Medium polarization effects and pairing interaction in finite nuclei

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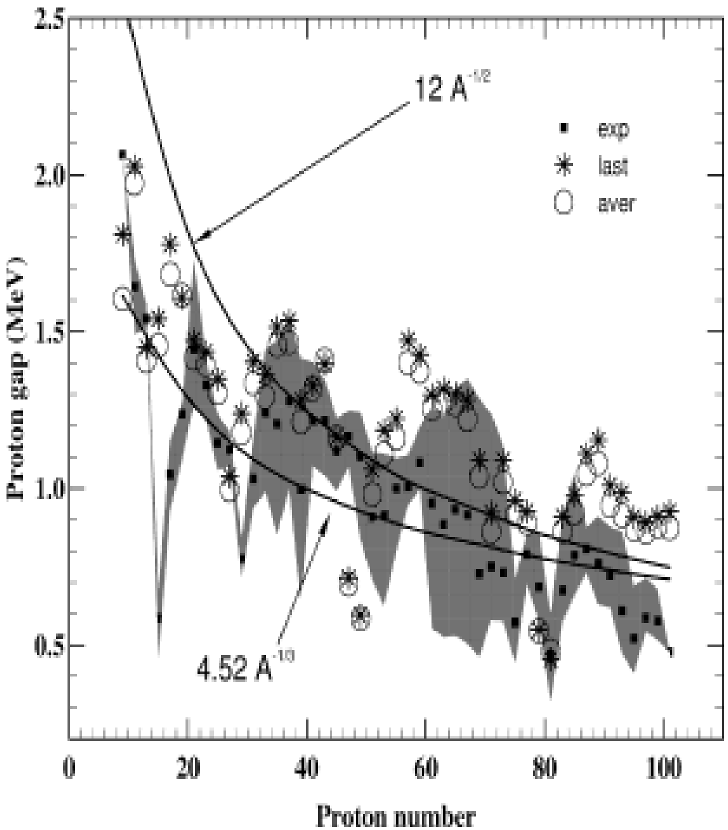


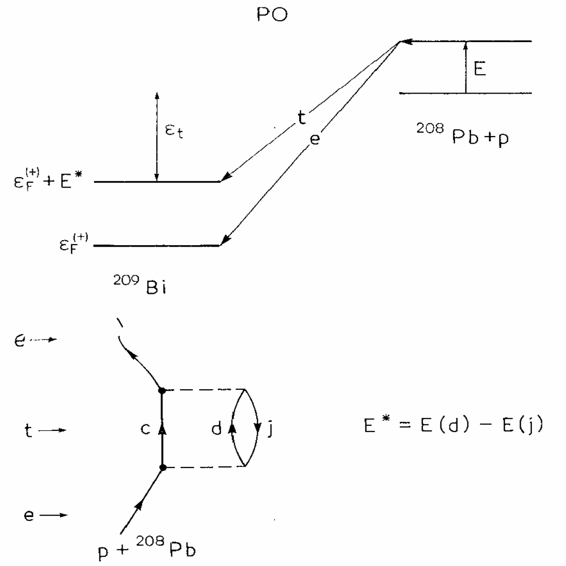
Fig. 1. Comparison between experimental and theoretical proton pairing gaps plotted as functions of the proton number Z. Squares represent experimental gaps extracted with the filter $\Delta^{(3)}$. Stars and circles are the corresponding theoretical values $\overline{\Delta^{\pi}_{\text{last}}}$ and $\overline{\Delta^{\pi}_{\text{aver}}}$ defined in the text. The shaded area represents the gap limits between which the experimental data are found before average. The lower curve corresponds to a least square fit on experimental data imposing an $A^{-1/3}$ law.

Hilaire et al., Phys. Lett B 531(2002)61

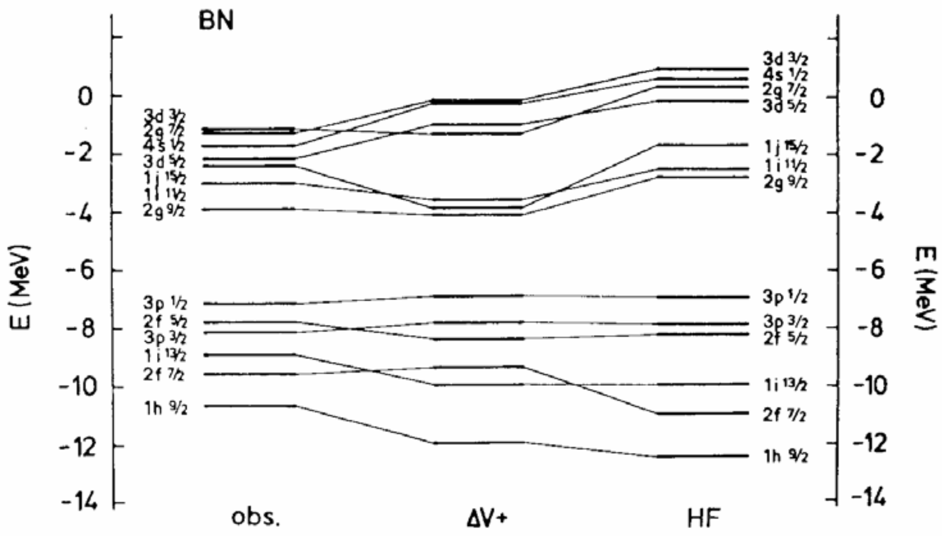
Commonly used mean-field approach: HFB theory with Gogny force

Overall agreement with 'experimental' $^1S_0$ pairing gaps, but many open questions remain.
Among them:
-connection with the bare force
- detailed isotopic dependence
- spectroscopic factors

C. Mahaux et al., Dynamics of the shell model



C. Mahaux et al., Dynamics of the shell model



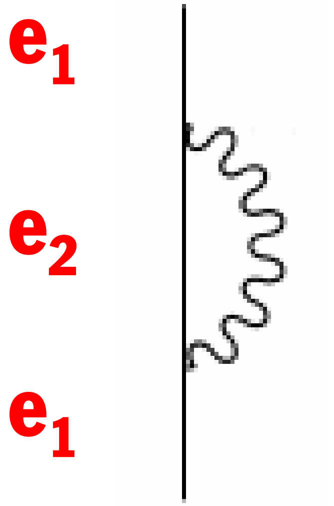
Coupling of vibrations to single-particle motion

↓

Effective mass m_{ω}

↓

Increased density at the Fermi energy

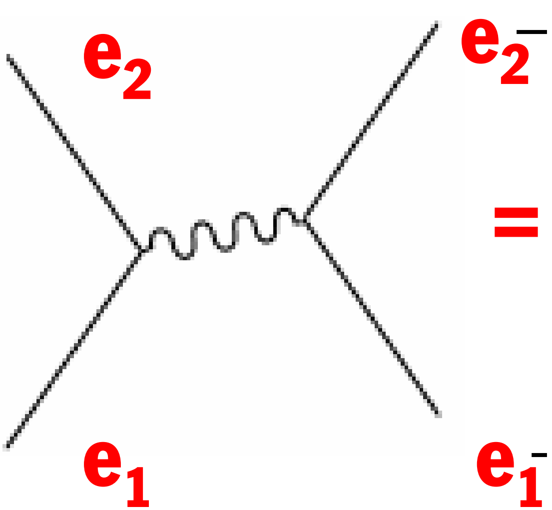


$$= \frac{V^2}{e_1 - (e_2 + \hbar\omega_\lambda)} \approx -\frac{V^2}{\hbar\omega_\lambda}$$

$$m_\omega \approx \left(1 + \frac{2N(0)V^2}{\hbar\omega_\lambda}\right) m$$

$$m_\omega \approx 1.5m$$
$$\hbar\omega_\lambda \approx 1\text{MeV}$$
$$N(0) \approx 3\text{MeV}^{-1}$$

$$\Rightarrow \quad V^2 \approx 0.1 \quad \text{MeV}^2$$

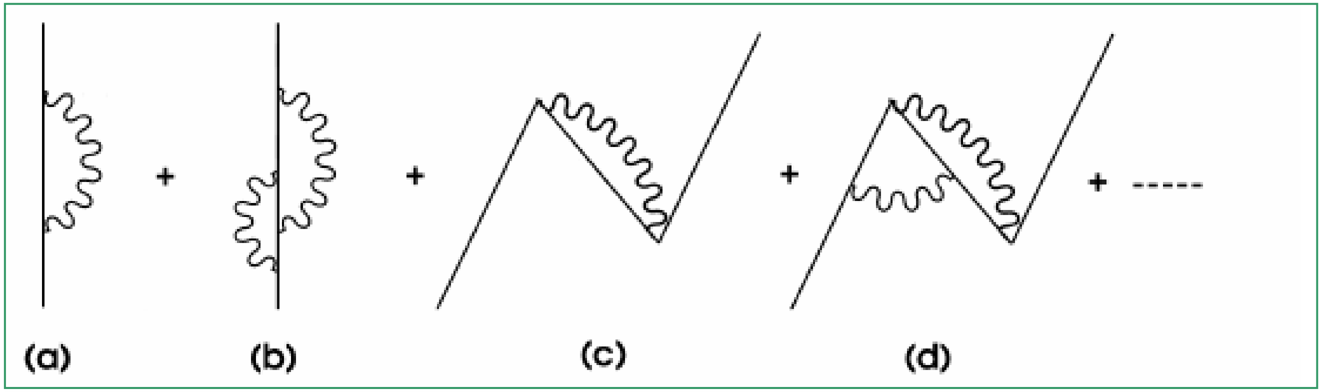


$$= \frac{V^2}{2e_1 - (e_1 + e_2 + \hbar\omega_\lambda)} = \frac{V^2}{e_1 - (e_2 + \hbar\omega_\lambda)} \approx -\frac{V^2}{\hbar\omega_\lambda}$$

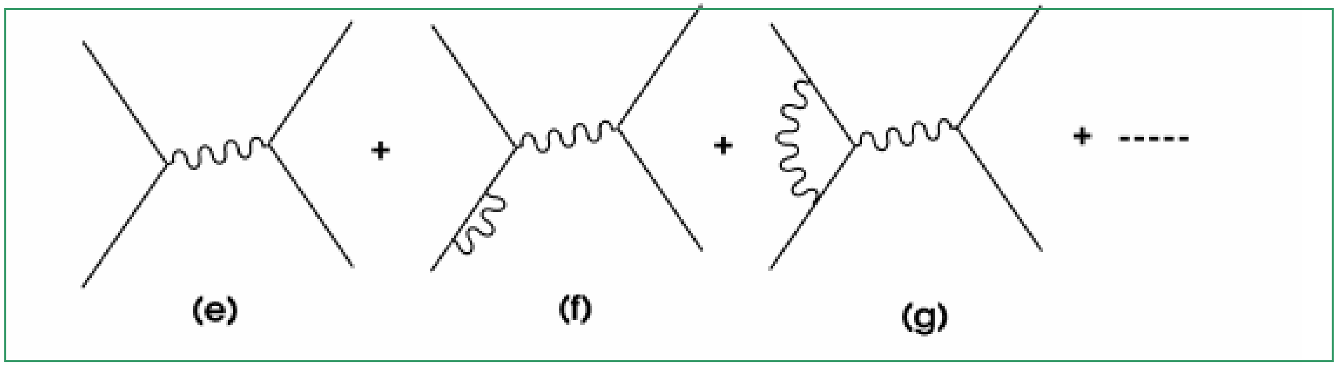
$$V_{ind} \approx -0.2\text{MeV}$$

Going beyond mean field: medium polarization effects

Self-energy



Induced interaction
(screening)

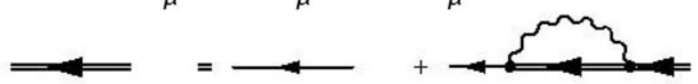


Going beyond the quasi-particle approximation

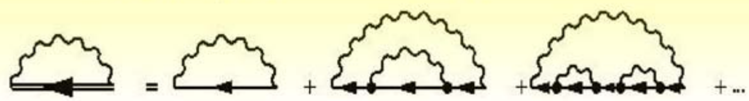
J. Terasaki et al., Nucl.Phys. **A697**(2002)126

by extending the Dyson equation…

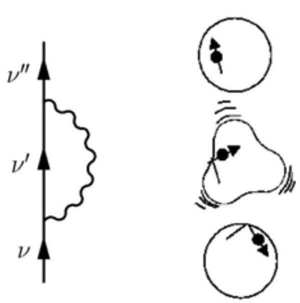
$$G_{\mu}^{-1} = (G_{\mu}^{o})^{-1} - \Sigma_{\mu}(\omega)$$



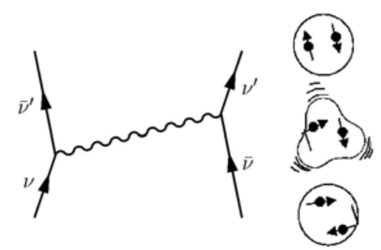
$$\Sigma_{\mu}(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \sum_{\mu'} \frac{1}{\hbar} G_{\mu'}(\omega') \sum_{\alpha} \frac{1}{\hbar} D_{\alpha}^{o}(\omega-\omega') * V_{\mu\mu',\alpha}^{2}$$



to the case of superfluid nuclei (Nambu-Gor'kov), it is possible to consider both



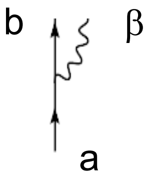
and



Coupling quasiparticle **a** to one quasiparticle-one phonon configurations **b**β, cγ .....

