LESS KNOWN ASPECTS of NUCLEAR PAIRING

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- *A. Volya, B.A. Brown and V. Zelevinsky,*

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- *A. Volya, V. Zelevinsky and B.A. Brown,*

  Phys. Rev. C 65 (2002) 054312
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  Prog. Th. Phys. Suppl. 146 (2002) 636
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  Phys. Atom. Nucl. 66 (2003) 1781
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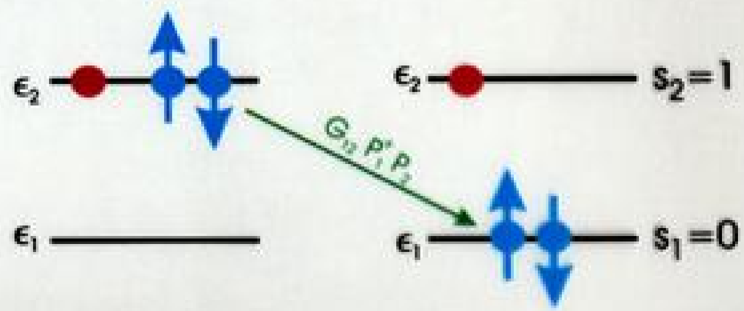
Topics for Discussion

- Exact solution of nuclear pairing
- Properties of the ground state
- Excited states
- New approximations
- Chaos and pairing
- Phase transitions
- Extending to continuum
- Ternary correlations

Pairing Hamiltonian

- Pairing on degenerate time-conjugate orbitals
- Pair operators *P* = (*a a*) (*J*=0, *T*=1)
- Number of unpaired fermions is seniority *s*
- Unpaired fermions are untouched by H

$$H = \sum_{1} \epsilon_1 N_1 - \sum_{12} G_{12} P_1^\dagger P_2$$



Approaching the solution of pairing problem

- *Approximate*
  - BCS - HFB theory
    + corrections + RPA
  - Iterative techniques
- *Exact solution*
  - Richardson solution (special choices of G)
  - Algebraic methods
  - Direct diagonalization + quasispin symmetry

SHORTCOMINGS of BCS

PARTICLE NUMBER NONCONSERVATION

$$|0\rangle = \prod_{\lambda\,(\text{doublets})} \{u_\lambda - v_\lambda P_\lambda^\dagger\}|\text{vac}\rangle$$

Projection correction: gauge freedom

$$v \Rightarrow v\, e^{i\phi}$$

(*losing the variational character of the solution...*)

NO SOLUTION for WEAK PAIRING

Simple model: $G_{\lambda\lambda'} = -G$ **const near Fermi-surface**

$$\Delta_\lambda = G \sum_{\lambda'} \frac{\Delta_{\lambda'}}{2E_{\lambda'}}$$

Nontrivial solution

$$1 = G \sum_{\lambda} \frac{1}{2E_\lambda}$$

Critical point, $G = G_c,\ \Delta = 0$

$$1 = G_c \sum_{\lambda} \frac{1}{2|\epsilon'_\lambda|}$$

Macroscopic superconductor:

$$1 = G\nu \int_0^{\hbar\omega} \frac{d\epsilon}{\sqrt{\epsilon^2 + \Delta^2}} \Rightarrow \Delta \approx 2\hbar\omega\ \exp\left(-\frac{1}{G\nu}\right)$$

Quasispin and exact solution of pairing problem (EP)

For each single j-level

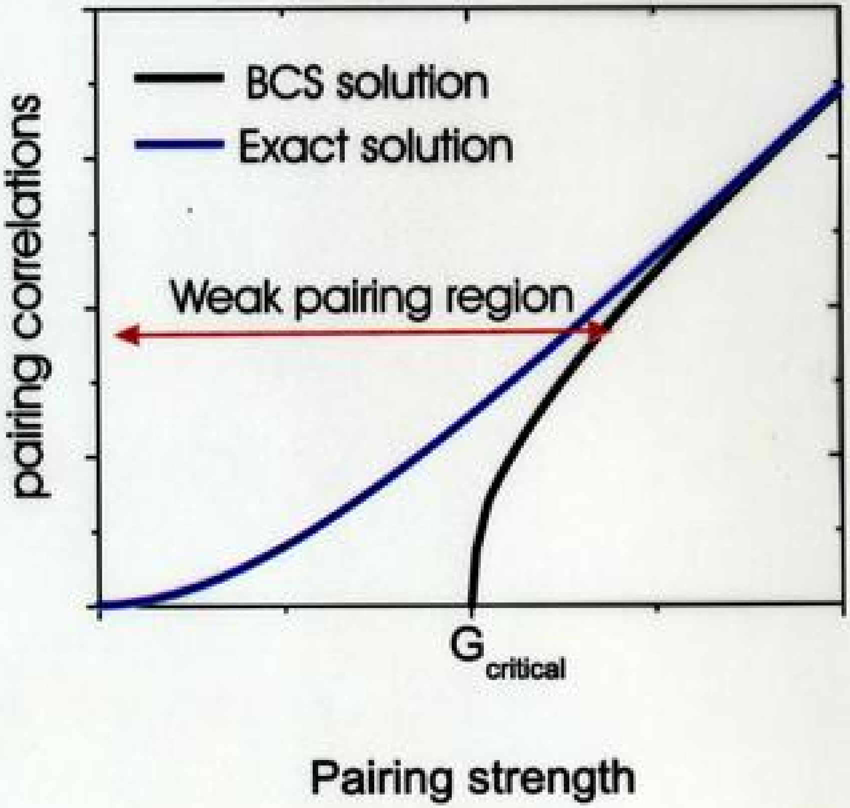
- Operators $P^{\dagger}_j$, P_j and N_j form a SU(2) group
  Quasispin L^2_j is a constant of motion,
  seniority $s_j=(2j+1) -2L_j$
- Each s_j is conserved but N_j is not
- Practically easy:
  Example: ^{116}Sn: 601,080,390 m-scheme states
  272,828 J=0 states
  110 s=0 states
- Generalization to isovector pairing, R_5 group

Drawbacks of BCS

- Particle number non-conservation
- Sharp phase transition
- No correlations beyond phase transition
- BCS fails for weak pairing
- Excited states and pair vibrations

Pairing phase transition

- BCS has a sharp phase transition



Occupation numbers and spectroscopic factors

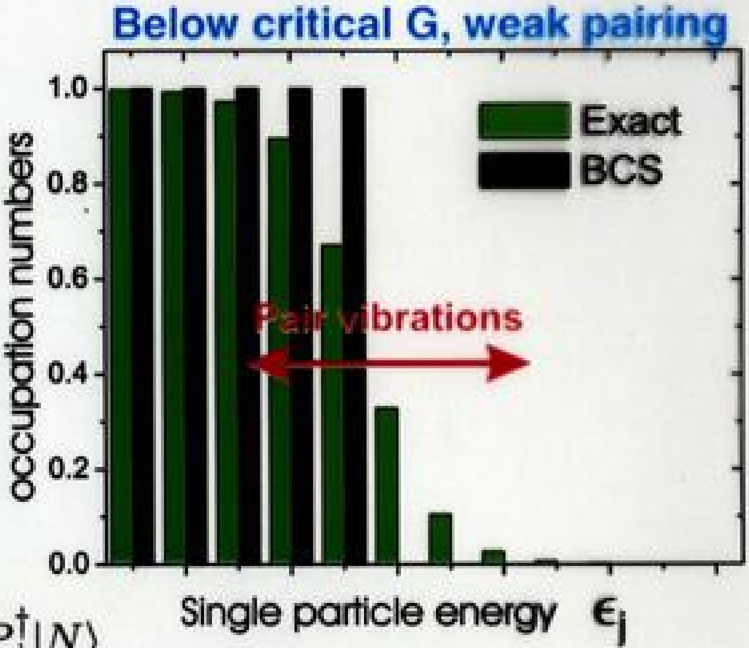
- Occupation numbers

  $n_j = \langle N | a_j^\dagger a_j | N \rangle$
- Spectroscopic factors

  $v_j = \langle N-1 | a_j | N \rangle$

  $u_j = \langle N+1 | a_j^\dagger | N \rangle$
- Two-body spectroscopic factors

$$\mathcal{P}_j(N) = \langle N-2 | P_j | N \rangle$$
$$\mathcal{P}_j^\dagger(N) = \mathcal{P}_J(N+2)^* = \langle N+2 | P_j^\dagger | N \rangle$$



BCS: $n_j = v_j^2 = 1 - u_j^2, \quad \mathcal{P}_j(\overline{N}) = \sqrt{n_j(1-n_j)} = u_j v_j$

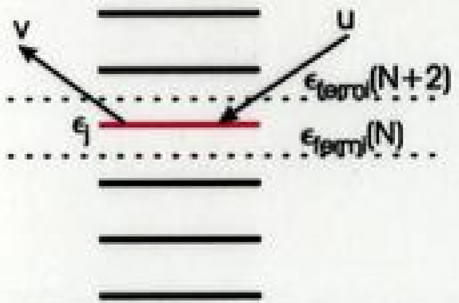
BCS works well: ^{114}Sn

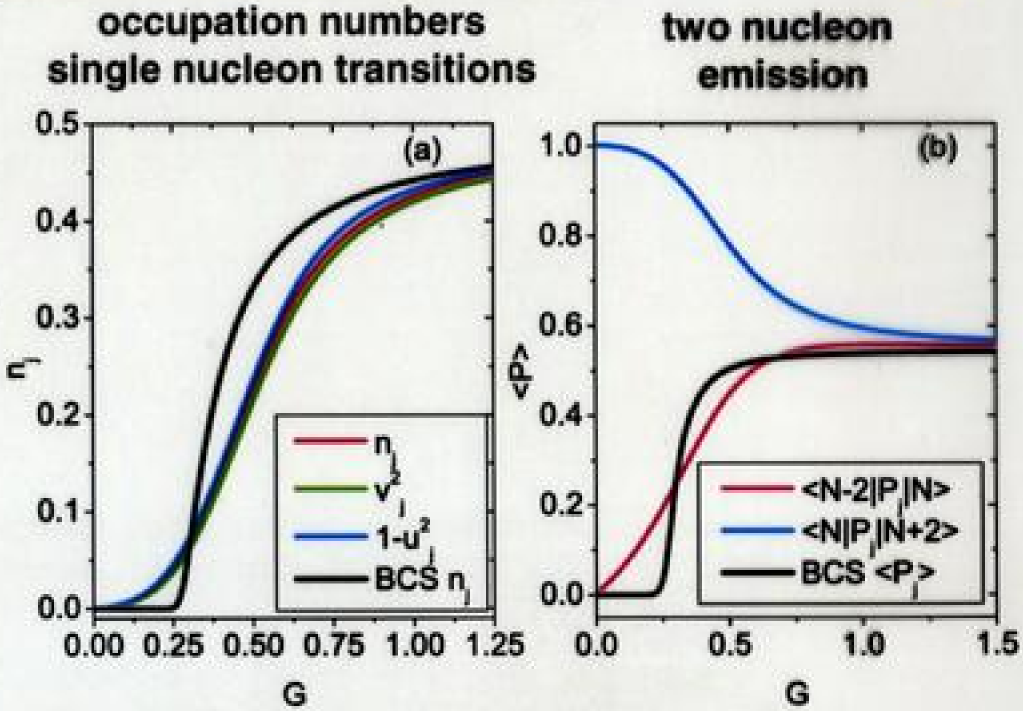
Exact calculation and BCS

Separation energy:
$S(N)=E(N-1)-E(N)$

j	$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$s_{1/2}$	$h_{11/2}$
N_j	6.96	4.46	0.627	0.356	1.60
N_j	6.71	4.14	0.726	0.507	1.91
n_j	0.870	0.744	0.157	0.178	0.133
$1-u^2_j$	0.872	0.748	0.162	0.183	0.137
v^2_j	0.865	0.736	0.155	0.177	0.131
n_j	0.839	0.690	0.181	0.254	0.159
S(N+1)	2.80	3.13	3.14	3.39	3.29
S(N+1)	2.89	3.21	3.11	3.21	3.26
S(N)	6.86	6.55	7.25	6.98	7.12
S(N)	6.89	6.64	7.20	7.03	7.06
P(N+2)	0.680	0.779	0.617	0.514	1.03
P(N)	0.810	0.930	0.524	0.396	0.845
P	0.734	0.801	0.545	0.435	0.896

