Continuum effects for many-body correlations in nuclei close to the neutron drip line

Masayuki Yamagami

RIKEN, Radioactive Isotope Physics Laboratory

Discussion

Novel features of ***many-body correlations*** among ***many loosely-bound neutrons*** in medium heavy mass neutron rich nuclei.

$^{86}Ni = {}^{78}Ni_{50} + 8n$

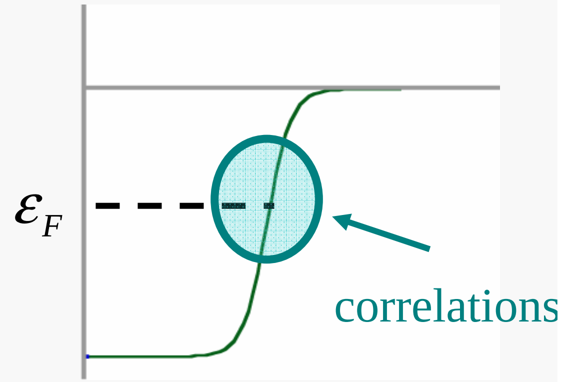
Cf. $^{11}Be = {}^{10}Be + n$, $^{11}Li = {}^{9}Li + 2n$

Zone of correlated states around the Fermi level

Stable nuclei

Pairing correlations (BCS approximation)

$$\left|\varepsilon_i - \varepsilon_F\right| < E_{cut}^{BCS} \approx 5 \text{ MeV}$$

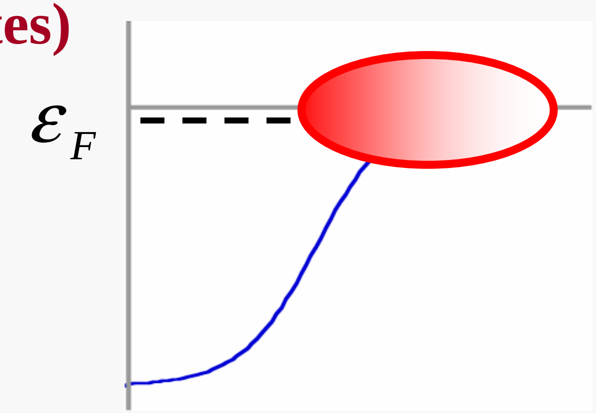


Low-lying excitations

$$0\hbar\omega + \text{effective charge}$$

⇨ **Correlations among *tightly-bound states***
(spatially localized states)

Nuclei close to the neutron drip line



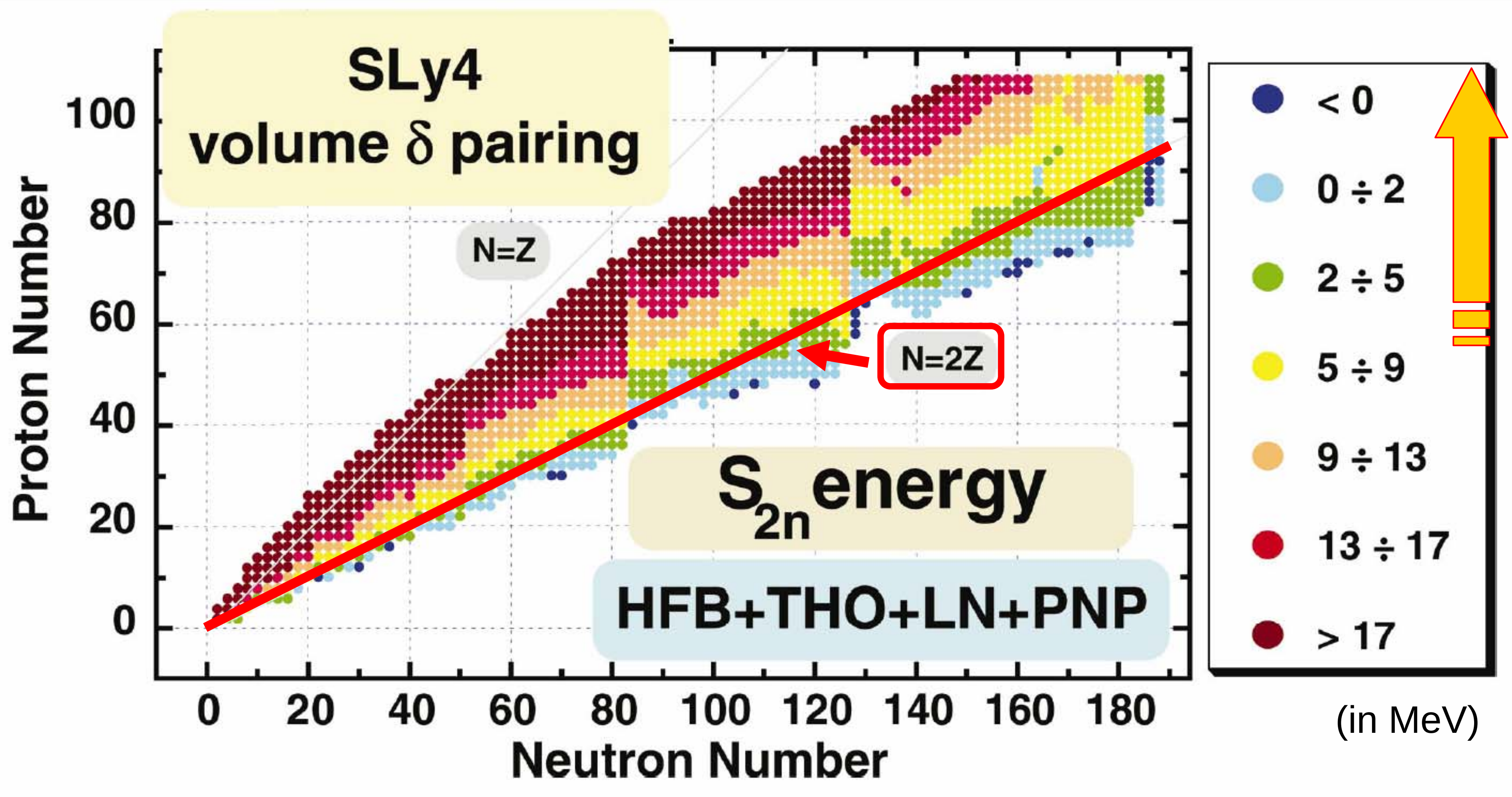
Correlations among *loosely-bound states*
and *continuum states*

⇨ **Variety of the spatial structure**

⇨ **New type collective motions ?**

New dimension !

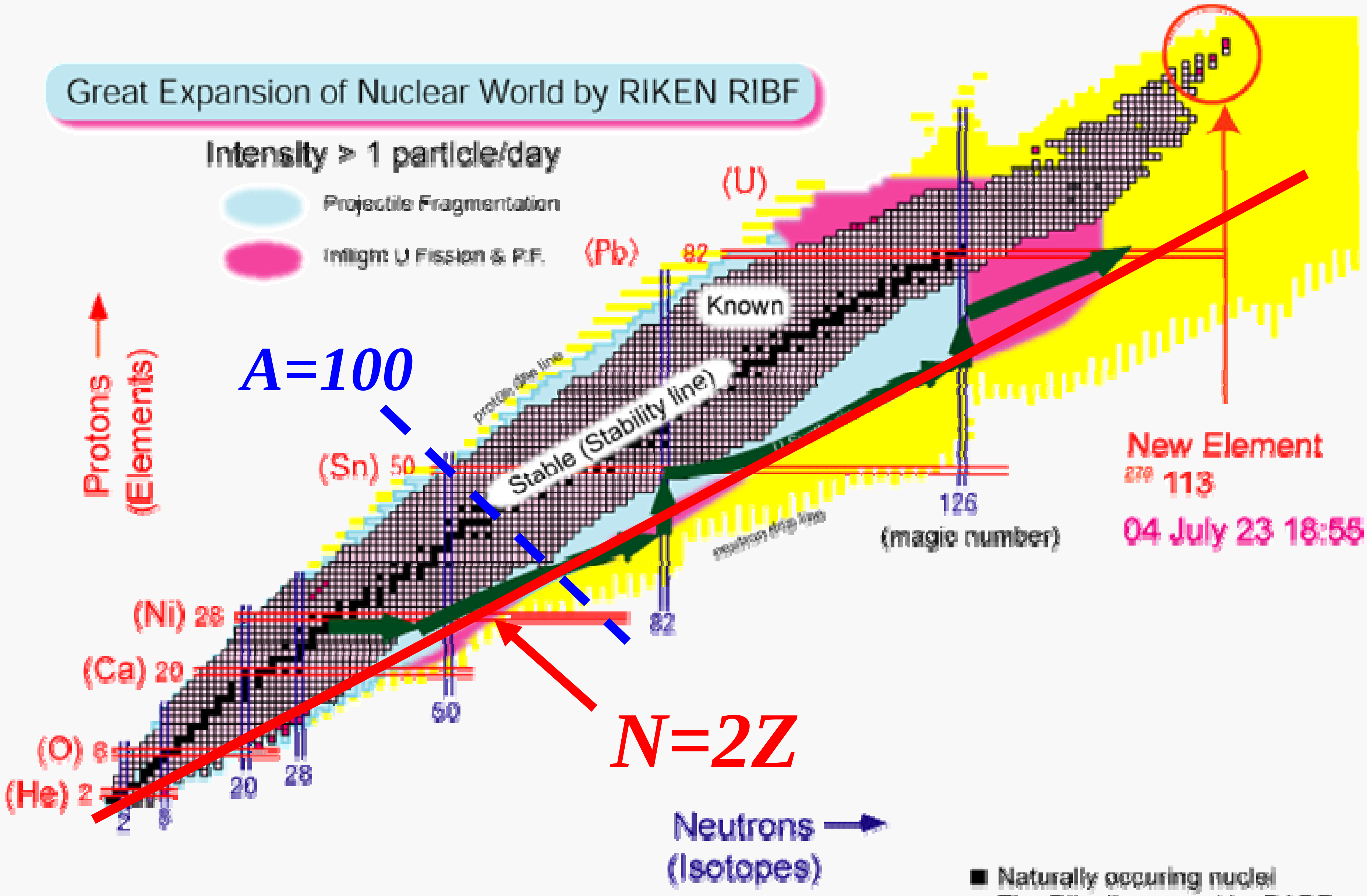
Two-neutron separation energy



M. V. Stoitsov, J. Dobaczewski, W. Nazarewicz, S. Pittel, D. J. Dean, Phys. Rev. C 68, 054312 (2003)

$$S_{2n} \approx 2|\lambda| \leq 2\Delta \approx 3\text{ MeV} \ll 2E_{cut}^{BCS} \approx 10\text{ MeV}$$

