Pairing Gaps and Polarization in Cold Fermions

Upper and Lower "Bounds" for Pairing Gap at Unitarity

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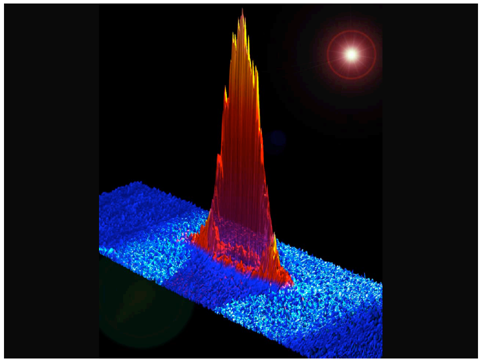
Lattice Methods: K. Schmidt (ASU) & Shiwei Zhang (W&M)
Original work w/ K. Schmidt (ASU), V. Pandharipande, S.Y. Chang (Ill)

Image from Randy Hulet



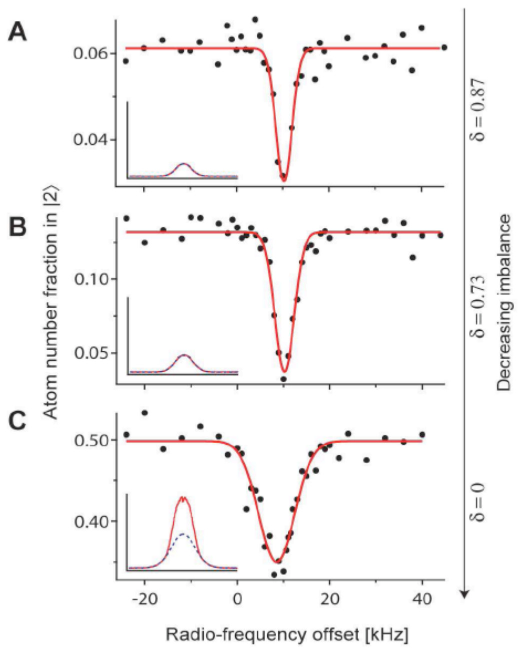
Simple (Universal) Interaction
Highly Tunable
Fundamental Studies of
strongly-paired systems
(nuclei and QCD)

`Benchmark' for Strongly-Coupled Fermions

$$\mathcal{H} = \sum_{k=1}^{A}(-\frac{\hbar^2}{2m_k}\nabla_k^2) + \sum_{i<j} v(r_{ij})$$

	Cold Fermi Atoms	Neutrons
scattering length	tunable	-18.5 fm
effective range	0	2.7 fm

Rich Set of Experimental Results

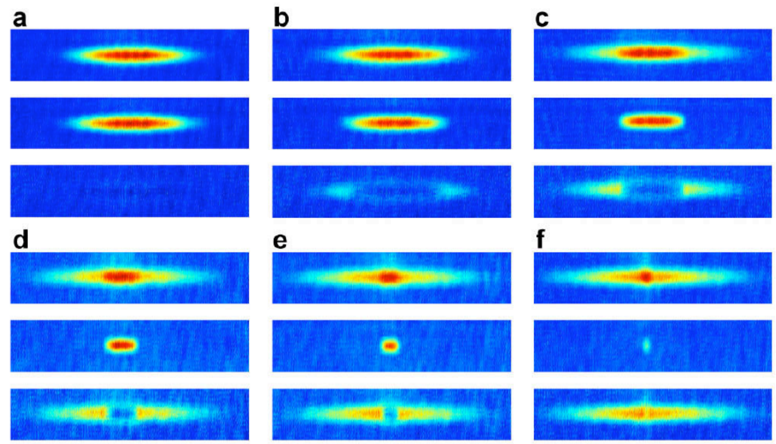


Radial Density

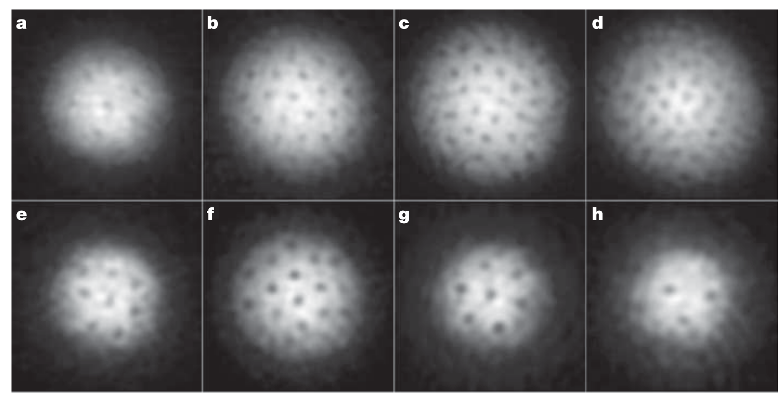
Polarization

Vortices

RF response



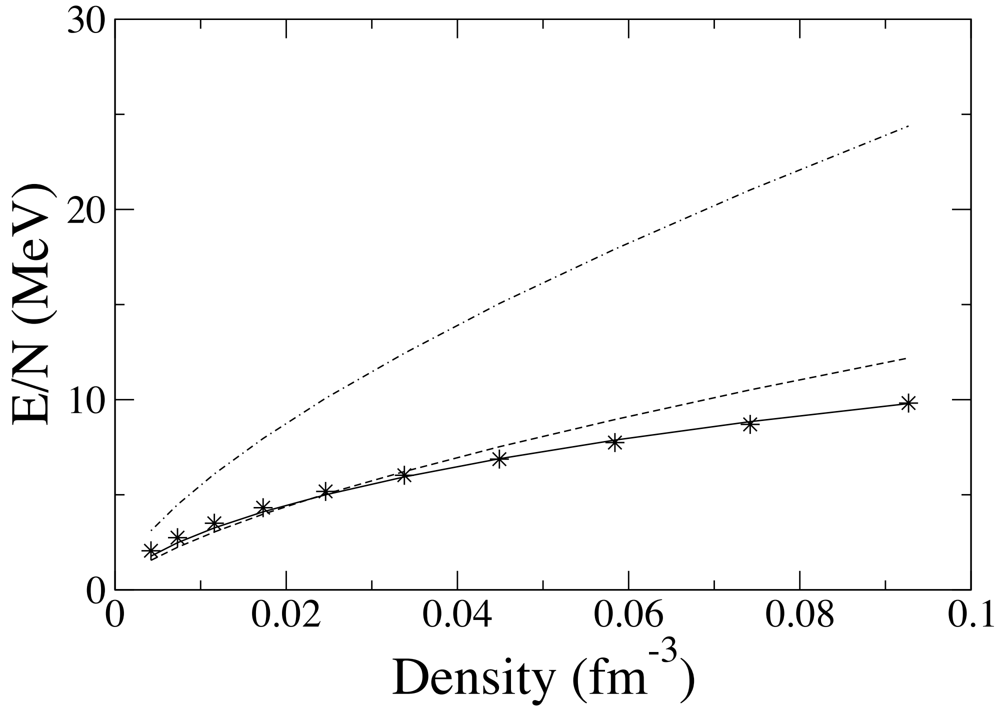
Rice



MIT

Neutron Matter

Neutron-Neutron interaction - dominantly s-wave (spin 0) at low energy
Large scattering length ~ -18 fm
Modest effective range ~ 2.7 fm



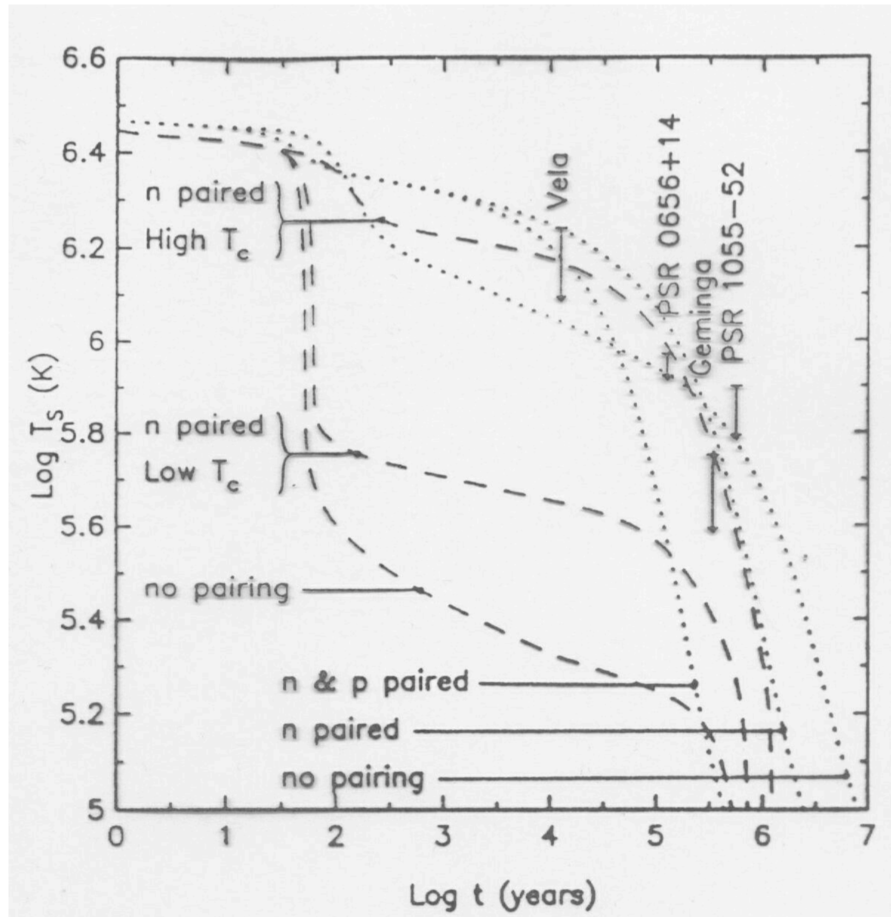
Zero Temperature Equation of State Difficult to get wrong --- at low density

