

# **Evolution Of Shell Structure, Shapes & Collective Modes**

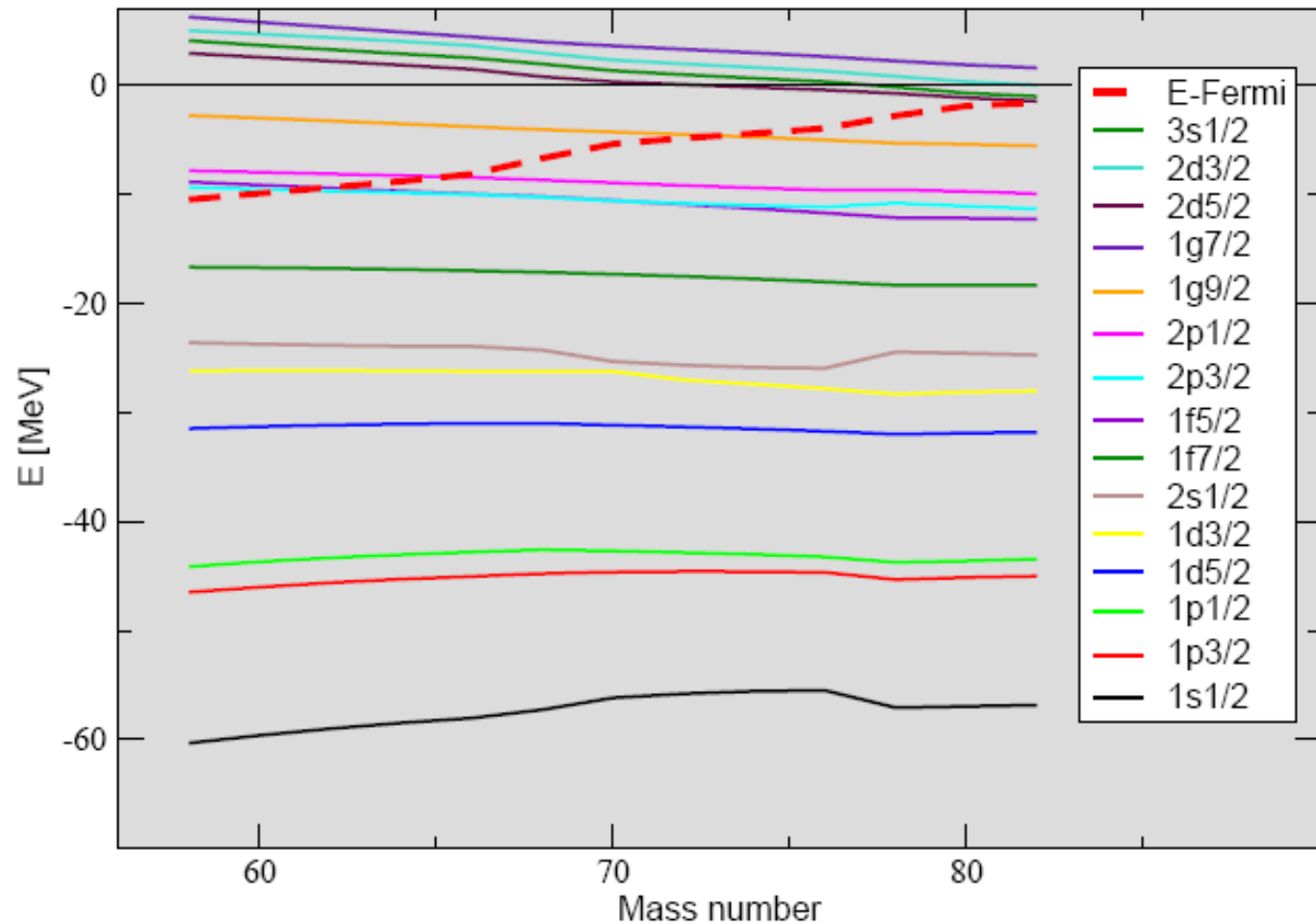
**Dario Vretenar**  
**vretenar@phy.hr**

# **1. Evolution of shell structure with N and Z**

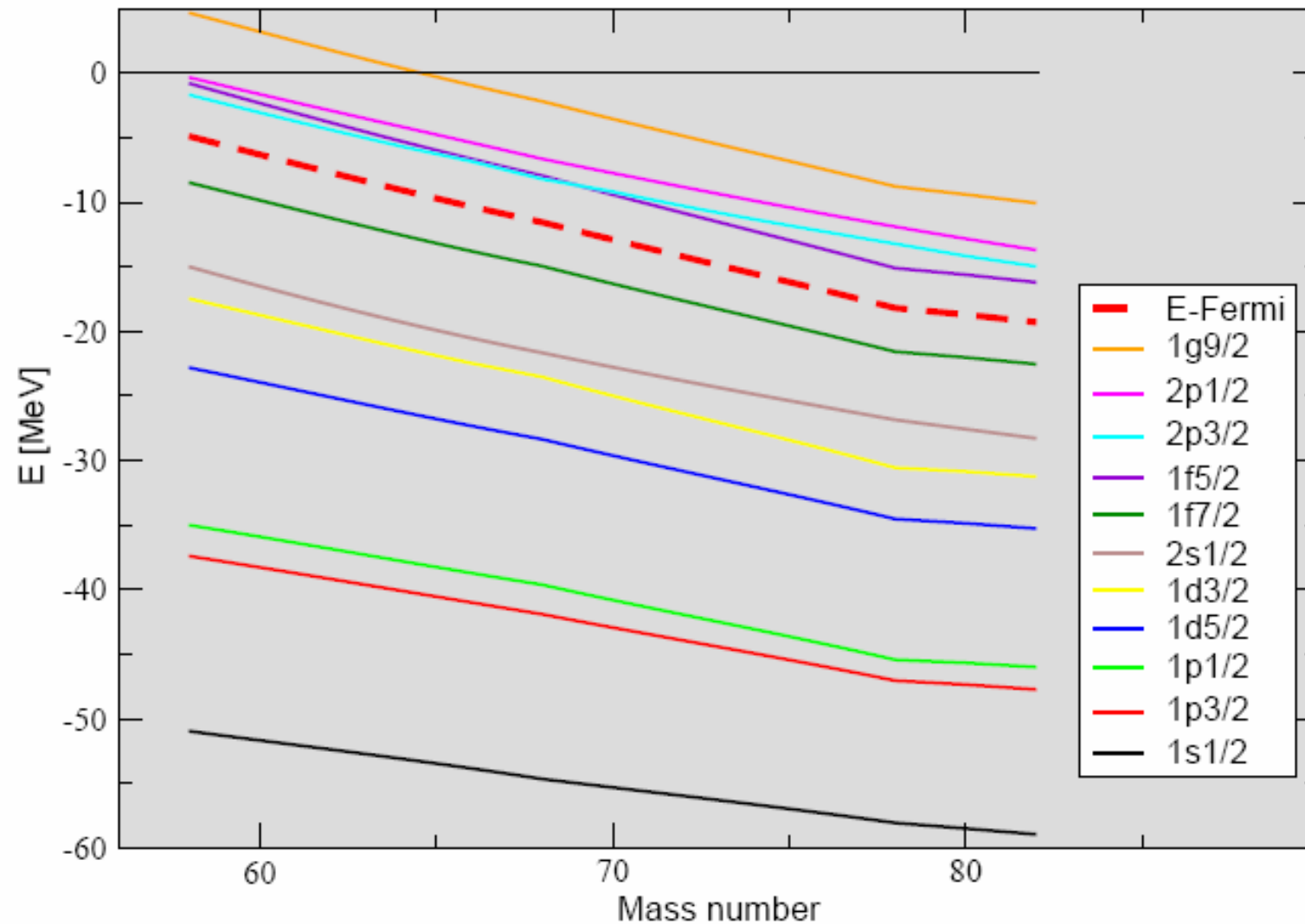
## A. Modification of the effective single-nucleon potential

Relativistic Hartree-Bogoliubov calculation:  
DD-ME1 effective interaction + Gogny D1S pairing

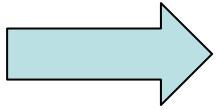
Neutron canonical  
single-particle levels  
in Ni isotopes.



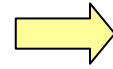
## Proton canonical single-particle levels in Ni isotopes



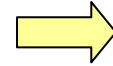
Most of the single-nucleon levels evolve in parallel, keeping the level spacing essentially unchanged!



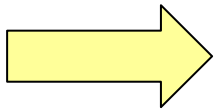
very different positions of the neutron and proton **Fermi levels:**



**fast beta-transitions**



**beta-delayed particle emission**



**Fermi level close to the continuum:**

neutron-rich nuclei:

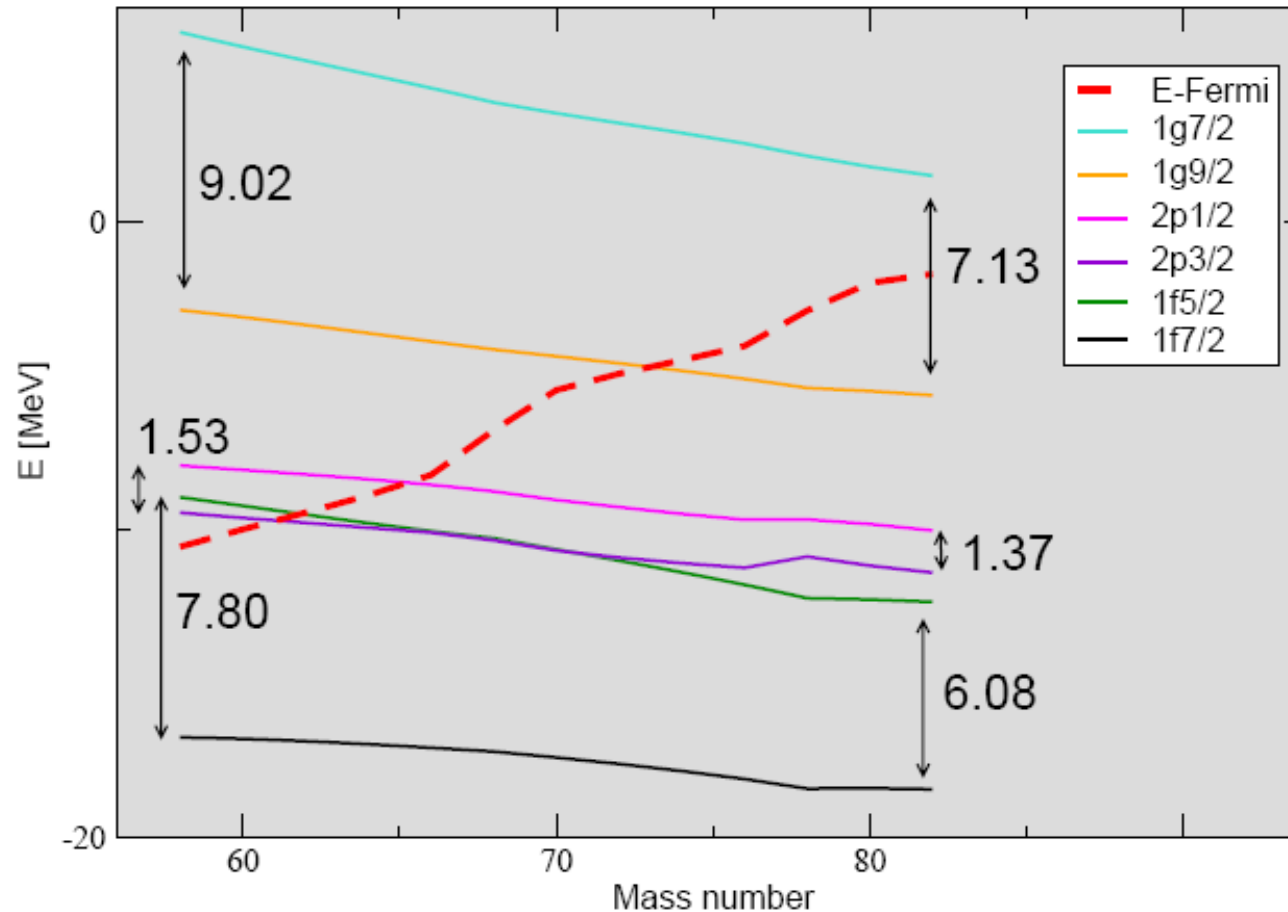
- coupling to resonances and scattering states
- formation of halo structures in light nuclei
- evolution of neutron skin in heavier systems

proton-rich nuclei:

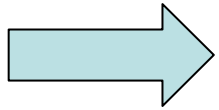
- occupied positive-energy levels => decay by direct one- and two-proton emission

## B. Isovector dependence of the effective spin-orbit interaction

Neutron s.p. levels in Ni isotopes

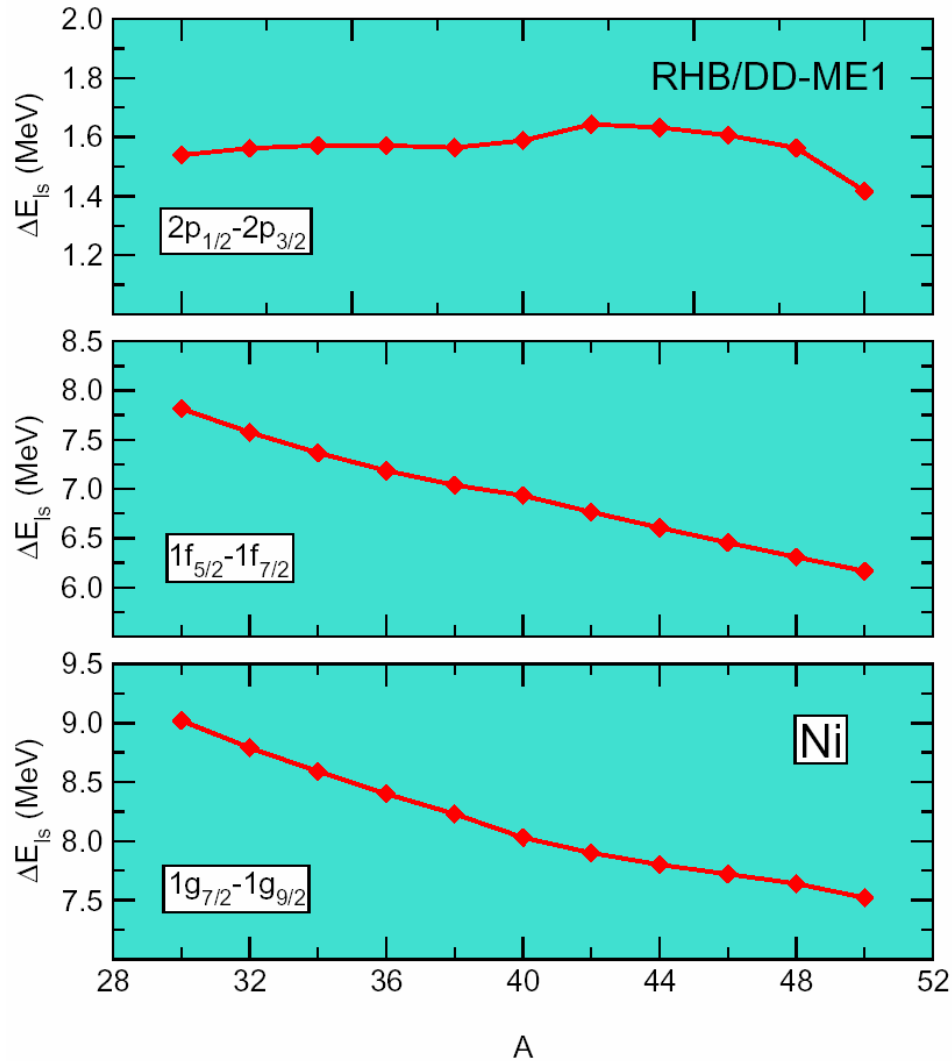


Relativistic Hartree-Bogoliubov calculation:  
DD-ME1 effective interaction + Gogny D1S pairing

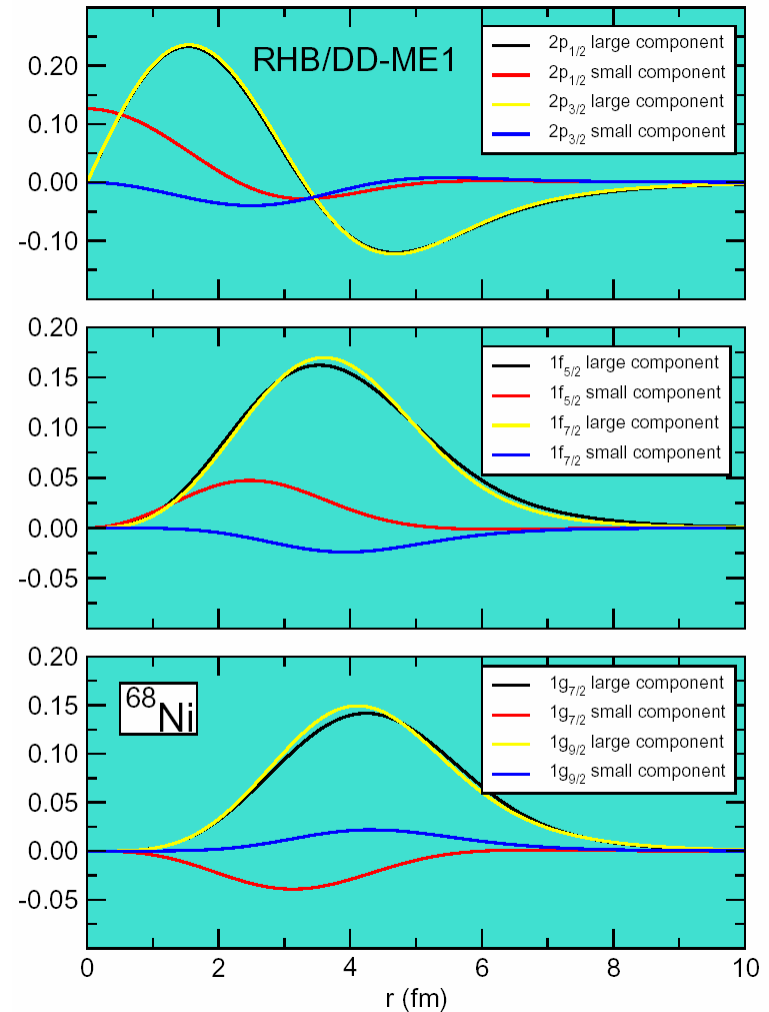


Reduction of the energy spacing between spin-orbit partner states.