RIKEN RIBF



http://www.rarf.riken.go.jp/RIBF/nuclearchart-e.htm

Contents

1. Spatial structure of single-(quasi)particle wave functions $\left(\varepsilon_i, l, \Delta, \text{continuum}\right)$

Consequently…

2. Enhancement of ***pairing correlations***

3. Strong transition strength of ***low-frequency vibrational excitations***

HFB equation in coordinate space

$$\begin{pmatrix} \hat{T} + V_{HF}(\vec{r}) - \lambda & \Delta(\vec{r}) \\ \Delta(\vec{r}) & -\hat{T} - V_{HF}(\vec{r}) + \lambda \end{pmatrix} \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix} = E \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix}$$

(zero-range interactions assumed)

A. Bulgac, FT-194-1980, CIP-IPNE, Bucharest Romania, 1980 (nucl-th/9907088)
J. Dobaczewski, H. Flocard, J. Treiner, Nucl. Phys. A422, 103 (1984)

$$v_k(E,\vec{r}) \qquad u_k(E,\vec{r})$$

$$\Psi_k(\vec{r}) = \begin{pmatrix} u_k(E,\vec{r}) \\ v_k(E,\vec{r}) \end{pmatrix} =$$

Asymptotic behavior

$$V_{HF}(\vec{r}), \Delta(\vec{r}) \to 0 \quad (r \to \infty) \quad \Rightarrow$$

$$\hat{T} u_k(E,\vec{r}) = (E + \lambda) u_k(E,\vec{r})$$

$$\hat{T} v_k(E,\vec{r}) = -(E - \lambda) v_k(E,\vec{r})$$

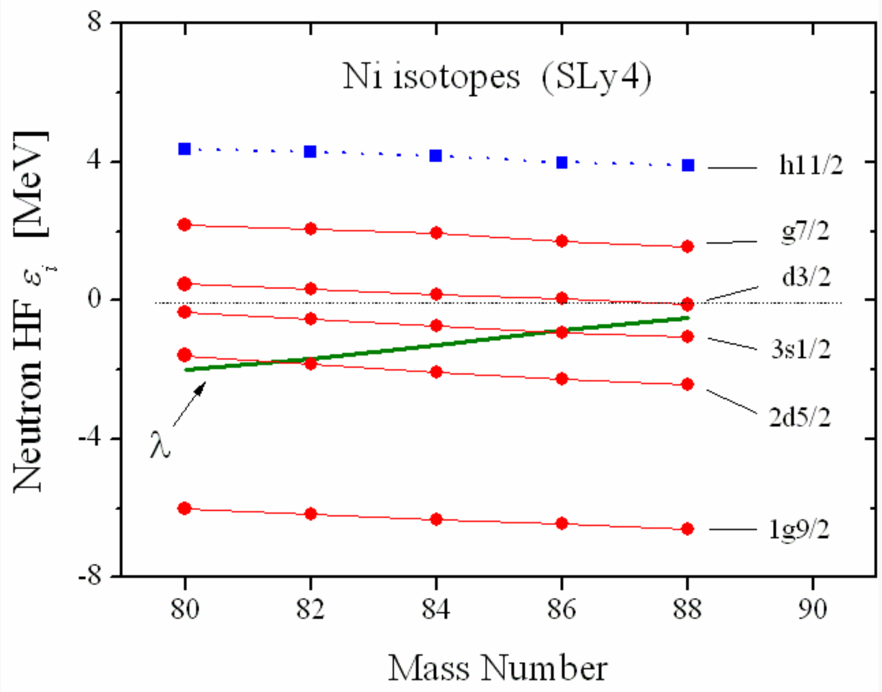
- Controlled by E
- $u_k(E,\vec{r}), v_k(E,\vec{r}) \Rightarrow$ different asymptotic behavior

Spatial structure of neutrons around ^{86}Ni

Single-particle energies

^{86}Ni is the neutron drip line nucleus without pairing

ν Fermi level → loosely-bound $3s_{1/2}$ state



Simplified HFB problem

- Solved only for neutrons
- Box boundary condition at R_{box}=75 fm
- Phenomenological Woods-Saxon potential for HF mean-field

$$R_{WS} = 5.5 \text{ fm}, \;\; a = 0.67 \text{ fm} \qquad V_{WS} = -41.4 \text{ MeV} \quad \left(V_{WS} \text{ dependence examined}\right)$$

- Self-consistent pairing correlations (surface type)

$$\Delta(r) = \frac{1}{2} V_{pair} \left(1 - \frac{\rho_n(\vec{r})}{\rho_c} \right) \tilde{\rho}_n(\vec{r})$$

$$V_{pair} = -680 \text{ MeV fm}^{-3} \quad \rho_o = 0.08 \text{ fm}^{-3} \approx \rho_n(r=0)$$

$$\Rightarrow \bar{\Delta} \approx 1.3 \text{ MeV} \;\; \approx 12/\sqrt{86}$$

Pairing anti-halo effect

K. Bennaceur, J. Dobaczewski, M. Ploszajczak, Phys. Lett. 496B, 154 (2000)

With pairing correlation

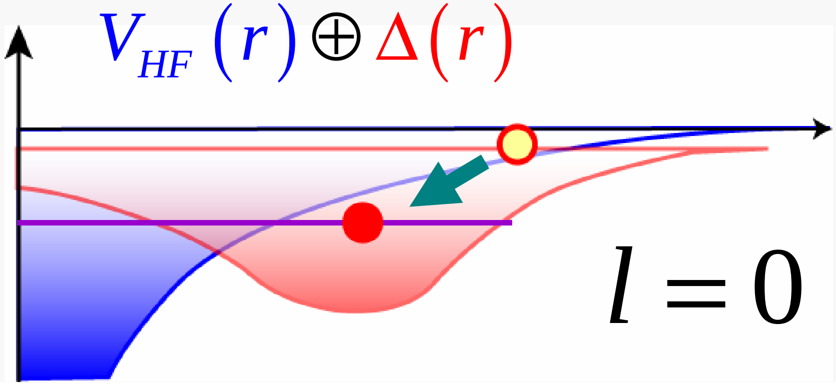
$$\hat{T}v(E,\vec{r}) = -(E-\lambda)v(E,\vec{r})$$

$(E-\lambda) > 0$

$$v(E,\vec{r}) \to \exp(-\beta r)/r$$

$$\beta = \sqrt{\frac{2m}{\hbar^2}(E-\lambda)} \geq \sqrt{\frac{2m}{\hbar^2}\Delta_E^{can}} \geq 0$$

$$\left(E \simeq E^{can} = \sqrt{(\varepsilon_E^{can}-\lambda)^2 + (\Delta_E^{can})^2}\right)$$



