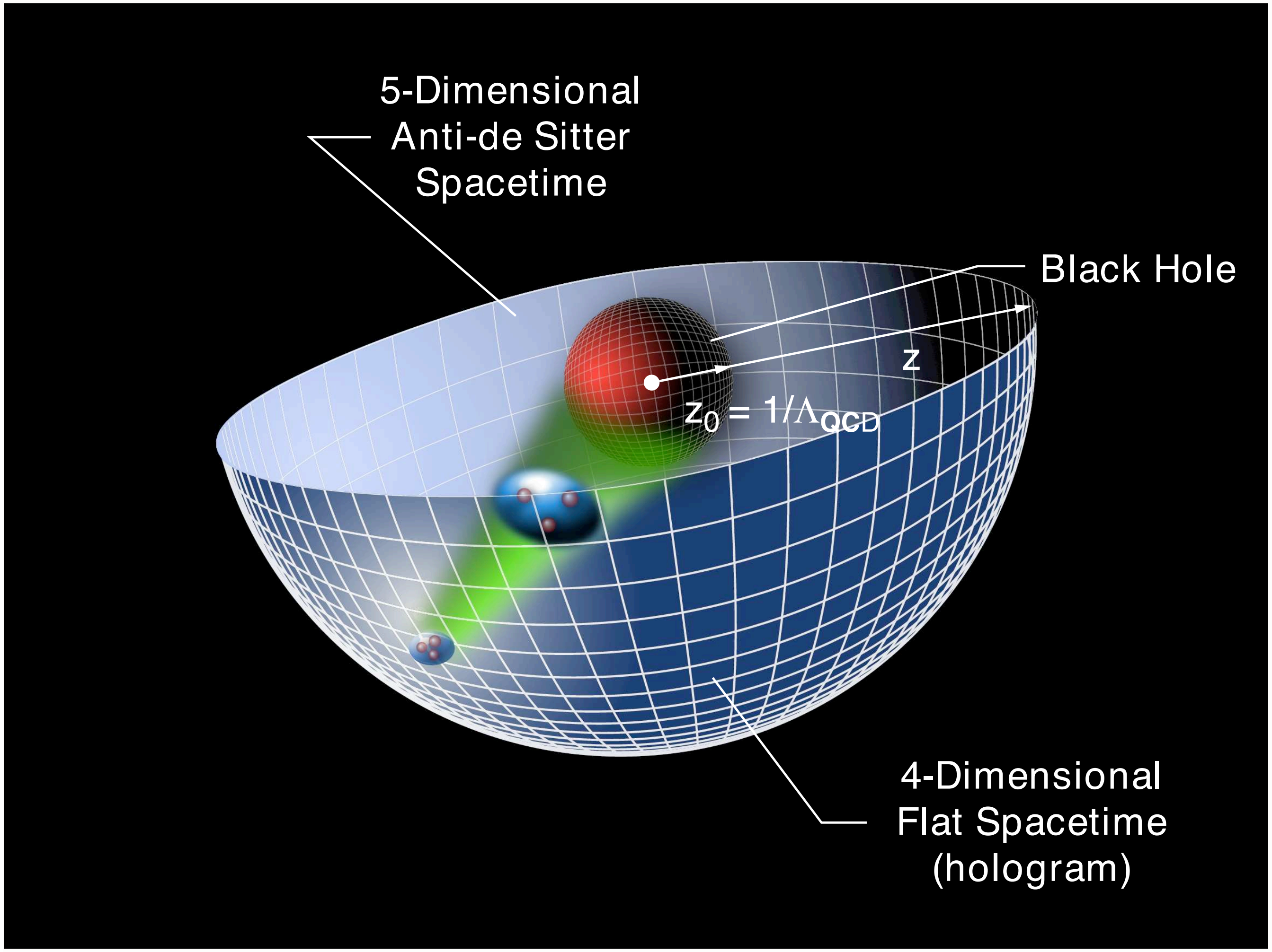
The Conformal Template, AdS/CFT, and QCD Phenomenology

Neutron Program

Application of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

AdS/CFT: Anti de Sitter Space/Conformal Field Theory

Maldacena:

map AdS_5 X S_5 to conformal N=4 SUSY

- QCD is not conformal; however, it has some manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- IR fixed point? $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- "Semi-classical" approximation to QCD
- Use mapping of conformal group SO(4,2) to AdS_5

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

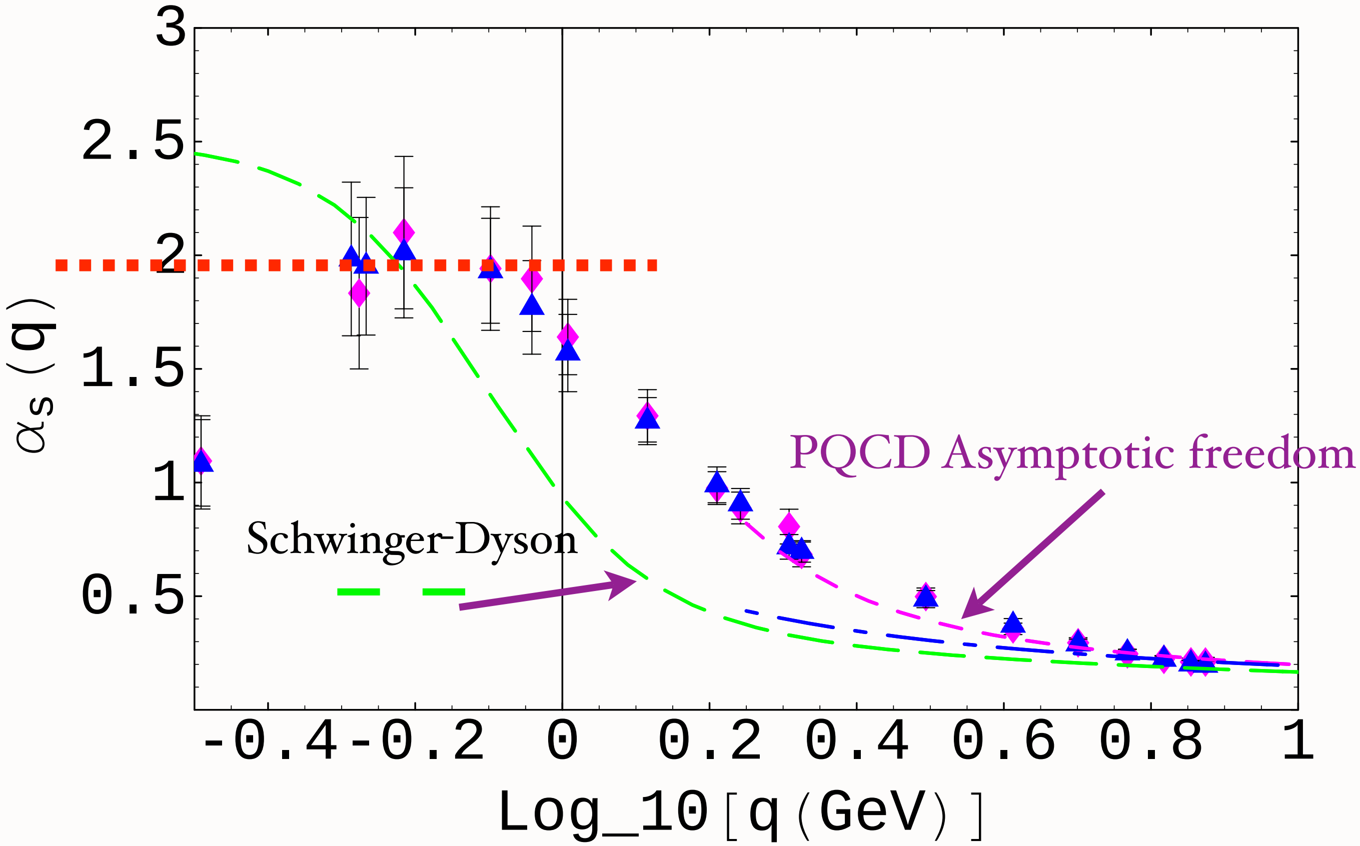
- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

Infrared-Finite QCD Coupling?



Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

Lattice simulation
(MILC)

Furui, Nakajima

DSE: Alkofer, Fischer, von Smekal et al.

IR Fixed Point for QCD?

- *Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point!*
  Alkofer, Fischer, von Smekal et al.

- *Evidence from Lattice Gauge Theory* Furui, Nakajima

- Define coupling from observable: indications of IR fixed point for QCD effective charges

- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2

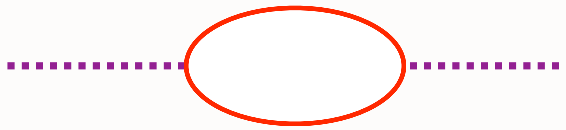
- Justifies application of AdS/CFT in strong-coupling conformal window

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge for QED

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}[\frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2})\sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|}]$$

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi} \frac{\log Q^2}{m^2} \qquad Q^2 >> 4M^2$$

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d \log Q^2} = \frac{4}{3}(\frac{\alpha}{4\pi})^2 n_\ell > 0$$

$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi} \frac{Q^2}{m^2} \qquad Q^2 << 4M^2$$

Serber-Uehling

$$\beta \propto \frac{Q^2}{m^2}$$

vanishes at small momentum transfer

$$\alpha_s(Q^2) \simeq \text{const at small } Q^2$$

Cornwall

Effective gluon mass: vacuum polarization vanishes at small momentum transfer

Analog of Serber-Uehling vacuum polarization in QED:

$$\Pi(Q^2) = \frac{\alpha}{15\pi}\frac{Q^2}{m_e^2} \qquad Q^2 << 4m_e^2$$

$$\Pi(Q^2) \propto \frac{Q^2}{m_g^2} \qquad Q^2 << 4m_g^2 \qquad \alpha_s(Q^2) \simeq \text{const}$$

$$\beta = 0$$

Decoupling of long wavelength gluonic interactions

Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

- Eigensolutions of ERBL evolution equation for distribution amplitudes

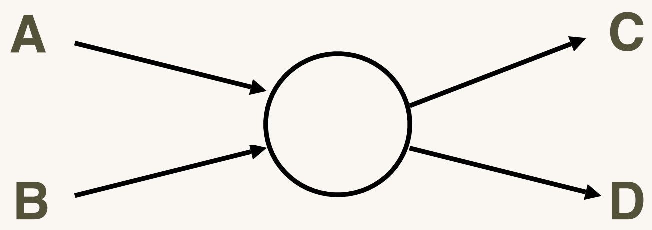
  V. Braun et al;
  Frishman, Lepage, Sachrajda, sjb

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Fix Renormalization Scale (BLM)

- Use AdS/CFT

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1} \qquad -t = Q^2$$

Farrar & sjb; Matveev et al

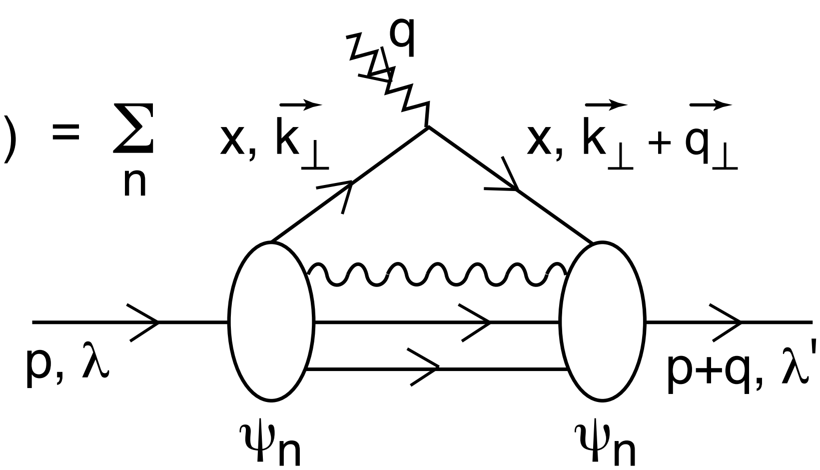
Conformal symmetry and PQCD predicts leading-twist power behavior

Characteristic scale of QCD: 300 MeV

New J-PARC, GSI, J-Lab, Belle, Babar tests

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+(0) | p \lambda \rangle$

$$F_{\lambda\lambda'}(Q^2) = \sum_n$$

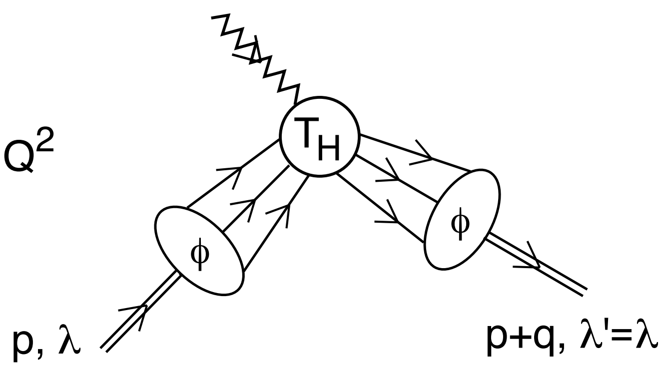


Lepage, Sjb
Efremov
Radyushkin

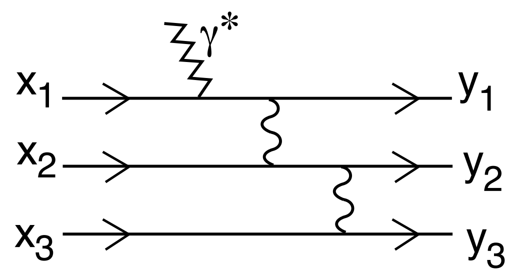
QCD Factorization

Scaling Laws from PQCD or AdS/CFT

⇨ Large Q^2



$T_H = \sum$

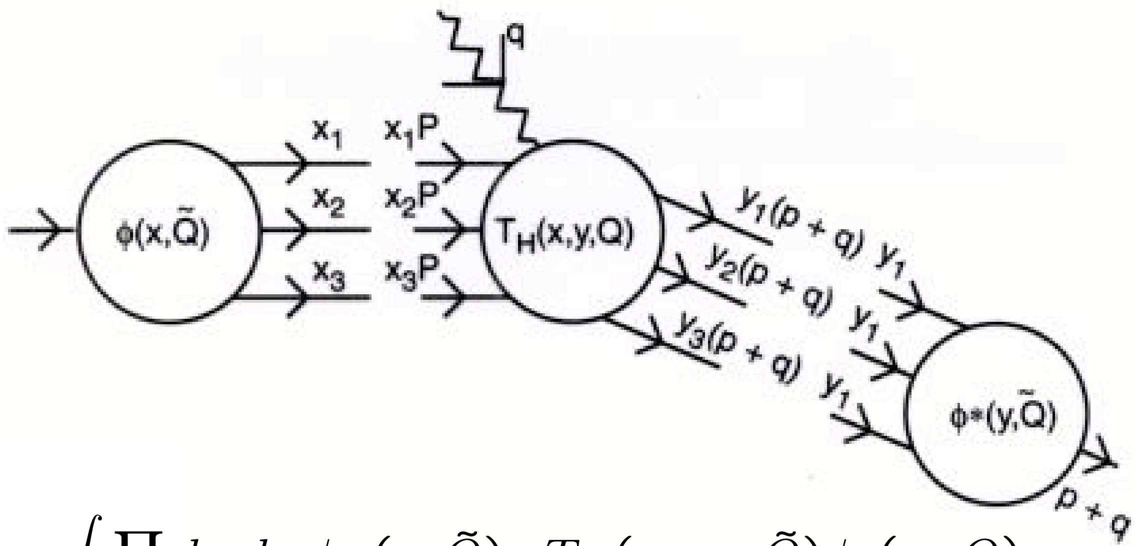


$+ \ \ldots$

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i,)$$

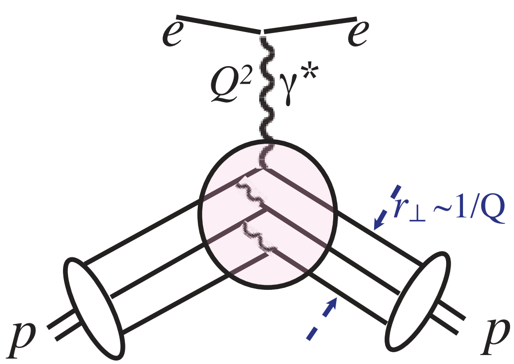
Leading-Twist PQCD Factorization

Lepage, sjb



$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

Exclusive



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

$Q^4 F_1(Q^2) \simeq$ constant

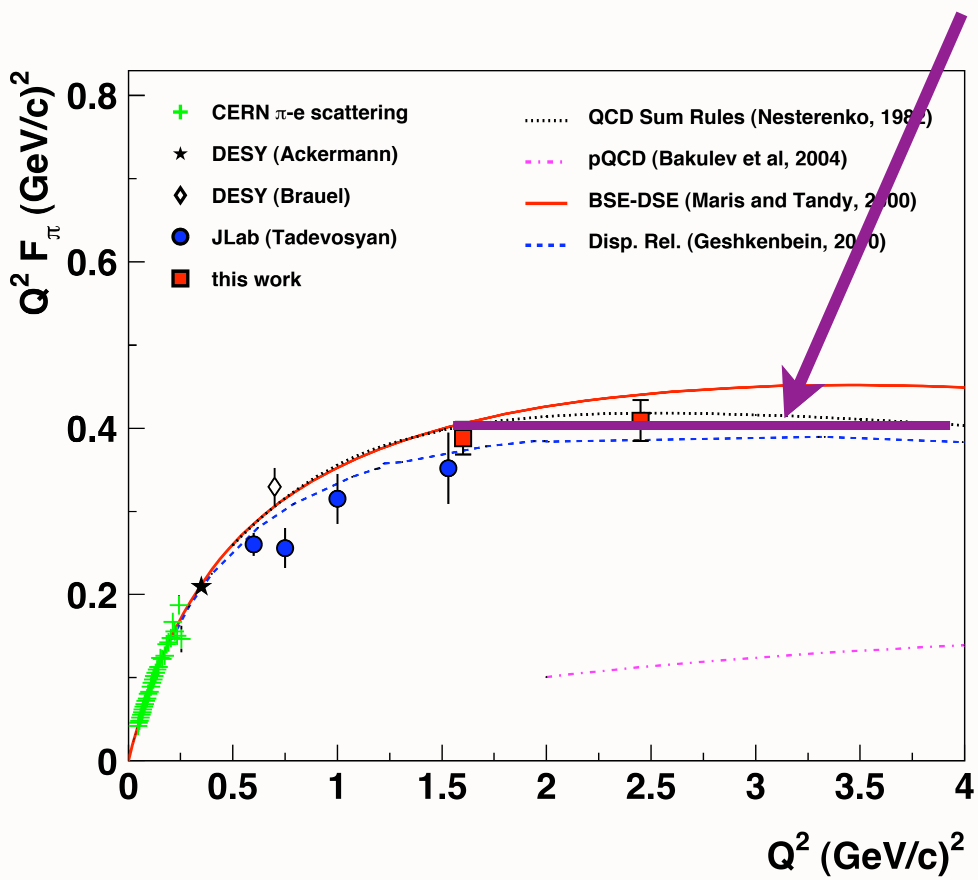
Features of Hard Exclusive Processes in PQCD

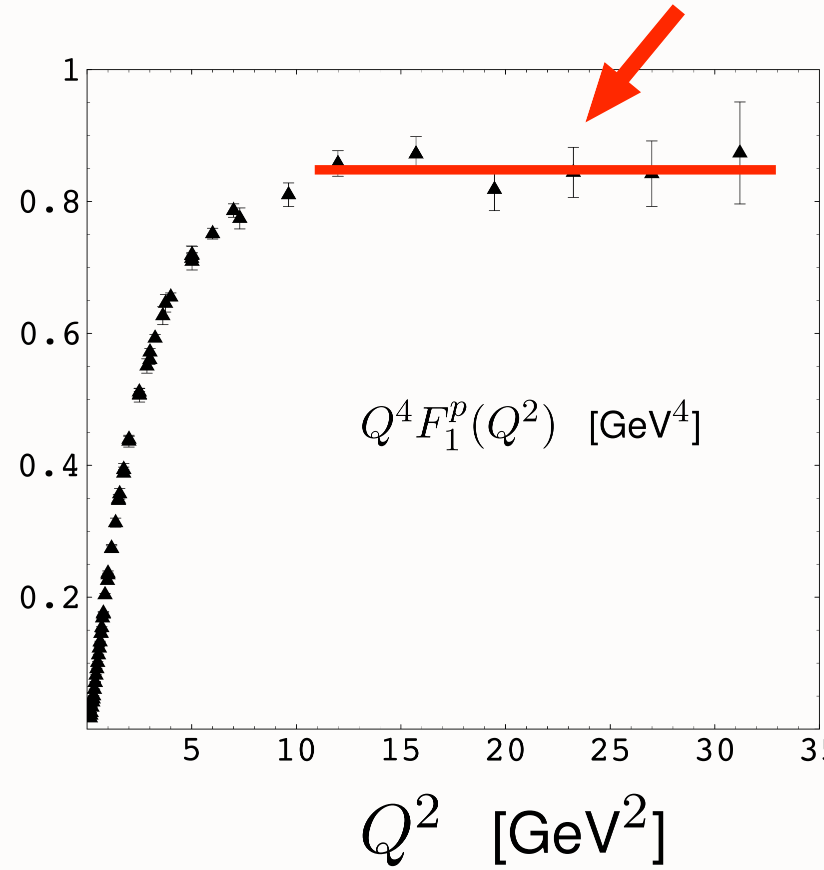
- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes $M = \int T_H \times \Pi \phi_i$
- Dimensional counting rules: short-distance dominance $M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}}$
- Hadron helicity conservation $\sum_{initial} \lambda_i^H = \sum_{final} \lambda_j^H$
- Color transparency $L = 0$ dominance $\frac{F_2}{F_1} \sim \frac{1}{Q^2}$
- Hidden color
- Evolution Equations

Lepage, sjb; Efremov, Radyushkin

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const

$Q^4 F_1(Q^2) \to$ const





Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

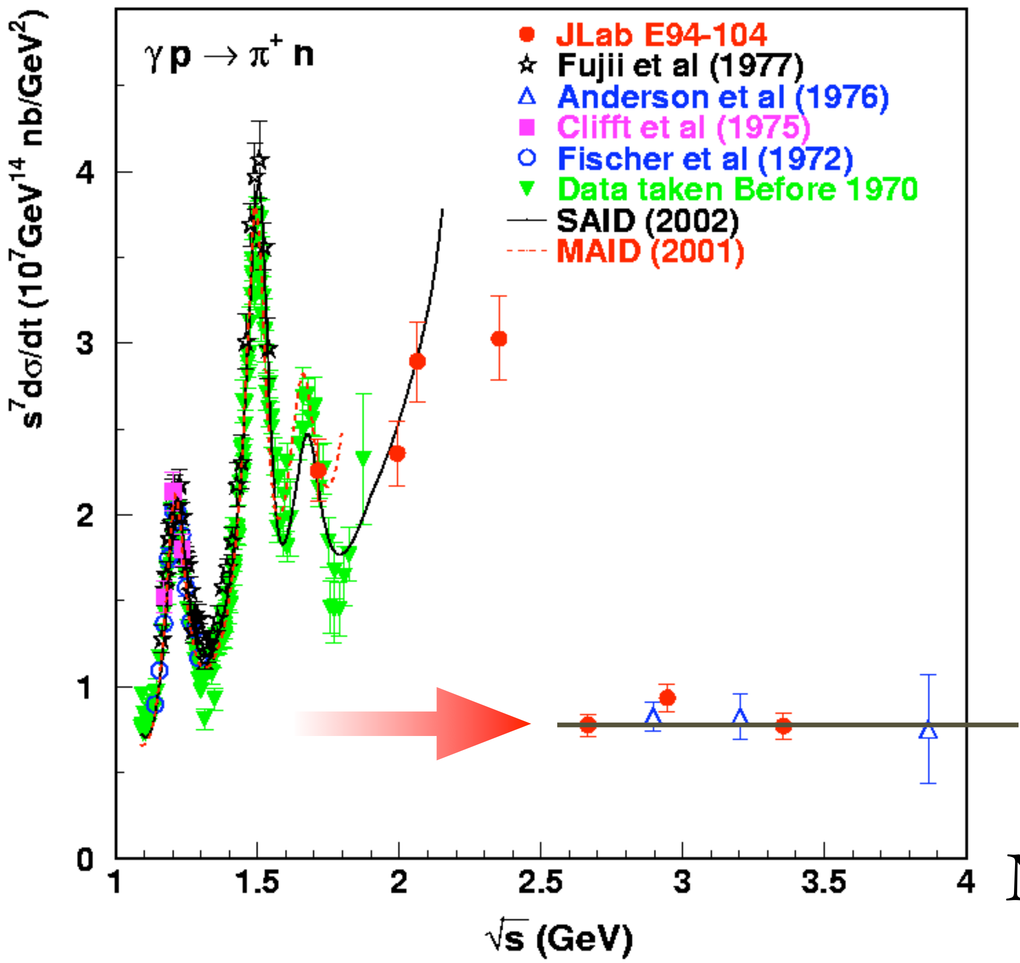
Generalized parton distributions from nucleon form-factor da
M. Diehl (**DESY**), **Th. Feldmann** (**CERN**), **R. Jakob**, **P. Kroll** (**W**
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

G. Huber

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

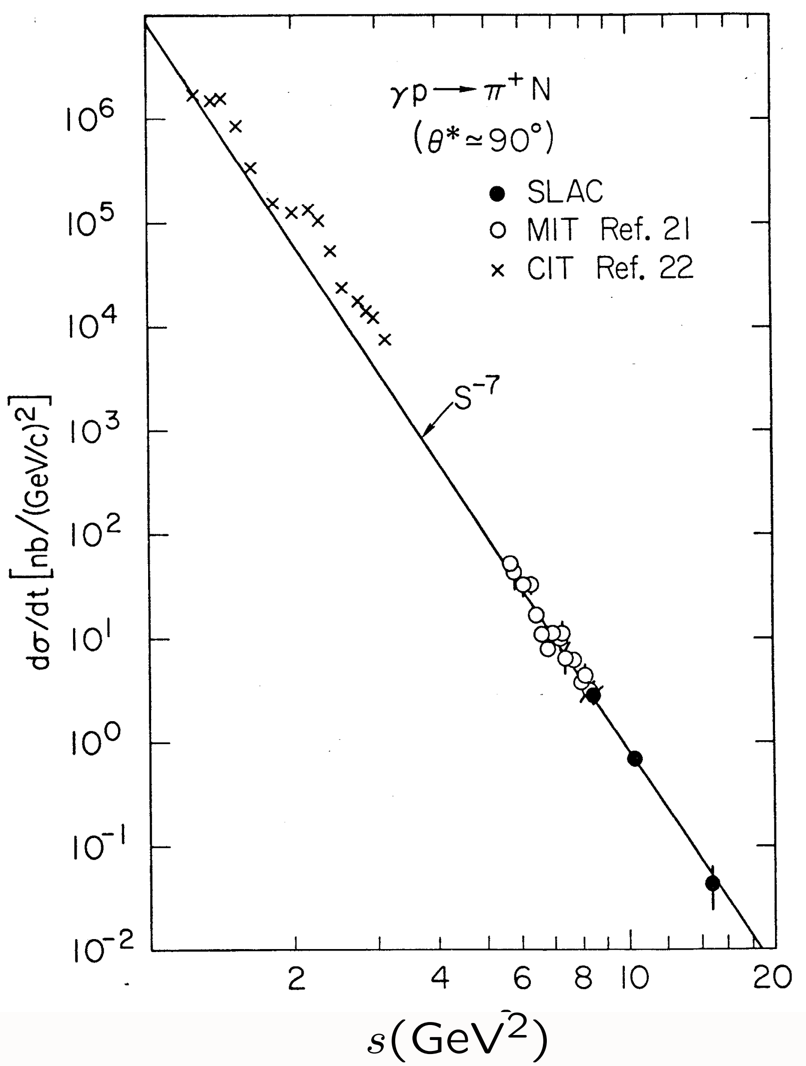
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

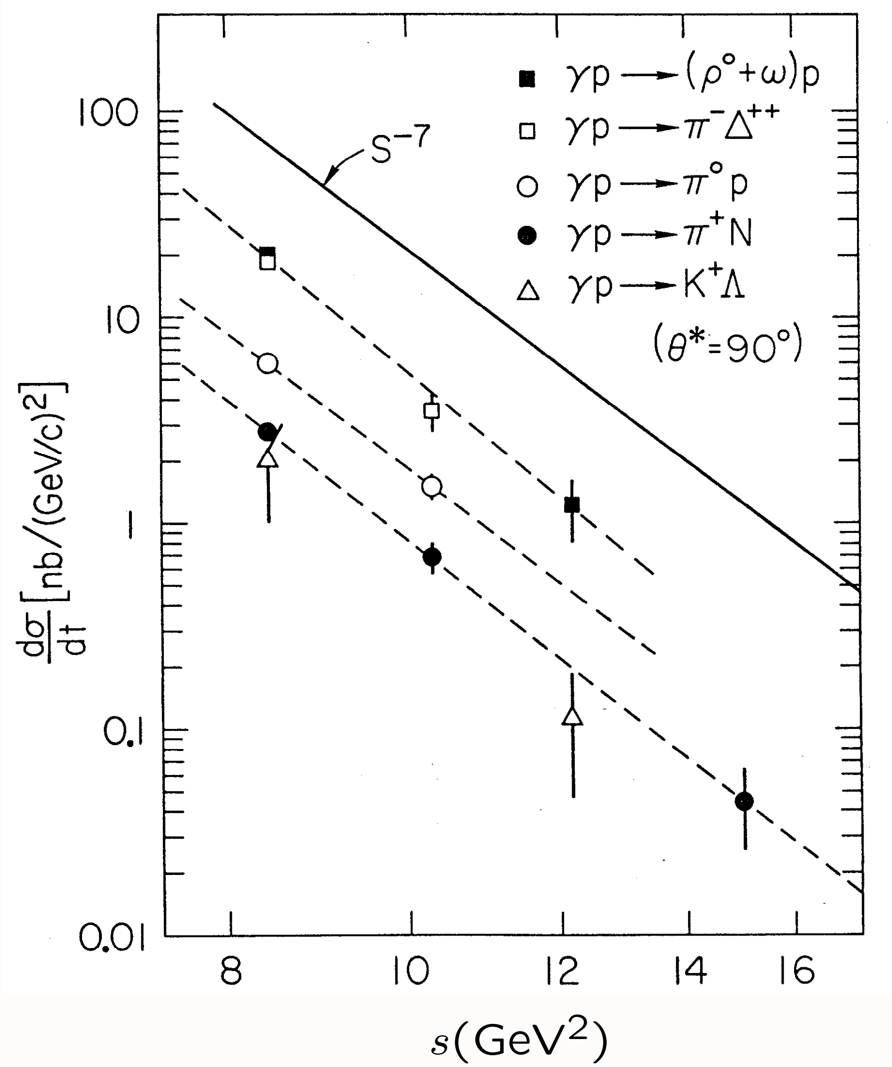
$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

Conformal invariance at high momentum transfer!