Even if no new phases, parameters including Superfluid gap Δ are important

Superfluid gap for low-density neutron matter affects cooling

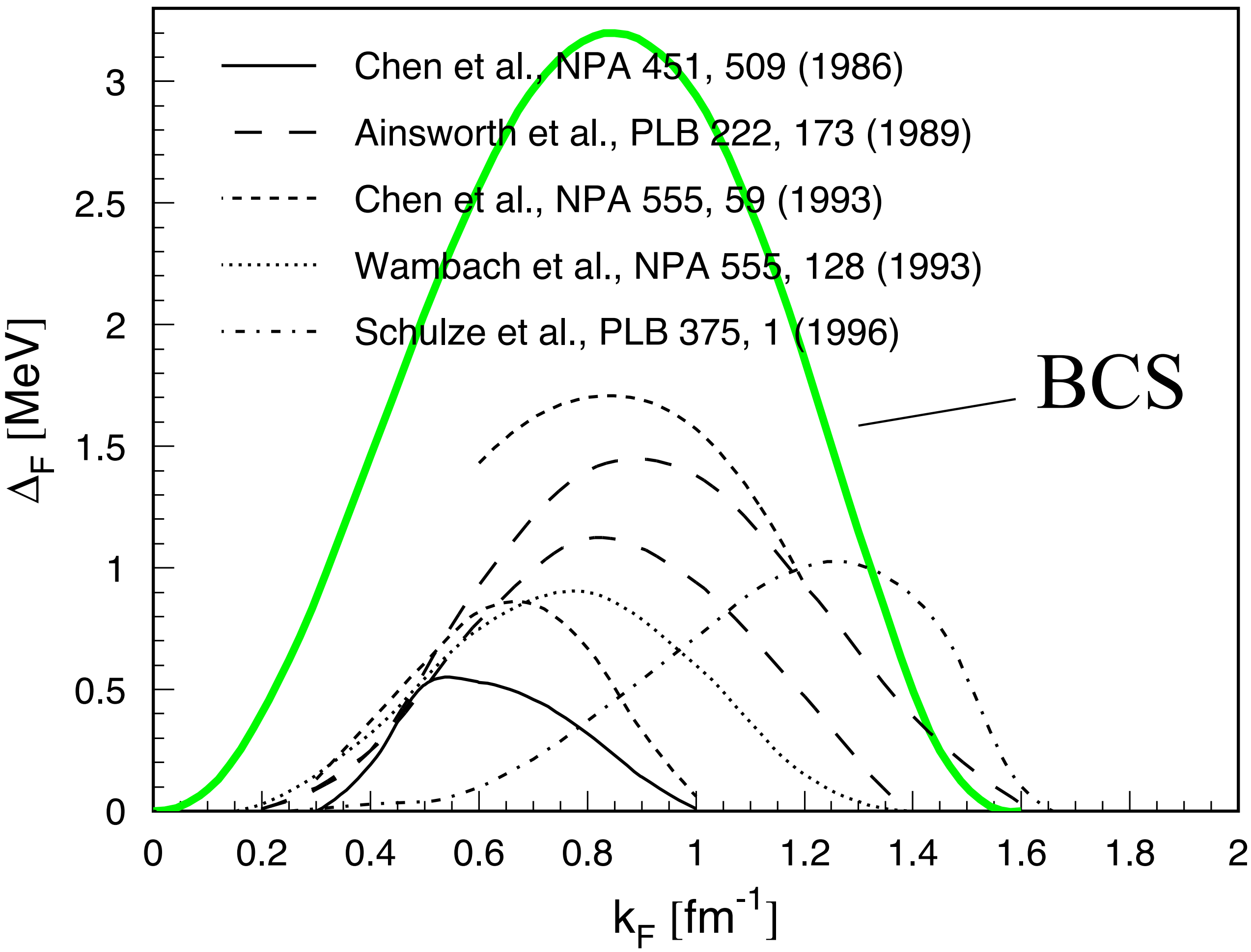
Benchmark for pairing in the strong-coupling QCD

QCD at high densities



Neutron star cooling curves

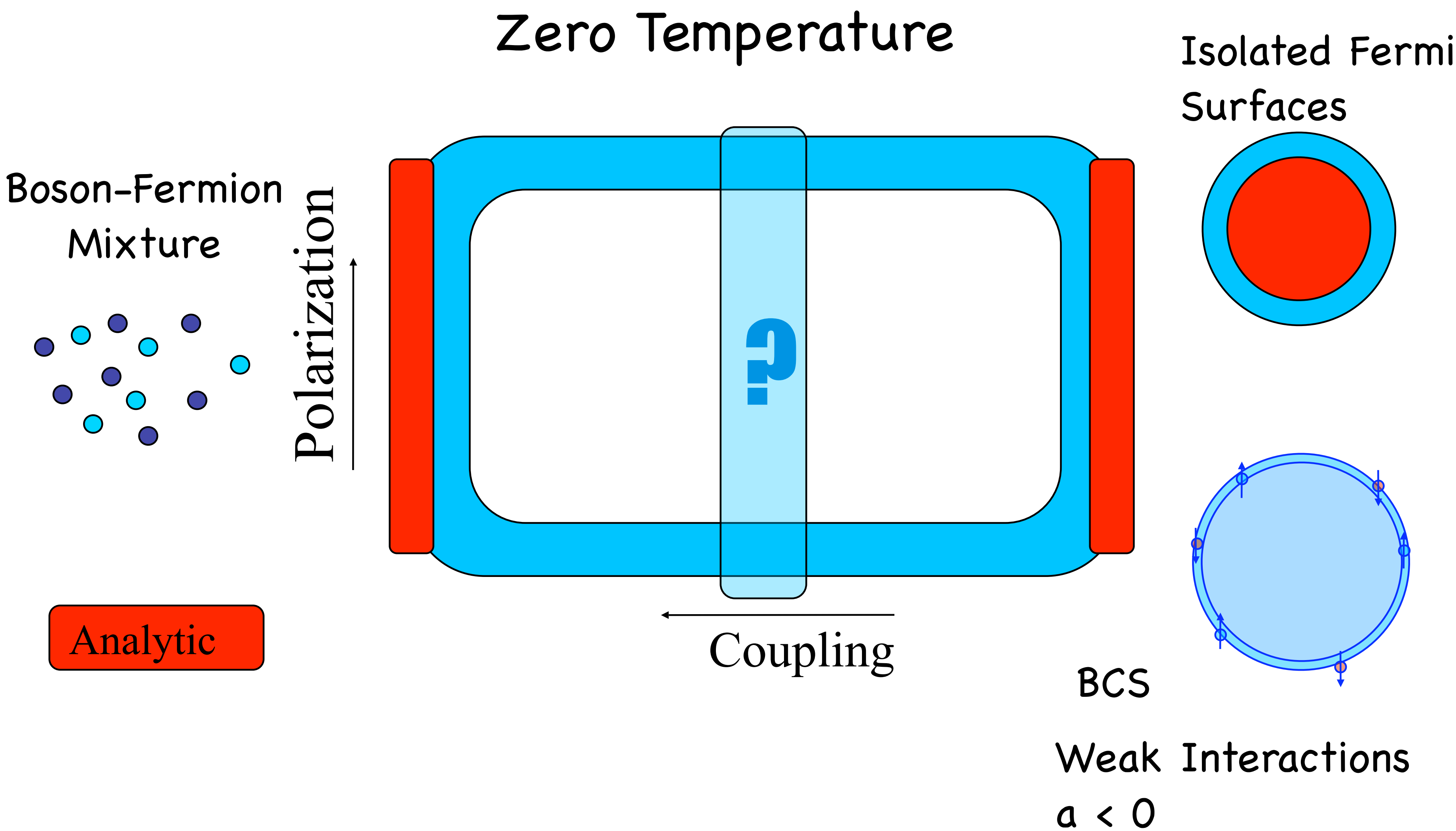
Superfluid (Pairing) Gap



Dean and Hjorth-Jenson RMP (2003)