Without pairing correlation

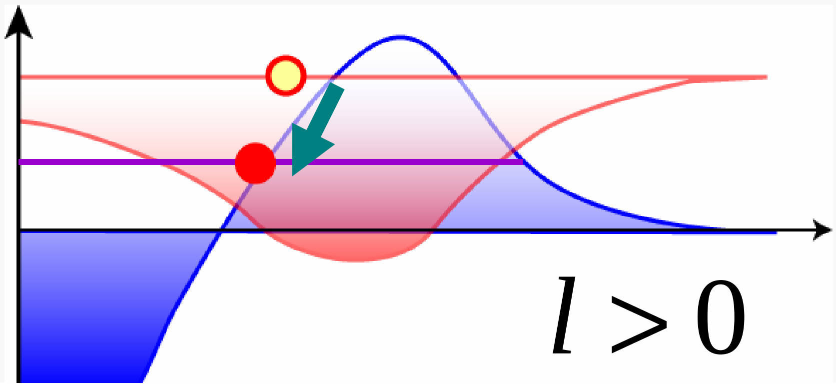
For $\varepsilon_i < 0$

$$\varphi(\varepsilon_i,\vec{r}) \to \exp(-\alpha_i r)/r$$

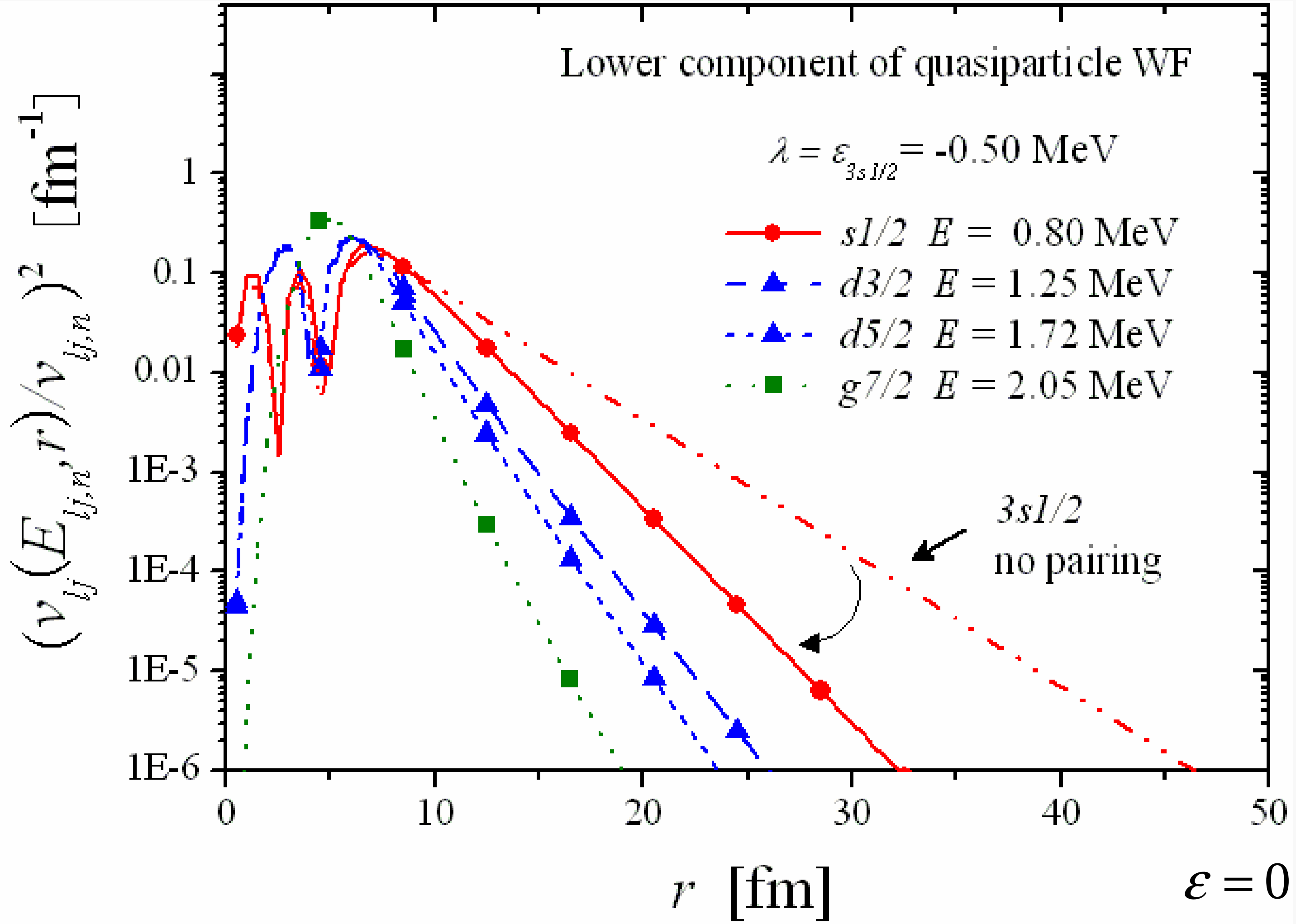
$$\alpha_i = \sqrt{-\frac{2m}{\hbar^2}\varepsilon_i} \to +0 \quad (\varepsilon_i \to -0)$$

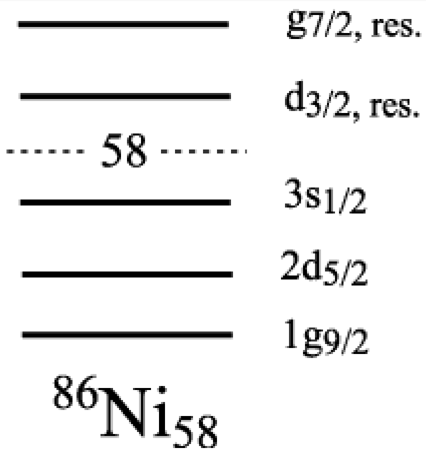
For $\varepsilon_i > 0$

$$\varphi(\varepsilon_i,\vec{r}) \to \sin(k_i r + \delta_i)/r$$



Lower components in ^{86}Ni





Spatial structure of upper components

The asymptotic equation

$$\hat{T}u_k\left(E,\vec{r}\right)=\underline{\left(E+\lambda\right)}u_k\left(E,\vec{r}\right)$$

< 0 *or* > 0

$$0<E<-\lambda \;\Rightarrow\; \text{bound state}$$

$$E>-\lambda \Rightarrow\; \text{unbound state}$$

Localization indicator for unbound states

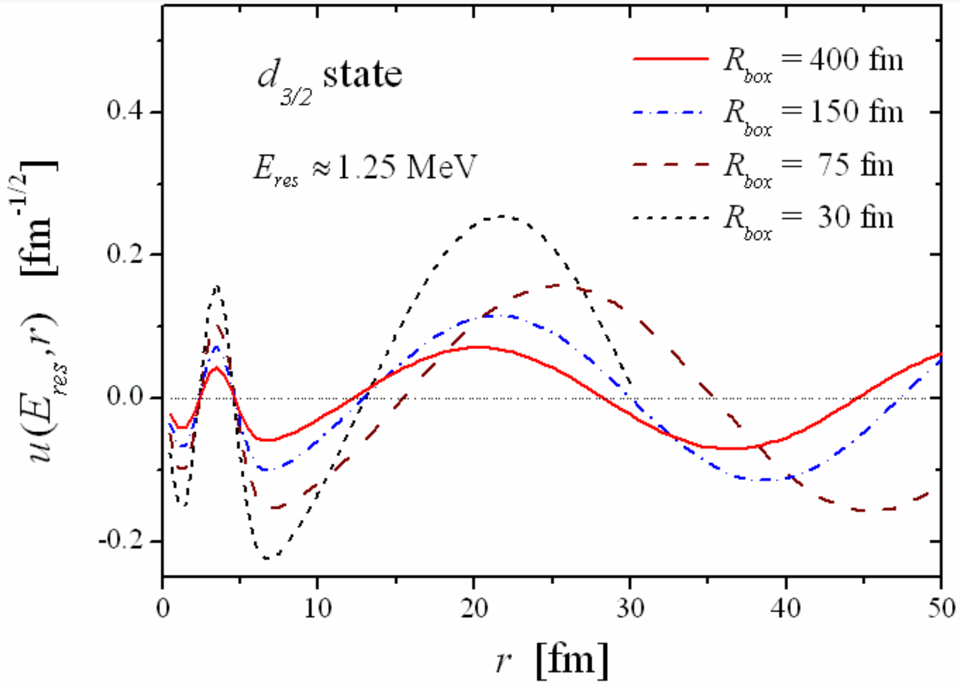
$$L_{lj}\left(r;\delta E\right)=\sum_{n\in I_{lj}^{(res)}(\delta E)}\left\{u_{lj}\left(E_{lj,n},r\right)\right\}^2$$

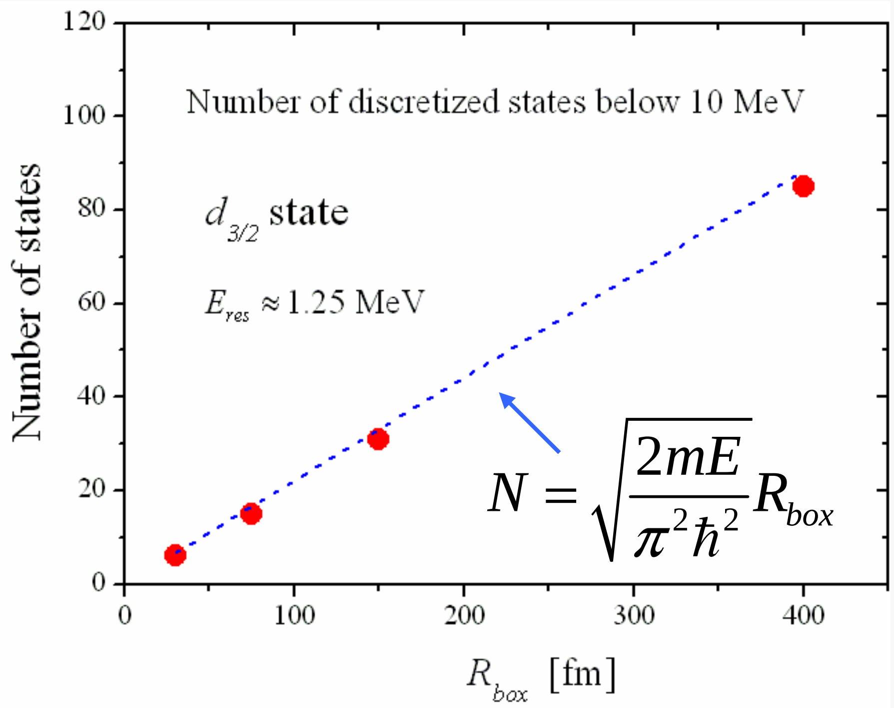
$$I_{lj}^{(res)}\left(\delta E\right)=\left\{n\,;\left|E_{lj,n}-E_{lj}^{(res)}\right|<\frac{\delta E}{2}\right\}$$