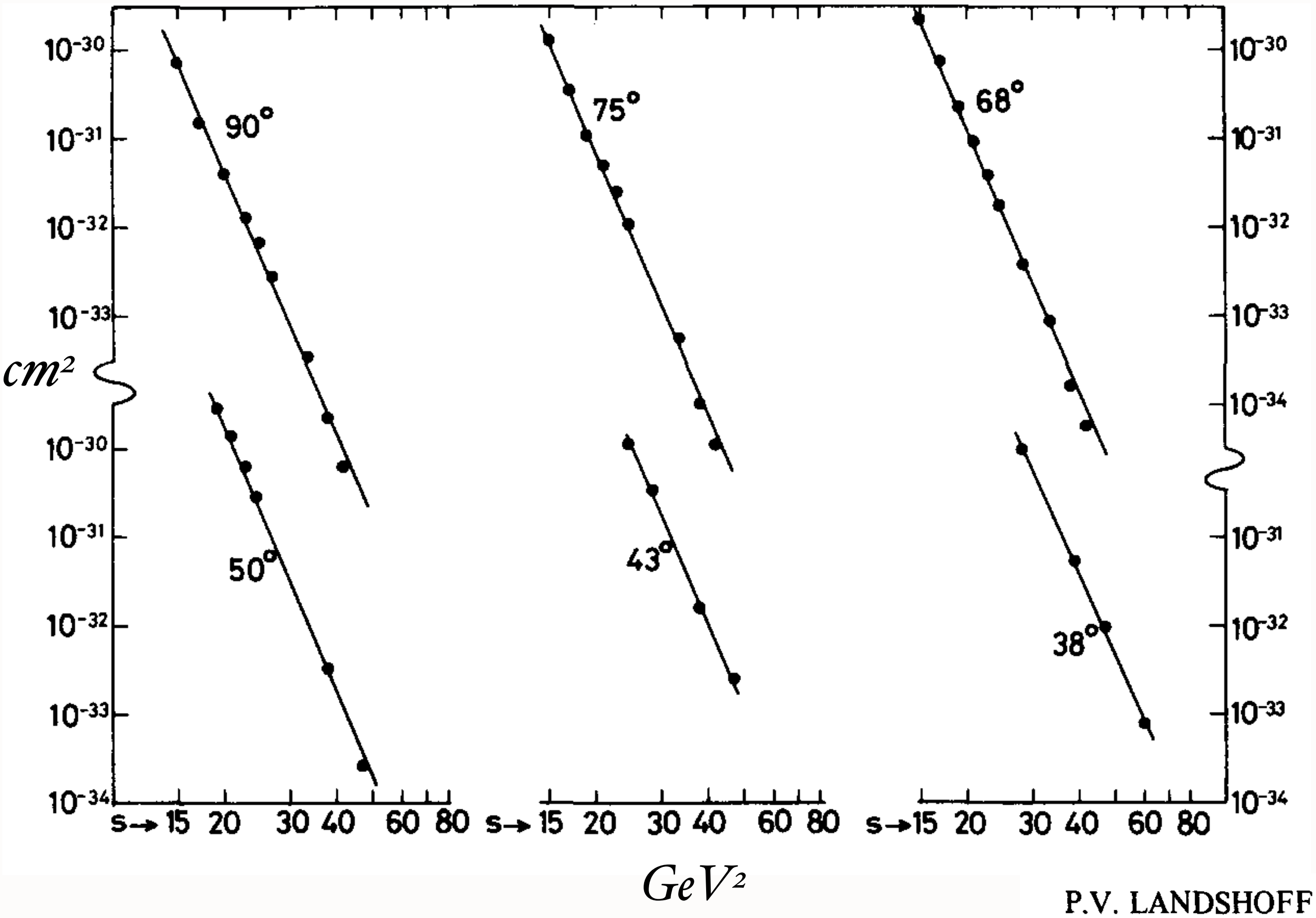




Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



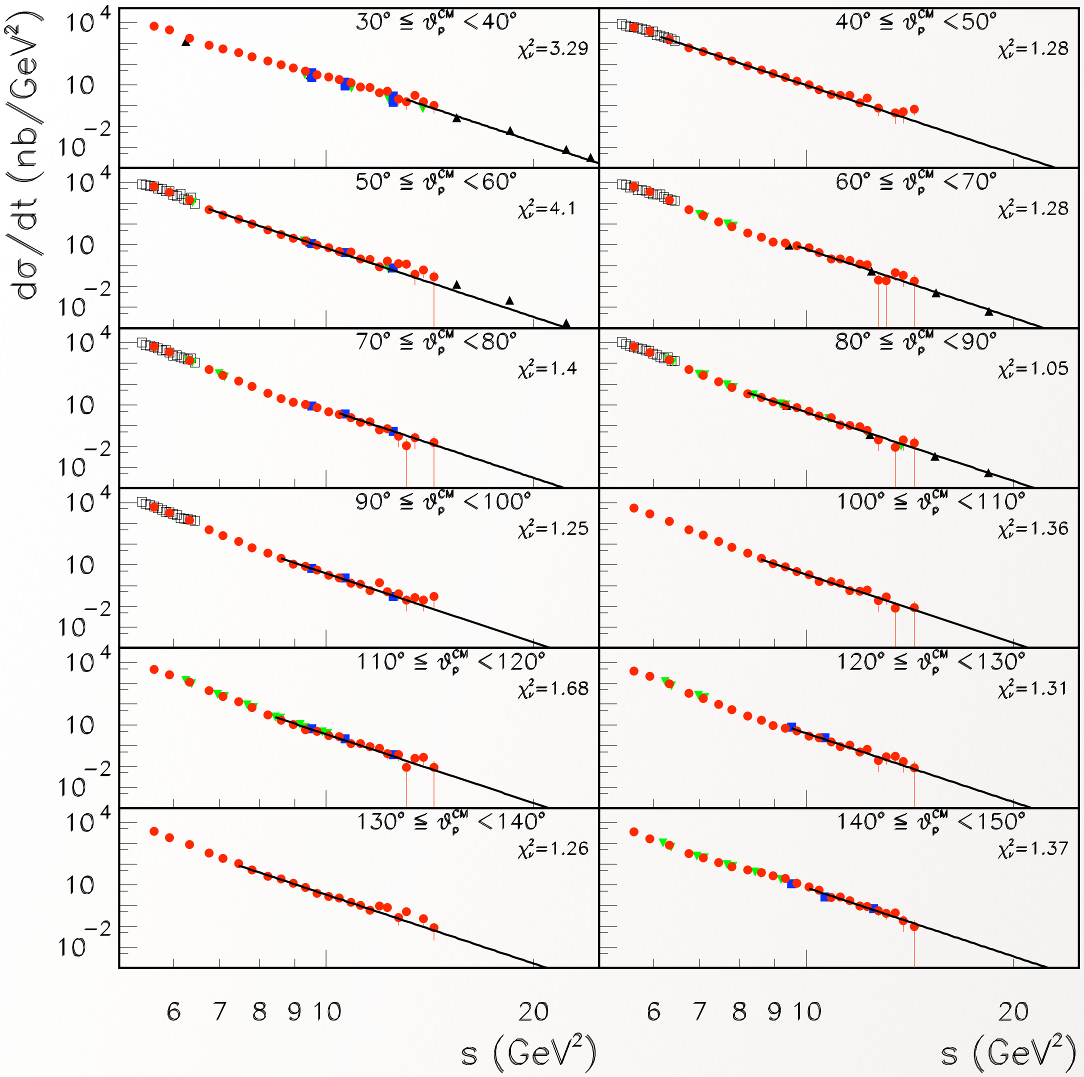
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

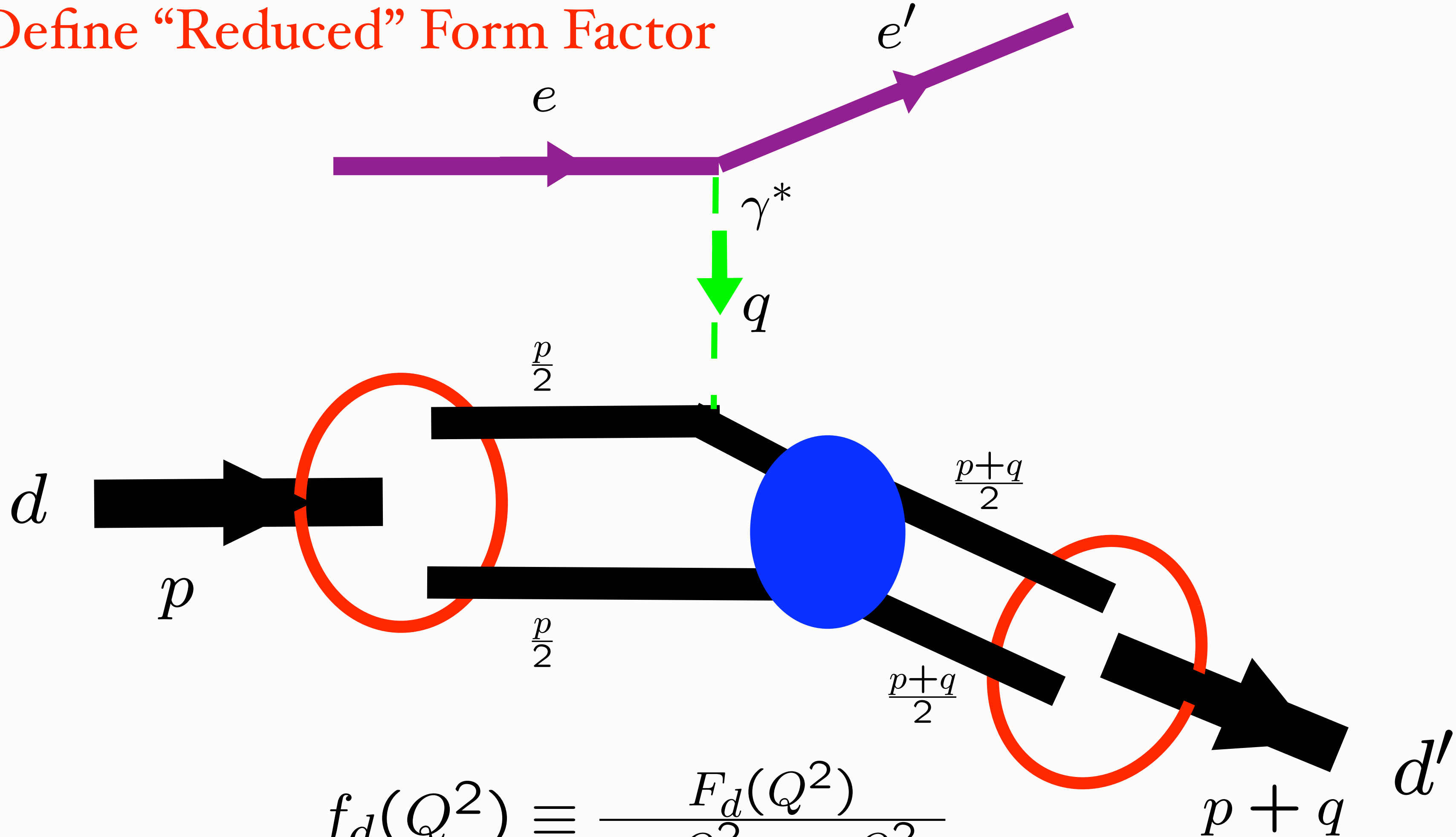
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$$n_{tot}-2 = (1 + 6 + 3 + 3) - 2 = 11$$

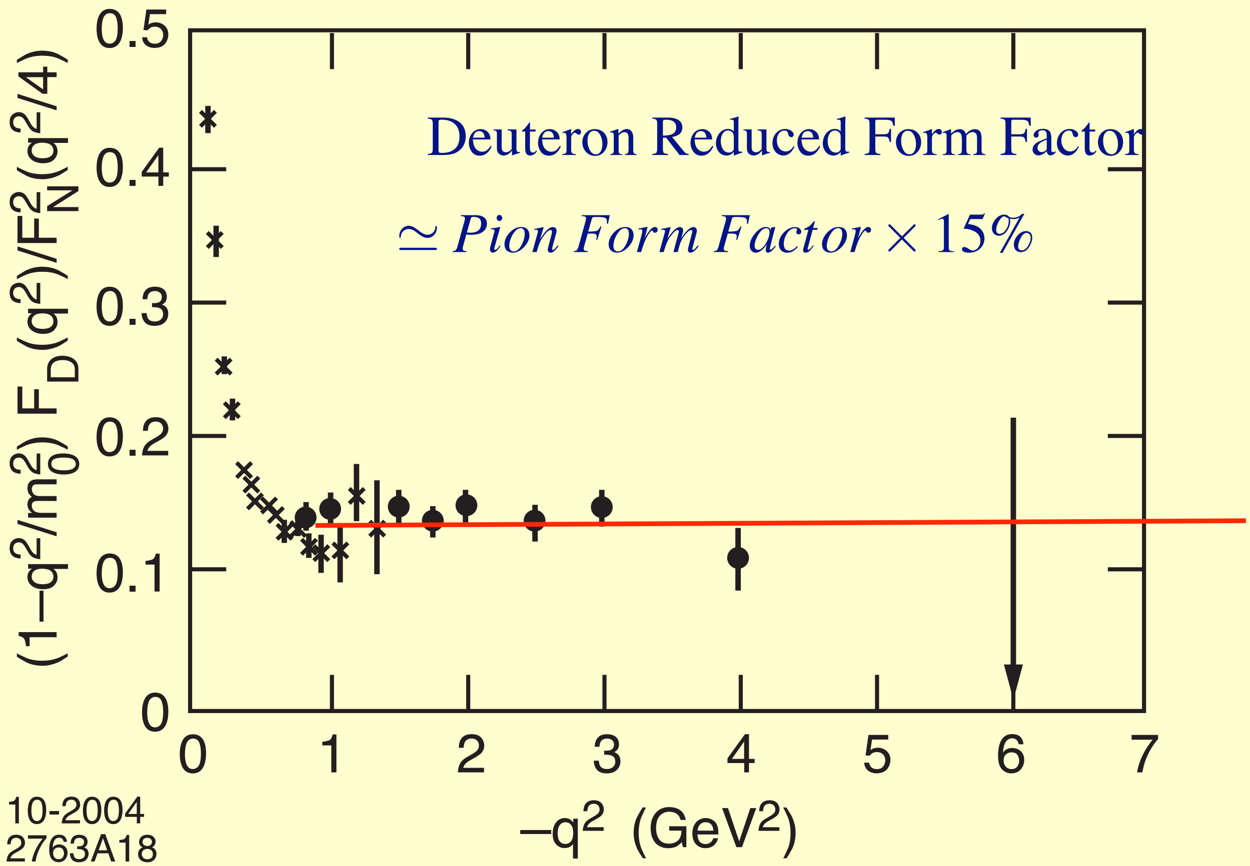
Conformal invariance at high momentum transfers!

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering



- Evidence for Hidden Color in the Deuteron

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is | n p
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin

- **DSE: QCD coupling (mom scheme) has IR Fixed point!** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^{μ}, D, K^{μ}, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops.

- Growing theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point:

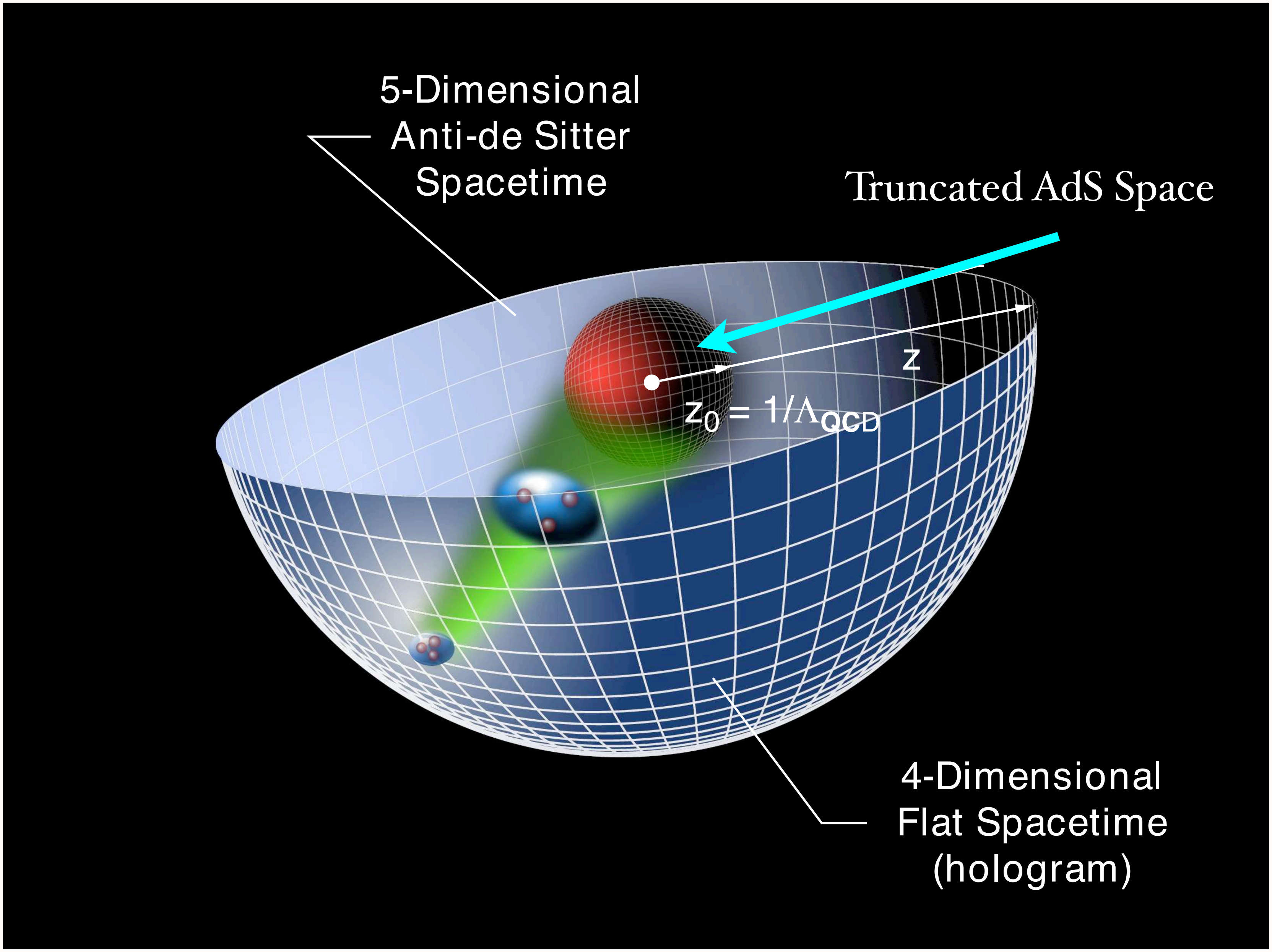
  von Smekal, Alkofer and Hauck, arXiv:hep-ph/9705242; Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Deur, Burkert, Chen and Korsch, hep-ph/0509113 . . .

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

  implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

  Brodsky and Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Matveev *et al.*, Lett. Nuovo Cim. **7**, 719 (1973).


5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

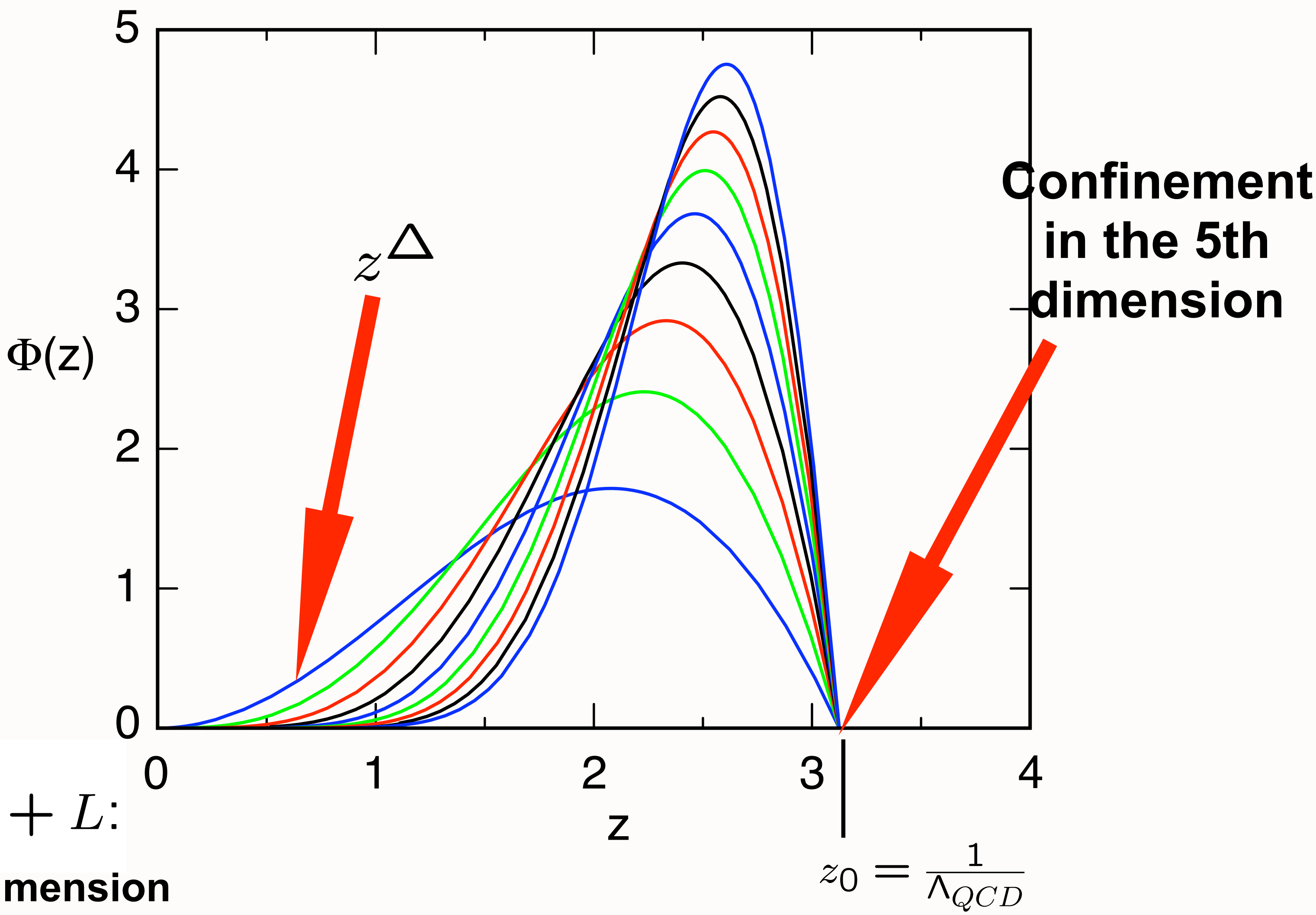
AdS/CFT

Guy de Teramond
SJB

- Use mapping of conformal group SO(4,2) to AdS_5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Identify hadron by its interpolating operator at z --> 0



$\Delta = 3 + L$:

Twist dimension of baryon

de Teramond, sjb

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

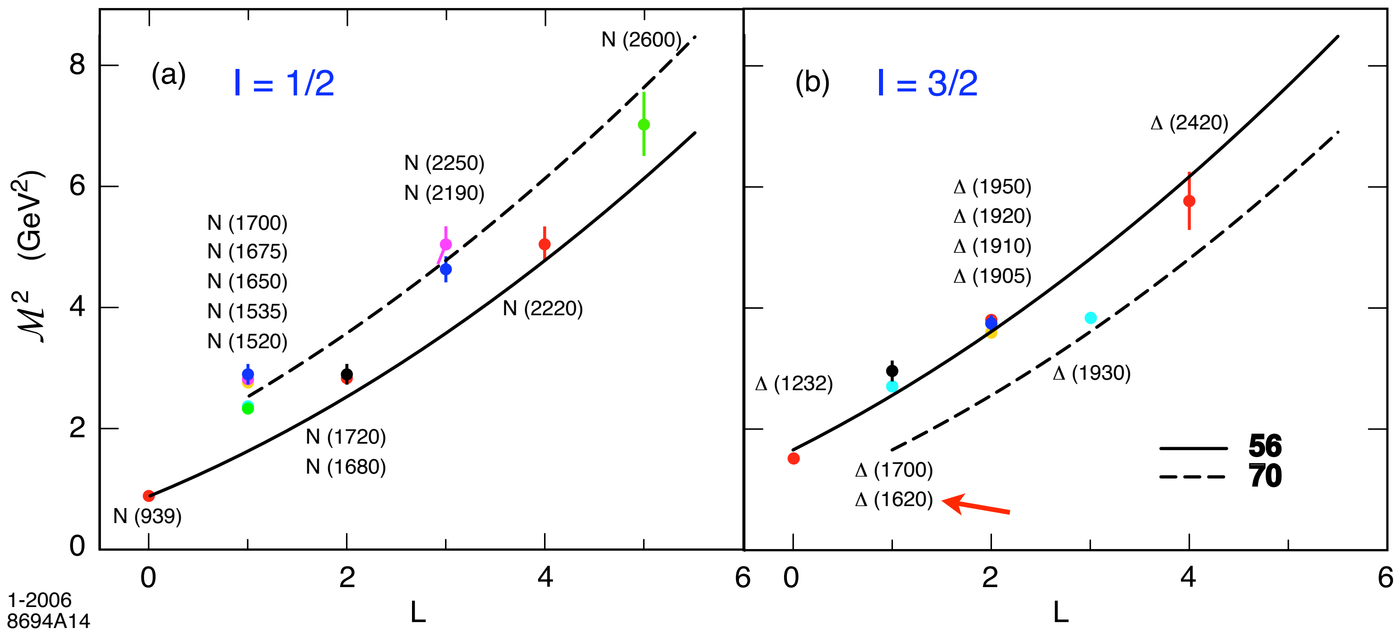


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Guy de Teramond
SJB

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Action for scalar field in AdS5

$$S[\Phi] = \kappa' \int d^4x dz \sqrt{g} \left[g^{\ell m} \partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi \right]$$

where $[\kappa'] = L^{-2}$ $\quad g^{\ell m} = \frac{z^2}{R^2} \eta^{\ell m} \quad \sqrt{g} = R^5 / z^5$

Action is invariant under scale transformations

$$x^\mu \to \lambda x^\mu, \quad z \to \lambda z.$$

$$\Phi(x^\ell) = \Phi(\lambda x^\ell)$$

Variation wrt. Φ

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0$$

Solutions of form: $\Phi(x,z) = e^{-iP\cdot x} f(z)$ $P_\mu P^\mu = \mathcal{M}^2$

$$S = -\kappa R^3 \int \frac{dz}{z^3} \left[(\partial_z f)^2 - \mathcal{M}^2 f^2 + \frac{(\mu R)^2}{z^2} f^2 \right]$$

Variation of S wrt f :

$$z^5 \partial_z \left(\frac{1}{z^3} \partial_z f \right) + z^2 \mathcal{M}^2 f - (\mu R)^2 f = 0.$$

$$\left[z^2 \partial_z^2 - 3z\partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] f = 0,$$

Introduce confinement, break conformal invariance

P-S Boundary Condition $f(z = \frac{1}{\Lambda_{QCD}}) = 0$

Normalization in truncated space $R^3 \int_0^{\Lambda_{\mathrm{QCD}}^{-1}} \frac{dz}{z^3} \, f^2(z) = 1$

Identify Orbital Angular Momentum $(\mu R)^2 = -4 + L^2$

- Wave equation in AdS for bound state of two scalar partons with conformal dimension $\Delta = 2 + L$

$$\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\,\mathcal{M}^2 - L^2 + 4\right]\Phi(z) = 0,$$

with solution

$$\Phi(z) = Ce^{-iP\cdot x}z^2 J_L(z\mathcal{M}).$$

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n} \sigma_i.$
- The twist τ is equal to the number of partons $\tau = n$.

