Neutron Beta Decay in Effective Field Theory

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Outline

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1. Introduction

Neutron β-decay

$$n \longrightarrow p + e + \bar{\nu}_e \, .$$

The hadronic current

$$J^\mu = \bar{u}_p \left[G'_V \gamma^\mu - G'_A \gamma^\mu \gamma_5 \right] u_n \, ,$$

and radiative corrections and weak magnetism.

“Standard calculations”:

Sirlin, Marciano, Towner, *et al.*’s works.

Observables of NBD

1. Decay rate (or lifetime)

$$\Gamma \quad = \quad \frac{G_V'^2}{4\pi^3}(1+3g_A^2)\int_{m_e}^{E_e^{max}} dE_e p_e E_e (E_e^{max}-E_e)^2 F(Z,E_e)\left[1+\frac{\alpha}{2\pi}g(E_e,E_e^{max})\right] ,$$

with

$$G_V'^2 = (G_F V_{ud})^2(1+\Delta_R^V)\,.$$

2. Correlation coeffi cients

$$\frac{d\Gamma}{dE_e d\Omega_{\hat{p}_e} d\Omega_{\hat{p}_\nu}} \propto 1 + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \hat{n}\cdot\left(A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e\times\vec{p}_\nu}{E_e E_\nu}\right) ,$$

where (at leading order)

$$a = \frac{1-g_A^2}{1+3g_A^3} , \quad A = 2\frac{g_A - g_A^2}{1+3g_A^2} , \quad B = 2\frac{g_A+g_A^2}{1+3g_A^2} , \quad D = 0\,,$$

with $g_A = G_A'/G_V'$.

Values of g_A

The neutron-spin and electron correlation coefficient A

$$A = \frac{2g_A(1-g_A)}{1+3g_A^2}\,, \qquad g_A = G'_A/G'_V\,,$$

and a recommended value by PDG2006 (the same as PDG2004)

$$g_A = 1.2695 \pm 0.0029\,.$$

This value is important for estimating the cross sections of the processes, *e.g.*, $pp \to de^+\bar{\nu}$ and $\nu d \to ppe(\nu d \to np\nu)$ and for test of the Goldberger-Treiman relation.

T-violating coefficient D

- The Standard Model, $D_{SM} < 10^{-12}$.
- Models beyond the SM (like MSSM), $D_{MSSM} \sim 10^{-7}$.
- The most recent experimental data

$$D_{exp.} = [-2.8 \pm 6.4(stat) \pm 3.0(sys)] \times 10^{-4}\,,$$

from Soldner et al. (2004).

- But from the final state interaction

$$D_{FSI} \simeq -2.3 \times 10^{-5}\,.$$

CKM Unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta$$

Using V_{ud} from $0^+ \rightarrow 0^+$ nuclear β-decay and V_{us} from PDG02, $\underline{\Delta = 2.2\sigma}$ (0.0032$\pm$ 0.0014). ($\Delta = 0.0008 \pm 0.0011$, PDG06.)
Suggested solution:
New V_{us} values from E865, KTeV K_{e3}, *etc.*

From the most recent data of neutron β-decay (A), however, $\underline{\Delta = 2.7\sigma}$ (0.0076$\pm$ 0.0028).
Suggested solution:
New τ value from ILL, but $\sim 6\sigma$ discrepancy from the former exp. values.

New features in NBD

Experiment:

1) New neutron facilities under construction, *e.g.*, at Oak Ridge and J-PARC,

2) A neutron source of “Hanaro” in Korea,

3) New experiment proposals and new detectors under investigation around the world.

Theory:

1) Model independent calculations,

2) Error estimations using EFT approach

2. NBD in EFT

Chiral Perturbation Theory: a low energy EFT of QCD

- SSB of chiral sym. of QCD (pions: Goldstone-bosons).
- A systematic perturbation scheme (renormalizable order by order)

$$\mathcal{L}_{SM} \rightarrow \mathcal{L}_\chi = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

$$M \sim \sum \left(\frac{Q}{\Lambda_\chi}\right)^\nu ,$$

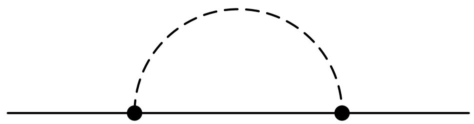
where $Q \simeq m_\pi$ or $|\vec{p}|$, $\Lambda_\chi \simeq 4\pi f_\pi \sim m_N \sim 1$ GeV.