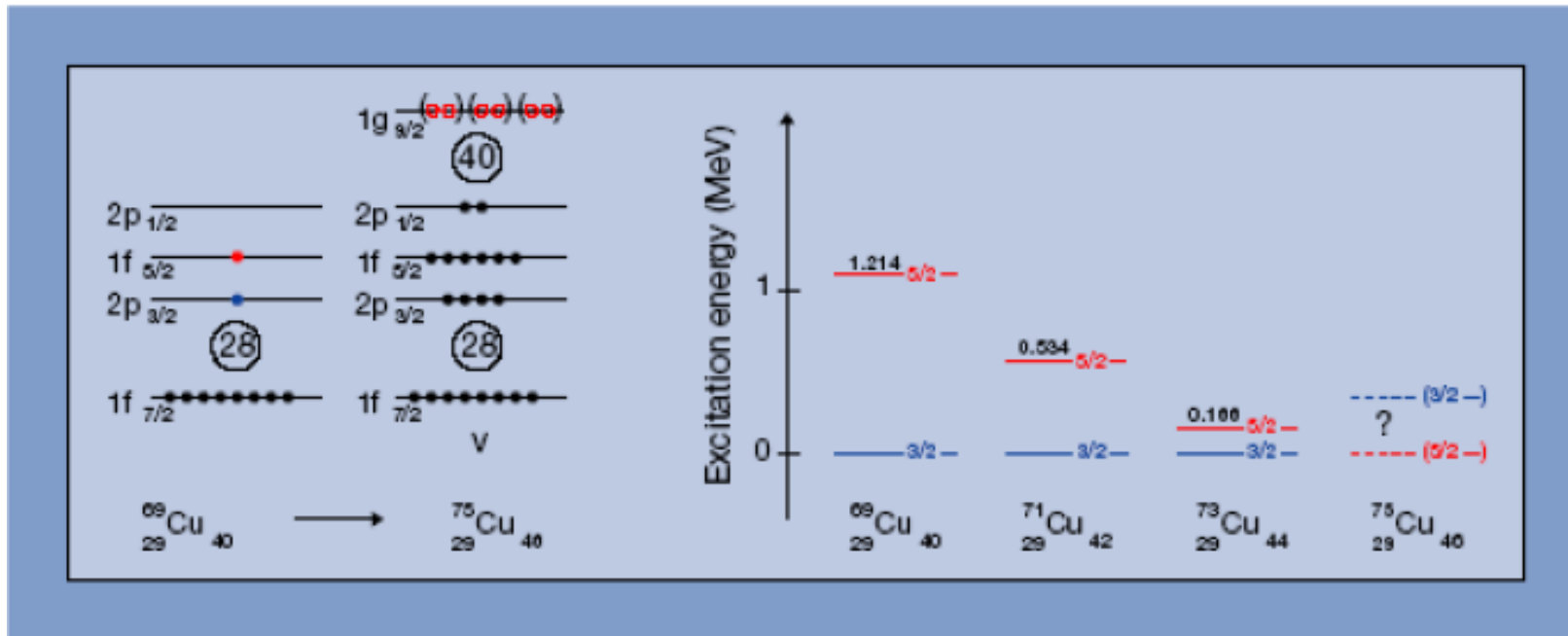
$$\Delta E_{ls} = E_{n,l,j=l-1/2} - E_{n,l,j=l+1/2}$$



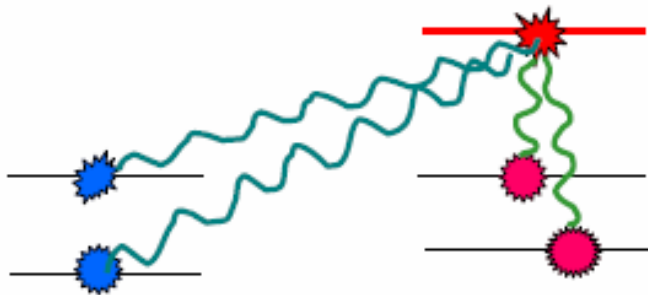
Radial Dirac wave-functions of spin-orbit doublets.



## C. Monopole migration

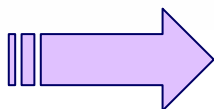


Odd-A copper isotopes: migration of the  $1f_{5/2}$  proton orbital as the  $1g_{9/2}$  neutron orbital is being filled.



Monopole part of the NN interaction between valence protons and neutrons:

$$V_{ab}^T = \frac{\sum_J (2J + 1) V_{abab}^{JT}}{\sum_J (2J + 1)}$$



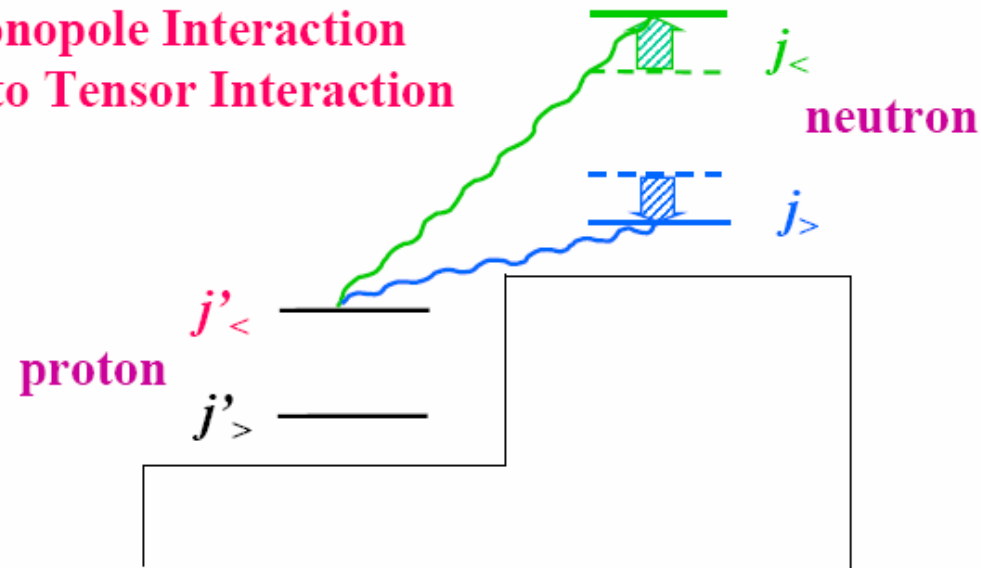
**Effective Single-Particle Energies (ESPE)** – depend on the monopole interaction between valence nucleons



# What is the microscopic mechanism that determines the monopole migration of effective single-particle energies?

1)

Monopole Interaction  
due to Tensor Interaction

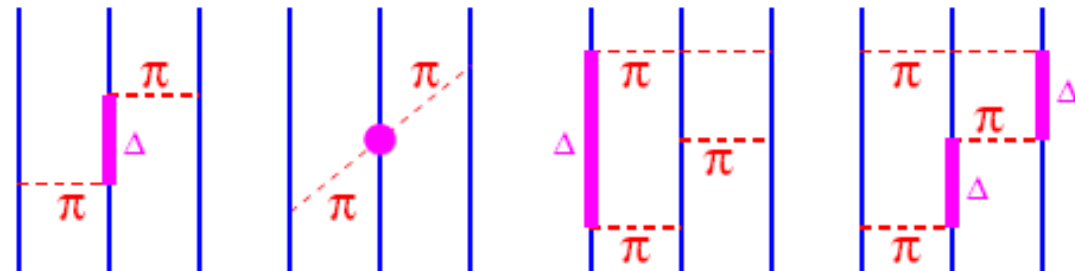


?

2)

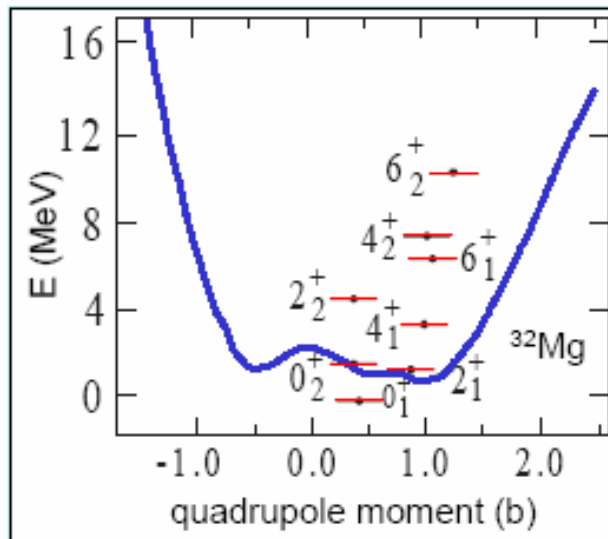
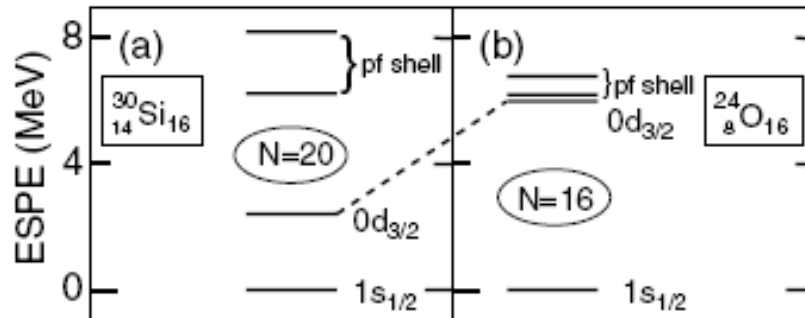
Monopole contribution  
from three-body forces?

?



## Evolution of shell structure in nuclei with $N, Z < 40$

Low-density of single-particle states!



- reduced spherical shell gaps
- reordering of nucleonic shells (occurrence of islands of inversion!)
- spherical magic gaps ( $N=8, 20, 28, \dots$ ) may disappear and new strong gaps can be found at  $N=6, 16, 32, \dots$
- onset of deformation and shape coexistence (could extend the neutron drip line!)

# Global Shell Model Description

## Advantages:

- the ability to describe simultaneously all spectroscopic properties of low-lying states with very different structure within a nucleus
- effective interactions connected with both two- and three-nucleon bare forces
- a description of collective properties in the laboratory frame

## Recent successful applications:

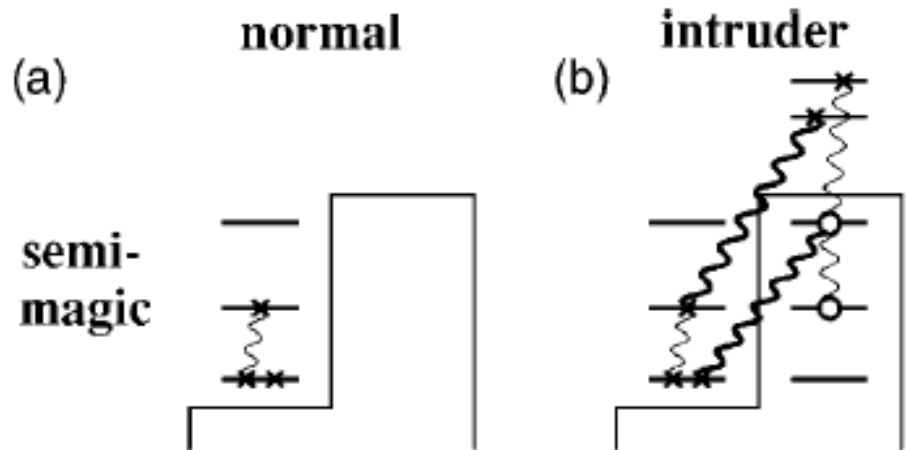
**Standard large-scale shell model studies (Strasbourg-Madrid)**

**Shell-Model Monte Carlo**

**Quantum Monte Carlo diagonalization – MCSM (Tokyo)**

**No-core Shell Model**

## Island of inversion around $Z \approx 11$ and $N = 20$



The ground states of these nuclei gain binding energy by promoting neutrons from the **sd-shell** to the **fp-shell** => gain in **deformation energy**.

Intruder configurations have lower energy than normal configurations.



**Weakening of the shell closure and eventually the disappearance of the  $N = 20$  magic number!**

**Issues:**

**Boundary of the island of inversion?**

**Degree of mixing between  $0p0h$  and  $2p2h$  configurations?**

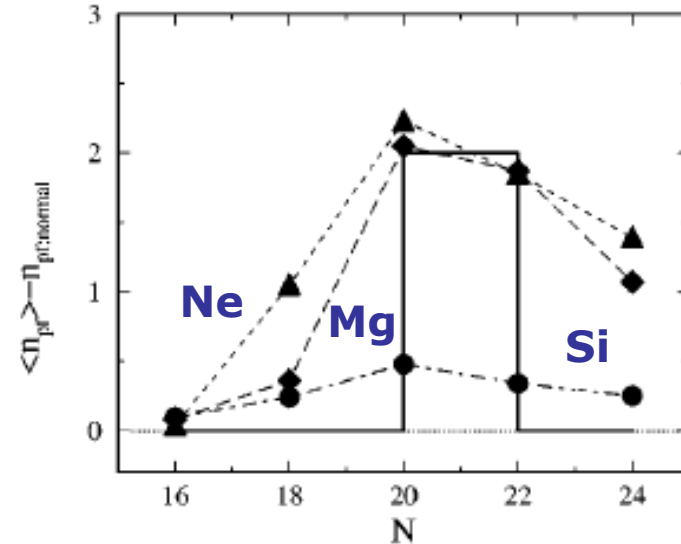
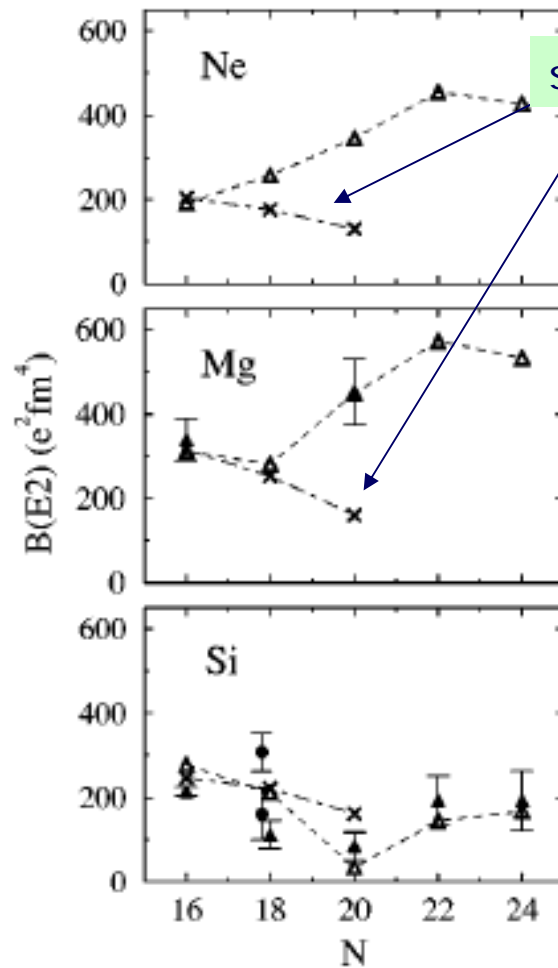
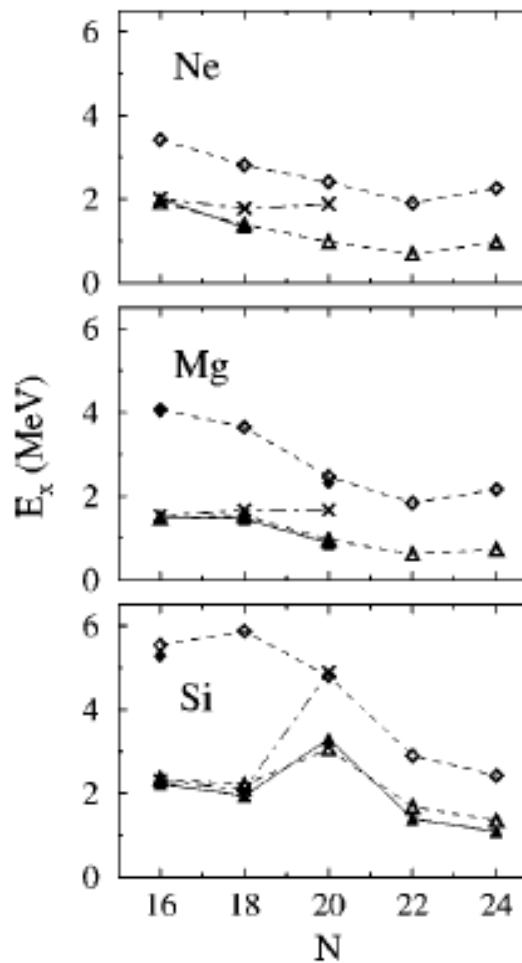
# Unrestricted SM calculations in the sd + f7/2 and p3/2 shells

## Quantum Monte-Carlo diagonalization based SM:

UTSUNO, OTSUKA, MIZUSAKI, AND HONMA  
 PHYSICAL REVIEW C **60** 054315

Yrast levels  $0^+, 2^+, 4^+$

$$B(E2; 0_1^+ \rightarrow 2_1^+)$$



Average number of neutrons in the pf-shell minus those in the normal configuration.