Pairing Gap (apparently) difficult to get right !
Situation now worse than shown

Cold (T=0) Fermions vs. Polarization



Method I: Diffusion (Green's function) Monte Carlo

Fixed Node - Variational Upper Bound

Vary parameters in nodal surfaces ~ different 'phases' (superfluid or normal)

Transient Estimation

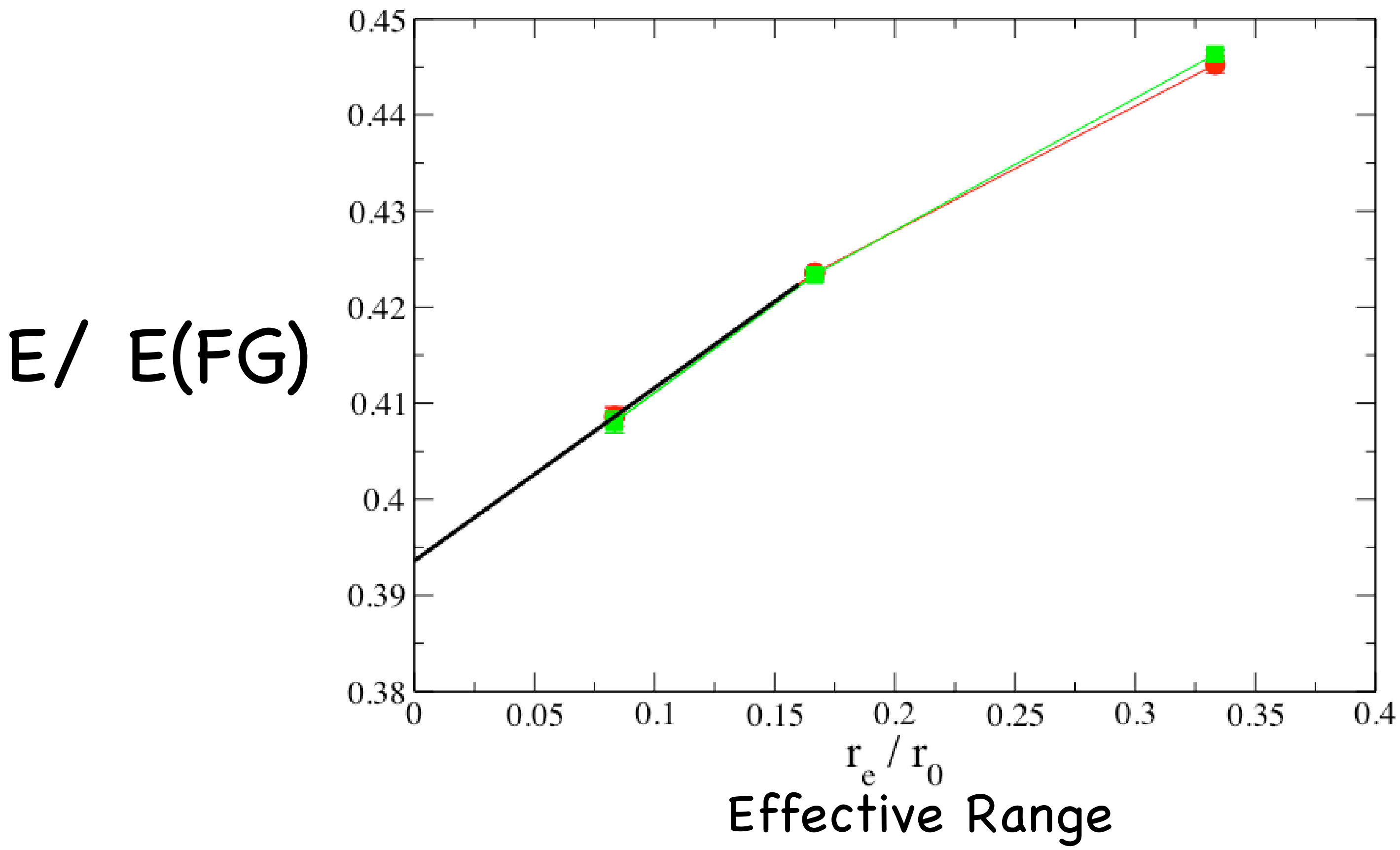
Comparisons to Lattice Methods at Equal Populations

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H}-E_T)\tau} \Psi_V$$

Variational wavefunction

$$\Psi_V(\mathbf{R}) = \prod_{i,j'} f(r_{ij'}) \Phi_{BCS}(\mathbf{R})$$

Energy vs. Potential Range

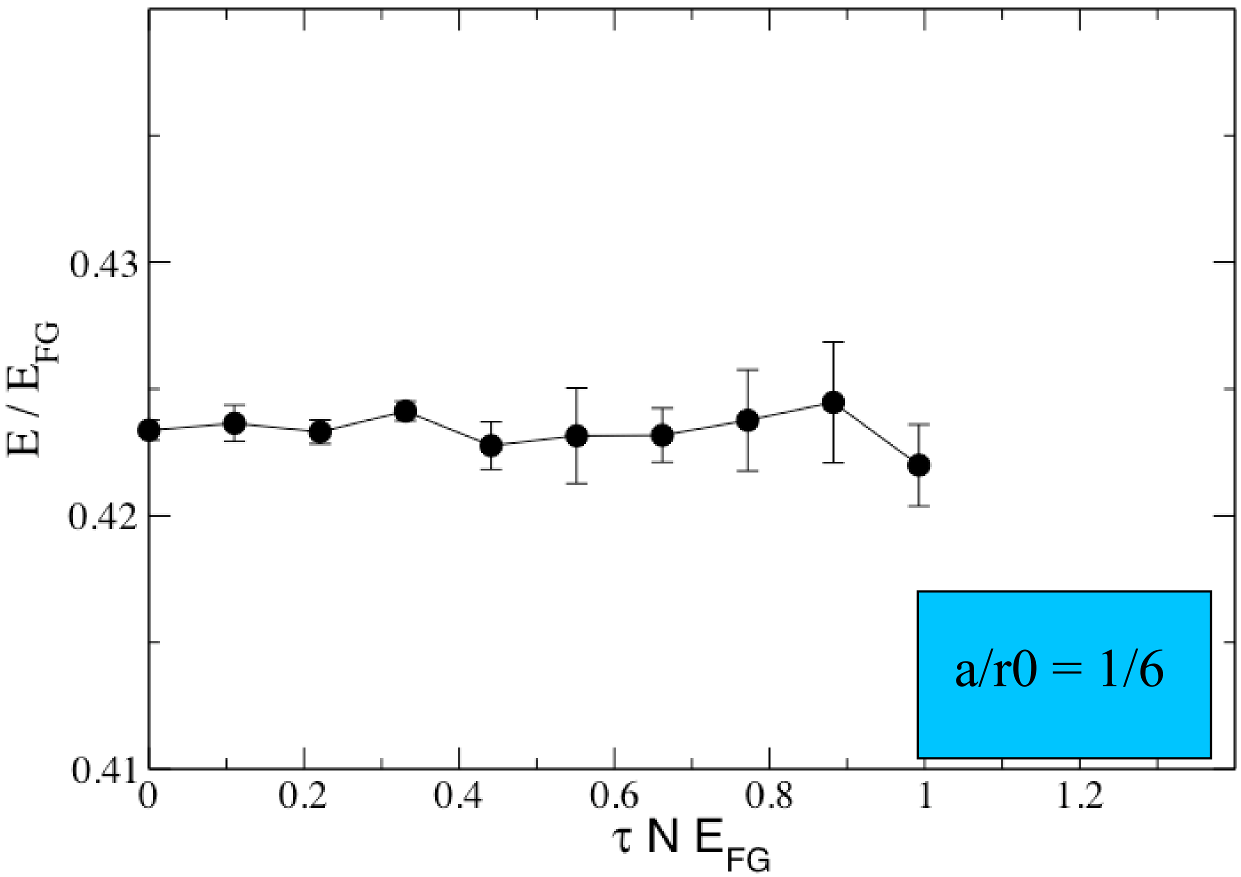


66 particles

Caution: states are metastable for potential range > 0

Transient Estimation

Releasing fixed-node constraint



Lattice Methods

Auxiliary Field QMC - evolve single particle orbitals
(Hirsch, Scalapino, Koonin, ...)

