$TeV for the first 2 generations
- OR EDM cancellations
- OR Flavor-off-diagonal CP violation

OR some kind of hybrid of above.

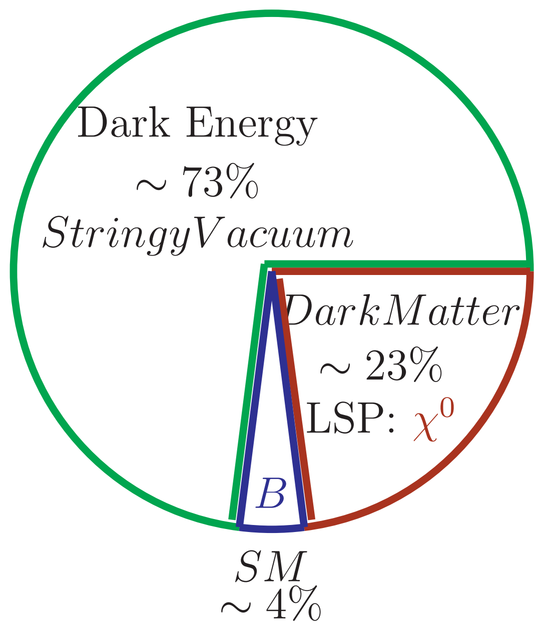
Dominant 2-loop EDMs have been studied by

D. Chang, W. Keung, A. Pilaftsis(1999), A. Pilaftsis(1999), D. Chang, W. Chang, W. Keung(2000), D. Chang, W. Chang, M. Frank, W. Keung(2000)...