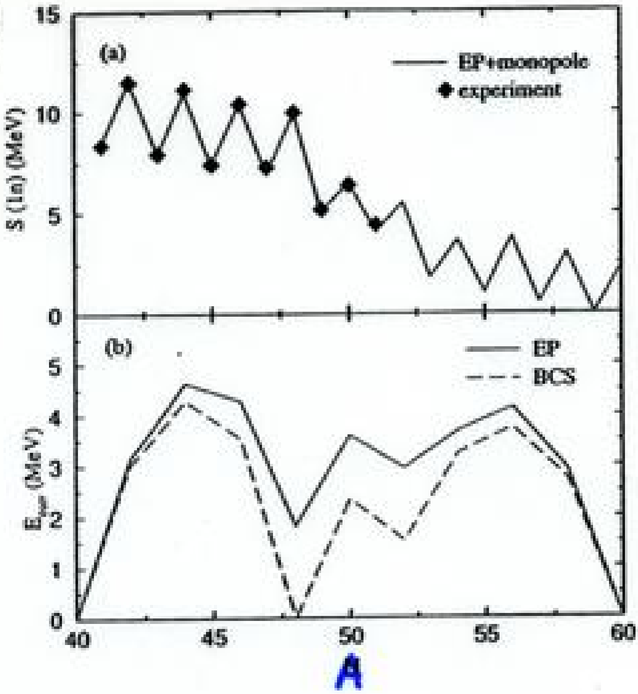
Ladder-system with constant G





Ladder with 12 levels, N=12

Ca isotopes



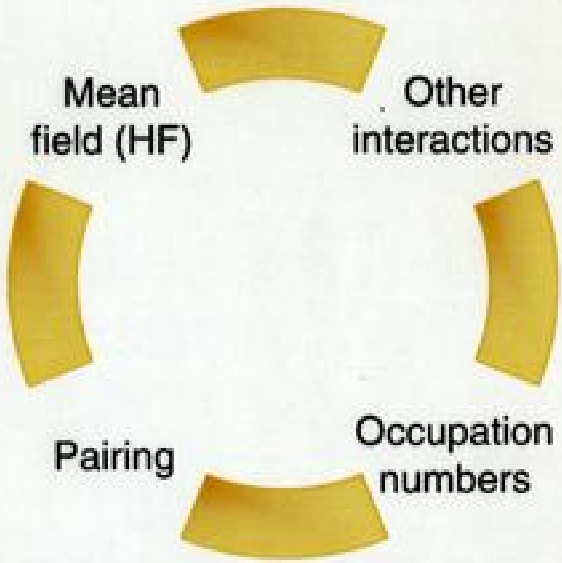
Monopole contribution: (FPD6)

$$\sum_j \bar{V}_{jj} \frac{N_j (N_j - 1)}{2} + \sum_{j \neq j'} \bar{V}_{jj'} N_j N_{j'}$$

$$\bar{V}_{jj} = \frac{1}{j(2j+1)} \sum_{L \neq 0} (2L+1) V_L(jj;jj), \quad \bar{V}_{jj'} = \frac{1}{(2j+1)(2j'+1)} \sum_{L \neq 0} (2L+1) V_L(jj';jj')$$

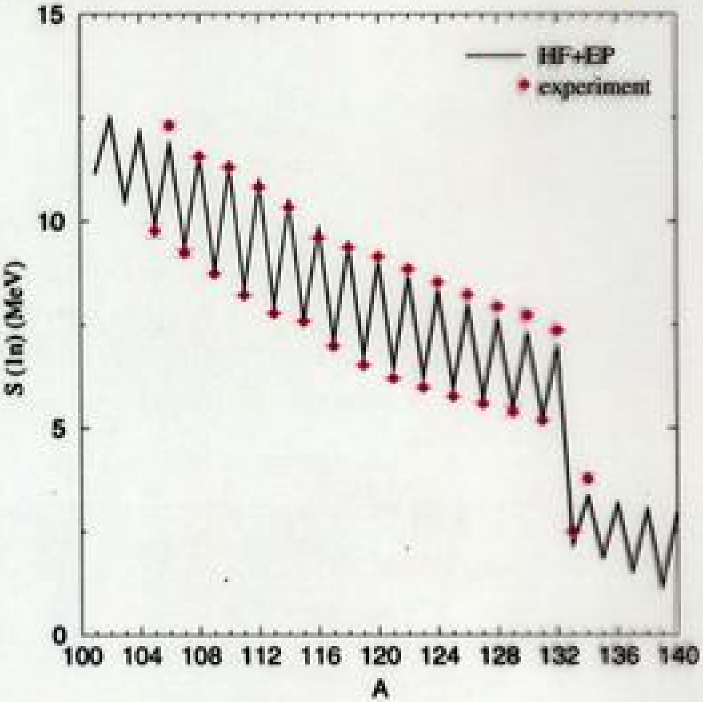
$(j \neq j')$

Hartree-Fock +EP



Hartree Fock +EP calculation: Sn isotopes.

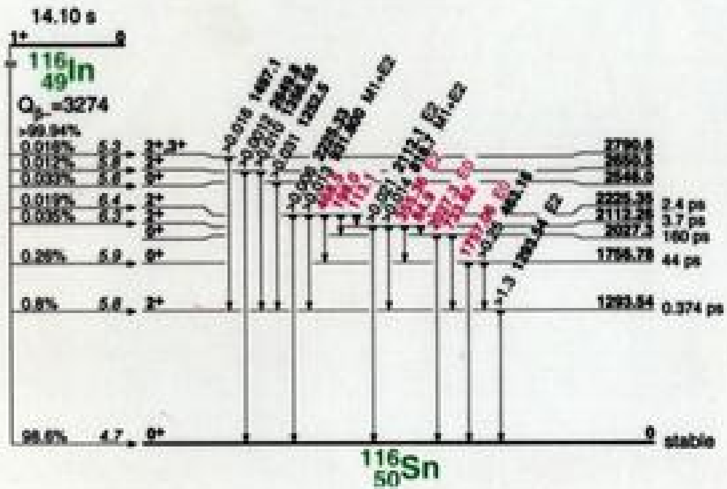
- We use Skyrme HF (SKX(1))
- Pairing matrix elements from G-matrix calculations (2)



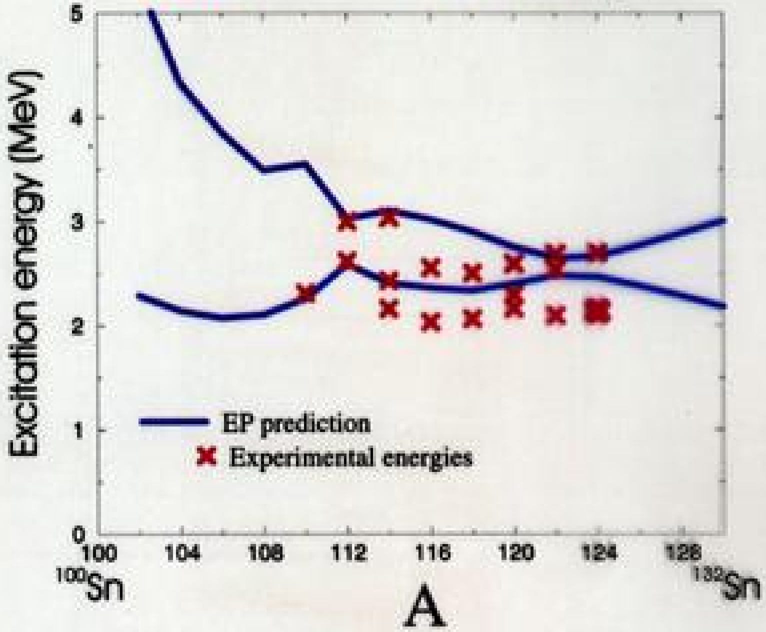
(1) B.A. Brown, Phys Rev. C **58** (1998) 220.
(2) A. Hold *et.al.*, Nucl. Phys. **A634** (1998) 41.

Pair vibrations in realistic cases

- J=0 pair-vibration states - "worsened" copies of the g.s.
- Less affected by other interactions
- Energy above 2 Δ

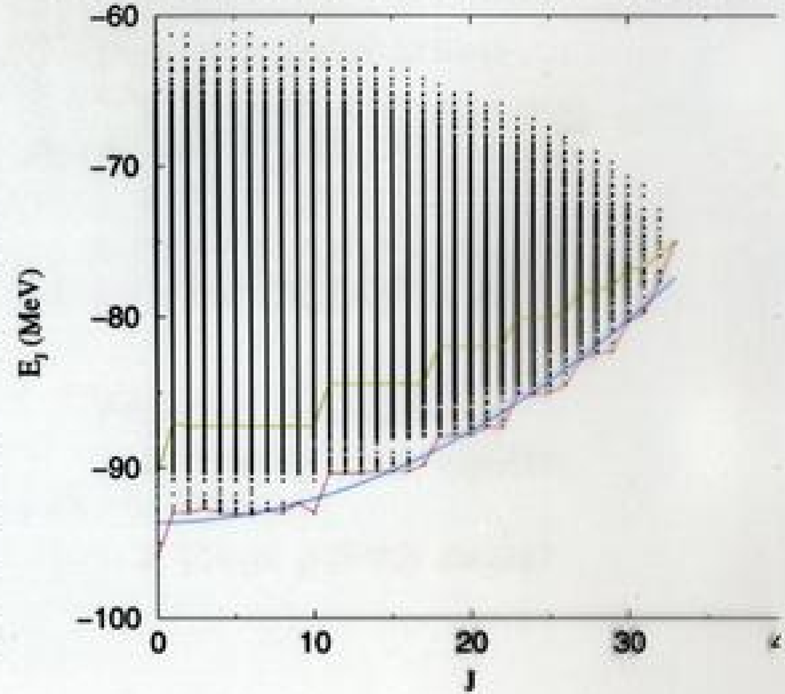


J=0 states in Sn isotopes



^{116}Sn example

Yrast line and moment of inertia



- Neutron system on $h_{11/2}, d_{3/2}, s_{1/2}, g_{7/2}, d_{5/2}$
- Realistic interactions Nijm-I G-matrix† (pairing part J=0, T=1)
- Exact solution determines all 601080390 m.b. states found in 420 representations, seniority=0 has 110 spin 0 states

Moment of inertia $E = \frac{\hbar^2 J(J+1)}{2I}$

EP I=32 MeV^{-1}, Rigid body I=35 MeV^{-1}, Experiment I=22 MeV^{-1}

† G.N. White *et al.* Nucl. Phys. A **644**, 277 (1998)

NEW ENTROPY

Response to noise - random parameter(s) λ:

$$|\alpha; \lambda\rangle = \sum_k C_k^\alpha(\lambda)|k\rangle$$

Density matrix of an individual state

$$(\rho_\alpha)_{kk'} = \langle C_k^\alpha(\lambda) C_{k'}^{\alpha *}(\lambda)\rangle_{\text{av over } \lambda}$$

Invariant correlational entropy (ICE)

$$S_\alpha = -\text{Trace}(\rho_\alpha \ln \rho_\alpha)$$

Sensitive to structural changes of wave functions

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$$t_j = \epsilon_j' \left(2y_j + \frac{\Omega_j}{2e_j} \Delta_j(y) \right). \tag{22}$$

This secular equation is also valid for the normal modes generated by pairing with no condensate present when we have only the trivial BCS solutions $X_j = 0$ and $\Delta_j = 0$, the vibrational amplitude z_j vanishes, and the single-particle energies are equal to $|\epsilon_j'|$. With the extension to higher orders of the same mapping procedure to the collective variables as in Eq. (11), one can study the anharmonic effects. However, if the anharmonicity is indeed important, as for example in the region of very low frequencies and correspondingly large amplitude collective motion, it is simpler (at least in the pure pairing problem) to switch to the exact solution.