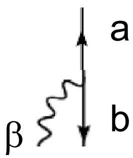
$$
\left(\begin{array}{ccc|ccc}
\mathbf{E_a} & V_{a,\tilde{b}\beta} & V_{a,\tilde{c}\gamma} & \mathbf{\Delta_a} & W_{a,\tilde{b}\beta} & W_{a,\tilde{c}\gamma} \\
V_{a,\tilde{b}\beta} & \tilde{E}_b + \hbar\omega_\beta + x_{b\beta,b\beta} & x_{c\gamma,b\beta} & W_{a,\tilde{b}\beta} & y_{b\beta,b\beta} & y_{c\gamma,b\beta} \\
V_{a,\tilde{c}\gamma} & x_{c\gamma,b\beta} & \tilde{E}_c + \hbar\omega_\gamma + x_{c\gamma,c\gamma} & W_{a,\tilde{c}\gamma} & y_{c\gamma,b\beta} & y_{c\gamma,c\gamma} \\
\hline
\mathbf{\Delta_a} & W_{a,\tilde{b}\beta} & W_{a,\tilde{c}\gamma} & -\mathbf{E_a} & -V_{a,\tilde{b}\beta} & -V_{a,\tilde{c}\gamma} \\
W_{a,\tilde{b}\beta} & y_{b\beta,b\beta} & y_{c\gamma,b\beta} & -V_{a,\tilde{b}\beta} & -\tilde{E}_b - \hbar\omega_\beta - x_{b\beta,b\beta} & -x_{c\gamma,b\beta} \\
W_{a,\tilde{c}\gamma} & y_{c\gamma,b\beta} & y_{c\gamma,c\gamma} & -V_{a,\tilde{c}\gamma} & -x_{c\gamma,b\beta} & -\tilde{E}_c - \hbar\omega_\gamma - x_{c\gamma,c\gamma}
\end{array}\right)
\left(\begin{array}{c}
\mathbf{u_a} \\ C_{a,b\beta} \\ C_{a,c\gamma} \\ \hline \mathbf{v_a} \\ D_{a,b\beta} \\ D_{a,c\gamma}
\end{array}\right)
= \tilde{E}_a
\left(\begin{array}{c}
\mathbf{u_a} \\ C_{a,b\beta} \\ C_{a,c\gamma} \\ \hline \mathbf{v_a} \\ D_{a,b\beta} \\ D_{a,c\gamma}
\end{array}\right)
$$

V(abβ)=



$$V_{a,\tilde{b}\beta} = -1/\sqrt{2j_a+1}\sqrt{\hbar\omega_\beta/2C_\beta} < b||Y_{L\beta}||a >< b|R_0\partial U/\partial r|a > (u_a^o u_b - v_a^o v_b)$$

W(abβ)=



$$W_{a,\tilde{b}\beta} = -1/\sqrt{2j_a+1}\sqrt{\hbar\omega_\beta/2C_\beta} < b||Y_{L\beta}||a >< b|R_0\partial U/\partial r|a > (u_a^o v_b + v_a^o u_b)$$

Coupling between configurations **b**β, cγ .....

→**leads to vertex corrections**

$$x_{c\gamma,b\beta} = \sum_d V_{\tilde{c},\tilde{d}\beta} \frac{1}{\tilde{E}_b/2 + \tilde{E}_c/2 - \tilde{E}_d - \hbar\omega_\beta/2 - \hbar\omega_\gamma/2} V_{\tilde{b},\tilde{d}\gamma} + \sum_d W_{\tilde{c},\tilde{d}\beta} \frac{1}{\tilde{E}_b/2 + \tilde{E}_c/2 - \tilde{E}_d - \hbar\omega_\beta/2 - \hbar\omega_\gamma/2} W_{\tilde{b},\tilde{d}\gamma}$$

$$y_{c\gamma,b\beta} = \sum_d W_{\tilde{c},\tilde{d}\beta} \frac{1}{\tilde{E}_b/2 + \tilde{E}_c/2 - \tilde{E}_d - \hbar\omega_\beta/2 - \hbar\omega_\gamma/2} V_{\tilde{b},\tilde{d}\gamma} + \sum_d V_{\tilde{c},\tilde{d}\beta} \frac{1}{\tilde{E}_b/2 + \tilde{E}_c/2 - \tilde{E}_d - \hbar\omega_\beta/2 - \hbar\omega_\gamma/2} W_{\tilde{b},\tilde{d}\gamma}$$

V. Van der Sluys et al., Nucl. Phys. A551(1993) 201

Projecting on the single-particle configuration,we obtain an equation for the Normal and abnormal energy-dependent self-energies:

$$\left[\begin{pmatrix} E_j^0 & \bar{\Delta}_j \\ \bar{\Delta}_j & -E_j^0 \end{pmatrix} + \begin{pmatrix} \Sigma_{11}(E_j) & \Sigma_{12}(E_j) \\ \Sigma_{12}(E_j) & \Sigma_{22}(E_j) \end{pmatrix}\right] \begin{pmatrix} x_j \\ y_j \end{pmatrix} = E_j \begin{pmatrix} x_j \\ y_j \end{pmatrix}$$

Fragmentation
$x_j^2+y_j^2<1$

$$\Sigma_{11}(E_j) = \sum_{j'\lambda'} \left[\frac{V^2_{j,j'\lambda'}}{E_j - (E_{j'} + \hbar\omega'_\lambda)} + \frac{W^2_{j,j'\lambda'}}{E_j + (E_{j'} + \hbar\omega'_\lambda)} \right]$$

$$\Sigma_{11}(E_j) = -\Sigma_{22}(-E_j)$$

$$\Sigma_{12}(E_j) = -\sum_{j'\lambda'} V_{j,j'\lambda'} W_{j,j'\lambda'} \left[\frac{1}{E_j - (E_{j'} + \hbar\omega'_\lambda)} - \frac{1}{E_j + (E_{j'} + \hbar\omega'\lambda)} \right]$$

$$V_{j,j'\lambda'} = h(jj'\lambda')(u_j^0 u_{j'} - v_j^0 v_{j'})$$

$$u_j = u_j^0 x_j + v_j^0 y_j \qquad v_j = v_j^0 x_j - u_j^0 y_j$$

$$W_{j,j'\lambda'} = h(jj'\lambda')(u_j^0 v_{j'} + v_j^0 u_{j'})$$

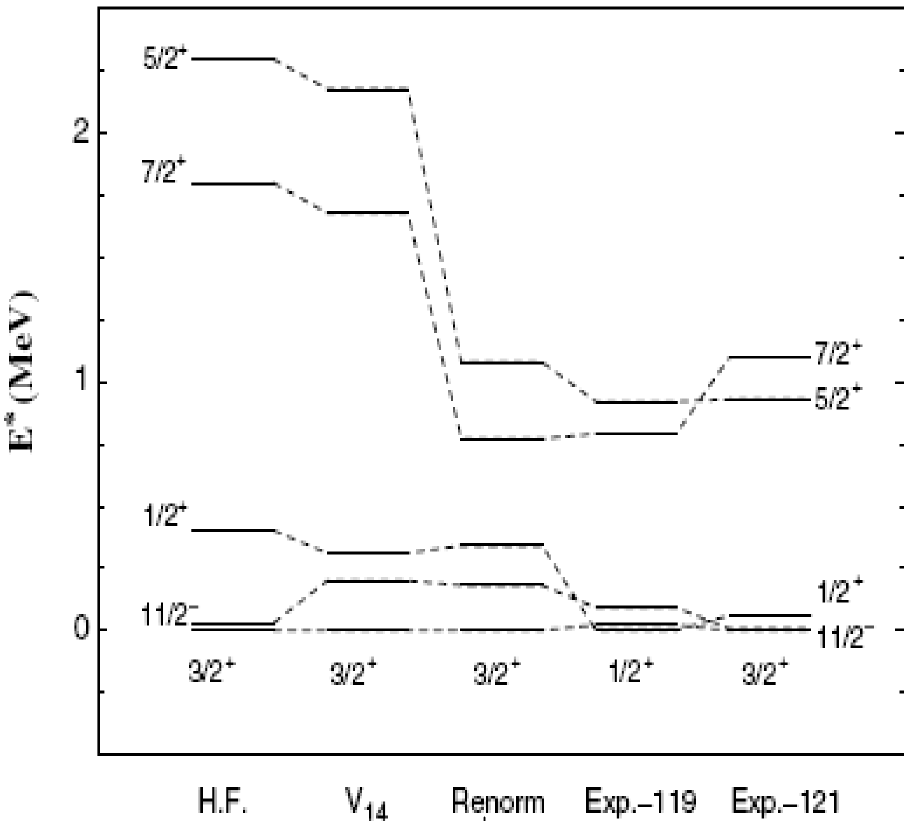
$\bar{\Delta}_j$=2(E_j) $u_j v_j$/(u_j^ 2+v_j^ 2)