Introduce confinement, break conformal invariance

$$f(z = \frac{1}{\Lambda_{QCD}}) = 0$$

Substitute $f(z) = (\frac{z}{R})^{\frac{3}{2}} \phi(z)$

$$S = \kappa \int_0^{\Lambda_{\rm QCD}} dz\, \phi \left[-\partial_z^2 - \mathcal{M}^2 - \frac{1-4\alpha^2}{4z^2} \right] \phi + \kappa \lim_{z \to 0} \phi \partial_z \phi,$$

$$z = \zeta$$

$$S = \frac{1}{\Lambda_{QCD}^2} \int_0^{\Lambda_{\rm QCD}^{-1}} d\zeta \left[(\partial_\zeta \phi)^2 - \mathcal{M}^2 \phi^2 - \frac{1-4\alpha^2}{4\zeta^2} \phi^2 \right]$$

Variation gives
$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta),$$

Conformal Kernel
$$V(\zeta) \to -(1 - 4\alpha^2)/4\zeta^2$$

Harmonic Oscillator model

Karch, et al.

$$S = \lambda \int_0^\infty d\zeta \left[(\partial_\zeta \phi)^2 - \mathcal{M}^2 \phi^2 - \frac{1 - 4\alpha^2}{4\zeta^2} \phi^2 + \kappa^4 z^2 \phi^2 \right]$$

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$V(\zeta) = -\frac{1 - 4\alpha^2}{4\zeta^2} + \kappa^4 \zeta^2$$

Solutions

$$\phi_\alpha(z) = \kappa^{\alpha+1} \sqrt{\frac{2n!}{(n+\alpha)!}}\, \zeta^{1/2+\alpha} e^{-\kappa^2 z^2/2} L_n^\alpha \left(\kappa^2 z^2 \right)$$

Eigenvalues $\mathcal{M}^2 = 2\kappa^2 (2n + \alpha + 1)$

Match fall-off at small z to Conformal Dimension of hadron state at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

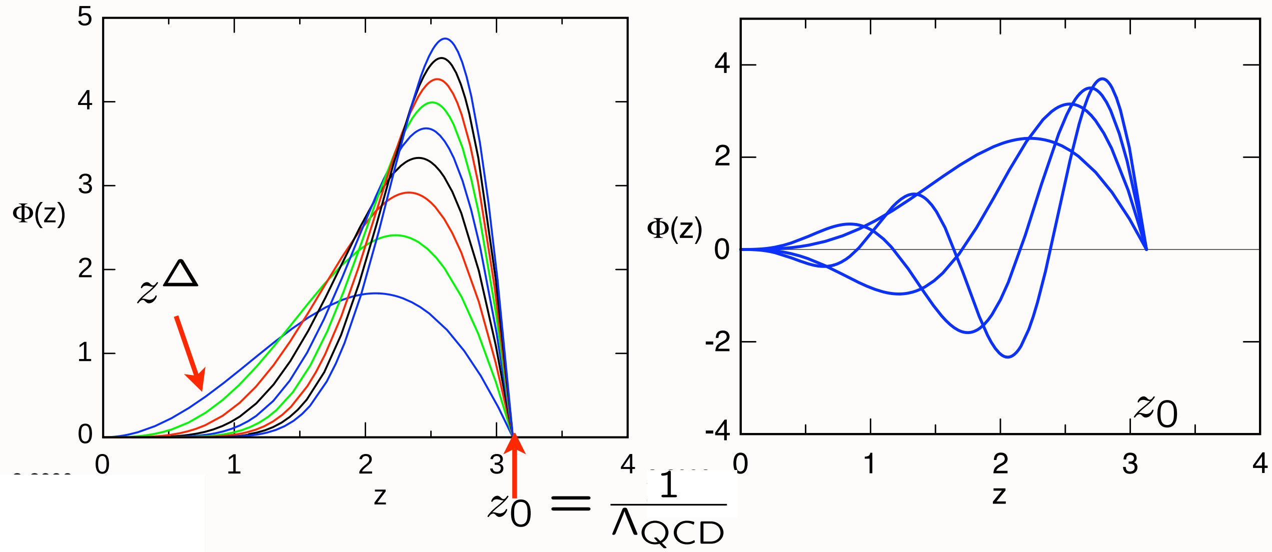
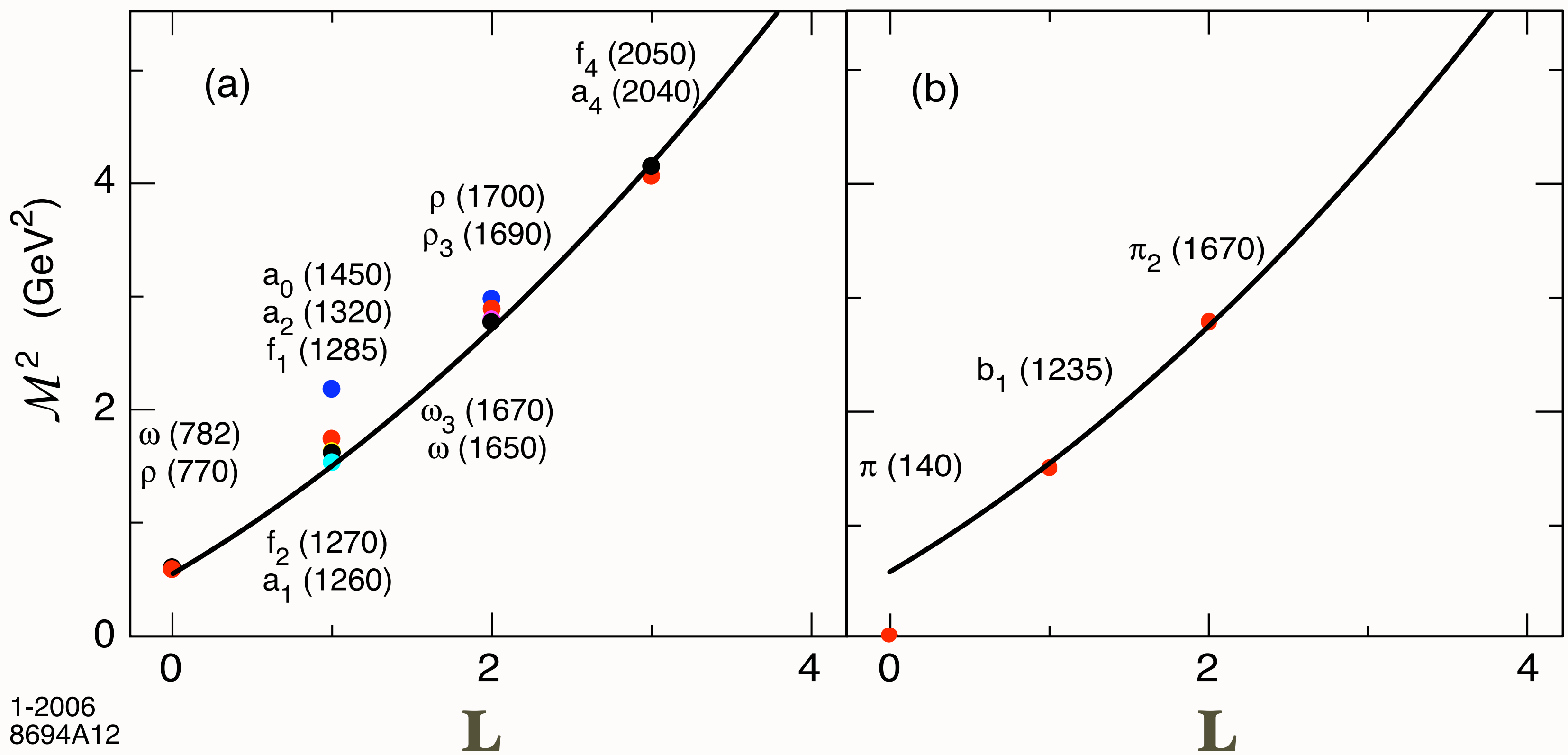


Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.



Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

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SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i .$$

Wave Equation: $\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow \quad$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^{+} = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^{-} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

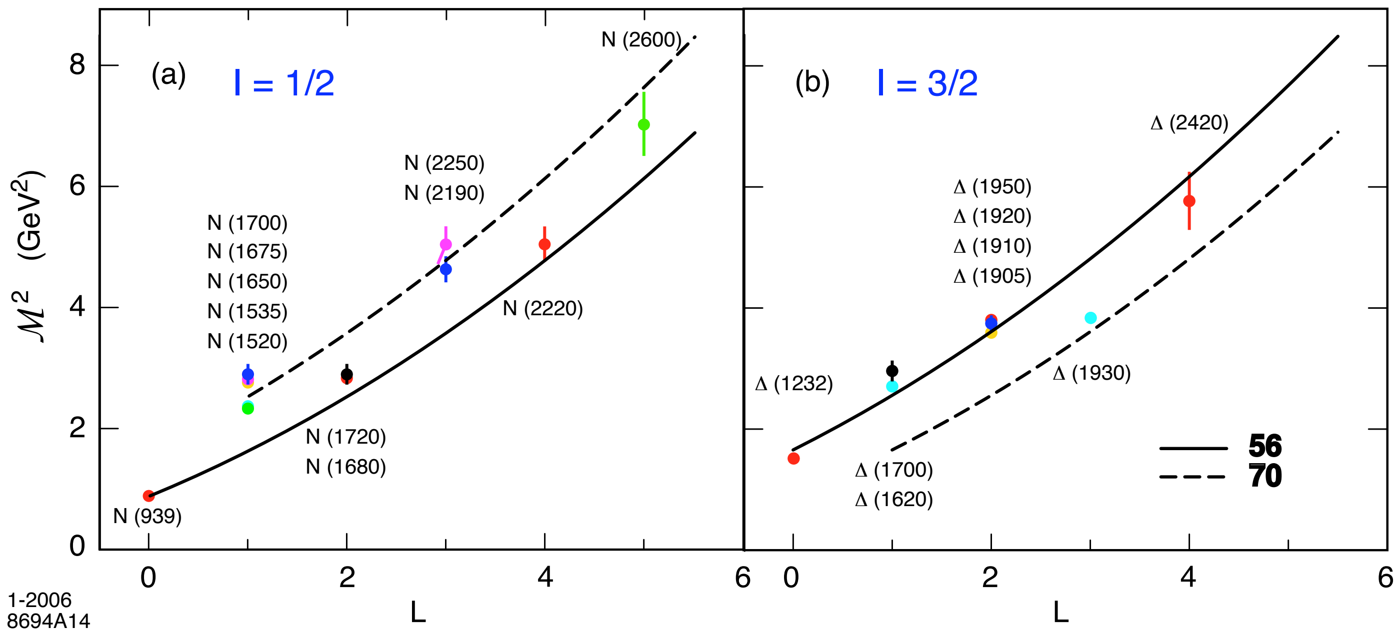

Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

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Glueball Spectrum

- AdS wave function with effective mass μ:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

where $\Phi(x,z) = e^{-iP\cdot x}\, f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

- Glueball interpolating operator with twist -dimension minus spin- two, and conformal dimension $4+L$

$$\mathcal{O}_{4+L} = F D_{\{\ell_1} \ldots D_{\ell_m\}} F,$$

where $L = \sum_{i=1}^{m} \ell_i$ is the total internal space-time orbital momentum.

- Normalizable scalar AdS mode (d = 4):

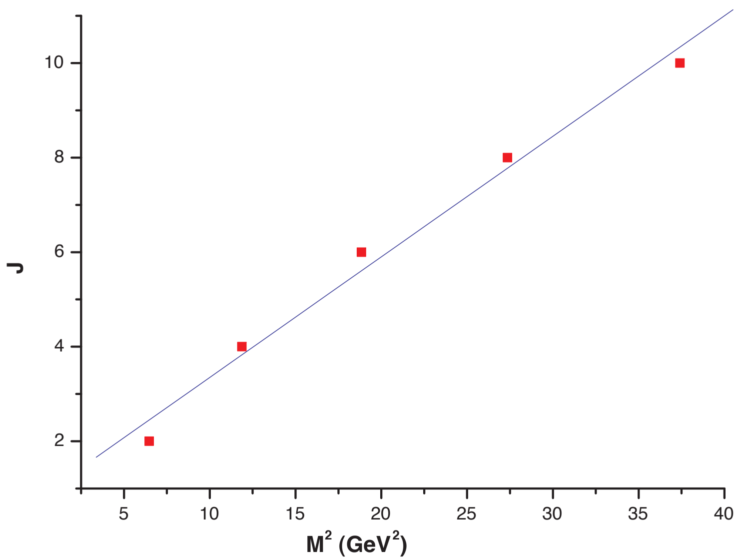
$$\Phi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,a}\Lambda_{QCD}\right)$$

with $\alpha = 2 + L$ and scaling dimension $4+L$.

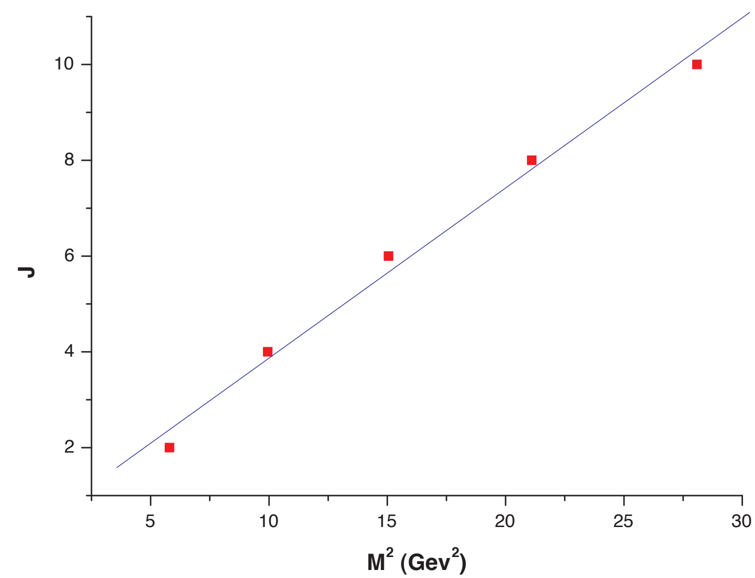
Glueball Regge trajectories from gauge/string duality and the Pomeron

Henrique Boschi-Filho,* Nelson R. F. Braga,† and Hector L. Carrion‡

Instituto de Física, Universidade Federal do Rio de Janeiro,



Neumann Boundary Conditions



Dirichlet Boundary Conditions

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$\begin{aligned} F(Q^2)_{I\to F} &= R^{3+2\sigma}\int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)}\Phi_F(z)\, J(Q,z)\, \Phi_I(z) \\ &\simeq R^{3+2\sigma}\int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z), \end{aligned}$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

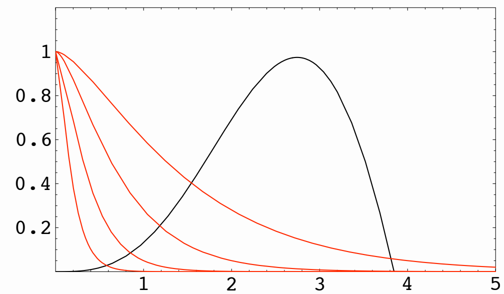
Hadron Form Factors from AdS/CFT

- Propagation of external perturbation suppressed inside AdS.

- At large Q^2 the important integration region is $z \sim 1/Q$.

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$



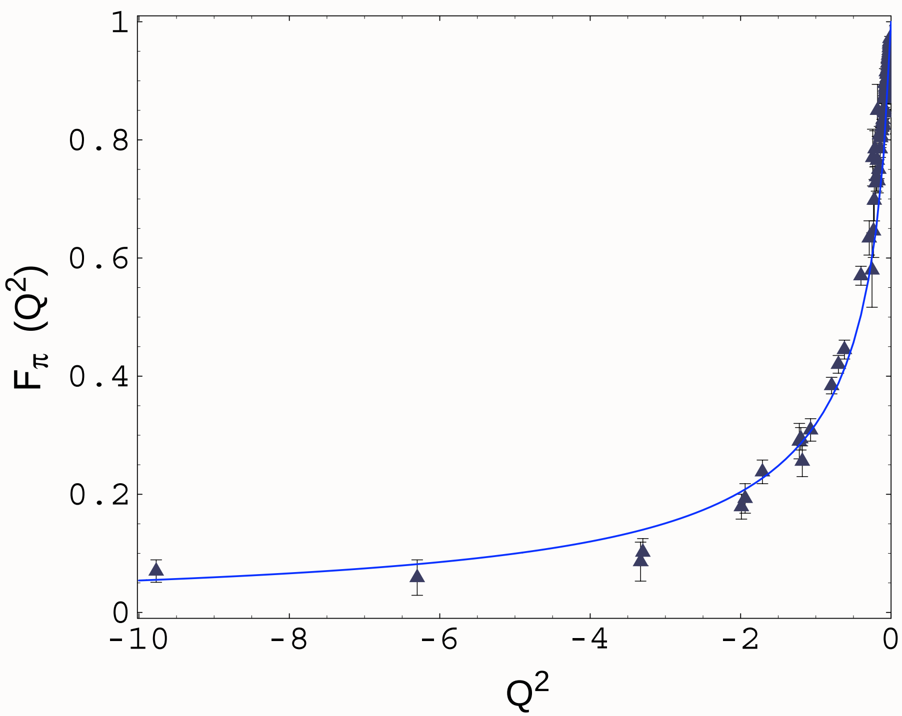
Polchinski, Strassler
de Teramond, sjb

- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Data Compilation from Baldini, Kloe and Volmer

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

$$\Phi(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}. \qquad J(Q,z) = zQK_1(zQ).$$

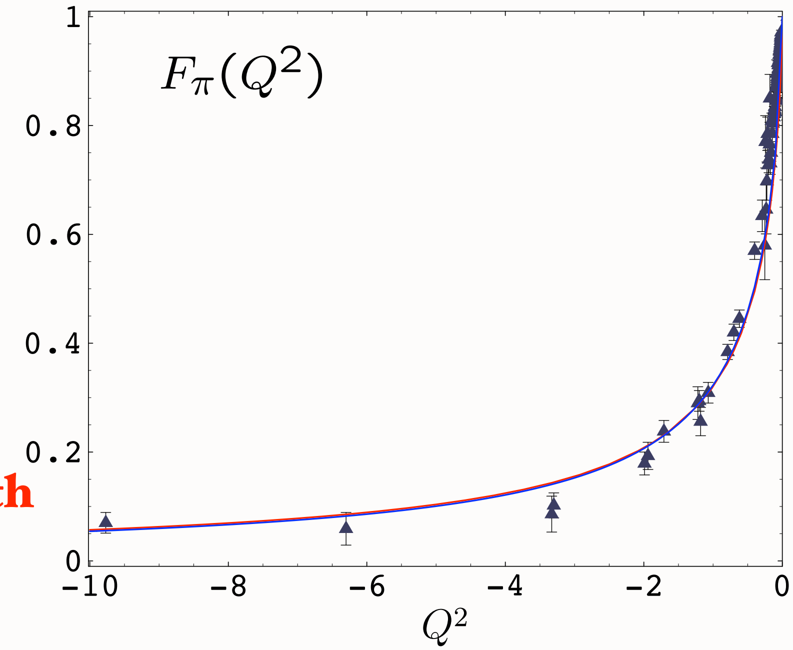
$$F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp\left(\frac{Q^2}{4\kappa^2}\right) Ei\left(-\frac{Q^2}{4\kappa^2}\right) \qquad Ei(-x) = \int_\infty^x e^{-t} \frac{dt}{t}.$$

Space-like Pion Form Factor

$\kappa = 0.4$ GeV

$\Lambda_{QCD} = 0.2$ GeV.

Identical Results for both confinement models



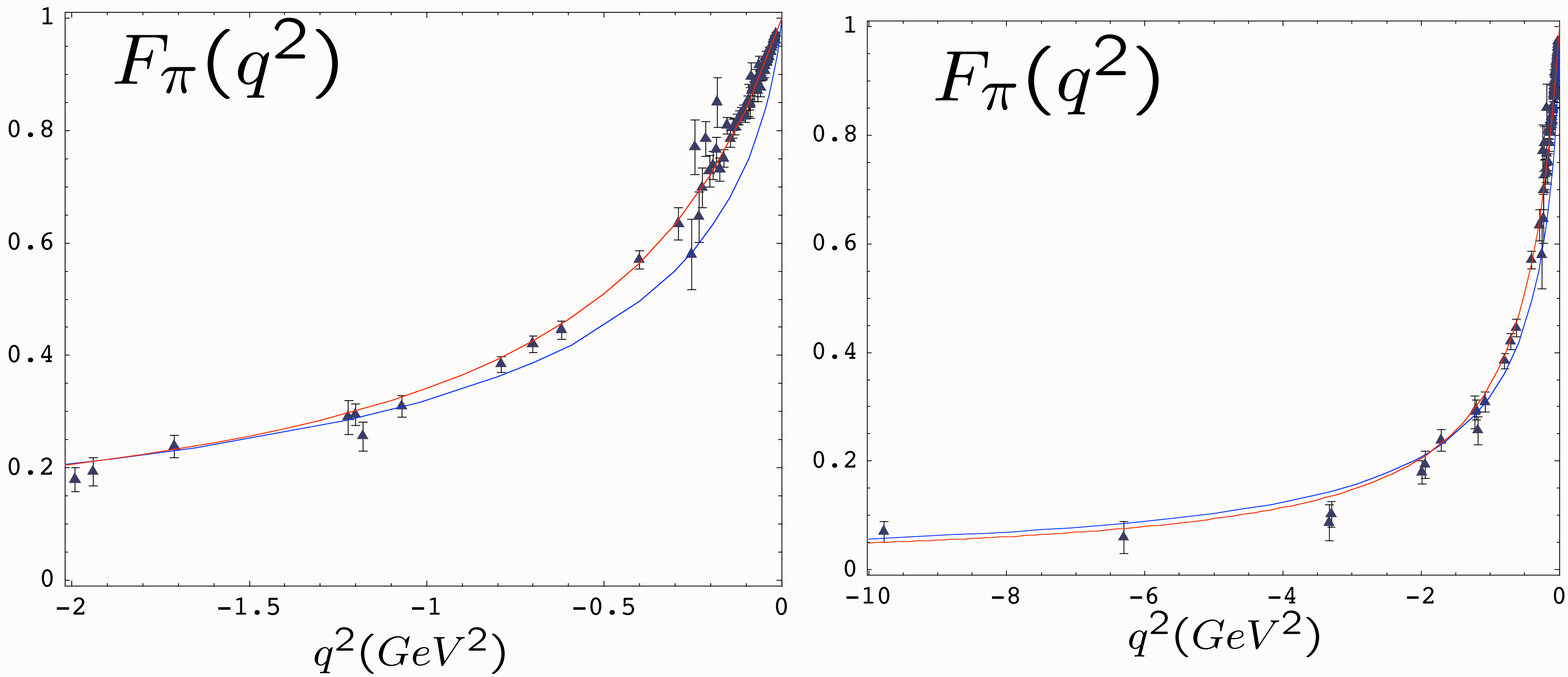
$F(Q^2) \to \frac{4\kappa^2}{Q^2}$

$\kappa = 2\Lambda_{QCD}$

High Q^2 from short distances

$z^2 = \zeta^2 = b_\perp^2 x(1-x)$
$= \mathcal{O}(\frac{1}{Q^2})$

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

One parameter - set by pion decay constant

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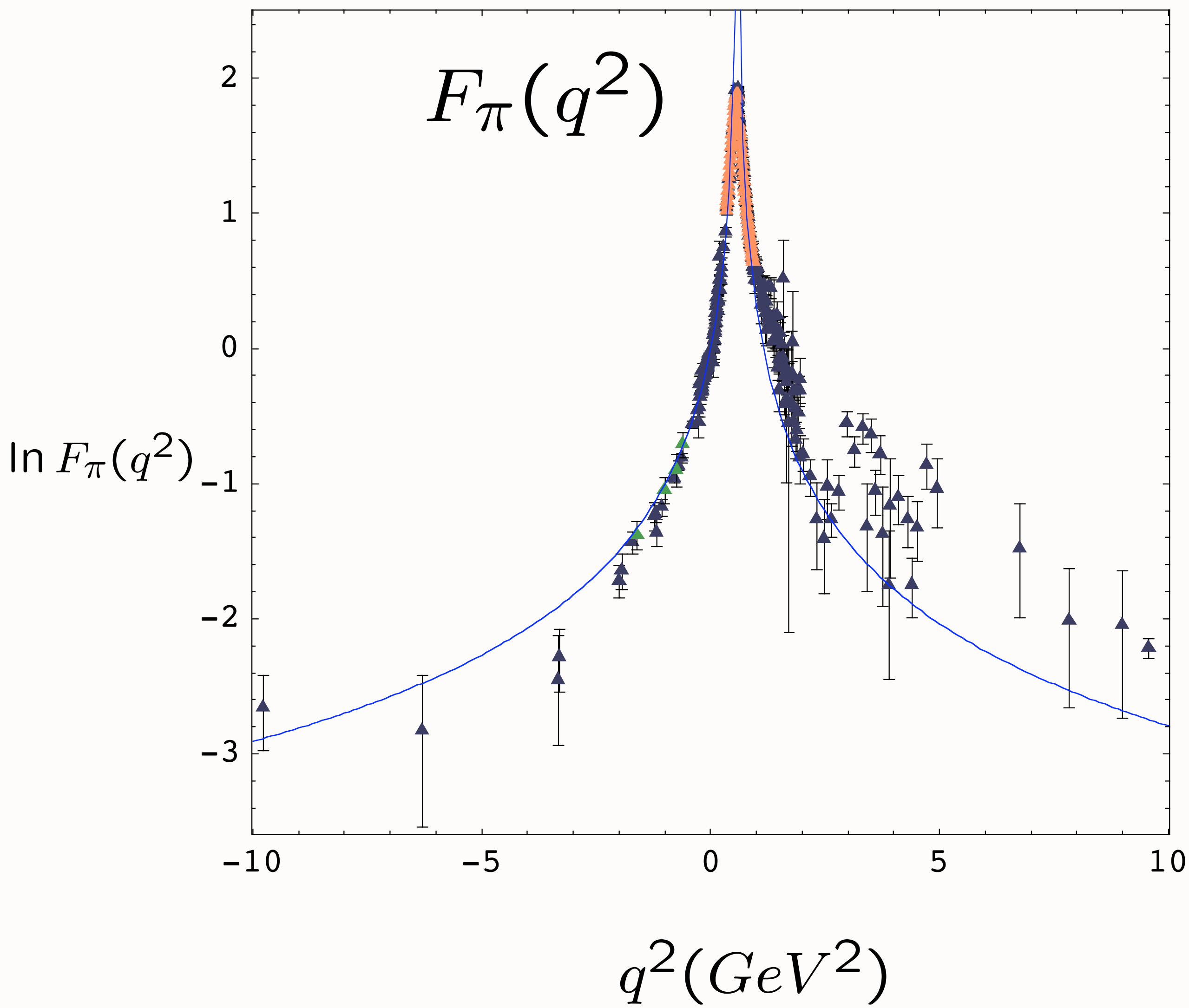
Spacelike and Timelike Pion form factor from AdS/CFT



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Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Spacelike and Timelike Pion form factor from AdS/CFT

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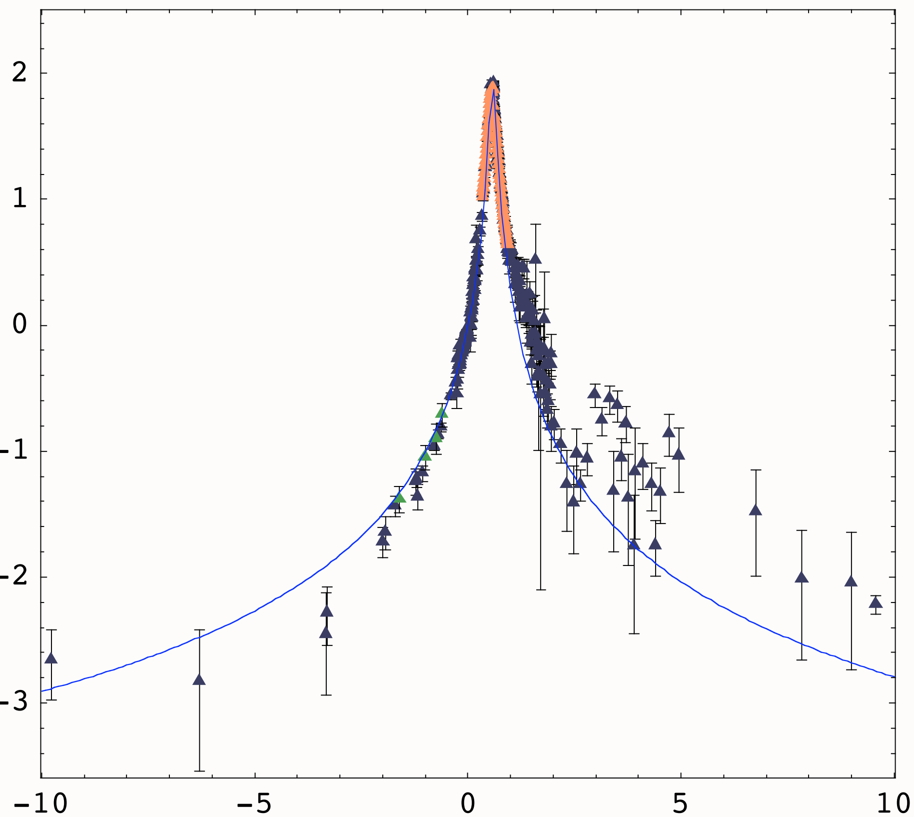
$F_\pi(q^2)$



$\ln F_\pi(q^2)$

$q^2(GeV^2)$

Harmonic Oscillator Confinement

$\kappa = 0.38$ GeV

Analytic continue to timelike momenta and introduce width

$$q^2 \rightarrow q^2 + i\epsilon \rightarrow q^2 + iM\Gamma$$

Fit to height, predict width

$\Gamma_\rho = 111$ MeV

$\Gamma_\rho^{exp} = 150.3 \pm 1.6$ MeV

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu\Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2J_1(zM), \qquad \psi_-(z) = Cz^2J_2(zM),$$

and

$$u(P)_\pm = \frac{1\pm\gamma_5}{2}\, u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

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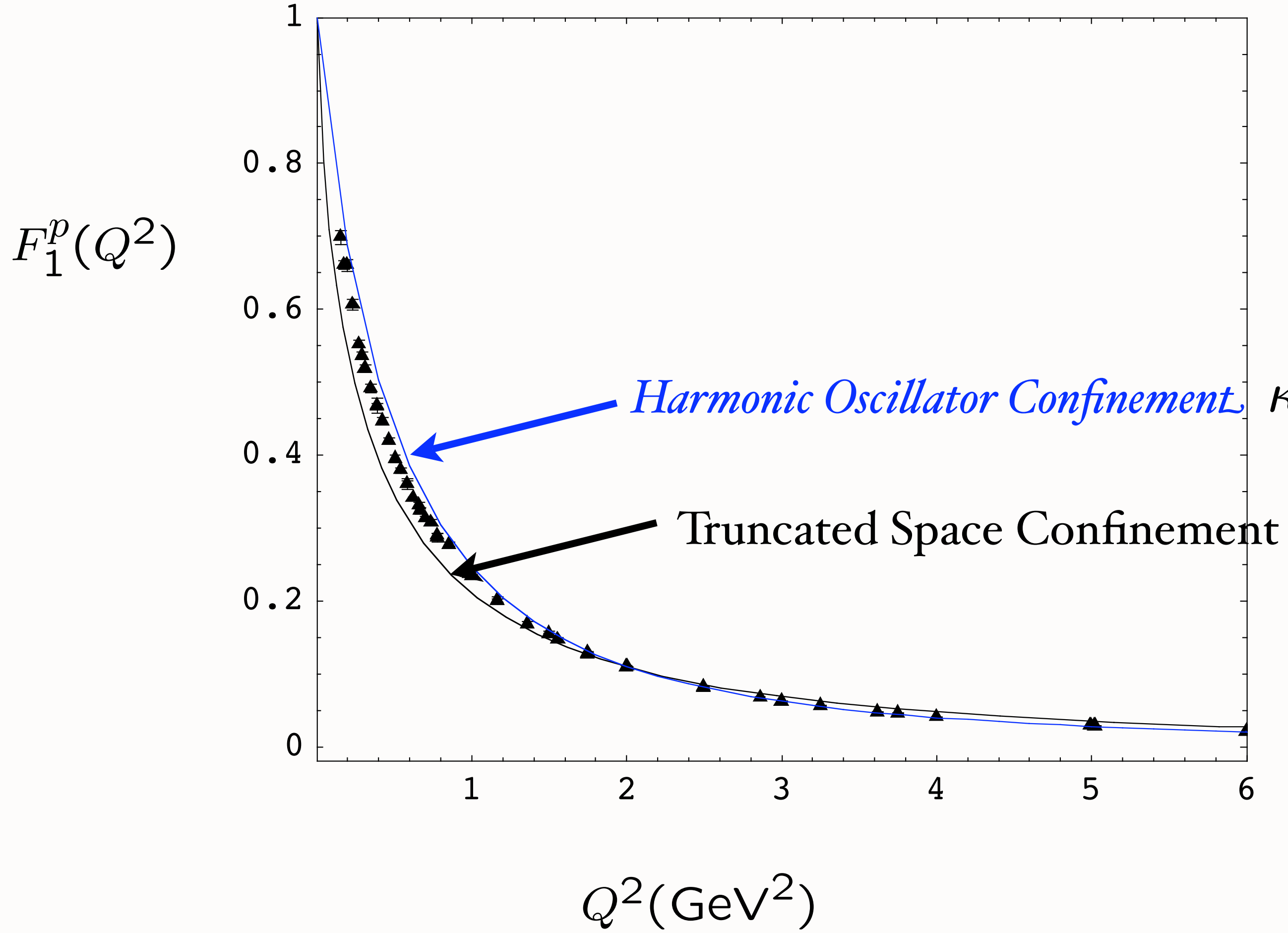
Preliminary



