Symmetry energy, masses and T=0 np-pairing

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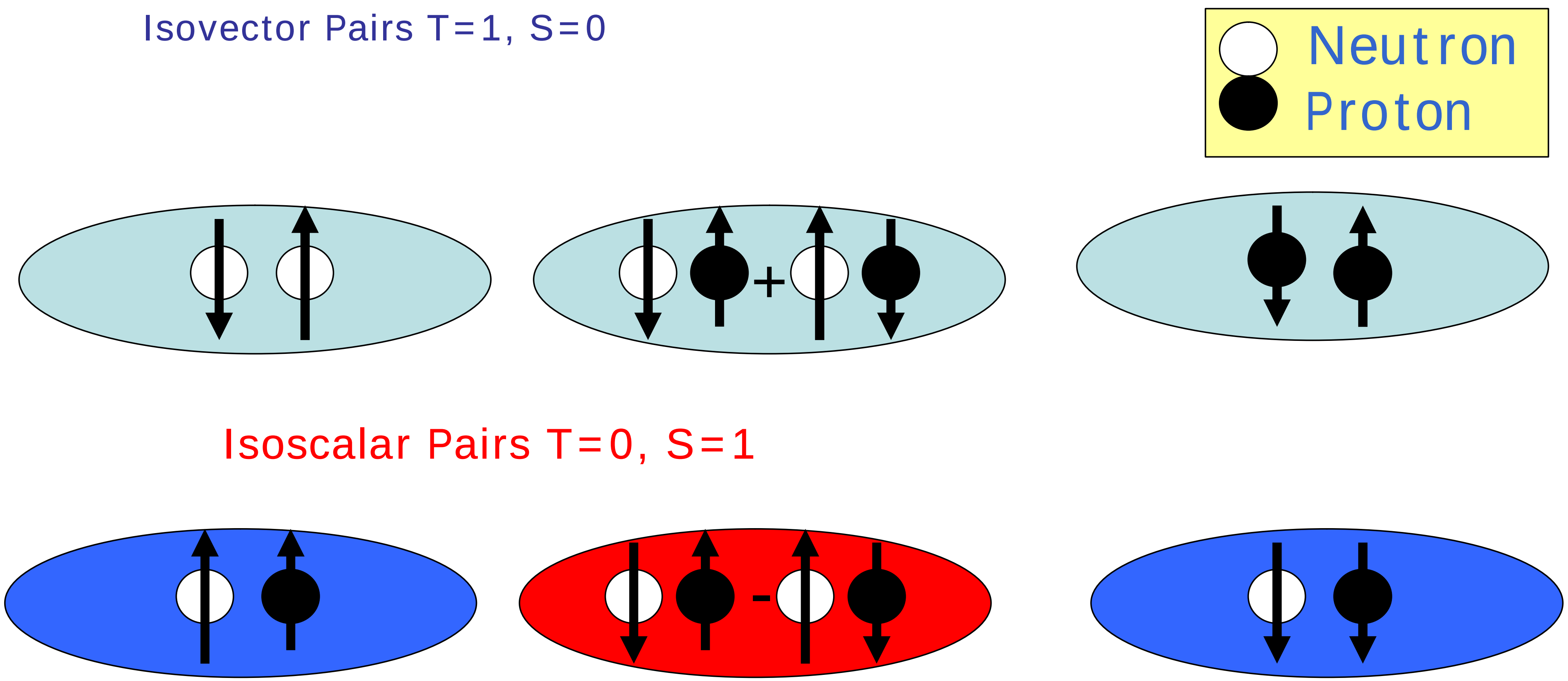
Can we measure the T=0 pair gap?
Do the moments of inertia depend on T=0 pairing?
Do masses evolve like T(T+1) or T^2 (N-Z)^2?
Origin of the linear term in mean field models
Investigation of the symmetry energy in Skyrme HF and RMF
New insight into the symmetry energy

$$\frac{E}{A} = -a_V + \frac{a_S}{A^{1/3}} + \left[a_{sym}^{(V)} - \frac{a_{sym}^{(S)}}{A^{1/3}} + \ldots\right]\left(I^2 + \lambda\frac{I}{A}\right) + \ldots,$$

Collaborator: W Satula, Univ. Warsaw, Shufang Ban, KTH

Structure of Nucleonic Pairs

- N=Z → (almost) identical wavefunctions
- particle particle interaction between pairs with identical orbits
- Pauli Principle



Generalised pairing interaction

- Start from a basis in which signature α is a good quantum number: $R_x(\pi)|\phi_j\rangle = +/- \mathbf{i}\, |\phi_j\rangle = e^{i\alpha}|\phi_j\rangle$, $a = +/- 1/2$,
- The standard pairing interaction scatters pairs in opposite signature orbits,

$$a\,\bar{a} \leftrightarrow a'\overline{a'} \qquad P^\dagger_{1\pm1} = \sum_{i>0} a^\dagger_{i\,{}^n_p} a^\dagger_{\tilde{i}\,{}^n_p}$$

- All possible couplings need to be present: T=1 nn, pp, **and** T=1 np

$$P^\dagger_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^\dagger_{i\,n} a^\dagger_{\tilde{i}\,p} + a^\dagger_{i\,p} a^\dagger_{\tilde{i}\,n})$$

- For the T=0 pairing, ***two*** different couplings are possible:
a) a T=0 np pair scatters between orbits of opposite signature,

$$a\,\bar{a} \leftrightarrow a'\overline{a'} \qquad P^\dagger_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^\dagger_{i\,n} a^\dagger_{\tilde{i}\,p} - a^\dagger_{i\,p} a^\dagger_{\tilde{i}\,n})$$

- b)a T=0 np pair scatters between orbits of the same signature,

$$a\,a \leftrightarrow a'a' \qquad \tilde{P}^\dagger_{0\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^\dagger_{i\,n} a^\dagger_{i\,p} + a^\dagger_{\tilde{i}\,p} a^\dagger_{\tilde{i}\,n})$$

Investigate the generalised pairing hamiltonian

$$\hat{H}^{\omega_\tau} = \hat{h}_{sp} - G_{t=1}\hat{\boldsymbol{P}}_1^\dagger \hat{\boldsymbol{P}}_1 - G_{t=0}\hat{\boldsymbol{P}}_0^\dagger \hat{\boldsymbol{P}}_0 - \vec{\boldsymbol{\omega}}_\tau \vec{\hat{\boldsymbol{t}}},$$

$$h_{\alpha\beta} = e_\alpha \delta_{\alpha\beta} - \omega j^{(x)}_{\alpha\beta} + \Gamma_{\alpha\beta},$$

Employ approximate number projection via L.N.

$$\hat{\mathcal{H}}^\omega = \hat{H}^\omega - \sum_\tau \lambda_\tau^{(1)} \Delta \hat{N}_\tau - \sum_{\tau\tau'} \lambda_{\tau\tau'}^{(2)} \Delta \hat{N}_\tau \Delta \hat{N}_{\tau'}$$

Investigate the BCS- and HFB solution as a function of strength

-BCS $G^{T=0}/G^{T=1}$ =? and HFB $G^{T=0}/G^{T=1}$ =?

BCS T=0,1 Pairing Hamiltonian

$$\tilde{P}^{\dagger}_{0\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{i\,p} + a^{\dagger}_{\tilde{i}\,p} a^{\dagger}_{\tilde{i}\,n})$$

p-n and p-ñ Pair

Pairs:

$$P^{\dagger}_{1\,\pm1} = \sum_{i>0} a^{\dagger}_{i\,{}^{n}_{p}} a^{\dagger}_{\tilde{i}\,{}^{n}_{p}}$$

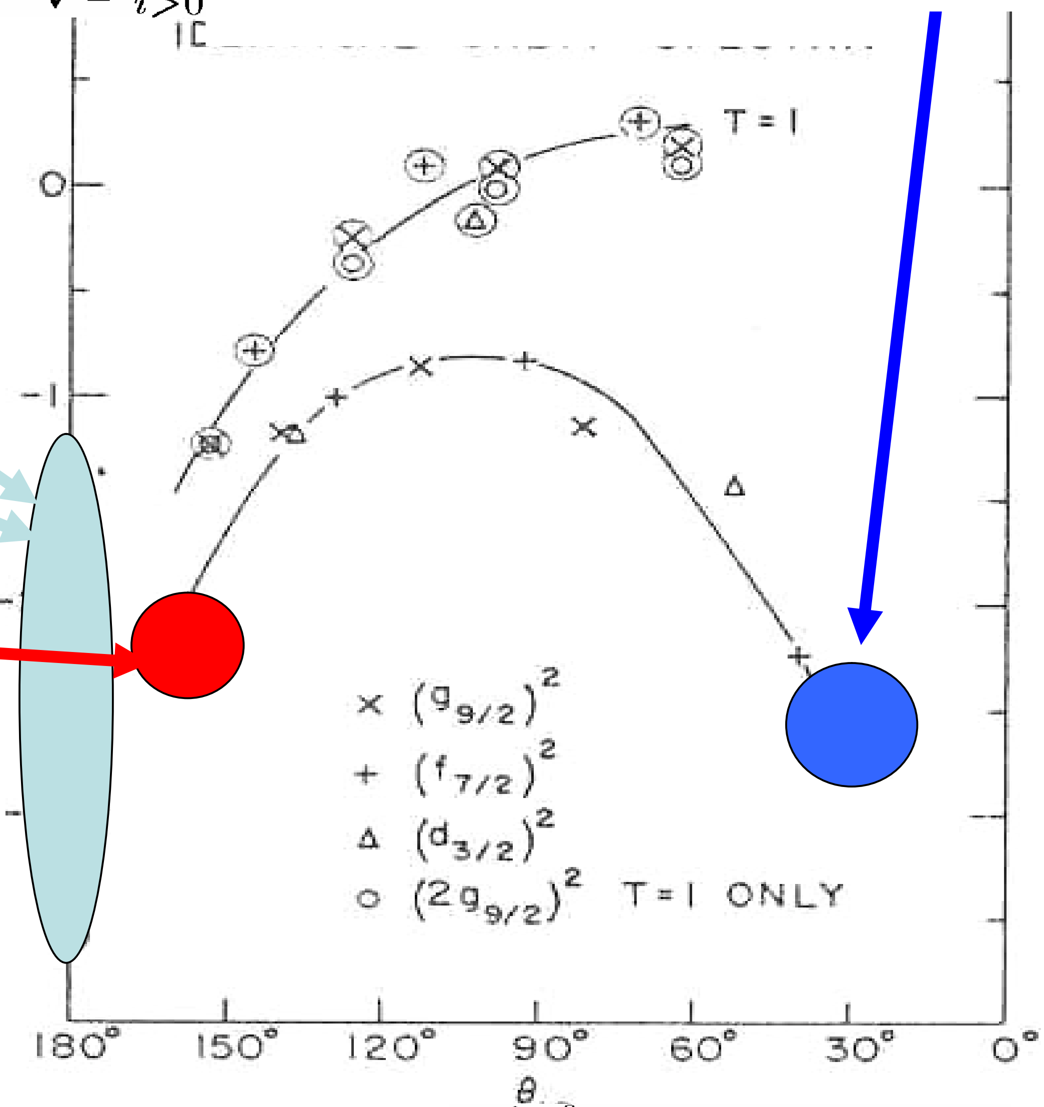
ñ-n and p-p « usũal » Pairs; T=1

$$P^{\dagger}_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{\tilde{i}\,p} + a^{\dagger}_{i\,p} a^{\dagger}_{\tilde{i}\,n})$$

p-ñ + n-p Pairs ; T=1

$$P^{\dagger}_{1\,0} = \frac{1}{\sqrt{2}} \sum_{i>0} (a^{\dagger}_{i\,n} a^{\dagger}_{\tilde{i}\,p} - a^{\dagger}_{i\,p} a^{\dagger}_{\tilde{i}\,n})$$

p-ñ – n-p Pairs; T=0



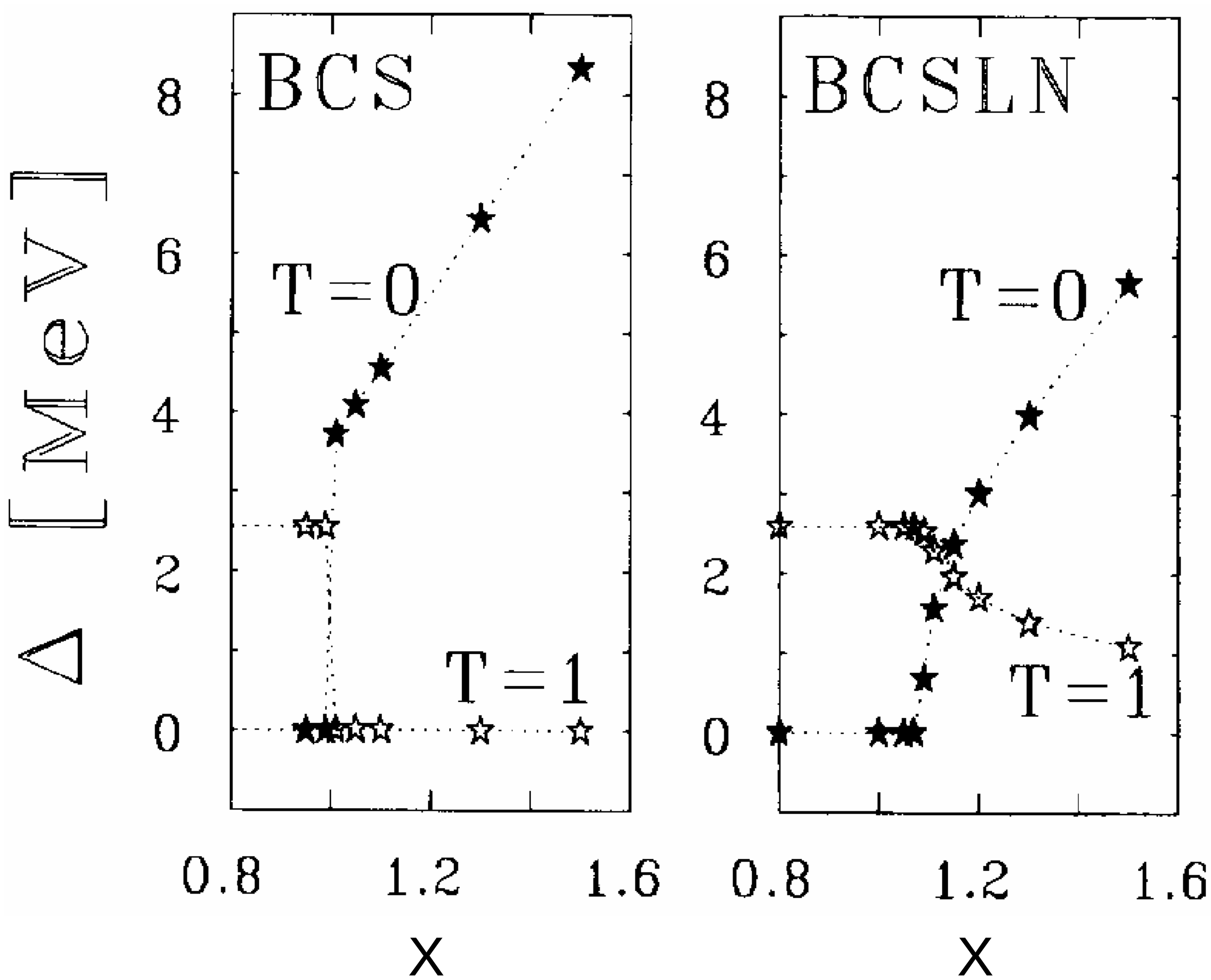
BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \hat{H}^{t=1} + \hat{H}^{t=0} + \hat{\tilde{H}}^{t=0}$$

$$\hat{H}^{t=1} = G^{t=1} \sum_{t_z=-1}^{+1} P^{\dagger}_{1\,t_z} P_{1\,t_z} \qquad \hat{H}^{t=0} = G^{t=0}\, P^{\dagger}_{0\,0} P_{0\,0} \qquad \hat{\tilde{H}}^{t=0} = \tilde{G}^{t=0}\, \tilde{P}^{\dagger}_{0\,0} \tilde{P}_{0\,0}$$