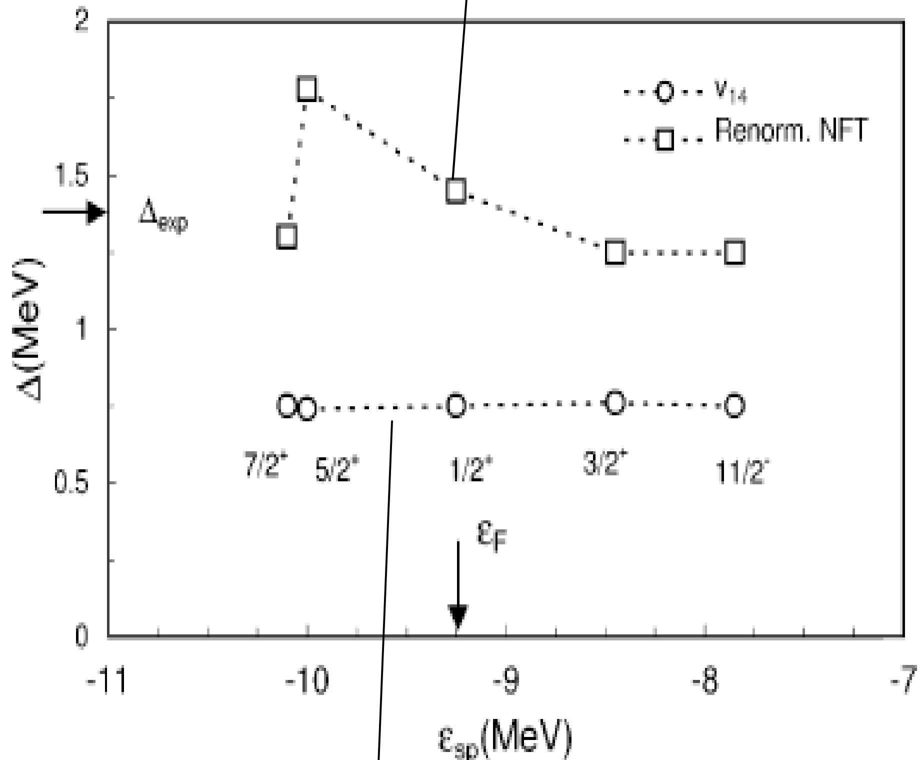
Renormalization of quasiparticles

120Sn

Renormalization of pairing gap

Argonne +
induced interaction





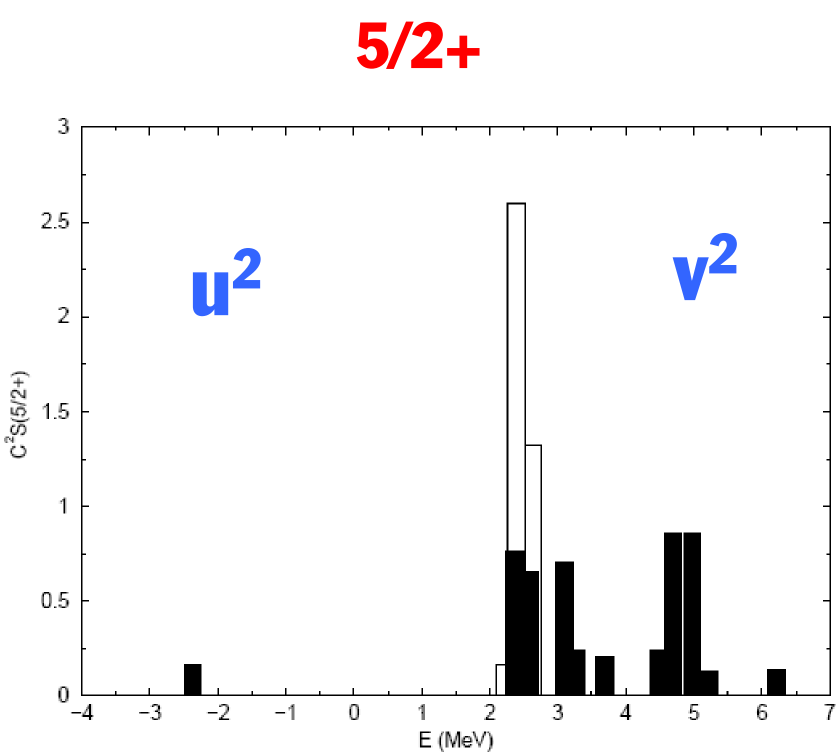
Bare Argonne force

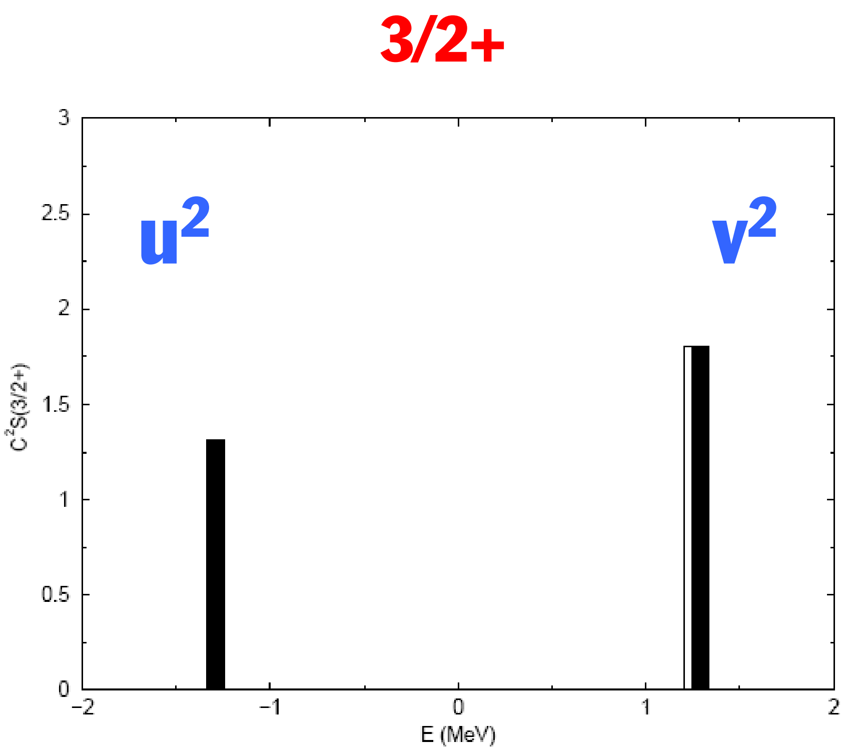
Argonne

Argonne + induced interaction

F. Barranco et al., EPJA21(2004)57

Fragmentation of quasi particle strength: comparison with spectroscopic factors from transfer reactions





A few selected questions:

- Pairing gaps obtained with bare, induced and effective interactions
- Dependence on the adopted mean field
- Role of spin modes
- Connection with infinite matter (neutron stars)
- How to calculate the phonons

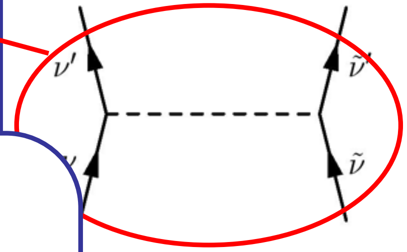
A simpler approach to calculate the pairing gap Δ due to the induced interaction

BCS equations

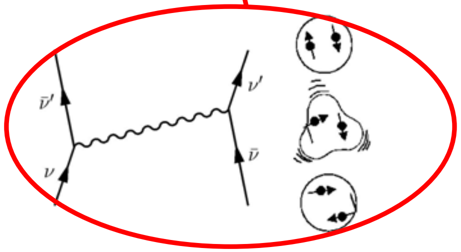
A) $$\Delta_k = -\frac{1}{2}\sum_{k'>0} V_{k\bar{k}k'\bar{k}'} \frac{\Delta_{k'}}{\sqrt{\varepsilon_{k'}^2 + \Delta_{k'}^2}}$$ gap Equation

B) $$\sum_{k>0}\left(1 - \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + \Delta_k^2}}\right) = N_0$$ Number Equation

One can include the matrix element of a 'bare' interaction



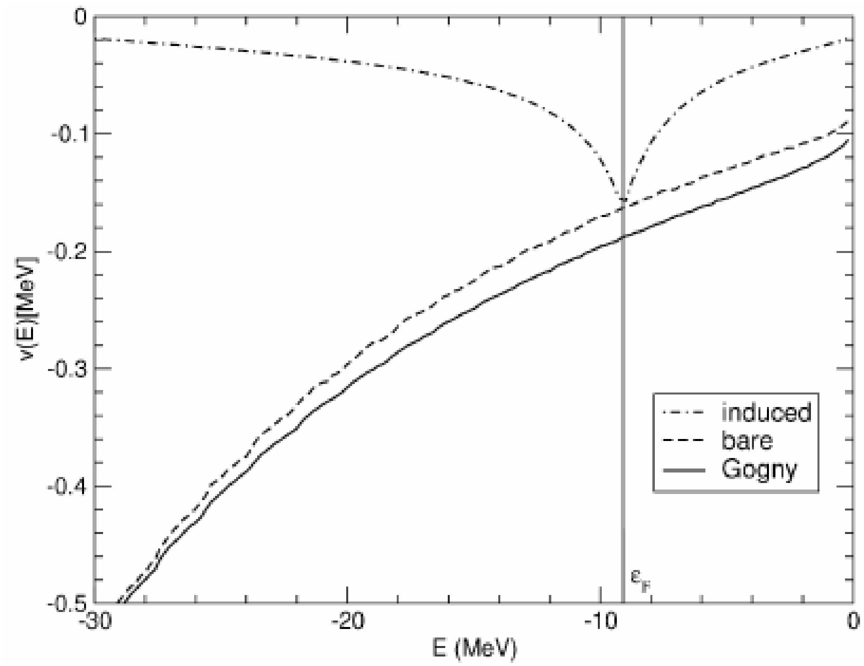
… and the matrix elements of the **interaction induced by phonons**

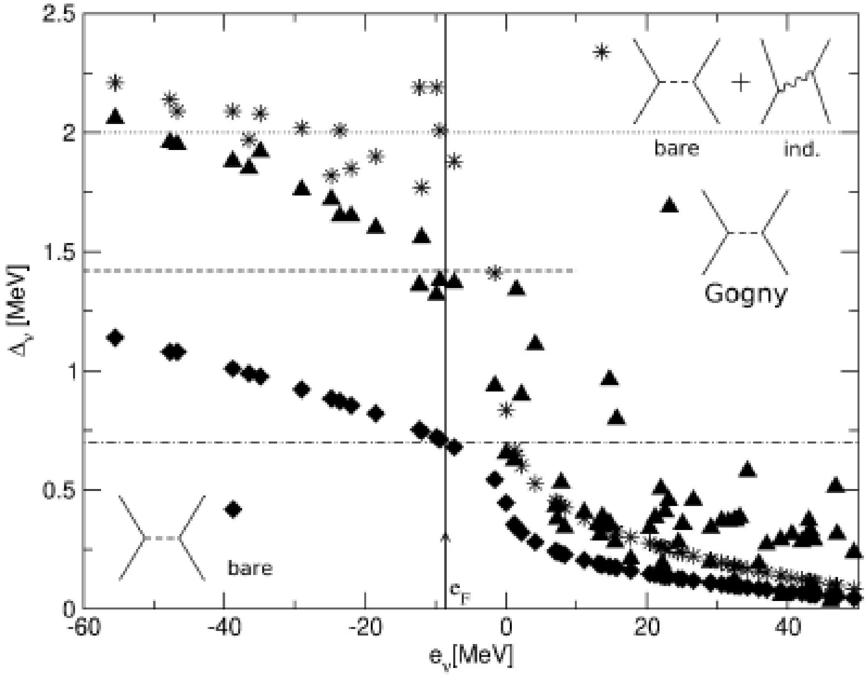


No explicit treatment of self-energy contribution:

single-particle levels result from a Hartree-Fock calculation.