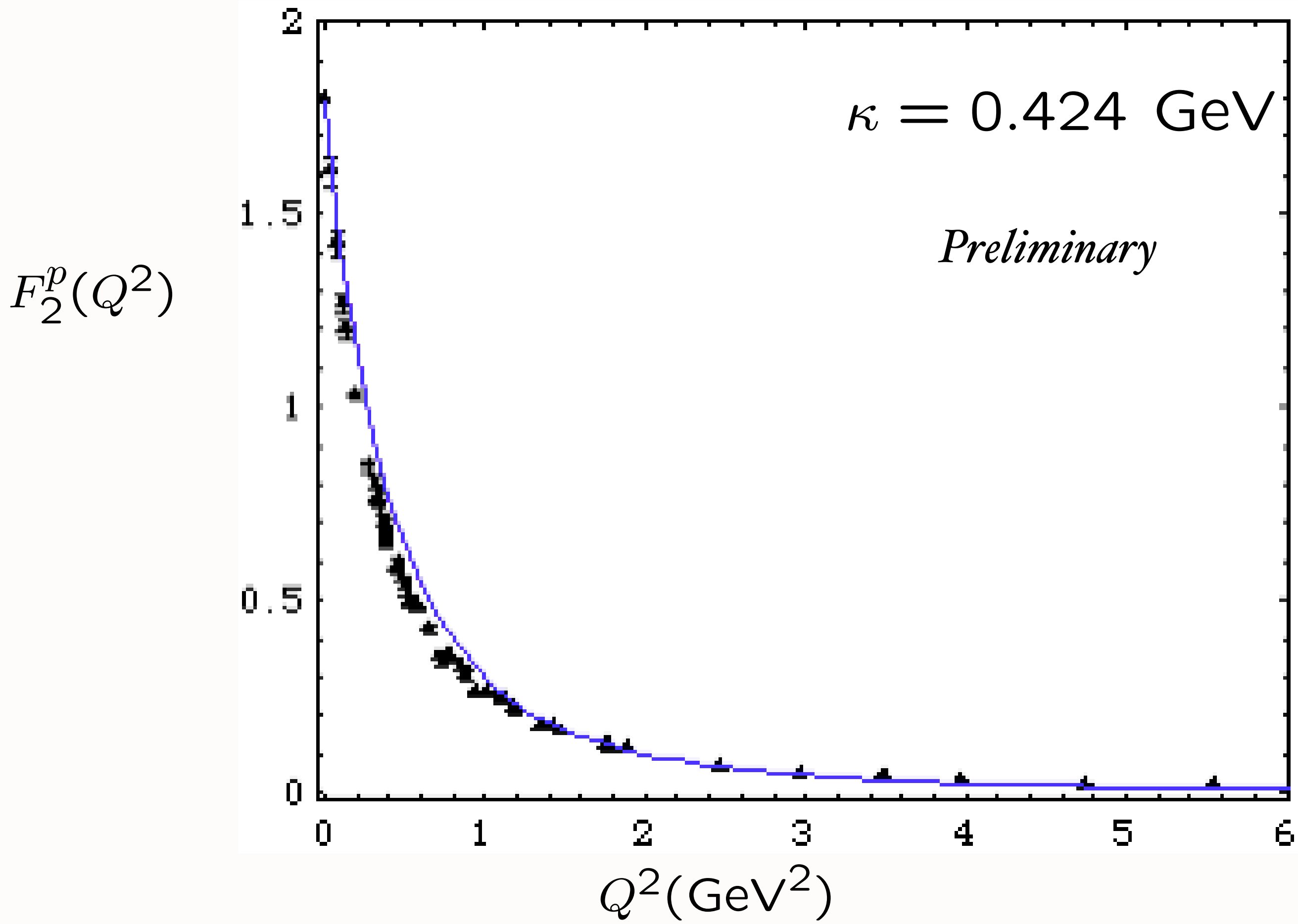
$\kappa = 0.424$ GeV

$\Lambda = 0.2$ GeV

Current modified by metric

$$F_1(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F^{\uparrow}(z) J(Q,z) \Phi_I^{\uparrow}(z)$$

Harmonic Oscillator Confinement



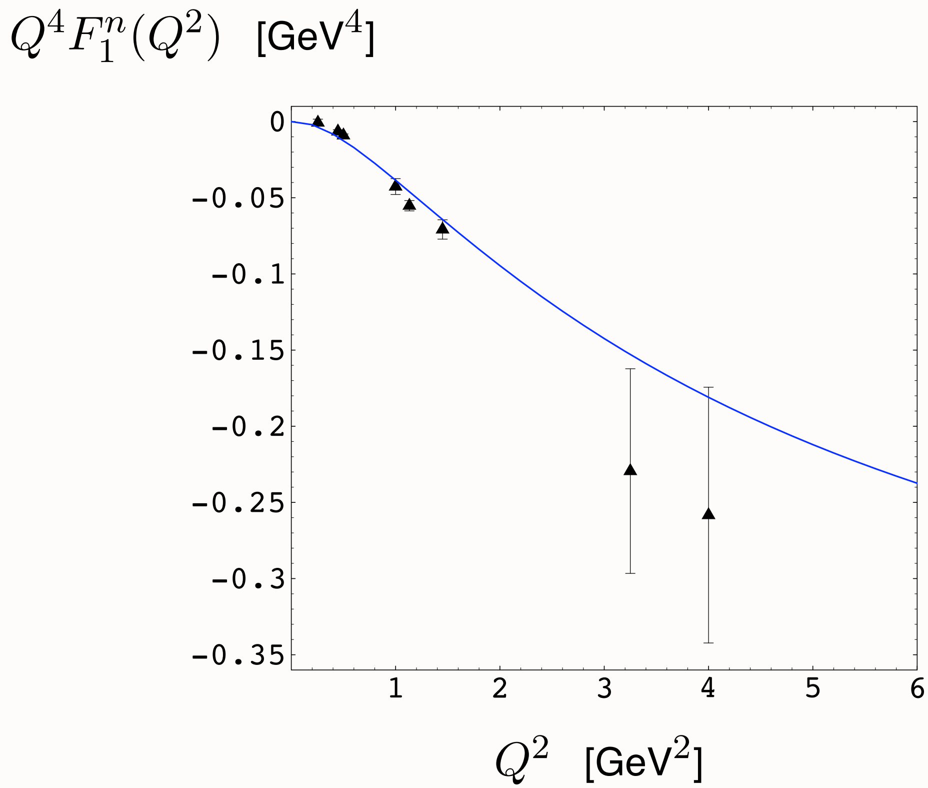
Current modified by metric

$$F_2(Q^2)_{I\to F} = \int \frac{dz}{z^2} \Phi^{\uparrow}_F(z) J(Q,z) \Phi^{\downarrow}_I(z)$$

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

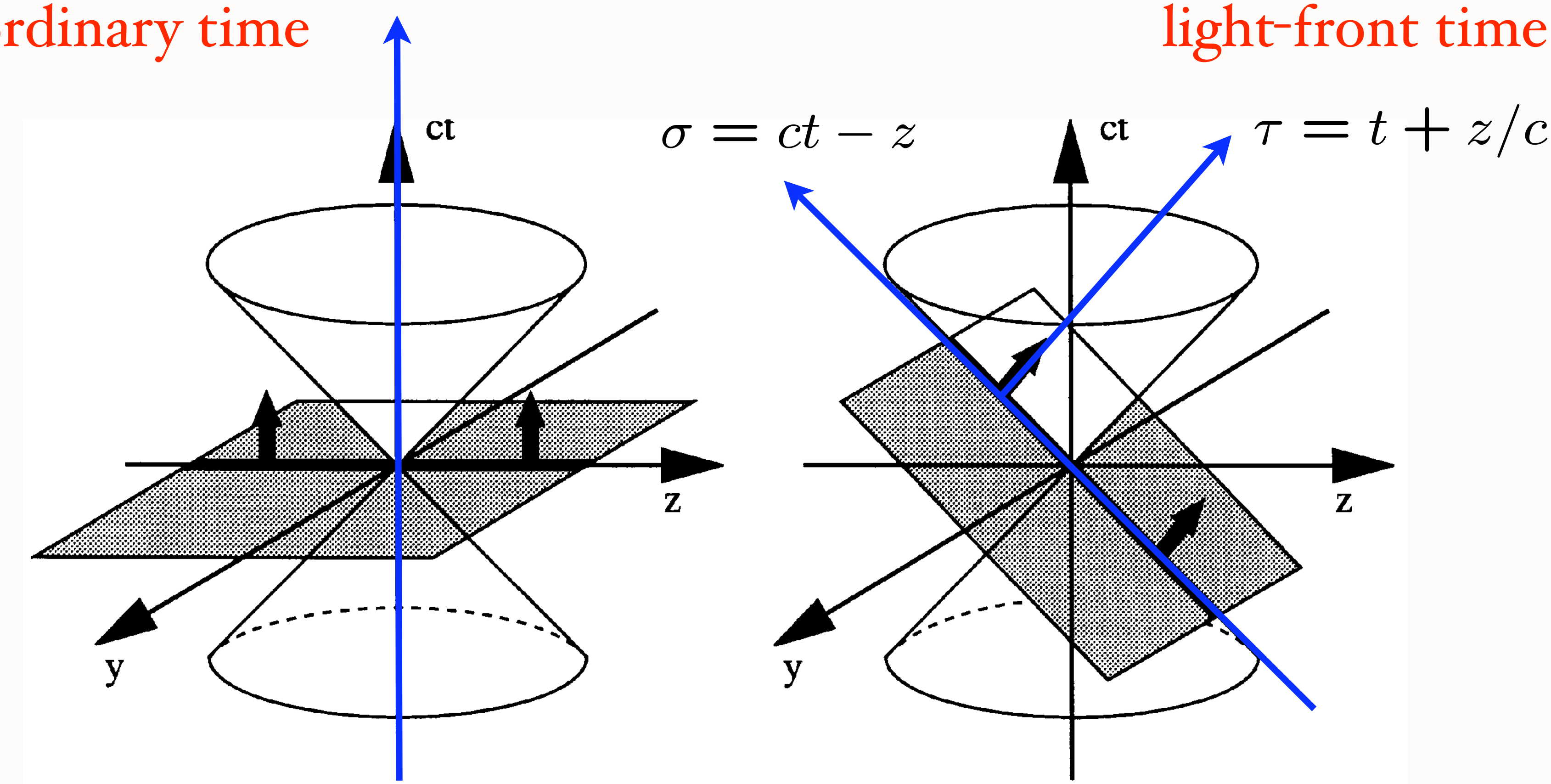


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Dirac's Amazing Idea: The "Front Form"

Evolve in ordinary time

Evolve in light-front time!



Instant Form

Front Form

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

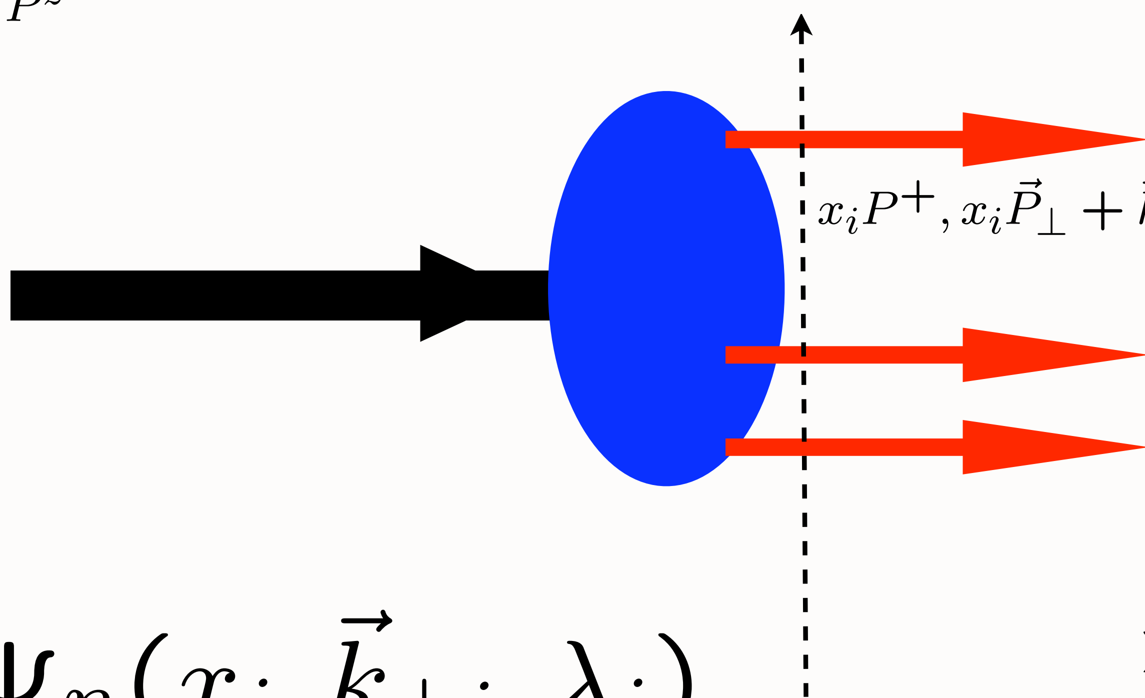
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+, \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

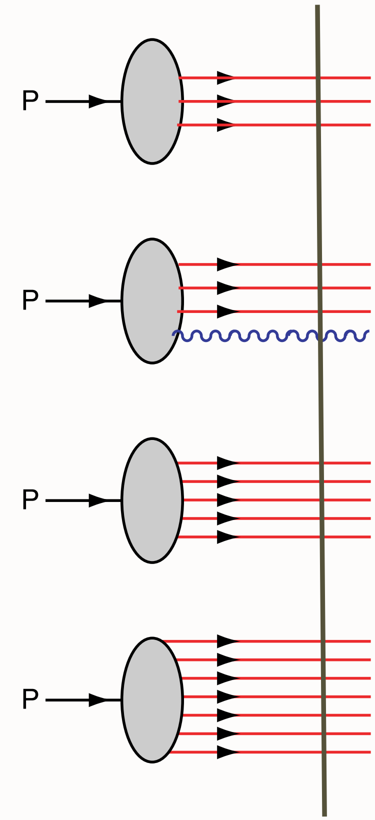
Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

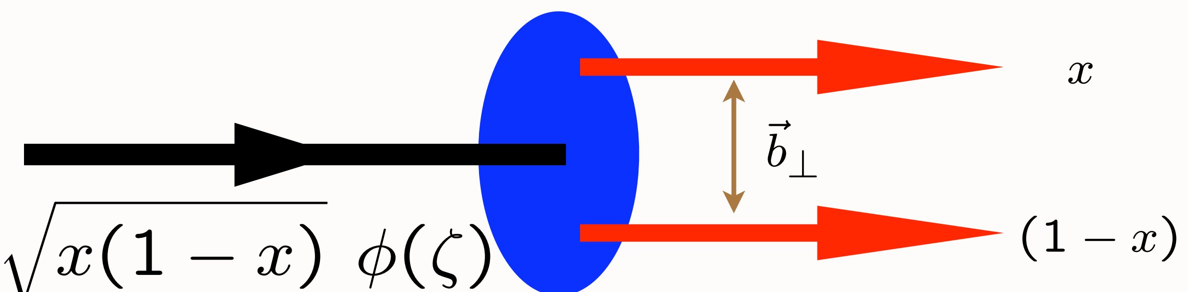


$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Intrinsic gluons, sea quarks, asymmetries

Mapping between LF(3+1) and AdS_5

LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	$\leftrightarrow$	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	$\leftrightarrow$	z



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

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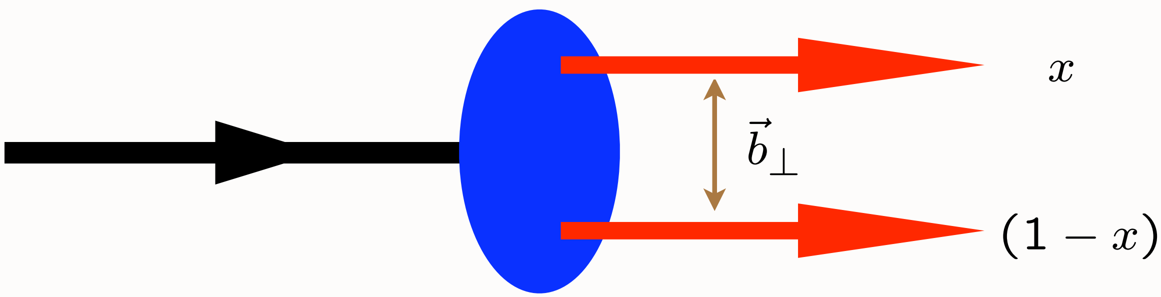
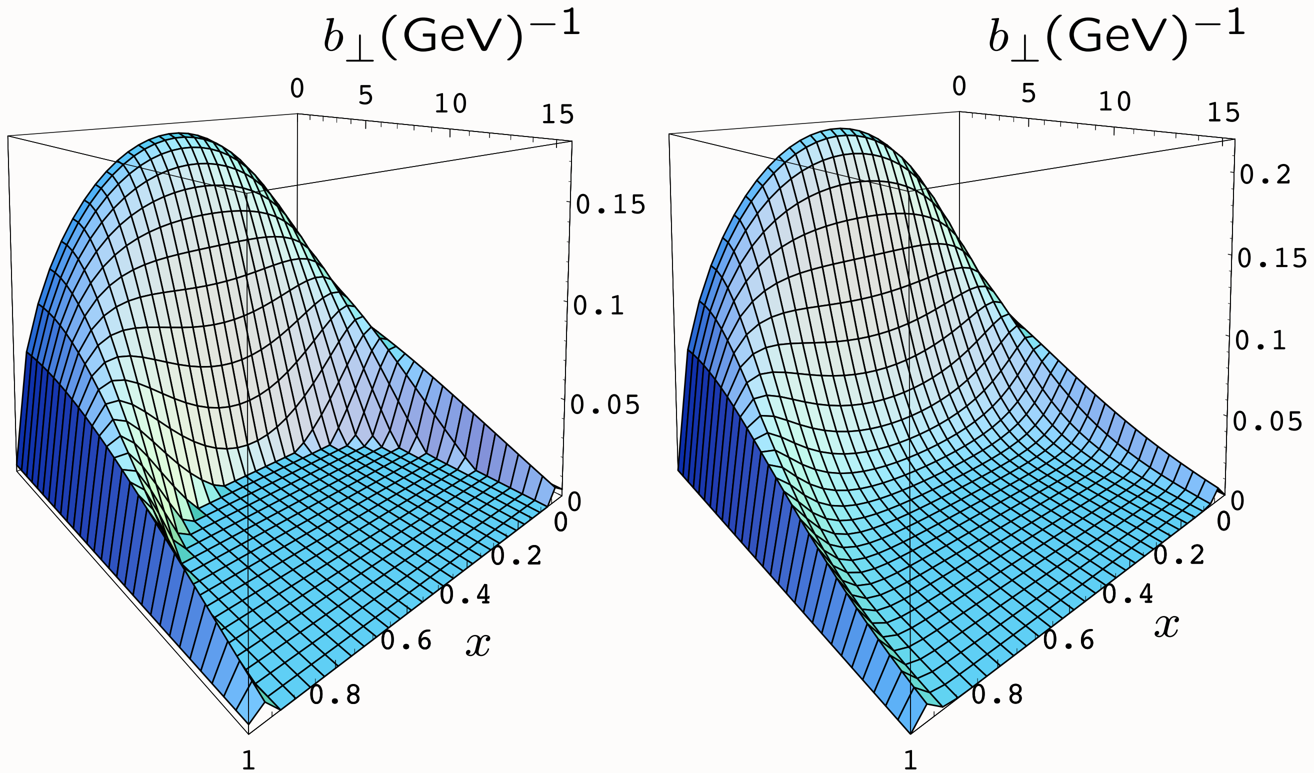
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2} .$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$

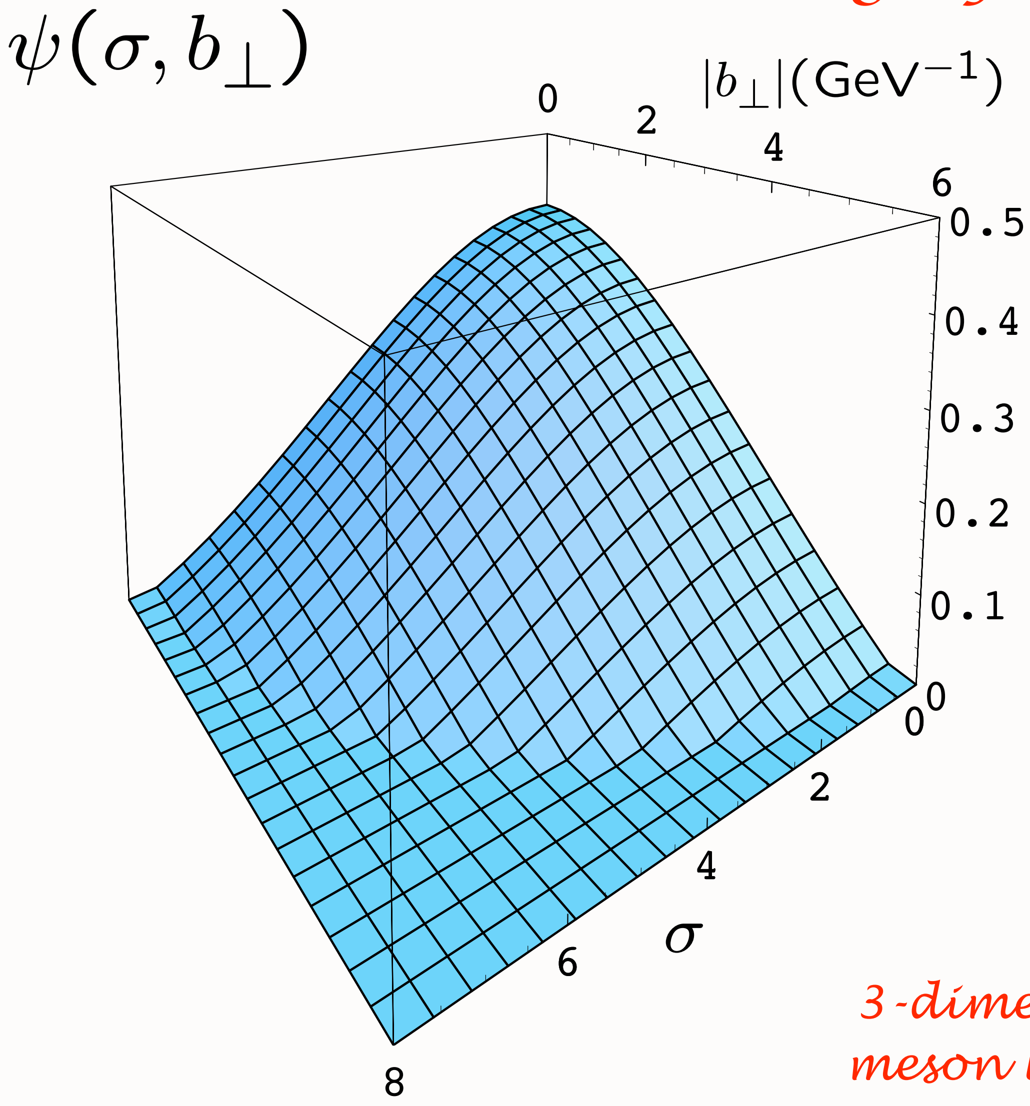


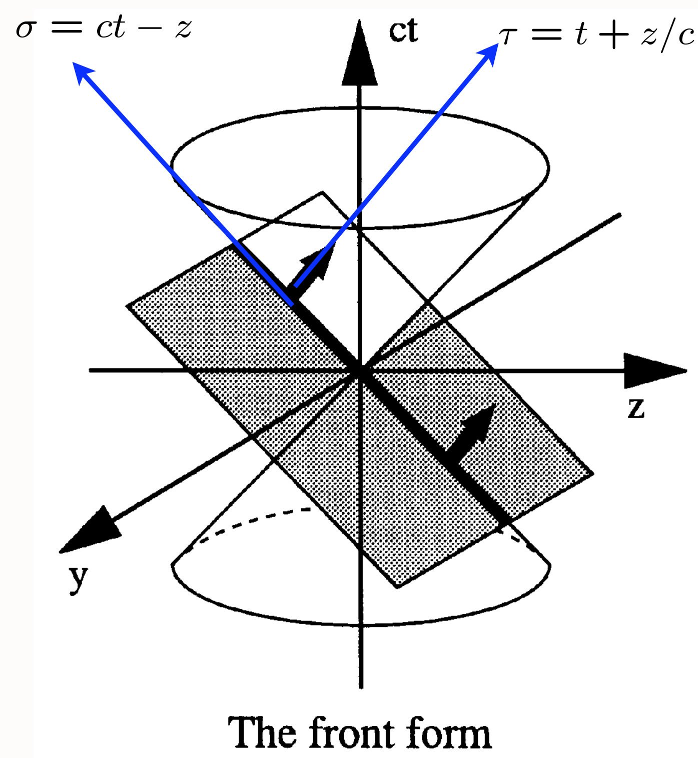
$\Lambda_{QCD} = 0.32$ GeV $\qquad \kappa = 0.76$ GeV

Truncated Space ***Harmonic Oscillator***

AdS/CFT Holographic Model

G. de Teramond
SJB





3-dimensional photograph: meson LFWF at fixed LF Time

Example: Evaluation of QCD Matrix Elements

Pion decay constant f_π defined by the matrix element of EW current J_W^+:

$$\langle 0 \left| \overline{\psi}_u \gamma^+ (1-\gamma_5) \psi_d \right| \pi^- \rangle = i\sqrt{2} P^+ f_\pi ,$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle .$$

Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky '80

Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\rm QCD}}{8 J_1(\beta_{0,1})} = 83.4 \,{\rm Mev},$$

for $\Lambda_{\rm QCD} = 0.2$ GeV (fixed from the pion FF).

Experiment: $f_\pi = 92.4$ Mev.

Pion Decay Constant in HO Model

$$f_\pi = 2\sqrt{N_C}\int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\,\psi_{\overline{q}q/\pi}(x,\vec{k}_\perp)$$

$$= 2\sqrt{N_C}\int_0^1 dx\;\phi(x,Q^2\to\infty),$$

$$\phi(x,Q^2) = \int^{Q^2}\frac{d^2\vec{k}_\perp}{16\pi^3}\psi(x,\vec{k}_\perp)$$

$$\psi_{\overline{q}q/\pi}(x,\vec{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}}\,e^{-\frac{\vec{k}_\perp^2}{2\kappa^2 x(1-x)}}$$

$$f_\pi = \frac{\sqrt{3}\kappa}{8} = 86.6 \text{ MeV}$$

$$\kappa = 0.4 \text{ GeV}.$$

$$f_\pi = 92.4 \text{ MeV}$$

Exp.

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

$$H_{LC}^{QCD} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}_\perp^2$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

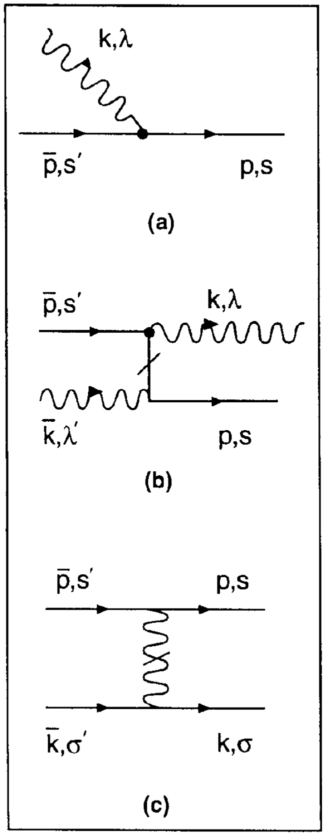
$$\sum_{n,\lambda_i} \int \ [dx_i\ d^2\vec{k}_{\perp i}]\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\times\ |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i\ d^2\vec{k}_{\perp i}]\ |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

Light-Front QCD Heisenberg Equation

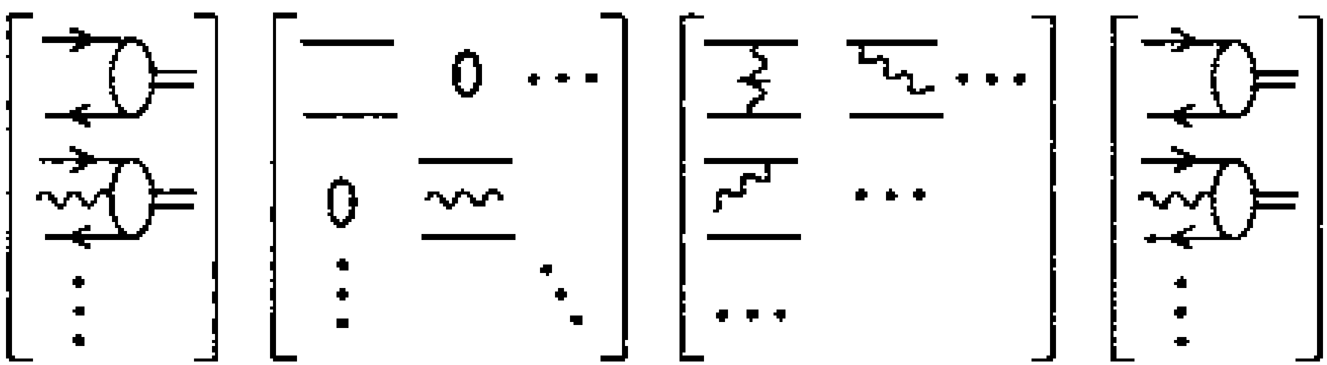
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\;|\Psi_h\rangle$$



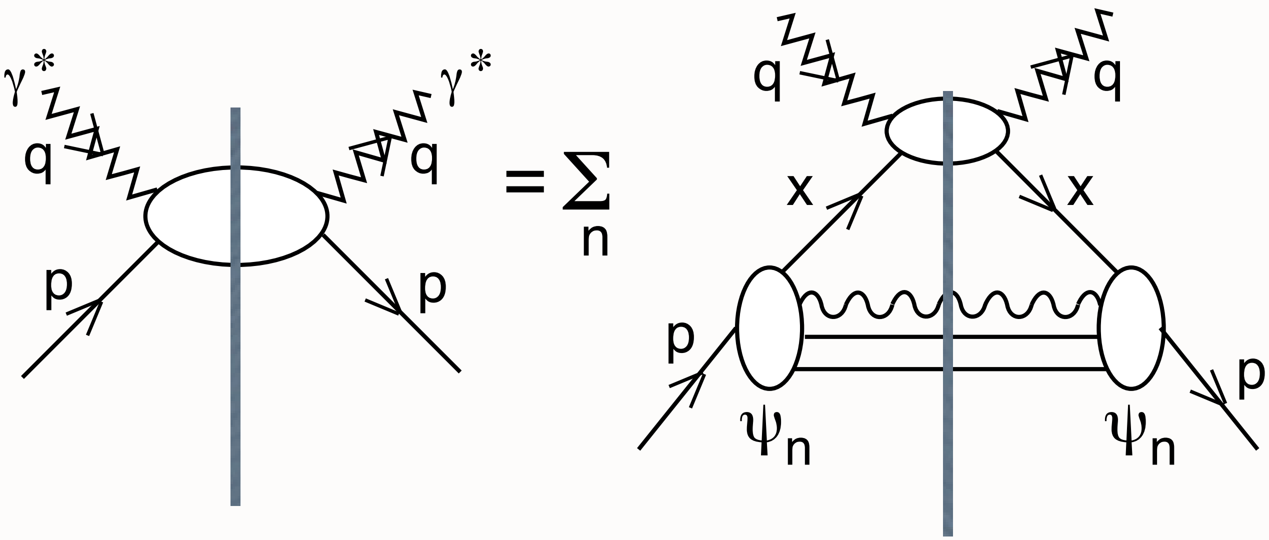
n	Sector	1 qq̄	2 gg	3 qq̄ g	4 qq̄ qq̄	5 gg g	6 qq̄ gg	7 qq̄ qq̄ g	8 qq̄ qq̄ qq̄	9 gg gg	10 qq̄ gg g	11 qq̄ qq̄ gg	12 qq̄ qq̄ qq̄ g	13 qq̄ qq̄ qq̄ qq̄
1	qq̄					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	qq̄ g								•	•		•	•	•
4	qq̄ qq̄		•			•				•	•		•	•
5	gg g	•			•			•	•			•	•	•
6	qq̄ gg								•				•	•
7	qq̄ qq̄ g	•	•			•				•				•
8	qq̄ qq̄ qq̄	•	•	•		•	•			•	•			
9	gg gg	•		•	•			•	•			•	•	•
10	qq̄ gg g	•	•		•				•				•	•
11	qq̄ qq̄ gg	•	•	•		•				•				•
12	qq̄ qq̄ qq̄ g	•	•	•	•	•	•			•	•			
13	qq̄ qq̄ qq̄ qq̄	•	•	•	•	•	•	•		•	•	•		