But it has a problem when a nucleon field is included.

[Gasser, Sainio, Švarc, NPB307(1988)779.]

No counting rules for loop diagrams with a relativistic nucleon.

e.g., Nucleon self-energy



$$\Sigma(p) = i\frac{3}{4}\frac{g_A^2}{f_\pi^2}\int \frac{d^4l}{(2\pi)^4}\frac{l\cdot\gamma\gamma_5[\gamma\cdot(l+p)+m_N]l\cdot\gamma\gamma_5}{[(l+p)^2-m_N^2](l^2-m_\pi^2)}$$

$$\sim \frac{m_N^3}{(4\pi f_\pi)^2} \sim m_N >> \frac{Q^2}{\Lambda_\chi^2} m_N\,.$$

Heavy-baryon formalism

$p^\mu = m_N v^\mu + k^\mu$, $N(x) \simeq e^{i m_N v\cdot x}\Psi_N(x)$, ...

then one has $\gamma^\mu \to v^\mu = (1,\vec{0})$, $\gamma^\mu\gamma_5 \to 2S^\mu = (0,\vec{\sigma})$, $1/(\gamma\cdot p - m_N) \to 1/v\cdot k$, and

$$\Sigma_v(k) = i\frac{3}{4}\frac{g_A^2}{f_\pi^2}\int \frac{d^4l}{(2\pi)^4}\frac{2S\cdot l 2S\cdot l}{v\cdot(l+k)(l^2-m_\pi^2)} \sim \frac{m_\pi^3}{(4\pi f_\pi)^2}\,.$$

Counting rules for NBD

A new scale: $\bar{Q} \simeq m_n - m_p - m_e << m_\pi$

- One-pion exchange diagram $(\bar{Q}/m_\pi)^2 \sim 10^{-5}$
- Weak-magnetism term $\bar{Q}\kappa_V/(2m_N) \sim 10^{-3}$

Modified counting rules:

- Expanding parameters; $\alpha/(2\pi)$, $\bar{Q}/(2m_N) \sim 10^{-3}$.
- τ, a, A, B up to NLO, $\alpha/(2\pi)$, $\bar{Q}/(2m_N) \sim 10^{-3}$
- Nonzero D appears in $\alpha\bar{Q}/(2m_N) \sim 10^{-5}$.
- Pion loops, $(m_\pi/\Lambda_\chi)^2$ corrections, in the renormalized coupling constants, g_A and κ_V,

Effective Lagrangian

$$\mathcal{L}_\beta \quad = \quad \mathcal{L}_{e\nu\gamma} + \mathcal{L}_{NN\gamma} + \mathcal{L}_{e\nu NN} \, ,$$

where

$$\mathcal{L}_{e\nu\gamma} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\xi} (\partial \cdot A)^2 + \left(1 + \frac{\alpha}{4\pi} e_1\right) \bar{\psi}_e (i\gamma \cdot D) \psi_e - m_e \bar{\psi}_e \psi_e + \bar{\psi}_\nu (i\gamma \cdot \partial) \psi_\nu \, ,$$