Pairing matrix elements and gaps of V_{low-k}, Gogny and induced interaction





$$v_{\rm ind}(E) = 2\sum_{nLM} c(E)\frac{\int d^3r_1 d^3r_2 f_{Ln}(r_1)Y_{LM}(\hat{r}_1) f_{Ln}(r_2)Y^*_{LM}(\hat{r}_2) j_0^2[k_E(R)s][E-U(R)]}{E_0 - 2|E-e_F| - \hbar\omega_{nL}},$$

$$v(E) = c(E)\int d^3R[E-U(R)]\theta[E-U(R)] \times v[k_E(R), k_E(R)],$$

F. Barranco, P. Schuck et al., Phys. Rev. C in press

Accuracy of the semiclassical matrix elements

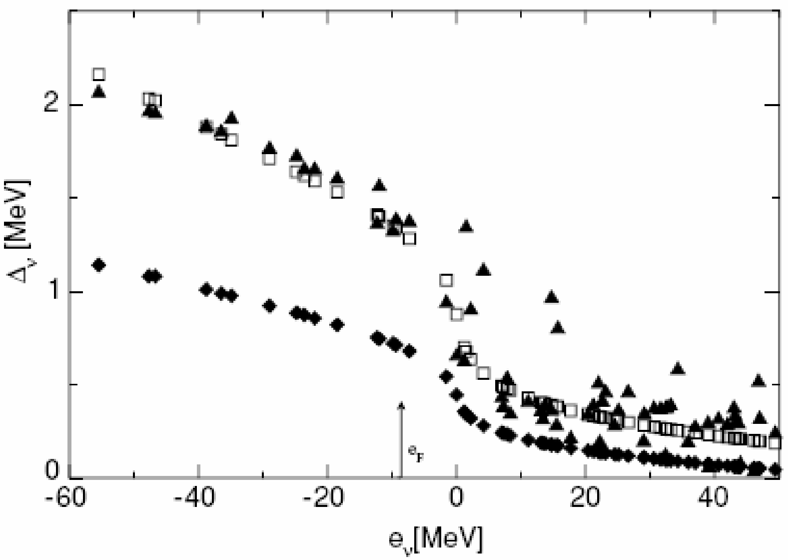
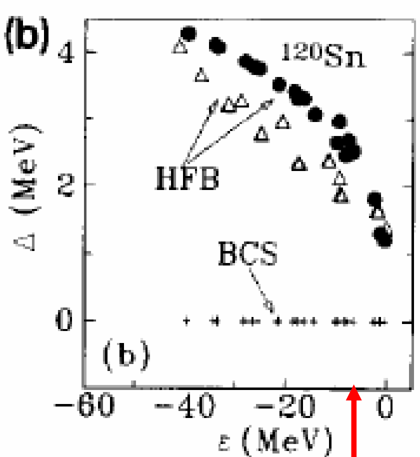


FIG. 8. State-dependent pairing gaps of ^{120}Sn calculated with a Woods-Saxon potential (with depth $V_0 = -64$ MeV, diffusivity $a = 0.65$ fm, and radius $R_0 = 6.17$ fm) as a function of the single-particle energy. The k mass m_k was set equal to $0.7\ m$. The Fermi energy is $e_F = -8.6$ MeV. Solid triangles (open squares) display the results of a HFB calculation with the Gogny interaction, with quantal (semiclassical) matrix elements. The solid diamonds refer instead to a HFB calculation using the semiclassical matrix elements of the $v_{\text{low-}k}$ potential.

$$v(E) = c(E) \int d^3R[E - U(R)]\,\theta[E - U(R)] \times v[k_E(R), k_E(R)],$$

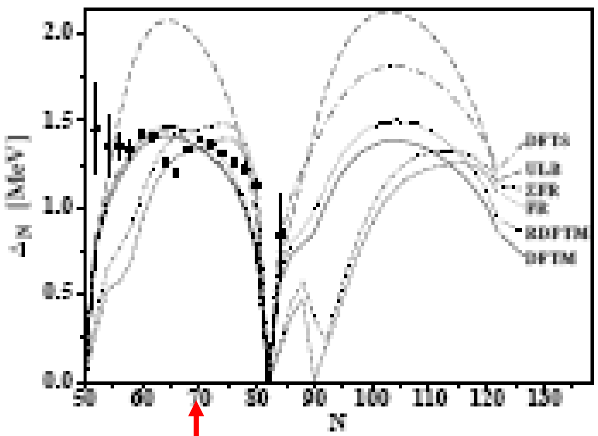
Pairing gaps of bare interactions in finite nuclei

V_{14} WS m*=1



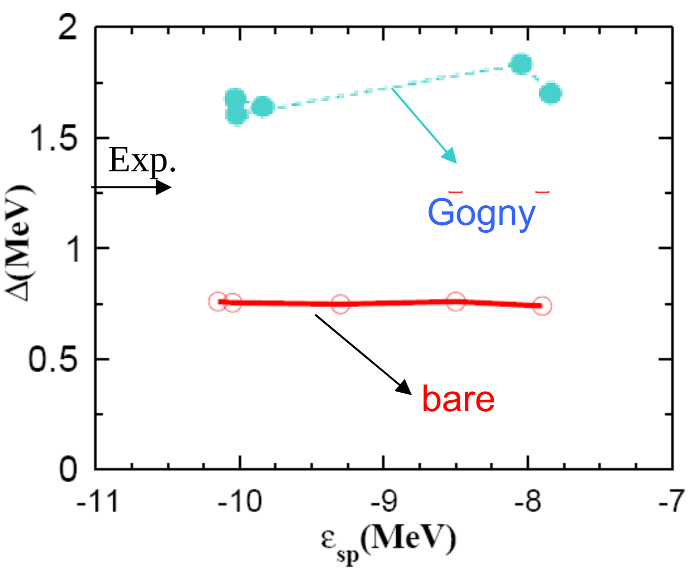
F. Barranco et al.,
PLB390(1997)13

V_{14} Sly5 m*=0.7



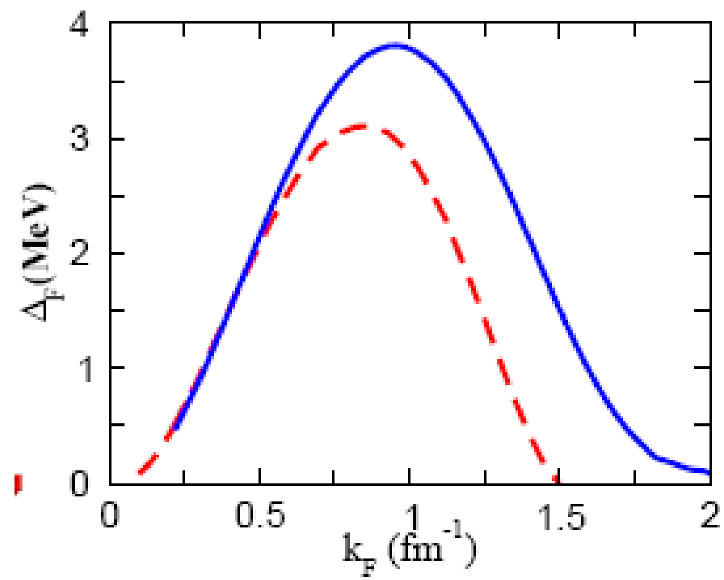
T. Duguet et al.,
nucl-th/050854

PAIRING GAP IN FINITE NUCLEI



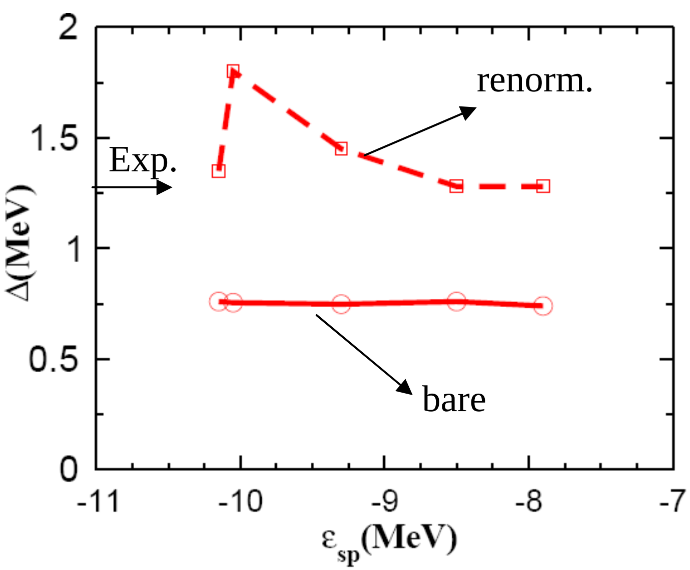
Gogny reproduces the gap (overestimates it slightly) , but the bare interaction yields only half of it.

PAIRING GAP IN NEUTRON MATTER



Gogny and Argonne interactions yield similar pairing gaps up to $k_F \sim 0.7\ fm^{-1}$, but at saturation density the repulsive core of Argonne suppresses the gap

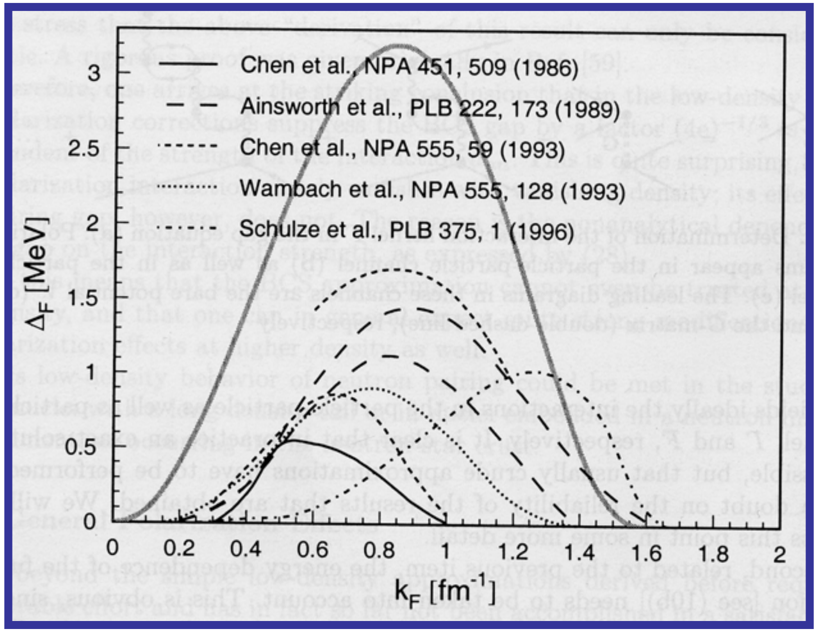
PAIRING GAP IN FINITE NUCLEI



Medium effects **increase** the gap in ^{120}Sn and bring it in agreement with experiment

F. Barranco et al., Eur. J. Phys. A21(2004) 57

PAIRING GAP IN NEUTRON MATTER



Medium effects **decrease** the gap

C. Shen et al., PRC 67(2003) 061302

Microscopic calculation of the matrix elements of the induced interaction in spherical open-shell nuclei including spin modes

Vibrations are calculated with QRPA and SKM* interaction

Particle-hole interaction:

Density channel ($S = 0$)

Skyrme interaction

Spin channel ($S = 1$)

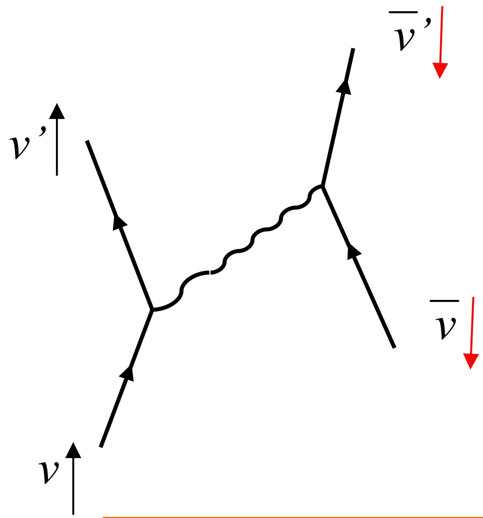
$$V(\vec{r},\vec{r'}) = \left[F_0(r) + F_0'(r)\vec{\tau}_1\cdot\vec{\tau}_2\right]\delta(\vec{r}-\vec{r'}) + \left[G_0(r) + G_0'(r)\ \vec{\tau}_1\cdot\vec{\tau}_2\right]\ \delta(\vec{r}-\vec{r'})\ \vec{\sigma}_1\cdot\vec{\sigma}_2$$