USD int. for the sd-shell

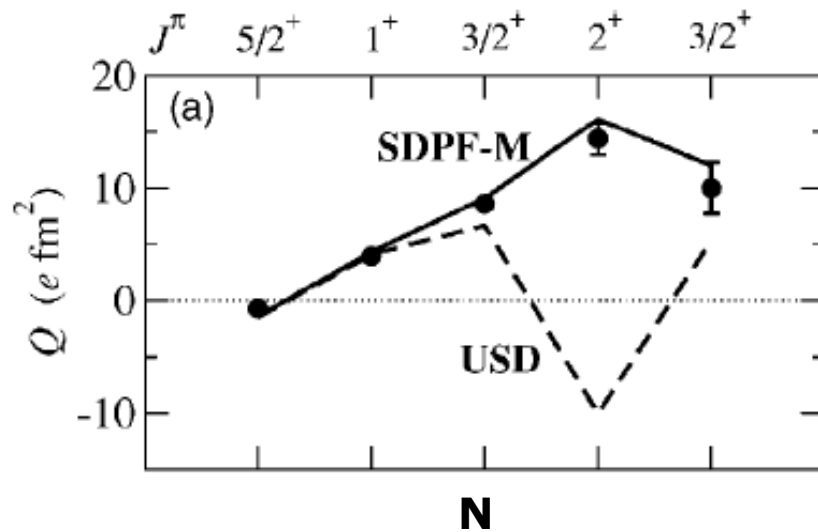
Kuo-Brown int. for the pf-shell

SDPF-M effective interaction

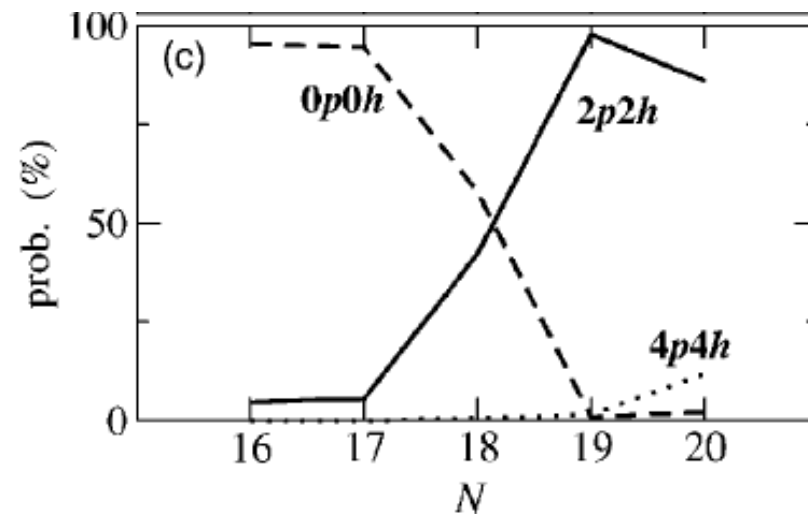
Modified Millener-Kurath int. for the cross-shell

monopole shifts

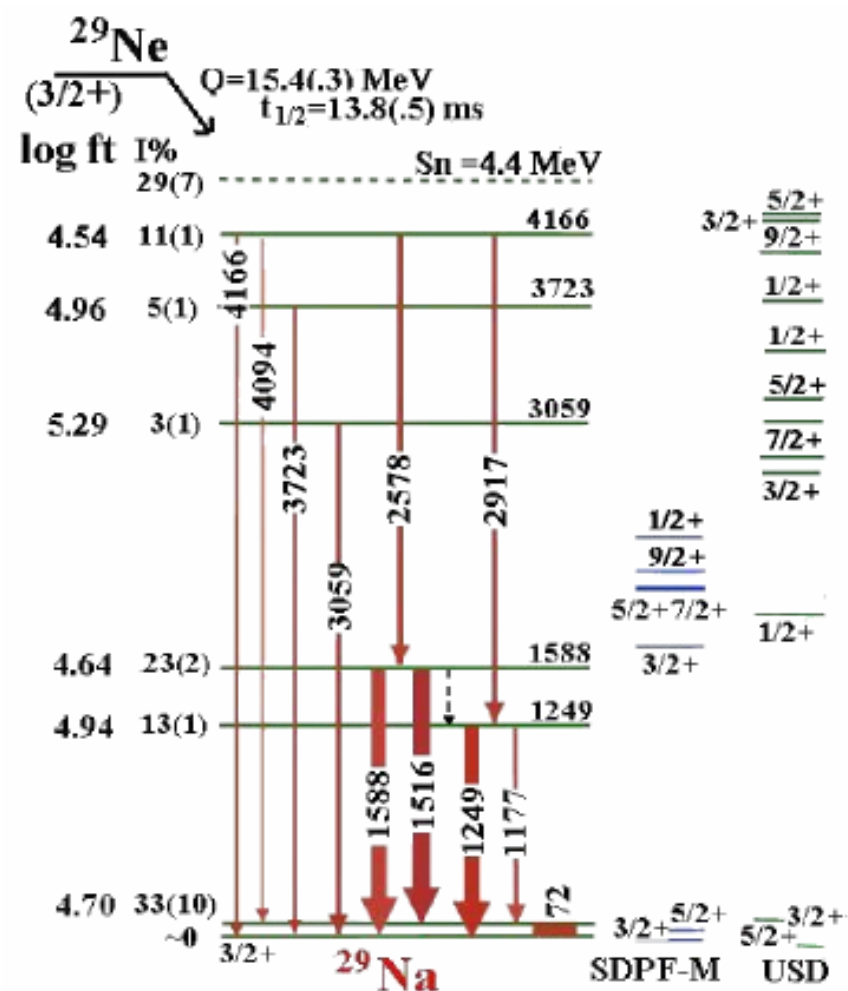
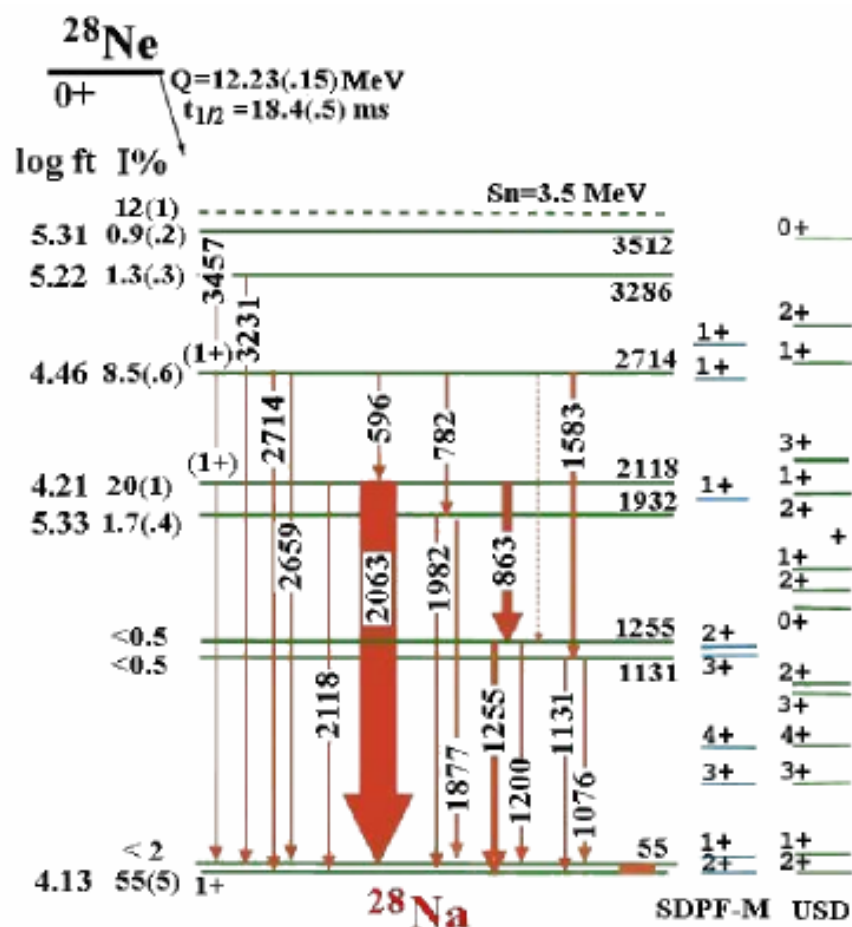
### Onset of intruder ground states in exotic Na isotopes



The transition from normal to the intruder ground state: between  $N=18$  and  $N=19$ .

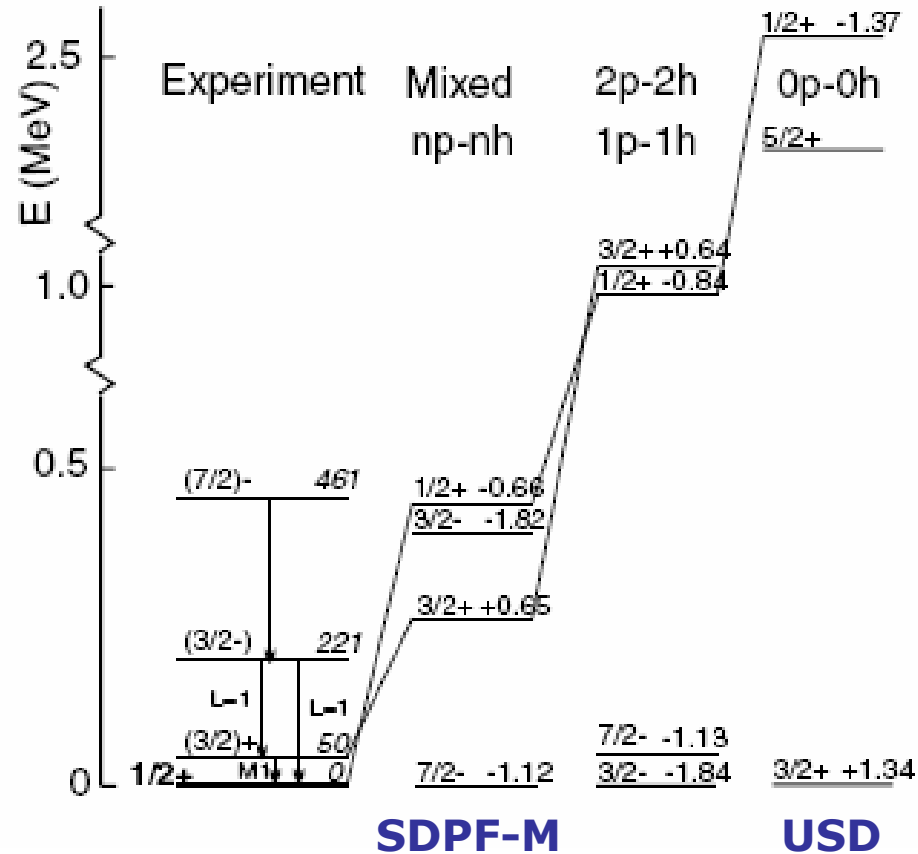


## $^{29}\text{Na}$ : Defining the Edge of the Island of Inversion for $Z=11$



Measurement of the Spin and Magnetic Moment of  $^{31}\text{Mg}$ :  
 Evidence for a Strongly Deformed Intruder Ground State

PRL 94, 022501 (2005)



The calculations do not reproduce the correct ordering of intruder levels!



## Large-scale shell-model calculation of the N=28 spin-orbit shell closure

configuration space ( $Z < 20$ ): **sd-shell for protons, pf-shell for neutrons**

**USD for the proton-proton int.**

**Kahana-Lee-Scott G-matrix for the cross-shell proton-neutron int.**

**SPDF-NR interaction**

**monopole adjustments**

**Kuo-Brown (modified) for the neutron-neutron int.**

N=28 isotones: quasiparticle neutron gaps, difference in correlation energy between the 2p-2h and the 0p-0h configurations and their relative positions.

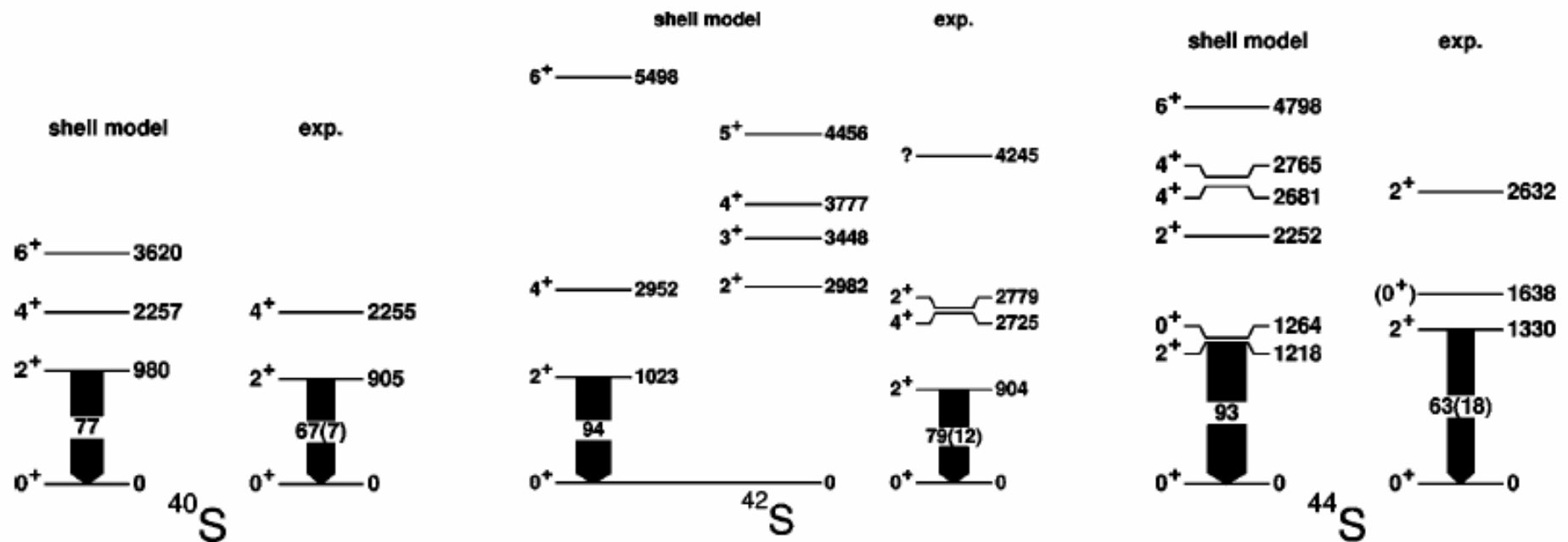
	<sup>40</sup> Mg	<sup>42</sup> Si	<sup>44</sup> S	<sup>46</sup> Ar	<sup>48</sup> Ca
gap	3.35	3.50	3.23	3.84	4.73
$\Delta E_{Corr}$	8.45	6.0	6.66	5.98	4.08
$E_{2p-2h}^*$	-1.75	1.0	-0.2	1.7	5.38

TABLE XII.  $N=28$  isotones: spectra, quadrupole properties, and occupancies.

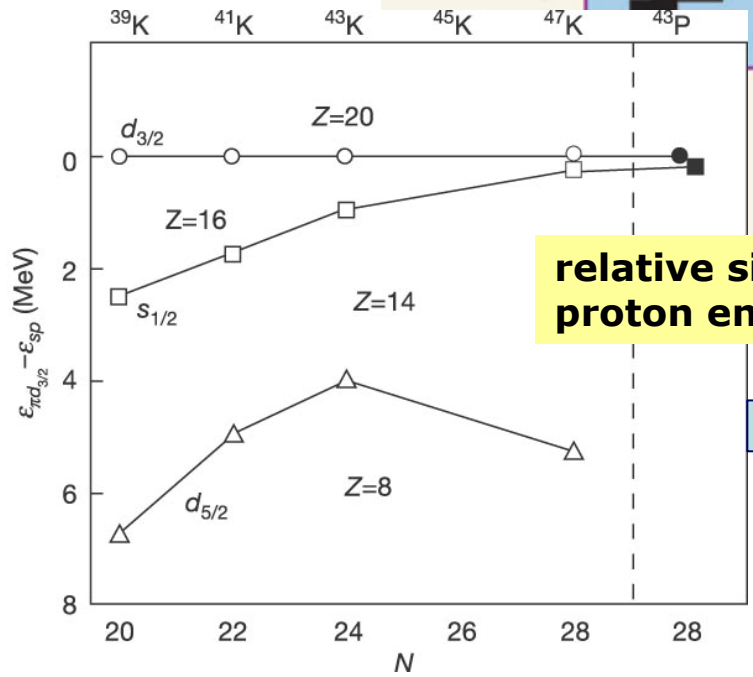
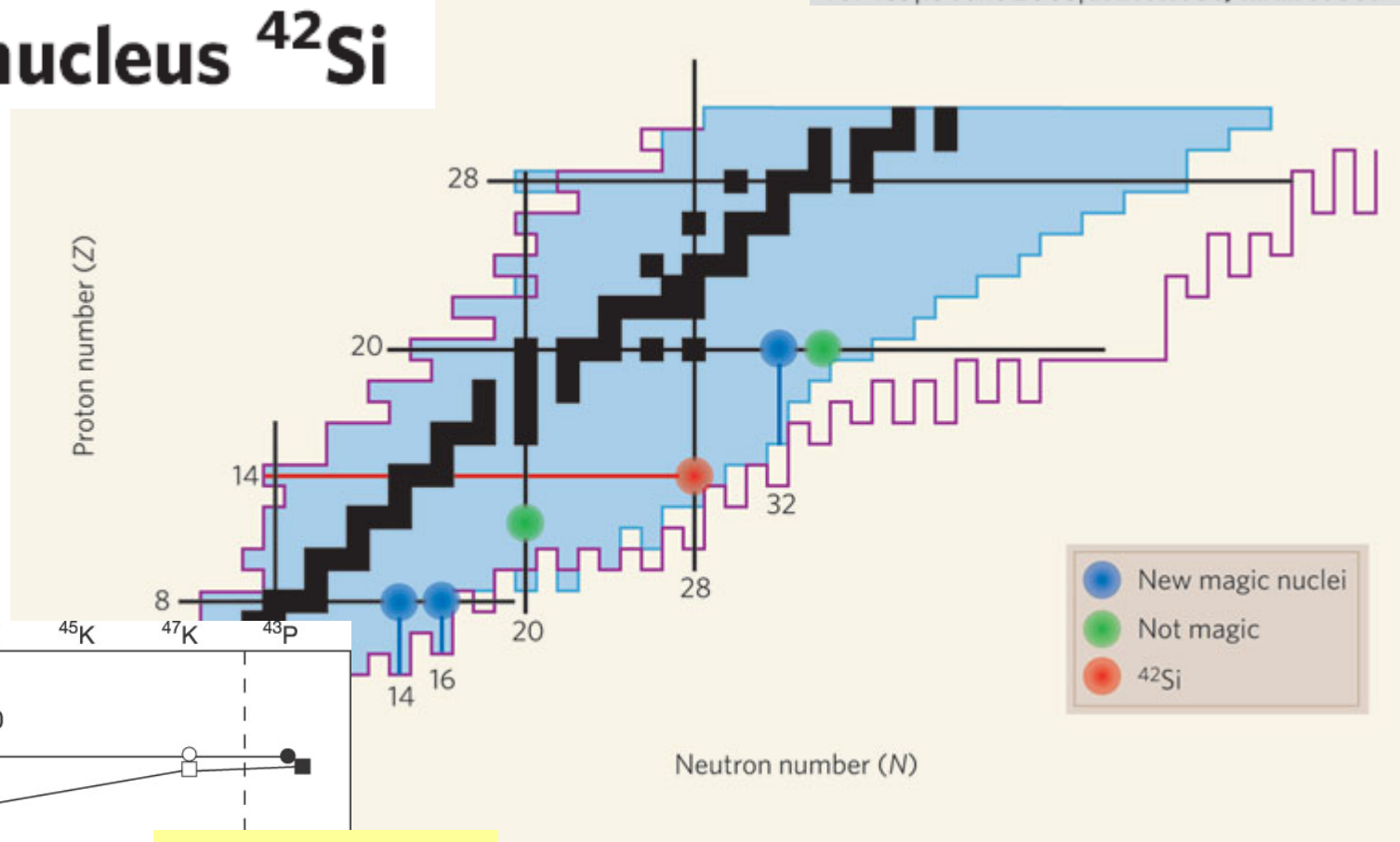
	$^{40}\text{Mg}$	$^{42}\text{Si}$	$^{44}\text{S}$	$^{46}\text{Ar}$
$E^*(2^+)$ (MeV)	0.81	1.49	1.22	1.51
$E^*(4^+)$	2.17	2.68	2.25	3.46
$E^*(0_2^+)$	1.83	1.57	1.26	2.93
$Q(2^+)$ ( $e\text{ fm}^2$ )	-21	16	-17	20
$B(E2)$ ( $e^2\text{ fm}^2$ )	108	71	93	93
$\langle n_{7/2} \rangle$	5.54	6.16	6.16	6.91
$(f_{7/2})^{8\%}$	3	28	24	45

Caurier *et al.*: The shell model as a unified view of nuclear structure

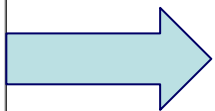
REVIEWS OF MODERN PHYSICS, VOLUME 77, APRIL 2005



# 'Magic' nucleus $^{42}\text{Si}$



relative single proton energies



The magic number  $N=28$  remains valid in this nucleus.  
Proton shell closure at  $Z=14$ .