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



Deep Inelastic Lepton Proton Scattering

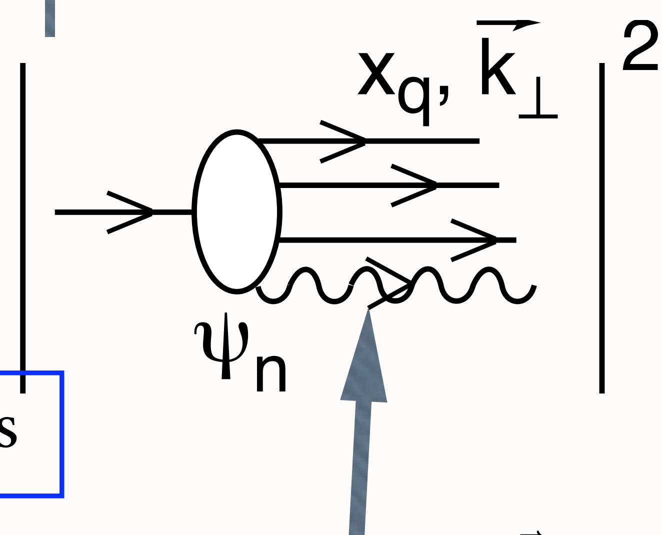


Imaginary Part of
Forward Virtual Compton Amplitude

$$q(x,Q^2) = \sum_n \int^{k_\perp^2 \le Q^2 \perp} d^2k_\perp \, |\Psi_n(x,k_\perp)|^2$$

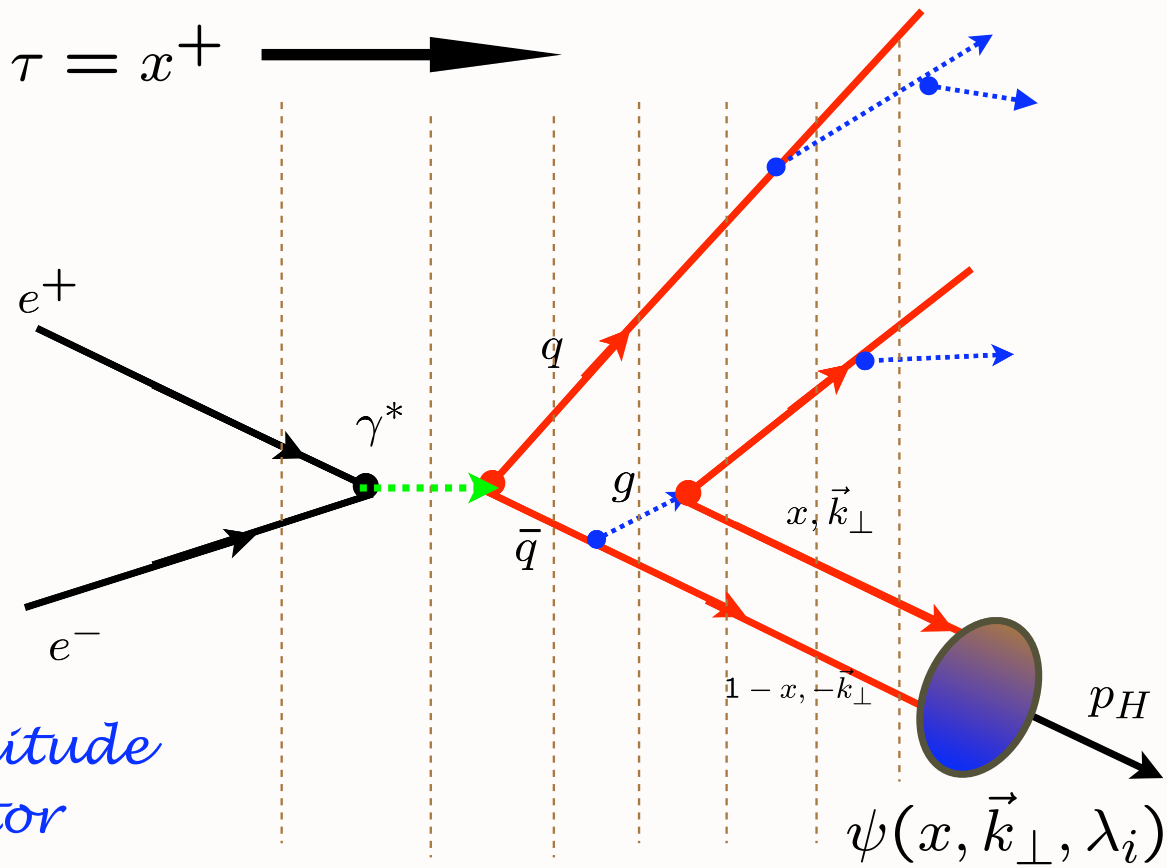
$x = x_q$

All spin, flavor distributions



Light-Front Wave Functions $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

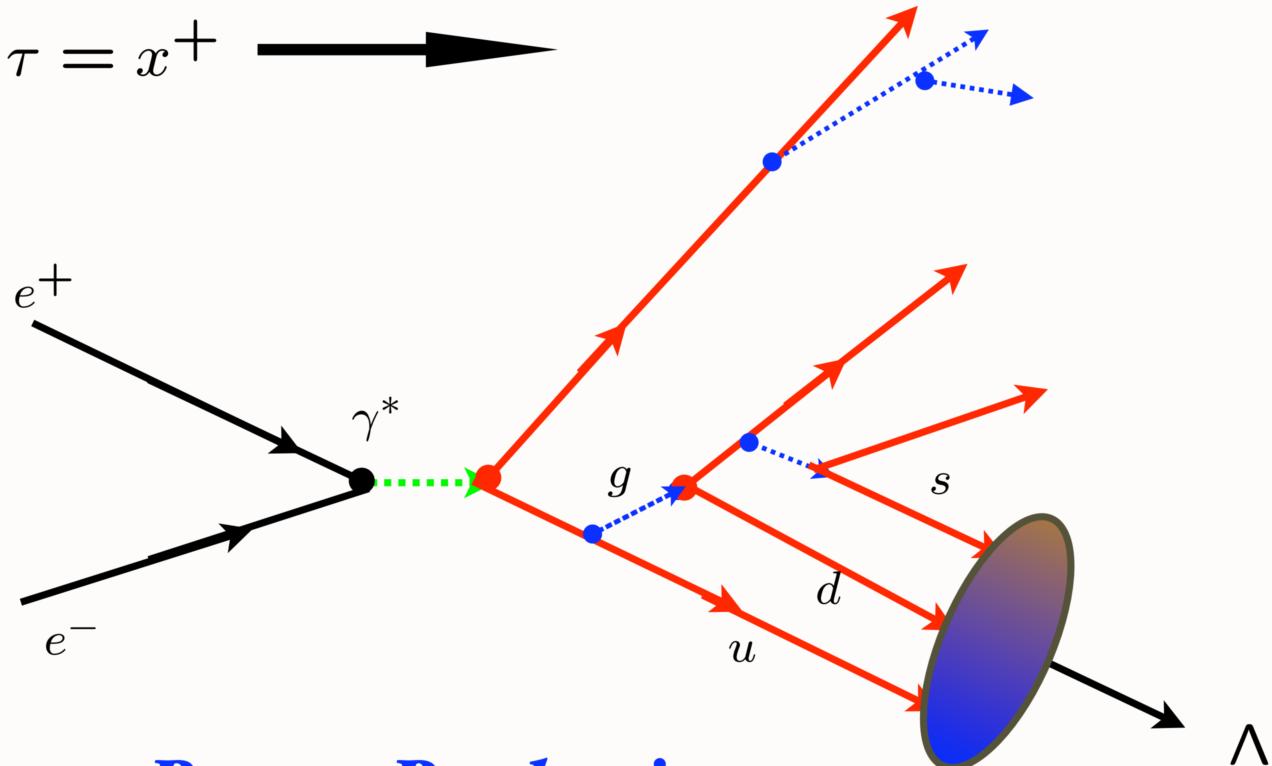
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Baryon Production

$\psi(x, \vec{k}_\perp, \lambda_i)$

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions

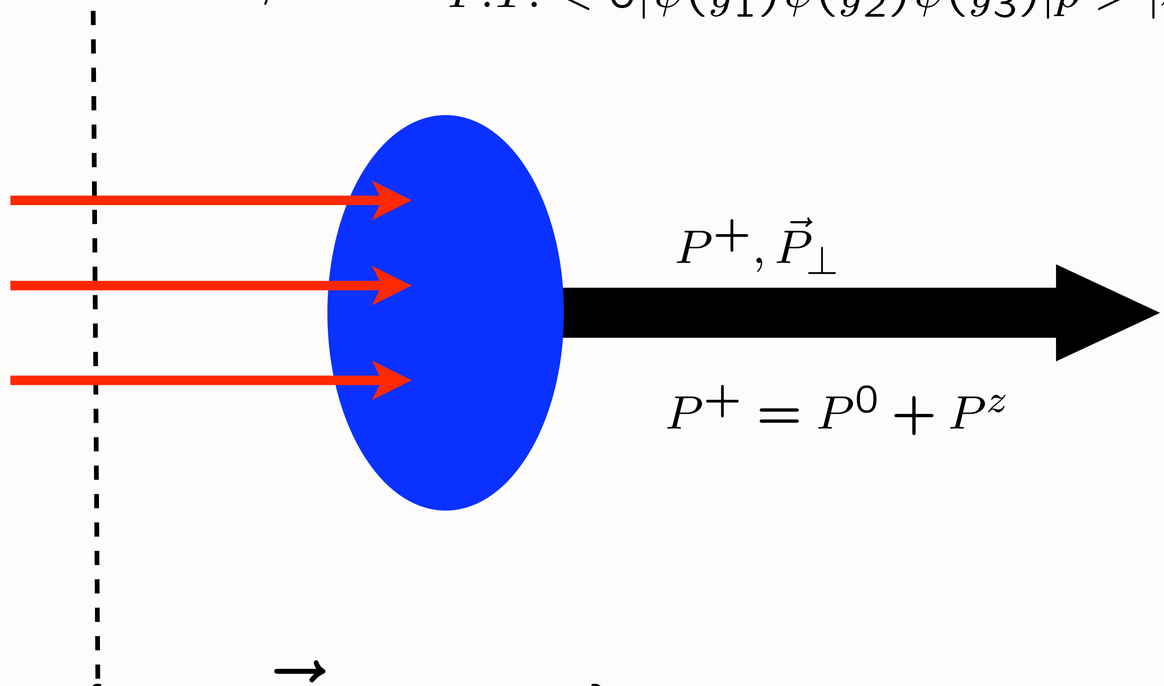
Fixed $\tau = t + z/c$

$F.T. < 0|\psi(y_1)\psi(y_2)\psi(y_3)|p > |_{\tau_i=0}$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$



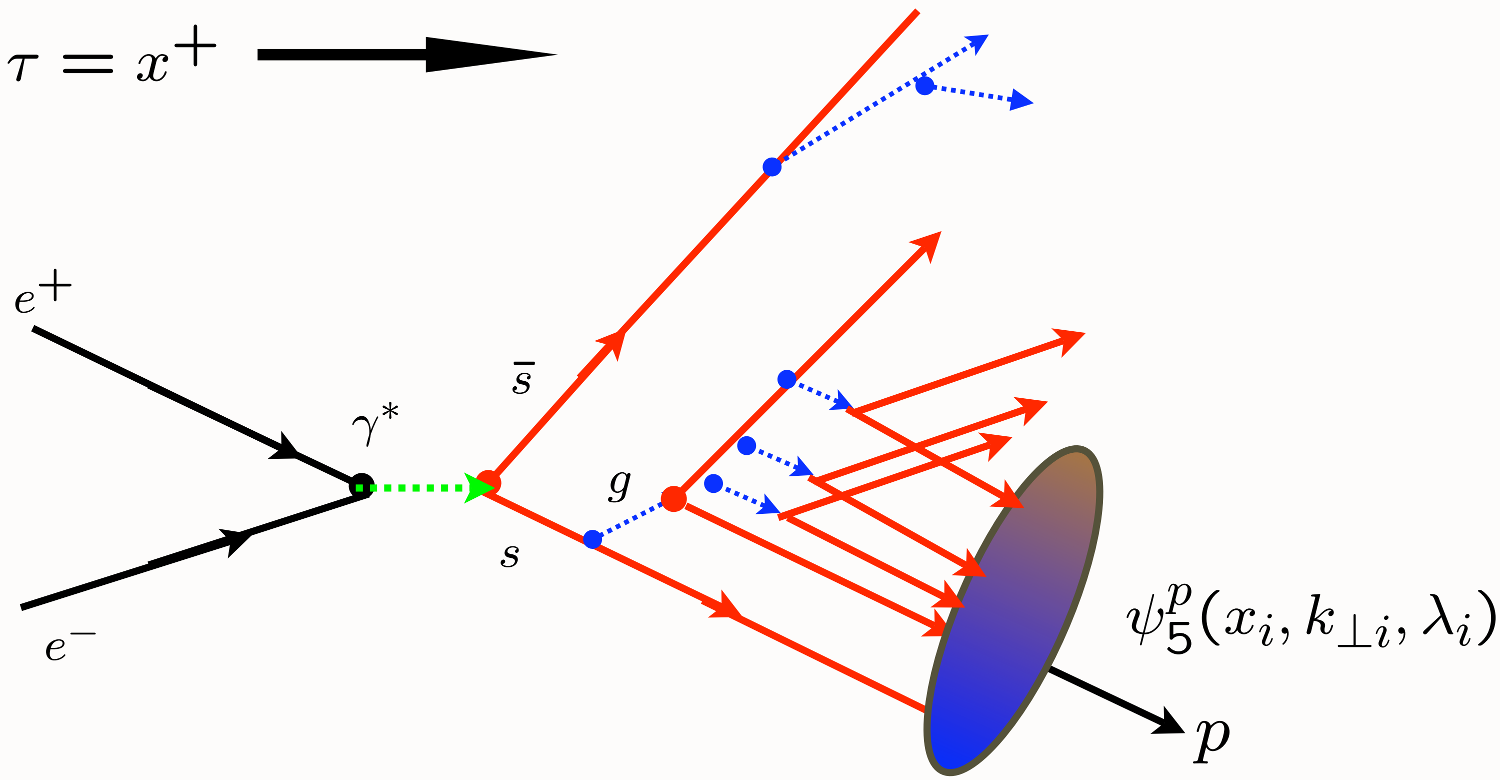
$P^+, \vec{P}_\perp$

$P^+ = P^0 + P^z$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



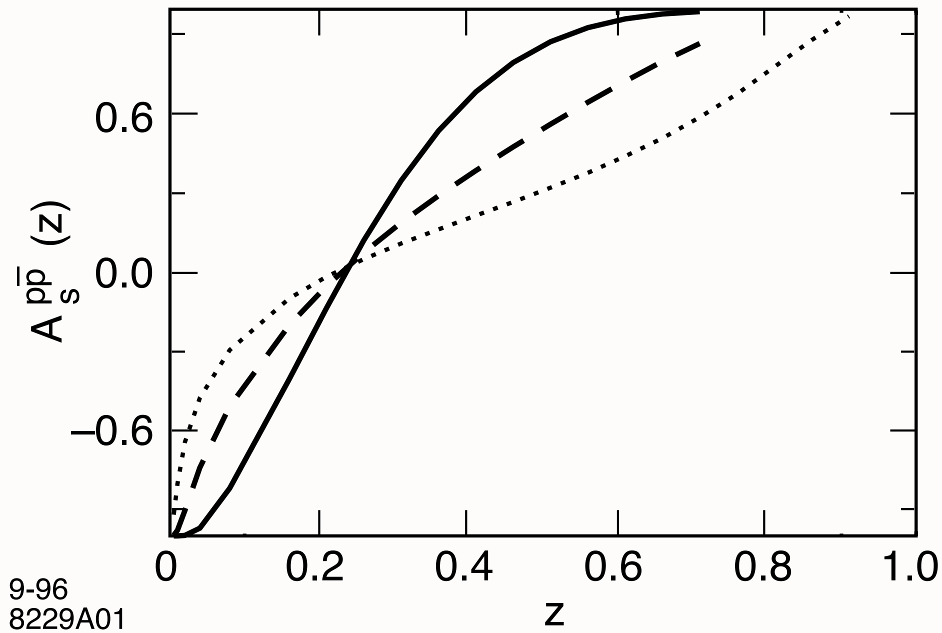
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\rightarrow p}(z) \neq D_{s\rightarrow \bar{p}}(z)$

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$$

B-Q Ma, sjb



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to \bar{p}}(z)}{D_{s\to p}(z) + D_{s\to \bar{p}}(z)}$$

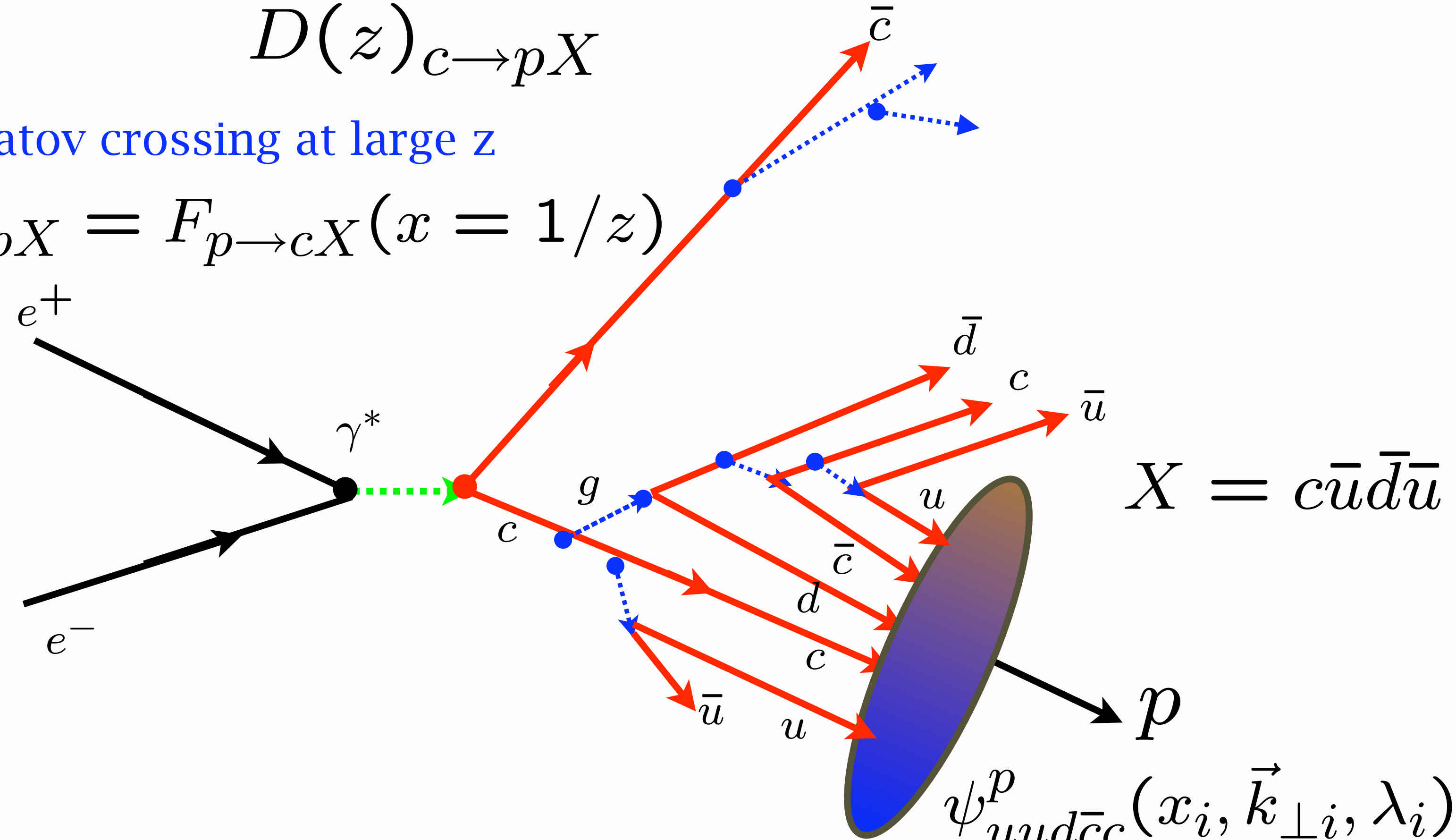
Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

Timelike Test of Charm Distribution in Proton

$$D(z)_{c\to pX}$$

Gribov-Lipatov crossing at large z

$$zD(z)_{c\to pX} = F_{p\to cX}(x = 1/z)$$



Intrinsic charm model:
predict proton at same rapidity as charm quark: high z

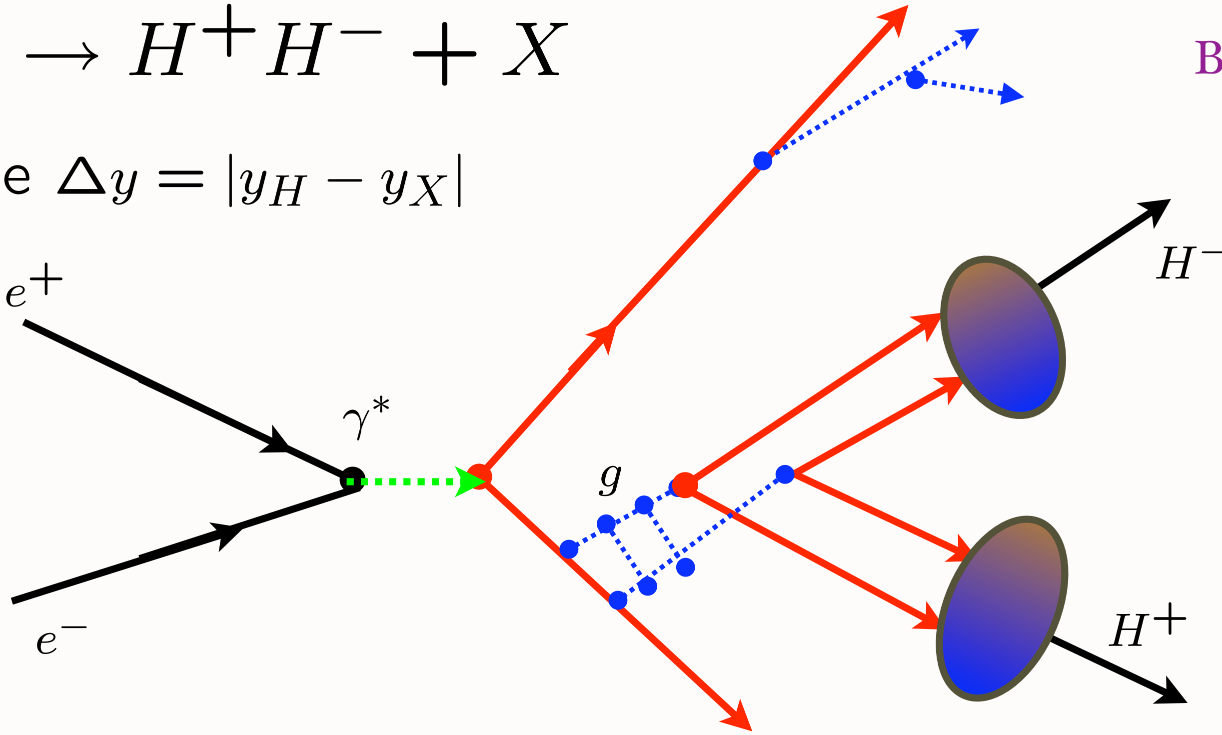
$$z_i \propto m_{\perp i} = \sqrt{m_i^2 + k_\perp^2}$$

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Large $\Delta y = |y_H - y_X|$

Bjorken, Lu, sjb
Kopeliovich, Schmidt, sjb



Timelike Pomeron   **C= + Gluonium Trajectory**

Large Rapidity Gap Events

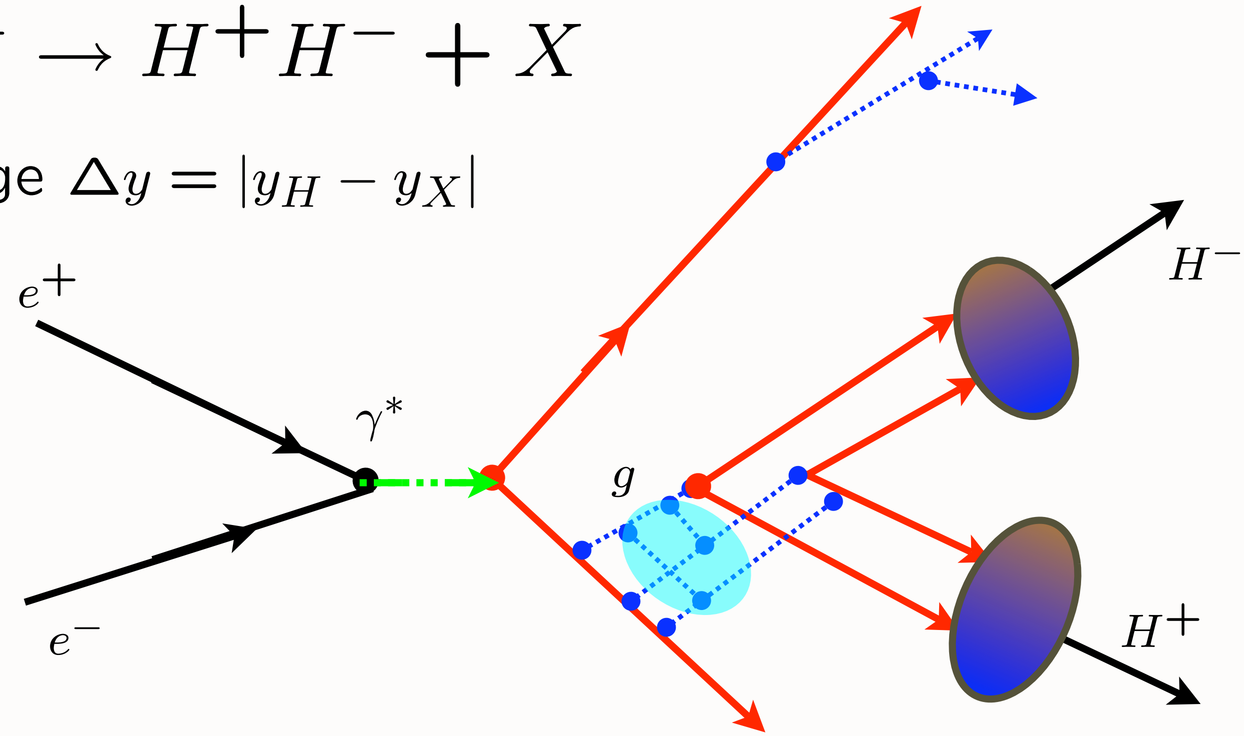
Crossing analog of Diffractive DIS $eH \to eH + X$

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Kopeliovich, Schmidt, sjb

Large $\Delta y = |y_H - y_X|$



Timelike Odderon

Large Rapidity Gap Events

C= - Gluonium Trajectory

H^+H^- asymmetry from Odderon-Pomeron interference

Angular Momentum on the Light-Front

A^+=0 gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment requires
Nonzero orbital angular momentum

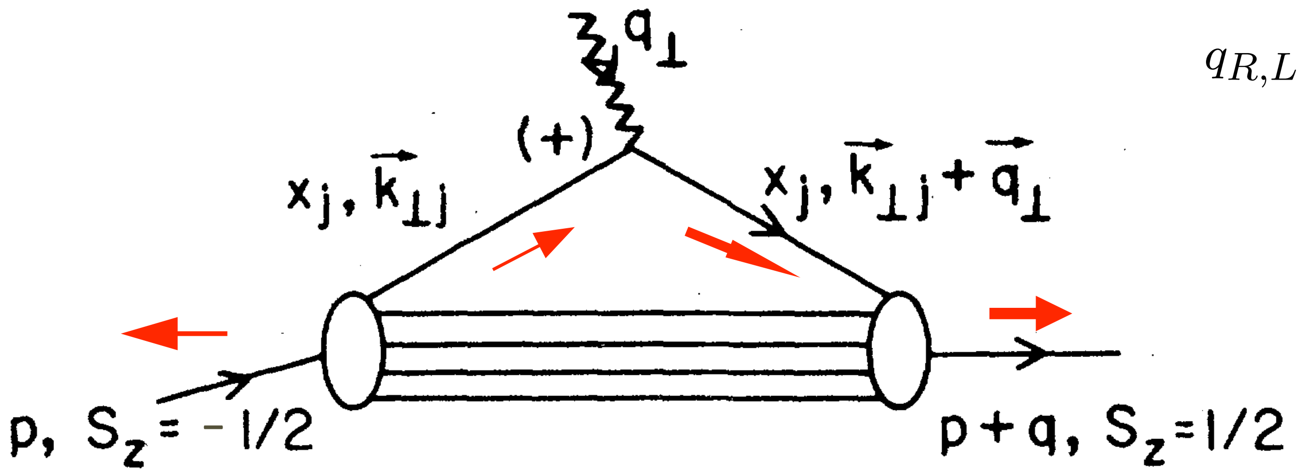
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \, \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