SSUSY

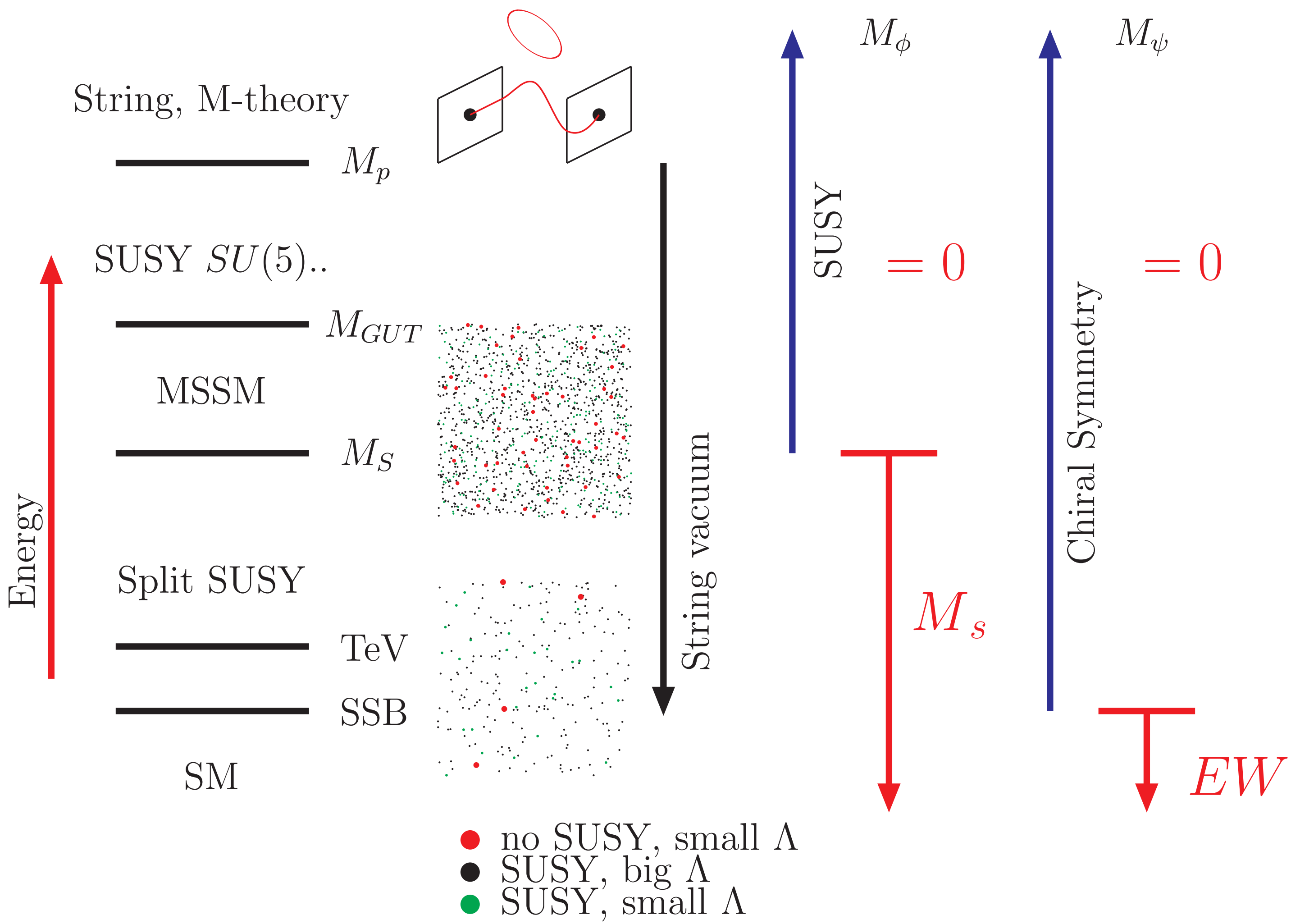
Arkani-Hamed, Dimopoulos 2004

- No principal for the small Cosmological constant, just fine tune to make it small. Same for the gauge hierarchy problem, the EW Higgs mass is no longer protected by SUSY but just fine tuning.
- All scalars, except the SM-like Higgs, are super heavy $\sim 10^{9-16}$ GeV.
- Gauginos and Higgsinos are $\sim 10^{2-3}$ GeV.



S = Split (or Schizophrenic)

How the SSUSY works?

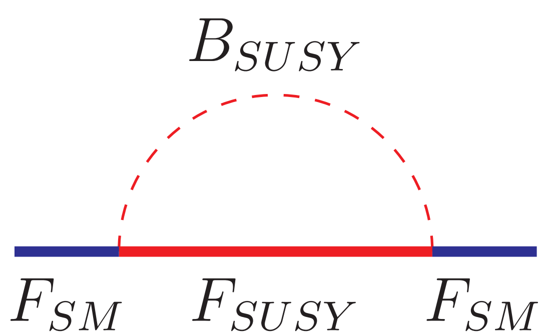


Characteristics of SSUSY

- All scalars, except the CP-even SM like Higgs, are super heavy $\sim 10^9$ GeV - M_{GUT}
- Gaugino and Higgsino masses are around the EW scale to TeV protected by R-symmetry and PQ symmetry.
- μ parameter is around the EW scale such that the lightest neutralino can annihilate effectively to give the dark matter density.
- Unification still works, mainly due to the gauginos contributions.

EDM starts at 2-loop Level

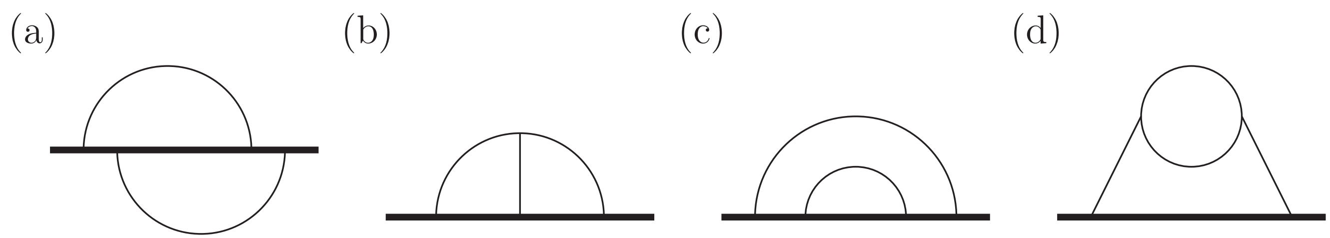
- There is no 1-loop EDM in SSUSY.
  How to see it?
- Rules:
  (1) Color: Red: Super particle, Blue: SM particles
  (2) R-parity requires the red line to form a close loop. (3) If a vertex contains a fermion, it must be two fermionic lines and one scalar line. This is due to the dim-4 $\bar{F}FB$ interaction.
- Here is the only 1-loop diagram we can draw:



Because all the superscalars are super heavy. The 1-loop EDMs are highly suppressed.