G. Gori et al., Phys. Rev. C72(2005)11302

Microscopic calculation of the matrix elements of the induced interaction

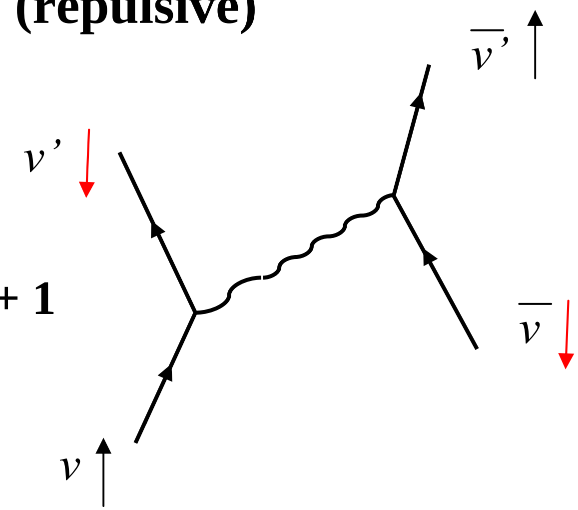
S=0 (attractive)

$S_z = 0$



S=1 (repulsive)

$S_z = +1$



$$2 \sum_{J^\pi M i} \frac{(f^2)^{\nu' m'}_{\nu m; J^\pi M i} - (g^2)^{\nu' m'}_{\nu m; J^\pi M i}}{E_0 - E_{int}}$$

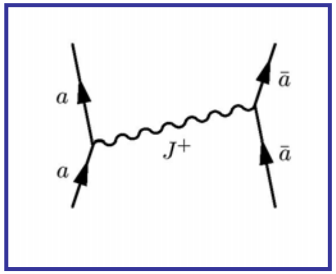
$$f^{\nu' m'}_{\nu m; J^\pi M i} = i^{l-l'} \langle j' m' | (i)^J Y_{JM} | jm \rangle \times$$
$$\int dr\, \varphi_{\nu'} \left[(F_0 + F_0')\, \delta\rho^i_{J^\pi n} + (F_0 - F_0')\, \delta\rho^i_{J^\pi p} \right] \varphi_\nu$$

$$g^{\nu' m'}_{\nu m J^\pi M i} = \sum_{L=J-1}^{J+1} i^{l-l'} \langle j' m' | (i)^L [Y_L \times \sigma]_{JM} | jm \rangle \times$$
$$\int dr\, \varphi_{\nu'} \left[(G_0 + G_0')\delta\rho^i_{J^\pi L n} + (G_0 - G_0')\, \delta\rho^i_{J^\pi L p} \right] \varphi_\nu$$

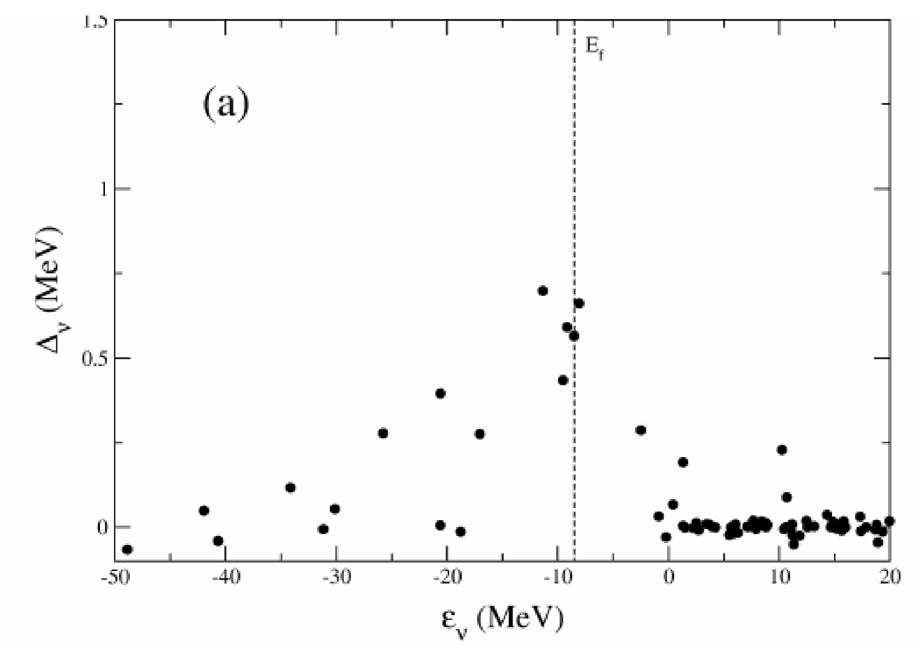
FINITE NUCLEI (^{120}Sn):

The induced interaction arising from the coupling to surface and spin modes is attractive and leads to a pairing gap of about 0.7 MeV (50 % of the experimental value). Excluding the coupling to spin modes, the gap increases to about 1.1 MeV.

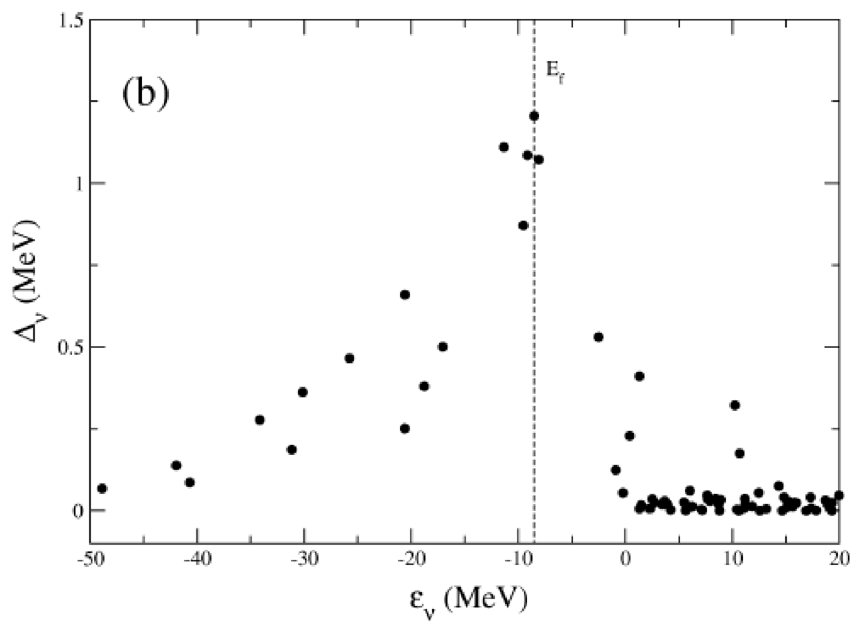
One must then add the bare interaction.



Surface + Spin modes



Surface modes only

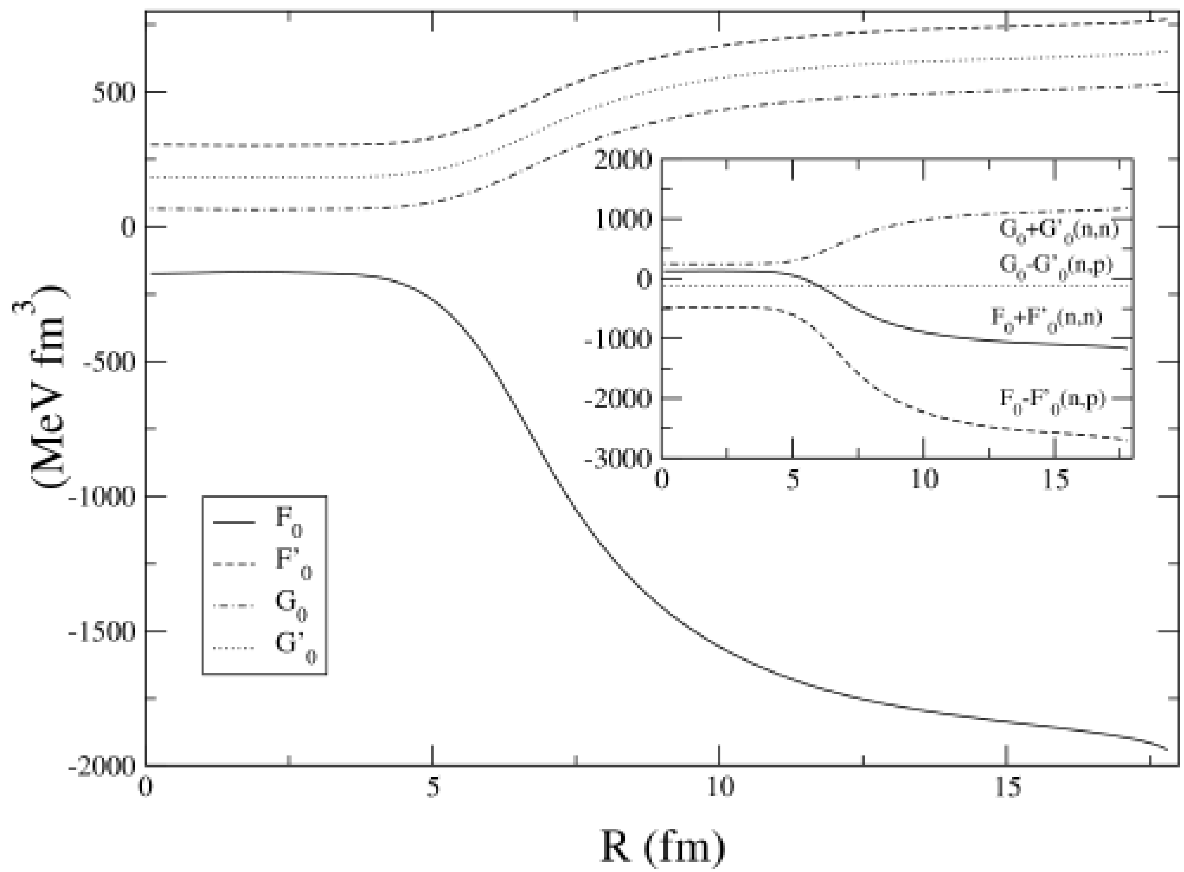


Why such a difference with neutron matter?

The proton-neutron interaction in the particle vibration coupling plays an essential role. If we cancel it, a net repulsive effect is obtained for the induced interaction.

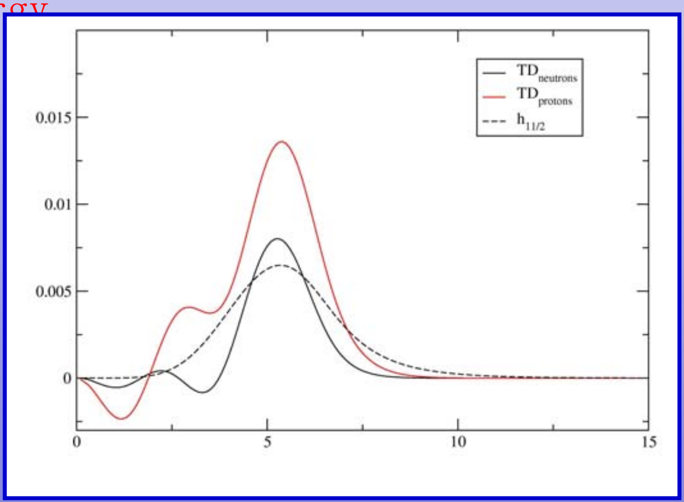
Strong difference between induced interaction in neutron and nuclear matter

Landau parameters of SkM* force in ^{120}Sn



Why such a difference with neutron matter?