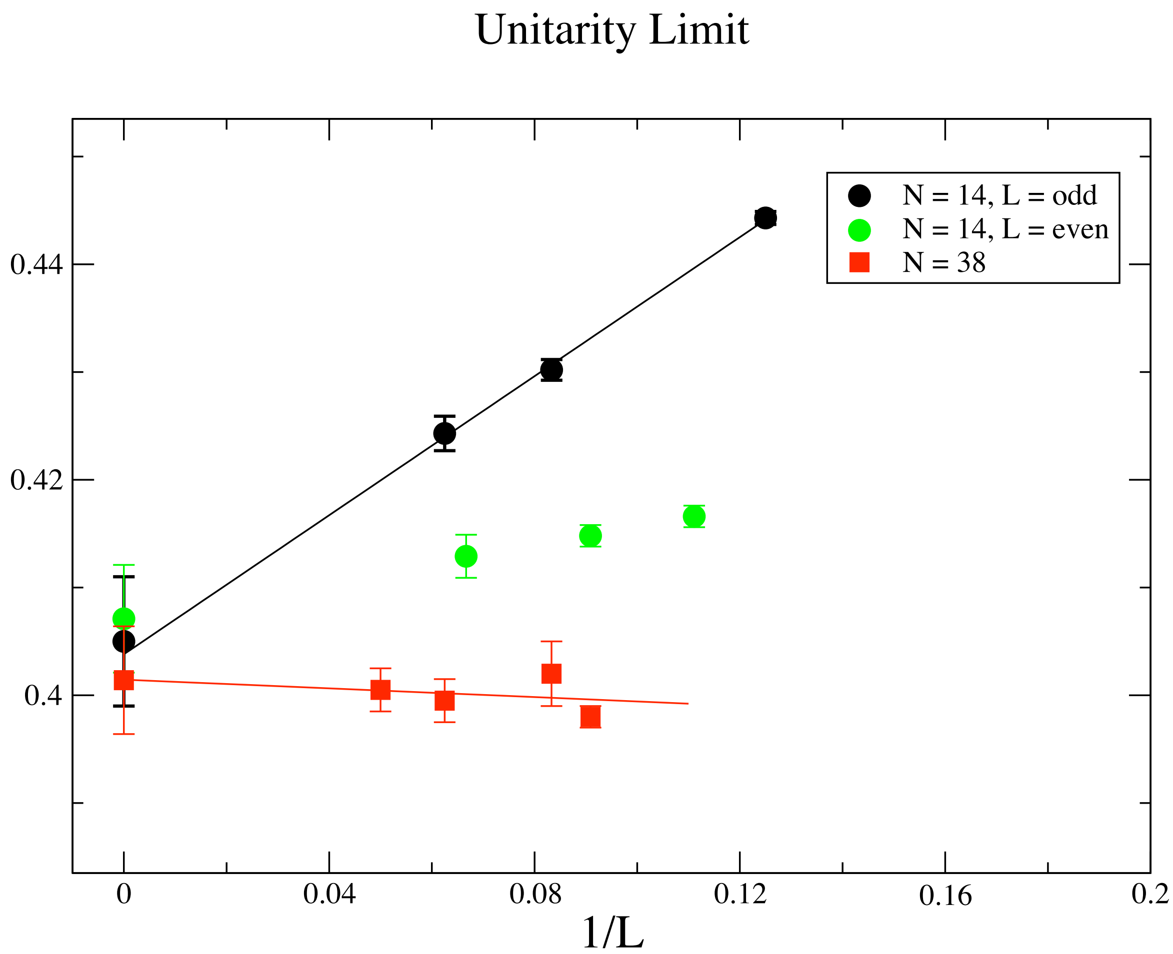
Continuum Limit = Limit of large # particles and dilute system

Fixed Particle number
BCS-like trial state used for importance sampling

Largest system: 38 particles on a 20x20x20 lattice : 0.25% filling

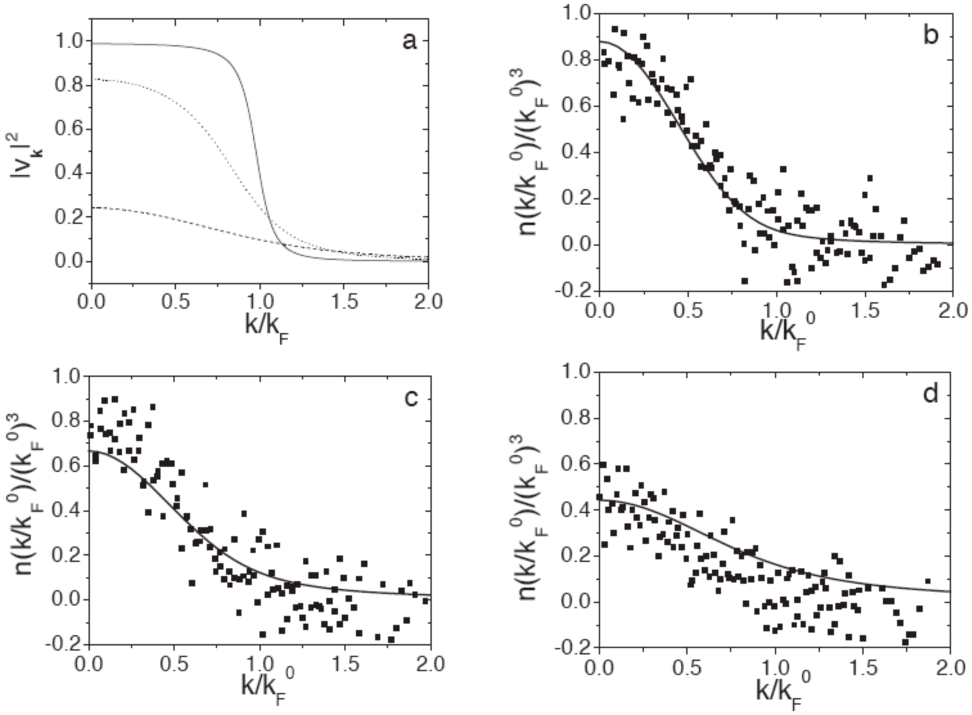
Exact (no sign problem) for zero polarization

Lattice Results at Unitarity

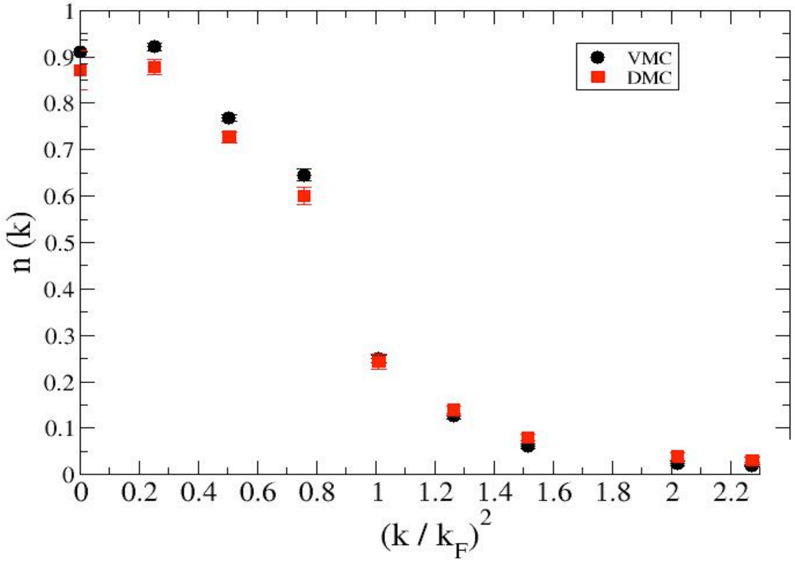


Measurements and EOS at a = infinity

0.51 (4)	Kinast, et al., Science (2005)
0.32 (+.13,-.1)	Bartenstein, et al., PRL (2004)
0.36(15)	Bourdel, et al., PRL (2004)
0.46(5)	Partridge, et al., PRL (2004)
0.45(5)	Stewart, et al., PRL (2006)
0.41(15)	Tarruell, et al., cond-mat/0701181