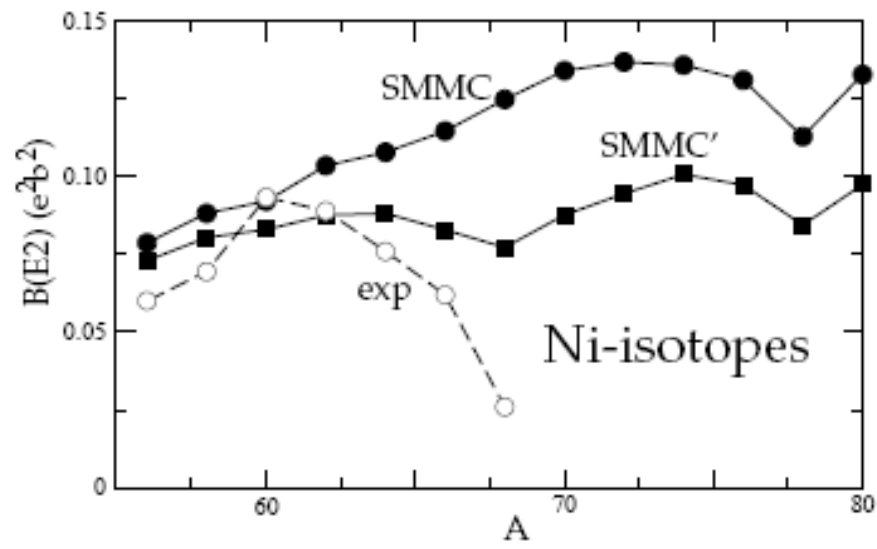
**This Si isotope is doubly magic!**

**N=40 – How magic is the magic  $^{68}\text{Ni}$  nucleus?**

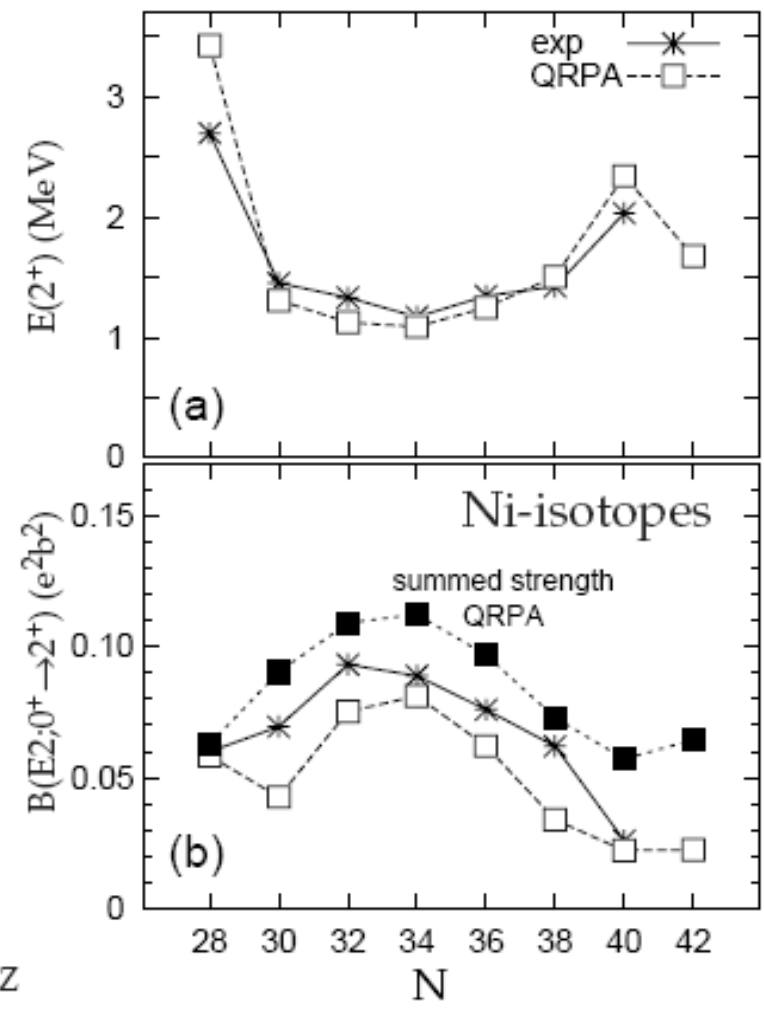
- low-lying  $0^+_2$  level
- higher energy of the  $2^+_1$
- small value of  $B(E2, 0^+_1 \rightarrow 2^+_1)$



**interpreted as evidence for magicity!**



Shell-model Monte-Carlo total summed B(E2) strength to the  $2^+$  excited states.



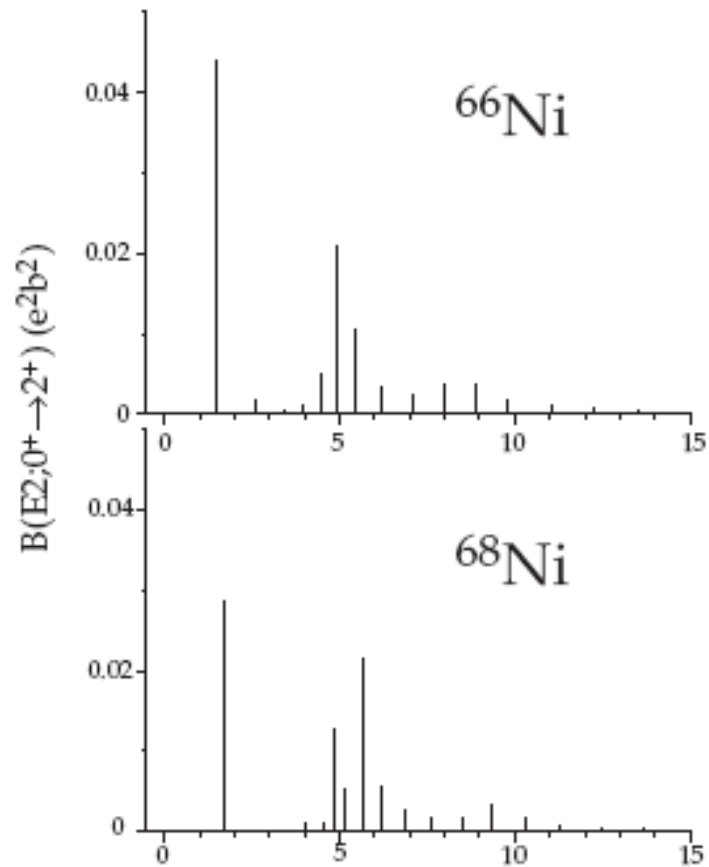


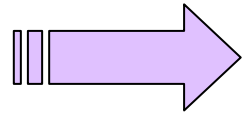
FIG. 4. Distribution of the  $B(E2; 0_{g.s.}^+ \rightarrow 2^+)$  strength in  $^{66,68}\text{Ni}$  calculated in the diagonalization shell model.

Shell-model Monte-Carlo, QRPA and large-scale diagonalization shell-model calculations have shown that the  $B(E2)$  transition to the first  $2^+$  state exhaust only a fraction of the low-lying  $B(E2)$  strength.



Small  $B(E2)$  value to the first  $2^+$  state is not a strong evidence for the doubly magic character of  $^{68}\text{Ni}$ .

# Microscopic description of weakly bound neutron-rich nuclei



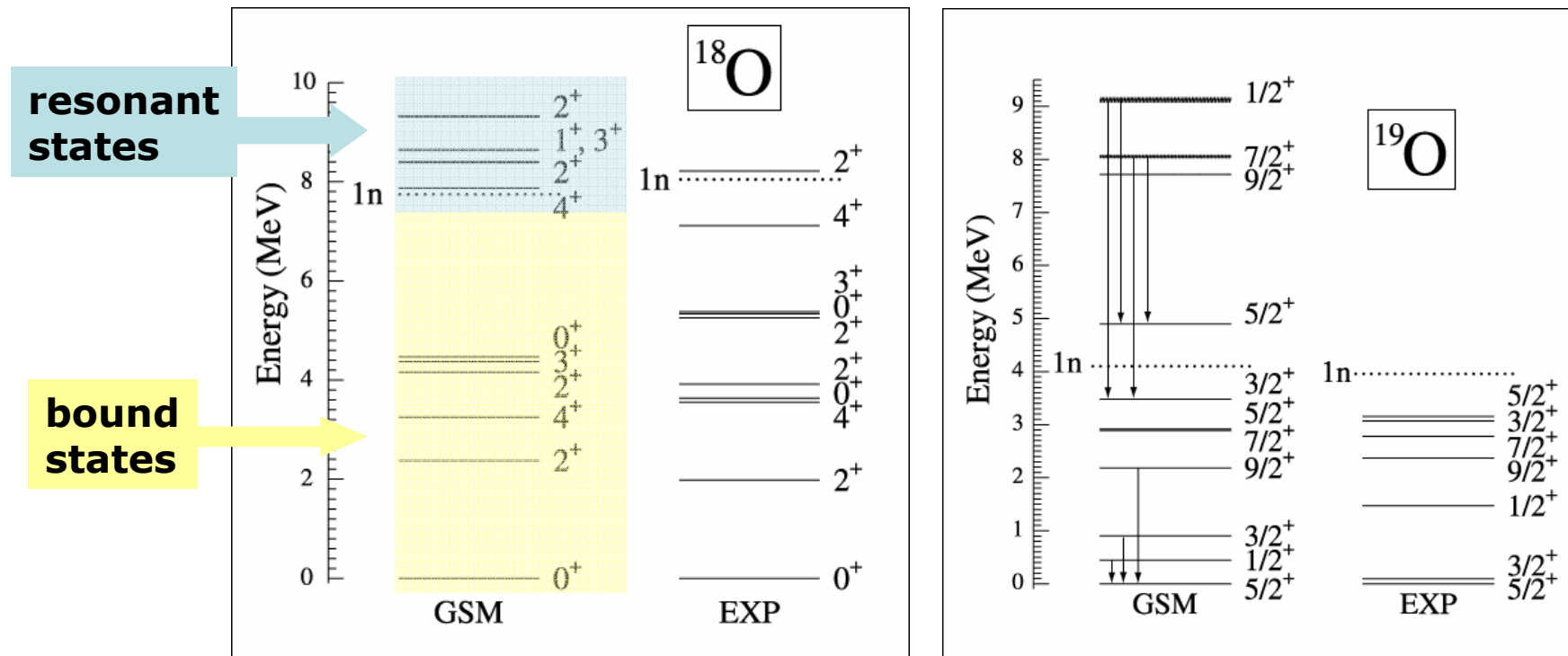
consistent treatment of both the many-body correlations and the continuum of positive energy states and decay channels

**GAMOW SHELL MODEL** -> description of bound states and the particle continuum (resonances and the nonresonant scattering background).

MICHEL, NAZAREWICZ, PŁOSZAJCZAK, AND OKOŁOWICZ

PHYSICAL REVIEW C 67, 054311 (2003)

Applications to systems with several valence neutrons!



# The shell model as a unified view of nuclear structure

REVIEWS OF MODERN PHYSICS, VOLUME 77, APRIL 2005

## Problems:

-the effective interactions strongly depend on the choice of the configuration space (active shells and the truncation scheme):  
**there is no universal effective SM interaction that can be used for all nuclei!**

-the effective interactions are adjusted starting from microscopic two-body forces. **3-nucleon effects?**


-a large number of two-body matrix elements ( $10^2 - 10^3$  ?) has to be adjusted to the data => **the effective interactions cannot be unique! Extrapolations to exotic nuclei not reliable!**

-nuclei very far from stability will require calculations with matrix dimension  $> 10^{10}$  => **far beyond the limits of current Shell Model variants!**

## 2. Evolution of shapes

High density of single-particle levels  
in medium-heavy and heavy nuclei



- 
- evolution of quadrupole collectivity
  - new regions of deformation: shape transitions and shape coexistence
  - nuclei with very diffuse neutron densities – neutron skin
  - exotic modes of collective excitations

Self-consistent mean-field methods:



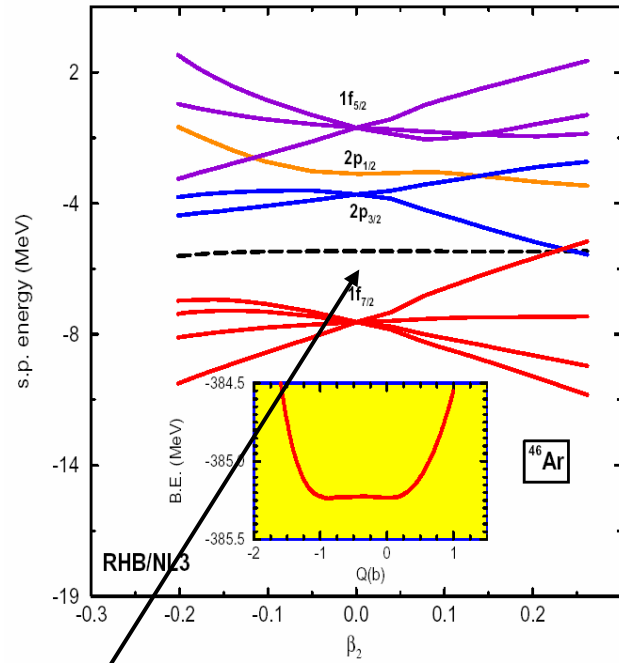
Non-relativistic HF/HFB  
based on Skyrme and  
Gogny interactions

Relativistic MF/HB  
finite-range meson-  
exchange and point-  
coupling models

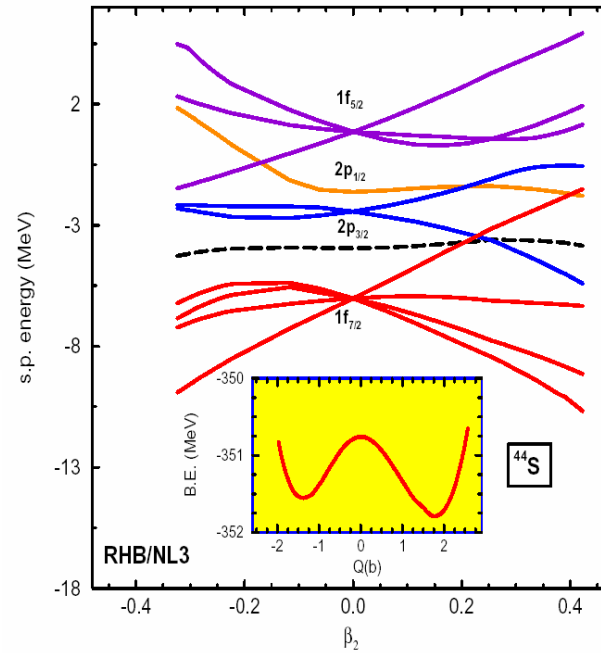
- use of global effective nuclear interactions
- description of arbitrarily heavy systems
- intuitive picture of intrinsic shapes



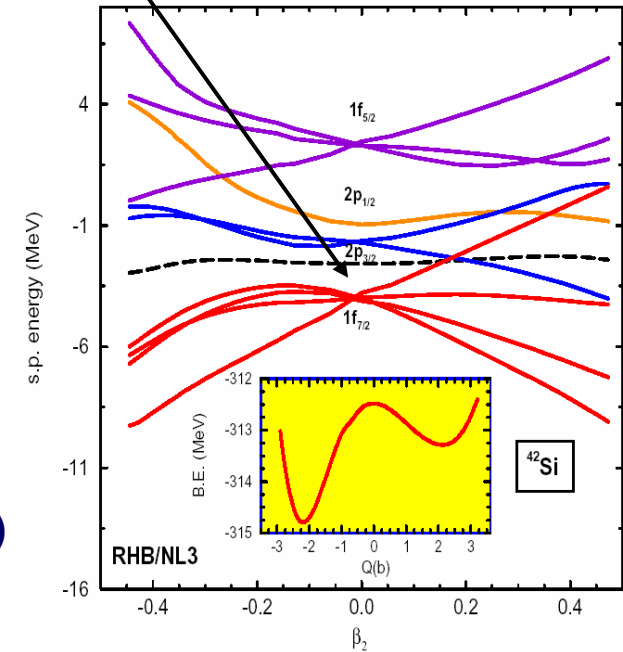
**Neutron single-particle levels for  $^{42}\text{Si}$ ,  $^{44}\text{S}$ , and  $^{46}\text{Ar}$  ( $N=28$ ) as functions of the quadrupole deformation (RHB/NL3 calculation).**



**large spherical gap**



**small spherical gap**



**For a quantitative description of shape transitions and shape coexistence => include correlations:**

- angular momentum projection (rotational energy)**
- quadrupole fluctuations (configuration mixing)**
- particle number projection**

...