Intensity T=0/T=1 ; Resultats (1)

48Cr Calculation 1) meanfield = W.S. 2) X= $\tilde{G}^{t=0}$ / $G^{t=1}$



W.Satula, R.Wyss
PLB 393 (1997) 1

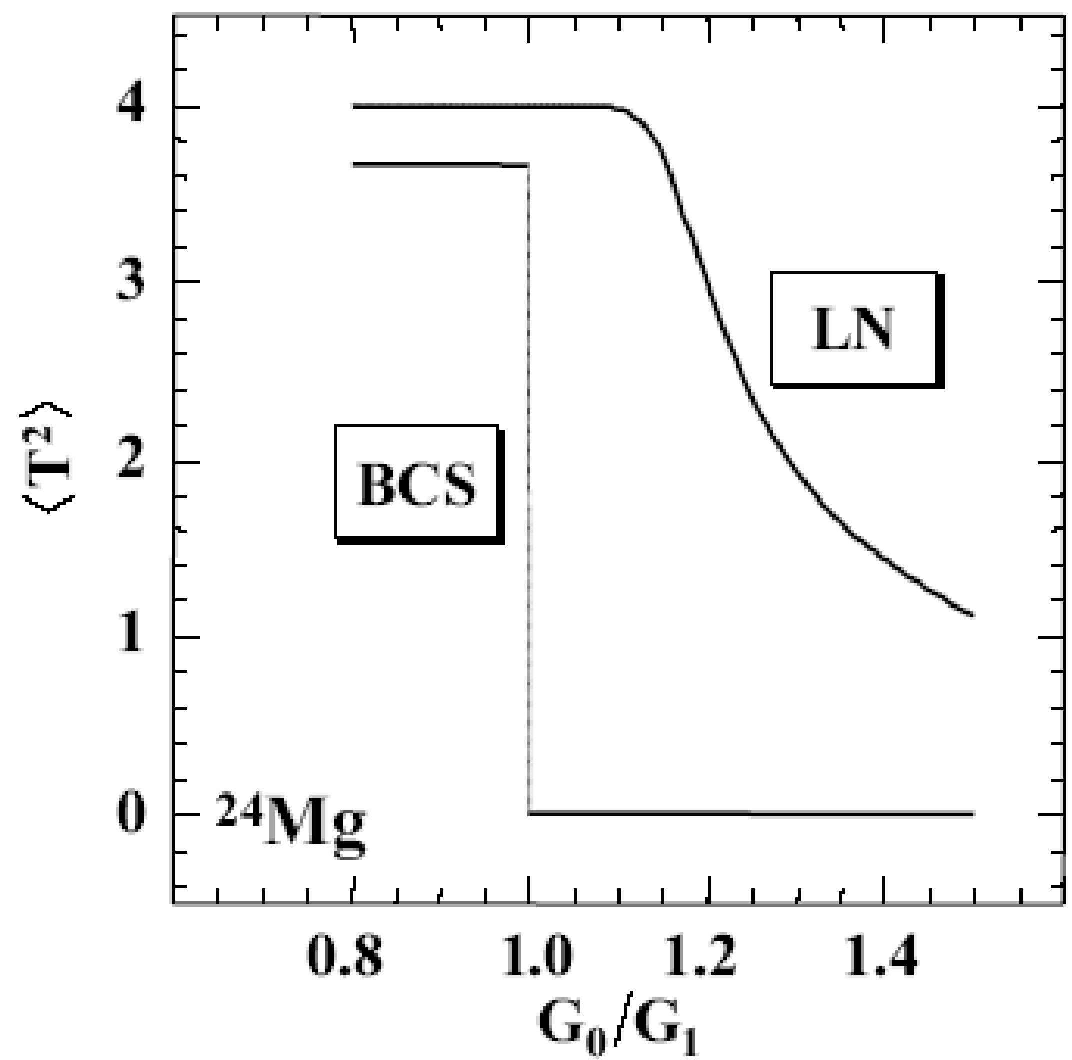
Incompatible?

-T=1, Tz=+1,-1 with Tz=0

-T=1 with T=0

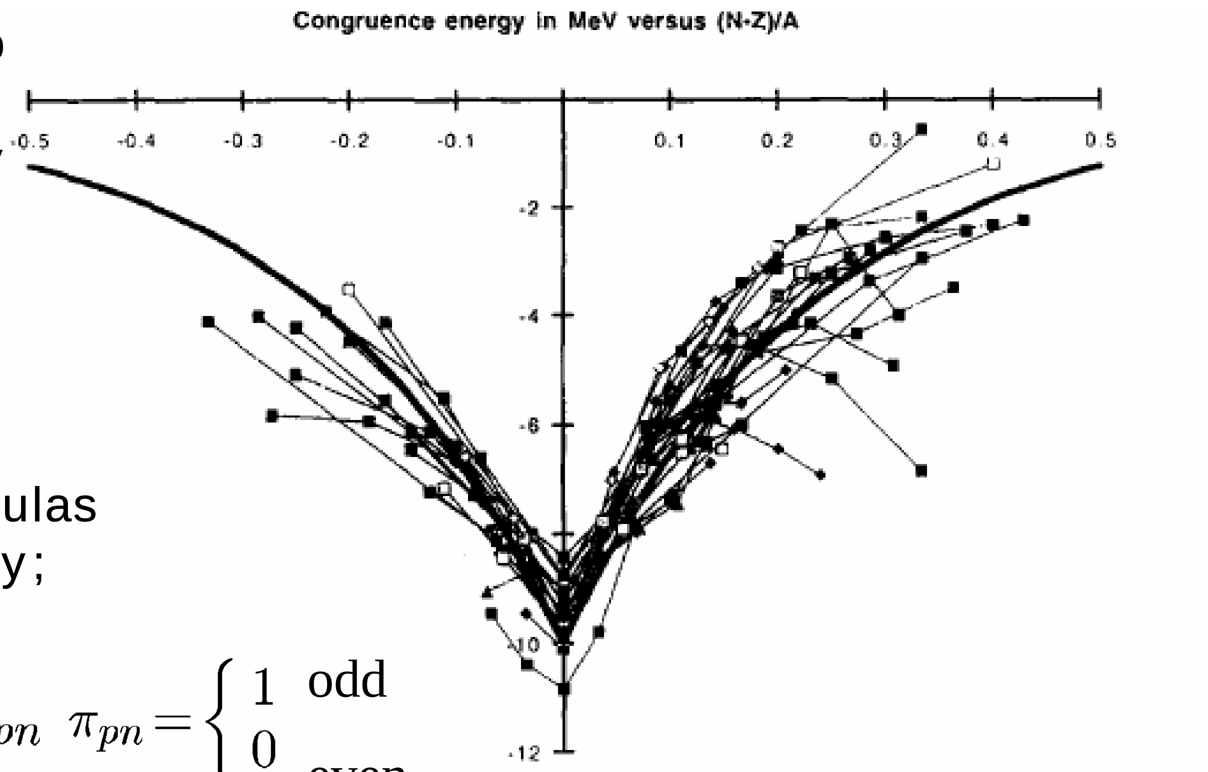
Iso spin mixing due to T=a pairing interaction

- T=1 pairing violates iso-spin – resulting in deformation in iso-space
- T=0 pairing restores iso spin (scalar in iso space)
- We need iso spin breaking to calculate iso spin excited states.



Mass excess due to Wigner energy

Mass defect with respect to the Thomas Fermi model.
The fitted curve is given by
$C(I)=10e^{(-4.2|I|)}$ /MeV,
I= N-Z = 1/2 Tz
(Myers Swiatecki, NPA612 (1997),249



In semiempirical massformulas one adds the Wigner energy;

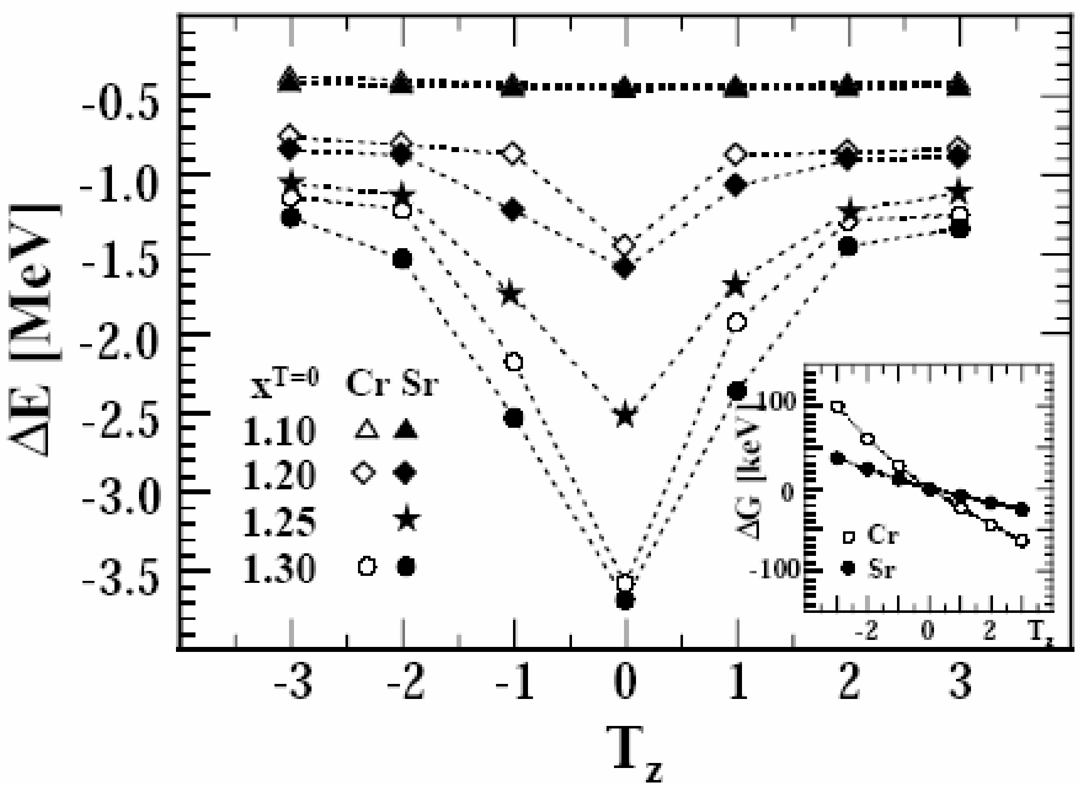
$$E_W = W(A)|N-Z| + d(A)\delta_{NZ}\pi_{pn} \quad \pi_{pn} = \begin{cases} 1 & \text{odd} \\ 0 & \text{even} \end{cases}$$

$$W(A) \sim 47/A \text{ MeV}$$

N=Z nuclei appear to be more bound, o-o have a repulsive term

Skyrme HF masscalculations S. Goriely et.al. PRC68 (2003) 054325, fully microscopic, rms=0.675, use a macroscopic Wigner Energy:

$$E_W = V_W \exp\left\{-\lambda\left(\frac{N-Z}{A}\right)^2\right\} + V'_W|N-Z|\exp\left\{-\left(\frac{A}{A_0}\right)^2\right\};$$

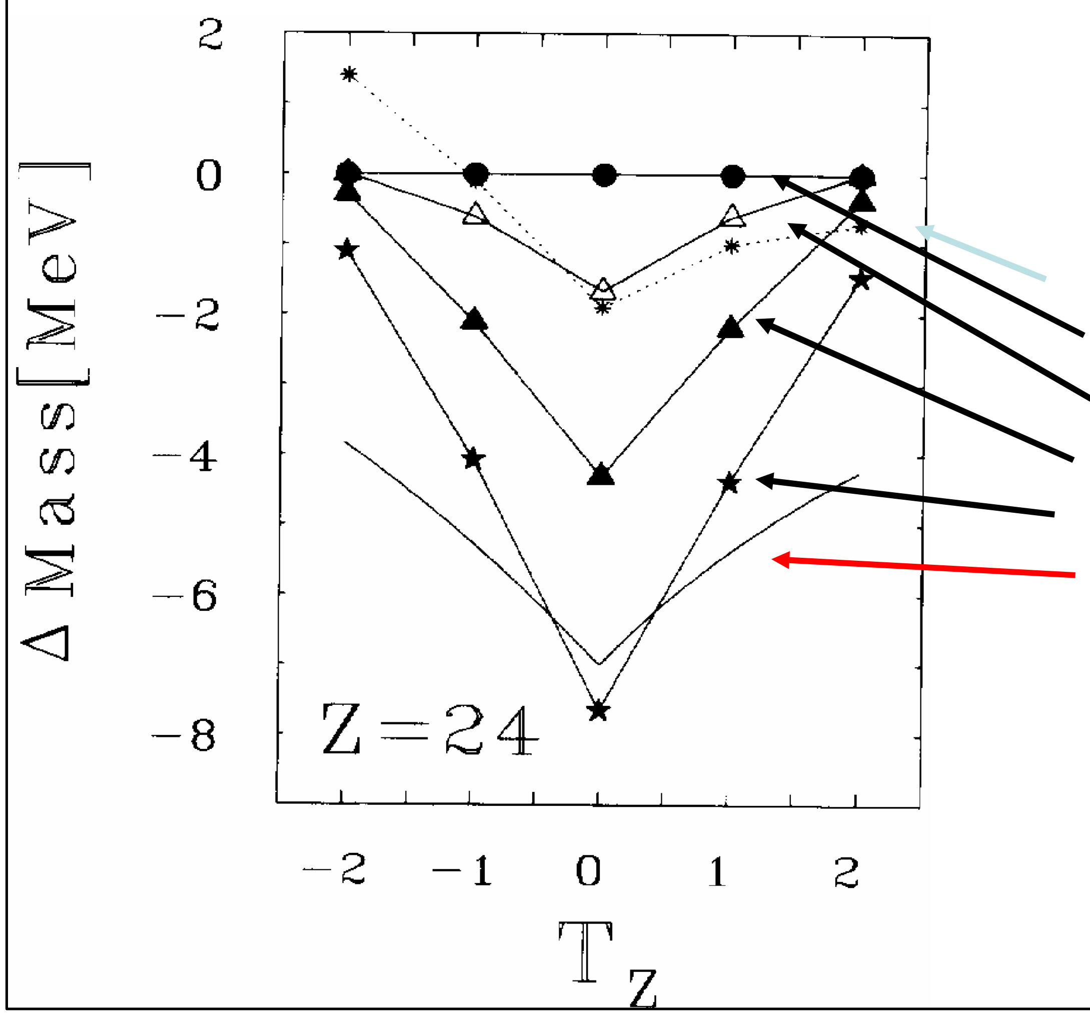


Additional binding from microscopic calculations due to T=0 pairing, W. Satula and R.W. NucPhA676 (2000) 120 and PLB393 (1997)1