B. Example: two-level system with off-diagonal pairing

As an example of the analytically solved RPA, we consider an interesting particular case of two levels with capacities $\Omega_{1,2}$ and single-particle energies $\epsilon_{1,2}$ when the pairing interaction has only the off-diagonal amplitude $V_{12} = V_{21} = g$ (in this case the sign of g does not matter). With the effective interaction parameters $\lambda_{1,2} = g\Omega_{1,2}/4$, the set of the BCS gap equations (17) takes the forms

$$\Delta_1 = \frac{\lambda_2}{e_2}\Delta_2, \quad \Delta_2 = \frac{\lambda_1}{e_1}\Delta_1, \tag{23}$$

which leads to the exact solutions

$$\Delta_1^2 = \frac{\lambda_1^2\lambda_2^2 - \epsilon_1'^2\epsilon_2'^2}{\lambda_1^2 + \epsilon_2'^2}, \quad \Delta_2^2 = \frac{\lambda_1^2\lambda_2^2 - \epsilon_1'^2\epsilon_2'^2}{\lambda_2^2 + \epsilon_1'^2}. \tag{24}$$

The corresponding quasiparticle energies are given by

$$e_1^2 = \lambda_1^2 \frac{\lambda_2^2 + \epsilon_1'^2}{\lambda_1^2 + \epsilon_2'^2}, \quad e_2^2 = \lambda_2^2 \frac{\lambda_1^2 + \epsilon_2'^2}{\lambda_2^2 + \epsilon_1'^2}, \tag{25}$$

with the useful identity

$$e_1 e_2 = \lambda_1 \lambda_2 \tag{26}$$

being valid. The BCS solution collapses, $\Delta_{1,2} \to 0$, at the critical coupling strength determined by $\lambda_1^2\lambda_2^2 = \epsilon_1'^2\epsilon_2'^2$, or at

$$g^2 \to g_c^2 = \frac{16|\epsilon_1'\epsilon_2'|}{\Omega_1\Omega_2}. \tag{27}$$

The secular equation [see Eqs. (21) and (23)], with the help of identity (26), gives, along with the spurious mode $\omega^2 = 0$, the physical root

$$\omega^2 = 4(e_1^2 + e_2^2 + 2\epsilon_1'\epsilon_2') = 4[(\epsilon_1' + \epsilon_2')^2 + \Delta_1^2 + \Delta_2^2]. \tag{28}$$

C. Exact solution versus BCS+RPA

The behavior of energies and entropy in the BCS phase transition region is illustrated in Fig. 3 for a two-level model, and in Fig. 4 for the realistic case. First we consider the two-level model, defined in Sec. IV B; see Fig. 3. In the case of half occupancy and two levels of equal capacity, $N = \Omega_1 = \Omega_2$, with energies $\pm\epsilon$ symmetric with respect to the chemical potential $\mu = 0$, the RPA predicts $\omega^2 = 8\Delta^2$, see Eq. (28). Note that in the case of many interacting levels with $V_{jj'} = \text{const}$ the spectrum of normal modes starts with the lower value $\omega^2 = 4\Delta^2$ (the threshold of pair breaking).

Panel (b) in the middle shows the energies of the lowest pair vibration state (thick solid line) and the lowest state with one broken pair (thick dashed line) as a function of the pairing strength g. The excitation energy of the pair vibration is compared with the RPA prediction shown by the thin dotted line. The BCS phase transition occurs, in the units corresponding to $|\epsilon_{1,2}| = 1$, at $g = g_c = 0.25$, where we see the breakdown of the RPA and the vanishing collective frequency. Below this point the RPA is constructed on the background of the normal Fermi distribution, while, above $g = g_c$ the RPA is built on the pairing condensate. The limits $g \to 0$ and $g \to \infty$ are described well within the RPA, while near the phase transition large fluctuations make the BCS +RPA description unreliable. The failure of the BCS is well discussed in the literature and studied using simple models [27]. In accordance with this instability, a sharp rise of en-

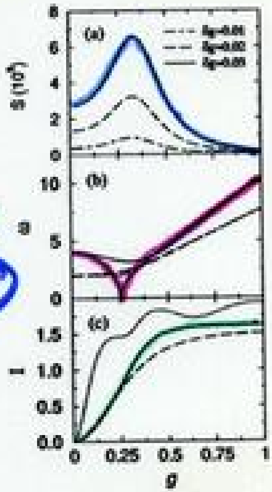
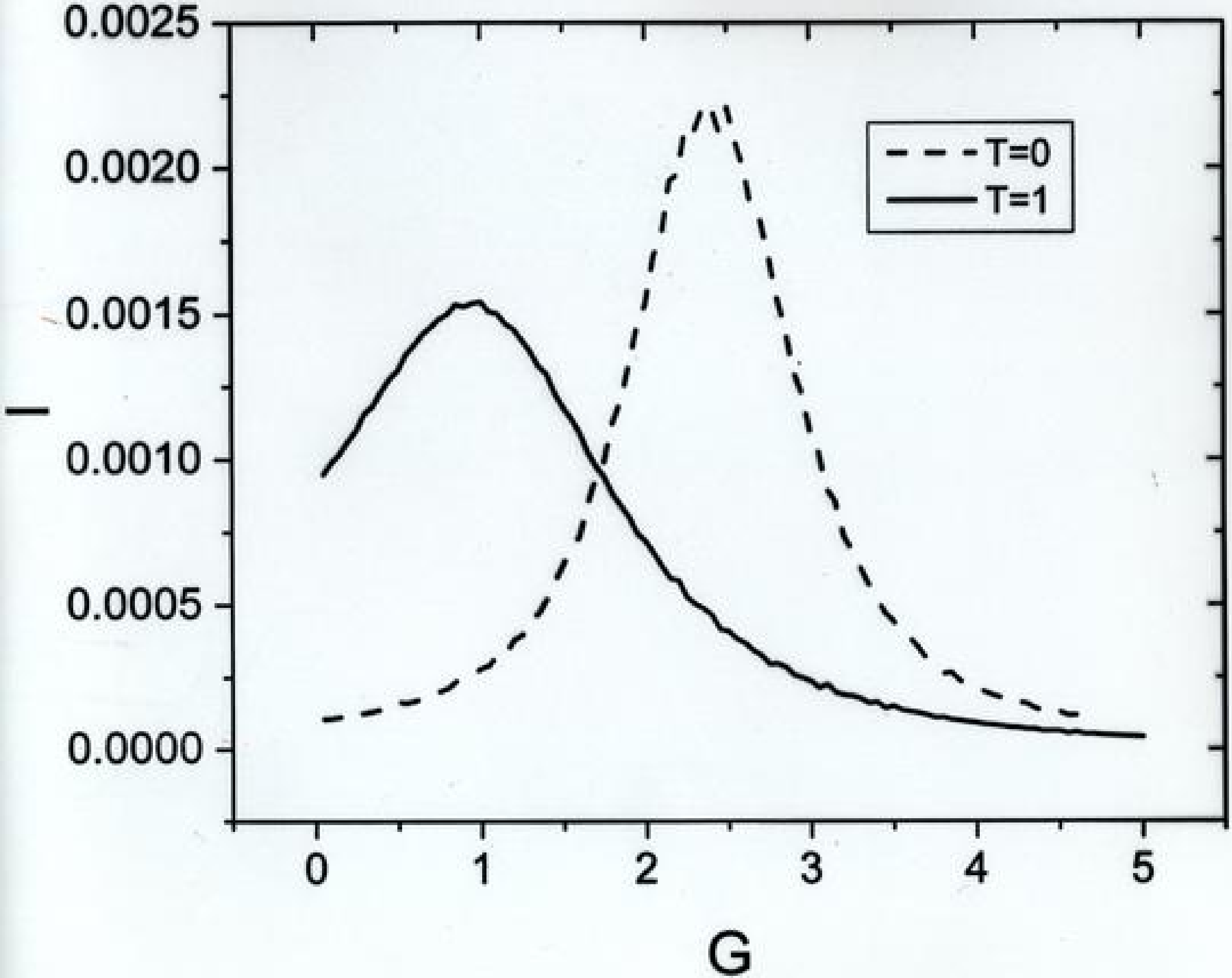
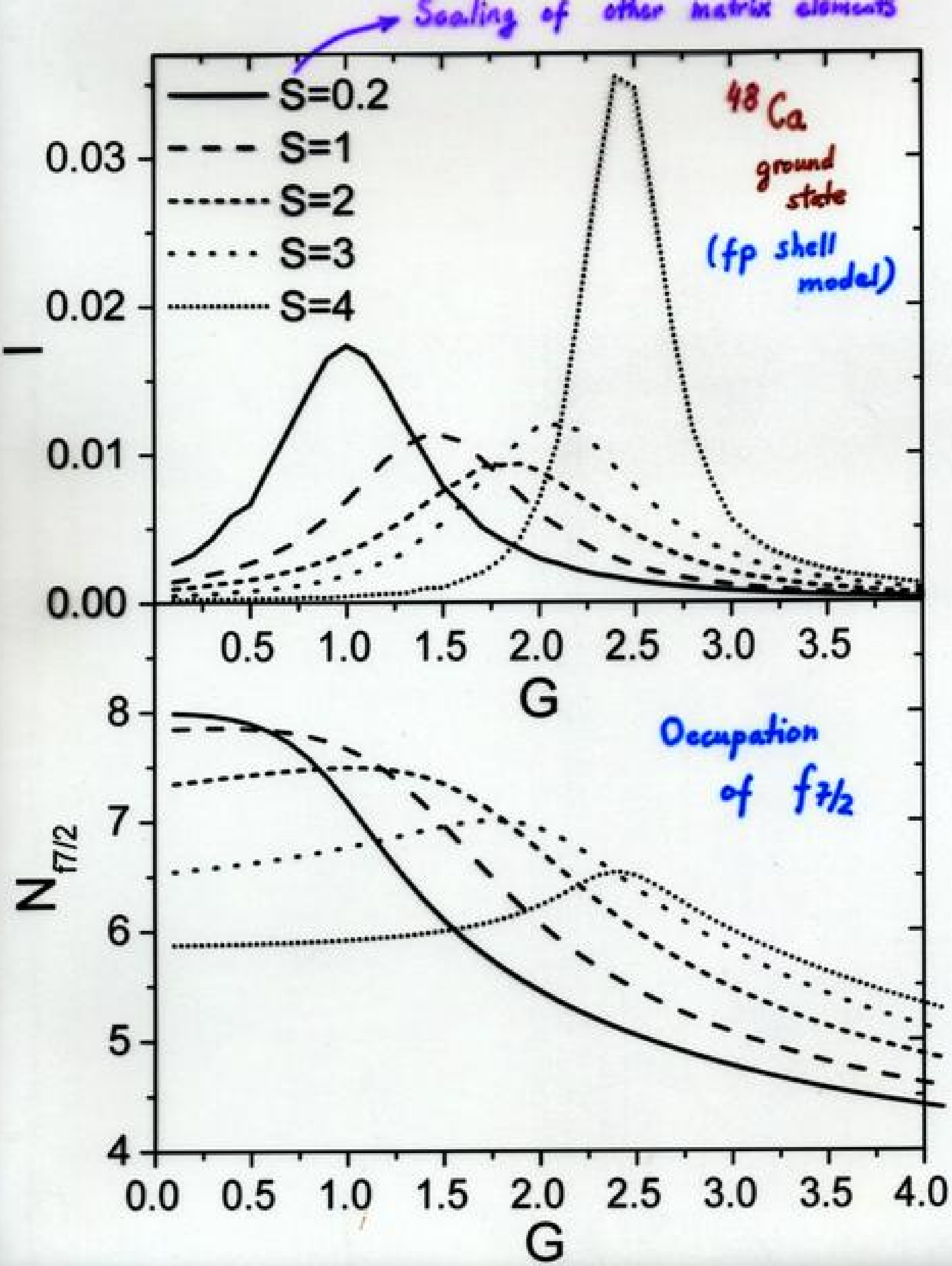