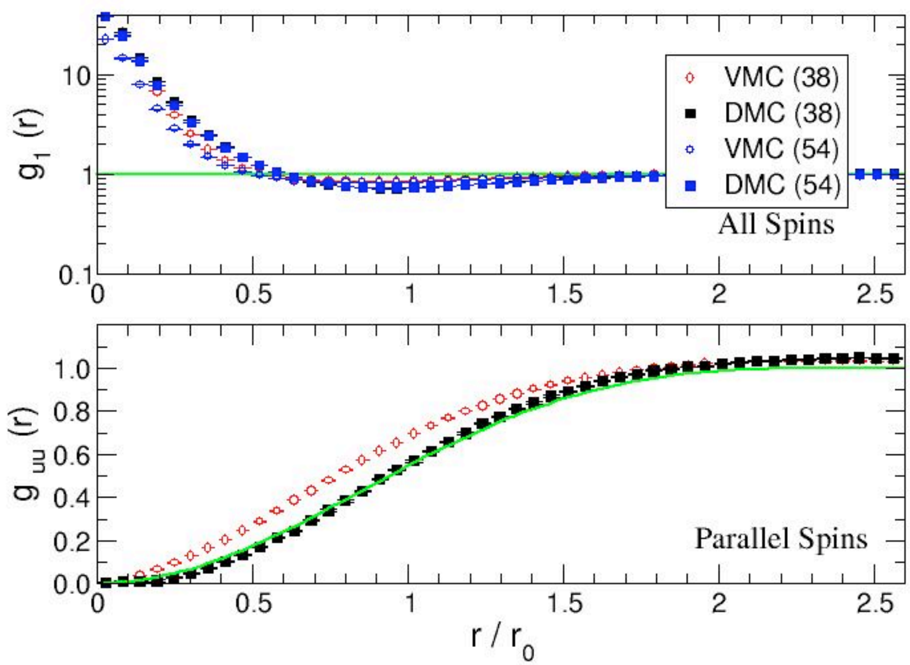


Momentum Distribution



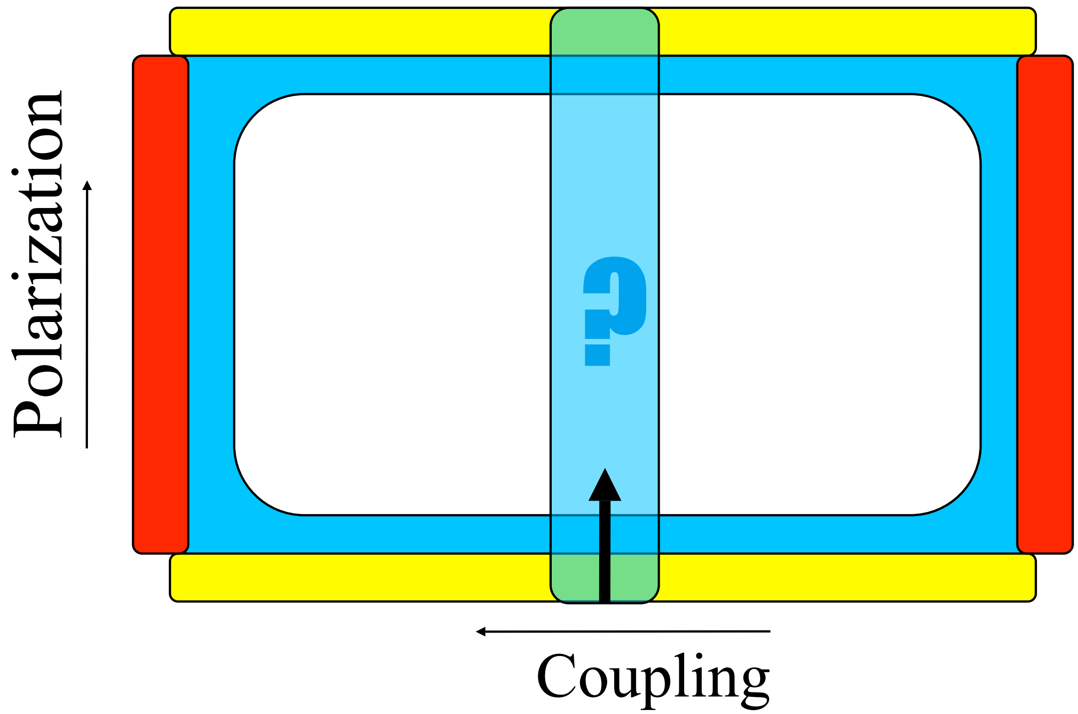
At unitarity
Very different from
Fermi Liquid