Mass excess in N=Z nuclei

- Nuclei along the N=Z are more bound
  B.E.=$\mathbf{a_{sym}T(T+1.25)}$
- Isovector pairing weakens ' $\mathbf{a_{sym}}$ '

T=0 pairing increases binding

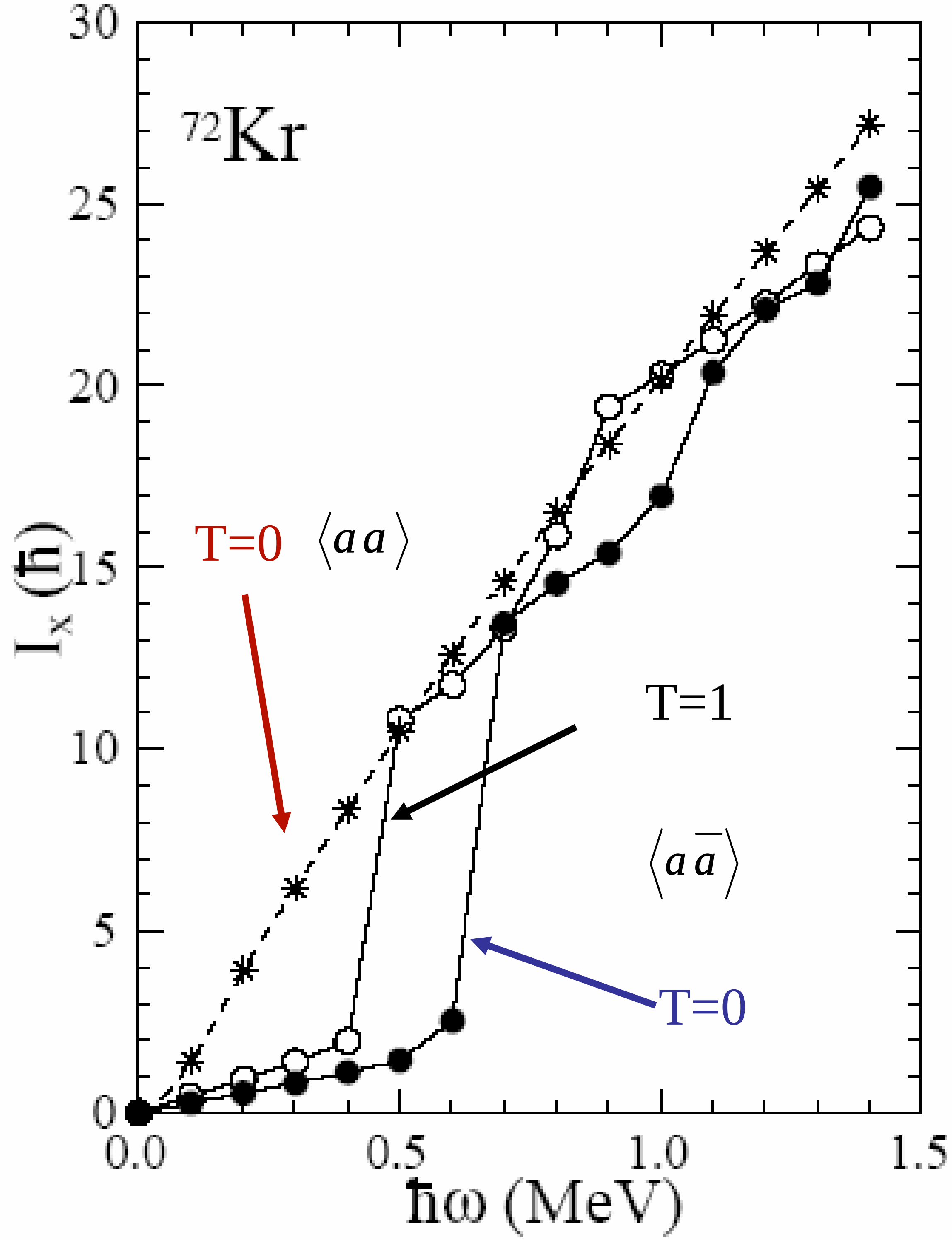


Gain in binding energy
E(T=0+1)- E(T=1)

Extended Thomas-Fermi
X=1.1
X=1.2
X=1.3
X=1.4
Wigner term(Myers-Swiatecki)

X= $\tilde{G}^{t=0}$ / $G^{t=1}$

Generic features of the alignment in the presence of the different T=0 and T=1 pairing modes



Effect of T=0 Pairing on MoI

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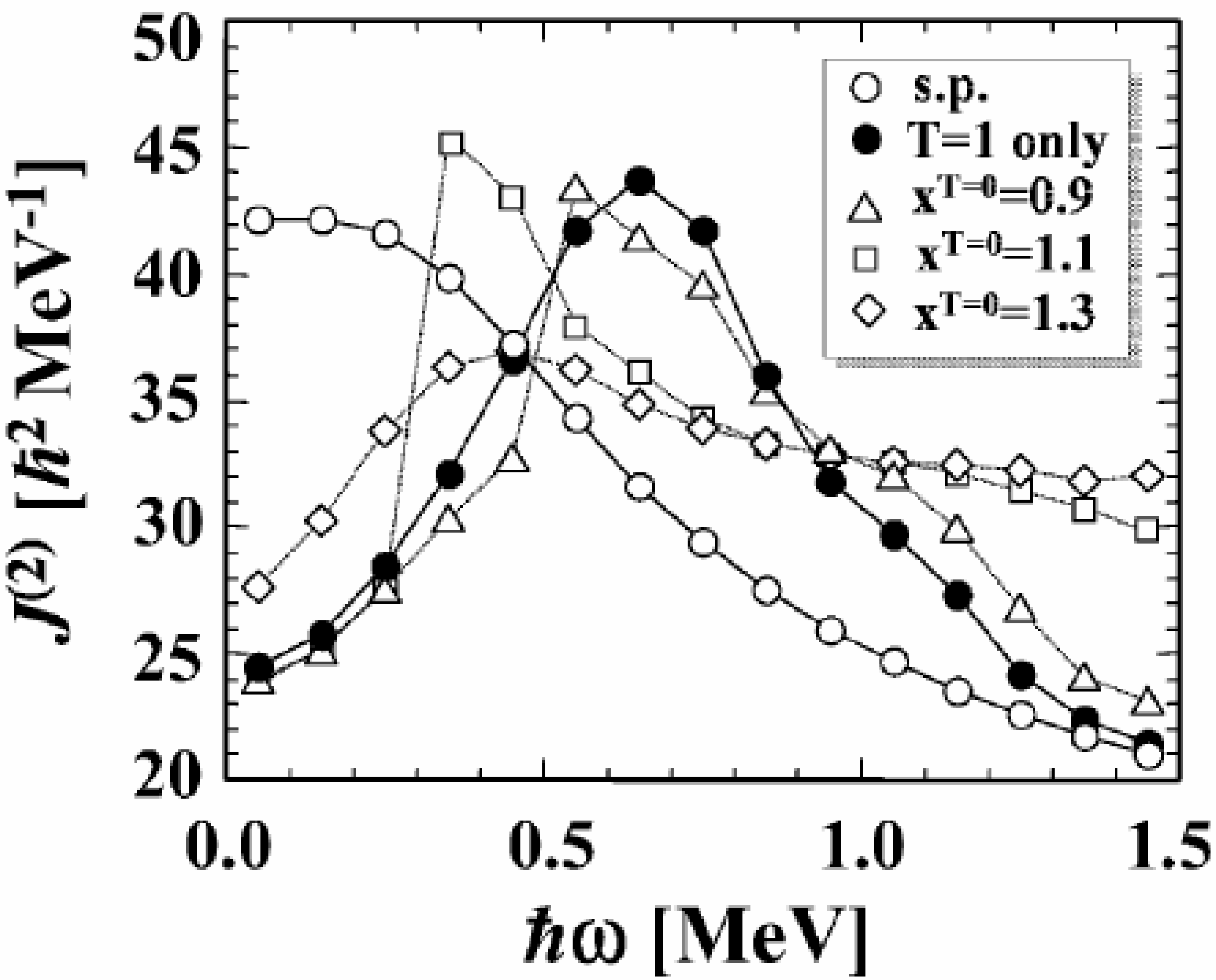
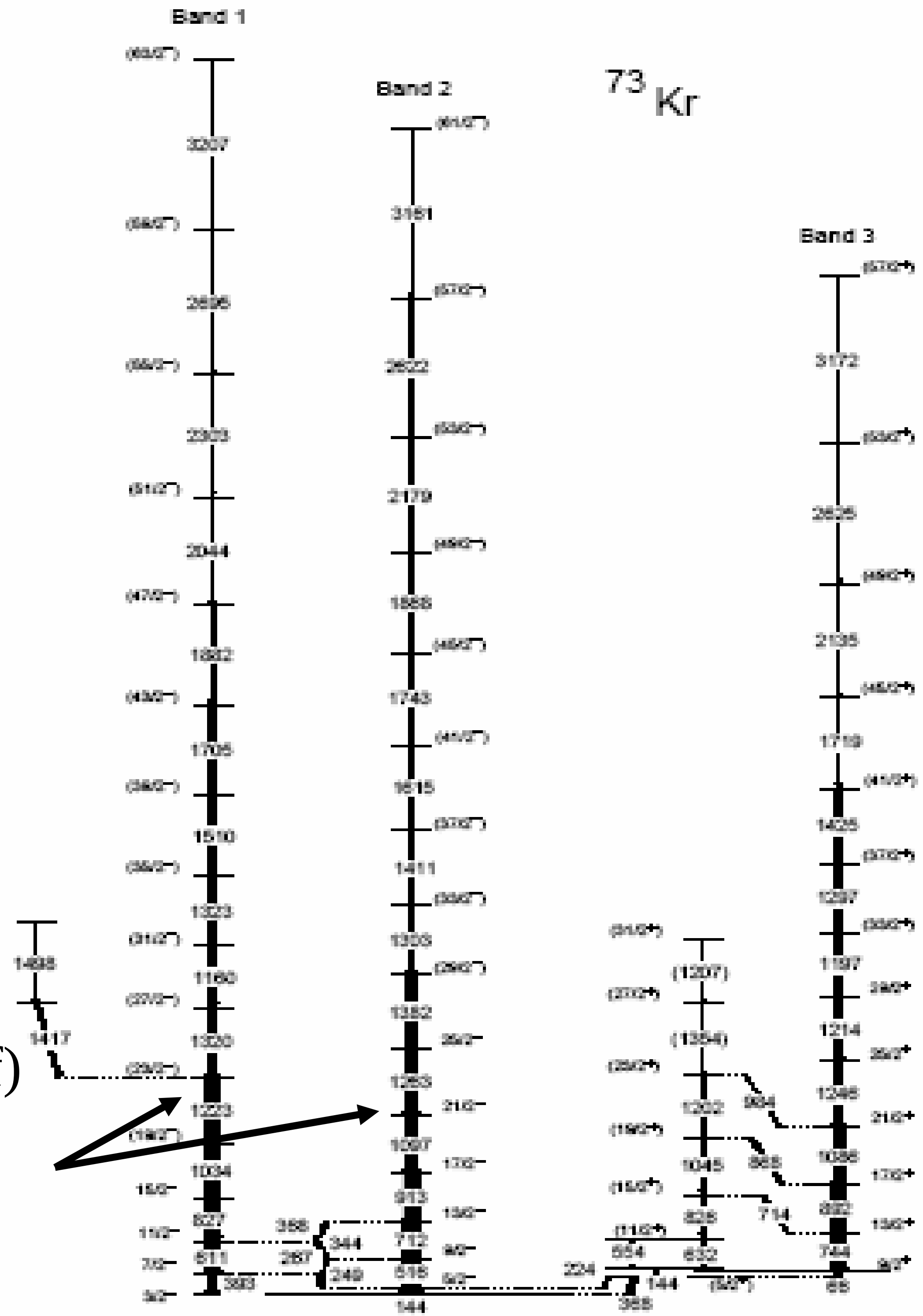


Level-scheme of 73Kr

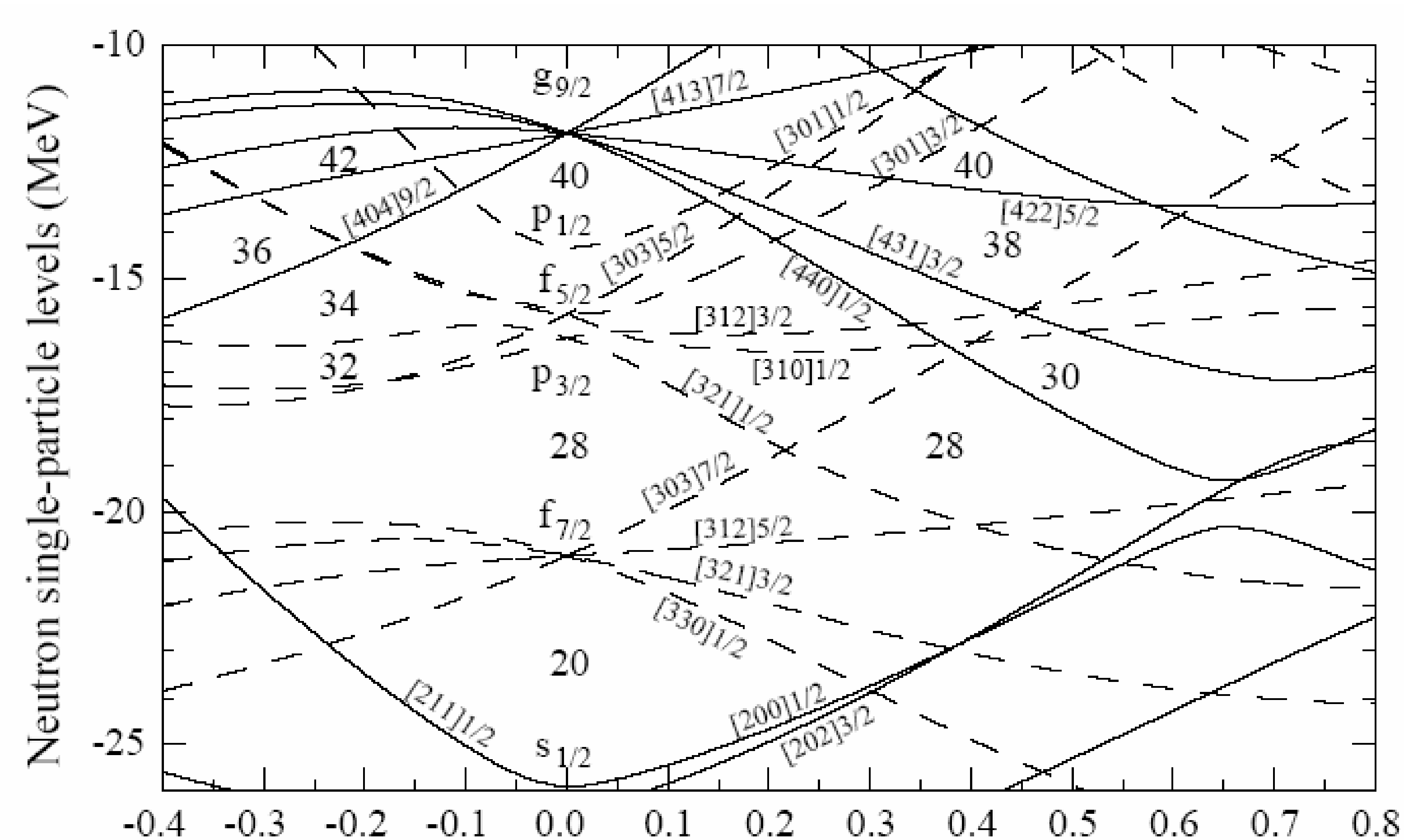
N.S.Kelsall et. al., Phys.Rev. C65, 044331 (2002)