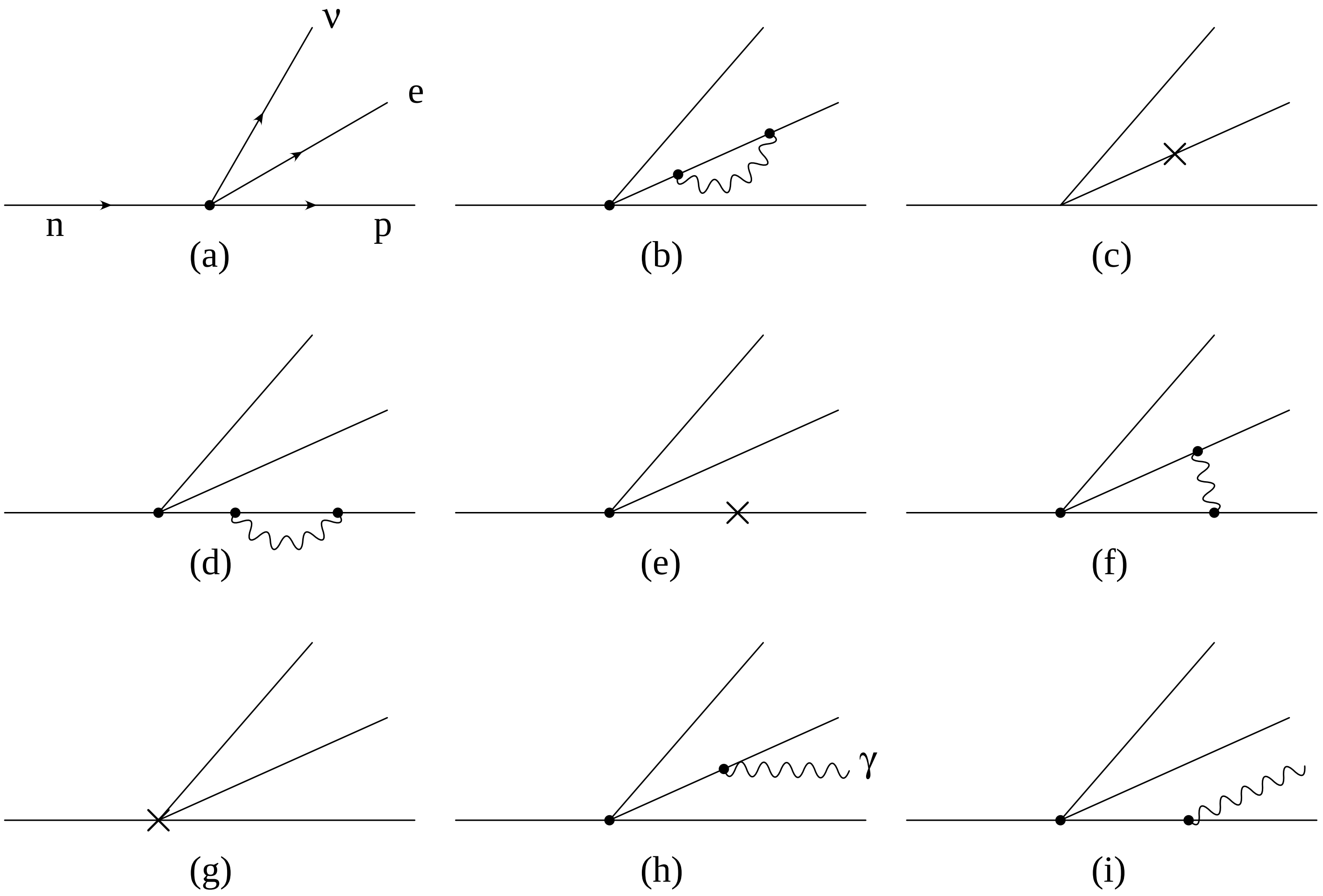
$$\mathcal{L}_{NN\gamma} = N^\dagger \left[1 + \frac{\alpha}{8\pi} e_2 (1 + \tau_3)\right] iv \cdot DN \, ,$$

$$\mathcal{L}_{e\nu NN} = -\frac{G_F V_{ud}}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu \left\{ N^\dagger \tau^+ \left[\left(1 + \frac{\alpha}{4\pi} e_V\right) v^\mu - 2 g_A \left(1 + \frac{\alpha}{4\pi} e_A\right) S^\mu \right] N \right.$$