FIG. 3. Two-level pairing model $N = \Omega_1 = \Omega_2 = 16$; only the off-diagonal pair transfer amplitude $V_{12} = V_{21} = g$ is taken into account. The upper plot (a) displays the invariant entropy (see the text), of the ground state for $\delta g = 0.01$, 0.02, and 0.03 averaging intervals. The middle part (b) shows the excitation energy of the lowest pair vibration state (thick solid line). The thin dotted line approximates this curve with the aid of the RPA built on the normal Fermi ground state on the left of the phase transition point at $g = 0.25$ and on the BCS ground state on the right of the critical point. The thick dashed line shows the excitation energy of the lowest state with broken pair $s = 2$. This curve is compared with $2e$ (thin dot-dashed curve), the BCS energy of a two-quasiparticle state. The lower panel (c) displays information entropy for the ground state (solid line), second pair vibration excited state $s = 0$ (dotted line), and the lowest $s = 2$ state (dashed line).



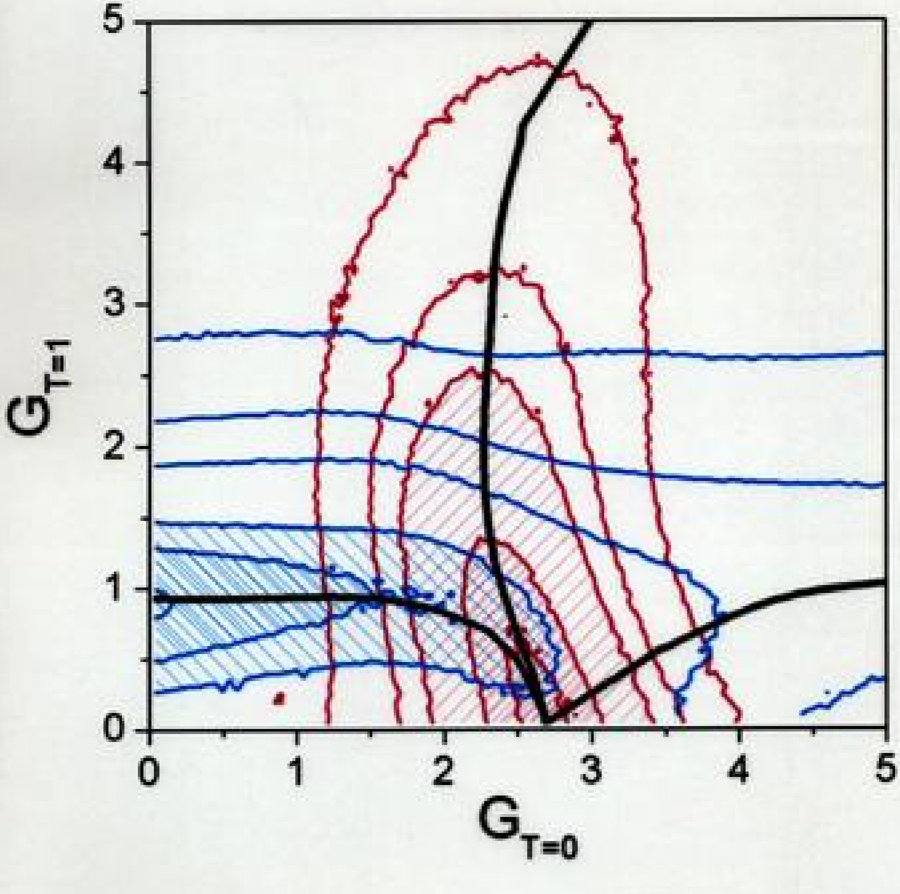
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
I
0
1
2
3
4
5
G
T=0
T=1

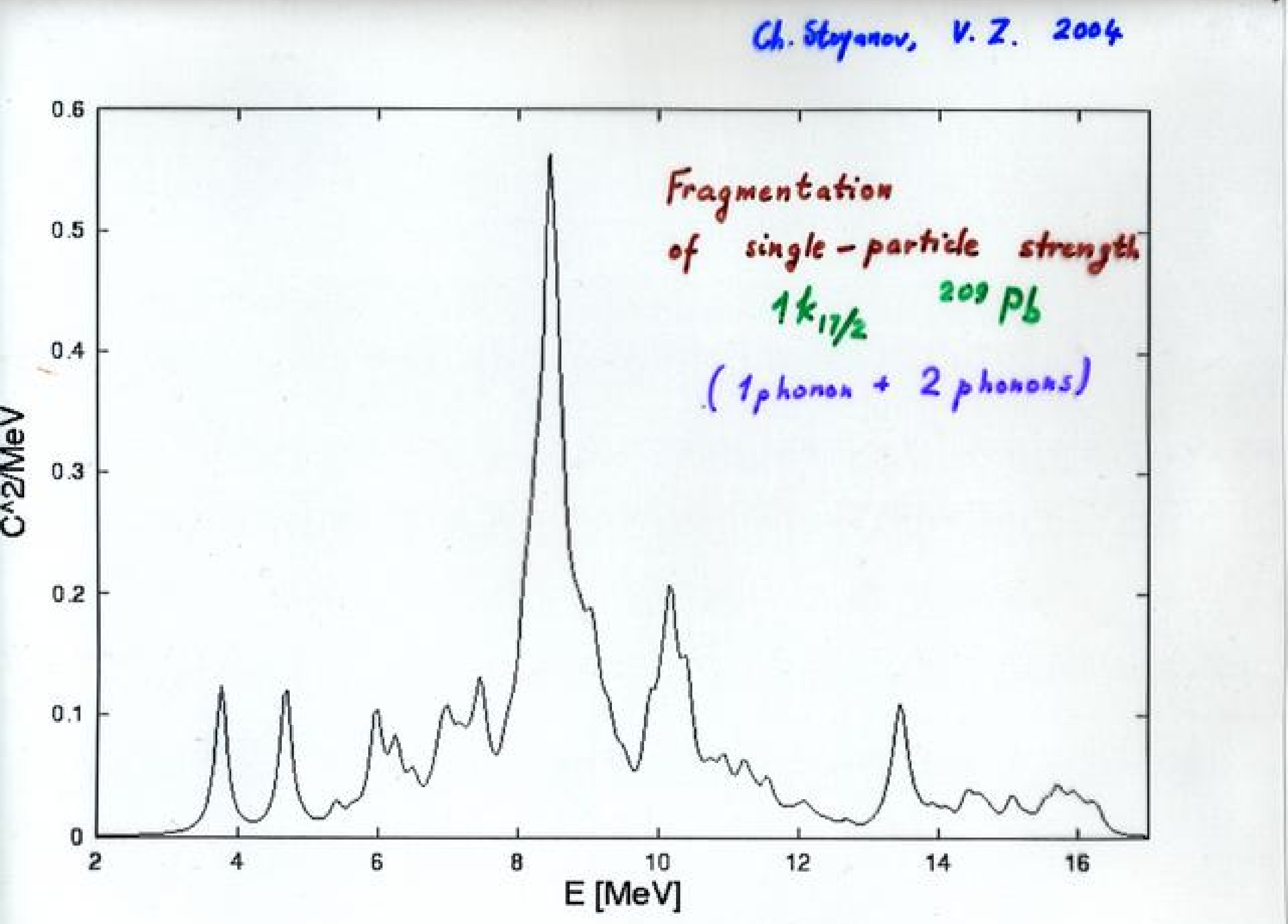


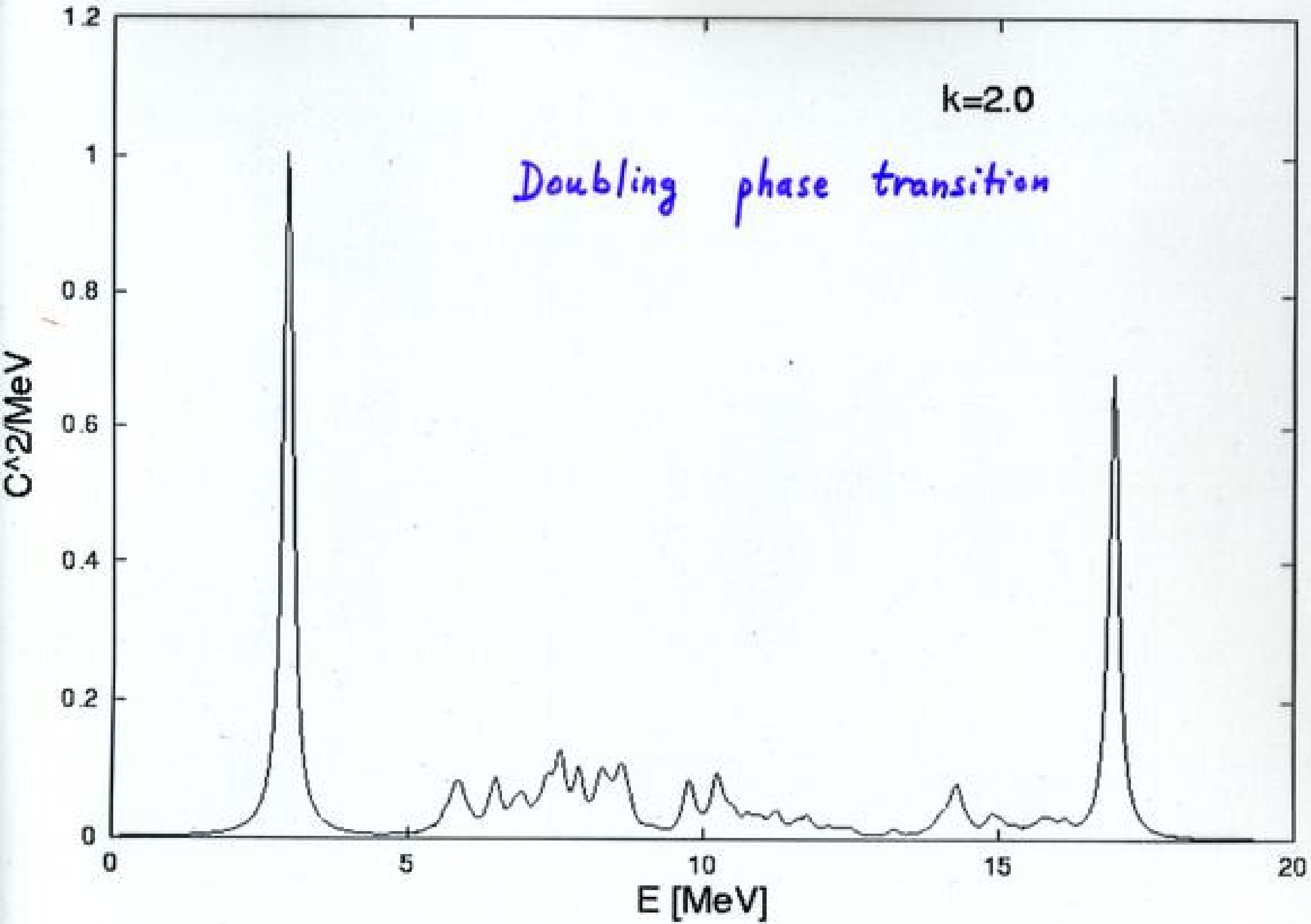
Scaling of other matrix elements
S=0.2
S=1
S=2
S=3
S=4
48 Ca
ground state
(fp shell model)
I
0.00
0.01
0.02
0.03
0.5
1.0
1.5
2.0
2.5
3.0
3.5
G
Occupation of f7/2
N f7/2
4
5
6
7
8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
G