Upper components with box boundary condition

Asymptotic behavior $(r \to \infty)$

$$E_{lj} > -\lambda(\approx 0) \Rightarrow u\left(E_{lj}, \vec{r}\right) \to \sin\left(\alpha_{lj} r + \delta_{lj}\right) \quad \alpha_{lj} = \sqrt{\frac{2m}{\hbar^2}\left(E_{lj} + \lambda\right)}$$

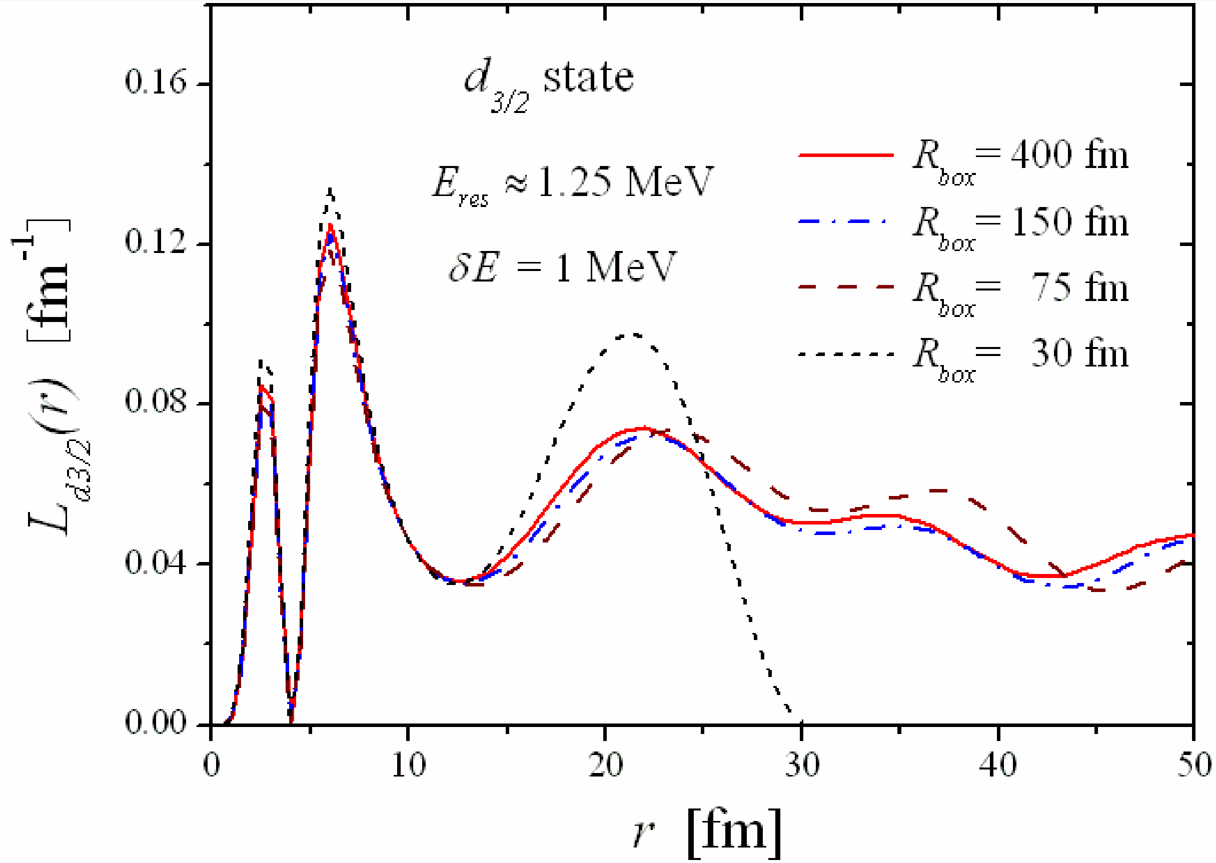




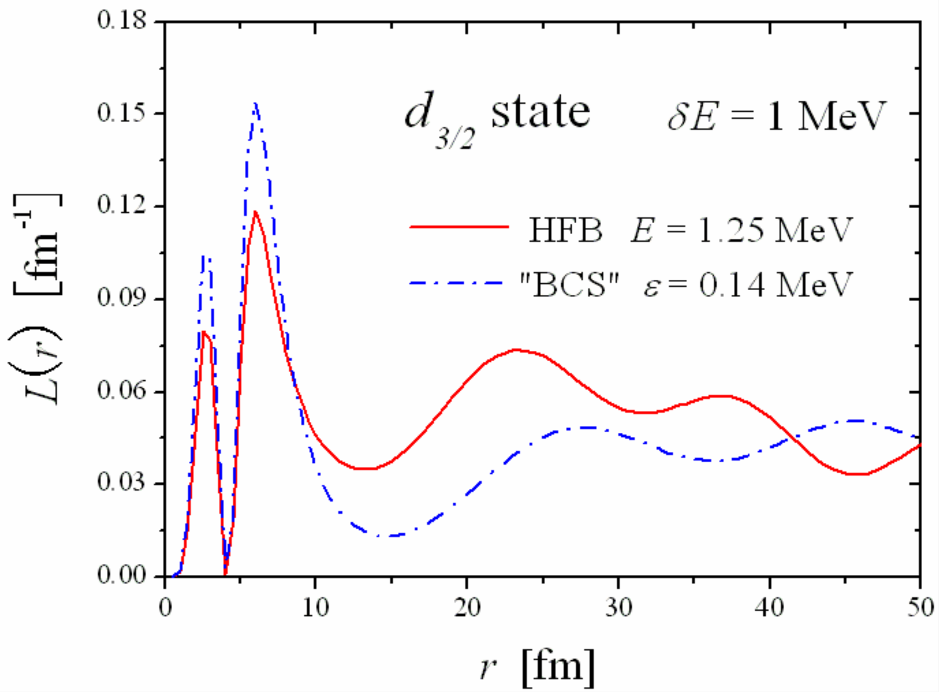
Localization indicator for unbound states

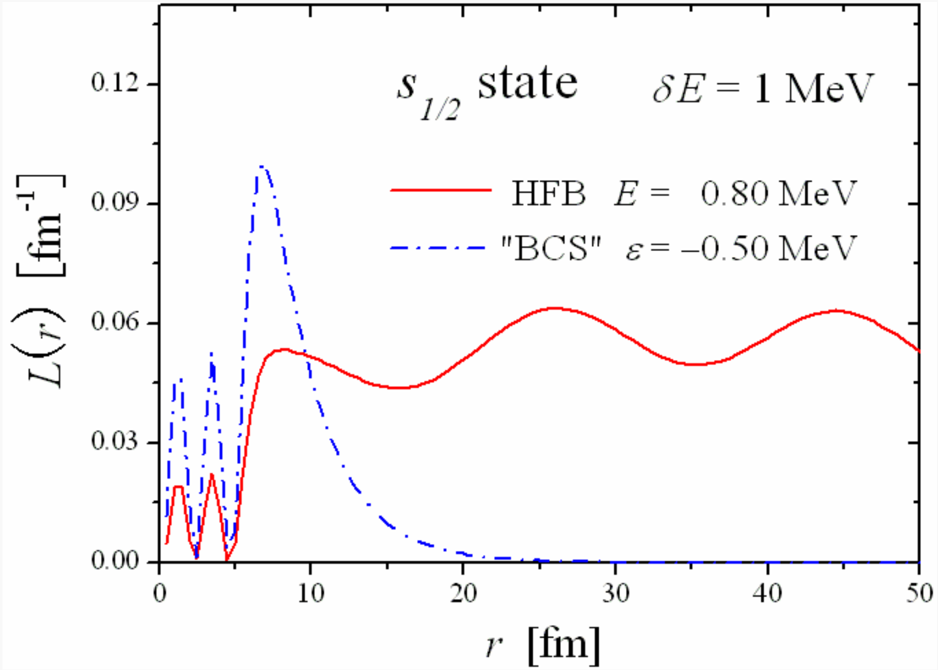
$$L_{lj}(r;\delta E) = \sum_{n \in I_{lj}^{(res)}(\delta E)} \left\{u_{lj}\left(E_{lj,n}, r\right)\right\}^2$$

Independent of R_{box} with arbitrary δE for $r \ll R_{box}$



Coupling to continuum in the upper components





$\varepsilon_{lj}^{res} > 0$

$\varepsilon_{lj,n} < 0$

Mixing of continuum states by pairing

- **Relaxation of the spatial localization**
- **Broadened (acquired) resonance width in $\varepsilon > 0$ ($\varepsilon < 0$) state**
- **This effect is stronger in lower-*l* states**