Pair Distributions



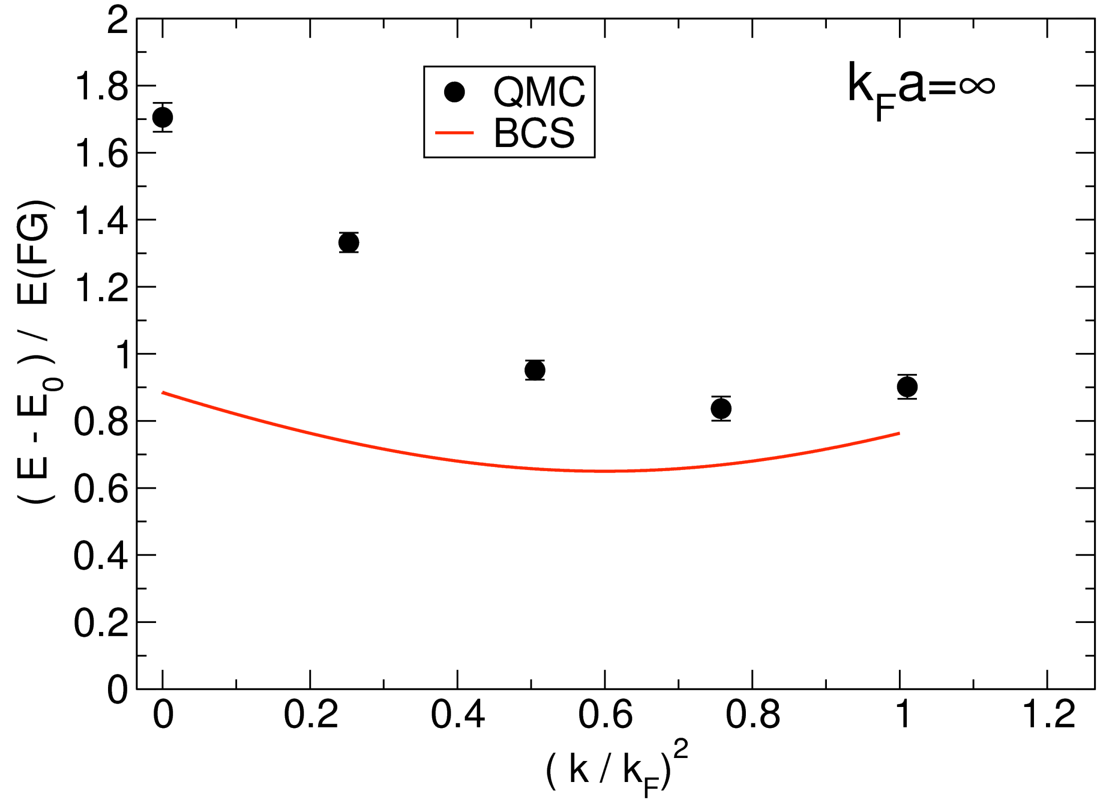
Strongly
Peaked
Pair distribution

Small Polarization



At Unitarity, expect large gap for unpaired particles

Excitation Energy/ E(FG) vs. k



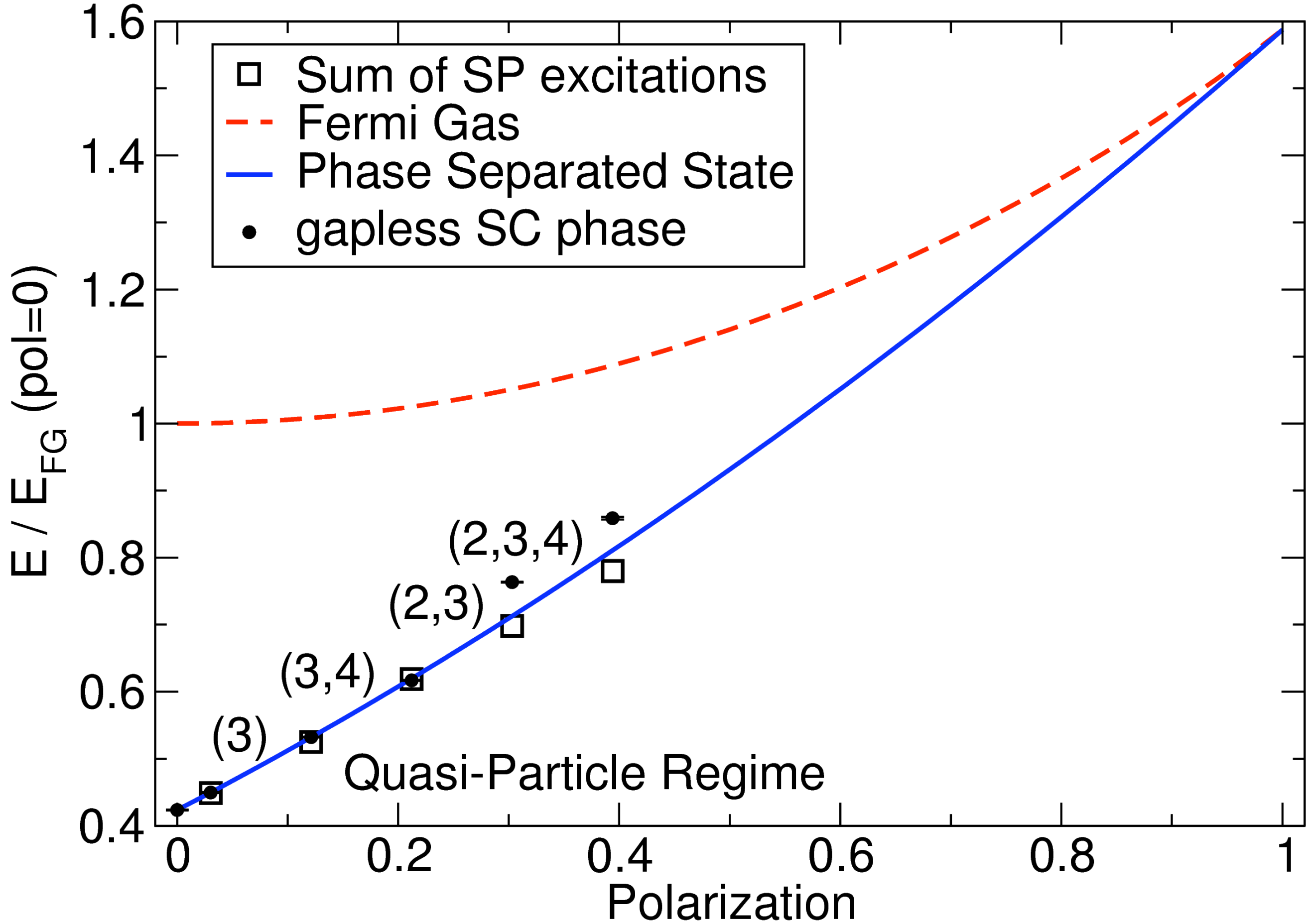
JC & Sanjay Reddy
PRL **95**, 060401 (2005)

In weak coupling (BCS): minimum near kf

In strong coupling (BEC), minimum at k=0

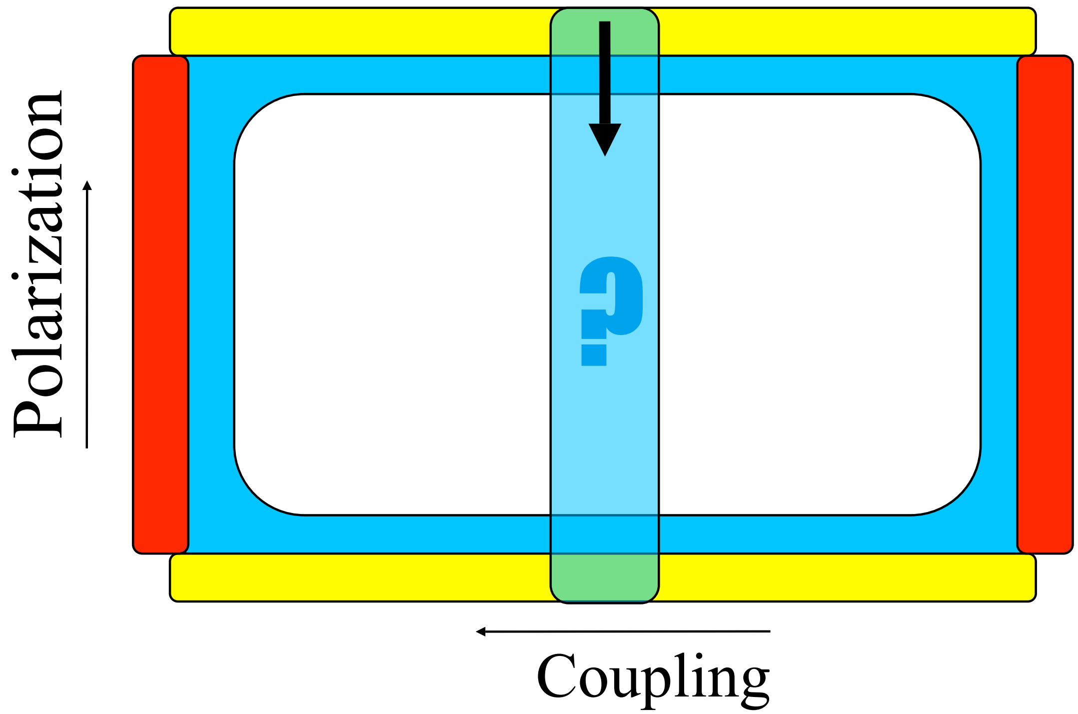
If background (unpolarized) superfluid is correct, get upper 'bound' for gap

Polarized Systems



Up to ~20% polarization, quasi-particles are nearly non-interacting

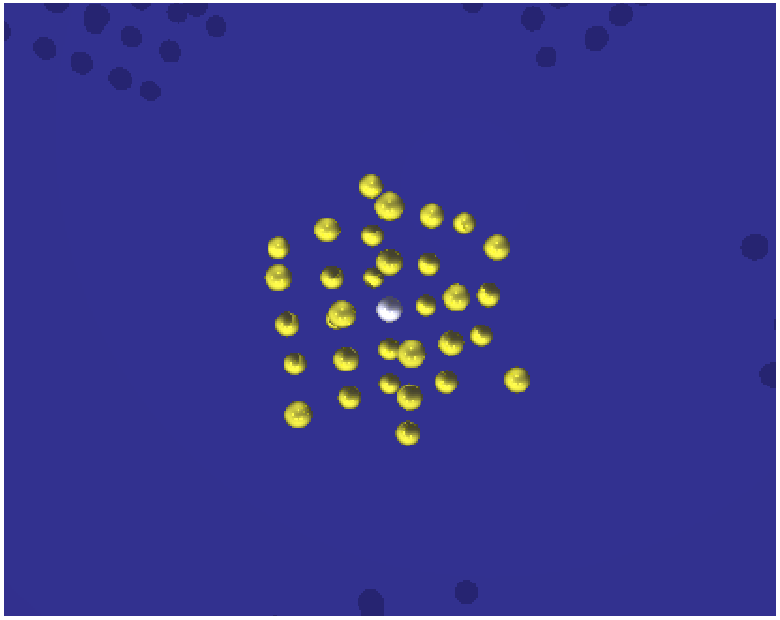
EOS vs. Polarization at Unitarity



At Unitarity, expect large gap for unpaired particles

Large Polarization

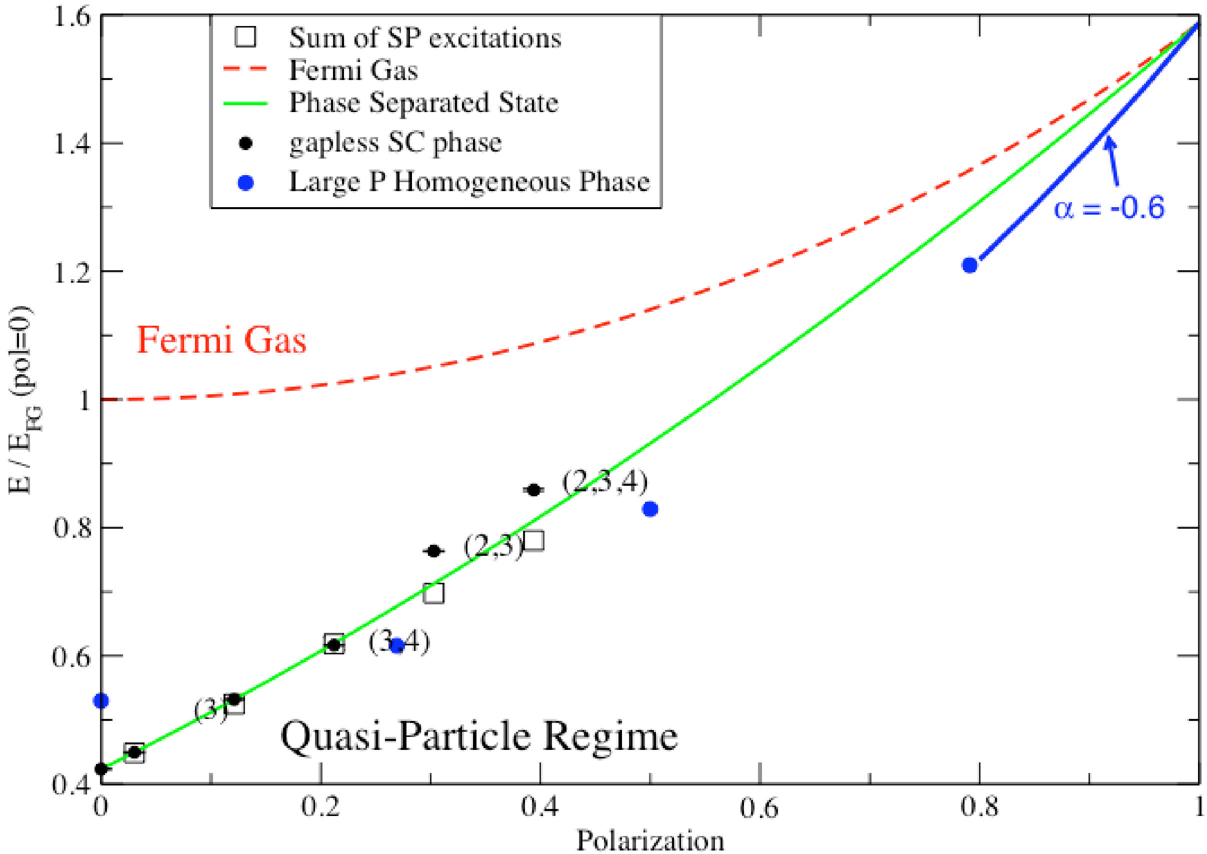
$$E_{N+1}(k) - (3/5)\ k_f^2/(2m)\ =\ \eta(k/k_f)\ k_f^2/(2m)$$