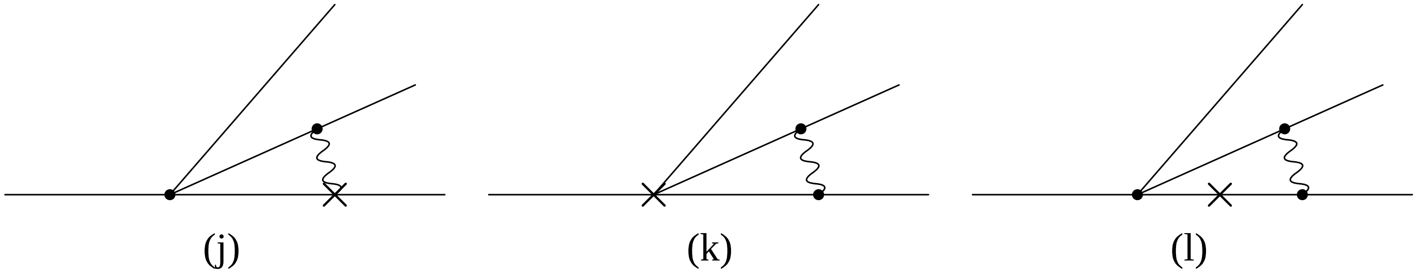
$$+ \frac{1}{2m_N} N^\dagger \tau^+ \left[i (v^\mu v^\nu - g^{\mu\nu}) (\overrightarrow{\partial} - \overleftarrow{\partial})_\nu - 2i\mu_V [S^\mu, S \cdot (\overrightarrow{\partial} + \overleftarrow{\partial})] - 2ig_A v^\mu S \cdot (\overrightarrow{\partial} + \overleftarrow{\partial}$$

and $v^{=}(1, \vec{0})$ and $2S^\mu = (0, \vec{\sigma})$.

Feynman diagrams for NBD



Feynman diagrams for the D coefficient



- Nonzero D appears from the imaginary part of the loop diagrams in $\alpha\bar{Q}/(2m_N)$ order ($\sim 10^{-5}$).

Results (1)

$$\frac{d\Gamma}{dE_e d\Omega_{\hat{p}_e} d\Omega_{\hat{p}_\nu}} = \frac{(G_F V_{ud})^2}{(2\pi)^5} \frac{F(Z,E_e)|\vec{p}_e|E_\nu}{m_n[E_p + E_\nu + E_e(\vec{\beta}\cdot\hat{p}_\nu)]}|M|^2 \,,$$

where

$$\begin{aligned}|M|^2 = m_n m_p E_e E_\nu &\left(1 + \frac{\alpha}{2\pi}e_V^R + \frac{\alpha}{2\pi}\delta_\alpha^{(1)}\right) C_0(E_e)(1+3\tilde{g}_A^2) \\ &\times \left\{1 + \left(1 + \frac{\alpha}{2\pi}\delta_\alpha^{(2)}\right) C_1(E_e)\vec{\beta}\cdot\hat{p}_\nu \right. \\ &\left. + \left(1 + \frac{\alpha}{2\pi}\delta_\alpha^{(2)}\right)\left[C_2(E_e) + C_3(E_e)\vec{\beta}\cdot\hat{p}_\nu\right]\hat{n}\cdot\vec{\beta} + \left[C_4(E_e) + C_5(E_e)\vec{\beta}\cdot\hat{p}_\nu\right]\hat{n}\cdot\hat{p}_\nu\right\} \,,\end{aligned}$$

and

$$\begin{aligned}\tilde{g}_A &= g_A\left[1 + \frac{\alpha}{4\pi}\left(e_A^R - e_V^R\right)\right] \,, \\ e_{V,A}^R(\mu) &= e_{V,A} - \frac{1}{2}(e_1+e_2) + \frac{3}{2}\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 1\right] + 3\ln\left(\frac{\mu}{m_N}\right) \,,\end{aligned}$$

where we have employed the dimensional regularization in $d = 4 - 2\epsilon$ for the loops.

