

Breakup and the Spectroscopy of Continuum States

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Which Continuum?

- The continuum appears in several ways:
 - Part of expansion of bound states;
 - eg needed in RPA for weakly bound states
 - Dominated by resonances;
 - These 'unbound states' identified eg with shell model eigenstates above threshold
 - In non-resonant continuum;
 - eg in breakup reactions, or low-energy capture.
- **ALL important parts of nuclear structure!!**

'Overlap' Challenges

- Reaction models need few-body degrees of freedom in structure models.
 - Solve a few-body model directly, or
 - Extract few-particle dof from microscopic model
 - Difficult for: GFMC,
 - for HFB, QRPA and RMF structure models
 - Do we transfer quasi-particles, or particles?

What a good theory needs:

- **Recoil & Finite Range of projectile vertex.**
- **Final-state (partial wave) interference**
- **Nuclear and Coulomb mechanisms**
- **Core excitation (initial and/or dynamic)**
- **Final-state interactions:**
 - between halo fragments (needed if resonances)
 - between fragments and target (needed if close in)
- **Multistep Processes (higher order effects)**

CDCC: Coupled Discretised Continuum Channels

Try CDCC:

Coupled Discretised Continuum Channels

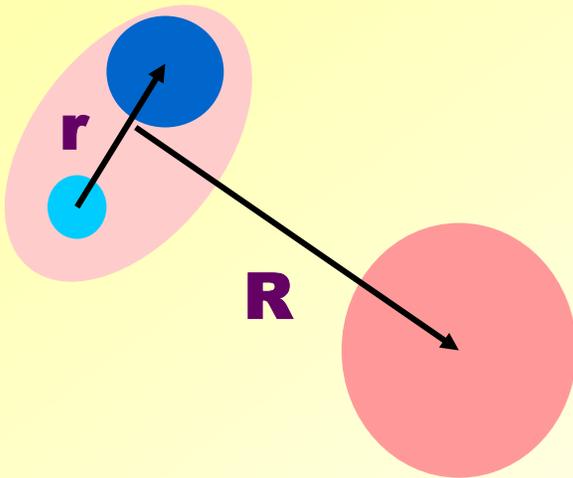
- Proposed by Rawitscher, developed by Kamimura group.
- Treat Coulomb and Nuclear mechanisms
 - Need to check convergence of long-range Coulomb process!
- All higher-order effects with a (r,R,L) reaction volume
- Can calculate fragment coincident angular distributions: Predict e.g. $d^3\sigma/dE_1d\Omega_1d\phi_{12}$ and fold with detector apertures & efficiencies

The Hamiltonian for the reaction of a projectile on a target

$$\mathbf{H} = \mathbf{h}_{\text{proj}} + \mathbf{h}_{\text{targ}} + \mathbf{T}_{\alpha} + \mathbf{V}_{\alpha}$$

$$\Rightarrow \mathbf{h}_{\text{proj}} = \mathbf{h}_{\text{core}} + \mathbf{h}_{\text{frag}} + \mathbf{T}_{\text{cf}} + \mathbf{V}_{\text{cf}}$$

$$\Rightarrow \mathbf{V}_{\alpha} = \mathbf{V}_{\text{core-targ}} + \mathbf{V}_{\text{frag-targ}}$$



$$\Psi_{\text{JM}}^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) = [\phi_0(\mathbf{r}) \otimes Y_L(\hat{\mathbf{R}})]_{\text{JM}} \chi_{0,L}^{\text{J}}(\mathbf{R}) + \sum_{l=0}^{l_{\text{max}}} \sum_L \sum_{i=1}^N [\phi_{i,l}(\mathbf{r}) \otimes Y_L(\hat{\mathbf{R}})]_{\text{JM}} \chi_{i,l,L}^{\text{J}}(\mathbf{R})$$

(neglect the internal structure of the target)

CDCC Formalism

The CDCC basis consists of scattering wavefunctions averaged over an energy interval