1 down spin in a sea
of up spins

QMC calculation
η(0) = -0.60(03)

Normal Phase at high Polarization



Checked for p-wave superfluidity,
Superfluid generalized from Fermi Gas,
Neither clearly preferred

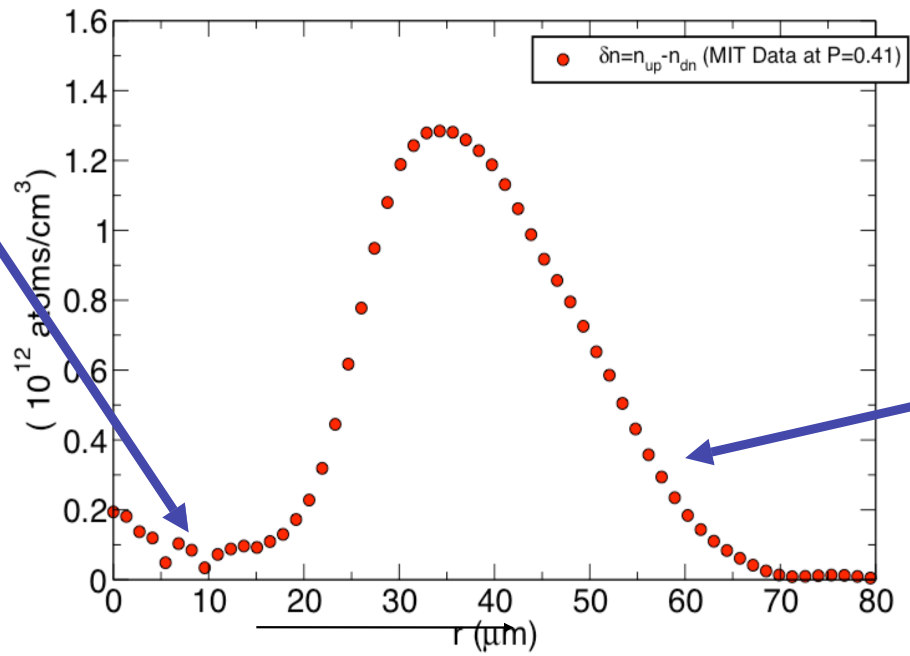
See results by
Lobo, Recati, Giorgini,Stringari
PRL 2006

Polarization vs. Radius : MIT data

MIT data P=0.41

unpolarized superfluid

$N\uparrow - N\downarrow$



Fully polarized Normal state

radius

At T = 0, assume 1st order phase transition at a local polarization of ~45%

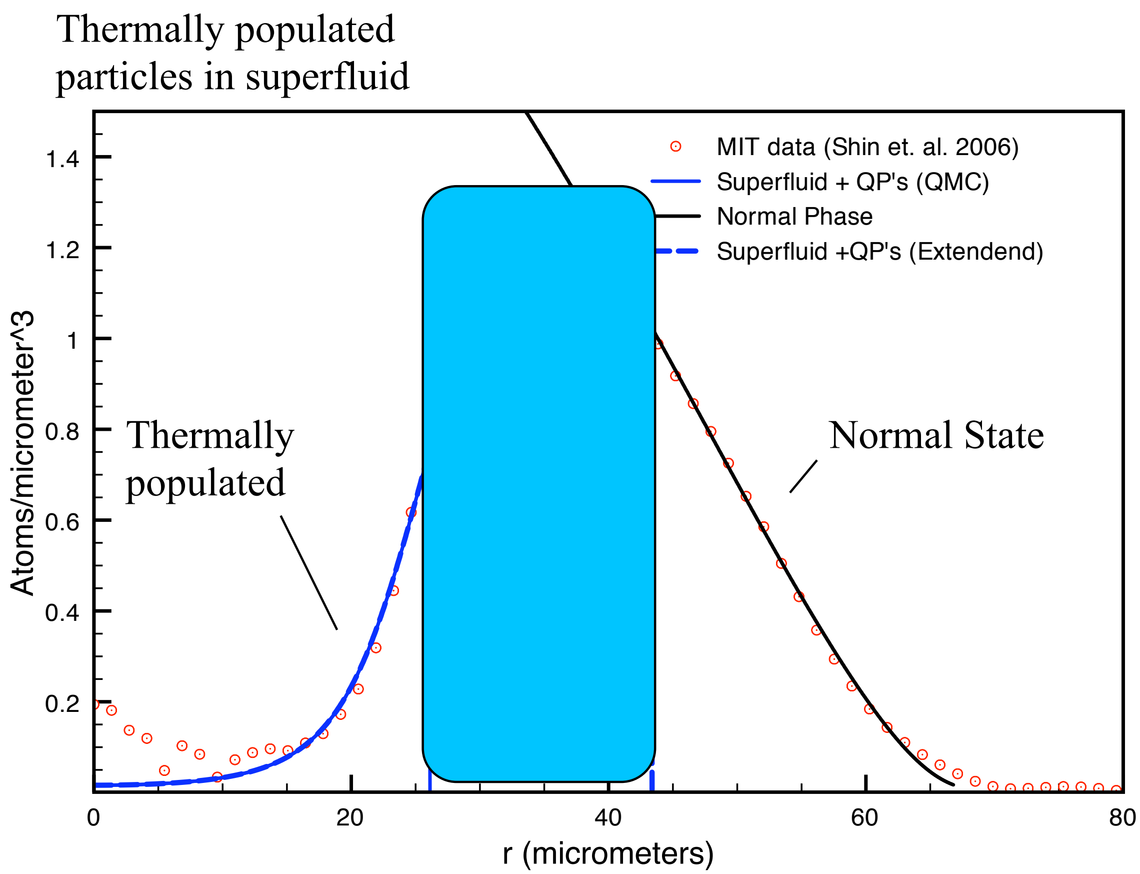
Calculated gap ≈ 0.5 (.05) Ef

If experiments say there is no polarization in the superfluid at T=0 :

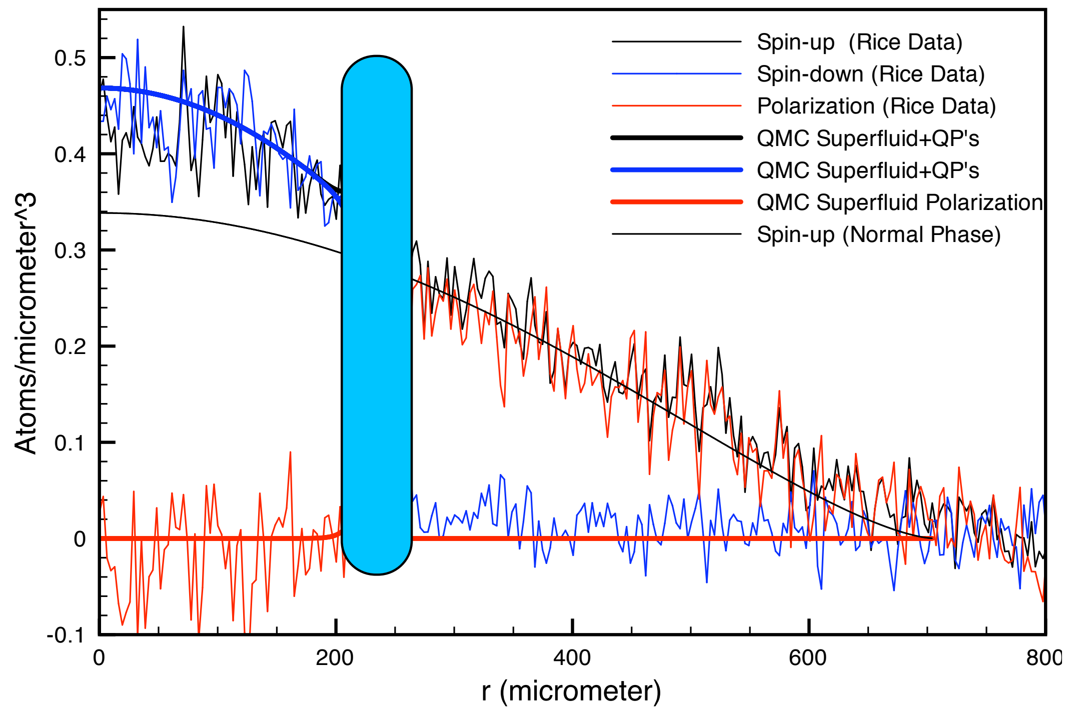
Equilibrium (chemical potentials, pressure) implies gap > 0.40(.02) Ef