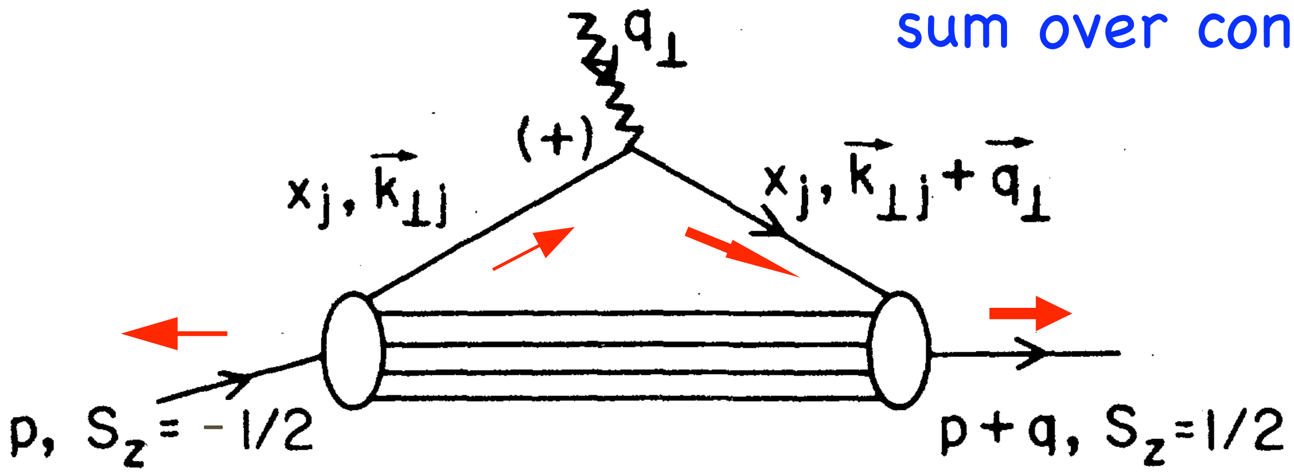
Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Okun et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents



Hwang, Schmidt, sjb;
Holstein et al

B(0) = 0

Each Fock State

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle =$$

$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

$$\kappa = \frac{e}{2M}\left[F_2(0)\right] , \qquad d = \frac{e}{M}\left[F_3(0)\right]$$

We will find a close connection between κ and d, as long anticipated. [Bigi, Uralstev, NPB 1991]

Gardner, Hwang, sjb,

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame,
imply ($q^{R/L} \equiv q^1 \pm iq^2$):

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big] ,$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big] ,$$

$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Gardner, Hwang, sjb,

CP-violating phase

$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$\begin{aligned} F(Q^2)_{I \to F} &= R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}} \, e^{(3+2\sigma)A(z)} \Phi_F(z) \, J(Q,z) \, \Phi_I(z) \\ &\simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}} \, \Phi_F(z) \, J(Q,z) \, \Phi_I(z), \end{aligned}$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0) = 1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ \cdot x} J(Q,z)$. Thus:

$$\lim_{Q \to 0} J(Q,z) = \lim_{z \to 0} J(Q,z) = 1.$$

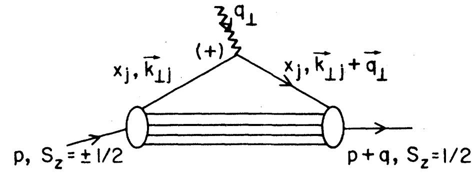
- Solution to the AdS Wave equation with boundary conditions at $Q = 0$ and $z \to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

Holographic Model for QCD Light-Front Wavefunctions

- Drell-Yan-West form factor



$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp) \, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp \, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp \, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Identical DYW and AdS5 Formulae: Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)}$$

the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

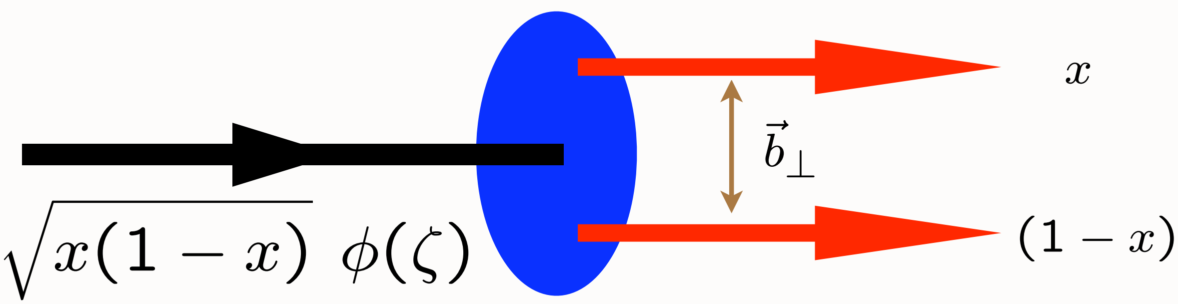
$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

Mapping between LF(3+1) and AdS$_5$

LF(3+1) | **AdS$_5$**

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

G. de Teramond and sjb

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

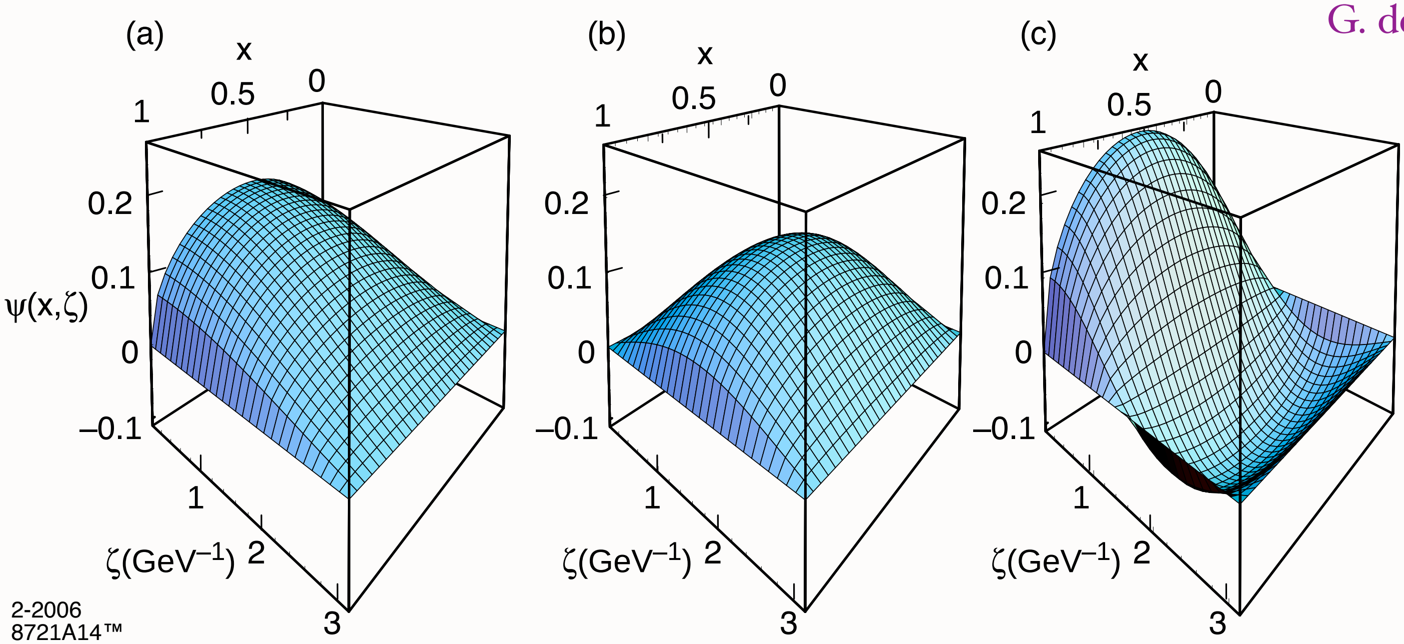
General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

AdS/CFT Prediction for Meson LFWF

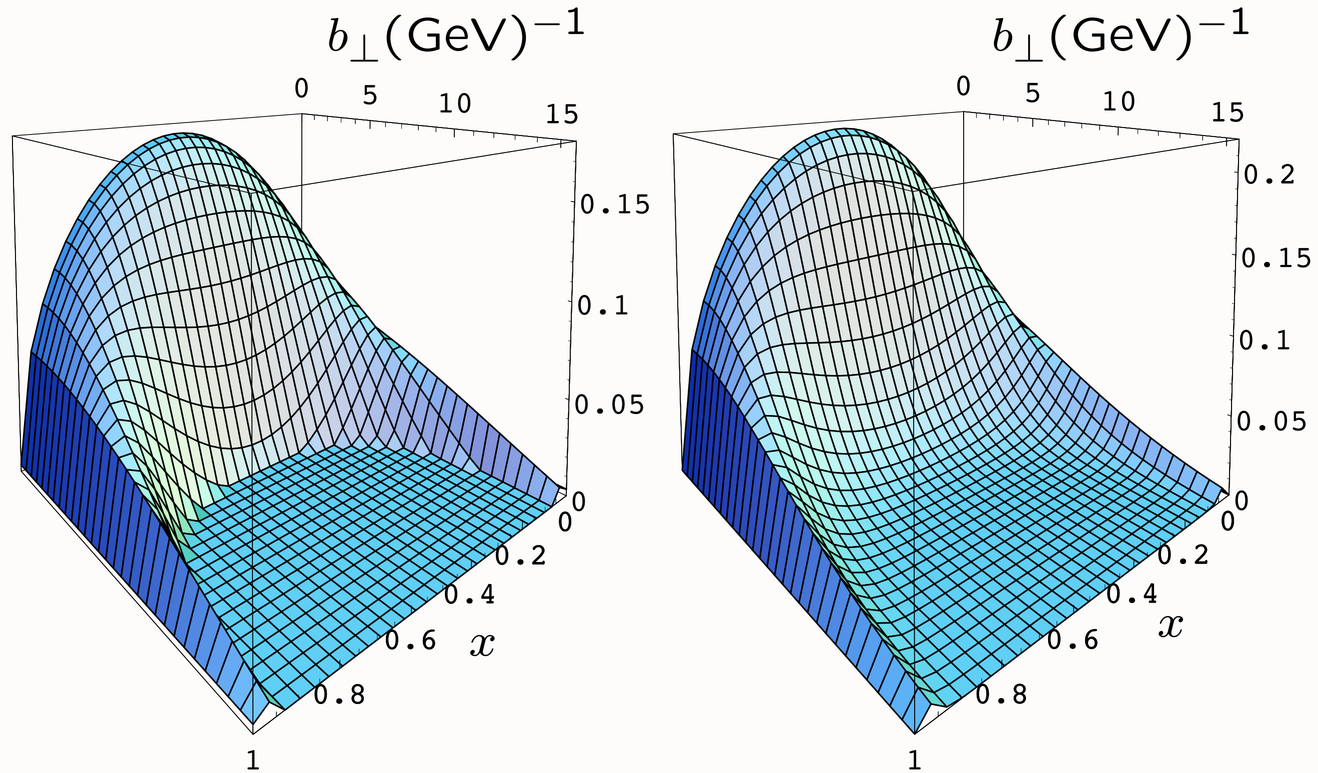
G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L=0,\ k=1$; (b) first orbital exited state $L=1,\ k=1$; (c) first radial exited state $L=0,\ k=2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{\mathrm{QCD}} = 0.32\ \mathrm{GeV}$ $\kappa = 0.76\ \mathrm{GeV}$

Truncated Space ***Harmonic Oscillator***

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

Karch, et al.

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^\nu_{LF}\phi_\nu = \mathcal{M}^2_\nu\phi_\nu$$ **Bilinear** $$H^\nu_{LF} = \Pi^\dagger_\nu \Pi_\nu,$$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $$b_\nu^\dagger = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad\qquad b_\nu^\dagger b_\nu = b_{\nu+1} b_{\nu+1}^\dagger$$

Ladder Operator $$b_\nu^\dagger|\nu\rangle = c_\nu|\nu+1\rangle$$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

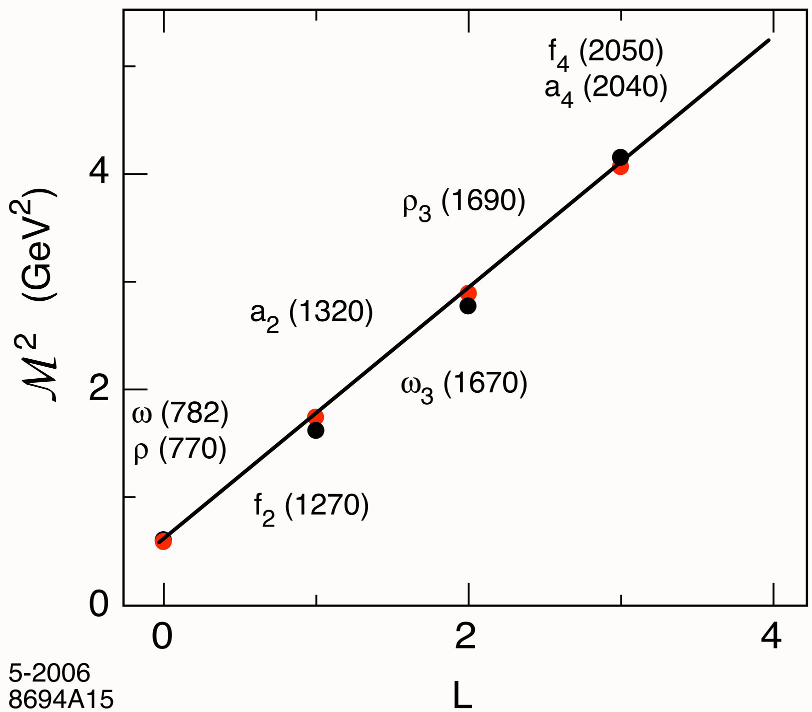
defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

Subtract Vacuum Energy

$$\mathcal{M}^2 \to \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$



$J = L + 1$ vector meson Regge trajectory for $\kappa \simeq 0.54$ GeV

Holographic Truncated Space Model: Baryons

$$\alpha \Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta);$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_\zeta\right)$$

$$\alpha^\dagger = \alpha, \quad \alpha^2 = 1,$$
$$\gamma_\zeta^\dagger = \gamma_\zeta, \quad \gamma_\zeta^2 = 1,$$
$$\{\alpha, \gamma_\zeta\} = 0.$$

$$\begin{pmatrix} 0 & -\frac{d}{d\zeta} \\ \frac{d}{d\zeta} & 0 \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} - \begin{pmatrix} 0 & \frac{\nu+\frac{1}{2}}{\zeta} \\ \frac{\nu+\frac{1}{2}}{\zeta} & 0 \end{pmatrix}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \mathcal{M}\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix},$$

Frame-Independent LF Dirac Equation

Holographic Harmonic Oscillator Model: Baryons

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0$$

Frame-Independent LF Dirac Equation

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)$$

$$\Pi_\nu^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 + \kappa^2\zeta\gamma_5\right)$$

Coupled Equations

$$\begin{pmatrix} 0 & -\frac{d}{d\zeta} \\ \frac{d}{d\zeta} & 0 \end{pmatrix}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} - \begin{pmatrix} 0 & \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \\ \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta & 0 \end{pmatrix}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} = \mathcal{M}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}$$

$$-\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - \kappa^2\zeta\psi_- = \mathcal{M}\psi_+,$$

$$\frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - \kappa^2\zeta\psi_+ = \mathcal{M}\psi_-.$$

HO due to Linear Potential!

$$V = -\beta\kappa^2\zeta$$

Holographic Harmonic Oscillator Model: Baryons

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

$$\begin{aligned}\Pi_\nu(\zeta) &= -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)\\ \Pi_\nu^\dagger(\zeta) &= -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 + \kappa^2\zeta\gamma_5\right)\end{aligned}$$

$$\left(H_{LF} - \mathcal{M}^2\right)\psi(\zeta) = 0, \qquad H_{LF} = \Pi^\dagger\Pi$$

Uncoupled Schrodinger Equations **Harmonic Oscillator Potential!**

$$\begin{aligned}\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2(\nu+1)\kappa^2 + \mathcal{M}^2\right)\psi_+(\zeta) &= 0,\\ \left(\frac{d^2}{d\zeta^2} + \frac{1-4(\nu+1)^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\nu\kappa^2 + \mathcal{M}^2\right)\psi_-(\zeta) &= 0,\end{aligned}$$

Solution

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^{\nu+1}(\kappa^2\zeta^2),\end{aligned}$$

Same eigenvalue!

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1)$$

Holographic Baryon Spectrum

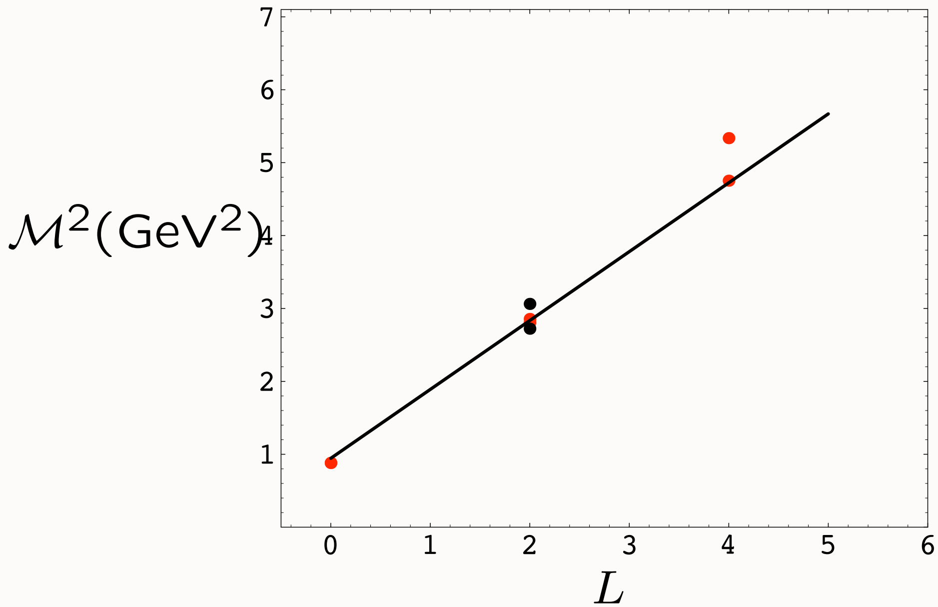
$$\psi(\zeta) = \kappa^{2+L}\sqrt{\frac{n!}{(n+L+2)!}}\zeta^{\frac{3}{2}+L}e^{-\kappa^2\zeta^2/2}\left[L_n^{L+1}\left(\kappa^2\zeta^2\right)u_+ + \frac{\kappa\zeta}{\sqrt{n+L+2}}L_n^{L+2}\left(\kappa^2\zeta^2\right)u_-\right.$$

$$\mathcal{M}^2 = 4\kappa^2(n+L+2).$$

Vacuum Energy Shift?

$$\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n+L+1).$$



$J = L + 1/2$ Regge trajectory

$\kappa \simeq 0.49$ GeV

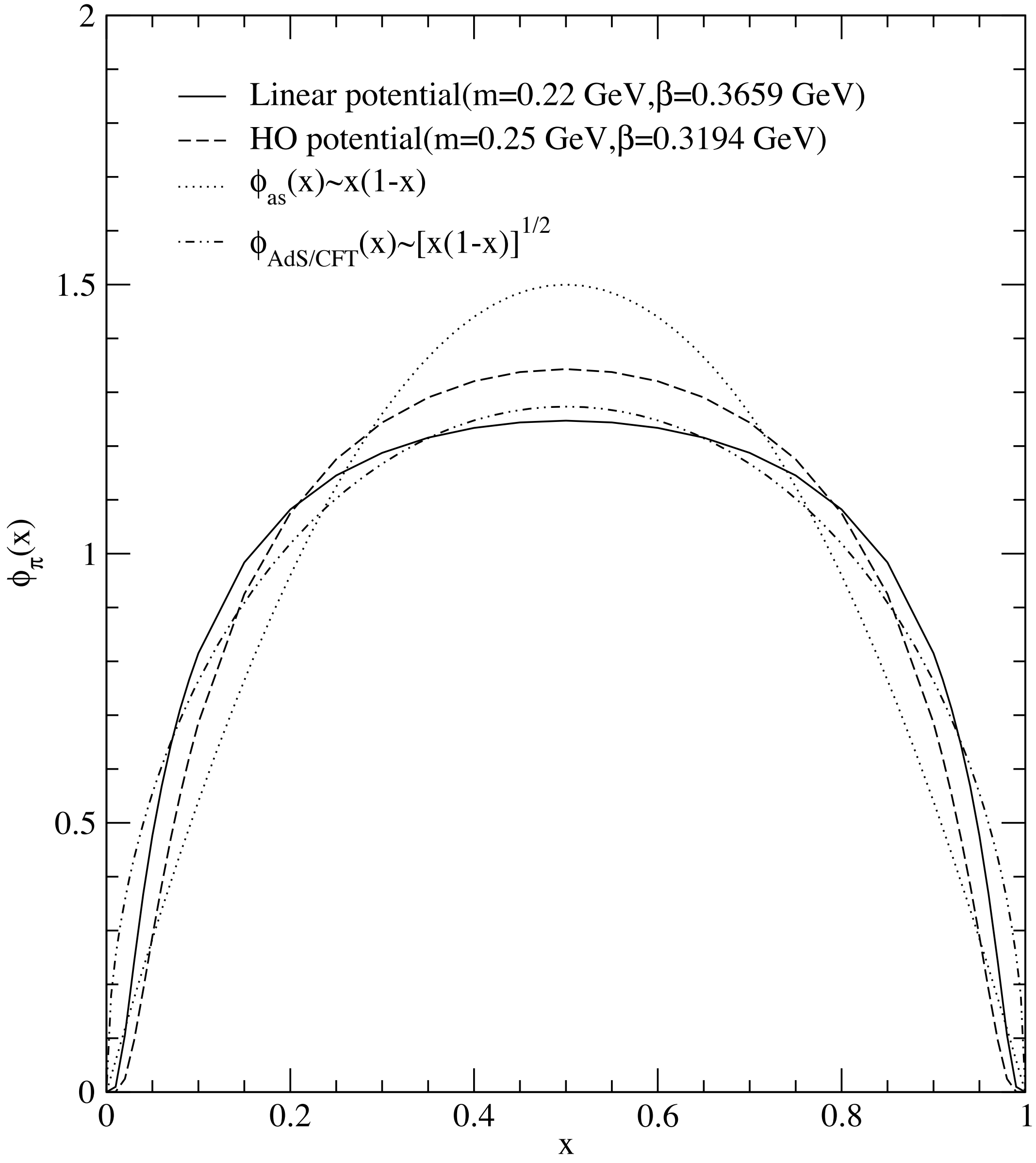
Same slope in L and n

Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^{Q} d^2\vec{k}_\perp \;\psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage, SJB



AdS/CFT:

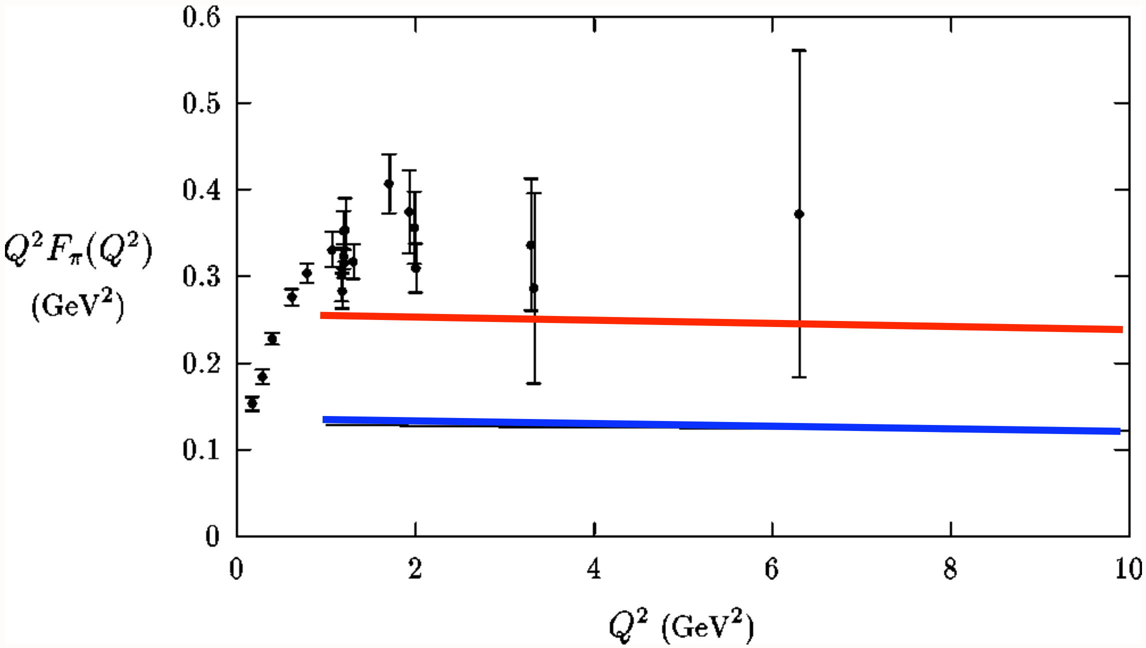
$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb

$$F_\pi(Q^2) = \int_0^1 dx\,\phi_\pi(x) \int_0^1 dy\,\phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$

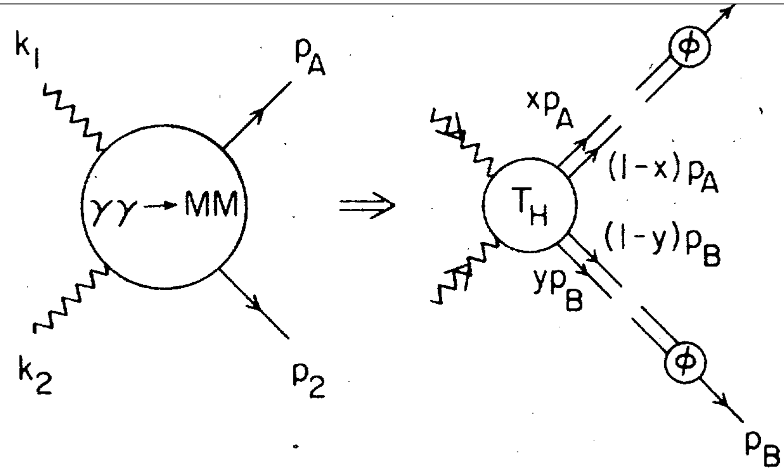


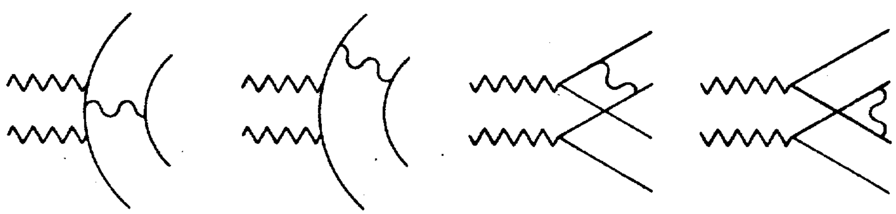
$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

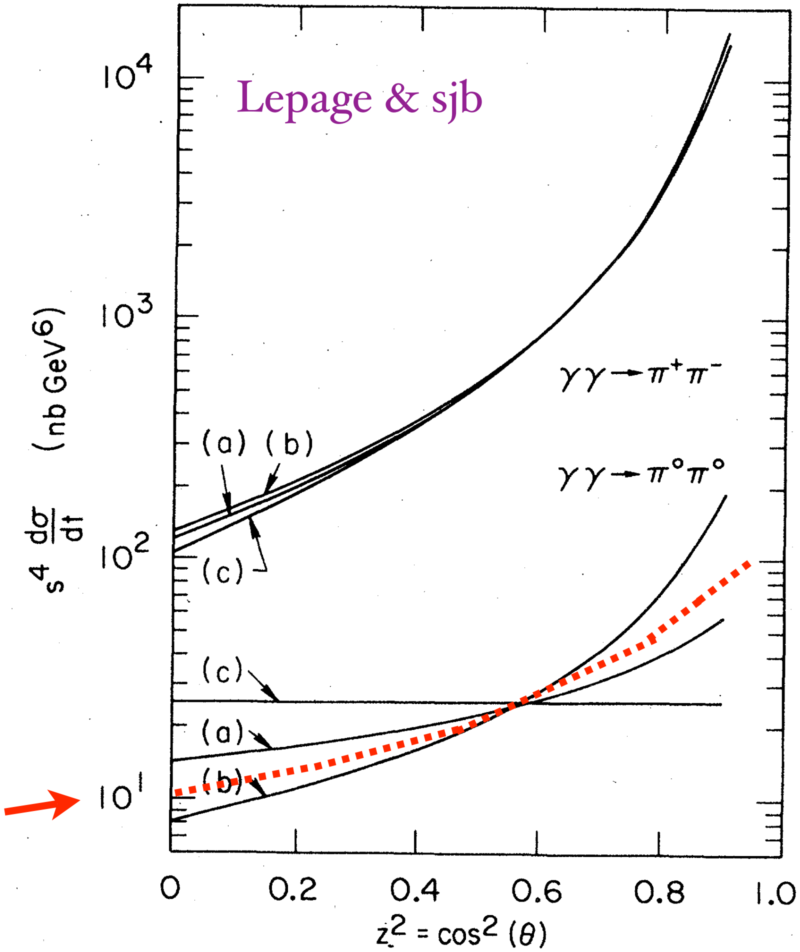
$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9







Neutral pair angular distribution sensitive to AdS/CFT distribution!

$$\phi_\pi^{AdS/QCD}(x) \propto [x(1-x)]^{1/2}$$

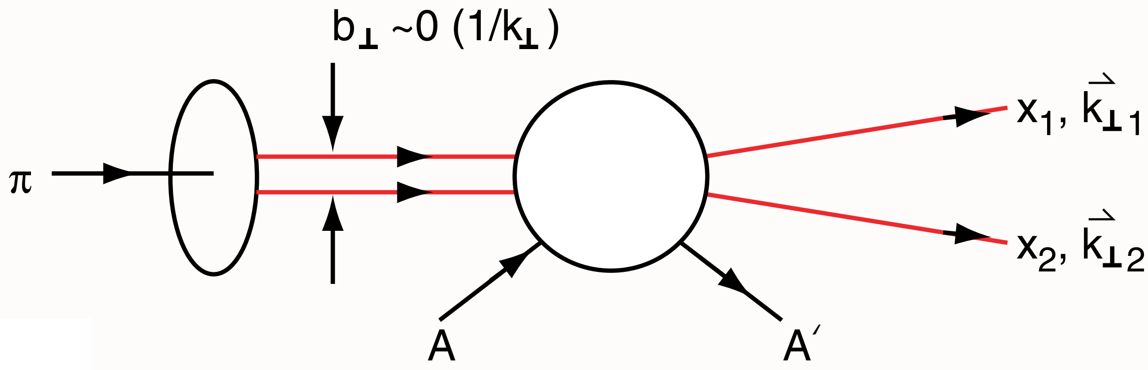
(a): $\phi_\pi(x) \propto x(1-x)$

(b): $\phi_\pi(x) \propto [x(1-x)]^{1/4}$

(c): $\phi_\pi(x) \propto \delta(x-1/2)$

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

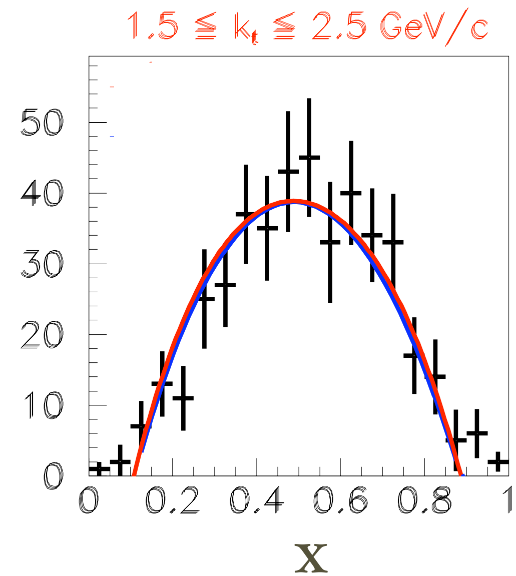
Nucleus left Intact!