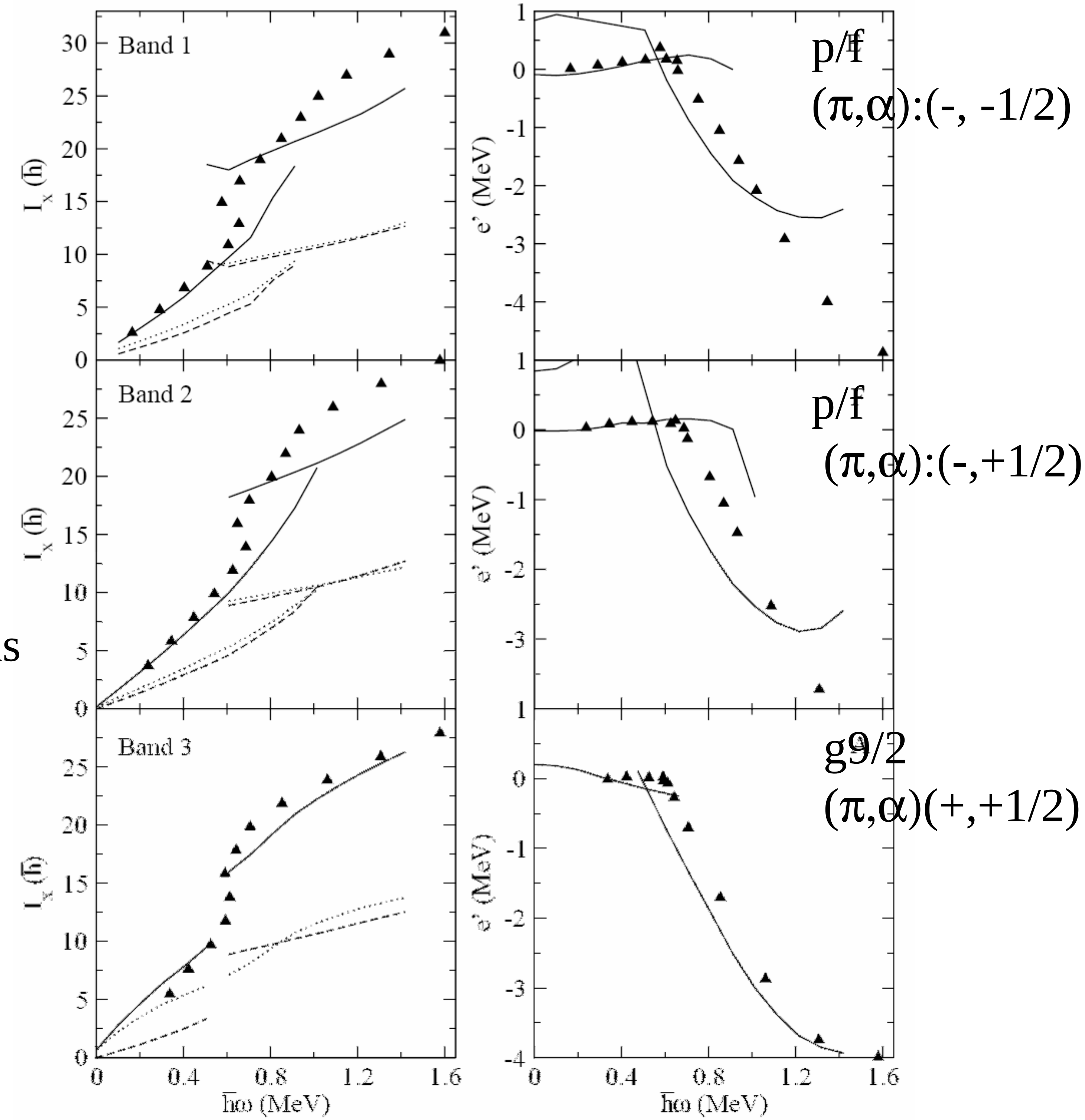
ν p3/2-f5/2 (p/f)
neg. par.

ν g9/2
pos. par.

WS sp diagramme

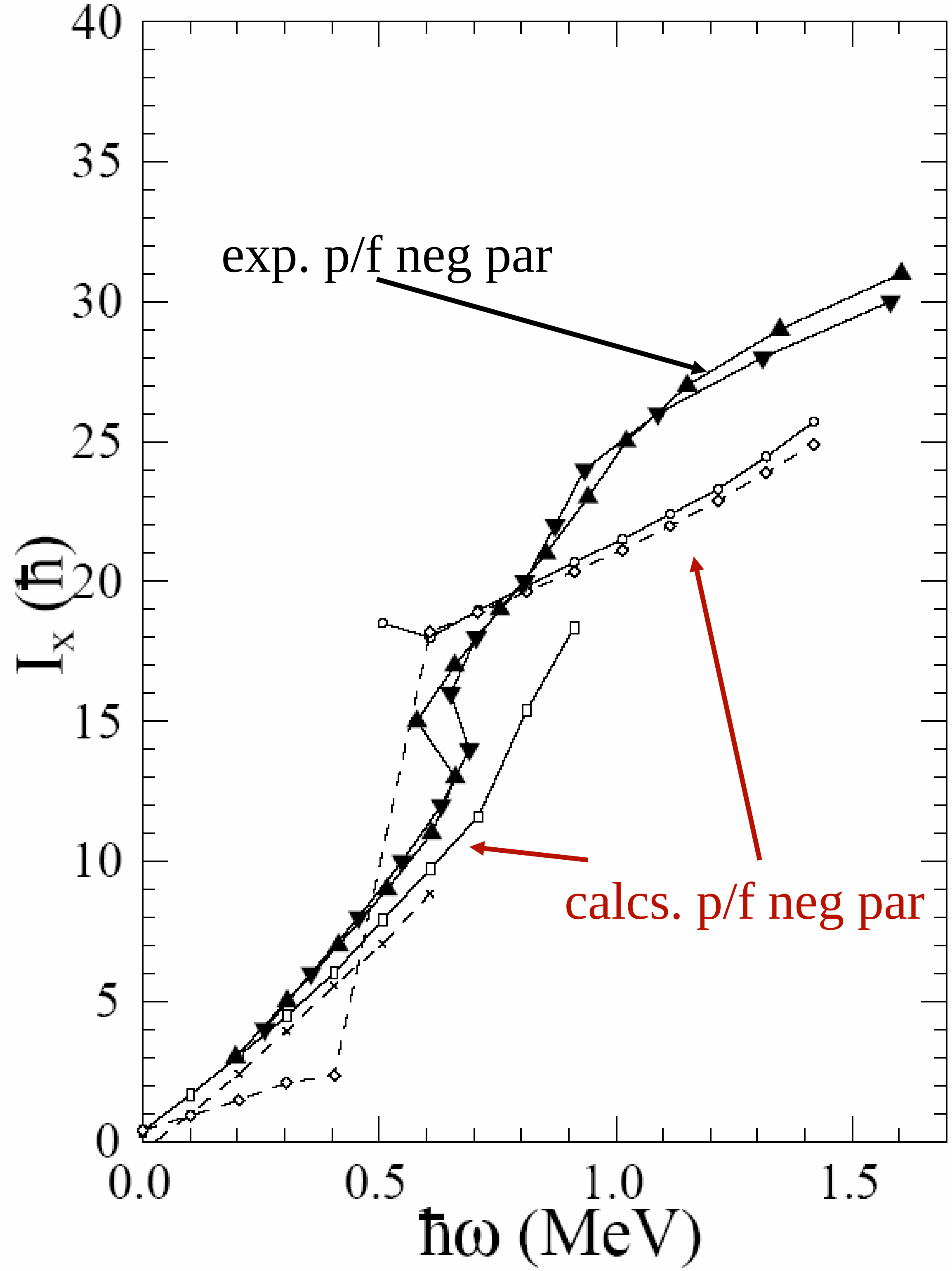


Alignments and Routhians for 73Kr



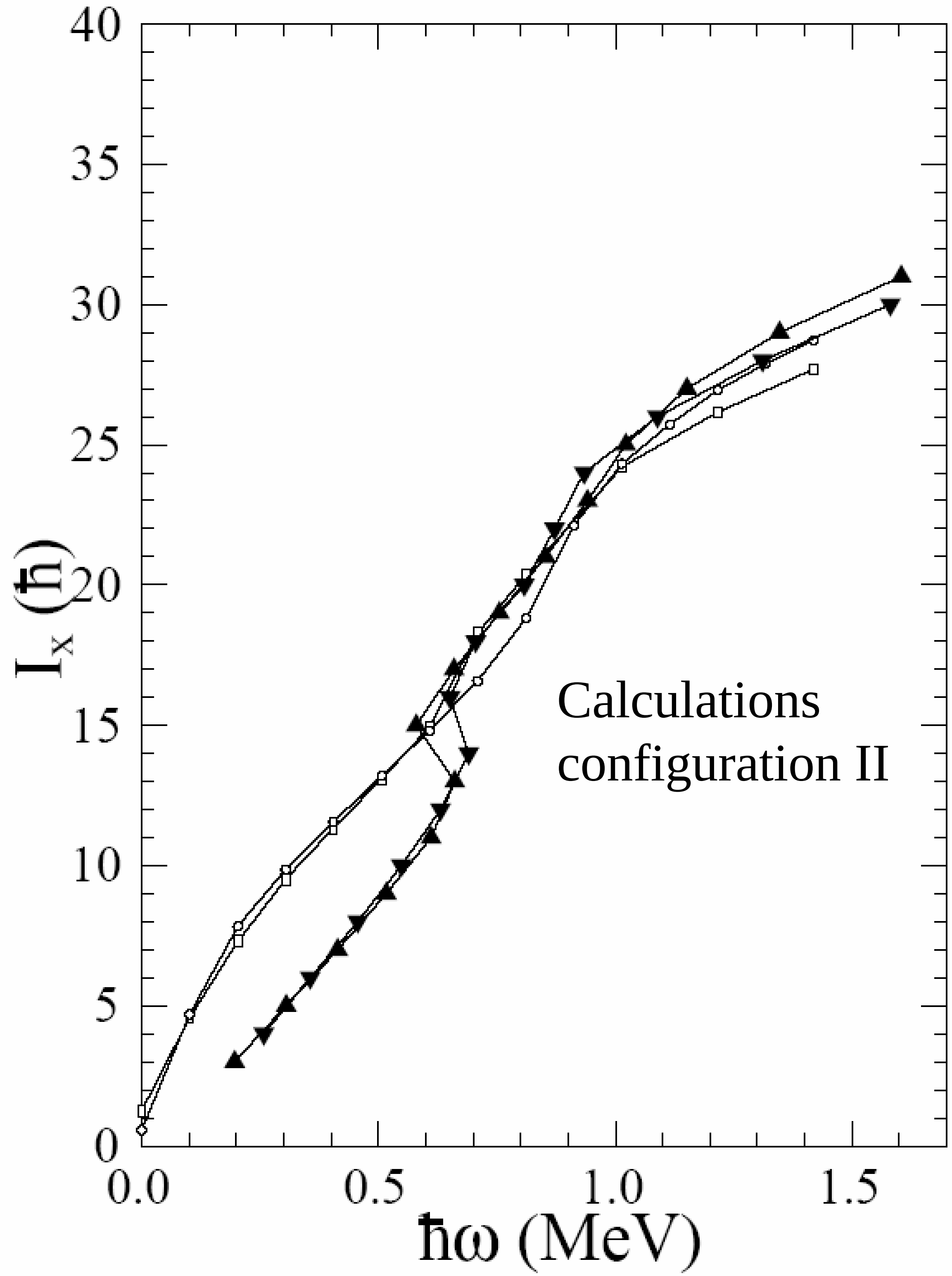
good agreement for low spin for the two neg. par. bands – disagreement at high spins

good agreement for the pos.par. band (g9/2) over the entire spin range

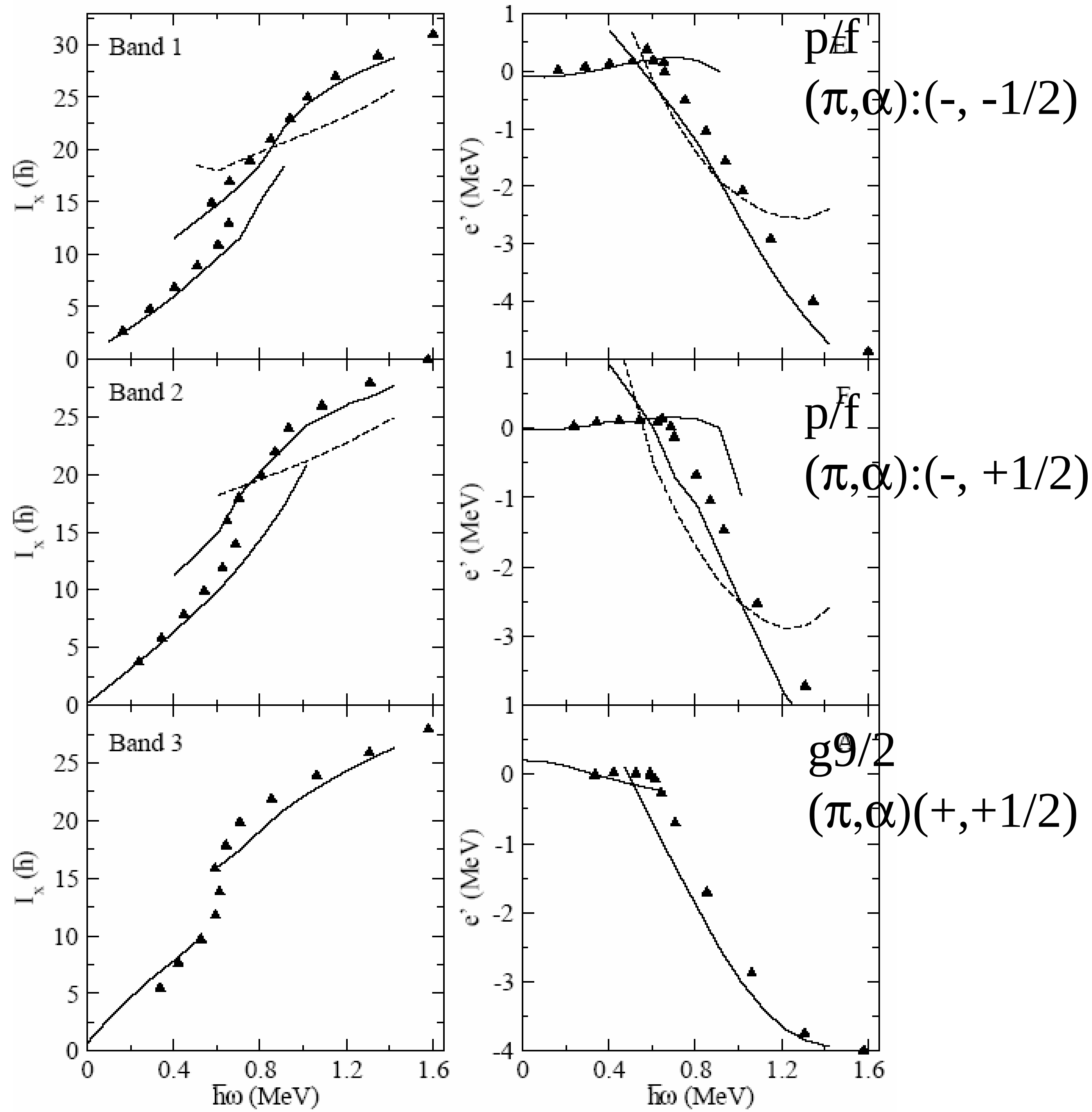

exp. p/f neg par
calcs. p/f neg par
I_x (ħ)
ħω (MeV)
0
5
10
15
20
25
30
35
40
0.0
0.5
1.0
1.5