EDM starts at 2-loop Level

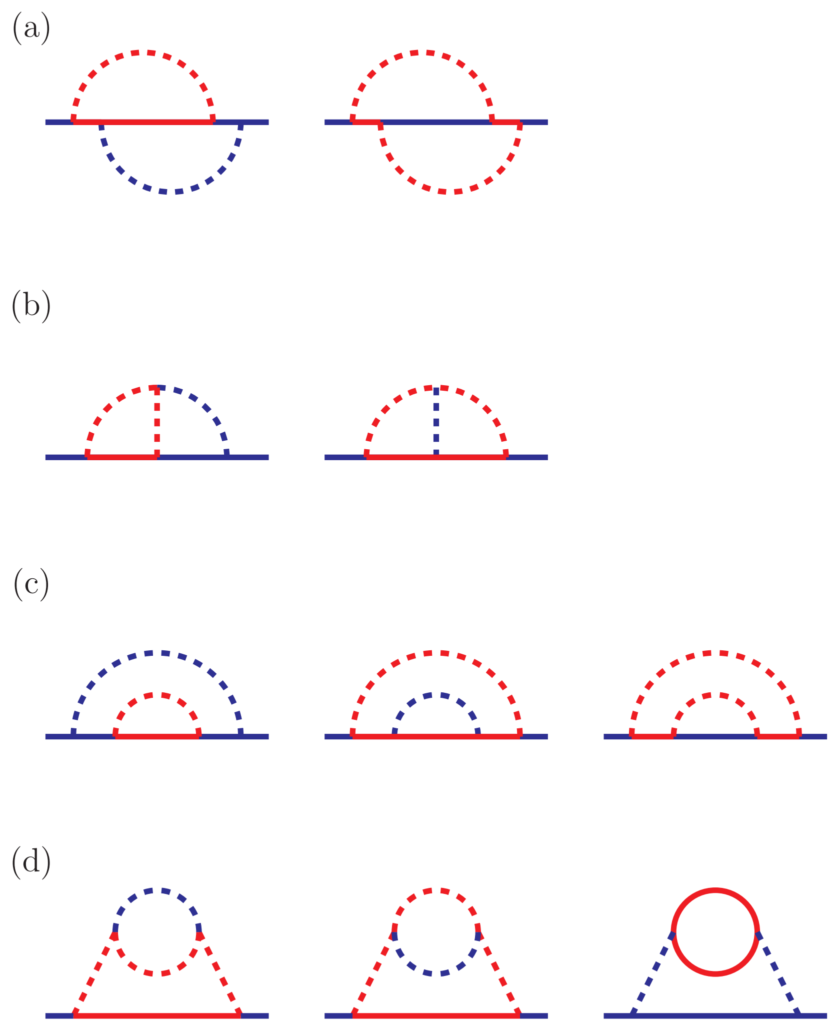
- As previously shown, 2-loop diagrams can be classified into four types of topologies:



- But how many?
- In SM, there are roughly ~ 2000 two-loop diagrams.
  Hopeless?
- In next page, I will prove to you that only the type-(d), Barr-Zee diagram, survives in SSUSY.
- And the number of diagrams turn out to be very small.

Digramatic proof of the Survival of BZ

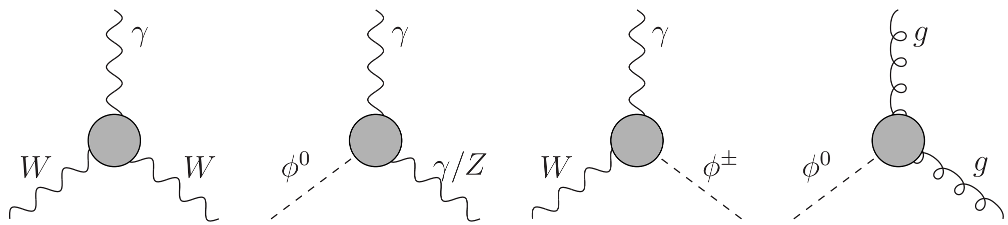
- Let's exhaust all possible ways of dressing color to all types of 2-loop diagrams:



Blue(Red) stands for SM (SUSY) particle. Solid (dash) line represents fermionic(bosonic) DOF.

Recipe for doing BZ

- If we were lucky, well, in most of the cases we are, the Barr-Zee type diagram is the most important EDM contribution.
- There are only few possible upper parts of Barr-Zee diagrams can generate sizable EDM. They are:



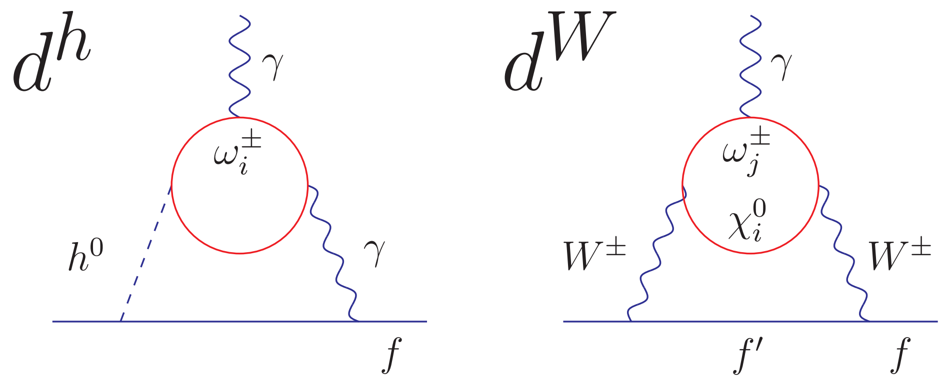
- You should first get the form factors for the upper loop. For example, for the $\gamma(k,\mu) \to \gamma(q,\nu)\phi(p)$ vertex, the most general gauge invariant form factor is:

$$\Gamma^{\mu\nu} = S[k^\nu q^\mu - k \cdot q g^{\mu\nu}] + P[i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta]$$

- Then it's easy, you just attach it to the electron or quark line to get the EDM. Be careful of the conjugated diagrams.
- I must warm you, the gauge independence is very important and usually the most tricky part. You should include every possible diagram to make them a gauge invariant set.

EDM in SSUSY

- Here are the two most important diagrams in SSUSY



- The EDM can be calculated to be:

$$\frac{d_f^{h^0}}{e} = \frac{Q_f \alpha^2 m_e}{4\sqrt{2}\pi^2 M_H^2 s_W^2} \sum_{i=1}^{2} Im O_i' \frac{m_{\omega_i}}{M_W} \mathcal{F}\left(\frac{m_{\omega_i}^2}{M_H^2}\right) \propto Im(\mu M_2)$$

$$\frac{d_f^W}{e} = \pm \frac{\alpha^2 m_f}{8\pi^2 s_W^4 M_W^2} \sum_{i=1}^{4} \sum_{j=1}^{2} \frac{m_{\chi_i} m_{\omega_j}}{M_W^2} Im(O_{ij}^L O_{ij}^{R*}) \mathcal{G}\left(r_i^0, r_j^\pm, r_{f'}\right)$$

The plus(minus) sign in front the RHS corresponds to the fermion f with weak isospin $+(-)1/2$ and Q_f is the charge of fermion f.

Some Details for the experts

- The relevant Lagrangian:

$$\mathcal{L} \supset +\frac{g}{\sqrt{2}}\overline{\omega_j^+}\gamma^\mu[O_{ij}^L P_L + O_{ij}^R P_R]\chi_i^0 W_\mu^+ - \frac{g}{\sqrt{2}}O_i'\overline{\omega_{iR}^-}\omega_{iL}^- h^0 + h.c.$$

- The coupling are

$$\begin{aligned} O_{ij}^R &= \sqrt{2}N_{2i}^* C_{1j}^L + N_{3i}^* C_{2j}^L \, , O_{ij}^L = \sqrt{2}N_{2i}C_{1j}^R - N_{4i}C_{2j}^R \\ O_i' &= (C_{1i}^R)^* C_{2i}^L \cos\beta + (C_{2i}^R)^* C_{1i}^L \sin\beta \end{aligned}$$

- The unitary matrices $C^{L,R}$ and N are defined to diagonalize the chargino and neutralino mass matrices with $C^{R\dagger}\mathcal{M}_C C^L = diag\{m_{\omega_1}, m_{\omega_2}\}$ and $N^T\mathcal{M}_N N = diag\{m_{\chi_1}, m_{\chi_2}, m_{\chi_3}, m_{\chi_4}\}$.

- The chargino mass matrix is

$$\mathcal{M}_C = \begin{pmatrix} M_2 e^{i\phi_2} & \sqrt{2} M_W c_\beta \\ \sqrt{2} M_W s_\beta & \mu e^{i\phi_\mu} \end{pmatrix}$$

- and the neutralino mass matrix is:

$$\mathcal{M}_N = \begin{pmatrix} M_1 e^{i\phi_1} & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 e^{i\phi_2} & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu e^{i\phi_\mu} \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu e^{i\phi_\mu} & 0 \end{pmatrix}$$

- The diagonized masses are positive and real. We use the convention that $m_{\omega_1} < m_{\omega_2}$ and $m_{\chi_1} < m_{\chi_2} < m_{\chi_3} < m_{\chi_4}$. Notation $s_W(s_\beta)$ stands for $\sin\theta_W(\sin\beta)$ and $\tan\beta = v_u/v_d$. The matrices $C^{L,R}$ are not uniquely defined. However the resulting EDM is basis independent.