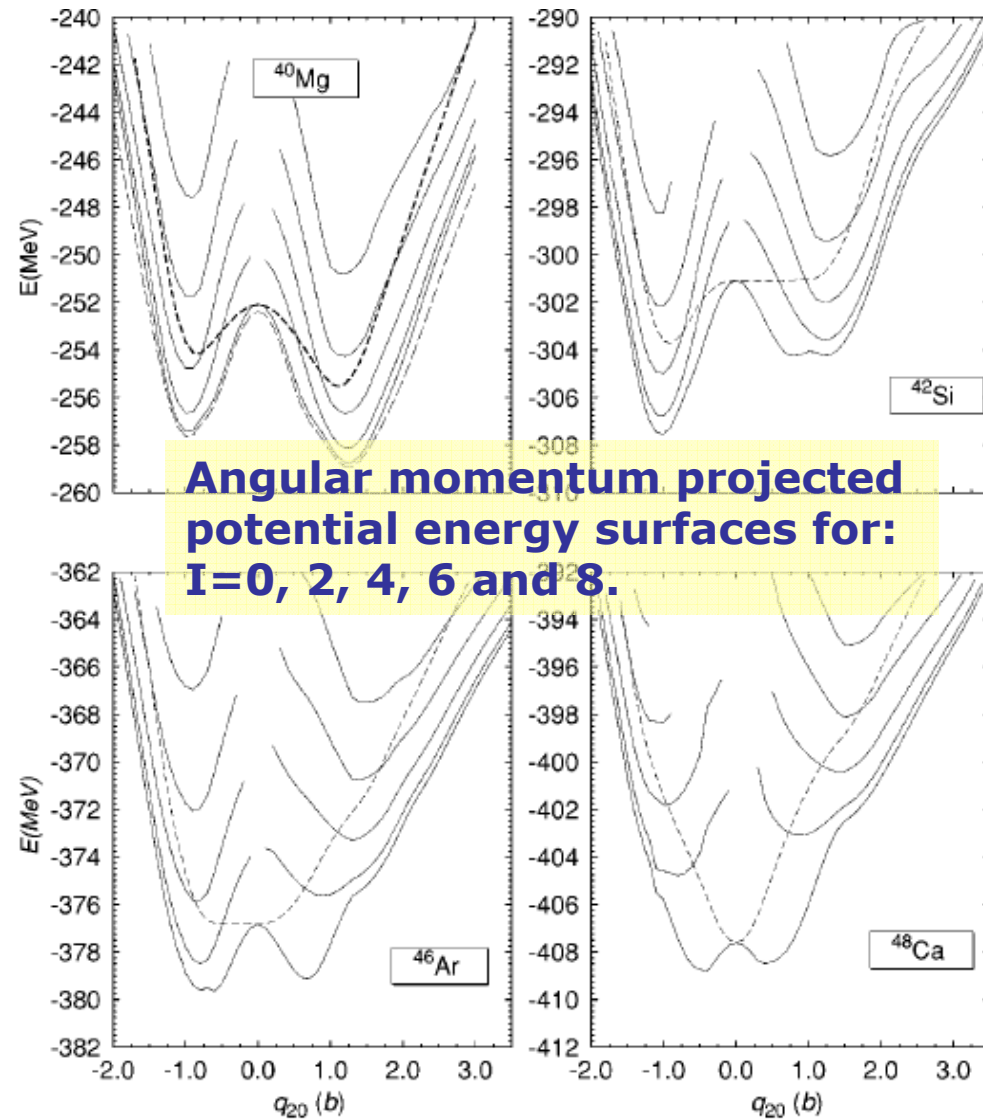
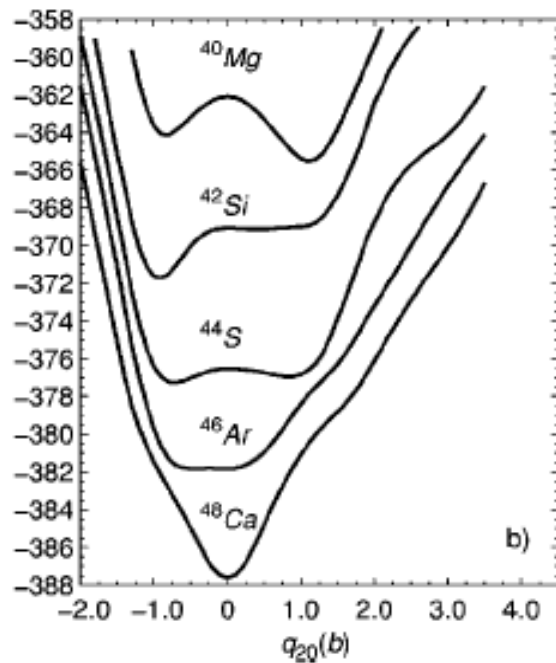
# Quadrupole collectivity in N=28 nuclei

angular momentum projection and configuration mixing

R. RODRÍGUEZ-GUZMÁN, J. L. EGIDO, AND L. M. ROBLEDO

PHYSICAL REVIEW C 65 024304

HFB -> Gogny D1S interaction:  
potential energy surfaces (PES)



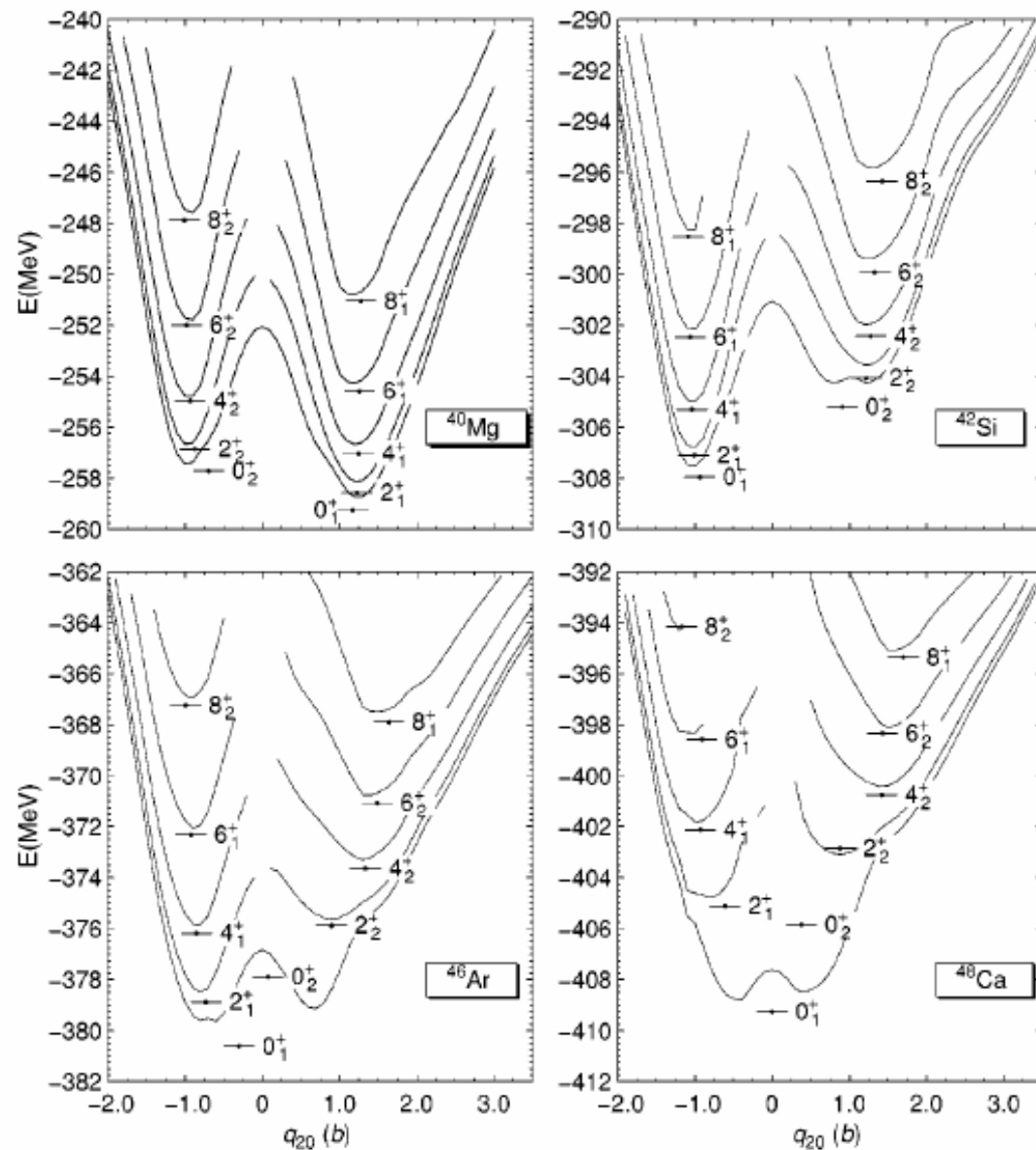
Small energy differences  
between coexisting minima:  
correlation effects beyond the  
mean-field level are important!

# Angular momentum projected configuration mixing:

**1. Constrained axially symmetric HFB calculations with the constraint on the mass quadrupole moment.**

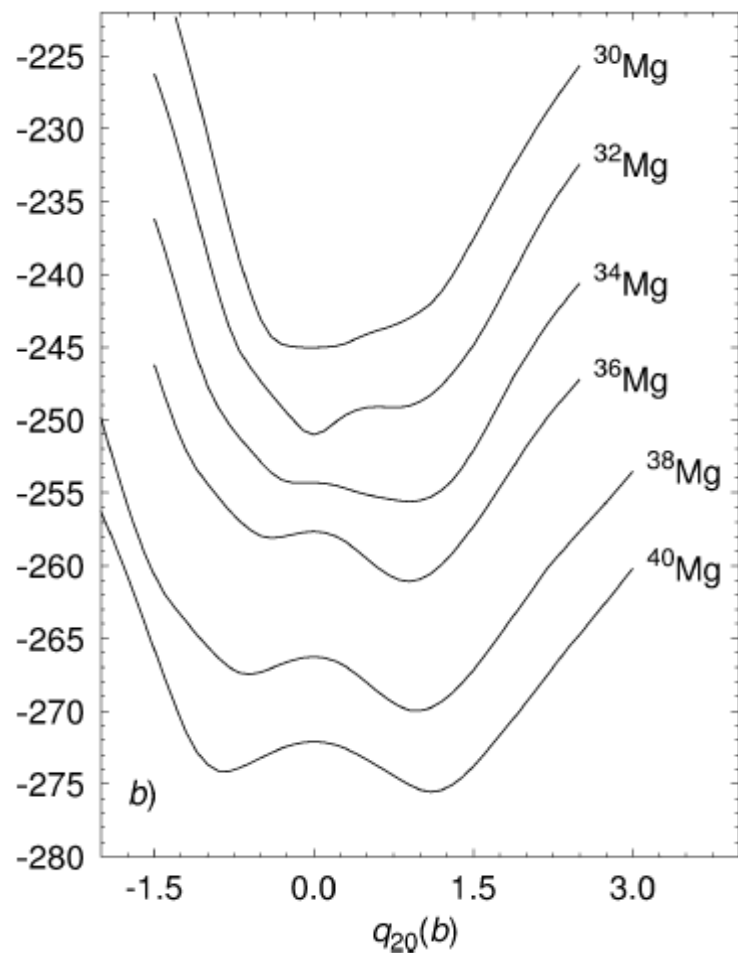
**2. Restoration of rotational symmetry  $\rightarrow$  rotational energy correction: energy gain  $\sim$  to the quadrupole deformation of the intrinsic state.**

**3. Configuration mixing  $\rightarrow$  Generator Coordinate Method**

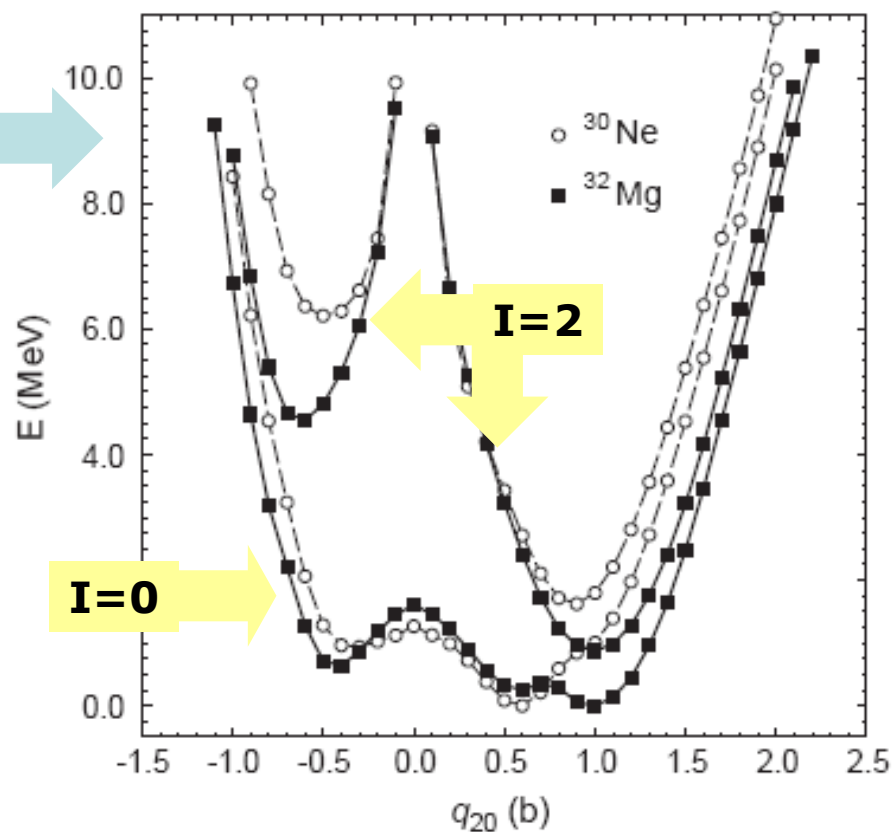


# CORRELATIONS BEYOND MEAN-FIELD IN NEUTRON-RICH Mg NUCLEI

HFB mean-field PES Gogny D1S interaction

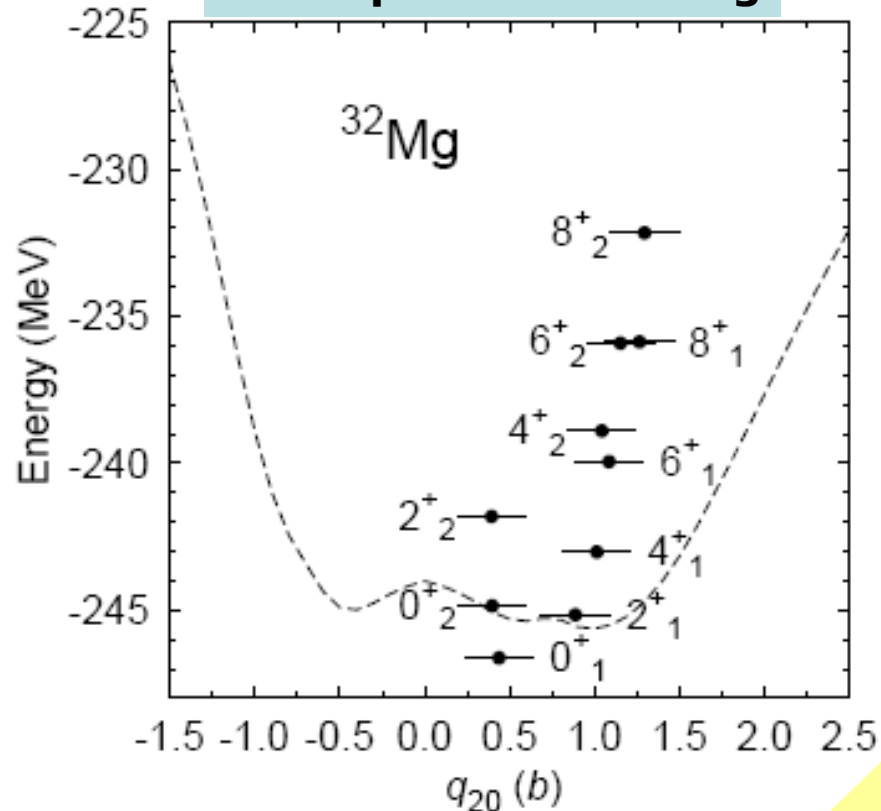


Nuclear Physics A 709 (2002) 201–235



Angular momentum projected PES

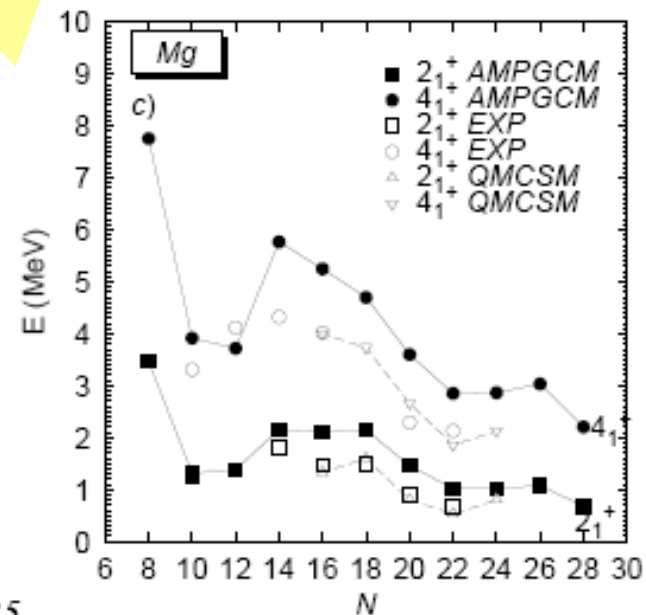
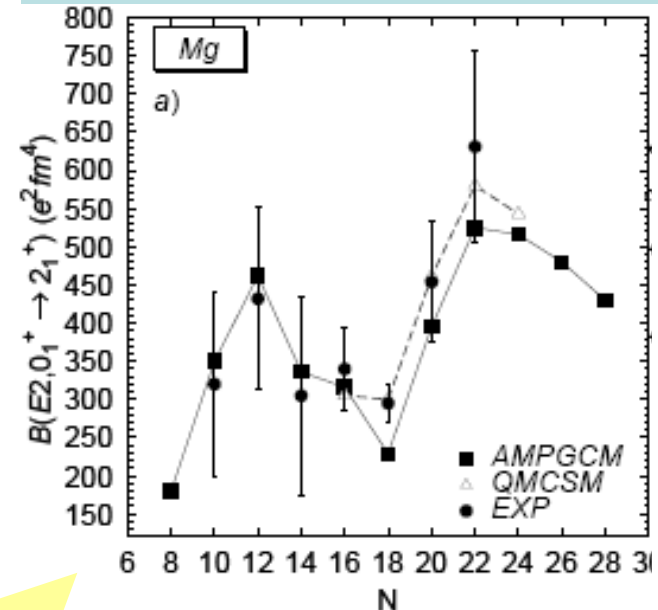
### GCM spectrum of $^{32}\text{Mg}$



Good agreement with exp. data and QMCSM calculations.