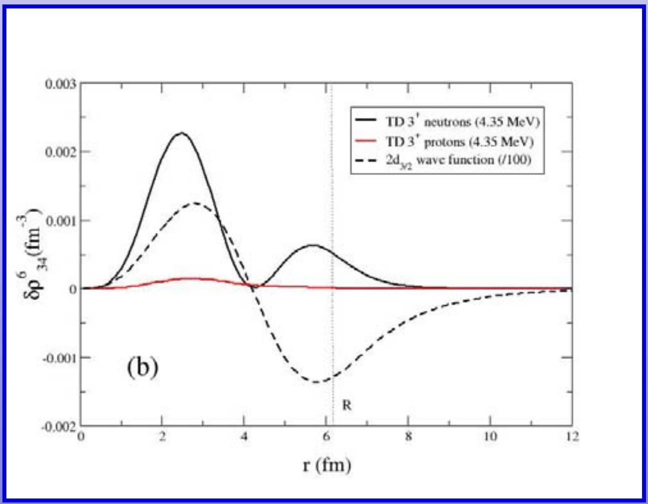
Crucial: the surface nature of density modes. This assures an important overlap between the transition density and the single-particle wave-function at the Fermi energy



$$\langle j'm'JM|V_{res}|jm\rangle = (-)^{M+J}\langle j'm'|(i)^J Y_{J-M}|jm\rangle \int \mathrm{d}r\varphi_{j'}(r)\varphi_j(r)\zeta_{JL}(r),$$

$$\zeta_{JL}(r) = [F_0(r)+F_0'(r)]\rho^{(1)\lambda}_{JL_n}(r) + [F_0(r)-F_0'(r)]\rho^{(1)\lambda}_{JL_p}(r).$$

Volume nature of Spin-modes



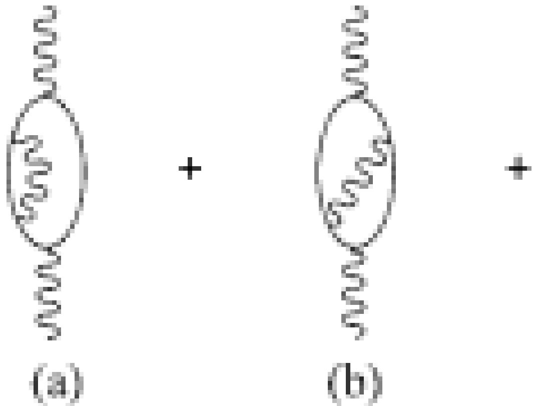
$$\langle j'm'JM|V_{res}|jm\rangle = (-)^{J+M+1}\sum_{L=J-1}^{J+1}\langle j'm'|(i)^L[Y_L\times\sigma]_{J-M}|jm\rangle \int \mathrm{d}r\varphi_{j'}(r)\varphi_j(r)\xi_{JL}(r),$$

$$\xi_{JL}(r) = [G_0(r)+G_0'(r)]\rho^{(1)\lambda}_{JL_n}(r) + [G_0(r)-G_0'(r)]\rho^{(1)\lambda}_{JL_p}(r).$$

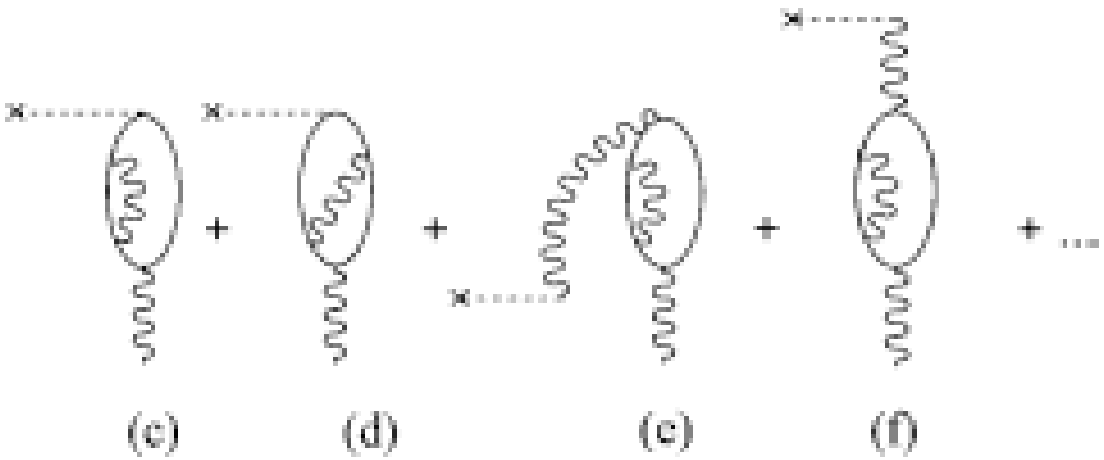
Renormalization of the properties of 2+ low-lying state due to mixing with more complex configurations

	$\hbar\omega_{2^+}$ (MeV)	$B(E2\uparrow)$ (e^2 fm^4)
RPA (Gogny)	2.9	660
RPA (Sly4)	1.5	890
RPA + renorm. [23]	0.9	2150
Exp.	1.2	2030

Energy correction



Transition amplitude correction



Can ground state correlations improve mean-field calculation of binding energies?

Mean field (MF)

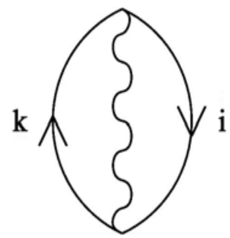
S. Goriely et al., At. Data 77(2001)311

- HF-BCS approximation
- Skyrme interaction MSk7 (**rmsd = 0.754 MeV**)
- *pp* channel:
  - T = 1 channel
  - δ-pairing force
  - energy cutoff at $\hbar\omega = 41 \cdot A^{-1/3}$

MF mass formula shows the largest deviations from experiment at shell closures

Surface vibrations

- (Q)RPA
- Skyrme interaction MSk7 with δ-pairing force
- 2^+ and 3^- multipolarities
- states with $\hbar\omega$ < 7 MeV and with B(Eλ) ≥ 2%



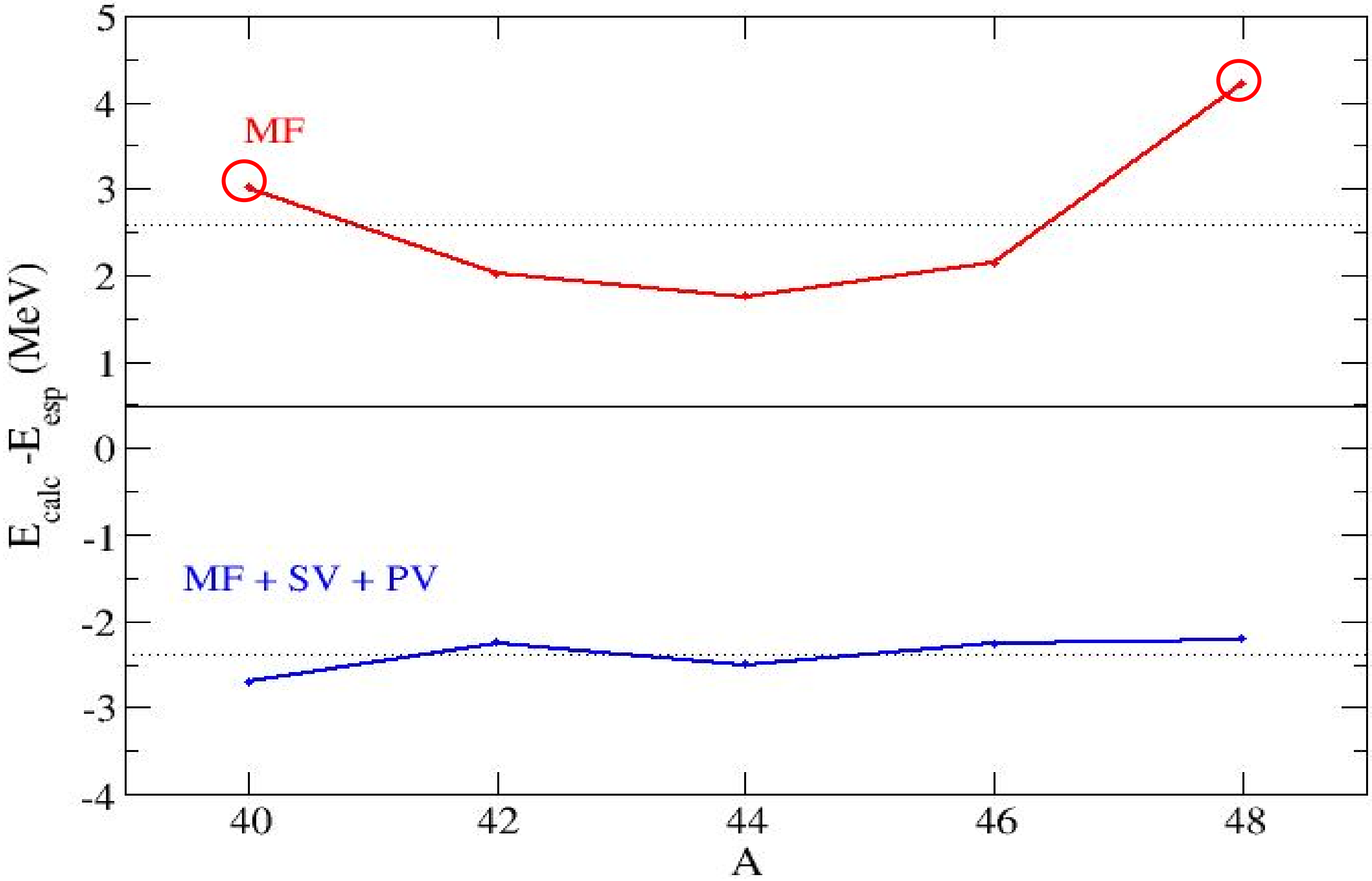
$$\leftrightarrow \quad -\sum_{\nu} \hbar\omega_{\nu} \sum_{ki} |Y_{ki}^{\nu}|^2$$

Pairing vibrations

- RPA
- on shell closures only
- separable interaction with constant matrix elements ≡ G
- 0^+, 2^+ multipolarities
- G calculated in double closed shell nuclei