Assume an entire different configuration:
Move the neutron from neg. par. f/p orbit into g9/2 and make a 2qp proton excitation from a f/p orbit into g9/2

ν g9/2 pos par (+,+1/2)
Π:[f/p❑g9/2] neg par (-,-/+1/2)



Alignment and Routhian for the new configuration



T=1 scenario:

conf I $a^{+}_{n(f/p)} \prod BCS_n > \prod BCS_p >$

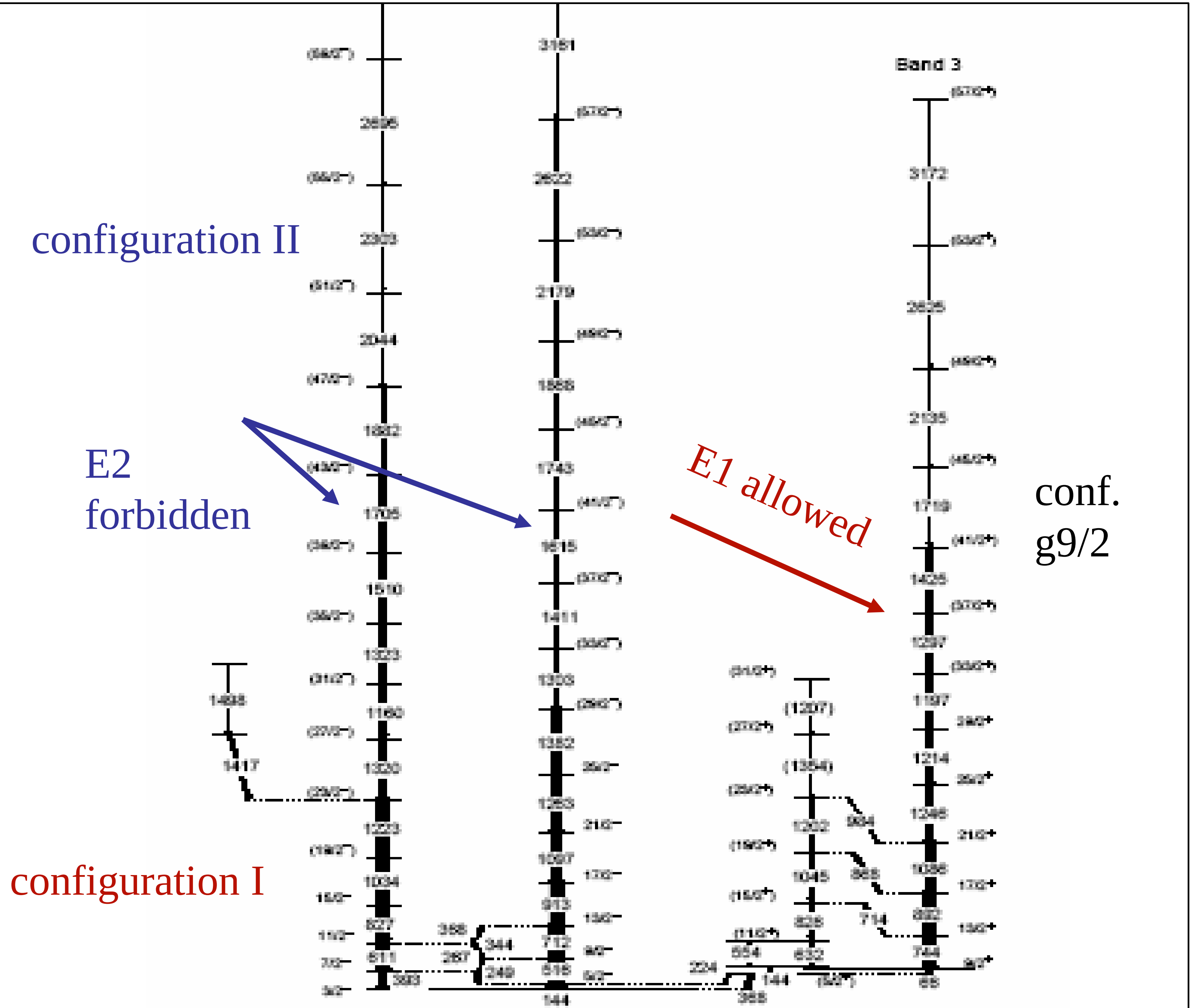
conf II $a^{+}_{n(g9/2)} \prod BCS_n > a^{+}_{p(g9/2)} a^{+}_{p(fp)} \prod BCS_p >$

conf g9/2 $a^{+}_{n(g9/2)} \prod BCS_n > \prod BCS_p >$

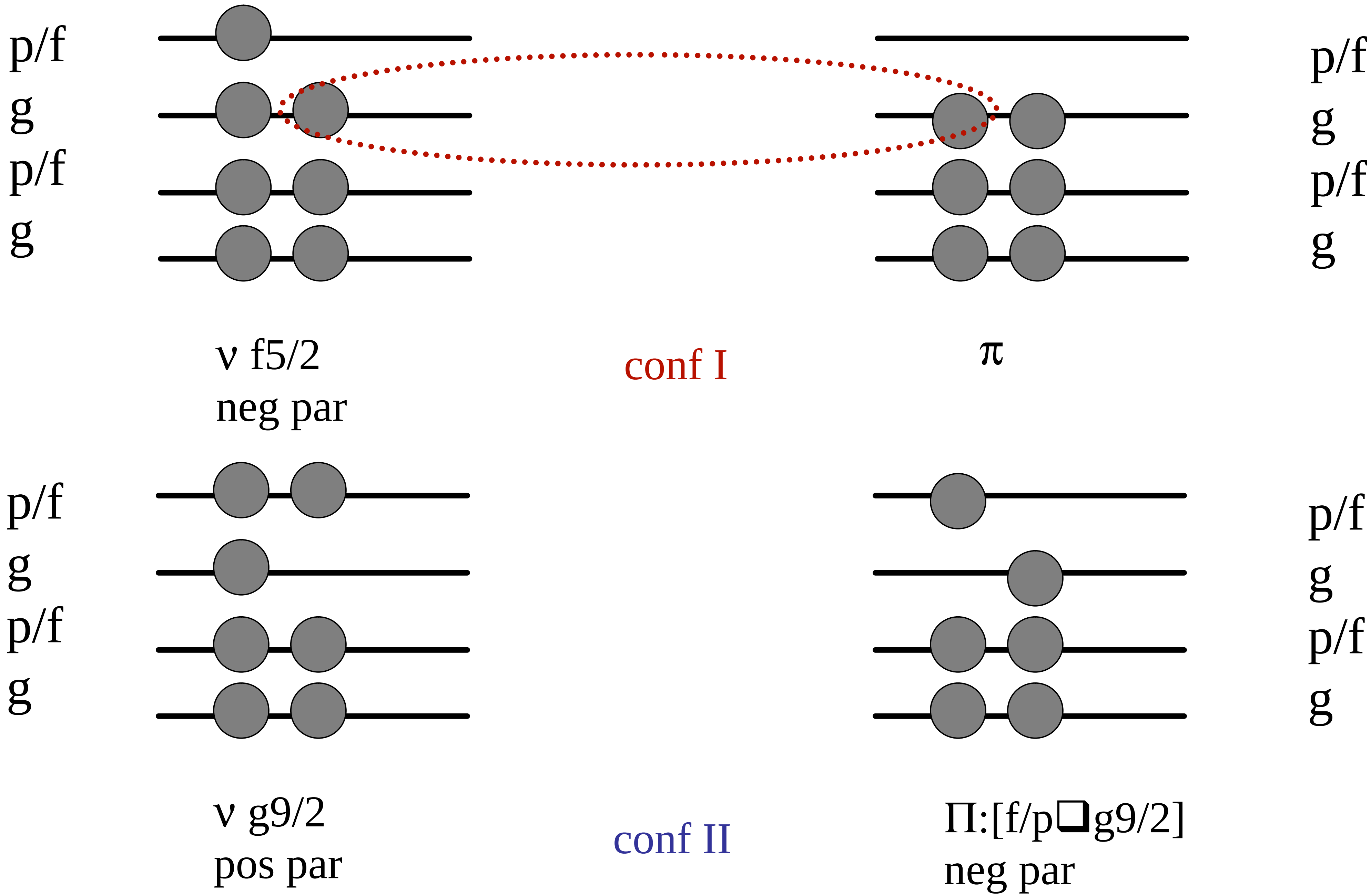
<conf I |O (E2) | conf II > forbidden

<conf II |O (E1) | conf g9/2> allowed

Level-scheme of 73Kr

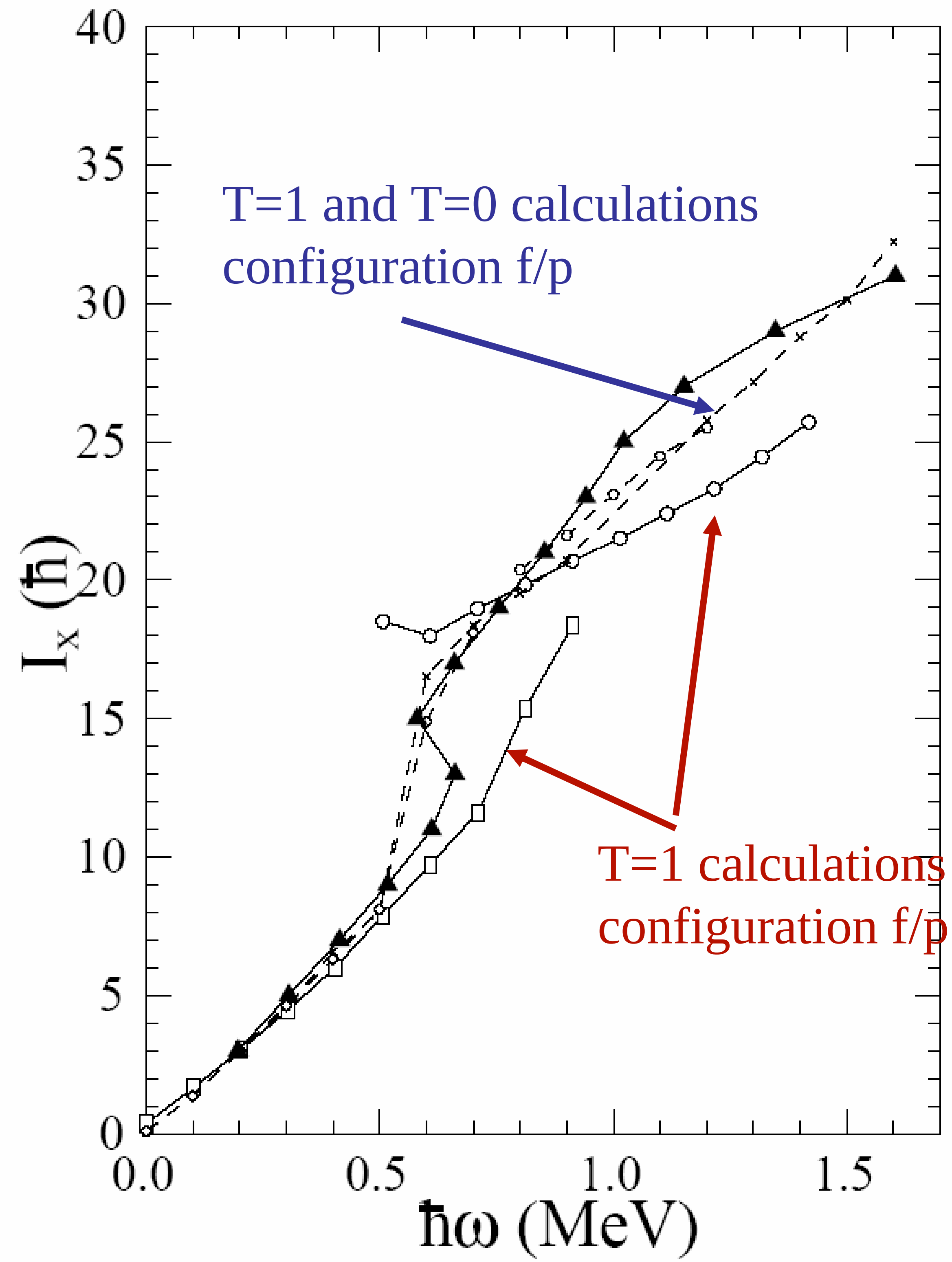


Scattering of a T=0 np pair



TRS calculations with T=0 and T=1 pairing

Same configuration blocked in both calculations – phase transition from T=1 to T=0 pairing



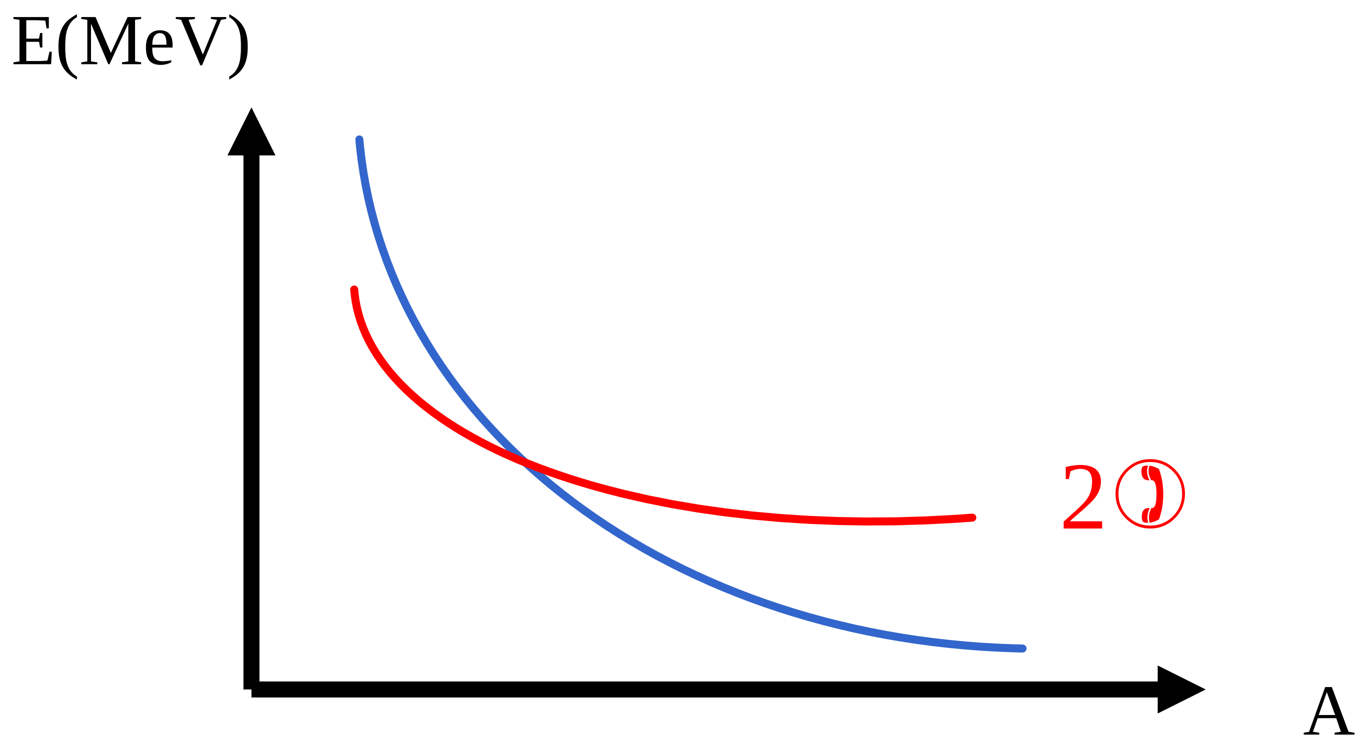
T=0 scenario

- conf I and conf II belong to the same band – become mixed via the T=0 pairing matrix element
- Phase transition from T=1 pairing at low spins to T=0 pairing at high spins
- Similar case in 75Rb (Tz=1/2)