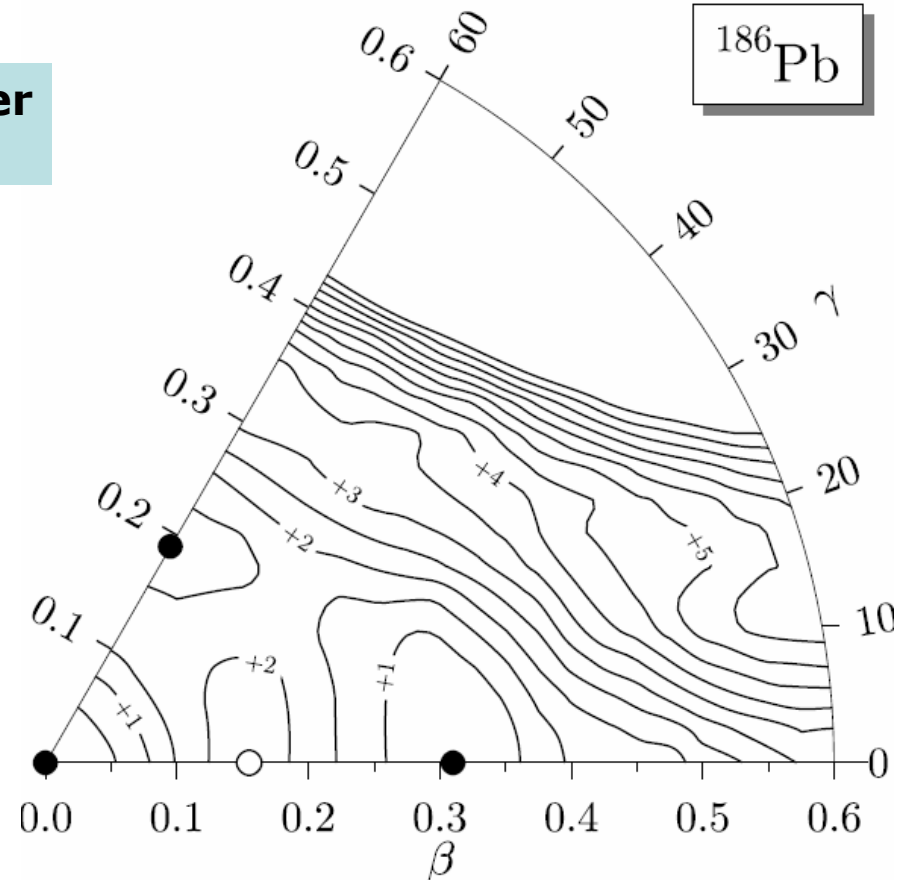
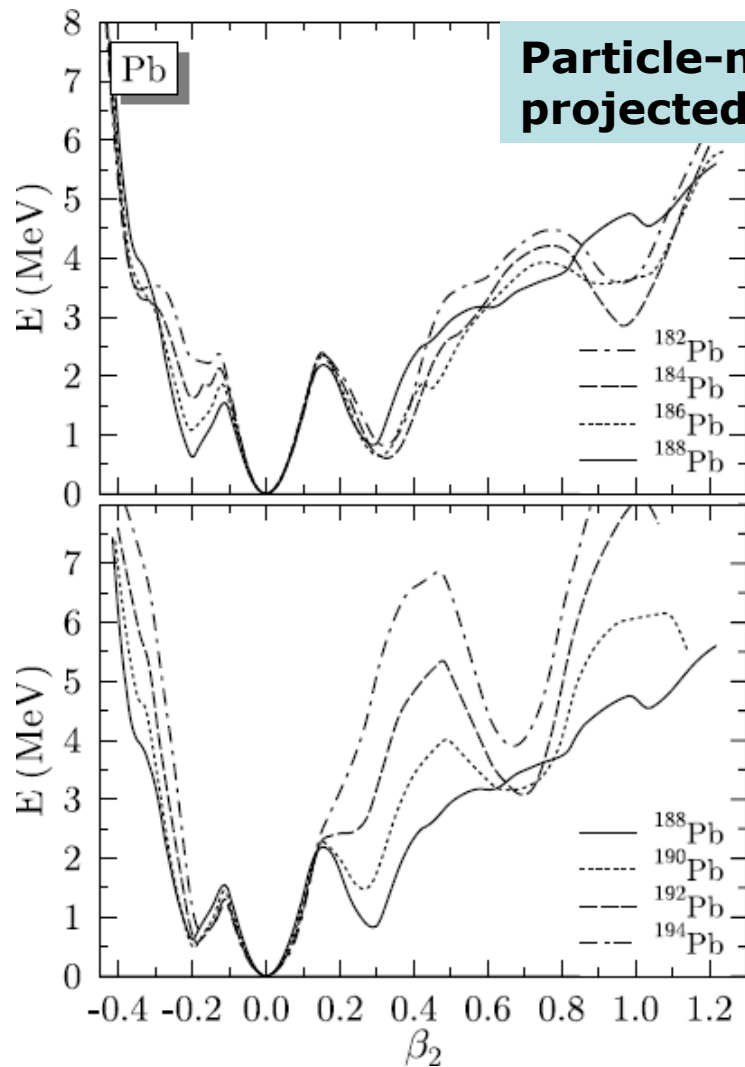
The effective interaction (Gogny) has not been adjusted to this mass region!  
No effective charges in the calculation of transition probabilities!

### $B(E2, 0_1^+ \rightarrow 2_1^+)$ in $^{20-40}\text{Mg}$

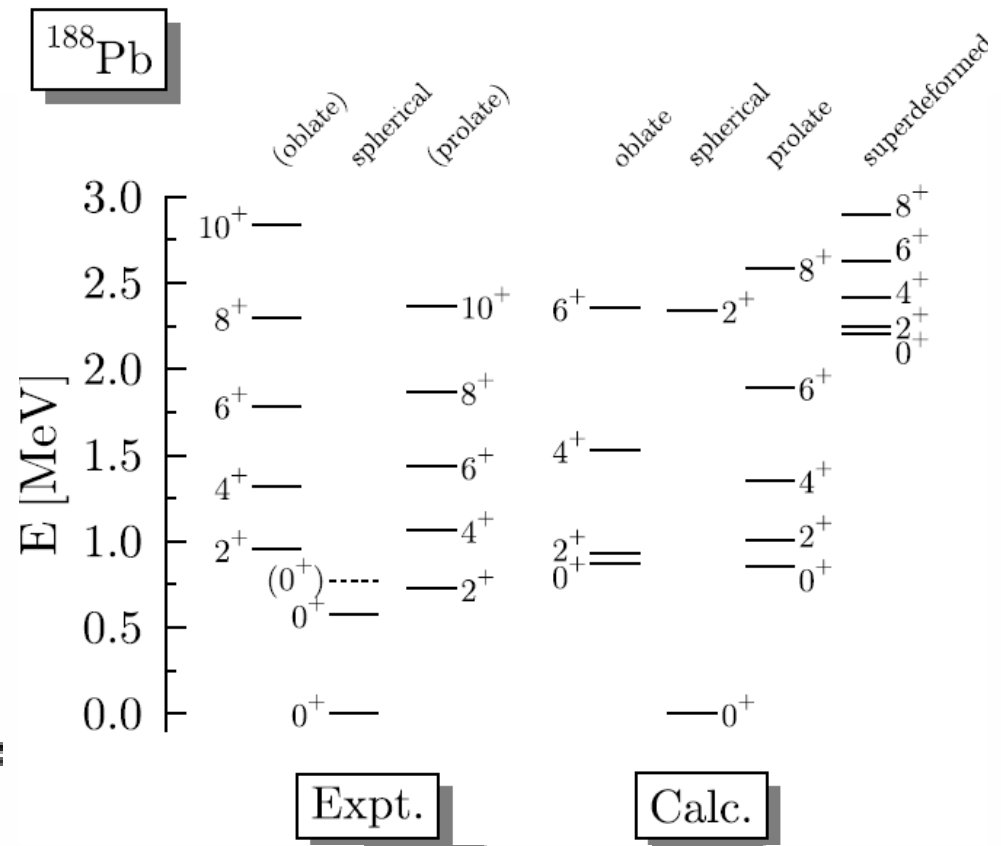
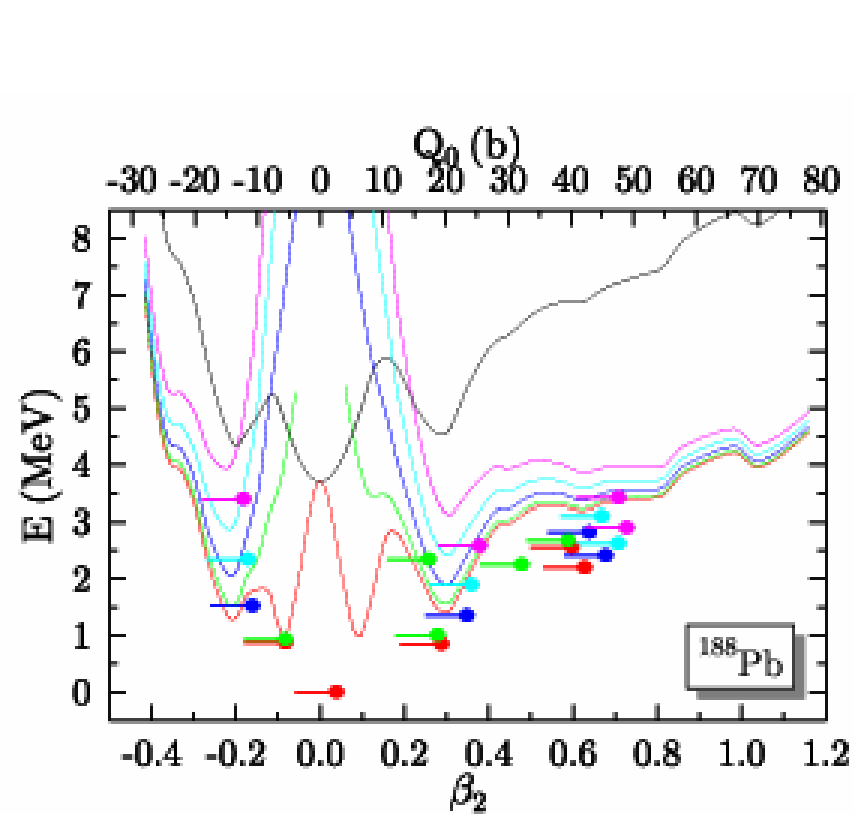


## SHAPE COEXISTENCE IN NEUTRON-DEFICIENT Pb ISOTOPES

GCM configuration mixing of angular-momentum and particle-number projected self-consistent HF+BCS states. Skyrme SLy6 interaction. Density-dependent zero-range pairing.



Unique example of a fermion system with the two lowest excited levels being  $0^+$  states!



Particle-number and angular-momentum projected PES for  $I=0^+, 2^+, 4^+, 6^+, 8^+$ .

GCM spectrum of the lowest positive parity bands with  $K=0$ .



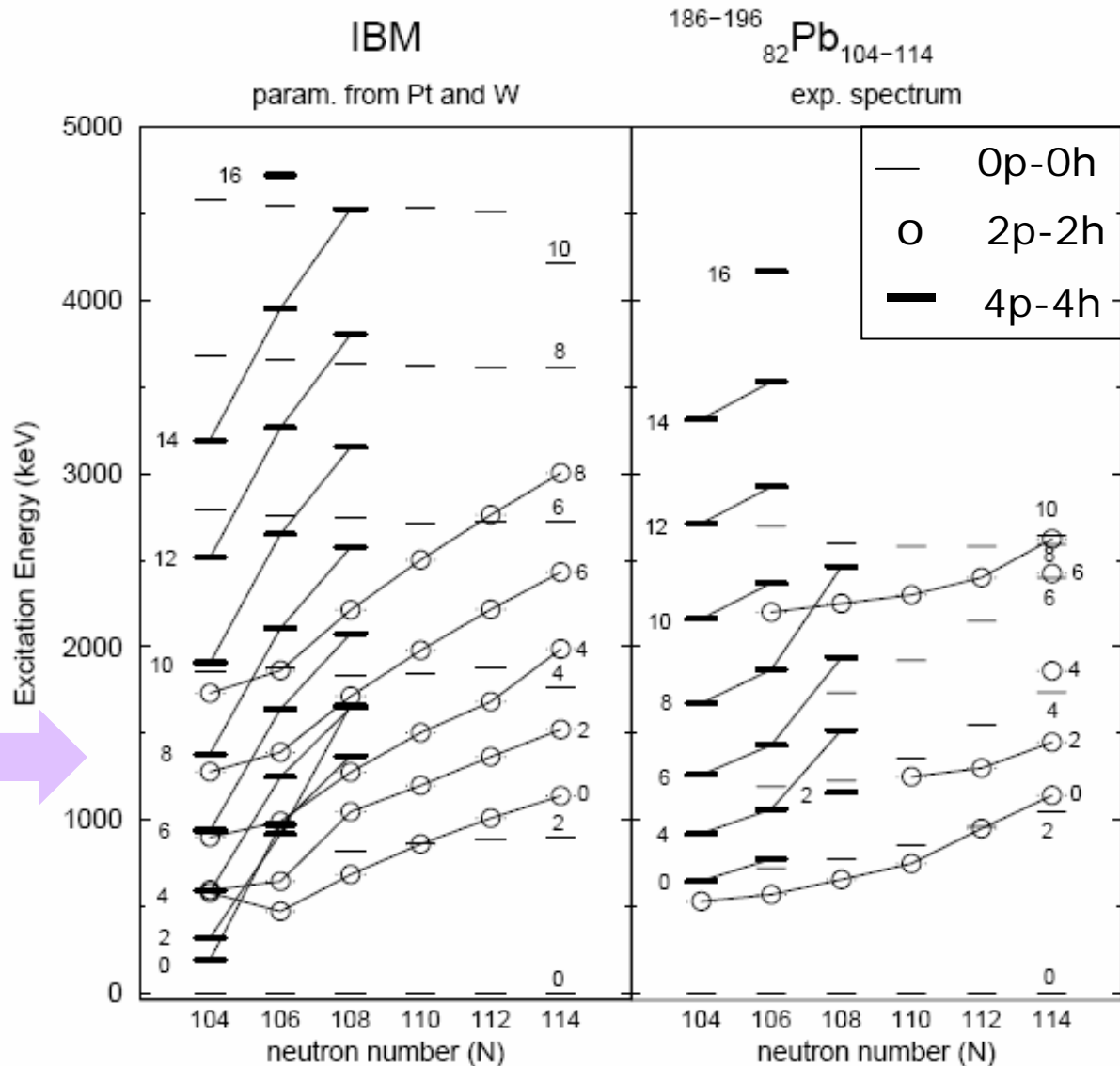
# Interacting Boson Model: Intruder bands in Pb isotopes

Multi-particle multi-hole excitations across the  $Z=82$  proton shell => calculations not feasible in the Shell Model.

## IBM framework:

**0p-0h** -> **N bosons**  
**2p-2h** -> **N+2 bosons**  
**4p-4h** -> **N+4 bosons**

IBM configuration mixing calculation



PHYSICAL REVIEW C 67, 024306 (2003)

R. FOSSION, K. HEYDE, G. THIAMOVA, AND P. VAN ISACKER



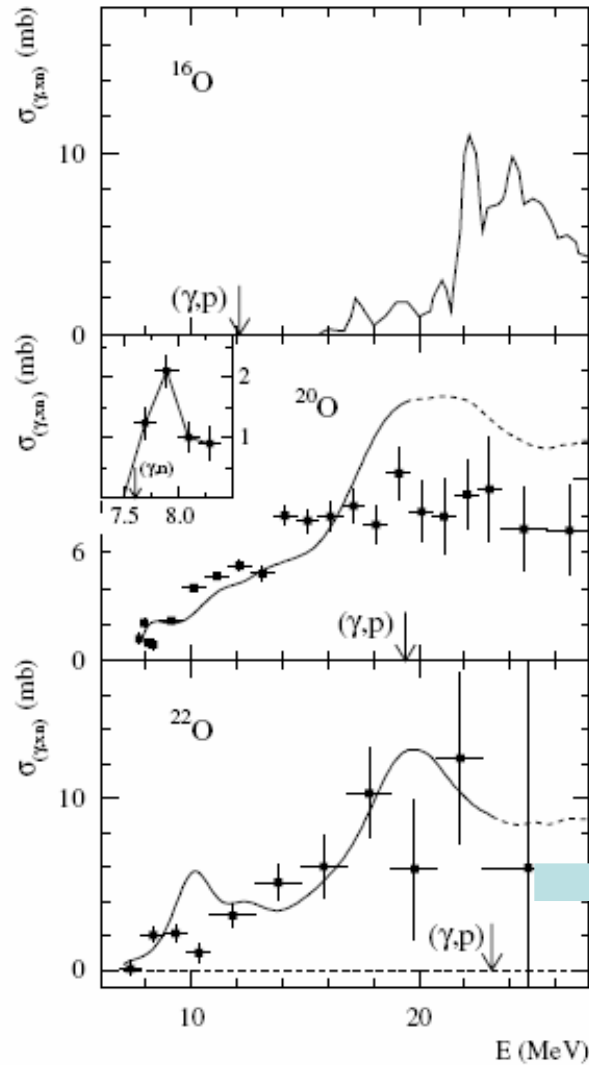
### **3. Evolution of low-lying collective modes**