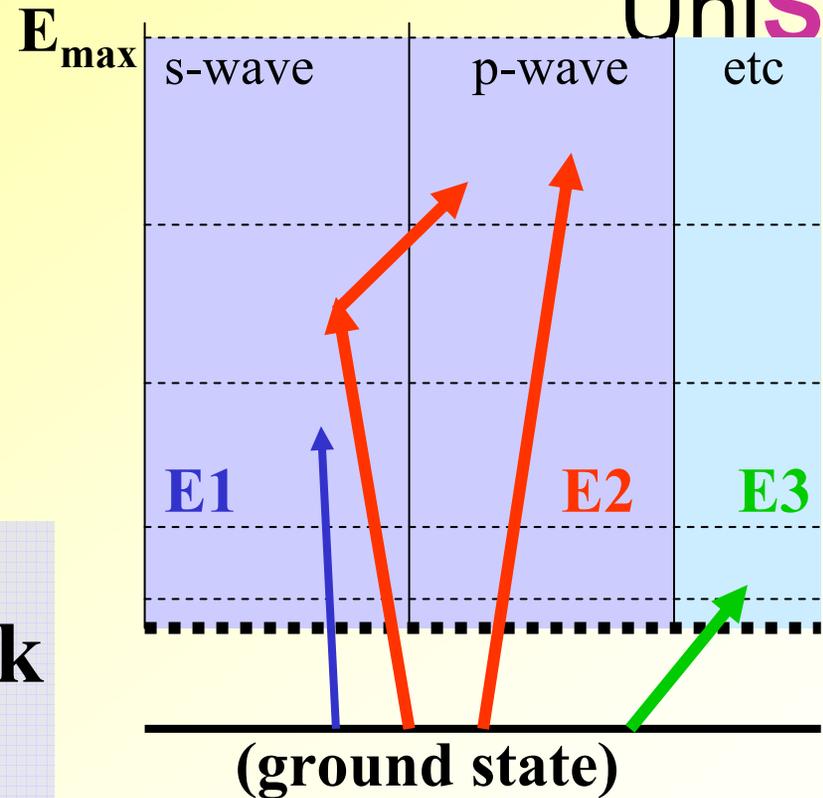
$$\mathbf{h}_{\text{proj}} \phi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} \phi_{\mathbf{k}}$$

$$\phi_{i,l} = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} \mathbf{w}_i(\mathbf{k}) \phi_{lm}(\mathbf{k}, \mathbf{r}) d\mathbf{k}$$