Competition between 2qp excitation and symmetry energy in o-o nuclei

T=0 states in o-o nuclei are 2qp excitations 📫1/sqrt(A)

T=1states have larger symmetry energy 📫1/A



Symmetry Energy

Standard text books:

$$E_{\text{sym}} = \frac{1}{2} a_{\text{sym}} T^2 = \frac{1}{2} (a_{\text{kin}} + a_{\text{int}}) T^2.$$

groundstate in nuclei have lowest T, <T> = Tz = ½ (N-Z)

Bethe-Weizsäcker massformula: $\sim(N\text{-}Z)^2$

$$E_{sym} = \tfrac{1}{2}\, a_{sym}\, T(T+1)$$

$$a_{sym} = \tfrac{1}{2}\, a_{vol}/A - \tfrac{1}{2}\, a_{surf}/A^{4/3}$$

$$= 134.4/A - 203.6/A^{4/3}$$

Duflo&Zuker, PRC 52(1995)R23

Physics Letters B 572 (2003) 152–158

Symmetry energy in the mean field

W. Satula and RW

$$E_{\text{sym}} = \frac{1}{2} a_{\text{sym}} T^2 = \frac{1}{2}(a_{\text{kin}} + a_{\text{int}}) T^2.$$

- Any bi-fermionic system is characterised by a symmetry energy, coming from the discretness of the s.p. levels (no assumption of any force!) This term is proportional to the average level spacing:

$$E = \frac{1}{2}\varepsilon T^2 \qquad \varepsilon \approx 2\frac{\pi^2}{3a} \approx 16\frac{\pi^2}{3A}\text{–}20\frac{\pi^2}{3A} \approx \frac{53}{A}\text{–}\frac{66}{A}\ \text{MeV}.$$

- The nuclear interaction differs between states of different iso-spin – resulting in an additional iso vector potential. This potential can be obtained e.g. from an interaction

$$V_{TT} = \frac{1}{2}\kappa \hat{T} \cdot \hat{T}.$$

- This interaction leads to a term E=k T^2, i.e. k Tz^2 = k ¼ $(N\text{-}Z)^2$ (Hartree approx) Taking into account the exchange term (Fock), E= k T(T+1) (see e.g B&M, vol 1)

$$E_{sym} = \frac{1}{2}(e + k)T^2 + \frac{1}{2}k\ T$$

Investigate this concept in Skyrme HF-BCS

- The Skyrme HF can be divided in an iso scalar G_0 and iso vector potential G_1. There are 5 isoscalar and 5 isovector densisties and related coupling constants.

$$\sum_{t=0,1} \int d^3\boldsymbol{r}\, \mathcal{H}_t(\boldsymbol{r}):$$

$$\mathcal{H}_t(\boldsymbol{r}) = C_t^{\rho}\rho_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^{\tau}\rho_t\tau_t + C_t^{J}\boldsymbol{J}_t^2 + C_t^{\nabla J}\rho_t\boldsymbol{\nabla}\cdot\boldsymbol{J}_t$$

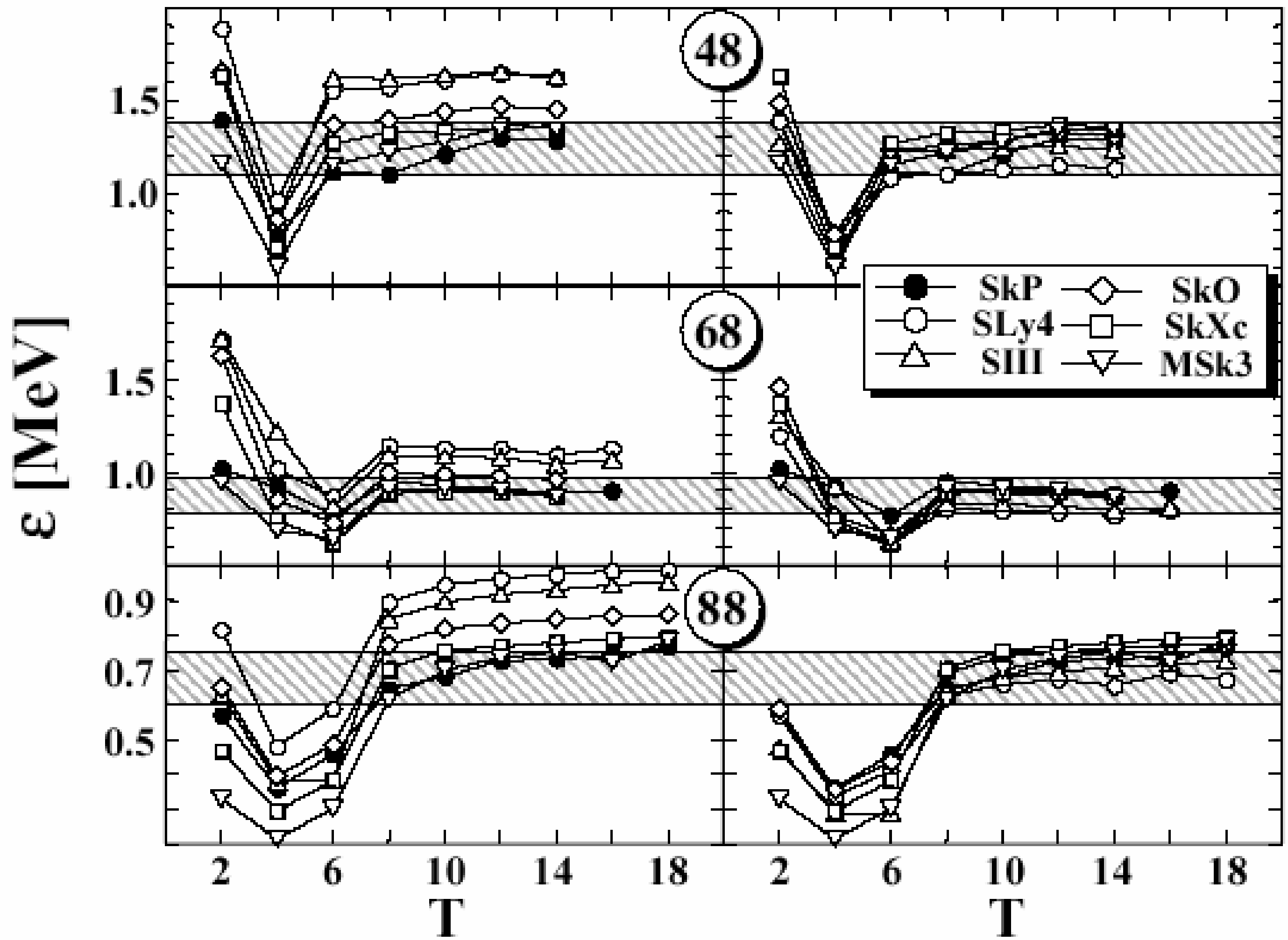
- Switch off the iso vector part and calculate the average spacing as a function of (N-Z)
- Determine k via calculating the full functional

Symmetry energy in Skyrme HF

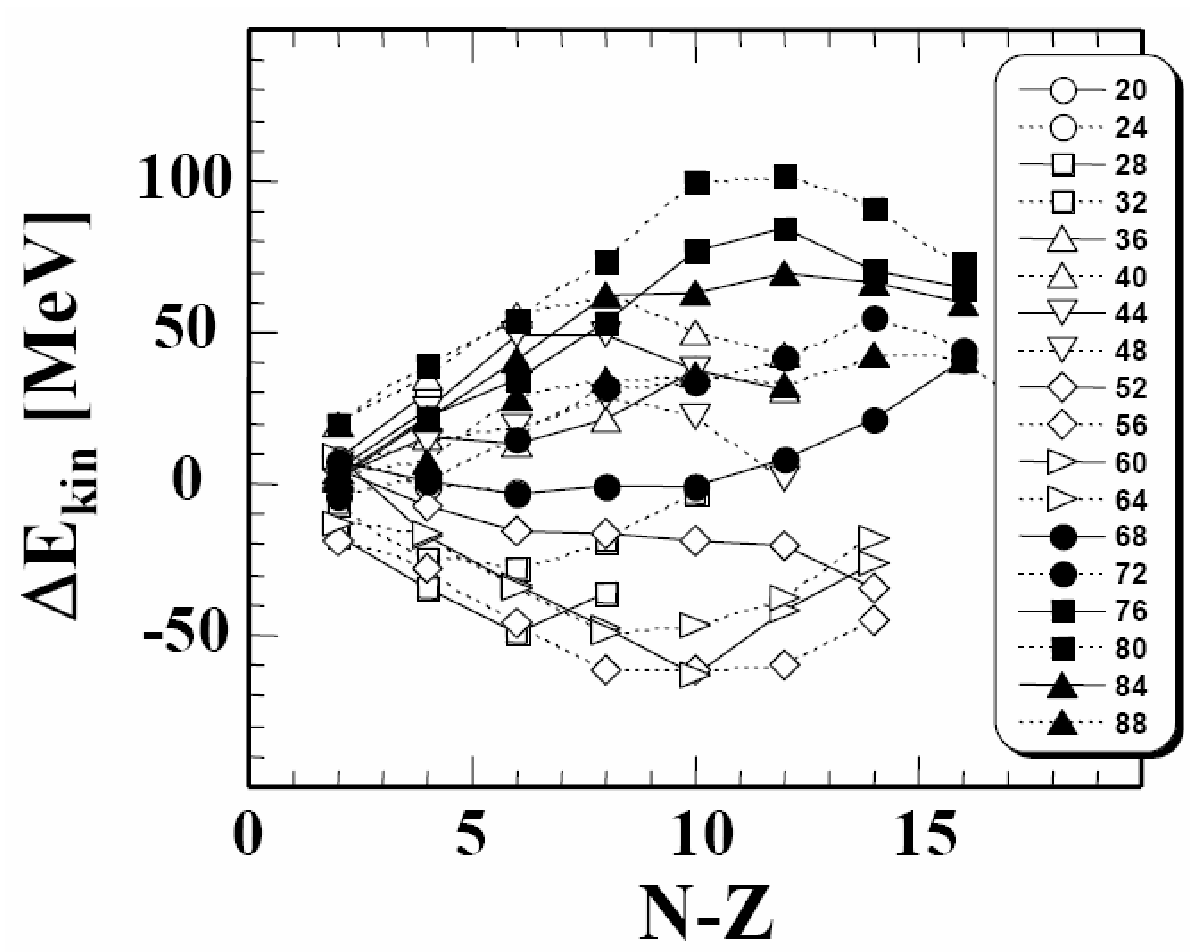
$$\Delta\tilde{E}_T^{(\mathrm{HF})} = \tilde{E}_T^{(\mathrm{HF})} - \tilde{E}_{T=0}^{(\mathrm{HF})} \approx \frac{1}{2}\varepsilon T^2,$$

The different Skyrme forces have different effective mass. Once the level spacing ε **is corrected for the effective mass,**
$\varepsilon' = m^*/m \; \varepsilon$
the coefficients become very similar. Shaded area corresponds to average spacing

No linear term!