Diffractive Dissociation of a Pion into Dijets

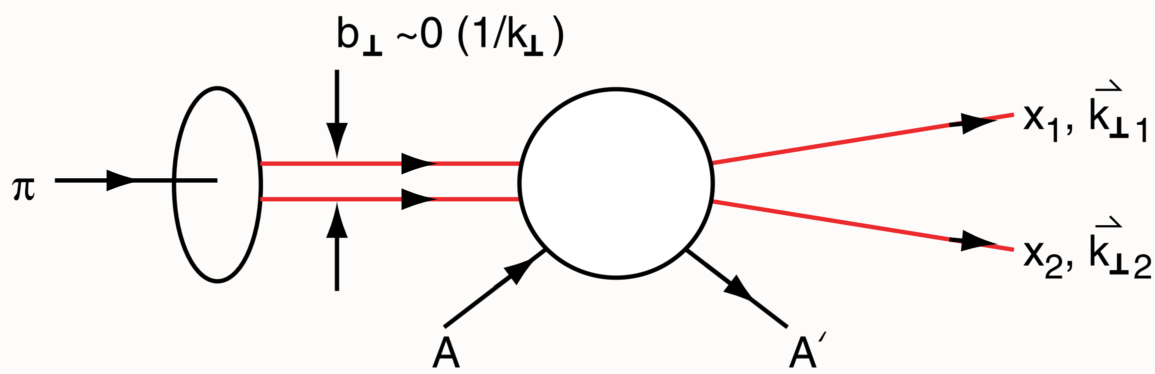
$$\pi A \rightarrow Jet\,Jet\,A'$$

$$\psi^{\pi}_{q\bar{q}}(x,\vec{k}_\perp)$$

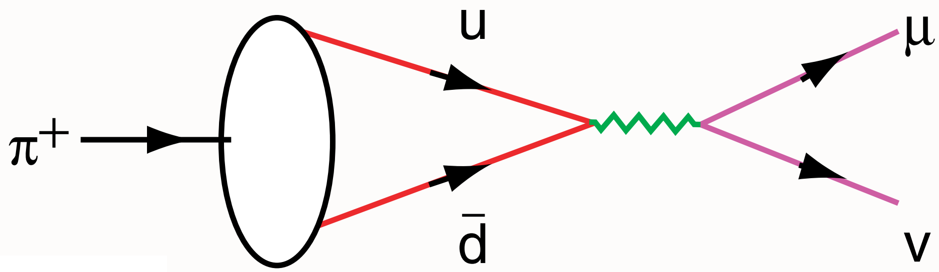
- E789 Fermilab Experiment Ashery et al
- 500 GeV pions collide on nuclei keeping it intact
- Measure momentum of two jets
- Study momentum distributions of pion LF wavefunction



Fluctuation of a Pion to a Compact Color Dipole State

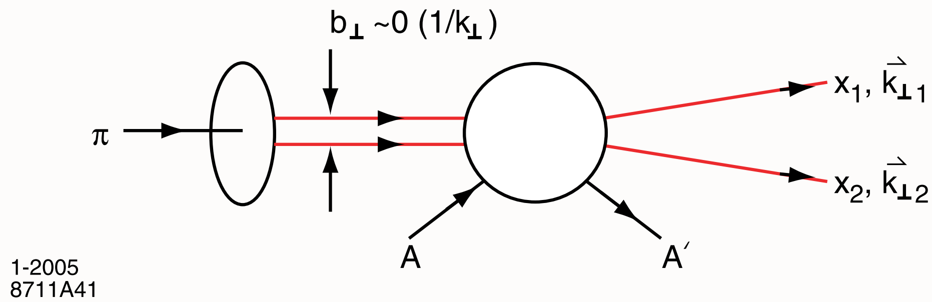


Color-Transparent Fock State For High Transverse Momentum Di-Jets

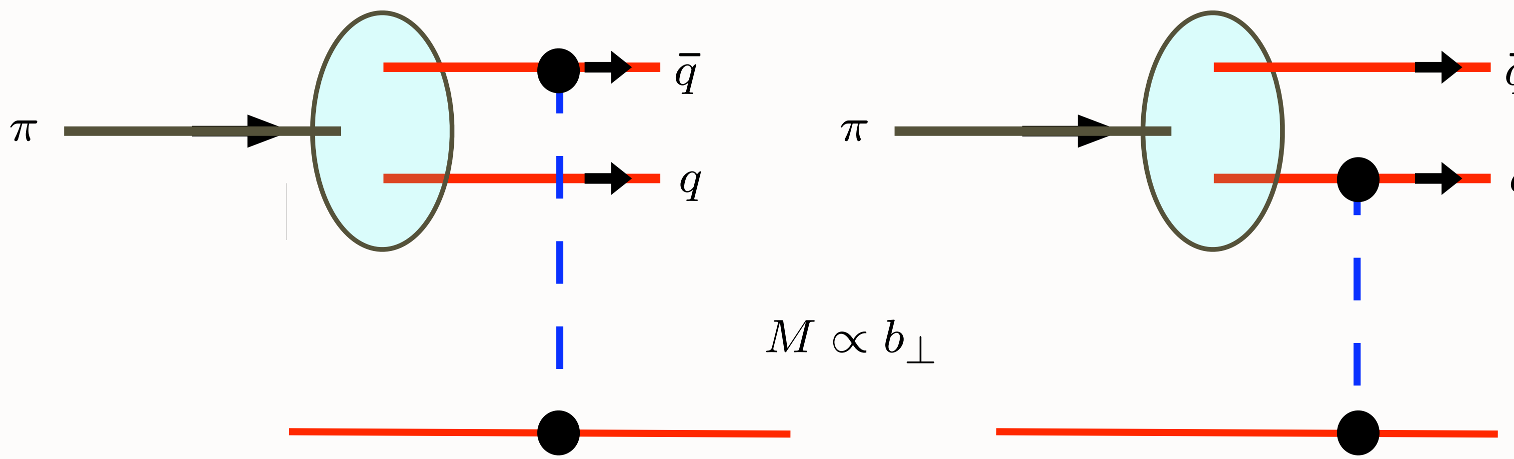


Same Fock State Determines Weak Decay

Key Ingredients in Ashery Experiment

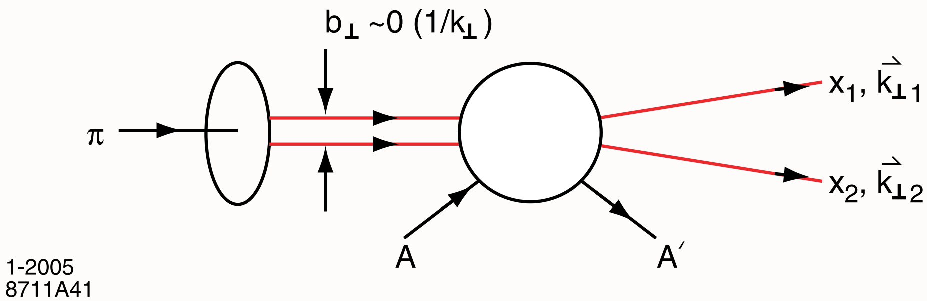


Local gauge-theory interactions measure transverse size of color dipole



AdS/QCD

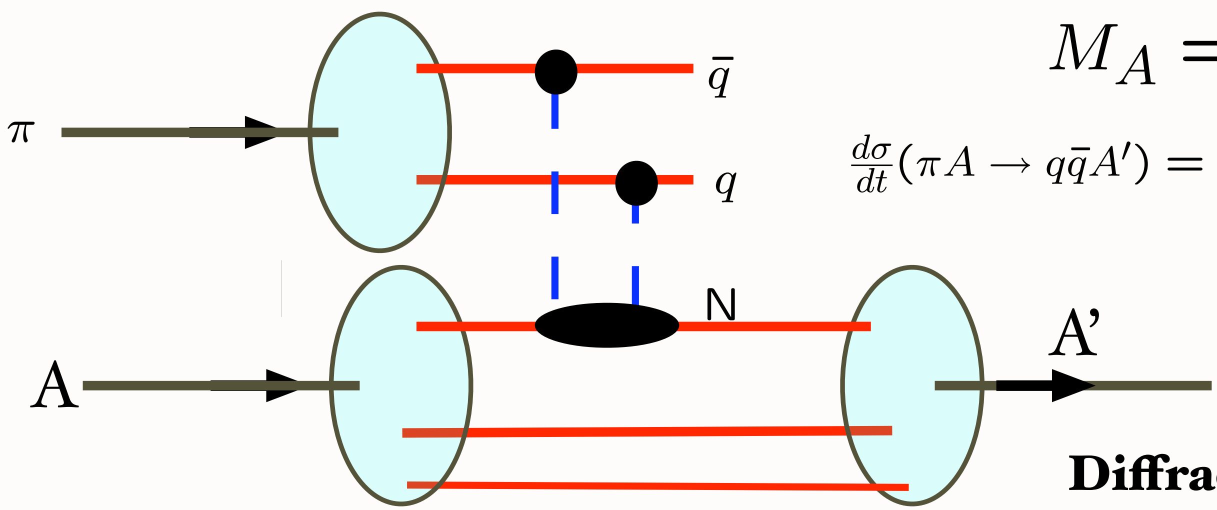
Key Ingredients in Ashery Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

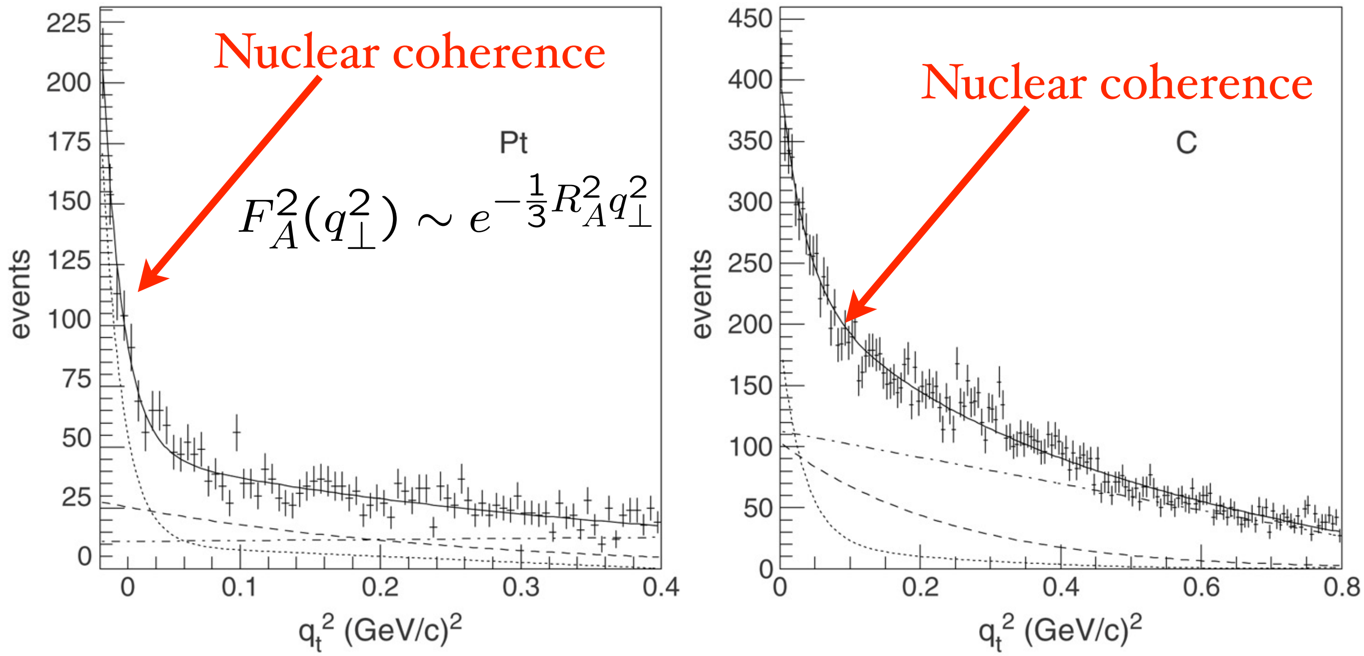
Diffraction, Rapidity gap

- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Ashery E791:
Measure of pion LFWF in diffractive dijet production
Confirmation of color transparency, gauge theory of strong interactions

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

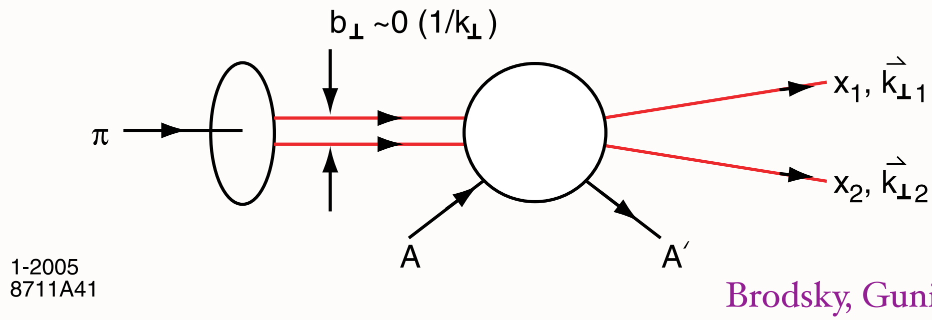
Factor of 7

Color Transparency

A. H. Mueller, sjb
Bertsch, Gunion, Goldhaber, sjb
Frankfurt, Miller, Strikman

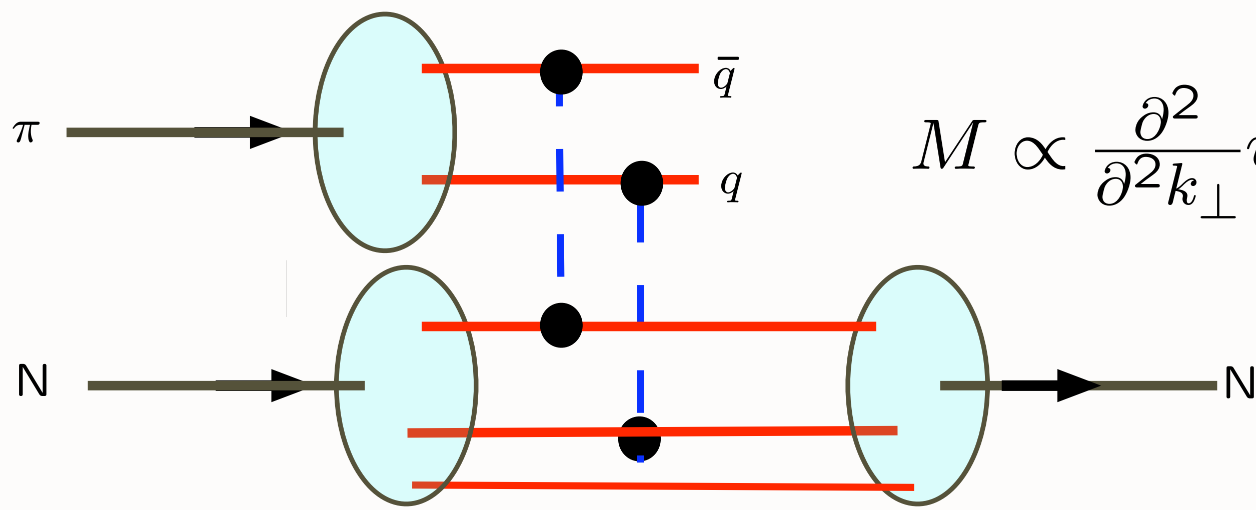
- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Key Ingredients in Ashery Experiment

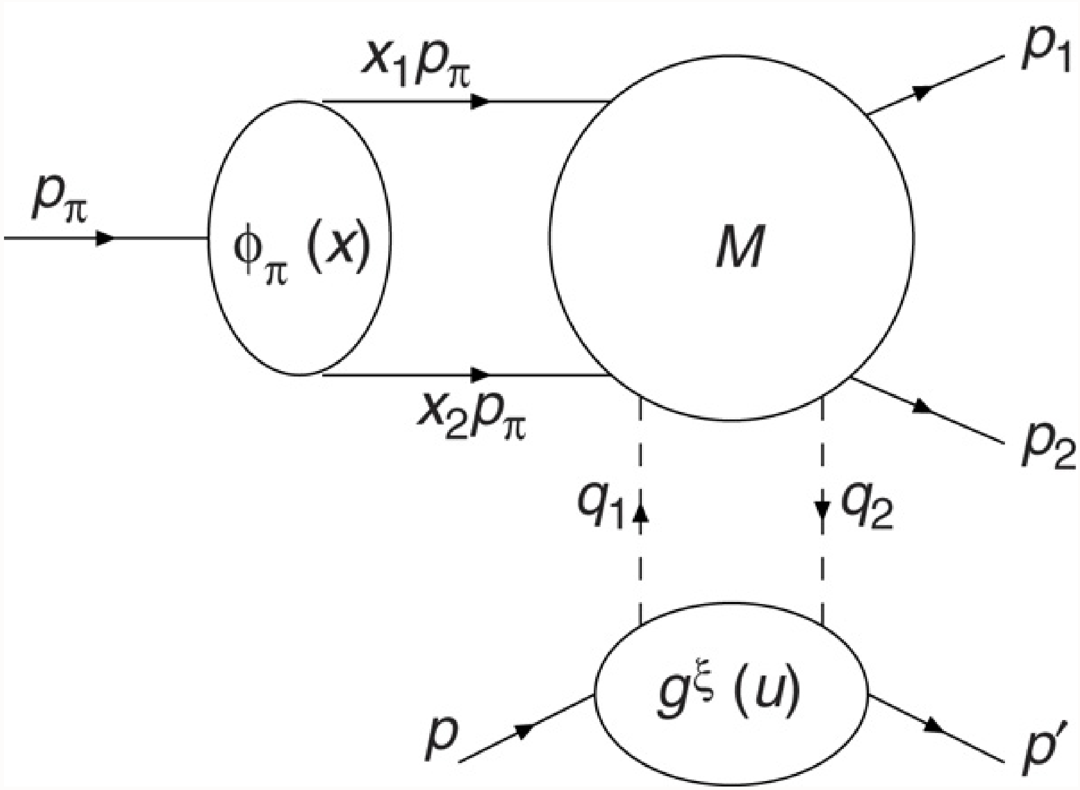


Brodsky, Gunion, Frankfurt, Mueller, Strikman
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp}\psi_\pi(x, k_\perp)$$



x

1-x

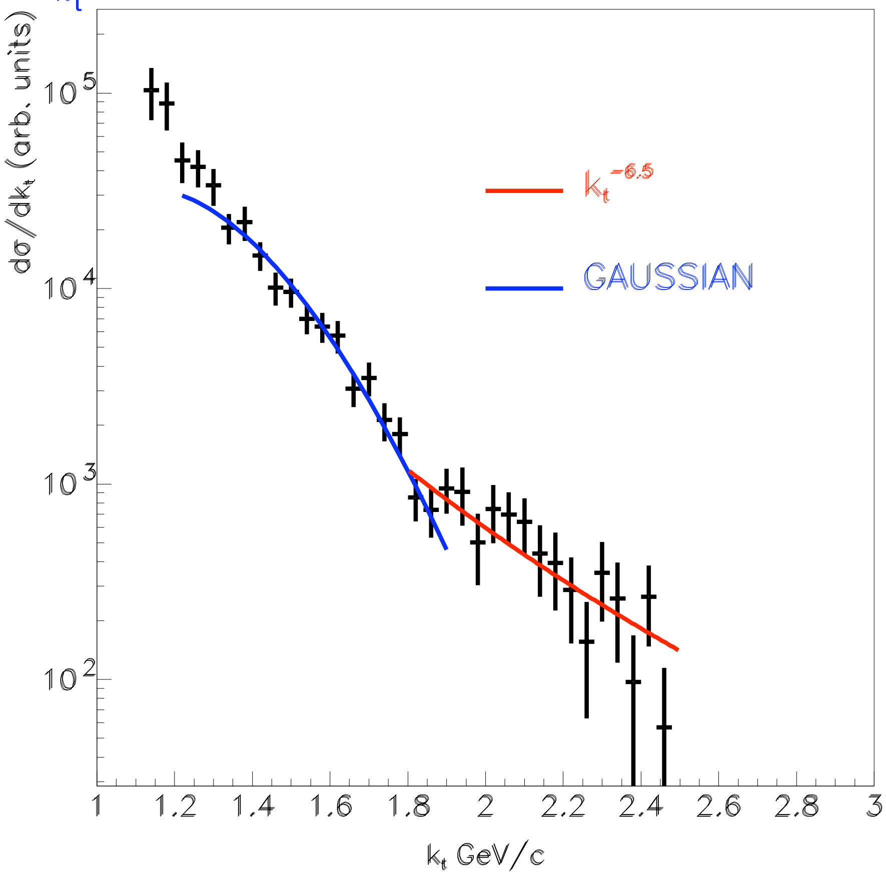
gluons measure size of color dipole

$$\frac{d\sigma}{dk_t^2} \propto |\alpha_s(k_t^2) x_N G(u, k_t^2)|^2 \left| \frac{\partial^2}{\partial k_t^2} \psi(x, k_t) \right|^2$$

THE k_t DEPENDENCE OF DI-JETS YIELD

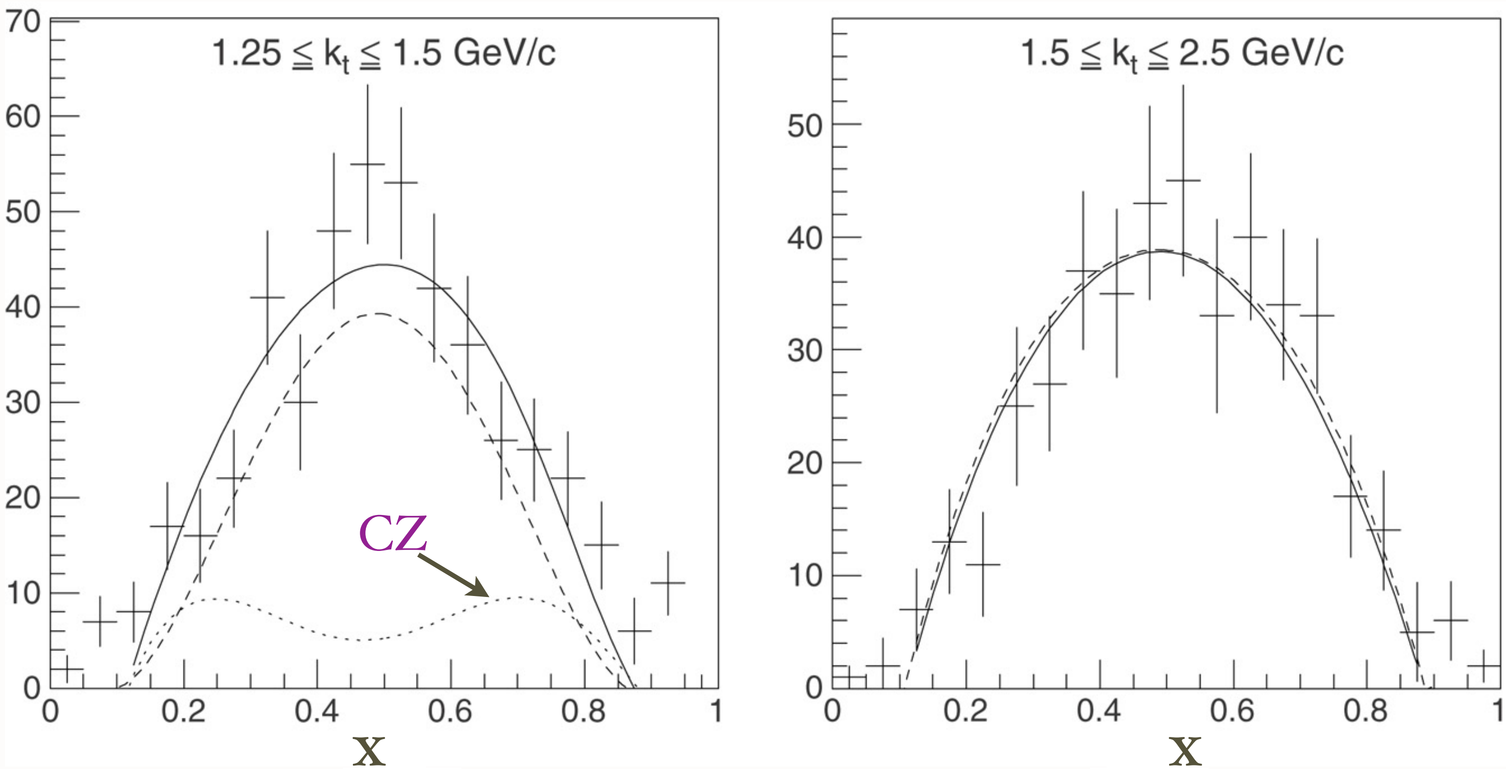
$$\frac{d\sigma}{dk_t^2} \propto \left|\alpha_s(k_t^2) G(x, k_t^2)\right|^2 \left|\frac{\partial^2}{\partial k_t^2}\psi(u, k_t)\right|^2$$

With $\psi \sim \frac{\phi}{k_t^2}$, weak $\phi(k_t^2)$ and $\alpha_s(k_t^2)$ dependences and $G(x, k_t^2) \sim k_t^{1/2}$: $\frac{d\sigma}{dk_t} \sim k_t^{-6}$



High Transverse momentum dependence consistent with PQCD, ERBL Evolution

Two Components?

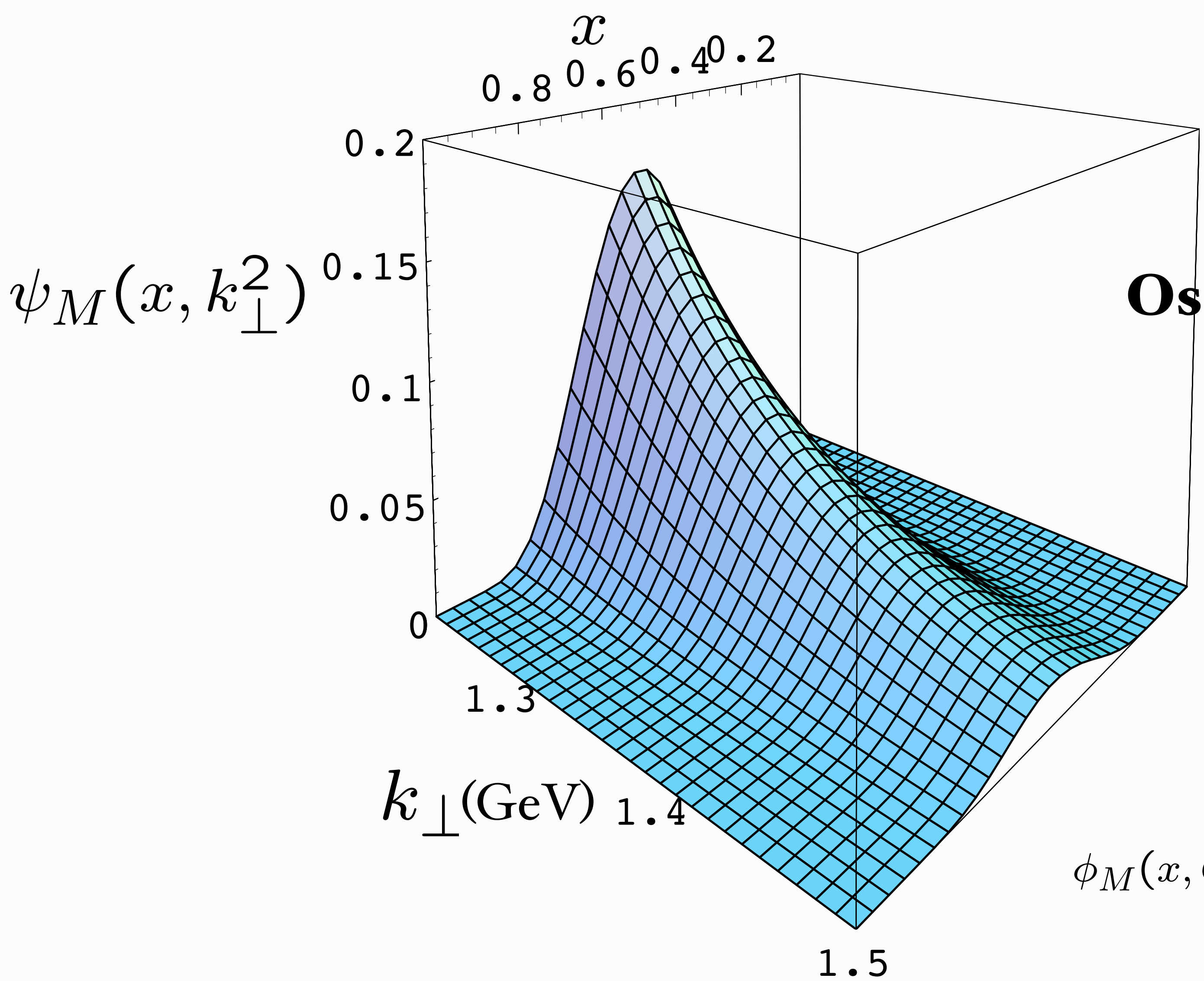


Narrowing of x distribution at higher jet transverse momentum

x: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative and Perturbative
(ERBL) Evolution

Prediction from AdS/CFT: Meson LFWF



Harmonic Oscillator model

$\kappa = 0.77\text{GeV}$

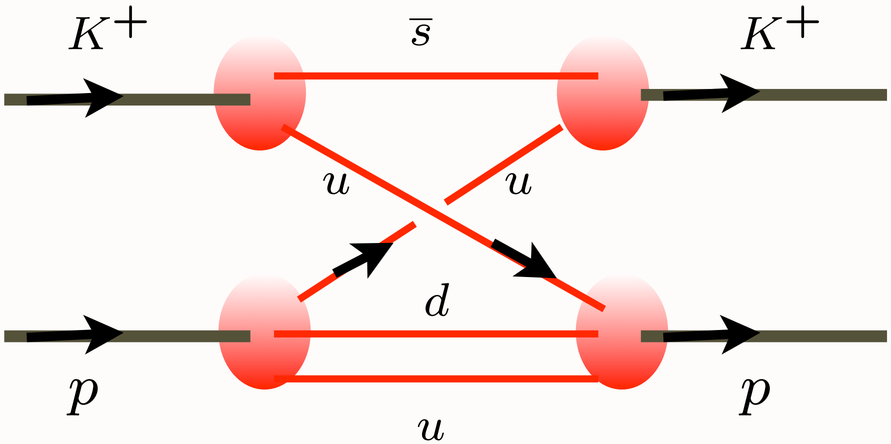
de Teramond, sjb

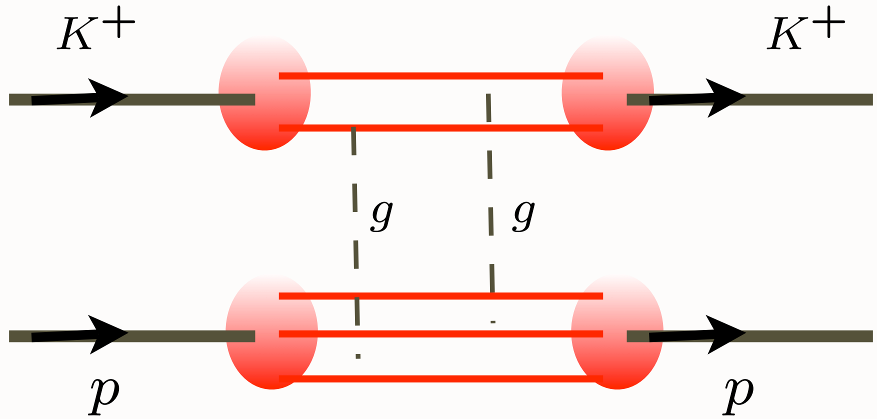
$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange
(Spin exchange in atom-atom scattering)