Results (1) (Cont.)

$$\delta_\alpha^{(1)} = 3\ln\left(\frac{m_N}{m_e}\right) + \frac{1}{2} + \frac{1+\beta^2}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{1}{\beta}\ln^2\left(\frac{1+\beta}{1-\beta}\right) + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right)$$
$$+4\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]\left[\ln\left(\frac{2(E_e^{max}-E_e)}{m_e}\right) + \frac{1}{3}\left(\frac{E_e^{max}-E_e}{E_e}\right) - \frac{3}{2}\right]$$
$$+\left(\frac{E_e^{max}-E_e}{E_e}\right)^2 \frac{1}{12\beta}\ln\left(\frac{1+\beta}{1-\beta}\right),$$

$$\delta_\alpha^{(2)} = \frac{1-\beta^2}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) + \left(\frac{E_e^{max}-E_e}{E_e}\right)\frac{4(1-\beta^2)}{3\beta^2}\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]$$
$$+\left(\frac{E_e^{max}-E_e}{E_e}\right)^2 \frac{1}{6\beta^2}\left[\frac{1-\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right],$$

where E_e^{max} is the electron maximum energy $E_e^{max} = (m_n^2 - m_p^2 + m_e^2)/(2m_n)$ and $L(x)$ is the Spence function

$$L(x) = \int_0^1 \frac{dx}{x}\ln(1-x)\,.$$

Results (1) (Cont.)

$$C_0(E_e) = 1 + \frac{1}{m_N(1+3\tilde{g}_A^2)}\Big\{(\tilde{g}_A^2 - 2\mu_V\tilde{g}_A + 1)E_e^{max} - \frac{m_e^2}{E_e}(1+\tilde{g}_A^2) + 2\mu_V\tilde{g}_A(1+\beta^2$$

$$C_1(E_e) = \tilde{a}\Big\{1 + \frac{1}{m_N}\Big[\frac{(\tilde{g}_A^2 + 2\mu_V\tilde{g}_A + 1)}{1+3\tilde{g}_A^2}\frac{m_e^2}{E_e} + \frac{(\tilde{g}_A^2+1)[8\mu_V\tilde{g}_A E_e - 4E_e^{max}\tilde{g}_A(\tilde{g}_A+\mu_V)]}{(\tilde{g}_A^2-1)(1+3\tilde{g}_A^2)}\Big]\Big\},$$

$$C_2(E_e) = \tilde{A}\Big\{1 + \frac{1}{m_N}\Big[\frac{(\tilde{g}_A^2-1)(\tilde{g}_A+\mu_V)}{2\tilde{g}_A(1+3\tilde{g}_A^2)}(E_e^{max} - E_e) + \frac{E_e(\mu_V - 1)}{\tilde{g}_A - 1} - \beta^2 E_e\frac{\tilde{g}_A^2 + 2\tilde{g}_A\mu_V + 1}{1+3\tilde{g}_A^2}\Big]\Big\},$$

$$C_4(E_e) = \tilde{B}\Big\{1 + \frac{1}{m_N}\Big[\frac{E_e\beta^2(\tilde{g}_A^2-1)(\tilde{g}_A-\mu_V)}{2\tilde{g}_A(1+3\tilde{g}_A^2)} + \frac{(\tilde{g}_A+\mu_V)(\tilde{g}_A-1)^2}{(\tilde{g}_A+1)(1+3\tilde{g}_A^2)}(E_e - E_e^{max})$$

$$C_3(E_e) = \tilde{A}\frac{E_e(\tilde{g}_A - \mu_V)}{2m_N\tilde{g}_A}, \quad C_5(E_e) = \tilde{B}\frac{(\tilde{g}_A+\mu_V)}{2m_N\tilde{g}_A}(E_e^{max} - E_e),$$

Results (2)

$$\begin{aligned} D_{FSI} &= \frac{1}{1+3g_A^2}\frac{\alpha E_e}{4m_N}\frac{1}{\beta}\Big\{(1+3g_A^2)[(\mu_V - g_A) - 3\mu_p(1-g_A)] \\ &\quad + \frac{m_e^2}{E_e^2}[(3+g_A)(\mu_V - g_A) + 3\mu_p(1-g_A)(1+3g_A)]\Big\} \\ &\quad + \frac{1}{1+3g_A^2}\frac{\alpha E_\nu}{4m_N}\frac{1}{\beta}8g_A(1-g_A)\,. \end{aligned}$$

- We reproduce the terms from Callan and Treiman (1961).
- We have a new term, but it does not appear in a relativistic calculation.