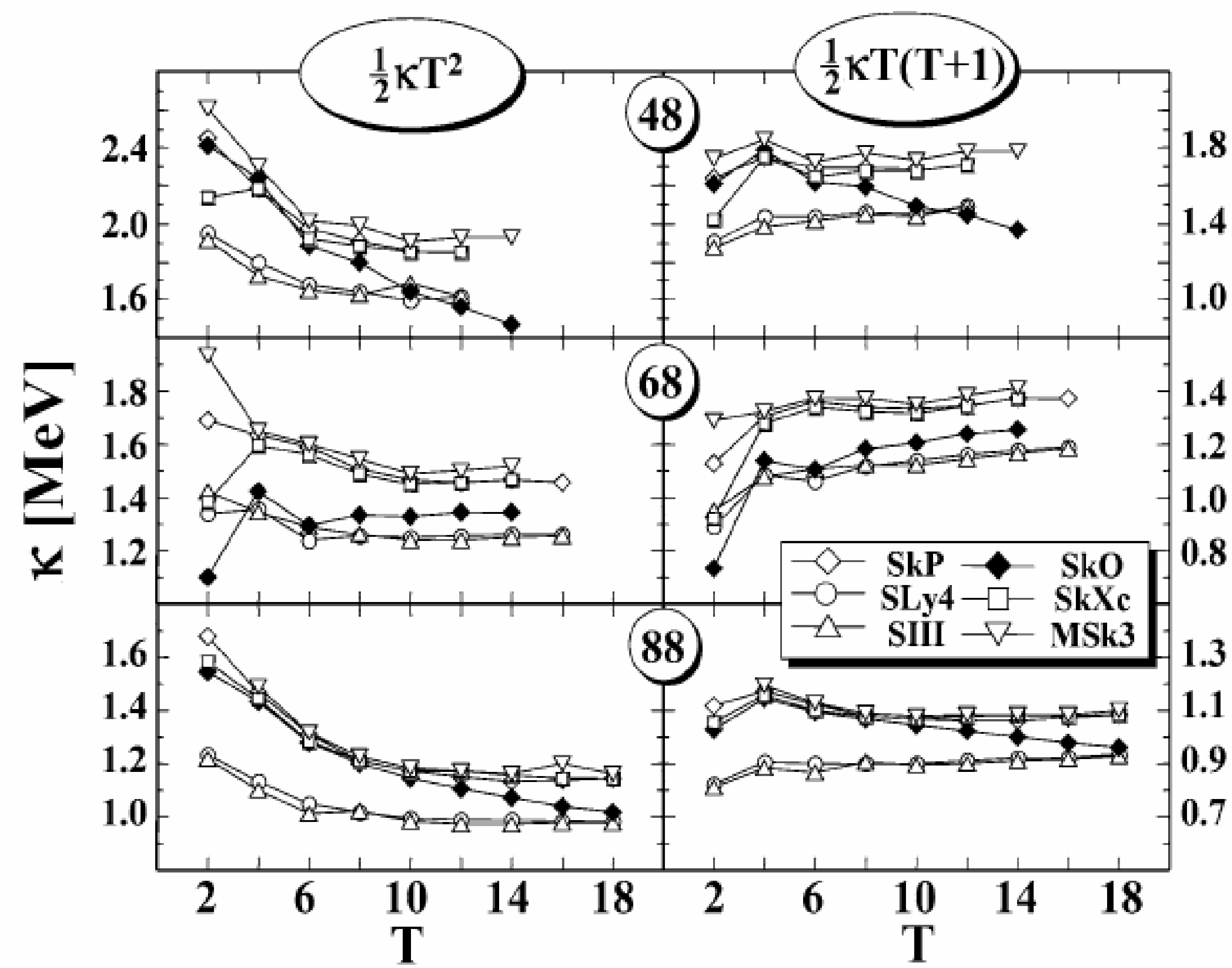


Spread in kinetic energy



Skyrme iso vector potential

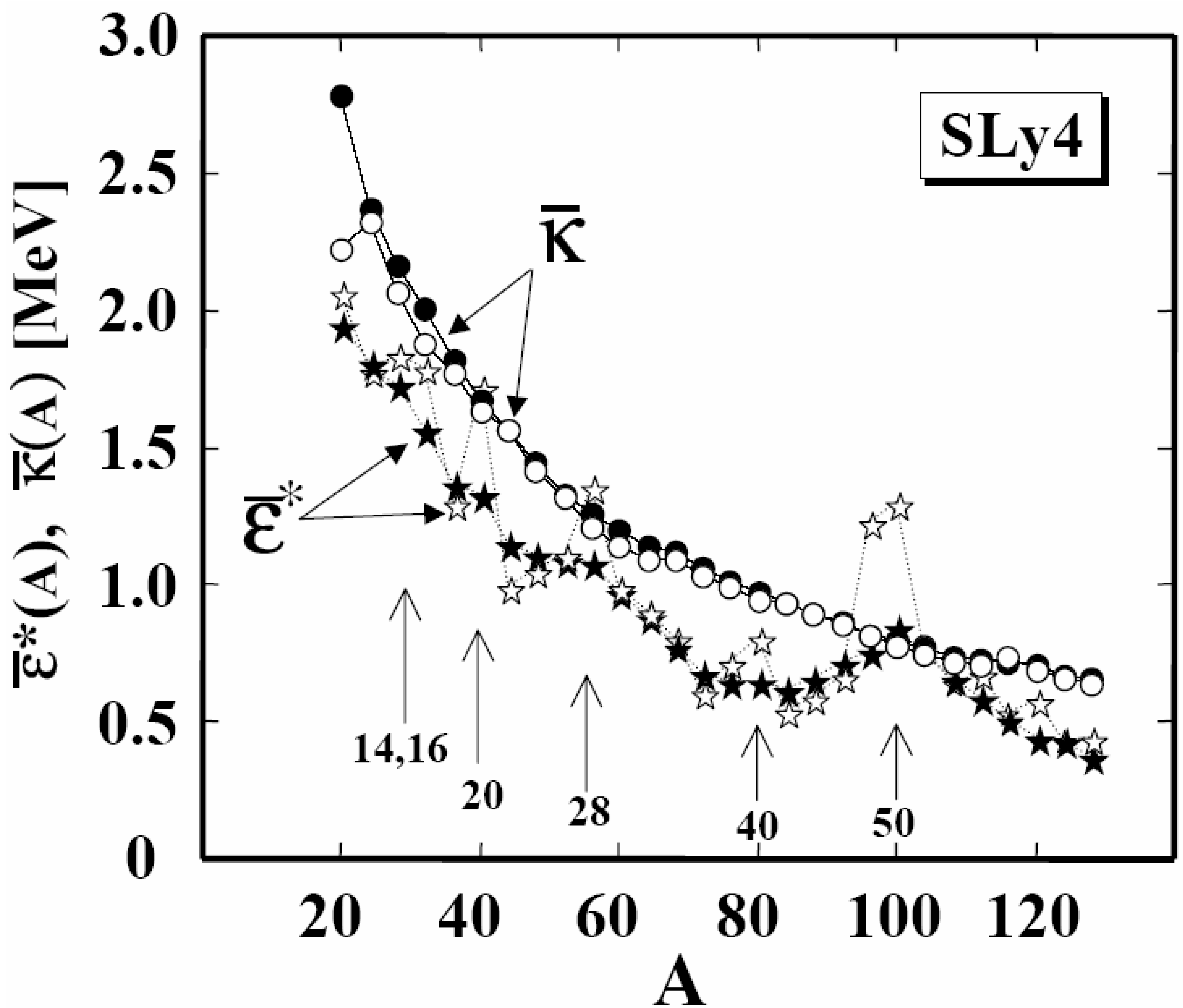
- The Skyrme functional has an iso vcector potential that is proportional to T(T+1) and can be characterised by a single coefficient, k
- The smaller the effective mass, the smaller the iso vector potential!



Global fit to ε and κ

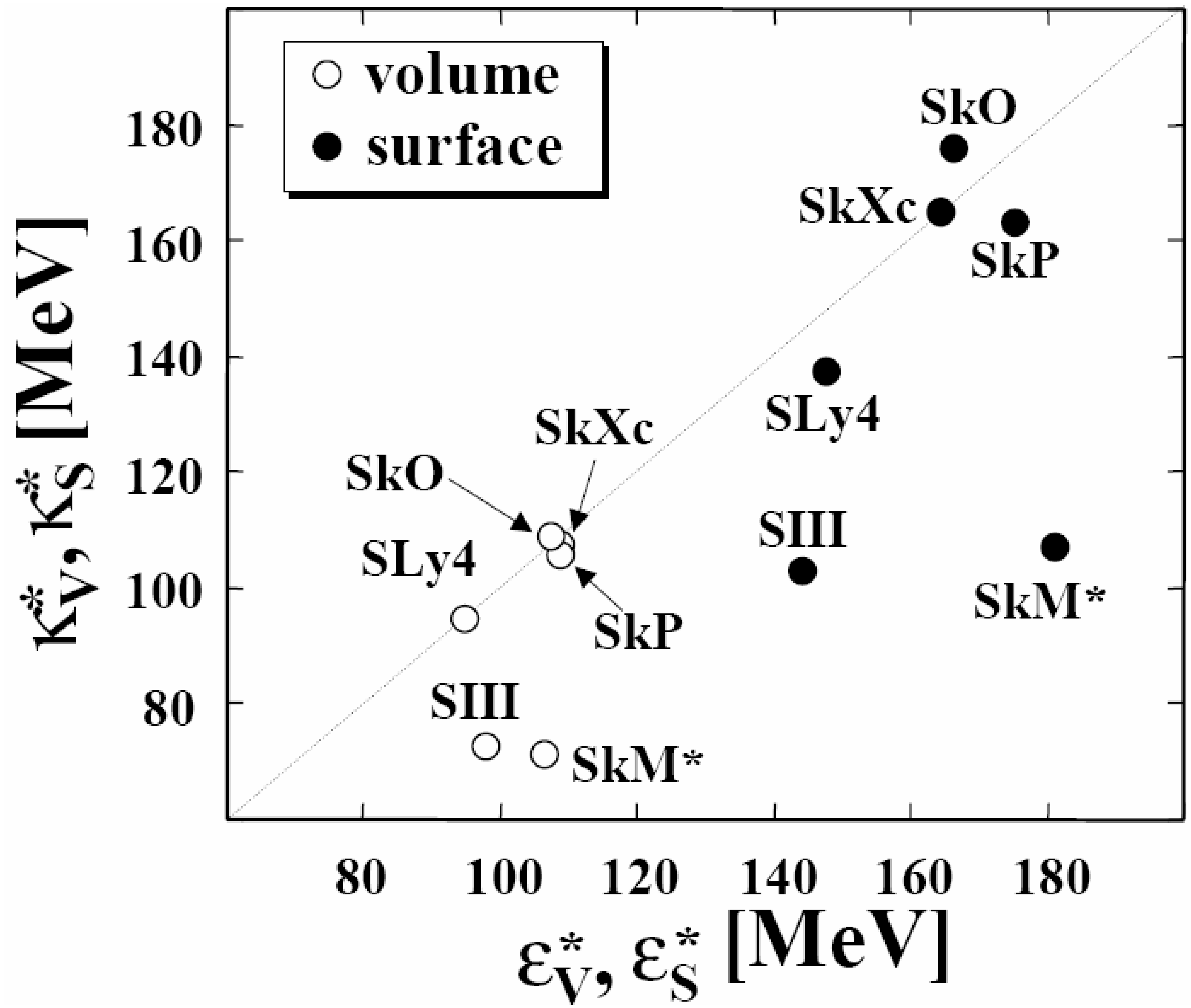
$$\varepsilon(A) = \frac{\varepsilon_V}{A} - \frac{\varepsilon_S}{A^{4/3}};$$

$$\kappa(A) = \frac{\kappa_V}{A} - \frac{\kappa_S}{A^{4/3}},$$



$$\frac{m_0^\star}{m}\varepsilon(A) \approx \frac{m_1^\star}{m}\kappa(A)$$

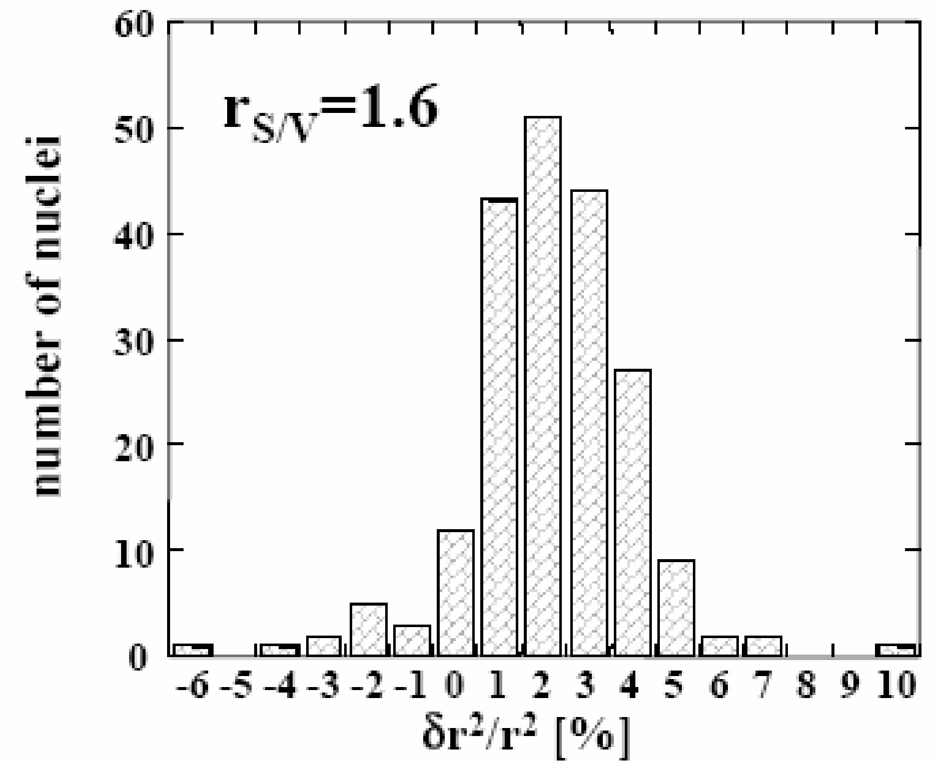
why is e*=k* ?



Surface to volume ration

Neutron skin thicknes determined by $r_{S/V}=a_S/a_V$

$$\frac{\delta r^2}{\langle r^2\rangle} \approx \frac{N-Z}{A}\left\{1+\frac{2}{3}\frac{r_{S/V}}{A^{1/3}}-\dots\right\}$$



Symmetry energy in SHF

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- Symmetry energy obtained as

$$E_{sym}^{(SHF)} = \frac{1}{2}\varepsilon(A,T_z)T^2 + \frac{1}{2}\kappa(A,T_z)T(T+1)\,,$$

- Value of a_V ($a_{sym}=a_V/A + a_S/A^{4/3}$) close to value from infinite nuclear matter

$$\begin{aligned} a_{sym}^{(\infty)} &= \frac{1}{8}\varepsilon_{FG}\left(\frac{m}{m_0^{\star}}\right) + \left[\left(\frac{3\pi^2}{2}\right)^{2/3} C_1^{\tau}\rho^{5/3} + C_1^{\rho}\rho\right] \\ &\equiv \frac{1}{8}\left[\varepsilon_{(\infty)} + \kappa_{(\infty)}\right], \end{aligned}$$

- Fundamental property that $e^* = k^*$?

Test the same concept in RMF

$$V_{tot} = V(\mathbf{r}) + \beta S(\mathbf{r}) = g_\omega \omega^0(\mathbf{r}) + g_\rho \vec{\tau} \cdot \vec{\rho}^{\,0}(\mathbf{r}) + \beta g_\sigma \sigma(\mathbf{r}).$$

$$\begin{aligned} V_{is}(\mathbf{r}) &= g_\omega \omega^0(\mathbf{r}) + \beta g_\sigma \sigma(\mathbf{r}), \\ V_{iv}(\mathbf{r}) &= g_\rho \vec{\tau} \cdot \vec{\rho}^{\,0}(\mathbf{r}). \end{aligned}$$

$$\tilde{E}_T(A, T_z) - \tilde{E}_{T=0}(A, T_z = 0) = \frac{1}{2}\varepsilon(A, T_z) T^2.$$

- Strong interaction – disregard Coulomb
- What is the size of the linear term?
- What are the values of $a_{sym} = a_V/A + a_S/A^{4/3}$