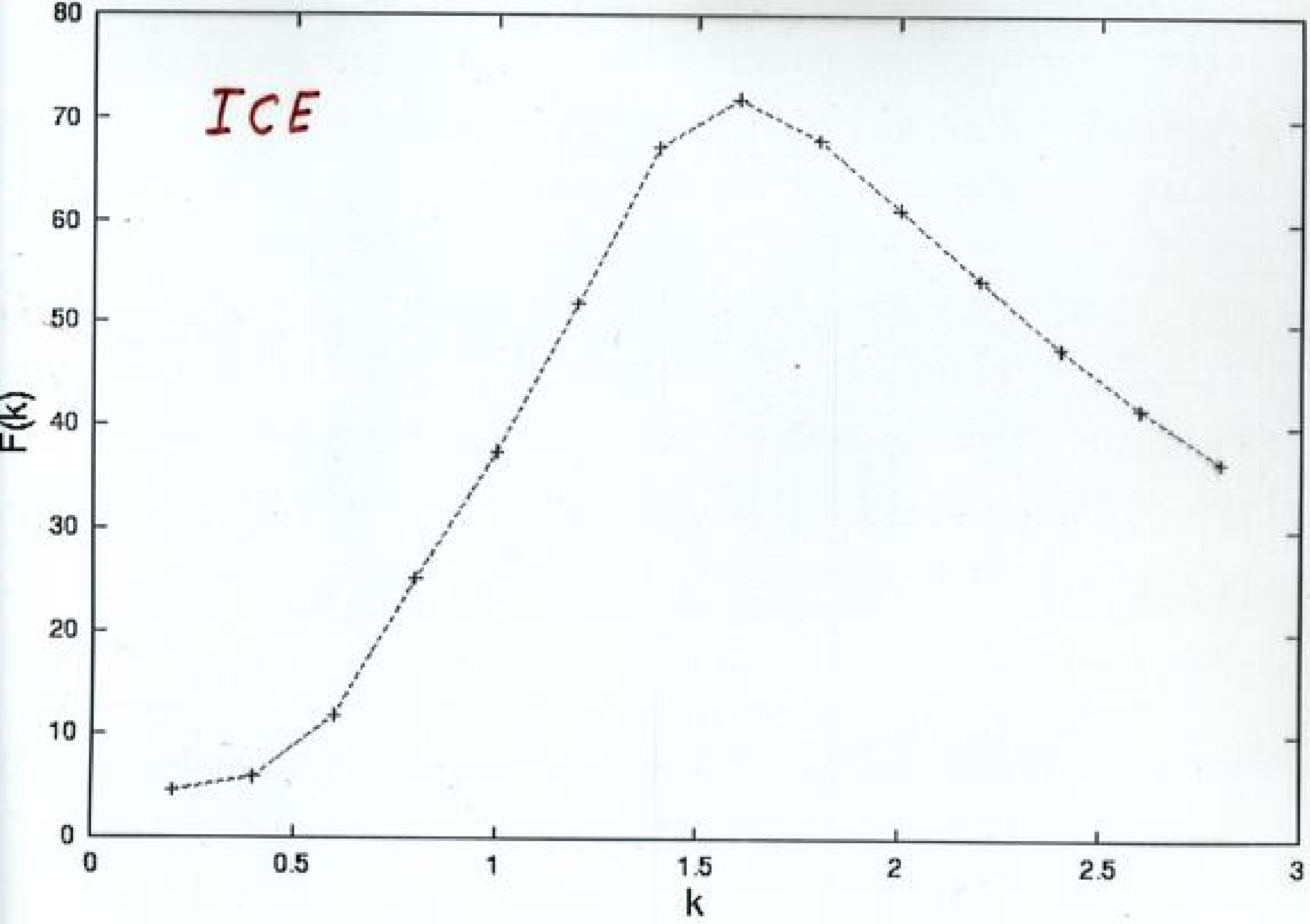
Phase diagram ^{24}Mg

(invariant entropy)




Ch. Stoyanov, V. Z. 2004
Fragmentation
of single-particle strength
$1k_{17/2}$ ^{209}Pb
(1phonon + 2 phonons)
C^2/MeV
E [MeV]
0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8
10
12
14
16


k=2.0
Doubling phase transition
C^2/MeV
E [MeV]
0
0.2
0.4
0.6
0.8
1
1.2
5
10
15
20



ICE
F(k)
k

sd-SHELL MODEL ^{24}Mg

(a) degenerate $\epsilon_{\text{s.p.}}$, 63 random m. e.
$J_0 = 0, T_0 = 0 \quad$ 59.1%; overlap 2%

(b) realistic $\epsilon_{\text{s.p.}}$, 63 random m.e.
$J_0 = 0, T_0 = 0 \quad$ 49.3%; overlap 5.3%

(c) realistic $\epsilon_{\text{s.p.}}$ and 6 pairing m.e., 57 random m.e.
$J_0 = 0, T_0 = 0 \quad$ 67.8%; overlap 10.6%

(d) degenerate $\epsilon_{\text{s.p.}}$, 6 random pairing m. e.
$J_0 = 0, T_0 = 0 \quad$ 92.2%; overlap 5.2%

Many spins 1/2: $J_0 = 0, T_0 = 0 \quad$ 99%
Quantum glass $J_0 \sim \sqrt{N}, \quad H = \sum_{12} J_{12}(\mathbf{s}_1 \cdot \mathbf{s}_2)$

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single-j level model that the ground-state wave function carries very little effect of pairing correlations. However, this overlap is still greater than one would expect in the case of extreme chaoticity when the components C_k of the wave function are uniformly distributed over the unit sphere in space of the corresponding dimension N and $\overline{|C|^2} = 1/N$ which gives rise to the so-called N-scaling [17,18]. In our case the dimension for the $J=0, T=0$ states is $N = 325$ which would give the average chaotic overlap factor 0.3%.

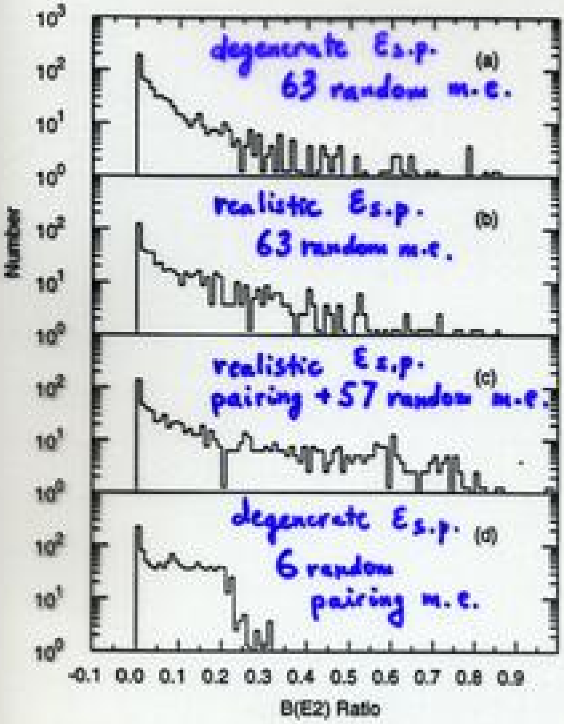

FIG. 2. Distribution of $B(E2)_R/B(E2)_W$ values from the first 2^+0 state to the 0^+0 g.s. for models (a-d), where $B(E2)_R$ are the values obtained from the random interactions and $B(E2)_W$ is the value from the W interaction.

The overlap is even greater in other models. The maximum of 11% is reached in model (c) because of the combined action of two effects. First, the presence of realistic pairing lowers the energy of a state with paired particles. On the other hand, basis states with large seniority (the number of unpaired particles) are now effectively removed from contributing considerably to the ground-state wave functions. This makes the effective dimension N smaller than the nominal one. This phenomenon was clearly seen for a simple $N = 3$ single-j case in Ref. [11]. The stabilizing presence of the mean-field orbitals, model (b), also increases the overlap with the realistic ground-state wave function.

In Fig. 1 it is seen that models (a) and (d) have overlaps which are strongly peaked at small values. Model (c) leads to a more smooth and uniform overlap distribution. As discussed for the average values, this means that the realistic mean-field orbitals (given by their single-particle energies) have a strong influence on the overlap. The overlap is more enhanced by realistic pairing (c), but it is still far from unity.

The predominantly chaotic nature of the low-lying states is confirmed by the weakness of multipole-multipole correlations produced by the random interactions. As an illustration, results for the quadrupole transition probabilities from the lowest 2^+ state to the ground 0^+ state are shown at the end of Table I and in Fig. 2. Typically, the $B(E2)$ value is by more than an order of magnitude weaker than obtained with the realistic interactions. The distribution of the $B(E2)$ values for model (a) is close to the Porter-Thomas as one expected for matrix elements of one-body operators between two complicated states [17,19]; the 2^+ state is even less ordered than the ground state. A trace of collective strength appears in the model (c). In this respect one can recall that low-lying collective vibrations, in contrast to high-lying giant resonances that are less sensitive to the residual interactions, emerge only in a superfluid Fermi-system. In a normal Fermi-system, the low-lying vibrations are not shifted outside the particle-hole continuum and have only a single-particle strength [20]. It means that again we see the pronounced pairing effects only if the residual interaction explicitly contains the pairing part.

In conclusion, with the aid of random rotationally- and isospin-invariant two-body interactions in the sd shell model we have studied the main features of the structure of the ground and low-lying eigenstates. We confirm the strong enhancement of the probability of the quantum numbers $J^\pi T = 0^+0$ for the ground states. However, the resulting ground-state wave functions have only a weak overlap with the realistic ground states that depends on the specific model of randomness. No considerable pairing effects generated by the random interactions were observed. The quadrupole transitions between the lowest 2^+ states and the ground states also do not reveal significant collectivity. We can claim that the apparent regular geometric pattern of the low-lying spectra coexists, in the case of the random two-body Hamiltonian, with mainly incoherent ("chaotic") structure of the eigenfunctions. Small hints of coherent components in the wave functions generated presumably by the off-diagonal pairing matrix elements and observed also in the earlier studies [11] require a more detail analysis.