# Low-lying dipole strength in neutron-rich nuclei

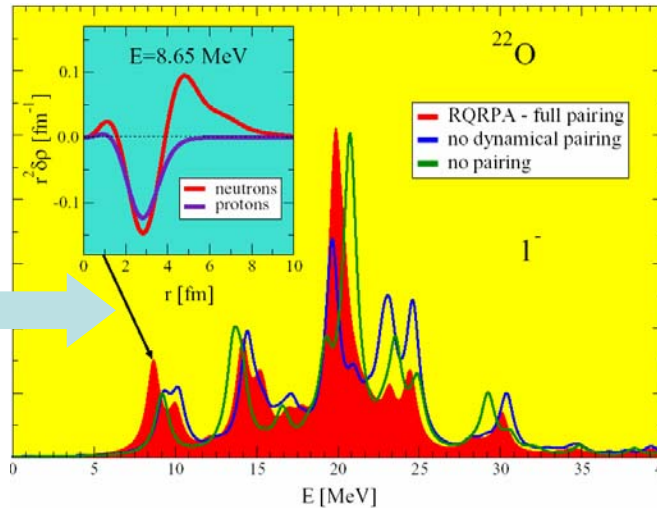
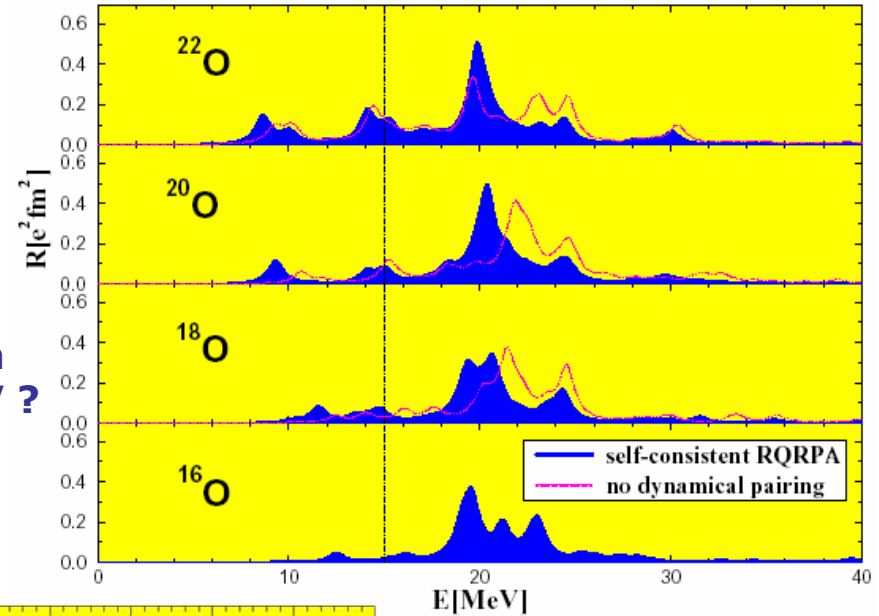
## Evolution of low-lying E1 strength in oxygen isotopes

Phys. Rev. Lett. 86, 5442 (2001)

Photon neutron cross section



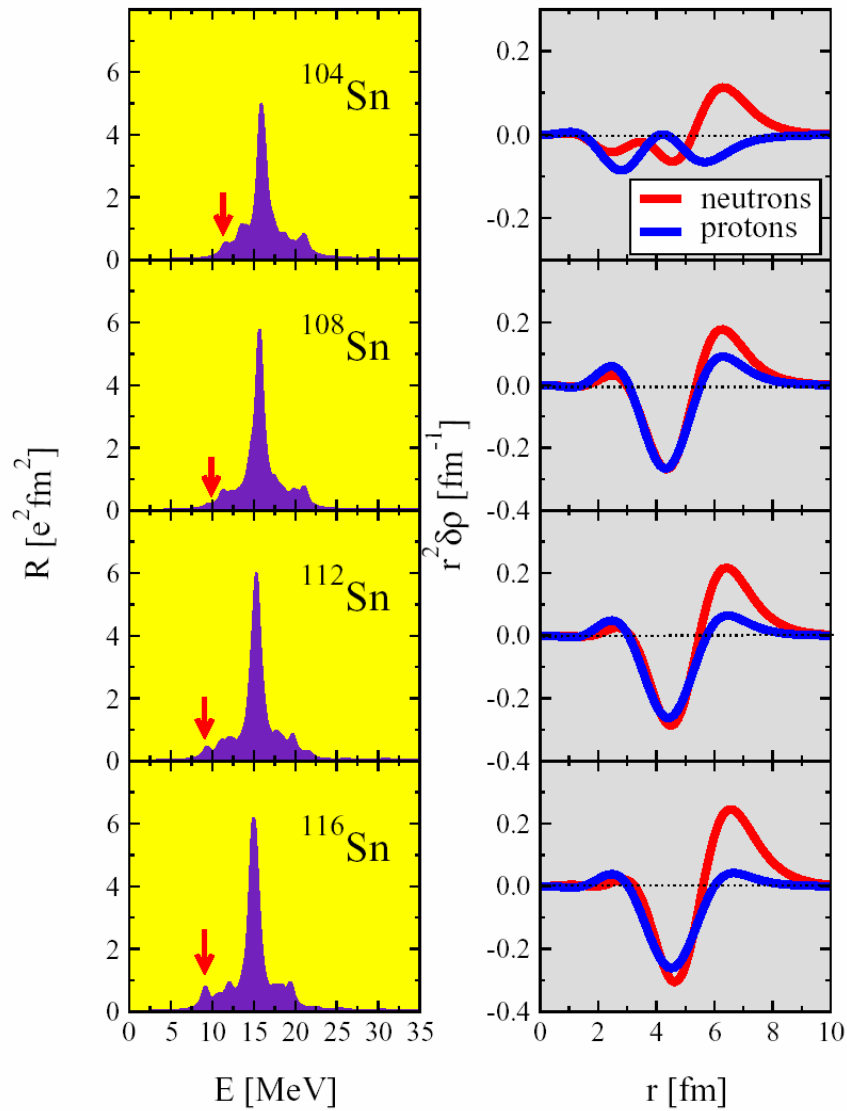
What is the structure of the low-lying strength below 15 MeV ?



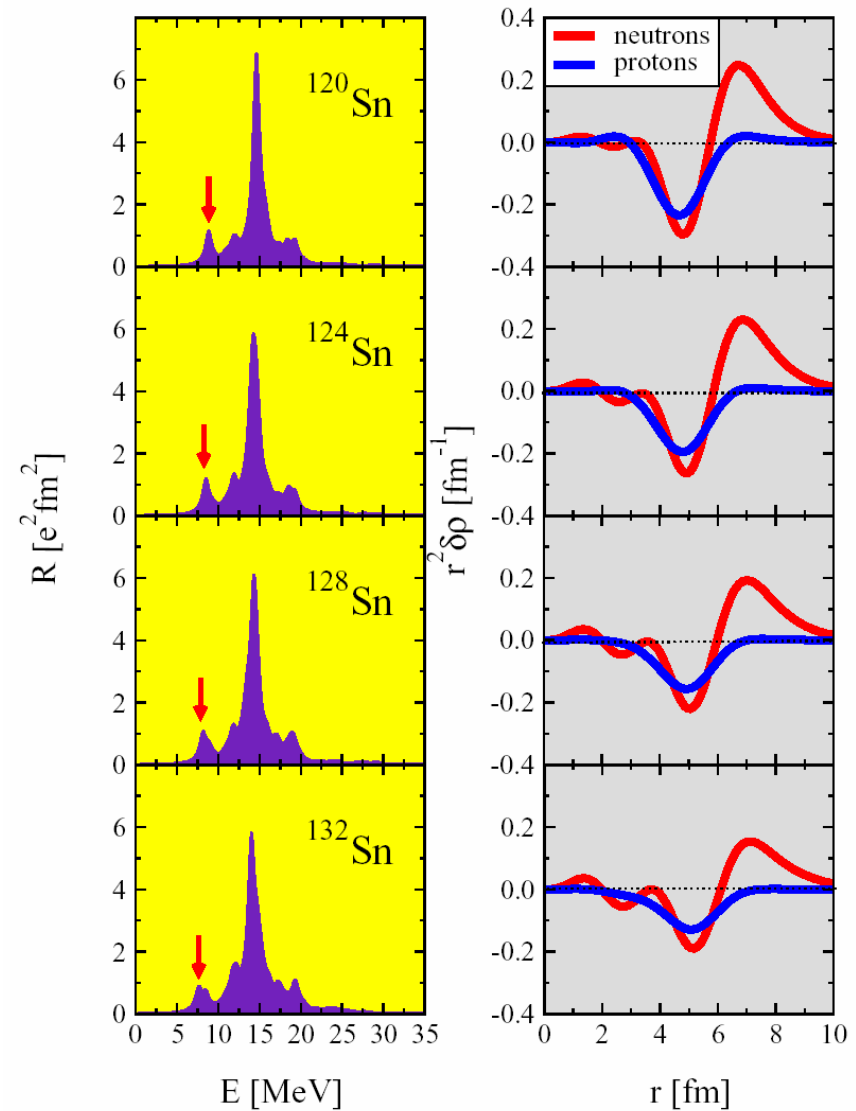
The dipole strength in the low-energy region is caused by non-resonant independent single particle excitations of the loosely bound neutrons!

# Evolution of isovector dipole strength in Sn isotopes:

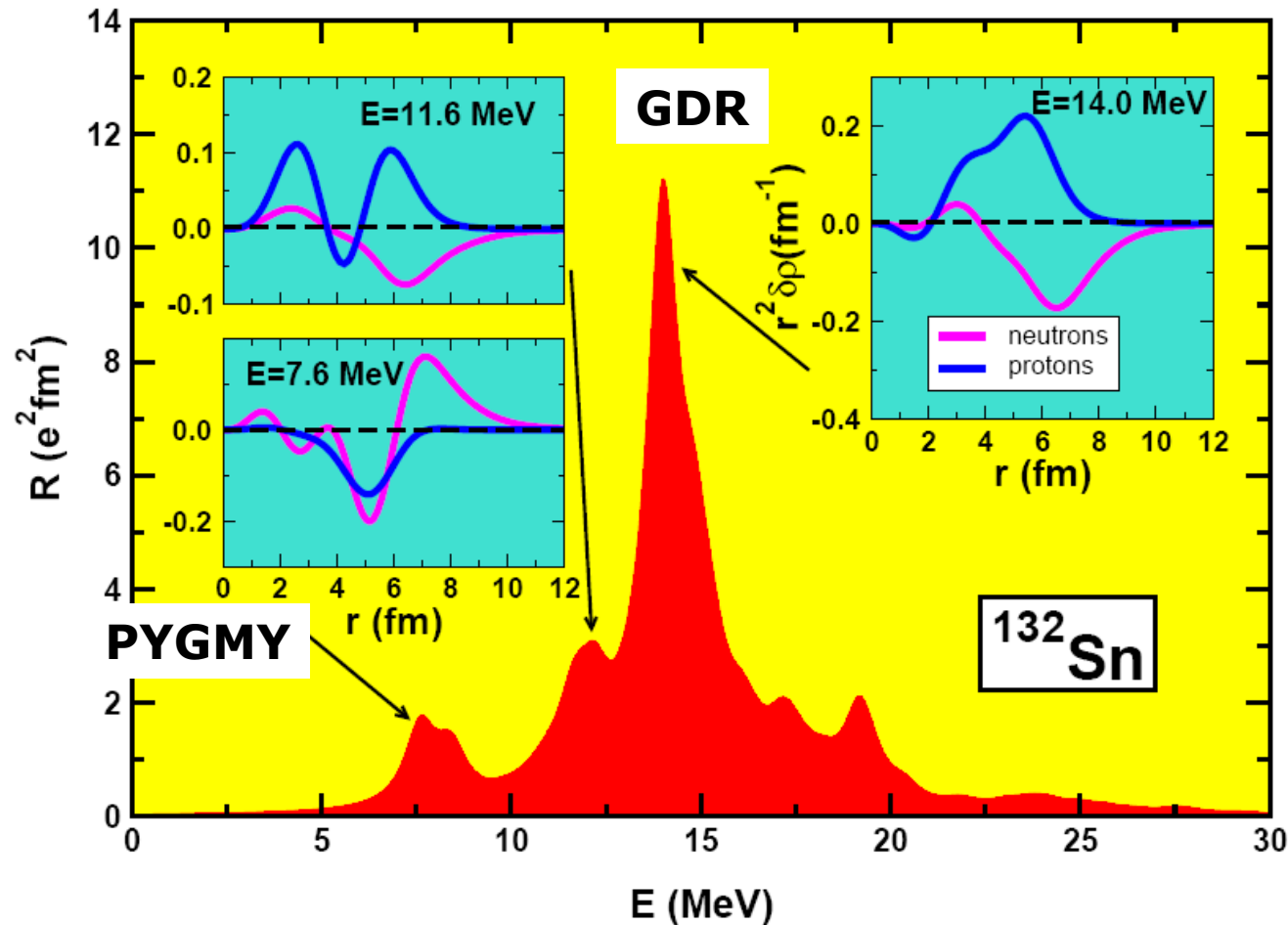
Transition densities



Transition densities



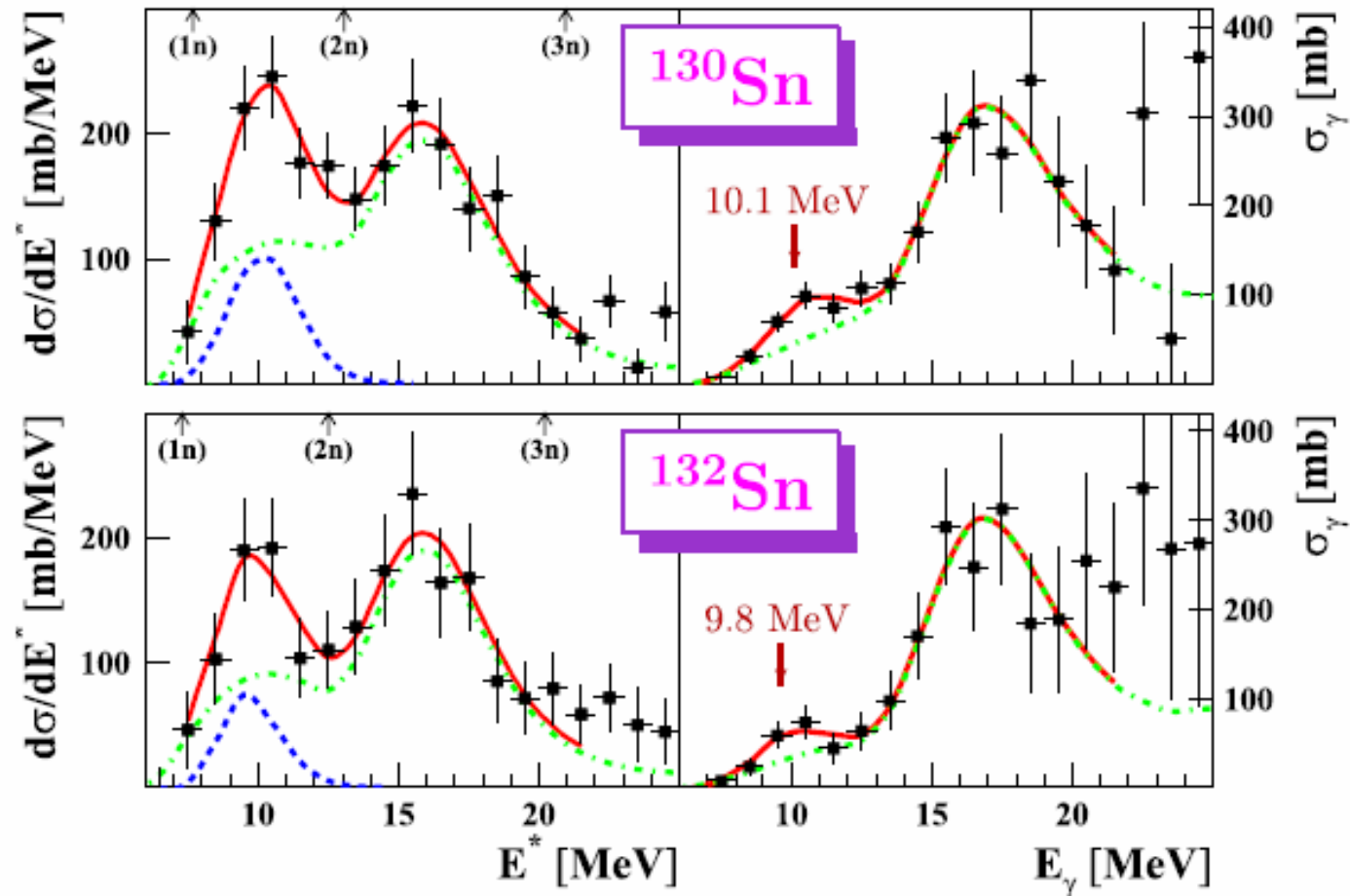
In heavier nuclei low-lying dipole states appear that are characterized by a more distributed structure of the QRPA amplitude.



Among several single-particle transitions, a single collective dipole state is found below 10 MeV and its amplitude represents a coherent superposition of many neutron particle-hole configurations.

# Evidence for Pygmy and Giant Dipole Resonances in $^{130}\text{Sn}$ and $^{132}\text{Sn}$

P. Adrich *et al.* (LAND-FRS Collaboration) **Phys. Rev. Lett. 95, 132501 (2005)**



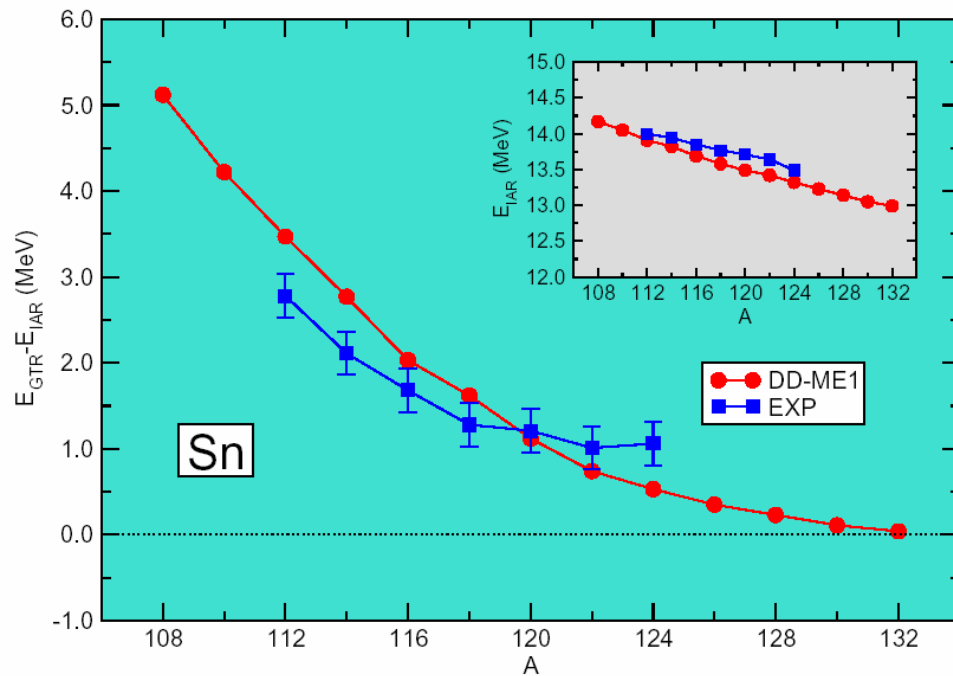
Energy differential electromagnetic dissociation cross section measured in  $^{130,132}\text{Sn}$ .

Deduced photo-neutron cross section.