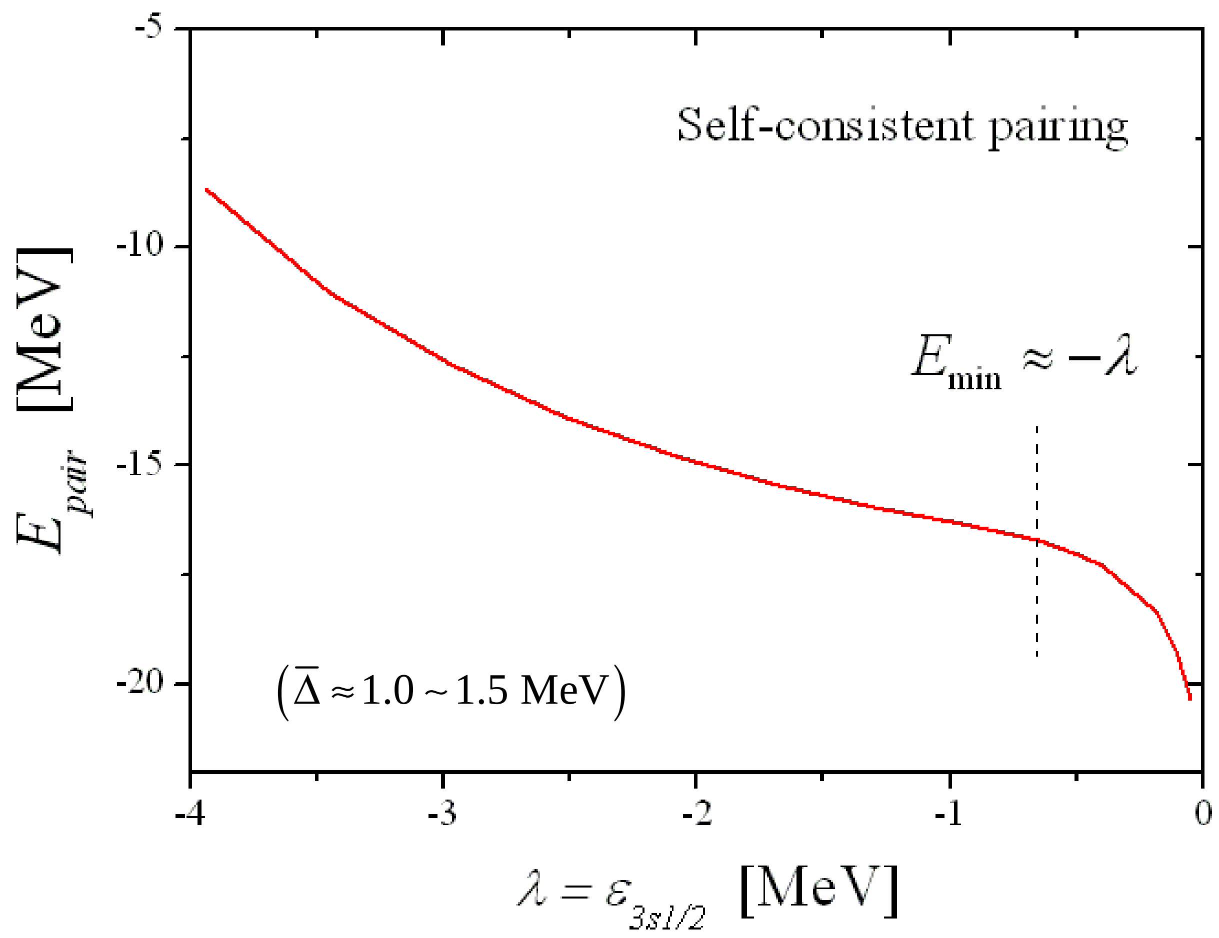
Spatial structure of single-quasiparticle wave functions

- $\varepsilon_{lj}\,,\, l$
  - **Pairing correlations**
  - **Coupling to continuum states**
  - **Difference between *v(r)* and *u(r)***

⇩ **?**

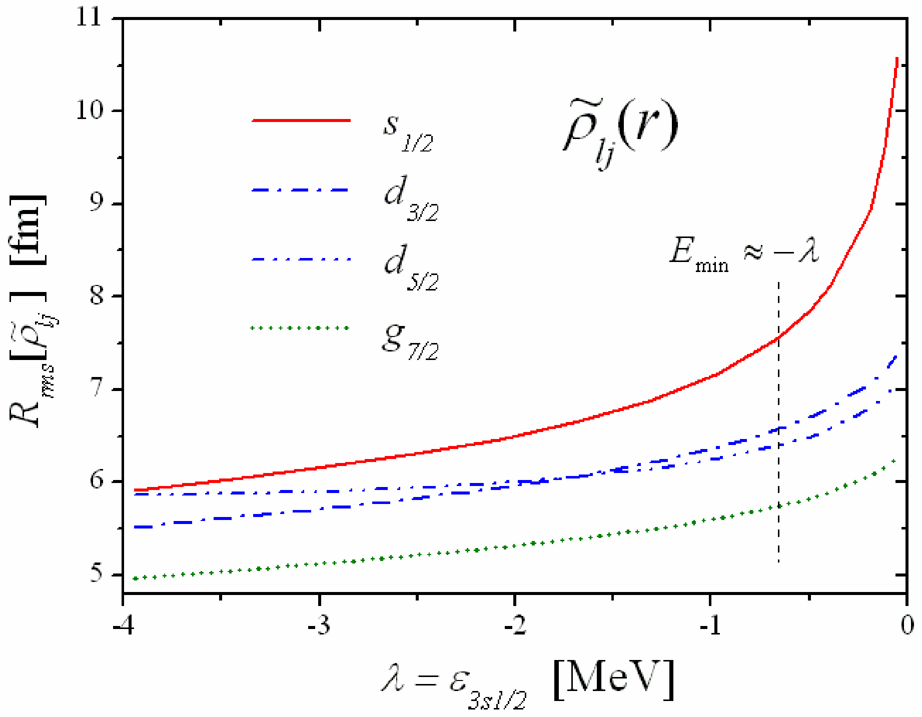
Pairing correlation for $\lambda = \varepsilon\,(3s_{1/2}) \rightarrow 0$

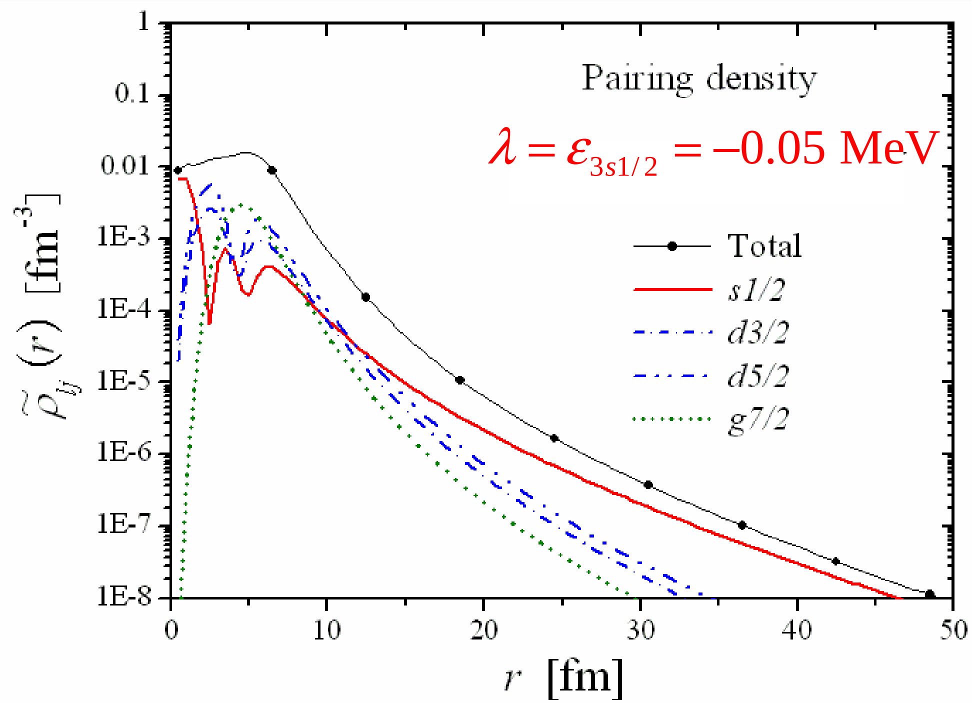
Pairing correlation for $\lambda = \varepsilon\,(3s_{1/2}) \rightarrow 0$



$$E_{pair} = \frac{1}{2}\int_0^{R_{box}} \Delta(r)\,\tilde{\rho}(r)\,4\pi r^2 dr$$

Spatial structure of pairing density





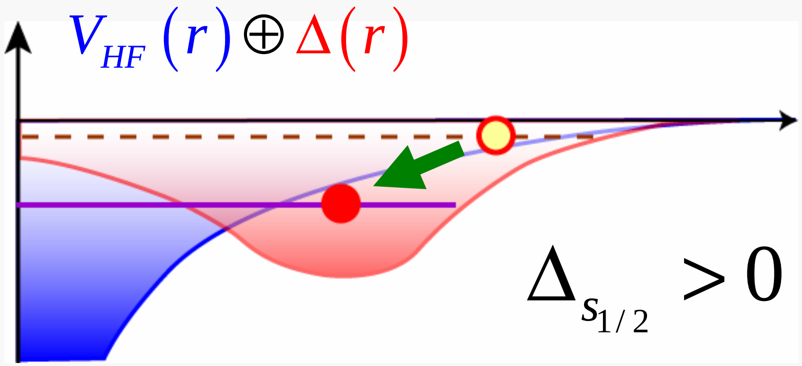
$$\tilde{\rho}(r) = \sum_{lj} \tilde{\rho}_{lj}(r)$$

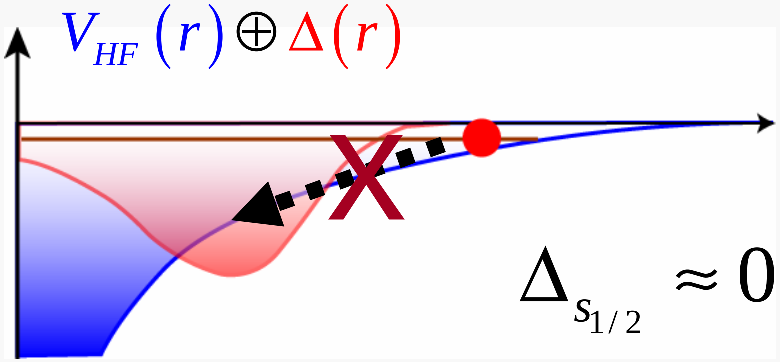
$$\tilde{\rho}_{lj}(r) = -\frac{2j+1}{4\pi r^2} \sum_n u_{lj}(E_{lj,n}, r) v_{lj}(E_{lj,n}, r)$$

Ingredients determining ***the spatial extent of*** $\tilde{\rho}_{lj}(r)$?