$$N_i = \int_{k_{i-1}}^{k_i} |\mathbf{w}_i(\mathbf{k})|^2 d\mathbf{k}$$

$$N_{\text{bins}} = \frac{k_{\text{max}}}{\Delta k}$$

UniS

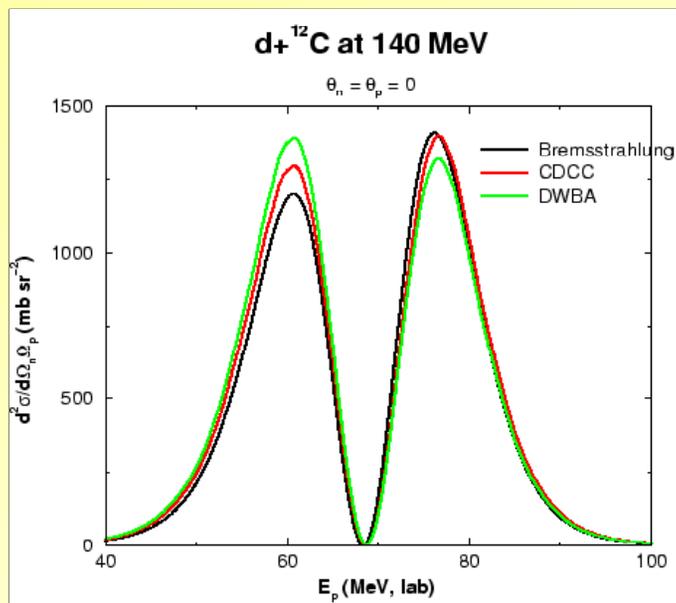


Coupling potentials

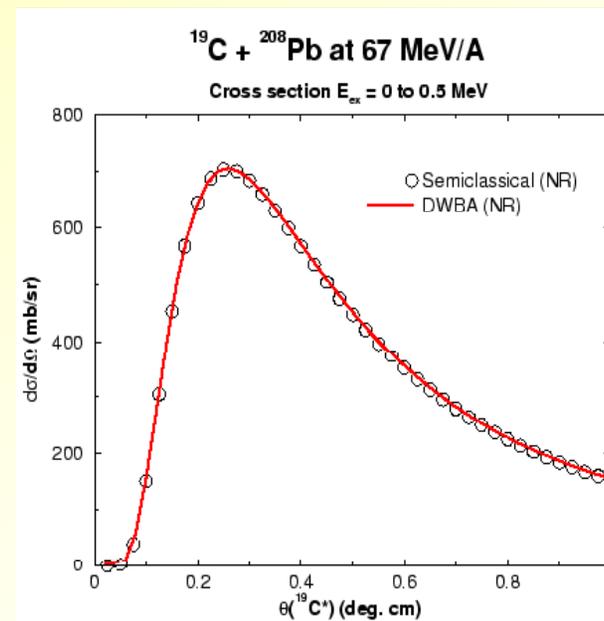
$$V_{il,i'l'}^{CDCC}(R) = \langle \phi_{il}(r) | V_{\alpha}(r, R) | \phi_{i'l'}(r) \rangle$$

Testing CDCC Convergence

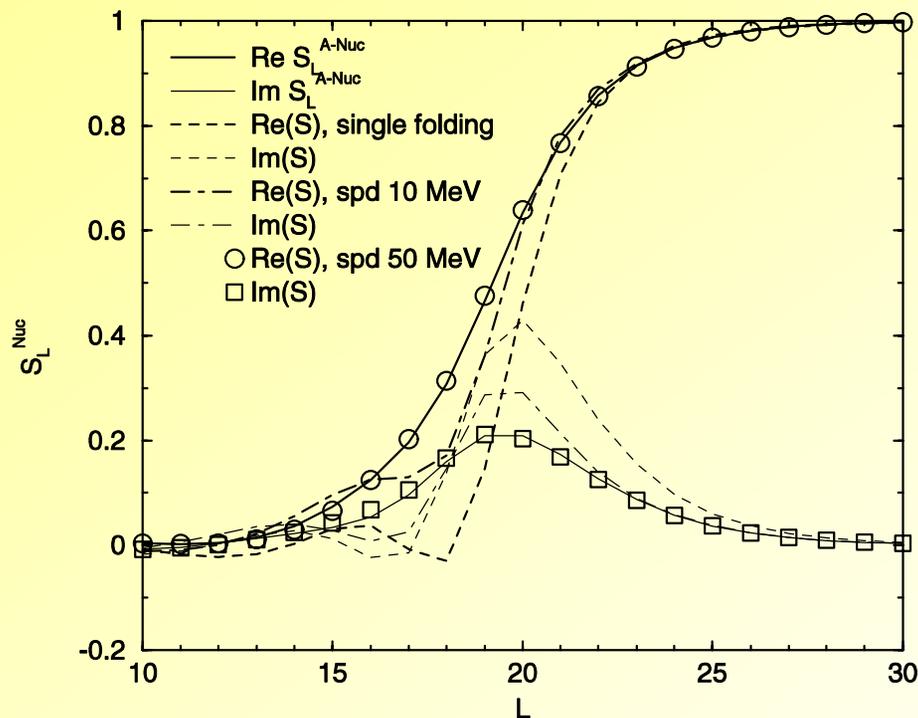
- Compare, in Adiabatic Few-Body Model, with Bremsstrahlung integral
- Compare, in first-order PWBA model, with semiclassical theory