Results

Calcium *Z = 20*



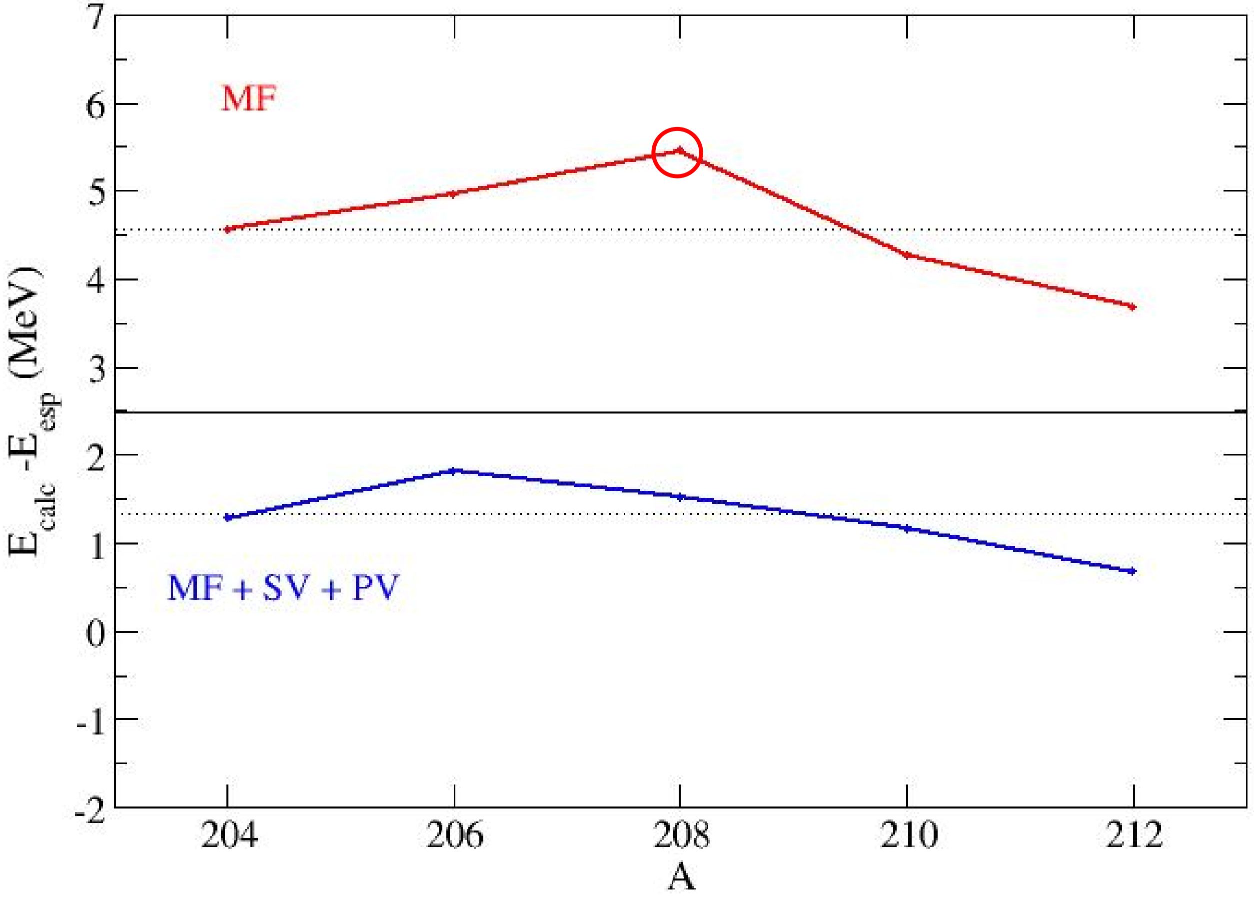
rmsd

0.899 MeV

0.187 MeV

Results

Lead ***Z = 82***



rmsd

0.602 MeV

0.381 MeV

Results

- ***clear reduction of rms errors in closed shell nuclei***

(all data in MeV)

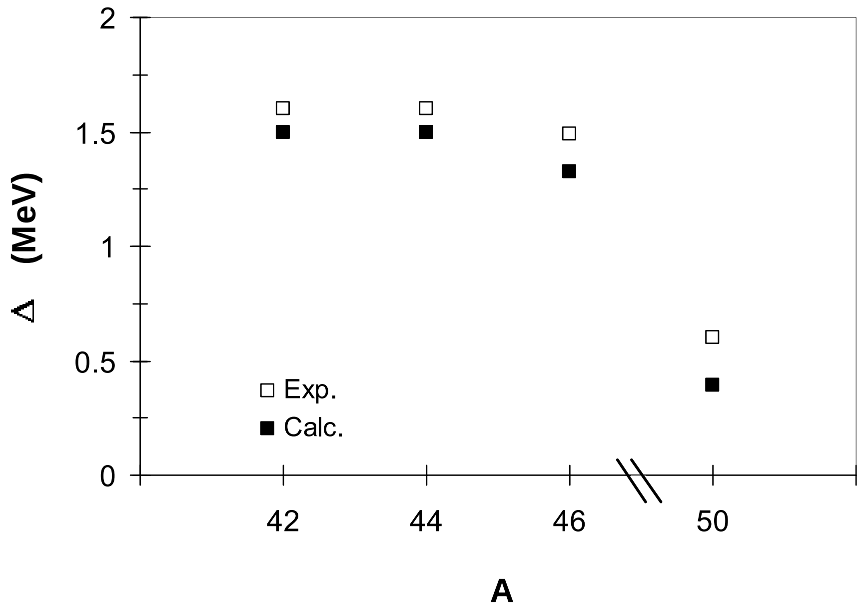
	# isotopes	MF	MF + SV + PV
closed shell nuclei	10	**1.440**	**0.554 (-62%)**
computed spherical nuclei	125	**0.562**	**0.486 (-14%)**

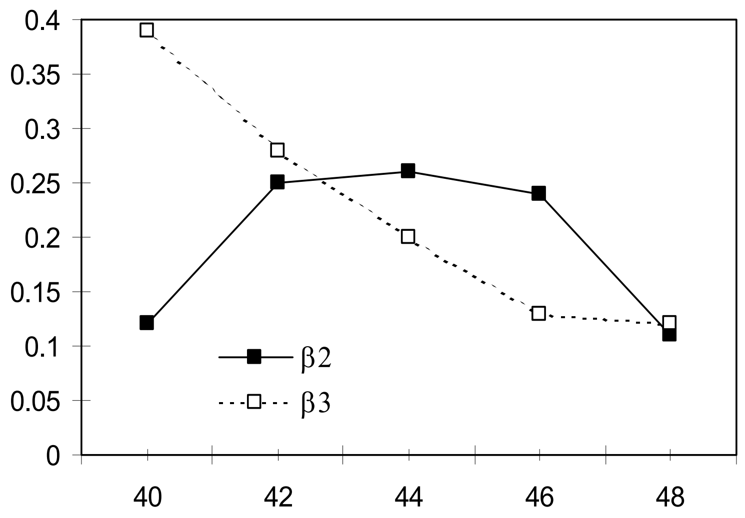
	MF	MF+SV+PV
^{40}Ca	**0.380**	**-0.313**
^{48}Ca	**1.589**	**0.173**
^{208}Pb	**0.868**	**0.226**
^{132}Sn	**3.575**	**0.266**
^{16}O	**0.873**	**0.139**
^{210}Po	**0.591**	**0.478**
^{42}Ti	**0.331**	**0.241**
^{50}Ti	**0.893**	**0.703**
^{38}Ar	**1.271**	**0.098**
^{46}Ar	**0.942**	**1.417**
total rmsd	**1.440**	**0.554**

Conclusions

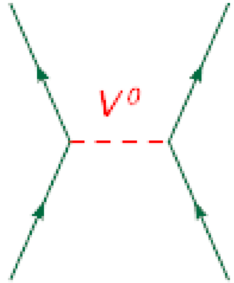
- The relation of our results with analagous approaches in uniform matter can be understood taking into account the nature of surface modes in finite nuclei
- It is possible to include on the same footing medium polarization effects in the particle-hole and in the particle-particle channel, based on a bare interaction in the pairing channel and an appropriate Hartree-Fock mean field, calculating at the same time the quasiparticle spectra and the pairing gaps.
- We have made progress towards a simple parametrization of the induced interaction

Isotopic effects in Calcium



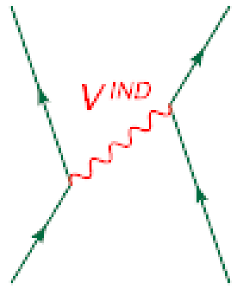


Bare gap



$$\Delta_0(k) = -\sum_{k'} \frac{V^0(k,k')}{2\sqrt{e(k')^2 + \Delta_0(k')^2}} \Delta_0(k')$$

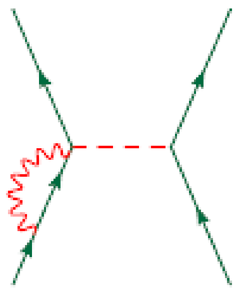
Induced interaction



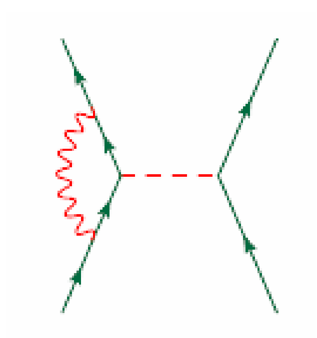
$k_F \to 0$: $\Delta = \Delta_0/(4e)^{1/3} \approx \Delta_0/2.2$

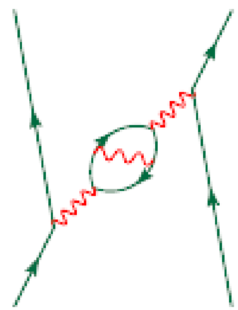
(neutron matter)

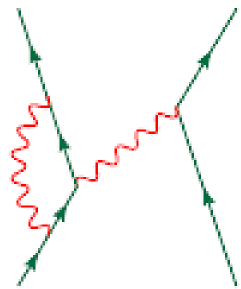
Self-energy

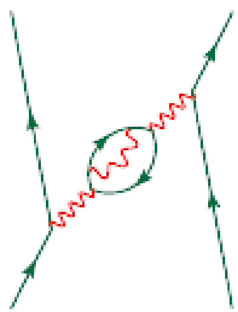


$$\Delta(k) = -\sum_{k'} \frac{Z(k)\, V^0(k,k')\, Z(k')}{2\sqrt{e^2(k') + \Delta(k')^2}} \cdot \Delta(k')$$

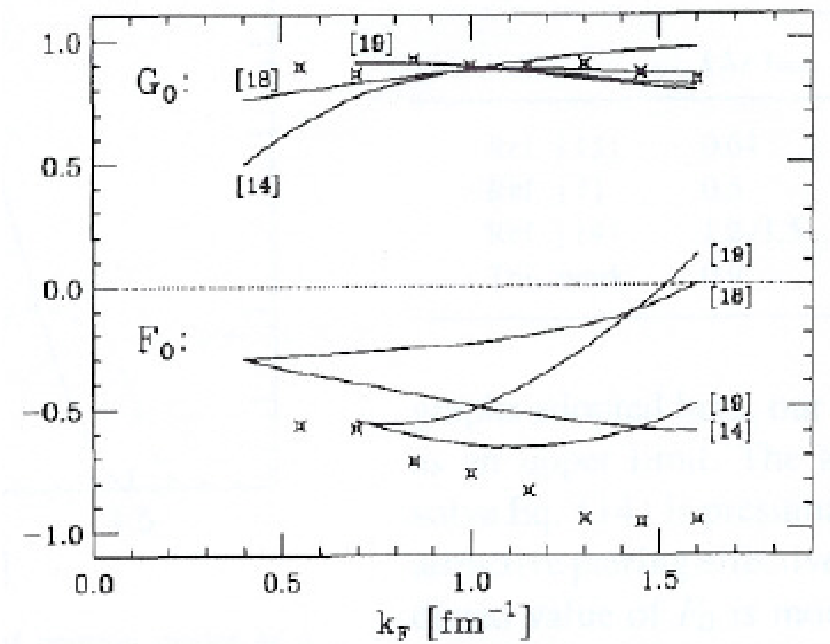




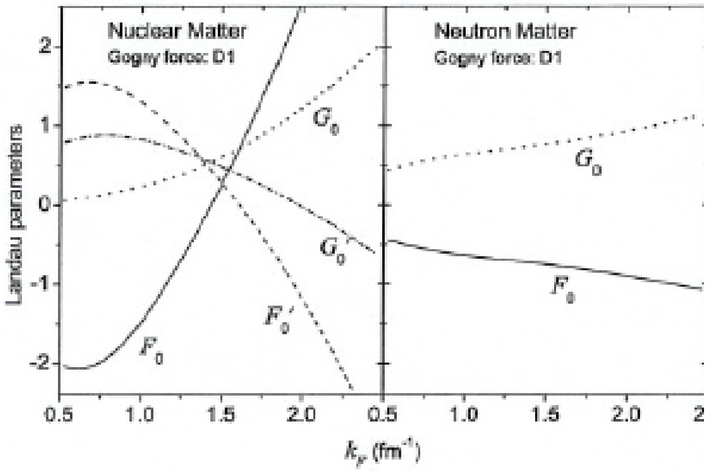




G-matrix

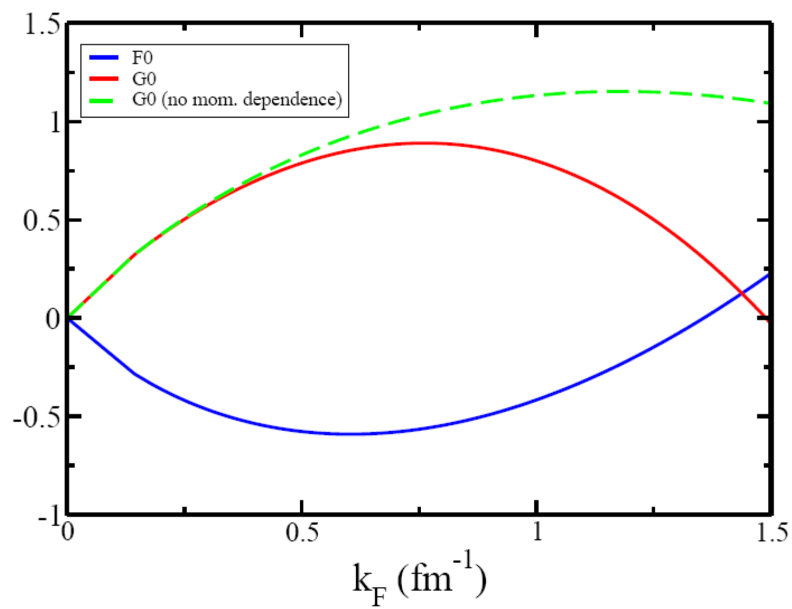


Gogny force

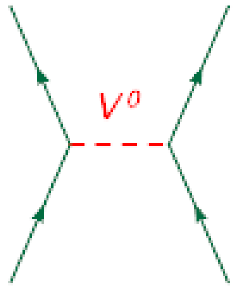


Density dependence of Landau parameters (at k=0)