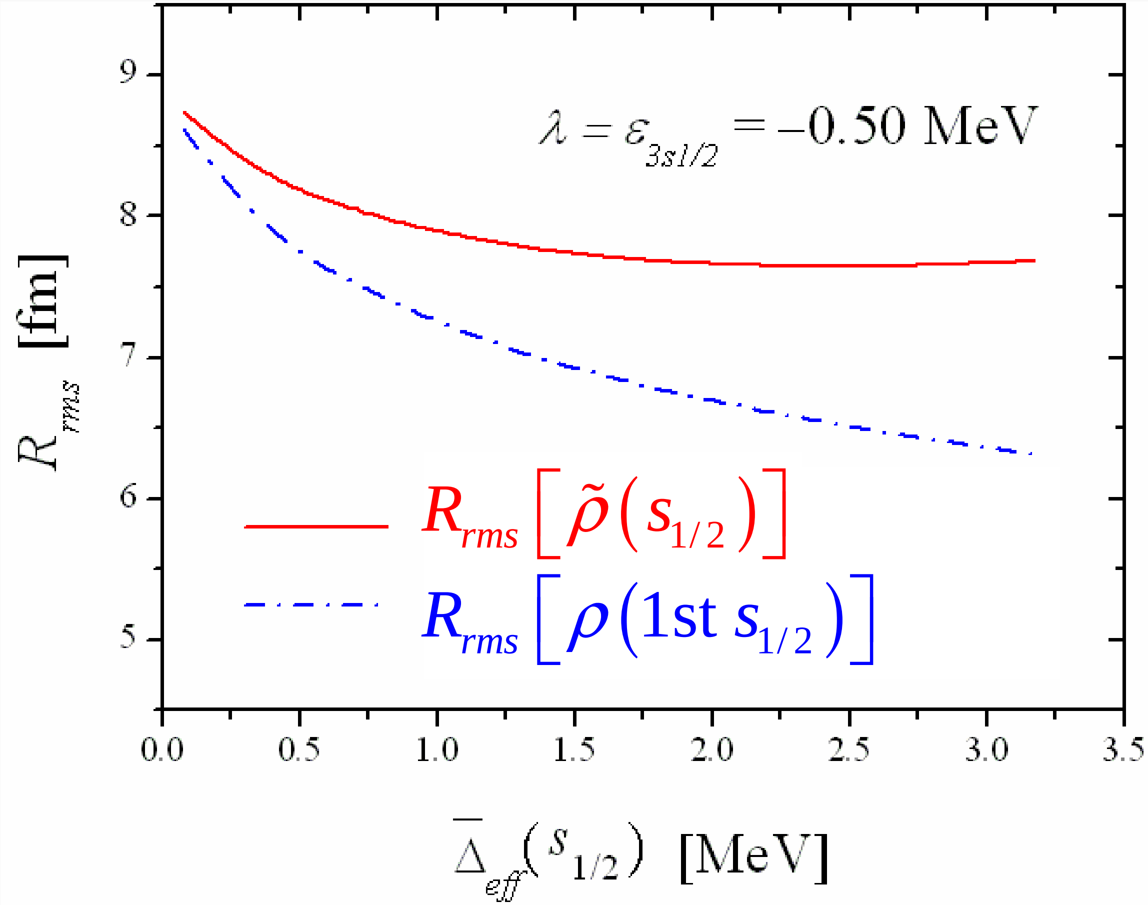
$$\tilde{\rho}_{lj}(r) = -\frac{2j+1}{4\pi r^2}\sum_n u_{lj}(E_{lj,n}, r)\, v_{lj}(E_{lj,n}, r)$$

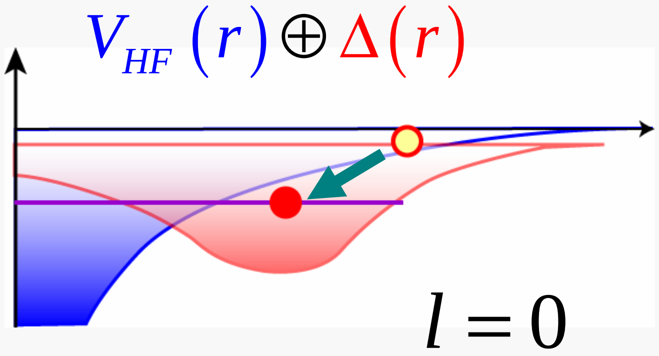
Coupling between ***$s_{1/2}$ halo state*** and ***the pairing potential survives*** or ***decouples*** for $\lambda = \varepsilon(3s_{1/2}) \rightarrow 0$?





Spatial extent of $s_{1/2}$ state and $\tilde{\rho}_{s_{1/2}}(r)$





- **Balance of the pairing effects in *v(r)* and *u(r)***
- $R_{rms}\left[\tilde{\rho}\left(s_{1/2}\right)\right] > R_{rms}\left[\rho\left(1\text{st } s_{1/2}\right)\right] = R_{rms}\left[v\left(s_{1/2}\right); E < 10 \text{ MeV}\right]$

Role of high-*l* states

Spatially extended distribution of pairing density of **low-*l* states**

⇩

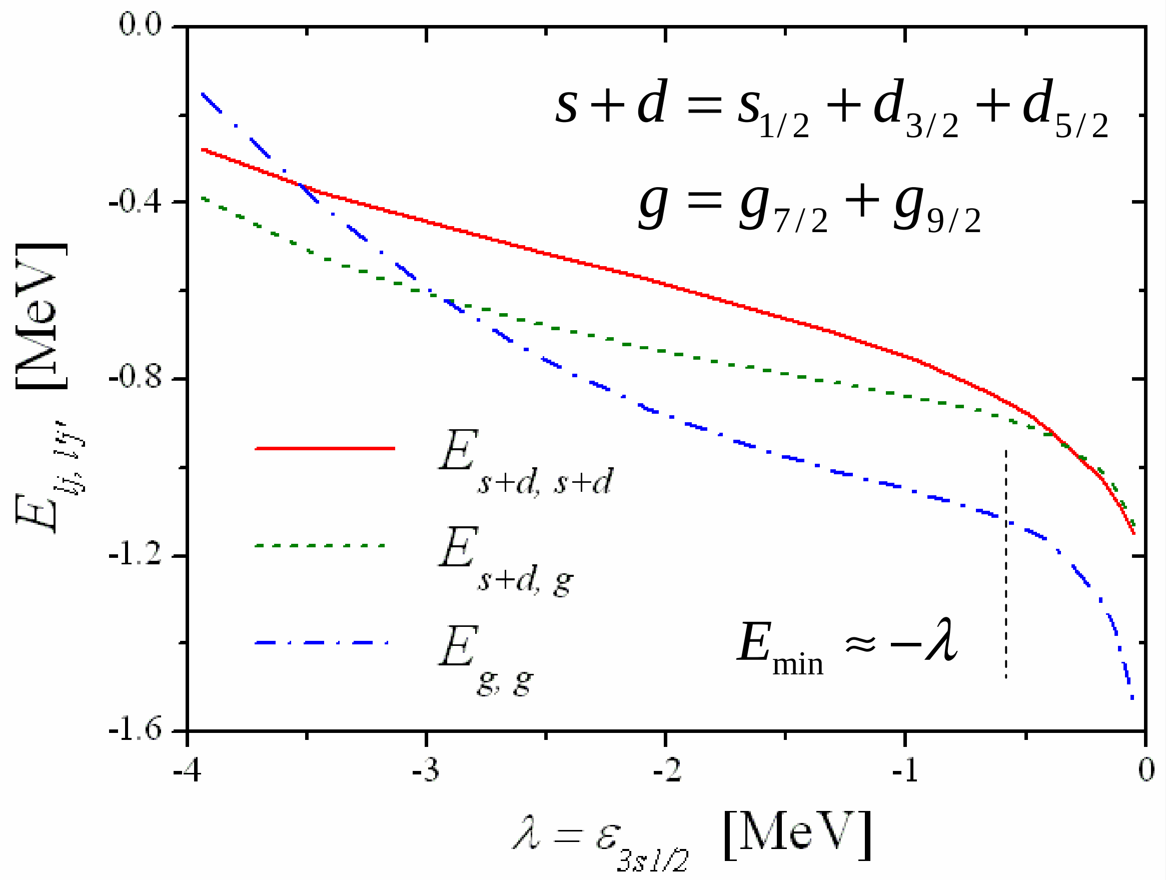
Enhancement of pairing energy
for $\lambda = \varepsilon(3s_{1/2}) \rightarrow 0$

⇧ ?