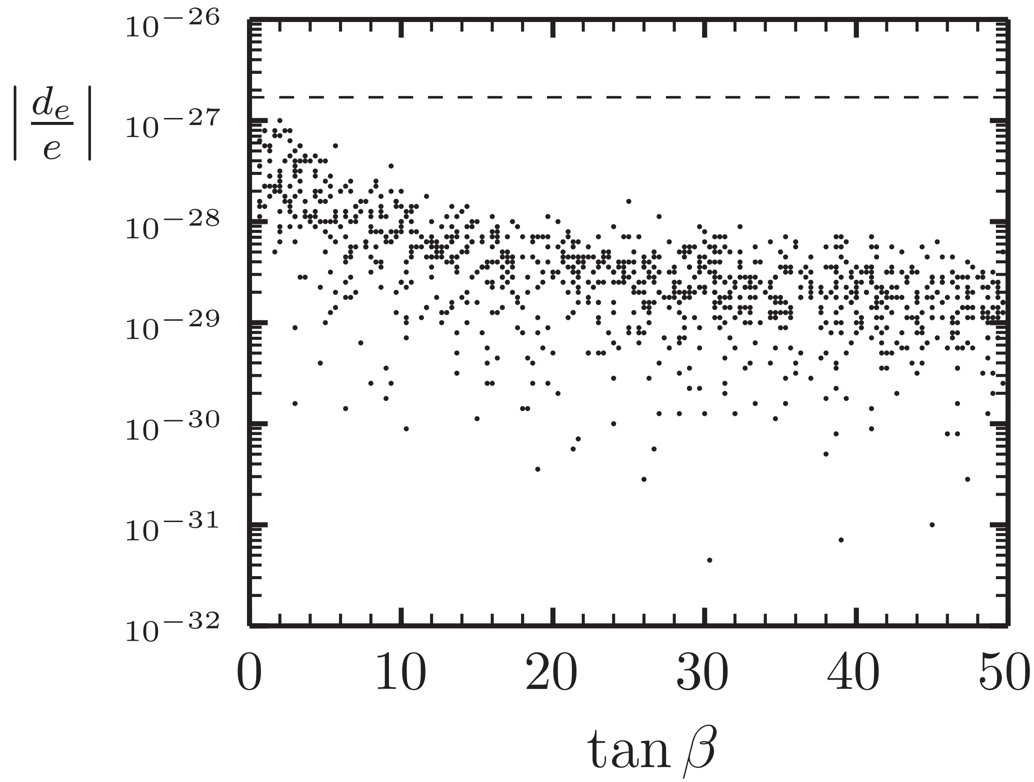
- There is no analytic solution for the 4×4 neutralino mass matrix. We evaluate it by numerical.
- we randomly scan the following parameter space,

$$200\ GeV < M_1, M_2, \mu < 1.0 TeV,$$

and let all three phases vary within $[0, 2\pi]$. (Note only 2 physical one)
Range for SM Higgs mass:

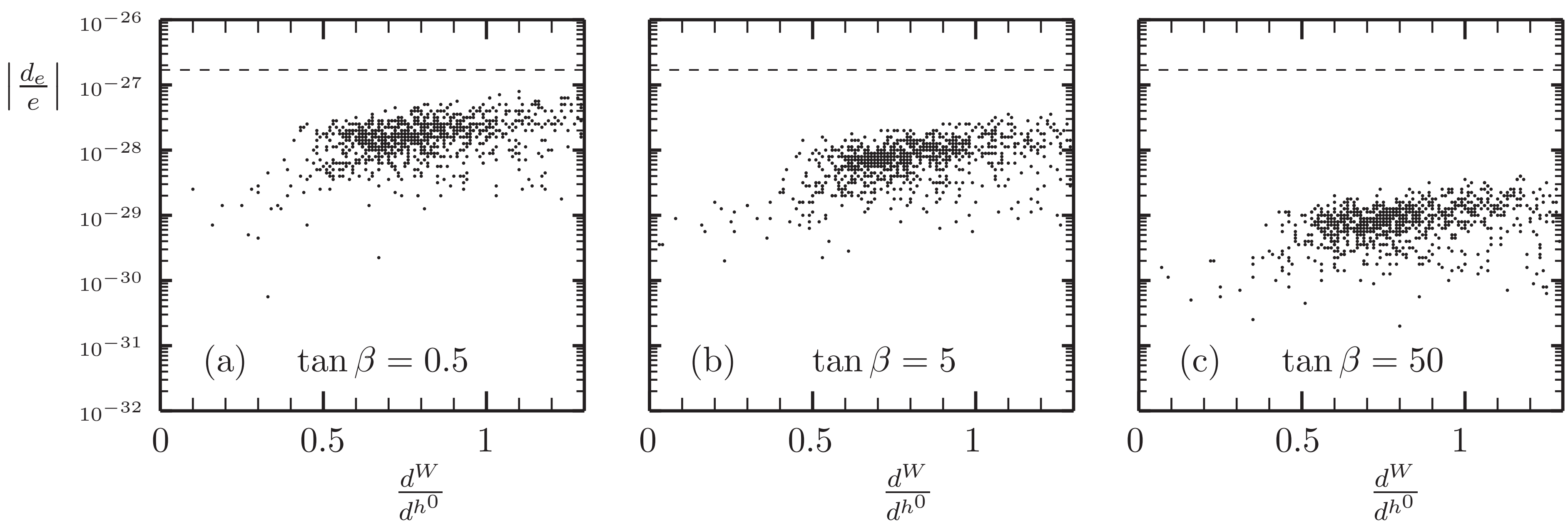
$$120\ GeV < M_H < 170\ GeV$$

Here is the electron EDM versus $\tan\beta$

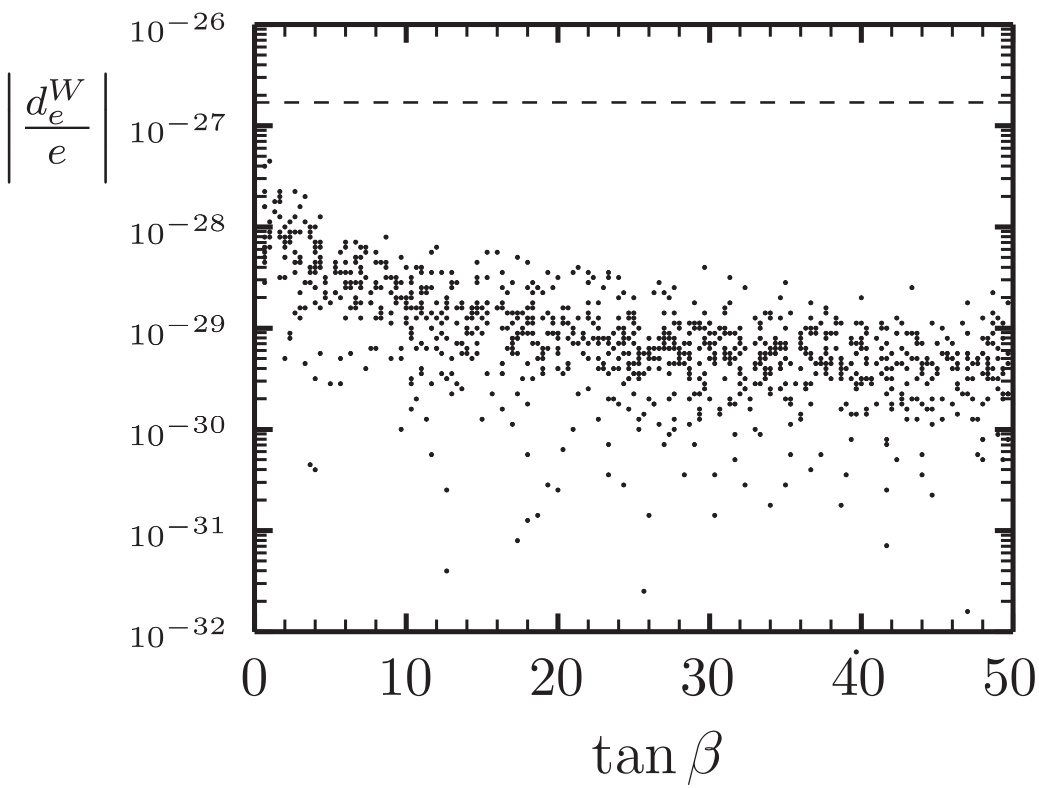


Based on the parameter scan, it seems very promising in the observation of the electron EDM by experiments with the sensitivity of 10^{-29} e-cm

- The above range of the Higgs mass was suggested by G. F. Giudice (2004), A. Arvanitaki(2004), S. P. Martin(2005). However, some variants allow the light Higgs to be as heavy as 400 GeV. (R. Mahbubani(2004), M. Binger(2004)
- As the lightest neutral Higgs becomes heavier, the d^W contribution becomes more and more important to the EDM of the charged SM fermion.
- Here is the same plots with 400 GeV$< M_H < 600$ GeV



- From the previous plot, we see that the d^{h^0} and d^W become roughly compatible when $M_H \sim 600$ GeV, and d^W becomes the dominate contribution when $M_H \geq 600$ GeV.
- One can also imagine a SUSY scenario, if phenomenologically plausible, in which the lightest neutral Higgs is super heavy or even without SM Higgs at all.
- In that extreme case, the d^W is the sole contribution to the EDM of SM fermions. And we show the d^W alone for those models.