Gluon Exchange
(Van der Waal -- Landshoff)

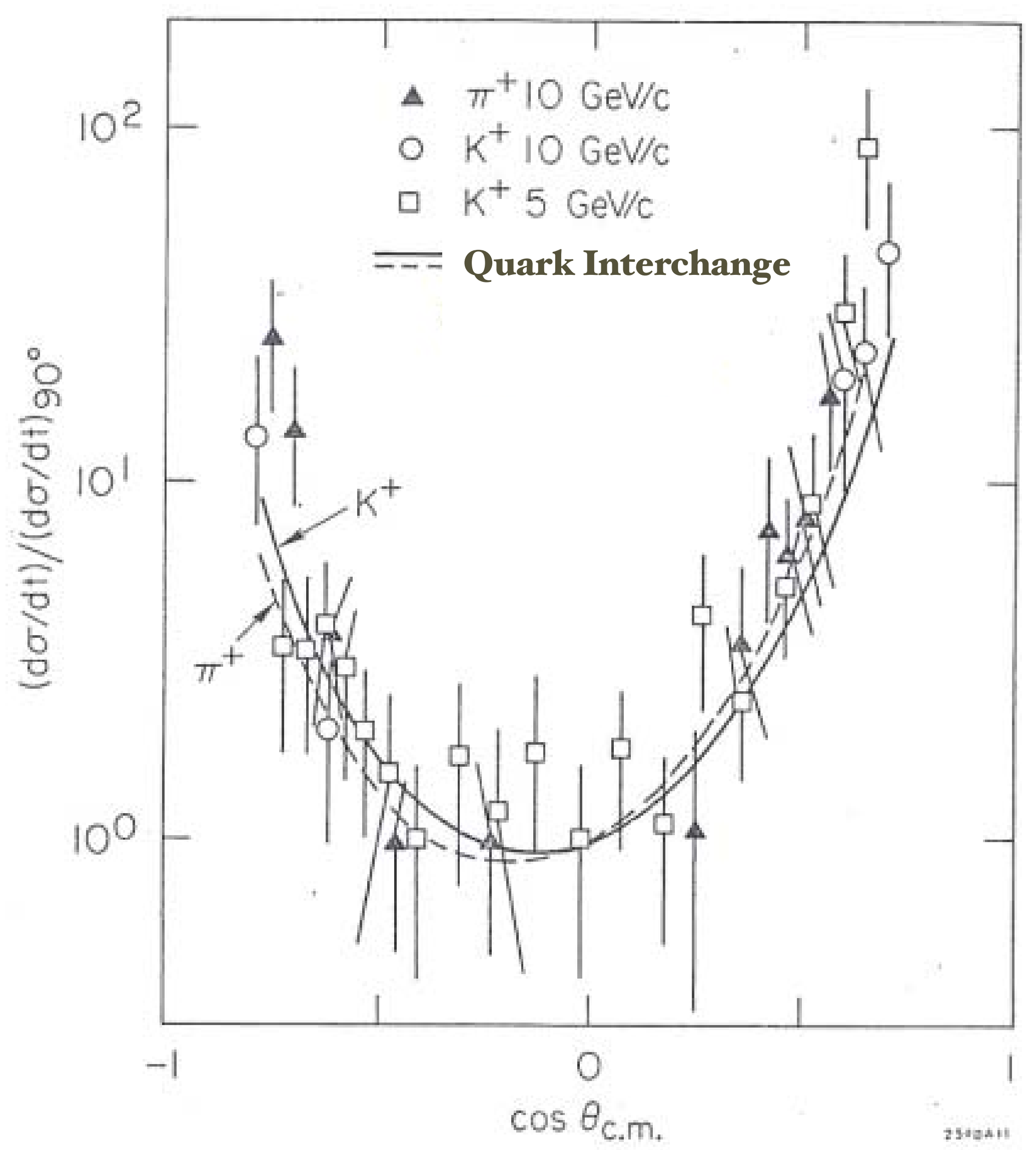
$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:

AdS/QCD



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \rightarrow p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \rightarrow pK^{\pm}$; $p^{\pm}p \rightarrow pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \rightarrow p\pi^{\pm}$,

$K^{\pm}p \rightarrow pK^{\pm}$,

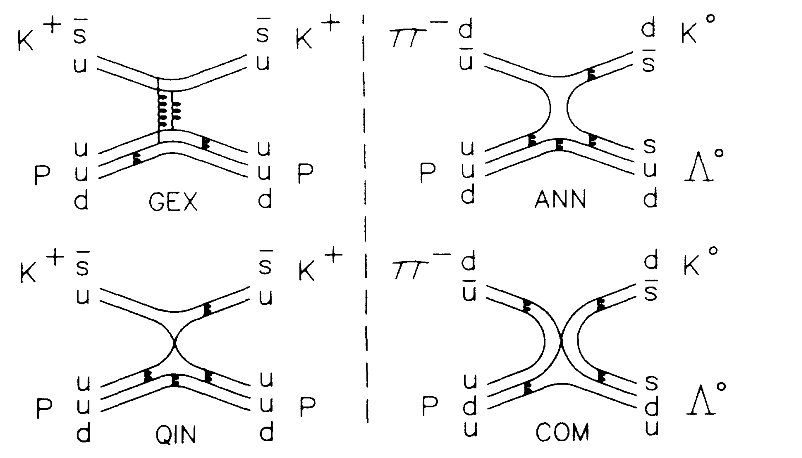
$\pi^{\pm}p \rightarrow p\rho^{\pm}$,

$\pi^{\pm}p \rightarrow \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \rightarrow K^{+}\Sigma^{\pm}$,

$\pi^{-}p \rightarrow \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \rightarrow pp^{\pm}$.



Hadron Dynamics at the Amplitude Level

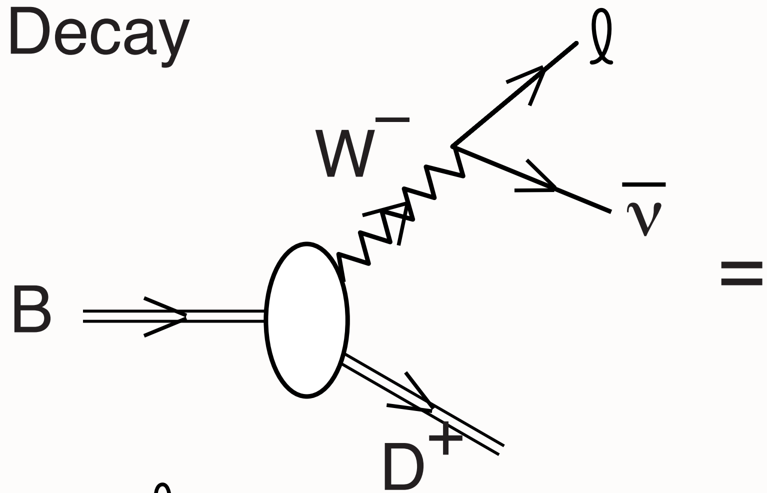
- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role if ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

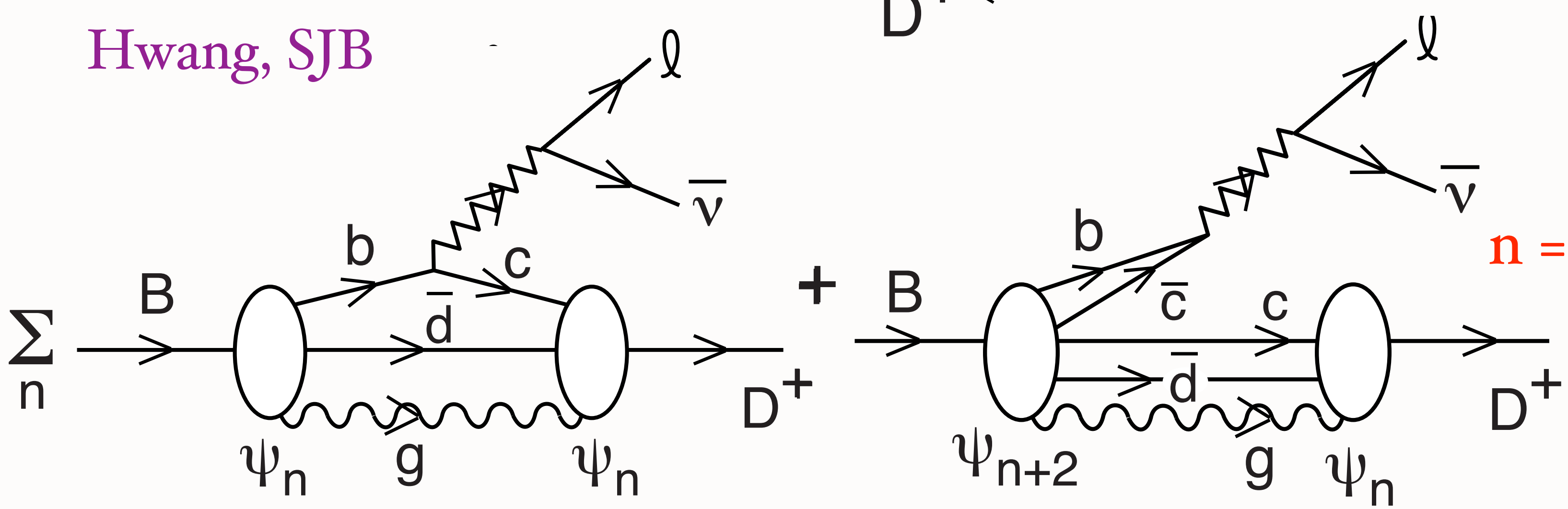
Weak Exclusive Decay

$\langle D | J^+ (0) | B \rangle$



Exact Formula

Hwang, SJB



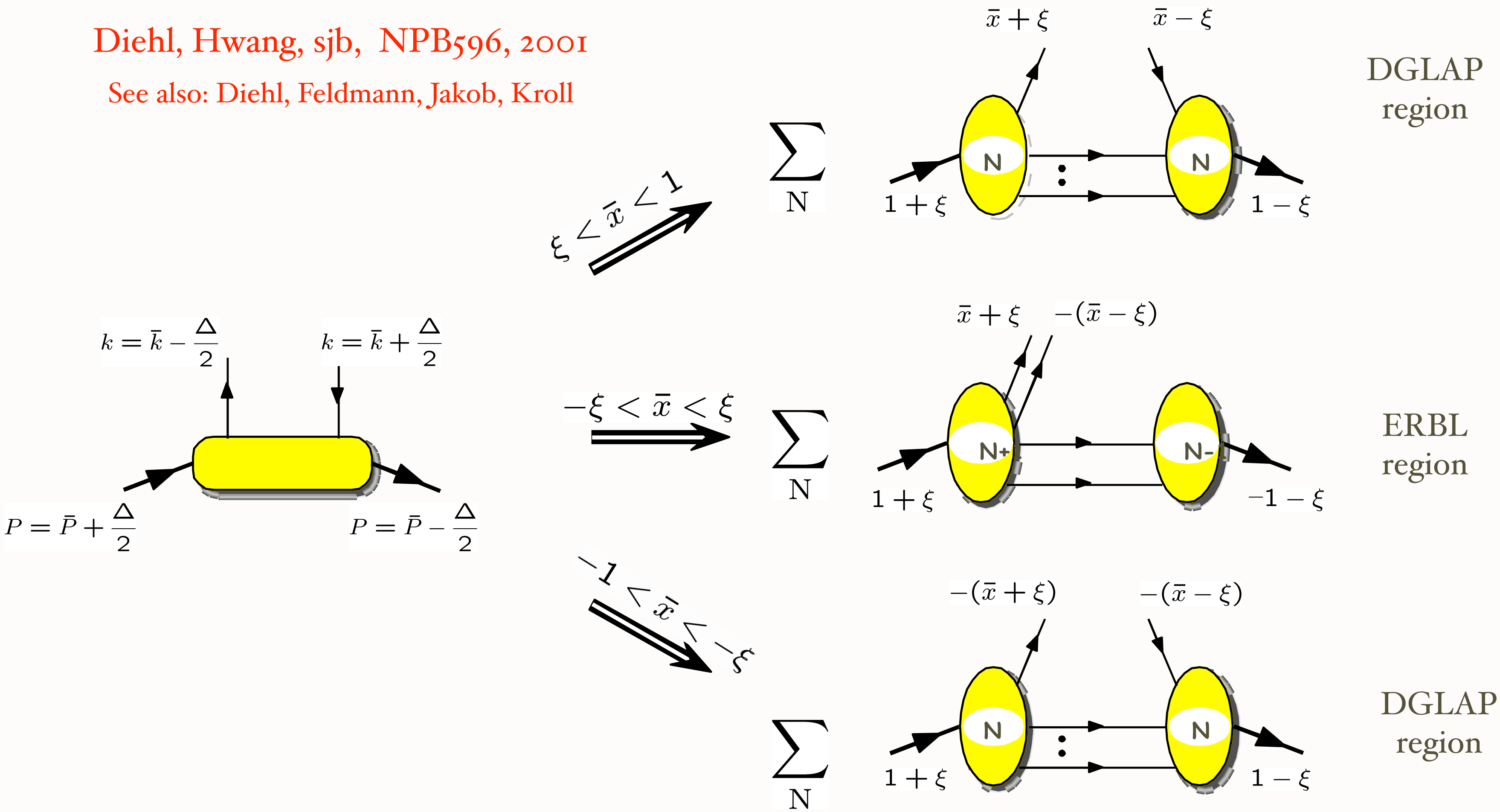
$n = n' + 2$

Annihilation amplitude needed for Lorentz Invariance

Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Pasquini

The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \mathbf{\Delta}_\perp, (t + \mathbf{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]
We find, under $\boldsymbol{q}_\perp \to \mathbf{\Delta}_\perp$, for $\zeta \leq x \leq 1$,

$$\frac{E(x,\zeta,0)}{2M} = \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x - x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \,,$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i.
The E distribution function is related to a $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}}$ matrix element at finite ζ as well.

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

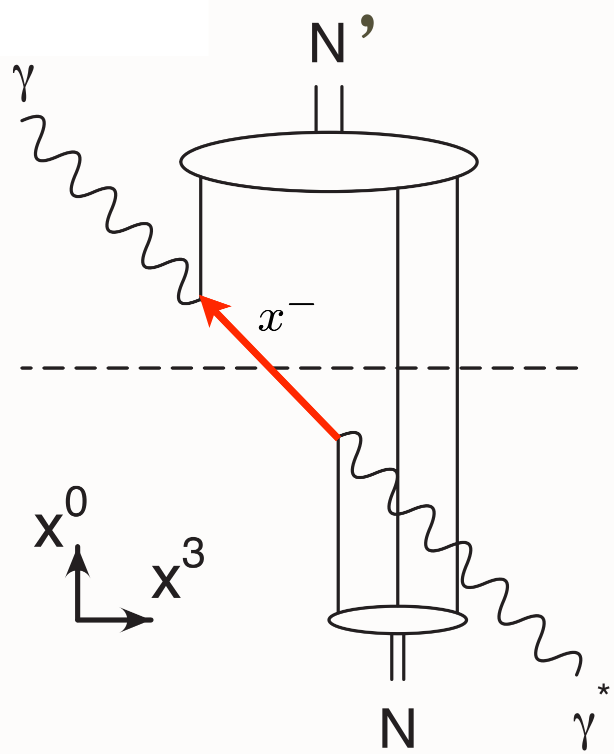
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Space-time picture of DVCS

P. Hoyer



$\sigma = \frac{1}{2} x^- P^+$

$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

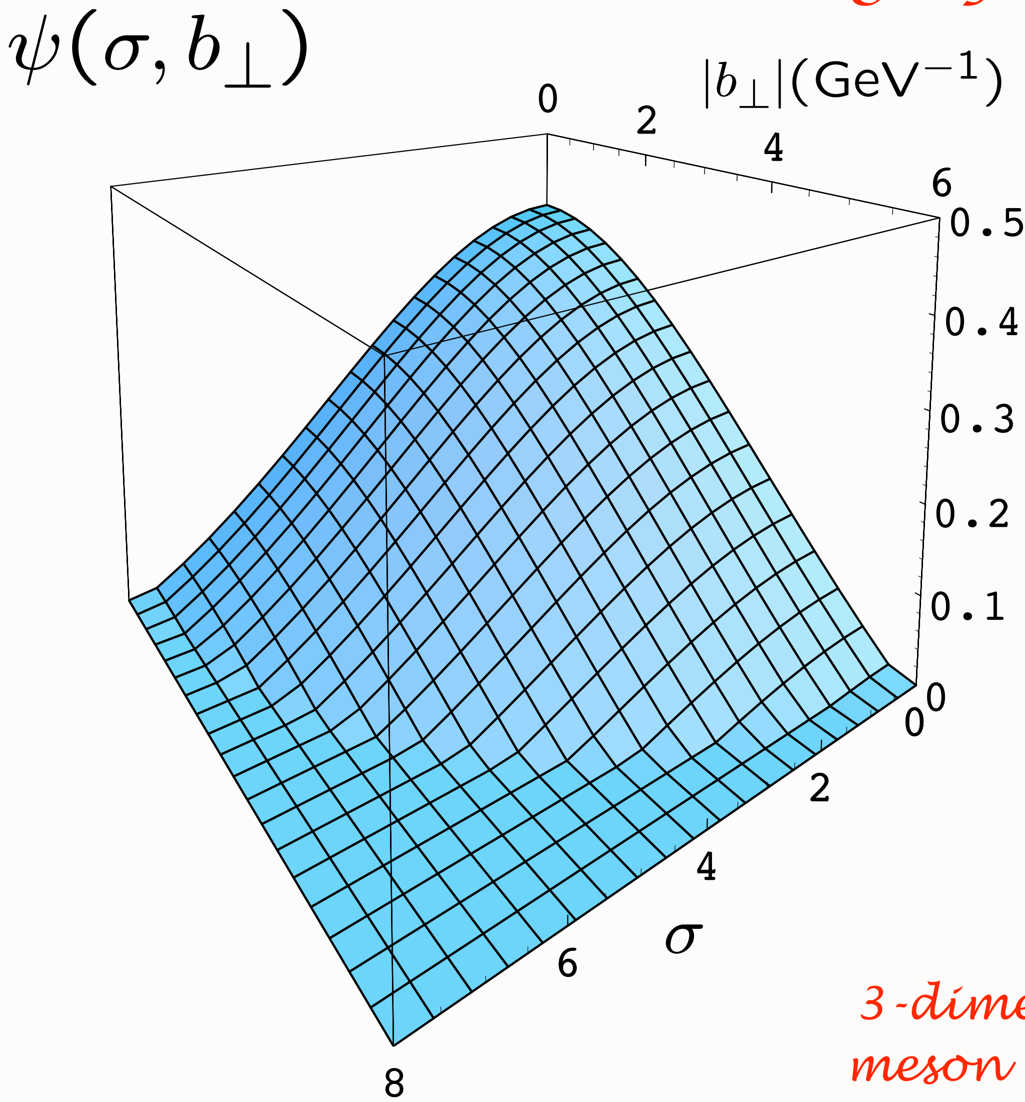
Measure x- distribution from DVCS:
Use Fourier transform of skewness,
the longitudinal momentum transfer

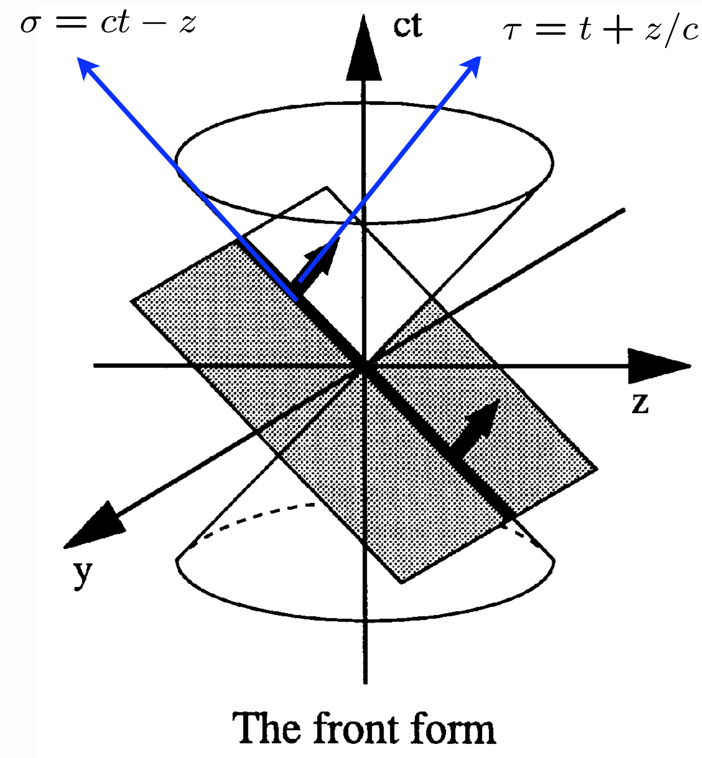
$$\zeta = \frac{Q^2}{2p \cdot q}$$

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

AdS/CFT Holographic Model

G. de Teramond
SJB





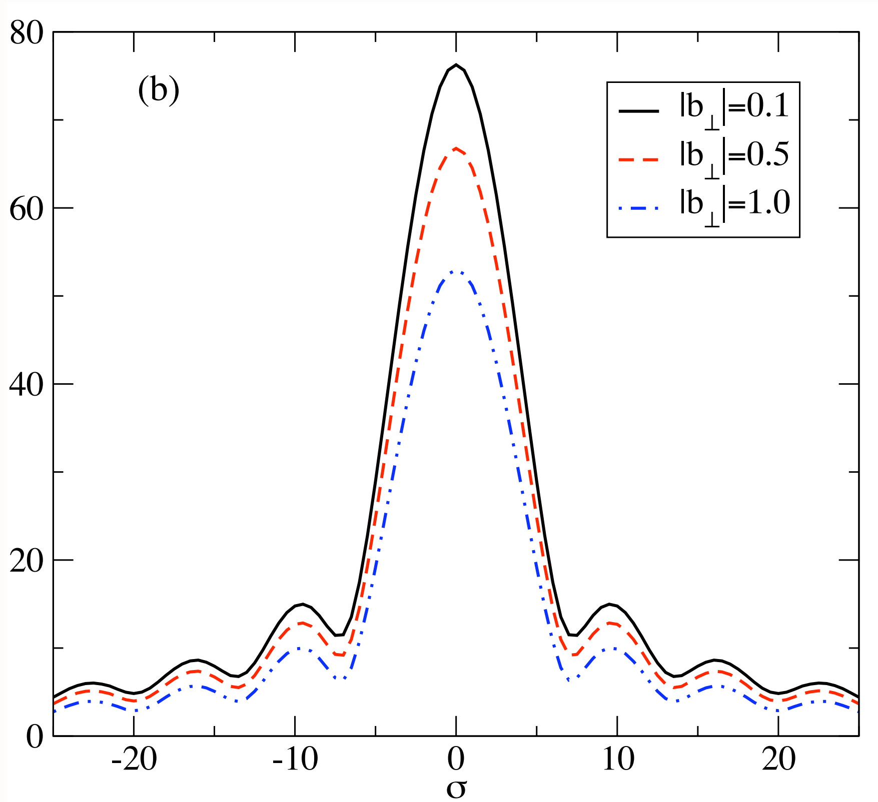
3-dimensional photograph: meson LFWF at fixed LF Time

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi}\int d\zeta e^{i\sigma\zeta}\tilde{A}(b_\perp, \zeta)$$

$$\sigma = \frac{1}{2}x^- P^+ \qquad \zeta = \frac{Q^2}{2p\cdot q}$$

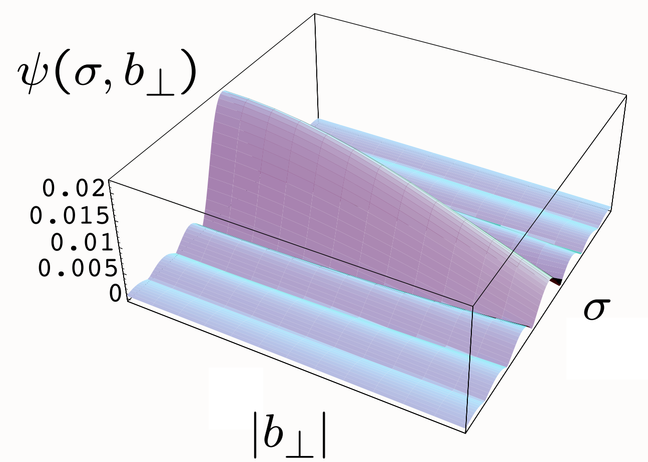


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$



Features of Light-Front Formalism

- *Hidden Color* Of Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- *Intrinsic glue, sea quarks, intrinsic charm*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

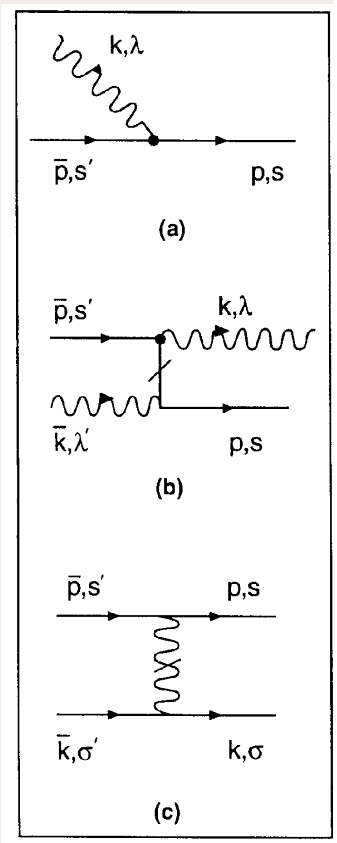
Integrable!

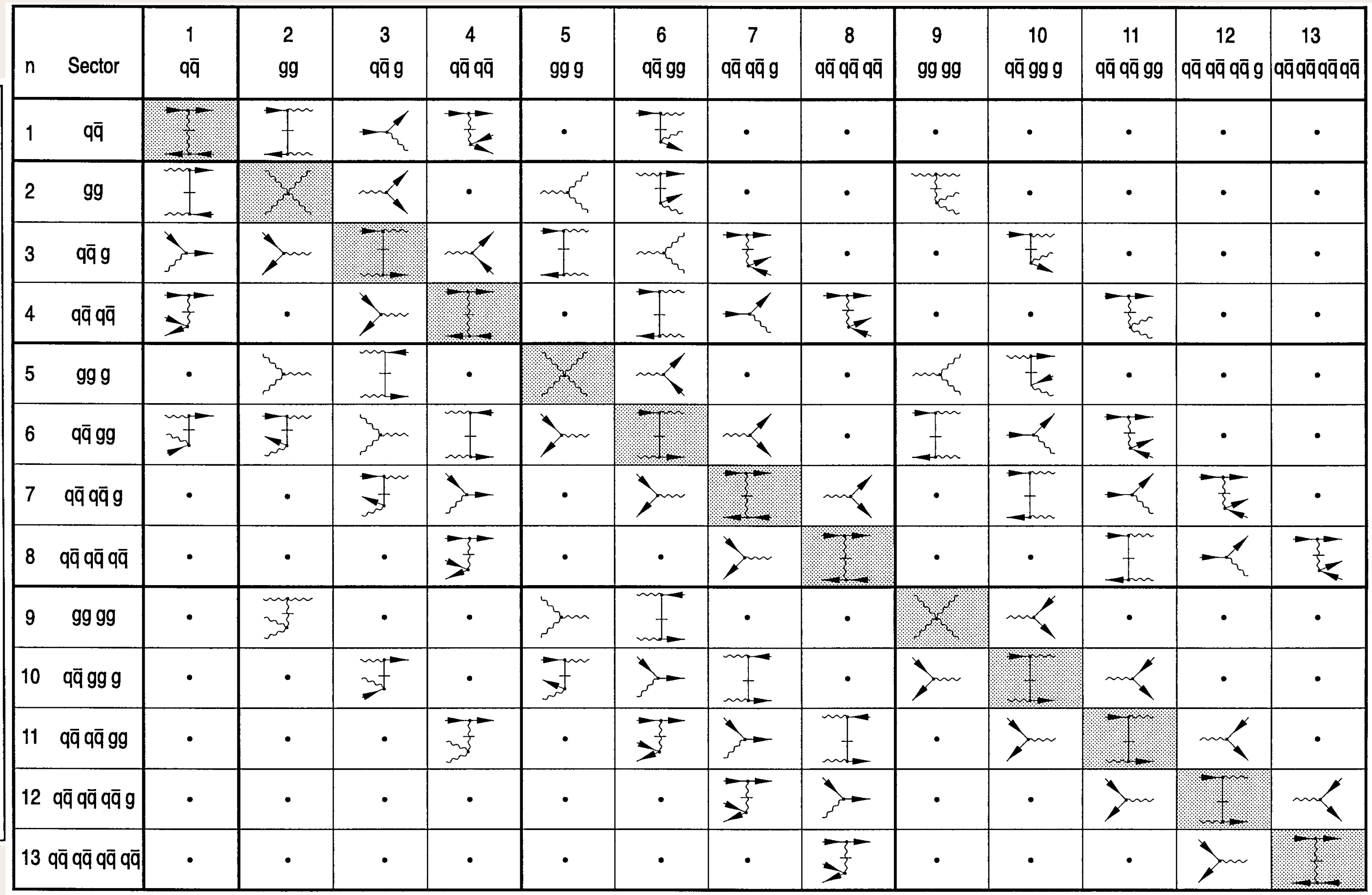
Hadron Spectra, Wavefunctions, Dynamics

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ





Use AdS/QCD basis functions

Pauli, Pinsky, sjb

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Vary, Harinandrath, sjb

AdS/QCD

- New initial approximation to QCD based on conformal invariance, and confinement
- Underlying principle: Conformal Template
- AdS5: Mathematical representation of conformal gauge theory
- Systematically improve using DLCQ
- Successes: Hadron spectra, LFWFs, dynamics
- QCD at the Amplitude Level

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

SCIENCE VOL 265 15 SEPTEMBER 1995

A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.