Role of **high-*l* states** ??

The clue → coupling between high-*l* and low-*l* states

Coupling between high-*l* and low-*l* states



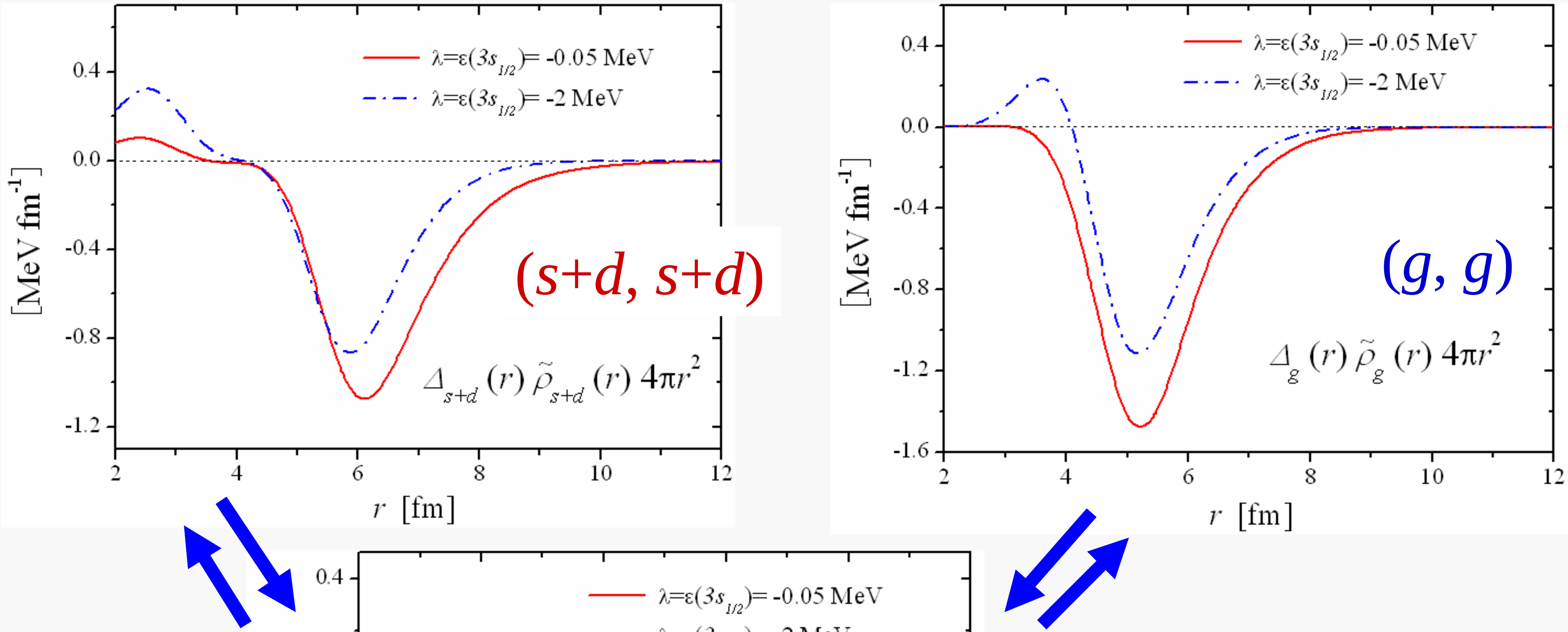
$$E_{pair} = \frac{1}{2}\int_0^{R_{box}} \Delta(r)\,\tilde{\rho}(r)\,4\pi r^2 dr$$

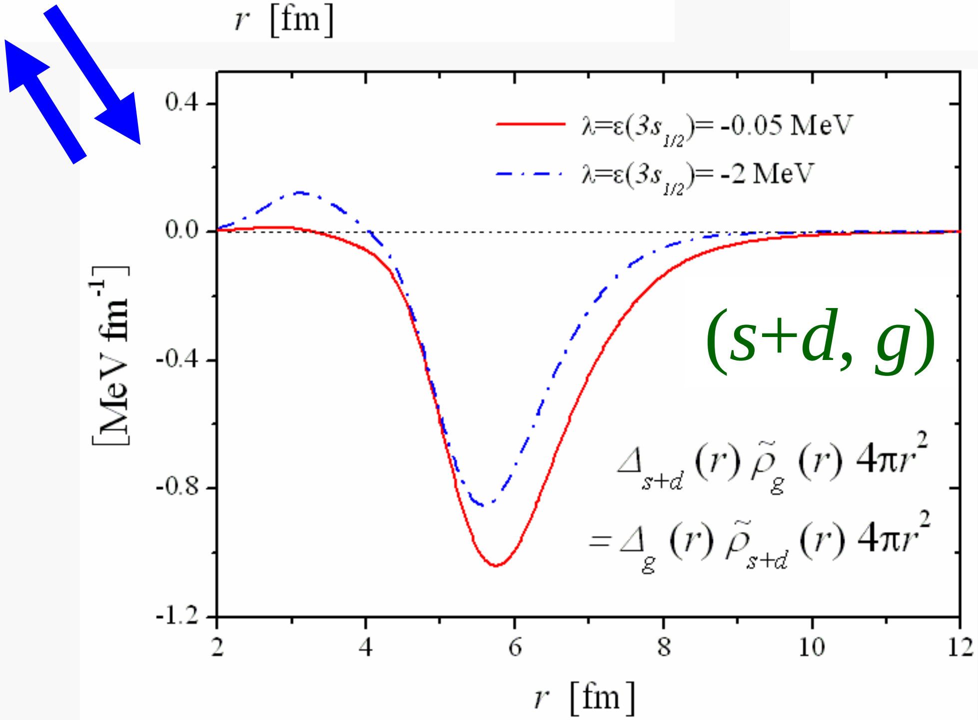
$$= \sum_{lj,l'j'} E_{lj,l'j'}$$

$$E_{lj,l'j'} = \frac{1}{2}\int_0^{R_{box}} \Delta_{lj}(r)\,\tilde{\rho}_{l'j'}(r)\,4\pi r^2 dr$$

$$\Delta_{lj}(r) = \frac{1}{2}V_{pair}\left(1 - \frac{\rho(\vec{r})}{\rho_c}\right)\tilde{\rho}_{lj}(r)$$

Self-consistency of pairing correlations





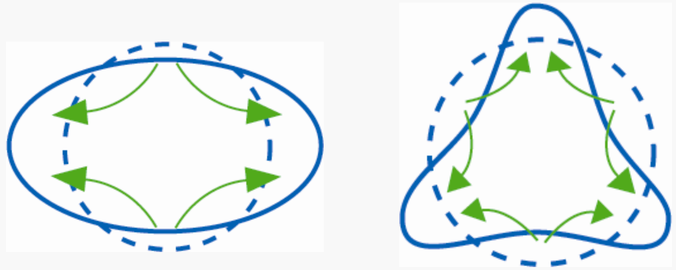
$$\Delta_{lj}(r)\,\tilde{\rho}_{l'j'}(r)\,4\pi r^2$$

$$\lambda = \varepsilon(3s_{1/2}) = -0.05\ \text{MeV}$$

$$\lambda = \varepsilon(3s_{1/2}) = -2\ \text{MeV}$$

Low-frequency vibrational excitations (2+,3-,…)

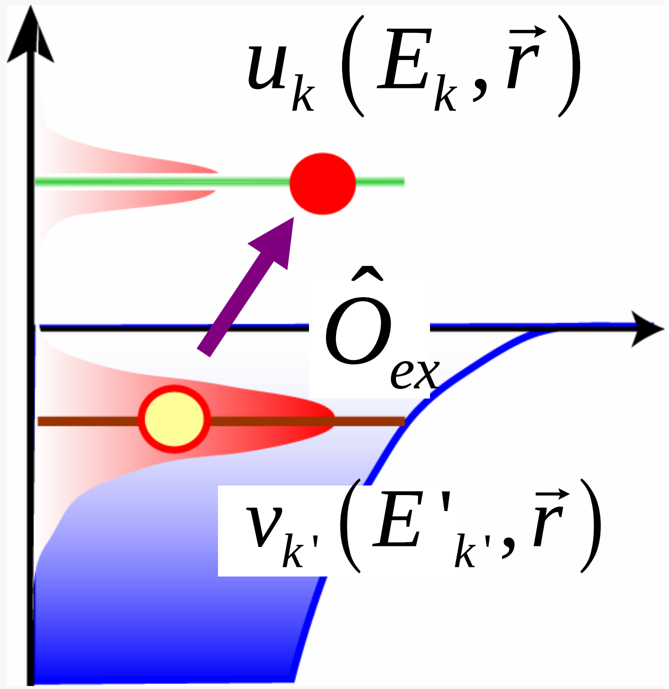
1 phonon state in Quasiparticle RPA:



$$|vib\rangle = \frac{1}{2}\sum_{kk'}\left(X_{kk'}\alpha_k^+\alpha_{k'}^+ - Y_{kk'}\alpha_{k'}\alpha_k \right)|0\rangle$$