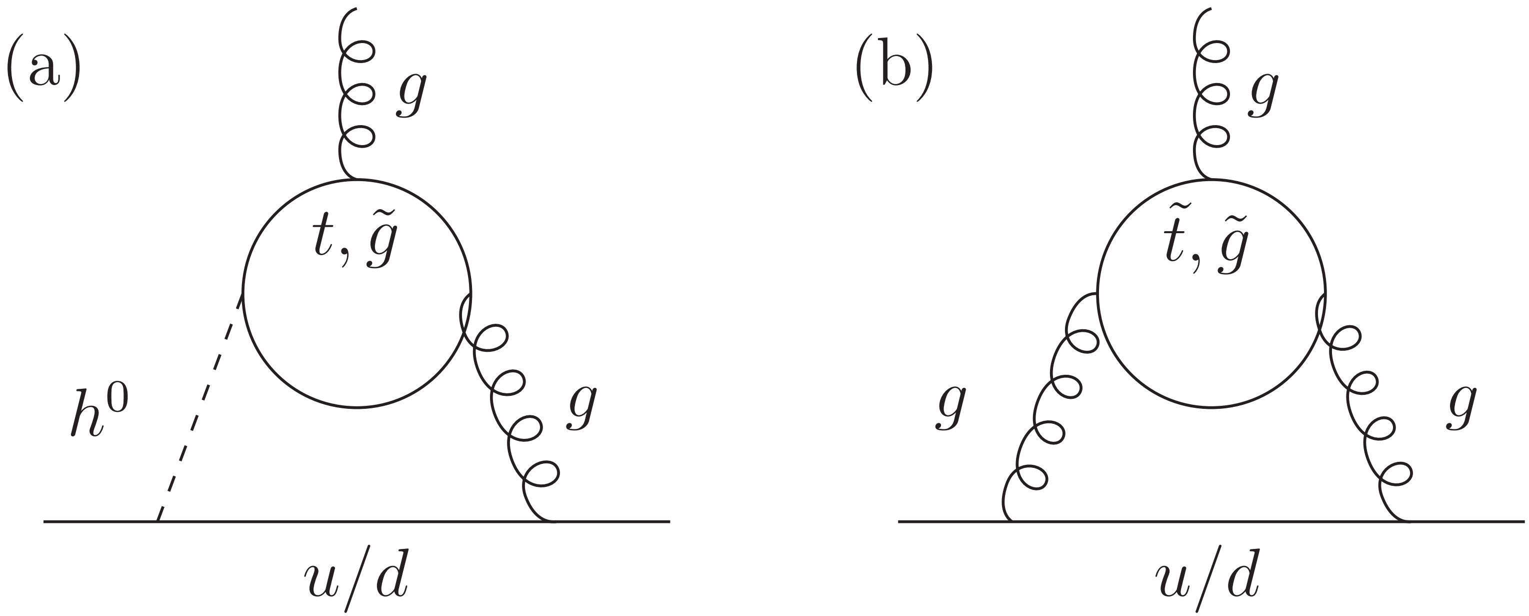
- Compared to the EDM with light Higgs mass within $120 - 170$ GeV, the EDM without SM Higgs is roughly half order smaller.
- However, it is still likely to see something in the 10^{-29}e-cm experiments.
- In SSUSY models, the charged lepton EDMs follow the simple mass scaling law

$$d_e : d_\mu = m_e : m_\mu$$

  which is quite different from some models, for example, L-R models, R-parity violating, low scale see-saw.
- Models can be distinguished by comparing d_e and d_μ.
  However, SSUSY predicts the d_μ to be roughly $10^{-24.5} - 10^{-27}$e cm, which is six to seven orders of magnitude lower than the current limit and it will be a great challenge for the newly proposed d_μ measurement.

Neutron EDM

- In MSSM, usually the chromo dipole moment and the 3-gluon operators are the dominant contribution to the neutron EDM.