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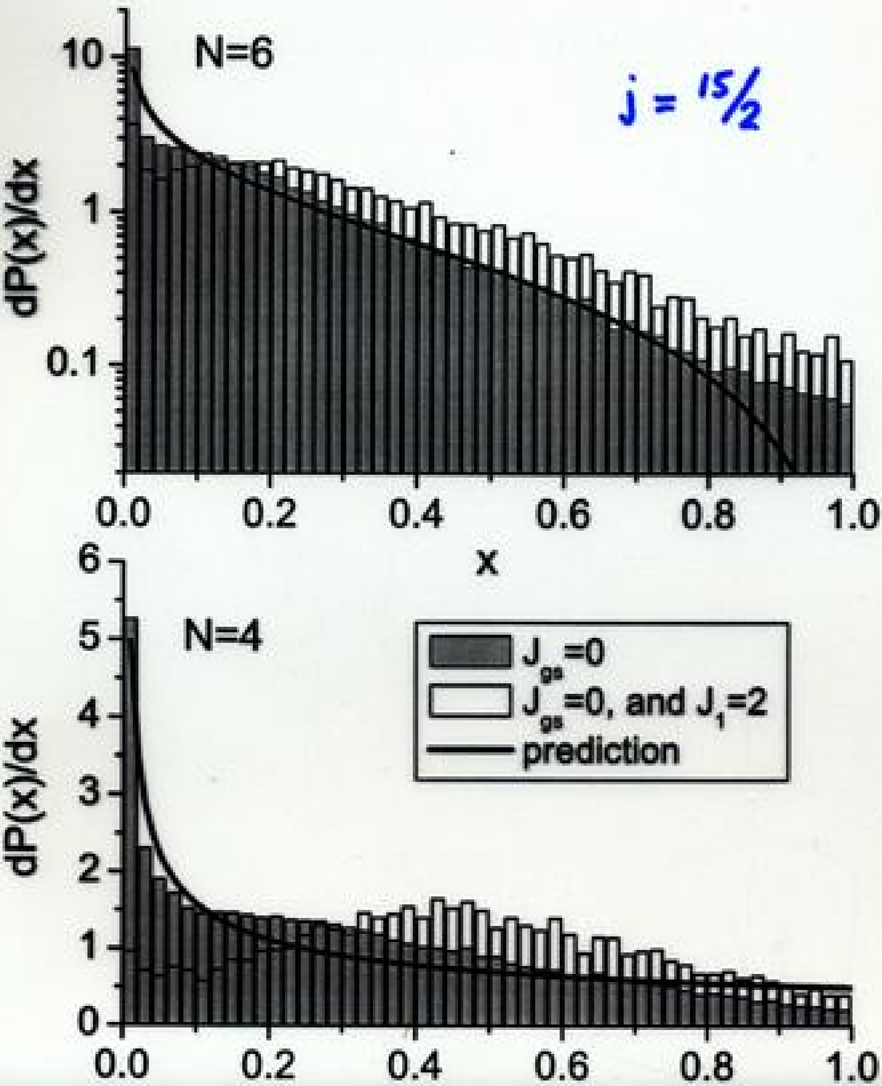
Overlap

$$x = \left| \langle \text{g.s. paired} \,|\, \text{g.s. random int.} \rangle \right|^2$$
$(s=0)$



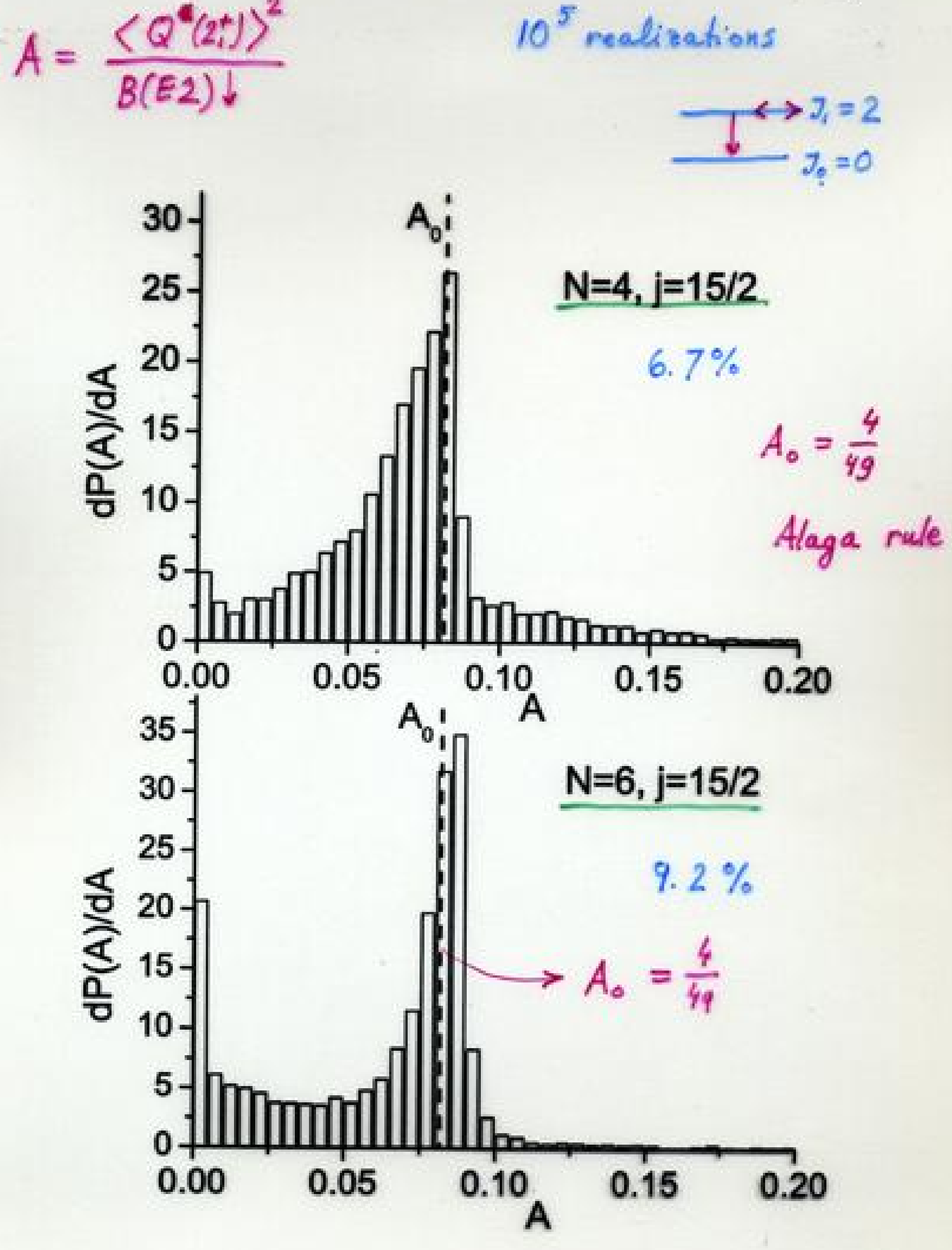
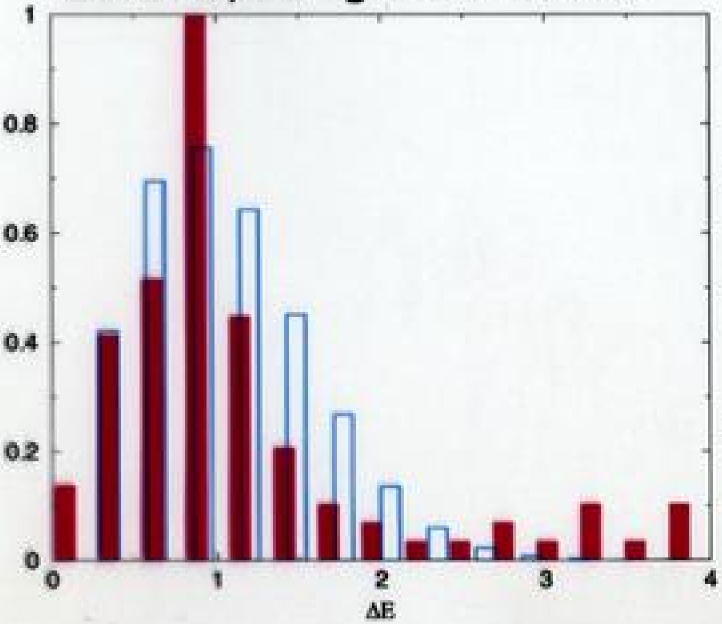


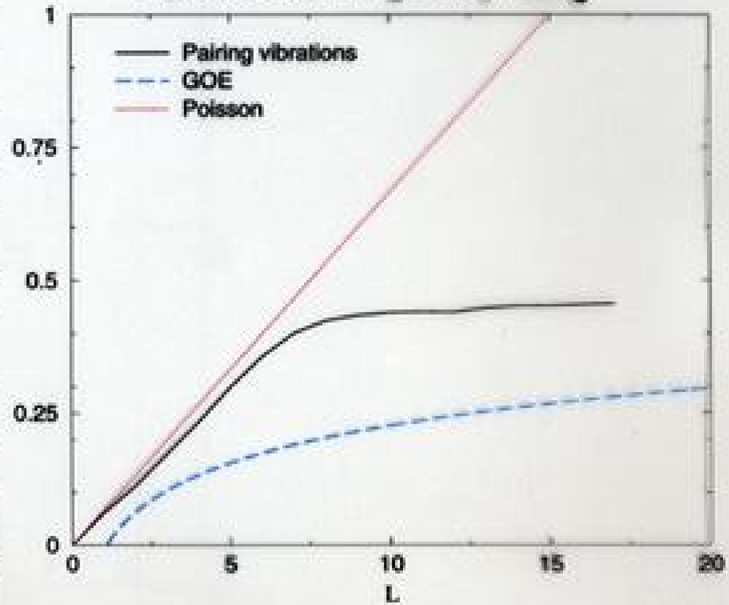
FIG. 1: Single-j system $j = 15/2$ is considered with $N = 4$ and $N = 6$ particles, upper and lower panels, respectively. Dynamics of these systems is driven by two-body random interactions, so that each matrix elements V_L has a gaussian distribution centered at zero and width 1. 100000 random realizations are considered in both cases. From all these realizations only those that result in the J=0 ground state and J=2 first excited state were selected. The number of such cases is 6650 (i.e. 6.7 %) for $N = 4$, and 9240 (9.2 %) for $N = 6$. The histogram shows the distribution of the ratio $A = Q^2/B(\mathrm{E2})$ for all selected cases. Here Q is the quadrupole moment of the first excited $J = 2$ state and $B(\mathrm{E2})$ is a transition strength for decay of this excited state

Is there chaotic in pairing

Level Spacing Distribution



Spectral Rigidity Δ_3



TERNARY CORRELATIONS

Remnants of three-body forces??

$$H^{(3)} = \sum_{1231'2'3'} g(123;1'2'3')a_1^\dagger a_2^\dagger a_3^\dagger a_{3'} a_{2'} a_{1'}$$

Possible collective effects

- **Monopole correction to pairing**

$$(\mathrm{Pair})_0(a^\dagger a)_0(\mathrm{Pair})_0$$

- **Cubic quadrupole anharmonicity**

$$[(\mathrm{Phonon})_2(\mathrm{Phonon})_2(\mathrm{Phonon})_2]_0$$

- **Shape fluctuations and giant dipole resonance**

$$[(\mathrm{Phonon})_1(\mathrm{Phonon})_2(\mathrm{Phonon})_1]_0$$

- **Renormalization of octupole mode**

$$[(\mathrm{Phonon})_3(\mathrm{Phonon})_2(\mathrm{Phonon})_3]_0$$

- ......