***p-h* transition matrix element:**

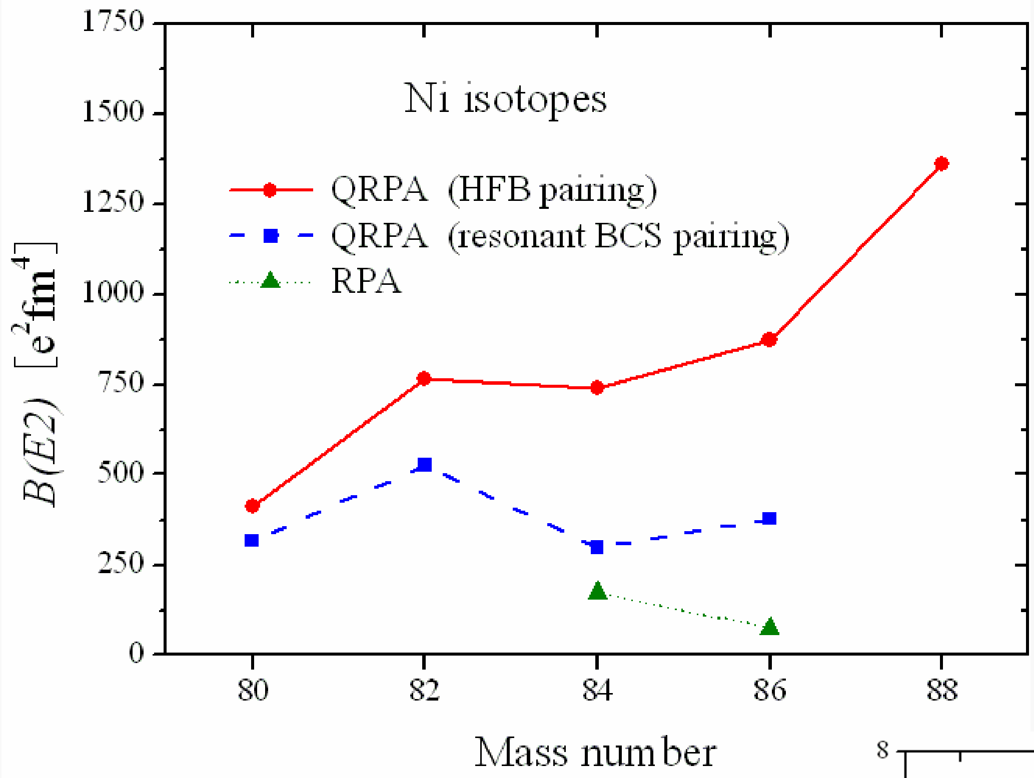
$$T_{kk'}^{(ph)} = \int d\vec{r}\; u_k\left(E_k,\vec{r}\right)\hat{O}_{ex}\left(\vec{r}\right)v_{k'}\left(E'_{k'},\vec{r}\right)$$

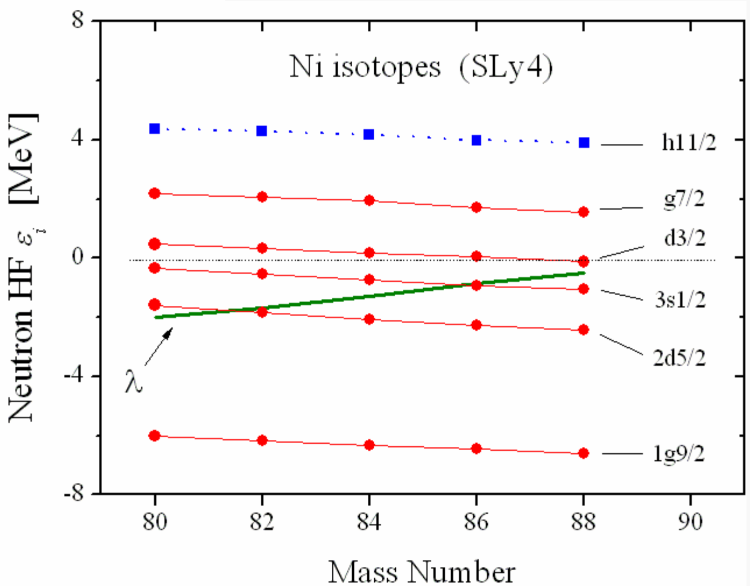


Spatial structure of two-quasiparticle state:

$$F_{kk'}^{(ph)}\left(\vec{r}\right) = u_k\left(E_k,\vec{r}\right)O_{ex}\left(\vec{r}\right)v_{k'}\left(E'_{k'},\vec{r}\right)$$

First 2⁺ states in neutron rich Ni isotopes





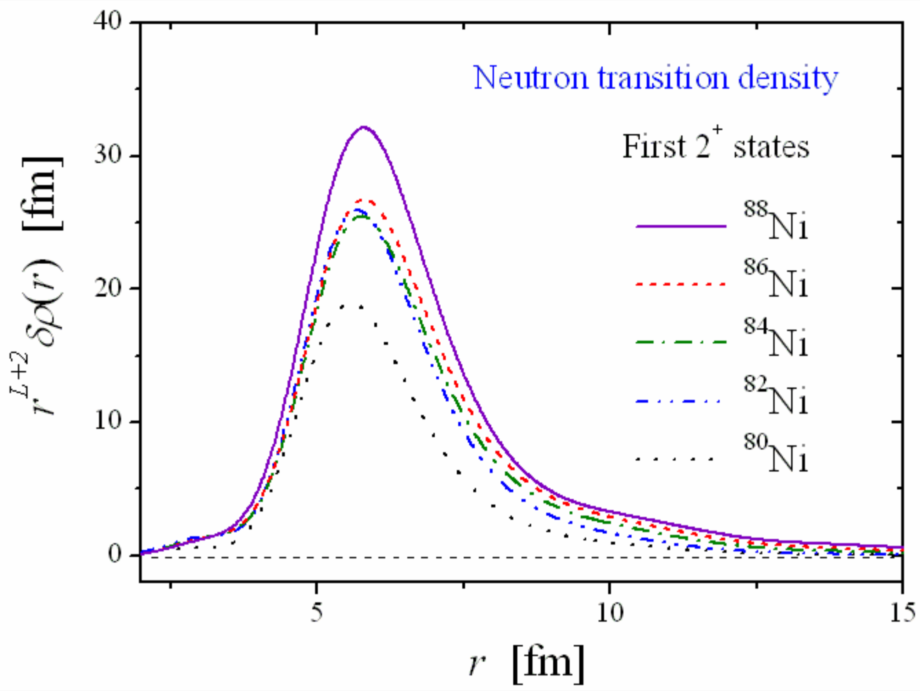
Comparison (Skyrme SLy4 force)

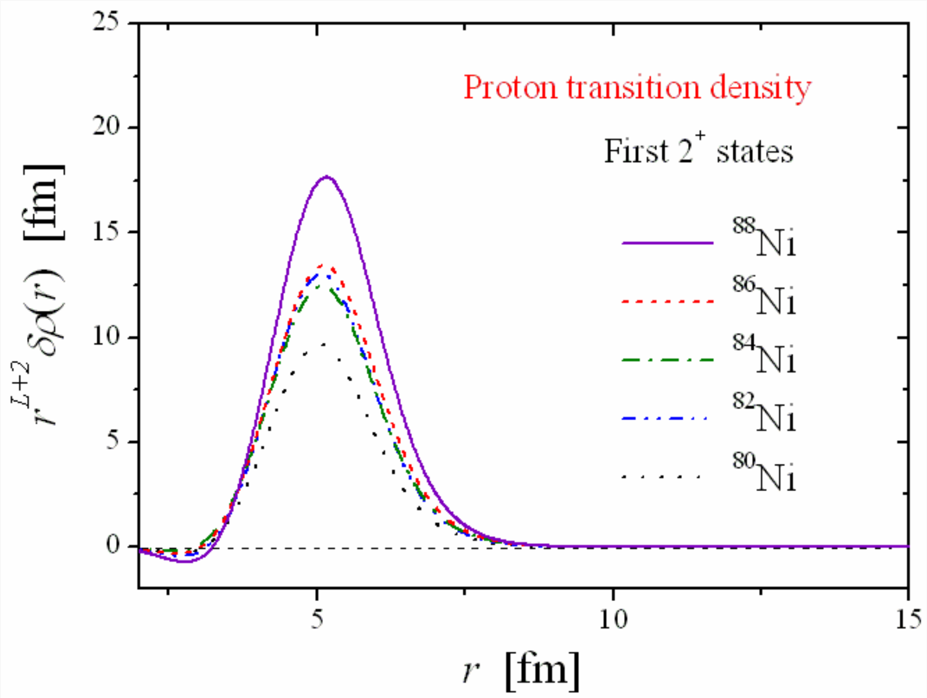
- HFB + QRPA
- HF-resonant BCS + QRPA
- HF + RPA

(These 2^+ states are below the thresholds)

$$E\left(2^+\right) \approx 1.0 - 1.5 \text{ MeV}$$

Transition densities of the first 2^+ states





- Spatially extended component of neutrons becomes sizable
- Correlations between ***neutrons*** and ***protons*** can occur only around the spatially narrow region

Conclusion

Pairing correlations and low-frequency vibrational excitations in nuclei close to the neutron drip line

Spatial structure of single-quasiparticle wave functions

$\left(\varepsilon_i, l, \Delta, \text{continuum}\right)$

Interplay of low-*l* and high-*l* states

Enhancement of pairing correlations

Strong transition strength of low-frequency vibrational excitations