Formalism to determine e and k in RMF

without isovector meson $g_s s \,,\; g_w w$

$\tilde{E}_T^{RMF}$ → $$\tilde{E}_T^{RMF} - \tilde{E}_{T=0}^{RMF} = \frac{1}{2} e T^2$$ → e

with isovector meson $g_s s \,,\; g_w w ,\; g_r r$

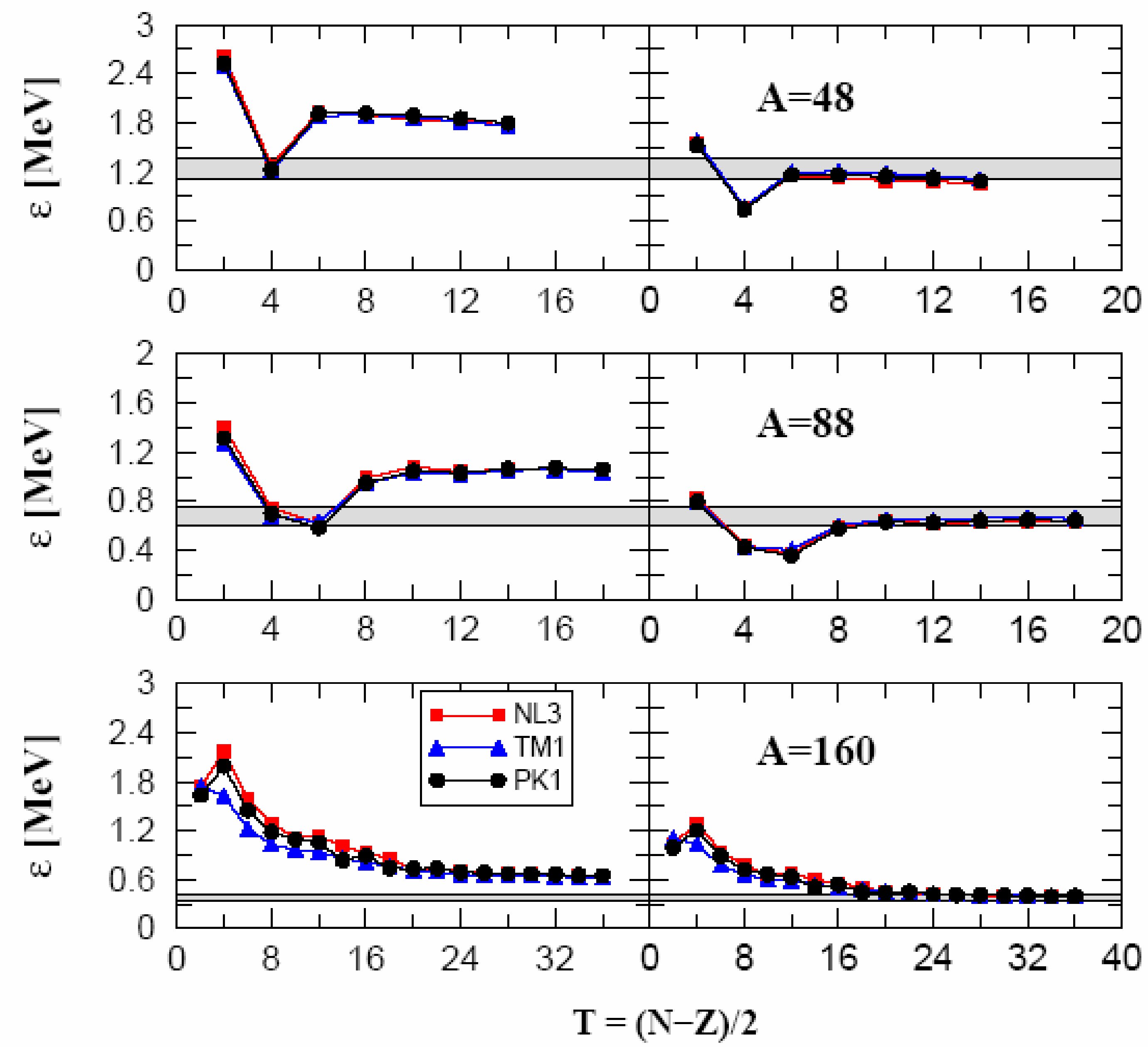
E_T^{RMF} → $$E_T^{RMF} - \tilde{E}_T^{RMF} = \frac{1}{2} k T^2 \quad or \quad \frac{1}{2} k T(T+1)$$ → k *i.e.,* k_{RMF}

Effect of pairing

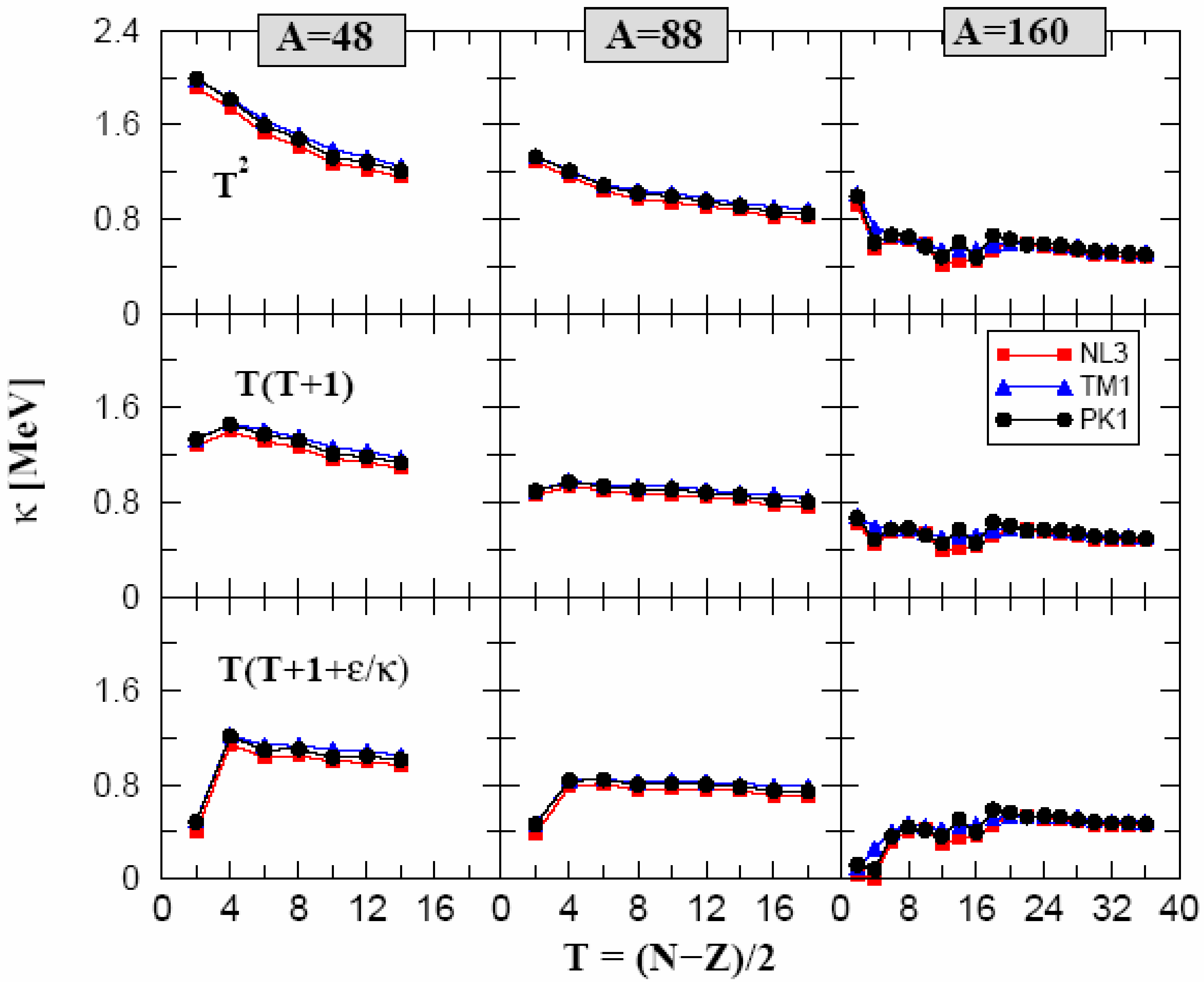
$E_T^{RMF+BCS}$, $\tilde{E}_T^{RMF+BCS}$ → $$E_T^{RMF+BCS} - \tilde{E}_T^{RMF+BCS} = \frac{1}{2} k T^2 \quad or \quad \frac{1}{2} k T(T+1)$$ → $k_{RMF+BCS}$

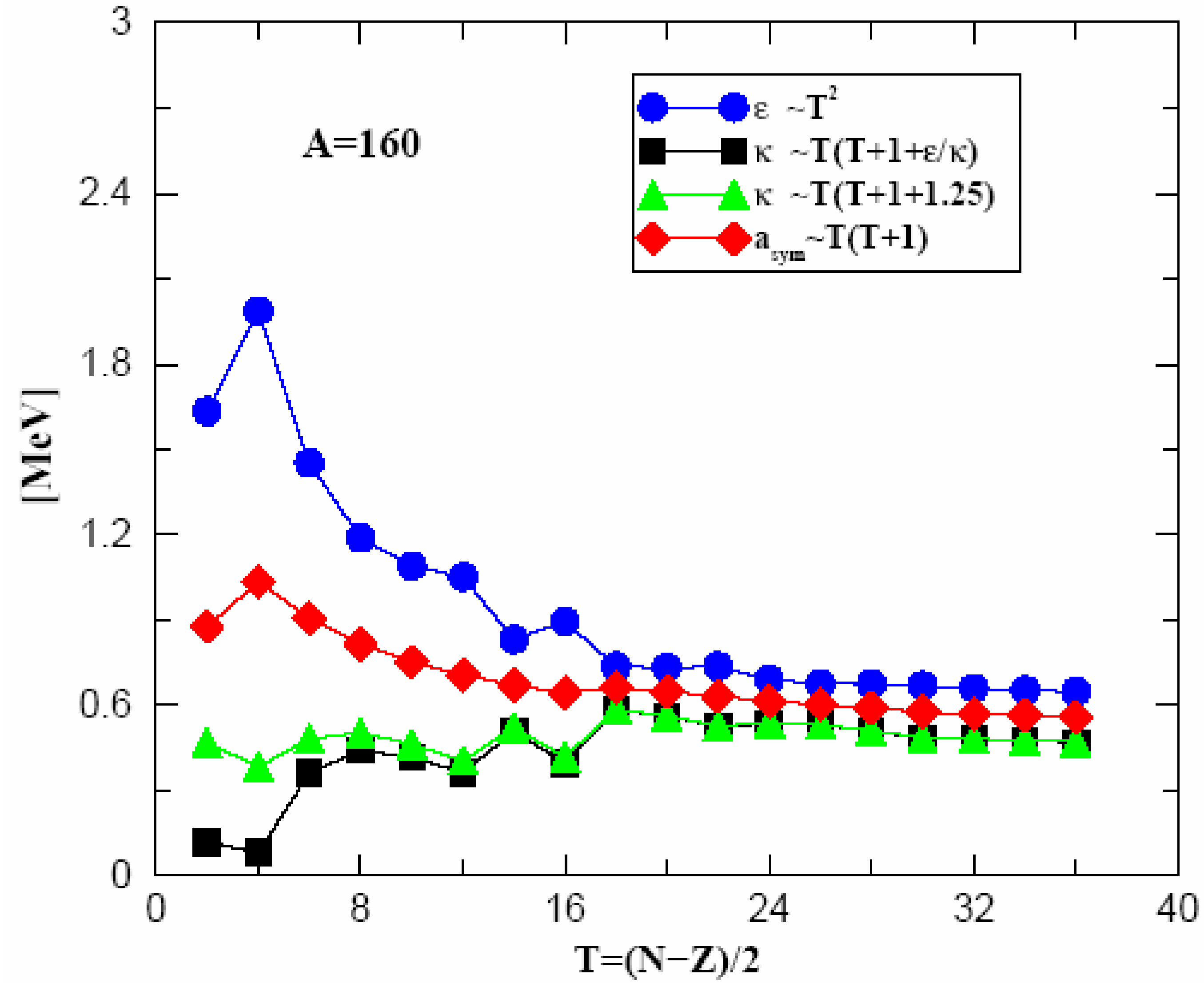
Symmetry energy in RMF from level spacing

- e is constant for large Tz
- After effective mass scaling m*/m within imperical limits



Determine k from the full Lagrangian, including the r-meson



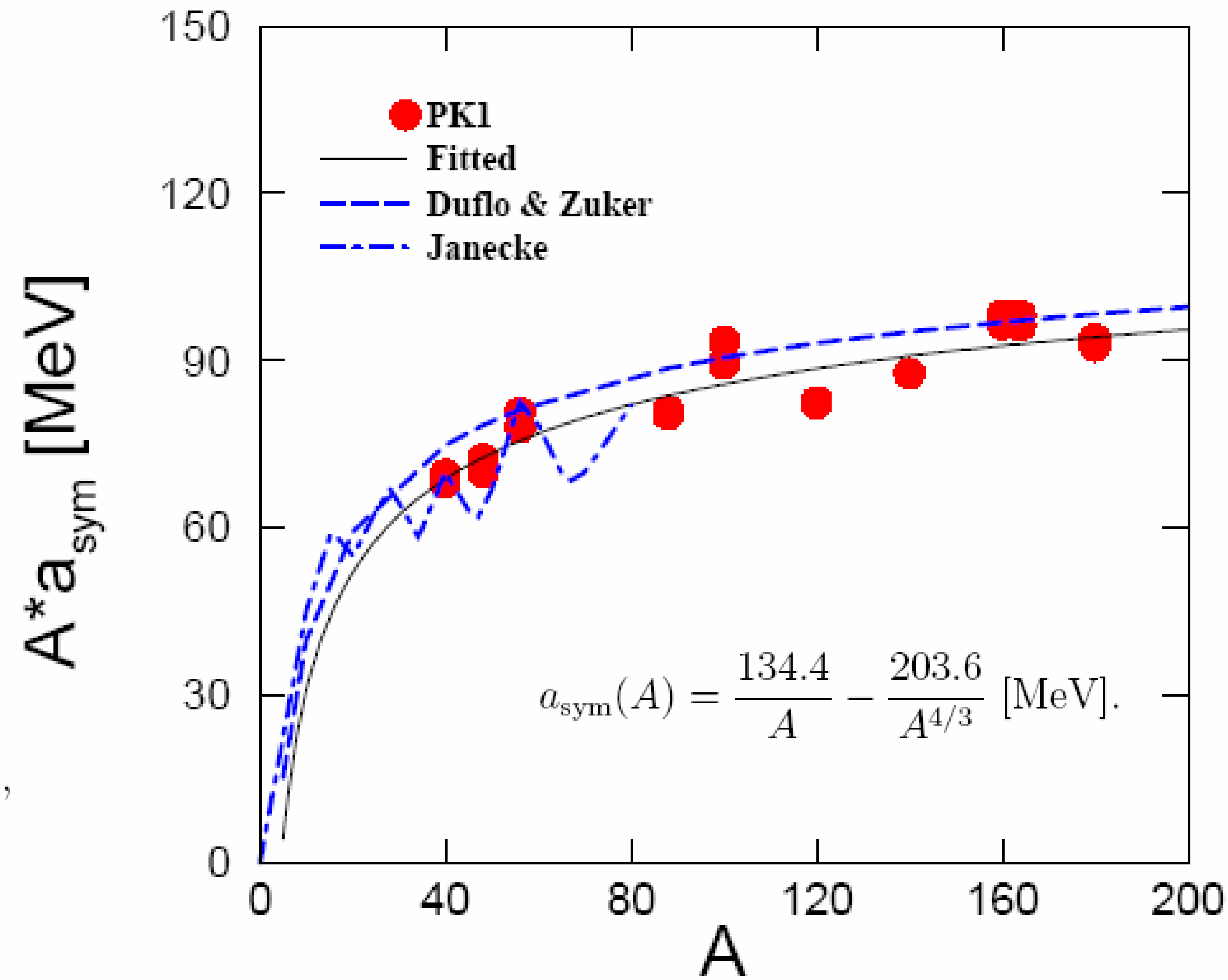


$$E_{\text{sym}} \approx \frac{1}{2}\varepsilon T^2 + \frac{1}{2}\kappa T(T+1+\varepsilon/\kappa) \approx \frac{1}{2}(\varepsilon+\kappa)T(T+1).$$

The nuclear symmetry energy in RMF, E_{sym}=a T(T+1)

The nuclear symmetry energy in RMF follows rather closely the values by Duflo Zuker

$$a_{\rm sym}^{\rm (RMF)} = \frac{133.20}{A} - \frac{220.27}{A^{4/3}}\ [\mathrm{MeV}],$$



Symmetry energy in RMF

- Concept of the symmetry energy composed by two terms, the average level spacing at the Fermi surface and an average potential with strength k
- The volume term of the symmetry energy a_V ($a_{sym}=a_V/A + a_S/A^{4/3}$) determined in finite nuclei is much smaller than the on in infinite nuclear matter a_V ($a_{sym}=a_V/A + a_S/A^{4/3}$)
- Surprisingly, the RMF theory which is a Hartree approximation generates a symmetry energy E_{sym} that is fitted nicely by a T(T+1) dependence.