MONOPOLE CORRECTION TO PAIRING

$$H = H_0 + H_P + H', \quad H_P = -\sum_{12} G_{12} P_1^\dagger P_2$$

$$H' = \sum_{12;3m} g_{12;3} T_{1;3m}^\dagger T_{2;3m}, \quad T_{1;3m} = a_{3m} P_1$$

Degenerate case:

$$H = \epsilon N - G P^\dagger P + g P^\dagger N P$$

Renormalization, $G \Rightarrow G - g(N-2)$.
Odd-even mass difference

$$\Delta = E_1(N+1) - \frac{1}{2}\left[E_0(N) + E_0(N+2)\right] = \frac{1}{4}\left[G\Omega - gN(\Omega - 2)\right]$$

Xe isotopes: 126-136

$$G \approx 0.5 \text{ MeV}, \quad g \approx 26 \text{ keV}$$

BCS-type solution:

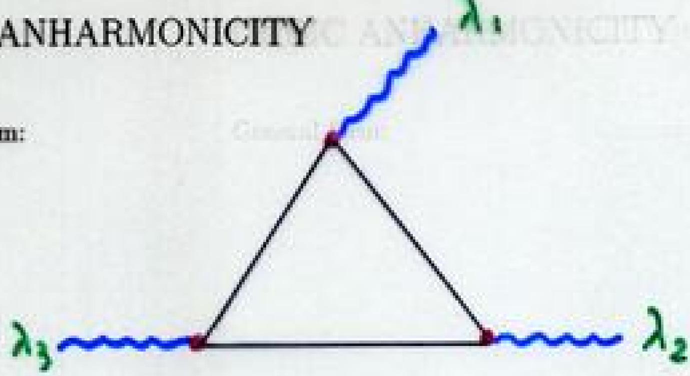
$$\Delta_1' = \sum_2 \Omega_2 \frac{\Delta_2'}{4E_2'} \left(G_{12} - \sum_3 g_{12;3} \langle n_3 \rangle \right)$$

$$\langle n_1 \rangle = \Omega_1 v_1'^2$$

$$\epsilon_1' = \epsilon_1 + \sum_{23} g_{32;1} \Omega_2 \Omega_3 \frac{\Delta_2' \Delta_3'}{16 E_2' E_3'}$$

CUBIC ANHARMONICITY

General form:



For <u>identical</u> phonons - suppressed by *Furry theorem*

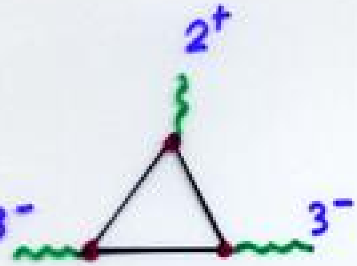
particle–hole symmetry $\sim (u^2 - v^2)$

Large contribution to the width of Isovector Giant Resonances (Quadrupole and Dipole);

- microscopic mechanism of shape fluctuations

$\lambda_2 = \lambda_3 = 2^+$ IVGQR

$\lambda_2 = \lambda_3 = 1^-$ IVGDR

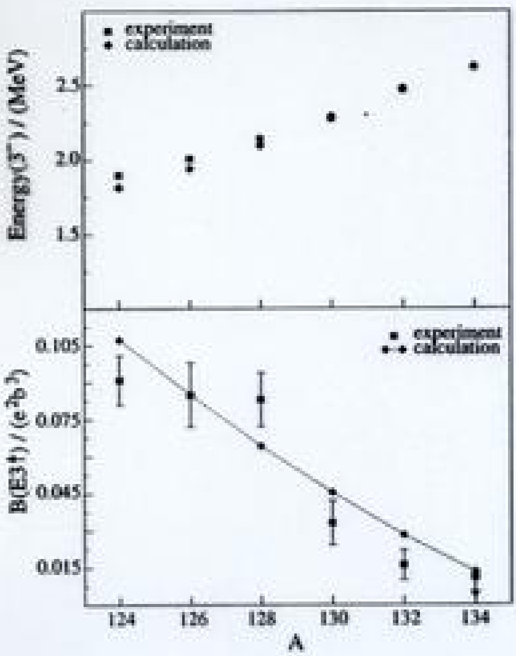


SOFT MODE DYNAMICS

Adiabatic adjustment to low-lying quadrupole mode

M.P. Metlay et al. Phys. Rev. C **52** (1995) 1801

New experiment *W.F. Mueller et al.* MSU (2005)



$$E(3^-) = E_0 - \frac{const}{E(2_1^+)}$$

$$B(E3\uparrow) = K \frac{Z^2 A^{1/3}}{E(3_-)} \frac{82-N}{12}$$

$h_{11/2}$ occupation

Xe isotopes

PARITY and TIME-REVERSAL VIOLATION

From Schiff moment to atomic EDM

$$\mathbf{S} = \frac{1}{10}\sum_a e_a \mathbf{r}_a \left[r_a^2 - \frac{5}{3}\langle r_{ch}^2 \rangle\right].$$

Expectation value

$$\langle \mathbf{S} \rangle = \langle (\mathbf{S} \cdot \mathbf{J}) \rangle \frac{\langle \mathbf{J} \rangle}{J(J+1)}$$

violates $\mathcal{P}$- and $\mathcal{T}$-invariance.

With static *quadrupole* and *octupole* deformation

$$\mathbf{S}_{intr} = S_{intr}\mathbf{n}$$

$$\langle n_z \rangle = 2\alpha \frac{KM}{J(J+1)}$$

$$\alpha = \frac{\langle J^+ | W(\mathcal{PT}) | J^- \rangle}{E_+ - E_-}$$

$$S_{intr} \propto \beta_2 \beta_3$$

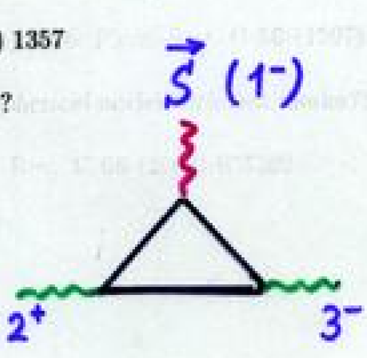
$$S \sim S_{intr} \frac{2\alpha J}{J+1} \propto \beta_2 \beta_3^2$$

N. Auerbach, V. Flambaum, V. Spevak

Phys. Rev. Lett. **76** (1996) 4316; Phys. Rev. C **56** (1997) 1357

Possible enhancement in *spherical* nuclei with soft modes??

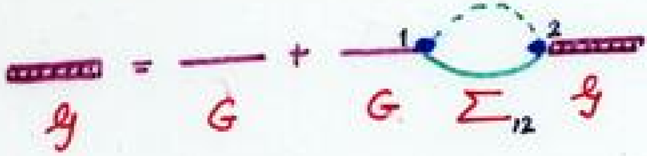
V. Flambaum, V.Z. Phys. Rev. C **68** (2003) 035502



W_{12} : Coupling through continuum

$$G = \frac{1}{E - H}$$ — Intrinsic propagator

$$\mathcal{G} = \frac{1}{E - \mathcal{H}}$$



$$\Sigma_{12}(E) \sim \sum_{\substack{\text{channels} \\ (\text{all!})}} \int \left(\prod d^3p\right)_c \frac{(c \to 1)\ (2 \to c)}{E - E_p^{(c)} + i0}$$

$$= Re\, \Sigma_{12} + i\, Im\, \Sigma_{12}$$

principal value (virtual processes) — closed and open channels

$\Downarrow$

$H \Rightarrow H'$

δ-function (real processes) — only open channels

$\Downarrow$

$$i\left(-\frac{1}{2} W_{12}\right)$$

$$W_{12} = \sum_{c} A_1^c A_2^{c*}$$

(open)

Channel variables are eliminated

EFFECTIVE HAMILTONIAN

$\mathcal{H}(E) = H - \frac{i}{2}W(E)$ - non-Hermitian

$$W_{12} = \sum_{c;\mathrm{open}(E)} A_1^c A_2^c$$

Internal representation: $H \rightarrow \epsilon_n$,

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)A_1^2 & -(i/2)A_1A_2 & -(i/2)A_1A_3 \\ -(i/2)A_1A_2 & \epsilon_2 - (i/2)A_2^2 & -(i/2)A_2A_3 \\ -(i/2)A_1A_3 & -(i/2)A_2A_3 & \epsilon_3 - (i/2)A_3^2 \end{pmatrix}$$

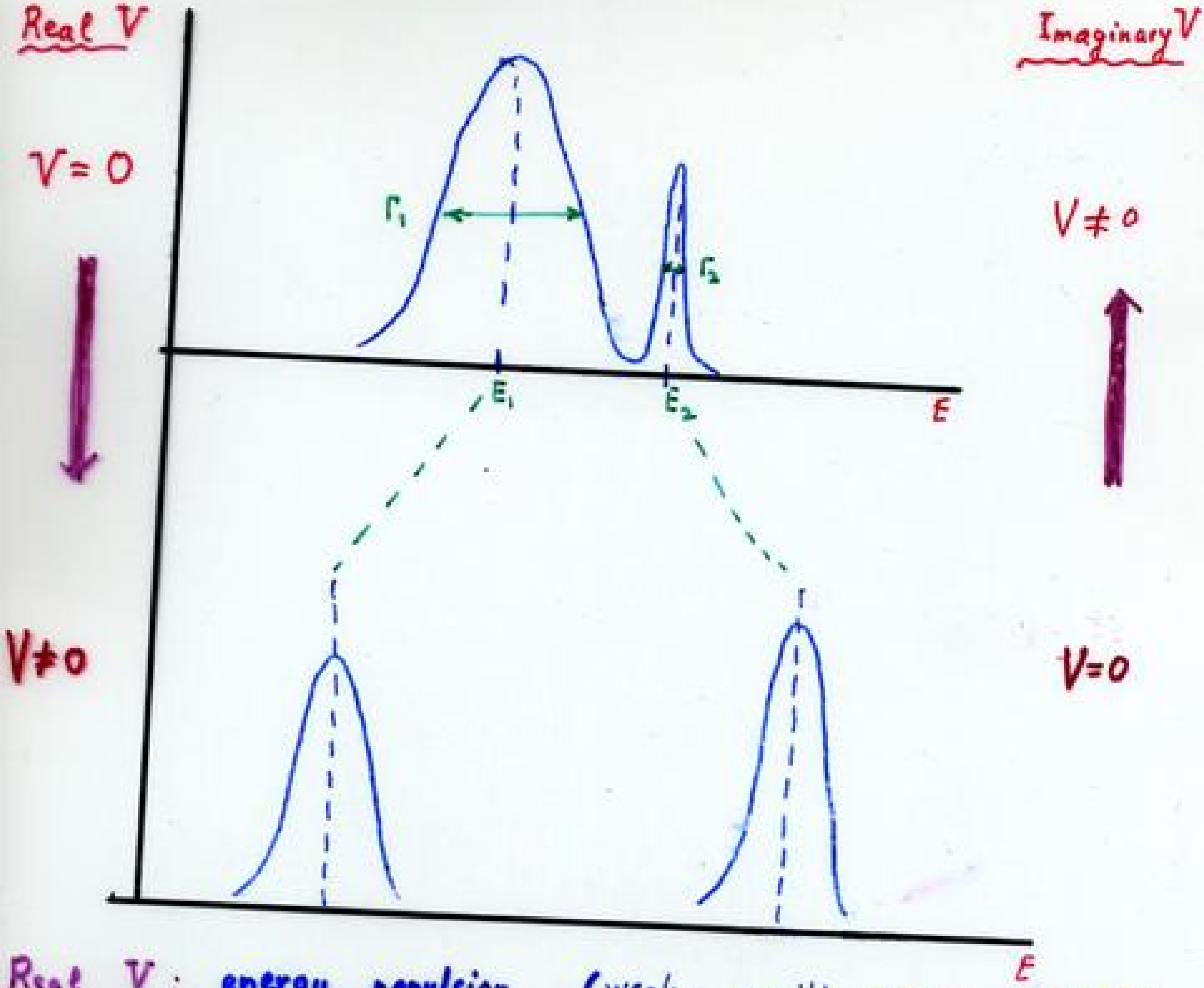
Weak coupling, $\kappa \ll 1$ – isolated resonances

$$\mathcal{E}_n = E_n - (i/2)\Gamma_n \approx \epsilon_n - (i/2)A_n^2$$

NSCL

Interaction between resonances

$$\mathcal{H} = \mathcal{H}^{\circ} + V$$



Real V: energy repulsion (weak, possible crossing at V≠0)
width attraction

Imaginary V: energy attraction
width repulsion broad resonance (Dicke)