SkM* force

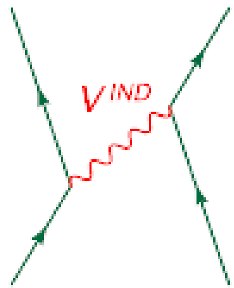


Bare gap



$$\Delta_0(k) = -\sum_{k'} \frac{V^0(k,k')}{2\sqrt{e(k')^2 + \Delta_0(k')^2}} \Delta_0(k'$$

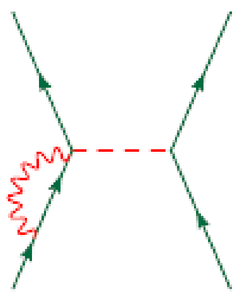
Induced interaction



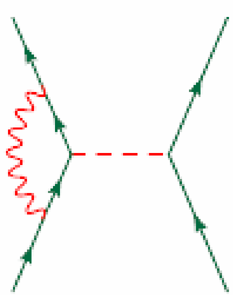
$k_F \rightarrow 0$: $\Delta = \Delta_0/(4e)^{1/3} \approx \Delta_0/2.2$

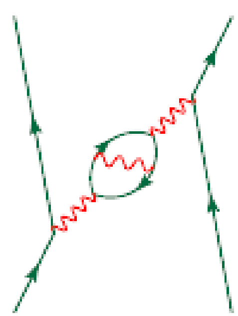
(neutron matter)

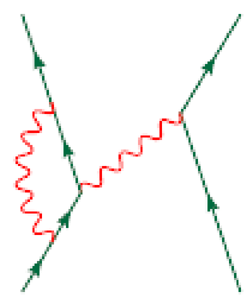
Self-energy

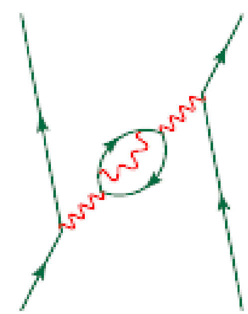


$$\Delta(k) = -\sum_{k'} \frac{Z(k)\, V^0(k,k')\, Z(k')}{2\sqrt{e^2(k') + \Delta(k')^2}} \cdot \Delta(k')$$

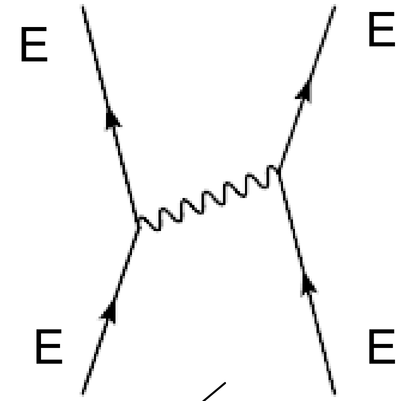


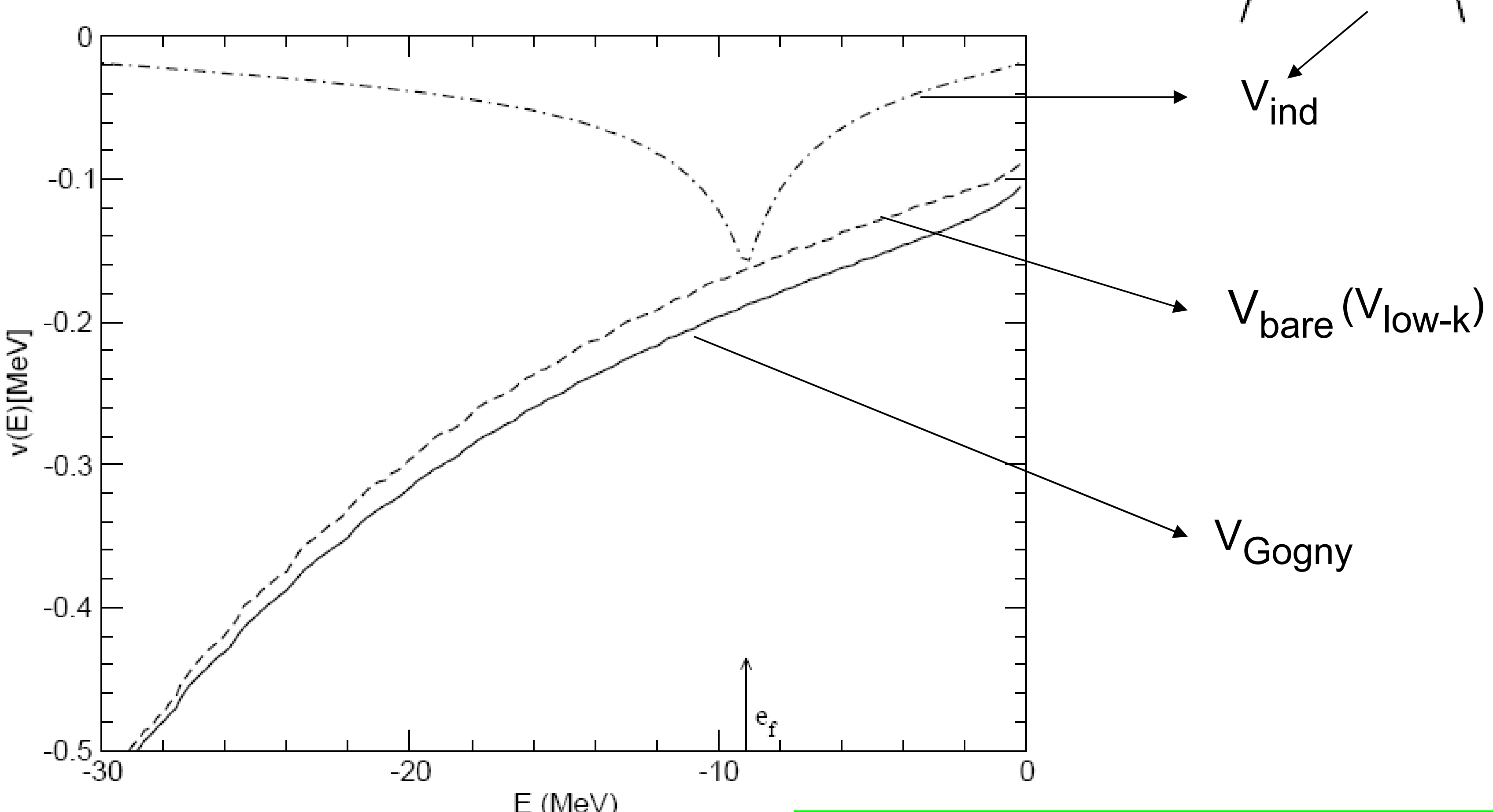






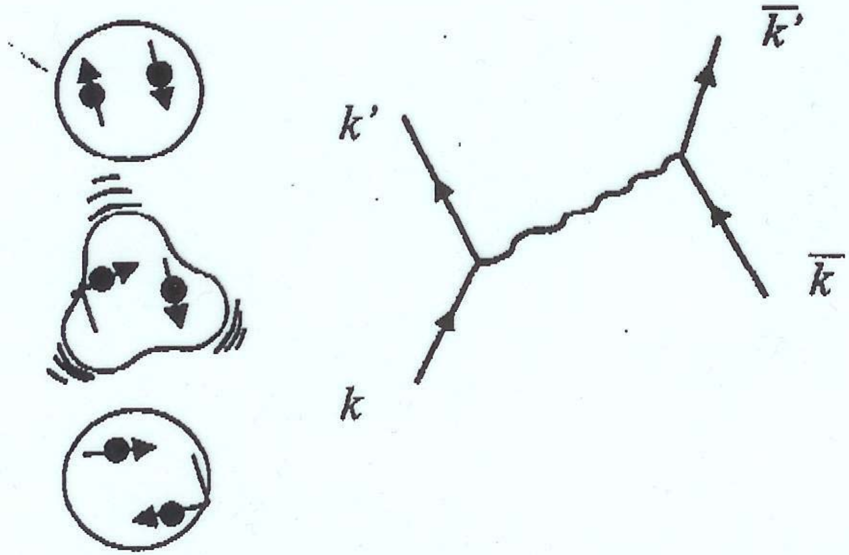
Semiclassical diagonal pairing matrix elements (120Sn)

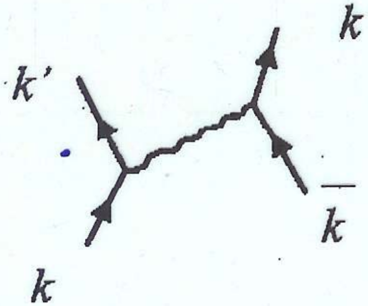




F. Barranco, P. Schuck et al., Phys. Rev. C, in press

Particle-vibration matrix elements are derived
from properties of experimental surface vibrations (2+,3-,4+,5-)





$$= \sum_{\lambda} \frac{2\beta_{\lambda}^{2}}{(2\lambda+1)} \frac{\left|\left\langle k \left\| R_0 \frac{\partial U}{\partial r} Y_{\lambda} \right\| k' \right\rangle\right|^{2}}{\sqrt{2j_k+1}\sqrt{2j_{k'}+1}}$$

$$* \frac{1}{E_0 - \left(\left|\varepsilon_k - \varepsilon_F\right| + \left|\varepsilon_{k'} - \varepsilon_F\right| + \hbar\omega_{\lambda}\right)}$$

Effect of surface vibrations upon single-particle motion → **Nuclear Field Theory**

Vector Space… $\left| \; \right\rangle = \prod_i a_i^+ \left| - \right\rangle_F \otimes \prod_\lambda \beta_\lambda^+ \left| - \right\rangle_B$

fermionic degrees of freedom

bosonic degrees of freedom

Nuclear Hamiltonian…

$$\hat{H} = \hat{H}_F + \hat{H}_B + \delta U$$

with

$$\hat{H}_F = \sum_i \varepsilon_i a_i^+ a_i \qquad \hat{H}_B = \hbar\omega_\lambda \sum_\lambda \left(\hat{\beta}_\lambda^+ \hat{\beta}_\lambda + \frac{1}{2} \right)$$

and $\delta\hat{U} = \sum_{\lambda\mu} \hat{\alpha}_{\lambda\mu} \hat{F}_{\lambda\mu}^+$ *..where..* $\hat{F}_{\lambda\mu} = -R_0 \frac{\partial U}{\partial r} Y_{\lambda\mu}$