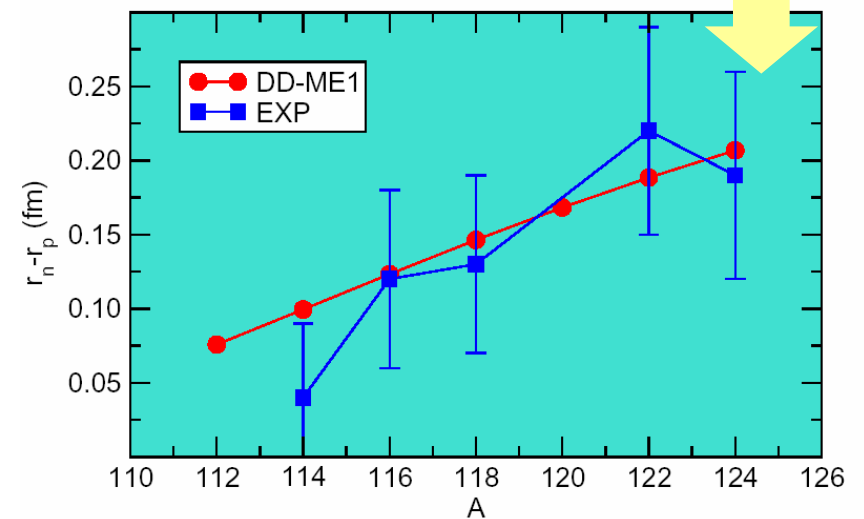
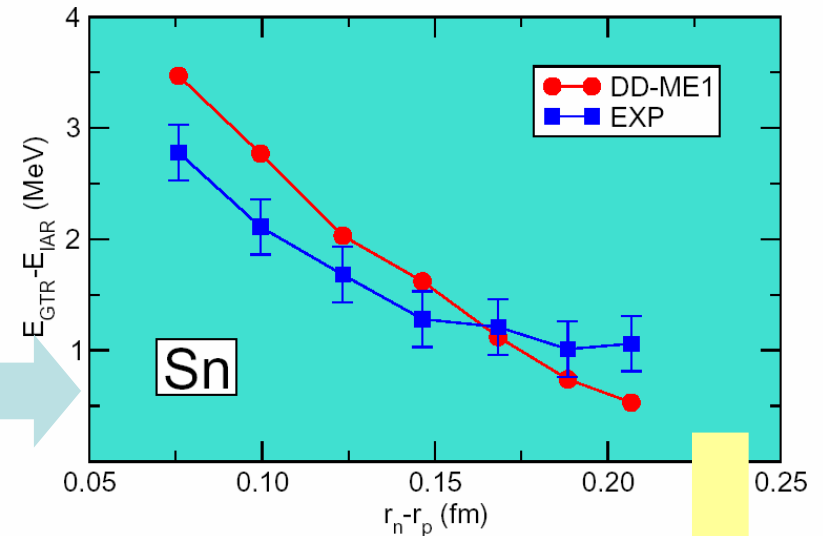
# Spin-Isospin Resonances and the Neutron Skin of Nuclei

The isotopic dependence of the energy spacings between the GTR and IAS

Proton-neutron RQRPA calculation



Information on the evolution of the neutron skin thickness along the Sn isotopic chain.

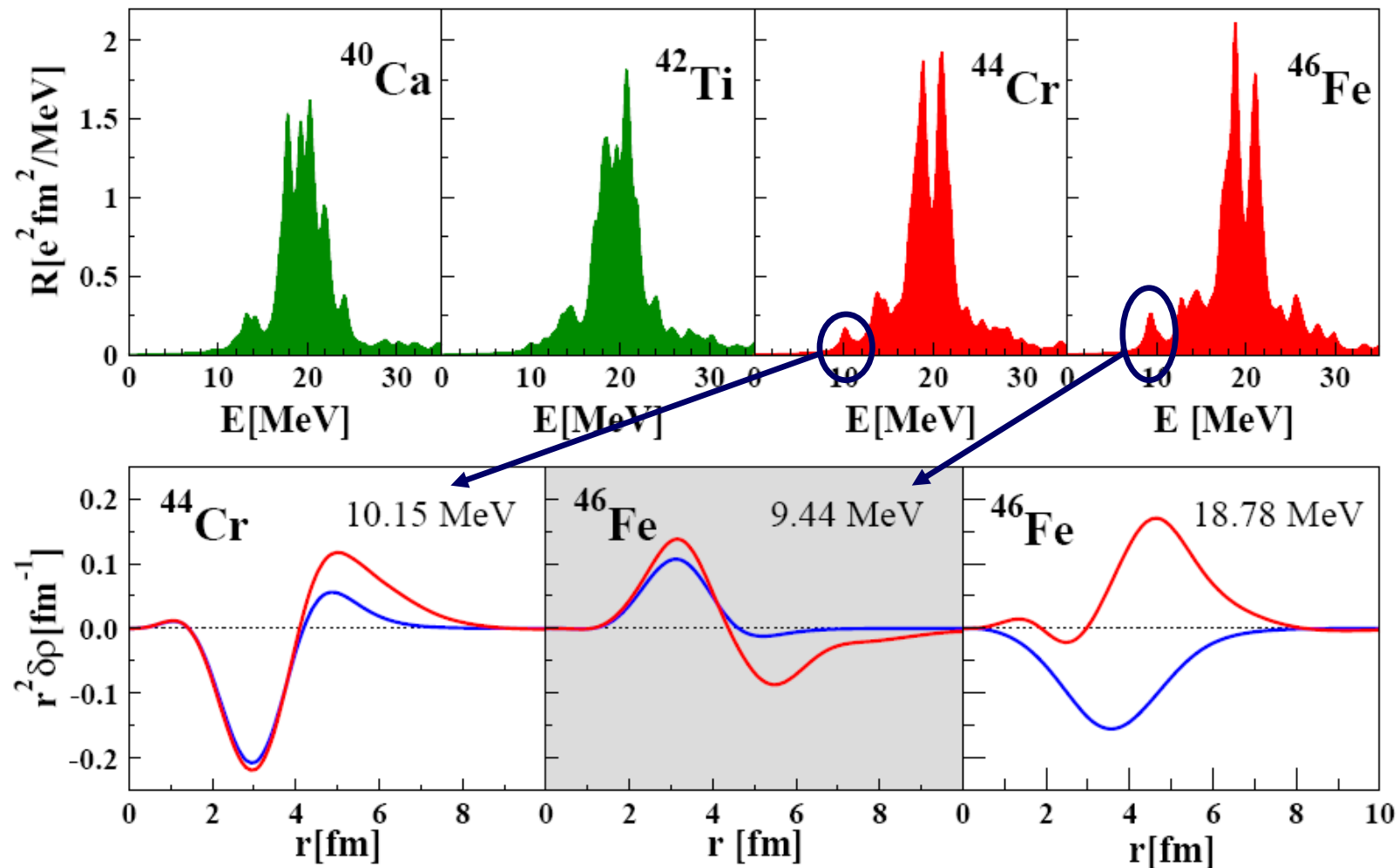


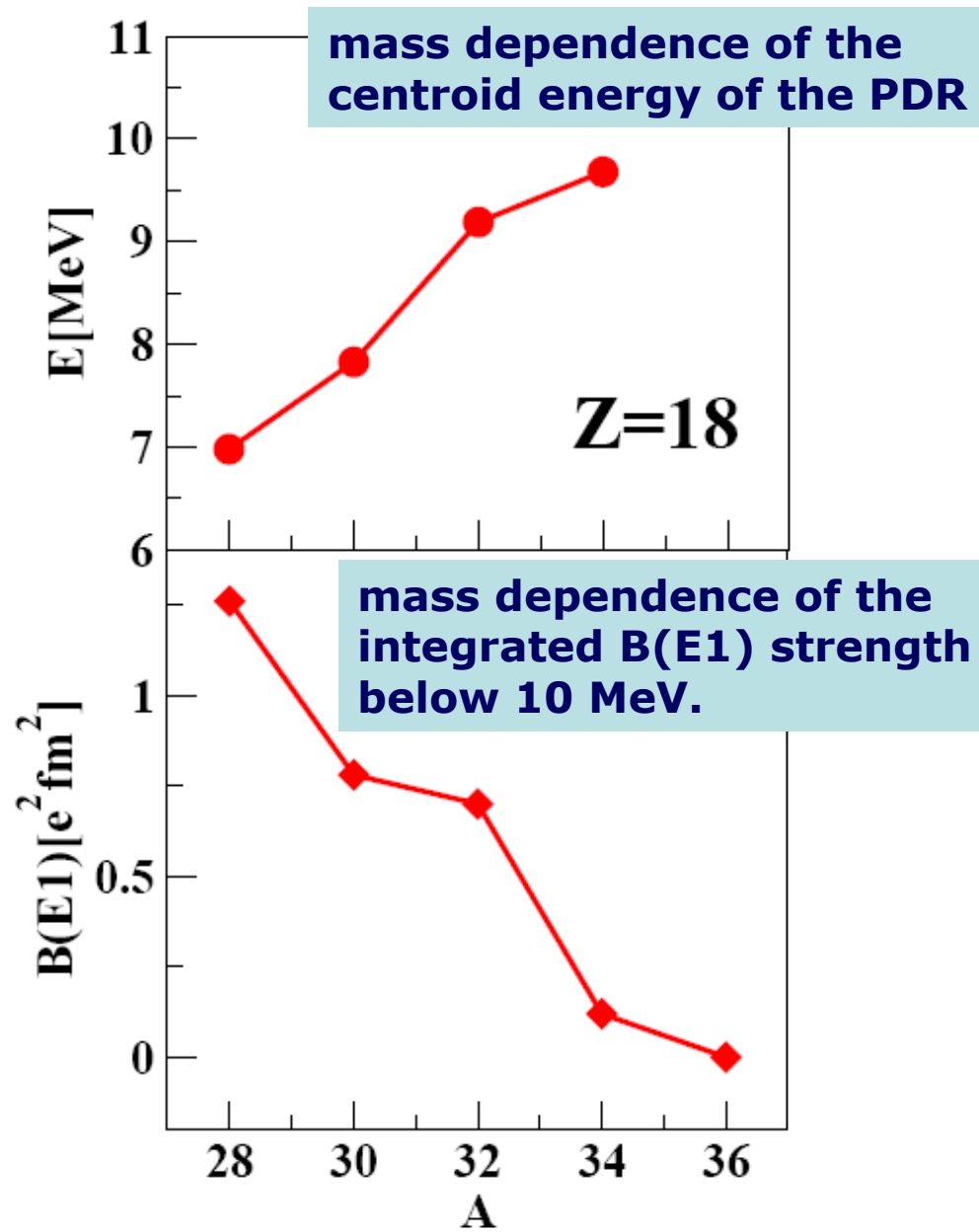
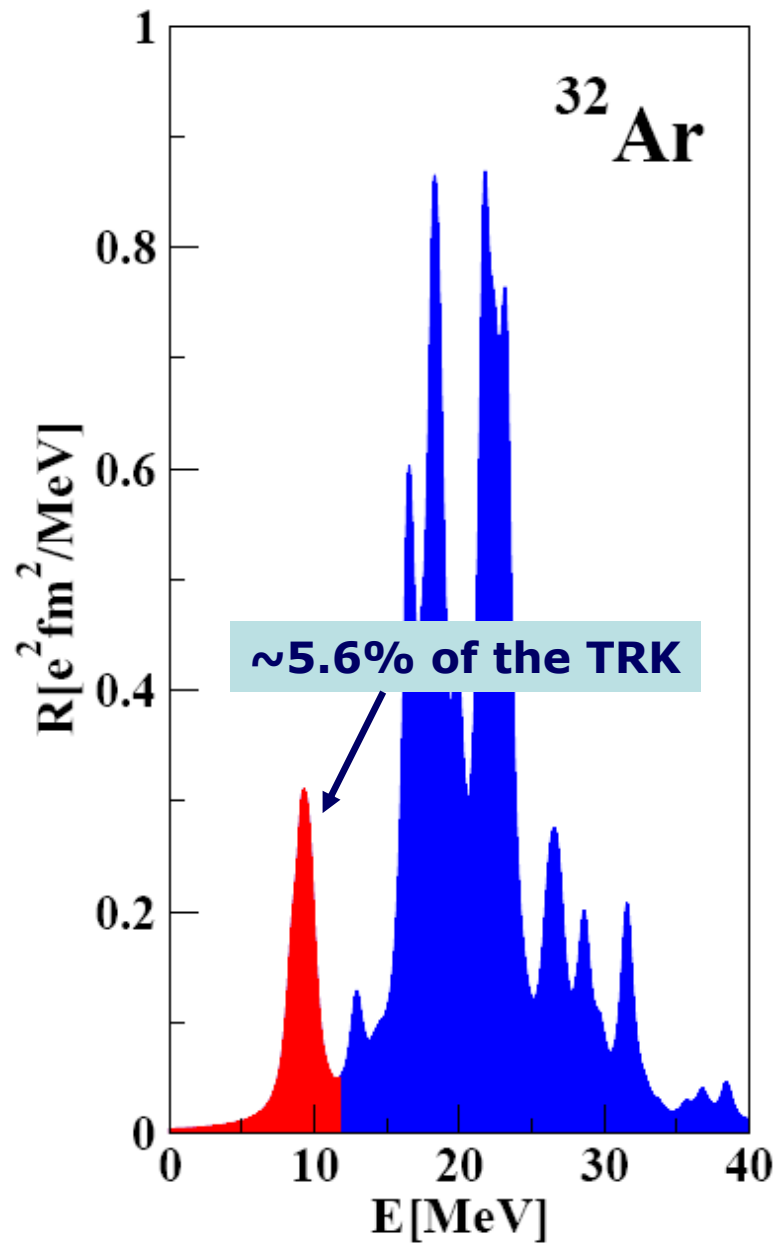
D. Vretenar, N. Paar, T. Nikšić, and P. Ring, Phys. Rev. Lett. 91, 262502 (2003).

# Evolution of low-lying E1 strength in proton-rich nuclei

Paar, Vretenar, Ring, Phys. Rev. Lett. 94, 182501 (2005)

RHB+RQRPA isovector dipole strength distribution in the N=20 isotones.  
DD-ME1 effective interaction + Gogny pairing.

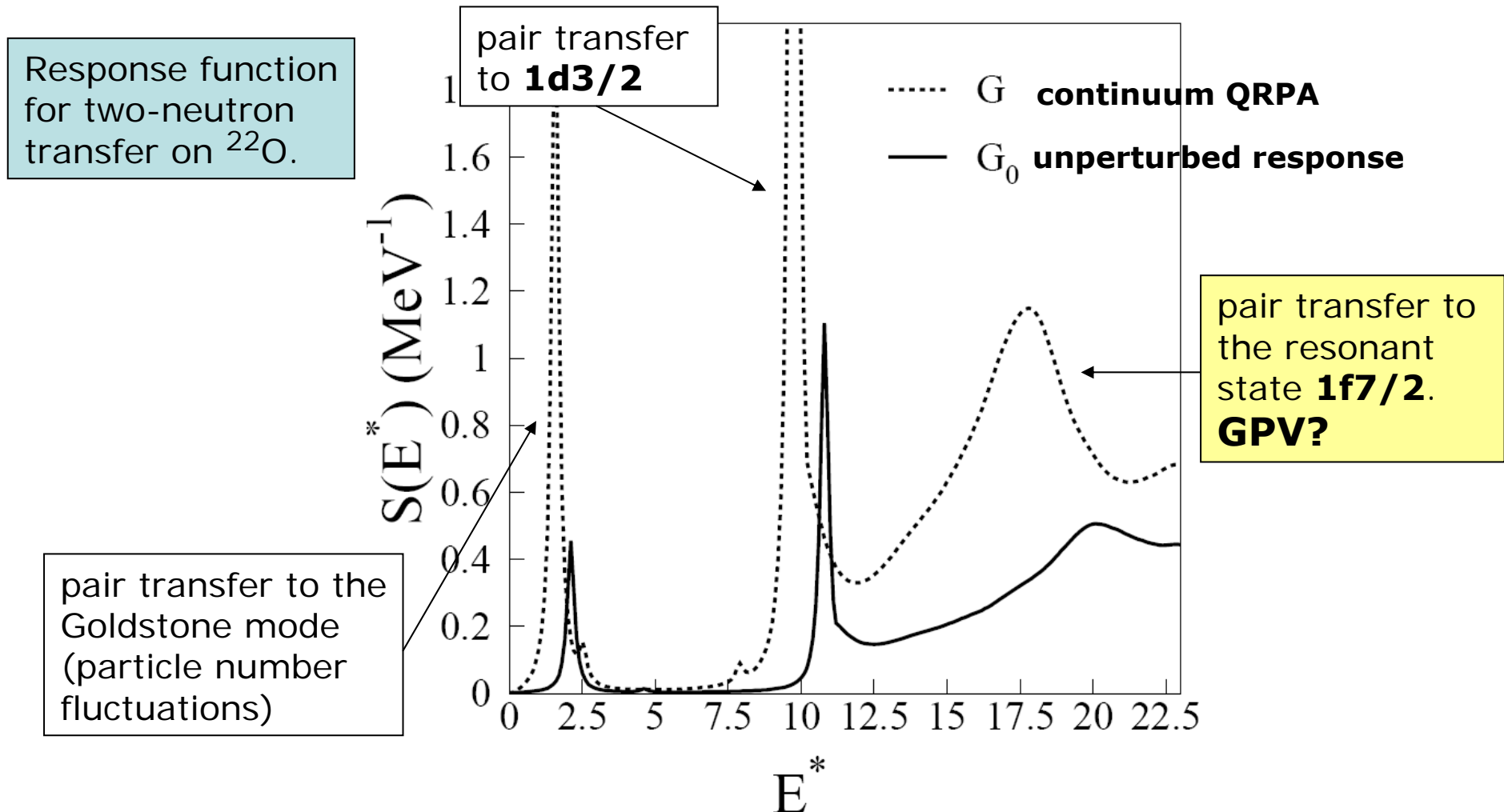






## Two-neutron Transfer In Nuclei Close To The Drip Line

- a) information on pairing correlations in nuclei far from stability
- b) chance of exciting the **GIANT PAIRING VIBRATION** mode



## **Modern Nuclear Structure Theory:**

**Unified microscopic description of structure and reactions for nuclei far from stability and reliable extrapolations toward the drip lines.**

**Microscopic global predictions for astrophysical applications: characteristics of strong, electromagnetic and weak interaction processes.**

**Next generation universal energy density functionals and effective shell model interactions -> based on the EFT representation of low-energy QCD.**

**Stringent constraints on the microscopic approach to nuclear dynamics, effective nuclear interactions, and nuclear energy density functionals, are obtained from studies of the structure and stability of exotic nuclei with extreme isospin values.**