Very close to Sarma phase at unitarity and T=0

MIT Data (P = 0.41)



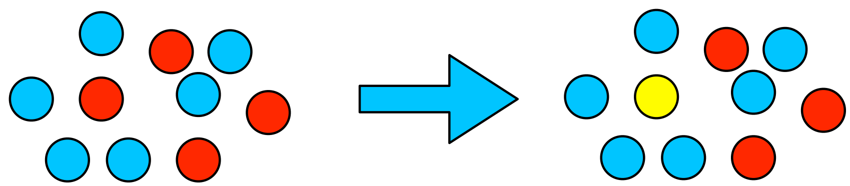
Rice Data



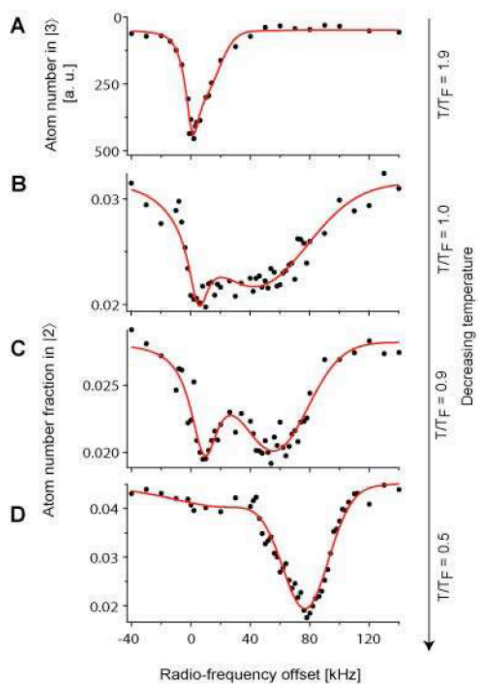
Spin-up (Rice Data)
Spin-down (Rice Data)
Polarization (Rice Data)
QMC Superfluid+QP's
QMC Superfluid+QP's
QMC Superfluid Polarization
Spin-up (Normal Phase)
Atoms/micrometer^3
r (micrometer)
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
200
400
600
800

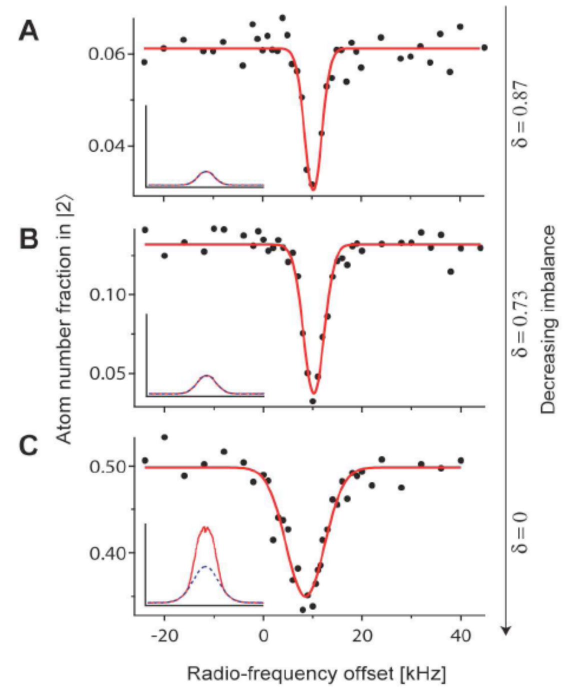
Is this consistent w/ RF response? measurement of 0.2 Ef claimed

Tune RF to specific transition: flip a minority spin to a 3rd (strongly-interacting) state - zero momentum transfer



Decreasing Temperature





Decreasing Polarization

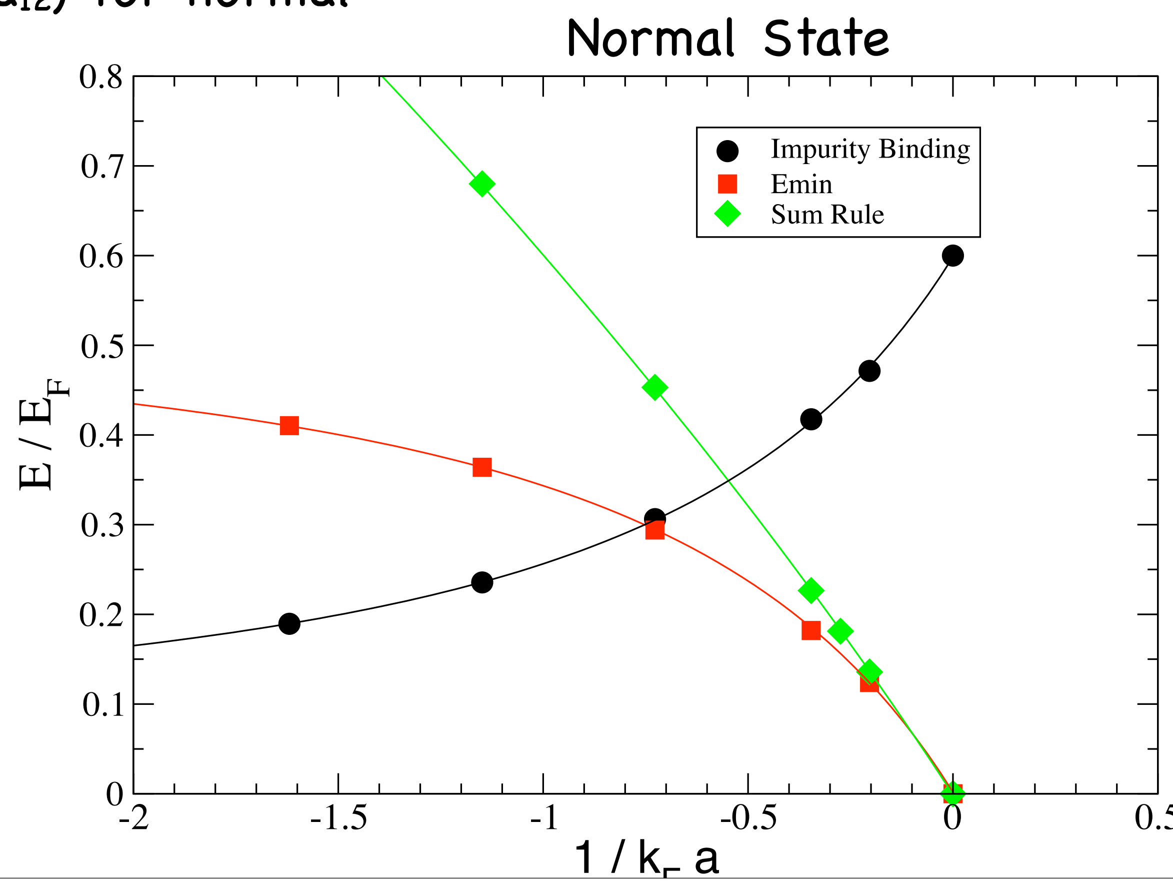
Entire Response Difficult to Calculate:
2 Simple Quantities: Sum Rule and 'Threshhold'

Sum Rule = <V13> - <V12> goes to zero as $a_{13} \Rightarrow a_{12}$

Threshold = BE (a_{13}) - BE (a_{12}) for normal

Width decreases as v13
becomes similar to v12
Sum Rule decreases also

Roughly consistent
w/ experiment



Conclusions / Future Directions

After a few years, we know the pairing gap at Unitarity
much better than we know the neutron superfluid gap
Delta / EF = 0.5 (0.1)

Fully Polarized state cannot exist in the bulk at finite polarization

Even small temperatures will polarize the superfluid state
near the transition, but not in the trap center

Experiment:

Experiments which measure both n, $n\uparrow - n\downarrow$ vs. r
for different Geometries, Polarizations and Temperatures

Theory

Calculations in different geometries
More accurate calculations of Gap and dispersion
Calculations of different possible phases