- However, in the SSUSY models, the CP phases associated with gluinos can always be shuffled off upon the squarks mass matrix by phase redefining of the gluino field. The chromo dipole moment therefore vanishes because all the squarks are decoupled from the low energy physics and d^{h^0} and d^W become the leading contribution to the neutron EDM
- As an order of magnitude estimation, the quark model prediction

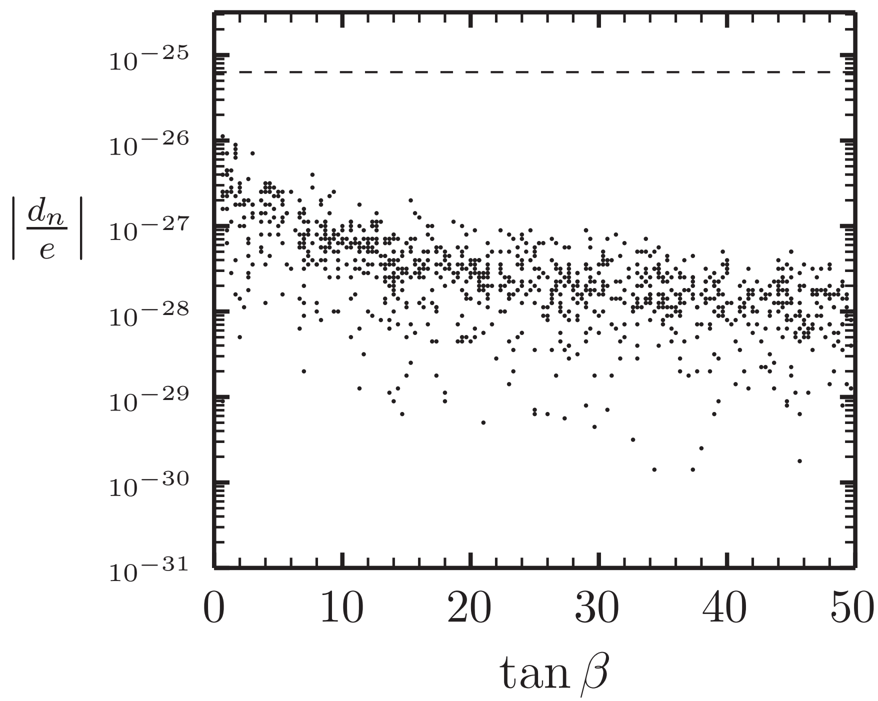
$$d_n = \frac{4d_u - d_d}{3}$$

can be used to give a rough estimation of the neutron EDM.

- By the scaling law and replacing the fermion charge accordingly, we can express the neutron EDM as

$$d_n^{h^0} = -\left(\frac{8m_u + m_d}{9m_e}\right) d_e^{h^0}, \, d_n^W = -\left(\frac{4m_u + m_d}{3m_e}\right) d_e^W$$

- The estimation of the resulting neutron EDM is displayed below



where the current quark masses, $m_u = 3$ MeV and $m_d = 6$ MeV, have been used.

- Our estimation of neutron EDM is conservative which could gain a few orders of magnitude enhancement due to the hardronic physics.

Recap for this part

- A high scale SUSY, dubbed SSUSY, was argued to be possible and can do away many phenomenological problems.
- Although all the scalar particles are super heavy, EDM still arises from 2-loop Barr-Zee type diagrams (with or without R-parity).
- For $130GeV < M_H < 170GeV$, $d_e^W \sim +0.4 d_e^h$ and $d_n^W \sim +0.7 d_n^h$.
  d^W is Higgs mass independent, it becomes crucial when SM Higgs is heavy or completely decoupled.
- Numerical survey indicates we shall see something with the electron EDM experiments with 10^{-29} e cm accuracy.
- In SSUSY, SM fermion EDMs follow mass scaling law: $d_\mu/d_e = m_\mu/m_e \sim 200$.
  If we have a muon EDM experiment with 10^{-27} e cm accuracy, we can distinguish SSUSY from other models.


SM Fermions Gauge
SUSY Fermions
SM
STRING

Message to take away

- On one hand, we need EXTRA CP violating source(s) beyond SM CKM phase to generate the matter anti-matter asymmetry in the universe. On the other hand, we have to mind the potentially too large EDM.
- For theorists, better arrange the 1-loop EDM to vanish in your model.
- For experimentalists, the 2-loop EDM from new physics with scale around electroweak - TeV is around the corner.
  Good luck and happy hunting.