3. How to fix the LECs ?

How to fi x the LEC e^R_V (usually fi xed by experiment)

$$\frac{\alpha}{2\pi} e^R_V \simeq \Delta^V_R ?$$

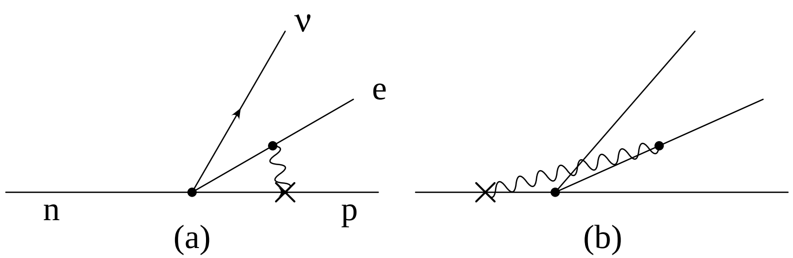
In the standard calculations (Marciano and Sirlin 86), one fi nds

$$\Delta^V_R \quad = \quad \frac{\alpha}{2\pi}\left[-4\ln\left(\frac{m_W}{m_Z}\right) + 3\ln\left(\frac{m_W}{m_N}\right) + \ln\left(\frac{m_W}{m_A}\right) + A_g + 2C\right],$$

- High energy part of $W\gamma$ box diagrams
- High energy part of $Z\gamma$ box diagrams
- High energy part of axialvector current induced $W\gamma$ box diagrams where m_A is a infrared cutoff
- pQCD correction A_g
- Low energy part of axialvector current induced diagrams C
  $\longrightarrow$ We may compare it with a result in the EFT calculation.

Model dependent term C: a difference

Low-energy axialvector current induced RC C in Δ_R^V



- In the standard (graphical) calculations, one has

$$C(Born)[\propto g_A(\mu_p+\mu_n)] = 0.881 \pm 0.030\,.$$

- In HBχPT calculation, they are higher order terms and

$$C(HB) \sim (\bar{Q}/m_N)^2 \sim 10^{-6}\,.$$

Other contributions to e_V^R

- Corrections due to $\rho - \omega$ mixing and isospin breaking from light quark mass difference are negligible. [Donoghue and Wyler, PLB241(1990)243.]
- Additional corrections: the resonances, $\Delta(1232)$, $N(1440)$, etc, in the C_{Born} diagrams.

What's wrong with the new term of D?

HB formalism

$$\frac{1}{i}\int \frac{d^4 l}{(2\pi)^4}\frac{l^\mu}{v\cdot(l+k)[(l-p_e)^2-m_e^2]l^2} = v^\mu f_1 + p_e^\mu f_2\,.$$

Relativistic formalism

$$\frac{1}{i}\int \frac{d^4 l}{(2\pi)^4}\frac{l^\mu}{[(l+p_p)^2-m_p^2][(l-p_e)^2-m_e^2]l^2} = p_p^\mu f_1' + p_e^\mu f_2'\,,$$

where $p_p^\mu = m_p v^\mu + k$ and $v^\mu = (1,\vec{0})$.

4. Discussion and conclusions

- The subleading results of EFT reproduce well low-energy model-independent terms for τ, a, A, B in the standard calculations, and thanks to the counting rules the higher order terms will be small, $\sim 10^{-5}$.
- The high-energy and low-energy model-dependent terms in the standard calculations are replaced by the two LEC's, e_V^R and $(e_A^R - e_V^R)$. We found that the estimations of the C term in the standard calculations and EFT are quite different.
- The D calculation is in progress.

Appendix: A problem of HBχPT

Nucleon propagator:

$$\frac{\gamma \cdot p + m_N}{p^2 - m_N^2} \quad \to \quad \frac{1}{v \cdot k} + \frac{1}{2m_N} \frac{(v \cdot k)^2 - k^2}{(v \cdot k)^2} + \cdots ,$$

where $p^\mu = m_N v^\mu + k^\mu$.

Up to a finite order, it does not reproduce analytic structure for on-shell nucleon.

$\longrightarrow$ Manifestly Lorentz invariant baryon ChPT with infrared and on-mass shell regularization schemes.