P. von Brentano. Phys. Lett. **B238** (1990), **B246** (1990)
Nucl. Phys. **A550** (1992)

One-body decay

- Continuum channel $|c;E\rangle_N = c_j^\dagger(\epsilon_j)|\alpha; N-1\rangle$
  - State α in N-1 nucleon daughter
  - Particle in continuum state j
  - Energy E=E_α+ϵ_j
- Transition Amplitude

s.p. decay amplitude

$$A_1^c(E_\alpha + \epsilon_j) = a^j(\epsilon_j)\ \langle \alpha; N-1 | b_j | 1; N\rangle$$

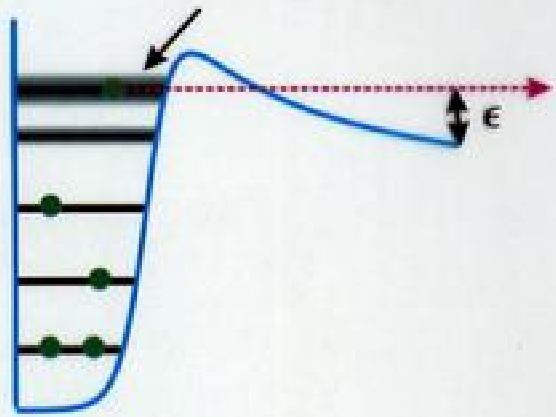
Shell model s.p. transition

Single-particle scattering problem

The same non-Hermitian eigenvalue problem

$$hu_l = \frac{1}{2\mu}\left\{-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + 2\mu\left[V(r) + \alpha\frac{Zz}{r}\right]\right\} u_l(r) = \epsilon u_l(r),$$

Internal states: $u_l(r)$



External states: $\epsilon = \frac{k^2}{2\mu}$

$$F_l(r) = krj_l(kr)$$

$$G_l(r) = krn_l(kr)$$

Single-particle decay amplitude

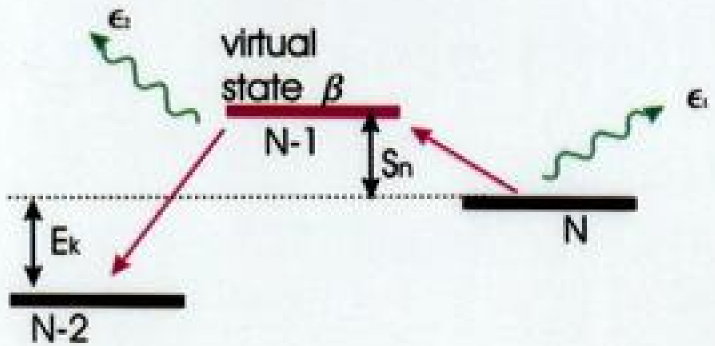
$$a^j(\epsilon) = \langle 0|c_j(\epsilon)Vb_j^\dagger|0\rangle = \sqrt{\frac{2\mu}{\pi k}}\int_0^\infty dr F_l(r)\, V(r)\, u_l(r)$$

Single particle decay width: (requires definition and solution for resonance energy)

$$\gamma_j(\epsilon) = 2\pi\left|a^j(\epsilon)\right|^2$$

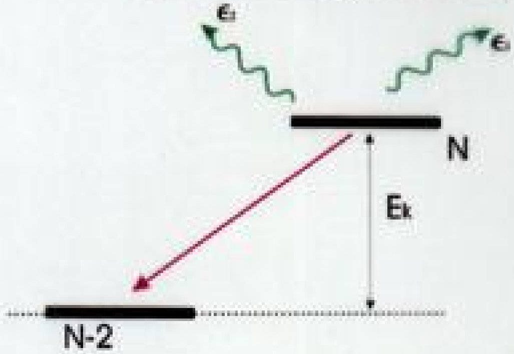
Two-body decays

Sequential process
via one-body interaction

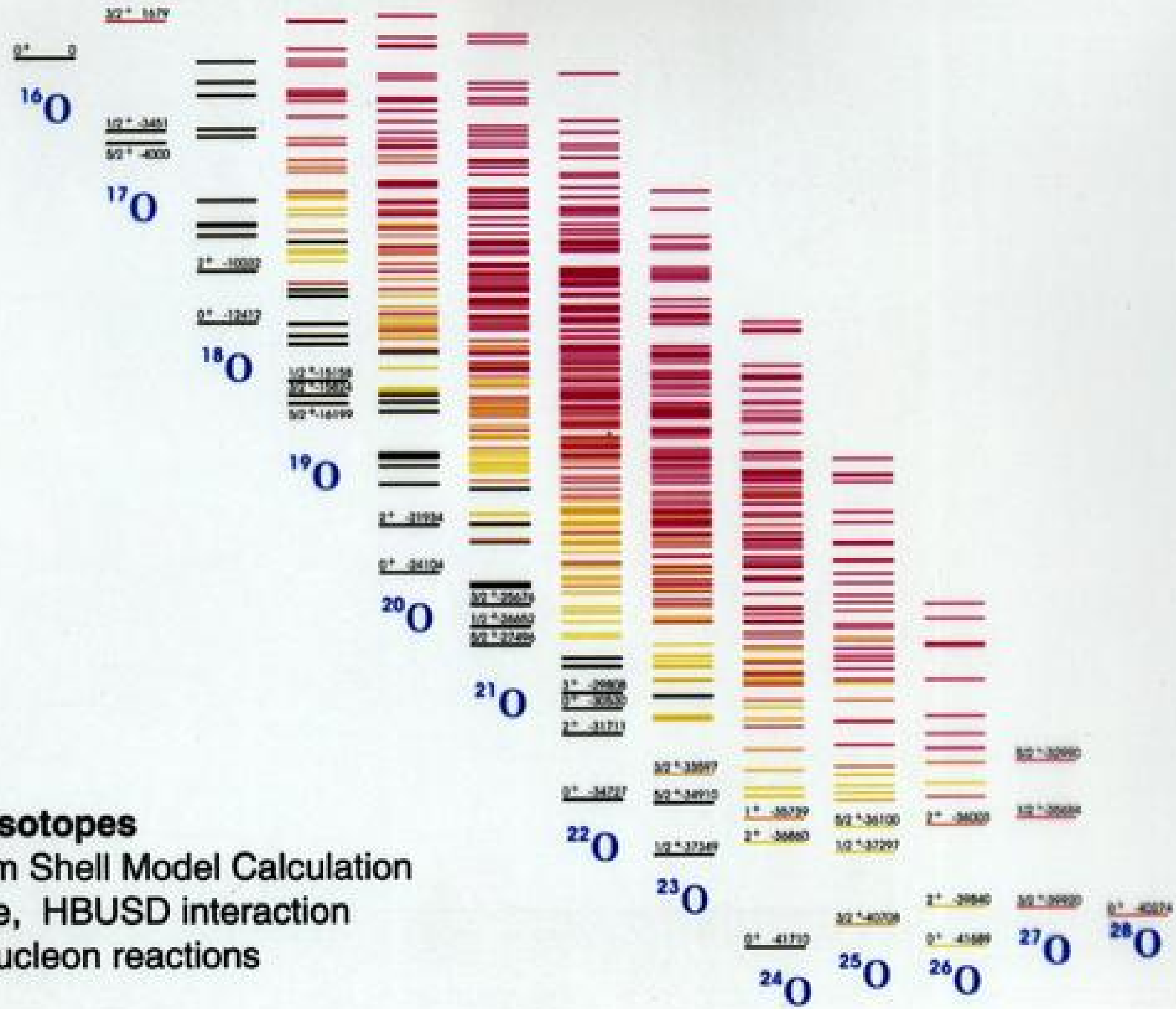


$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{s.p.} | 1; N \rangle$$

Direct process
via two-body interaction



$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{\text{2body}} | 1; N \rangle$$



Oxygen Isotopes

Continuum Shell Model Calculation

- sd space, HBUSD interaction
- single-nucleon reactions

Pairing in two-body decay

(even) (odd) (even)

Example: ^{4}He ^{5}Li ^{6}Be

Pairing energy: $2\Delta = 0.57 - (-1.97) = 2.54$ MeV

Single-particle level $\quad \varepsilon_r = \varepsilon_0 - i\frac{\gamma}{2}$

$$f(\varepsilon) = \frac{1}{2\pi}\frac{\gamma}{(\varepsilon-\varepsilon_0)^2 + \gamma^2/4}$$

Single-particle states in continuum

$$\psi(\vec{r};\varepsilon) = \sqrt{f(\varepsilon)}\;\psi^0(\vec{r};\varepsilon)$$

↓ scattering function (real ε)

Two-body problem (Cooper phenomenon)

$$[h(1) + h(2) + V(1,2)]\,\Psi(1,2) = E\,\Psi(1,2)$$

$$h\psi^0(\vec{r},\varepsilon) = \varepsilon\psi^0(\vec{r},\varepsilon)$$

Pairing interaction $\langle \varepsilon_1 \varepsilon_2 | U | \varepsilon_2' \varepsilon_1' \rangle = -G\sqrt{f_1 f_2 f_{2'} f_{1'}}$

$$G = -\int dr_1 dr_2 \, U(1,2) \left(\psi^0(r_1, \varepsilon_0) \psi^0(r_2, \varepsilon_0) \right)^2$$

Solution $\Psi(1,2) = \int d\varepsilon_1 d\varepsilon_2 \, C(\varepsilon_1, \varepsilon_2) \psi(r_1; \varepsilon_1) \psi(r_2; \varepsilon_2)$

$$(\varepsilon_1 + \varepsilon_2 - E) C(\varepsilon_1, \varepsilon_2) = G\sqrt{f(\varepsilon_1) f(\varepsilon_2)} \; C_0$$

$$C_0 = \int d\varepsilon_1' d\varepsilon_2' \; C(\varepsilon_1', \varepsilon_2') \sqrt{f(\varepsilon_1') f(\varepsilon_2')}$$

Secular equation $\dfrac{1}{G} = \int d\varepsilon_1 d\varepsilon_2 \dfrac{f(\varepsilon_1) f(\varepsilon_2)}{\varepsilon_1 + \varepsilon_2 - E - i0}$

Analytic continuation: Resonance $\mathcal{E} = E - \frac{i}{2}\Gamma$

$$\mathcal{E} \simeq 2\varepsilon_0 - G - \frac{i}{2}\Gamma = E_0 - \frac{i}{2}\Gamma$$

$$\Gamma \simeq G^2 F(E_0)$$

$$F(E) = 2\pi G \int d\varepsilon_1 d\varepsilon_2 \, f(\varepsilon_1) f(\varepsilon_2) \, \delta(\varepsilon_1 + \varepsilon_2 - E)$$