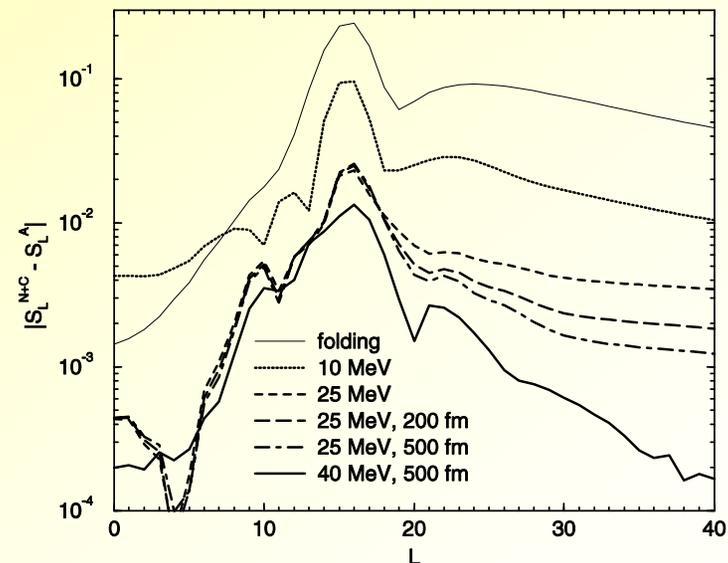
Note the 'post-acceleration'



Adiabatic CDCC: compare with Exact 3-body model



$d+^{208}\text{Pb}$ at 50 MeV, nuclear only



Absolute errors in CDCC for $d+^{208}\text{Pb}$ at 50 MeV, Nuclear+Coulomb

$^{15}\text{C} + ^9\text{Be}$ breakup at 54 MeV/u

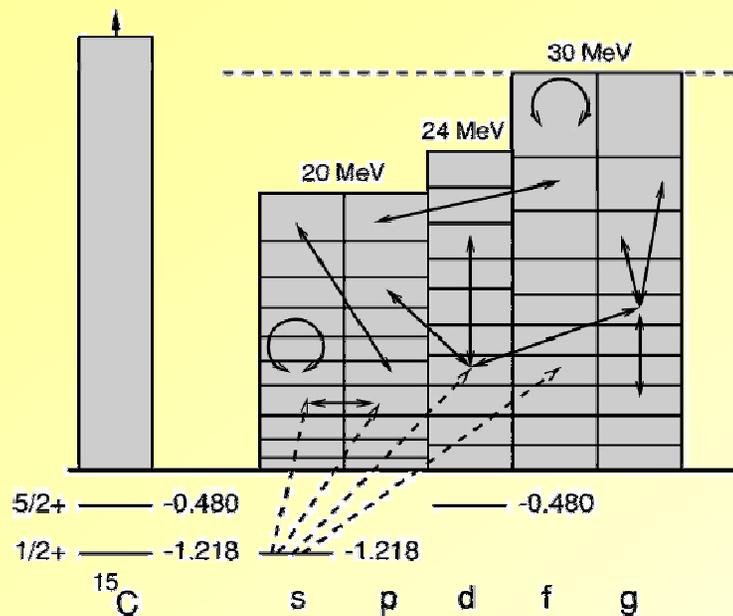


FIG. 4. Diagrammatic representation of the CDCC model space calculation for ^{15}C . The left side shows the physical bound states and continuum and the right hand side the included continuum bins (10) in each $n + ^{14}\text{C}$ partial wave. The dashed arrows are representative of the one-way couplings included in the DWBA. The solid arrows show representative couplings for the full CDCC calculations which connect all bins, including diagonal bin couplings, with two-way couplings to all orders. Relative l waves were found to make negligible contributions.

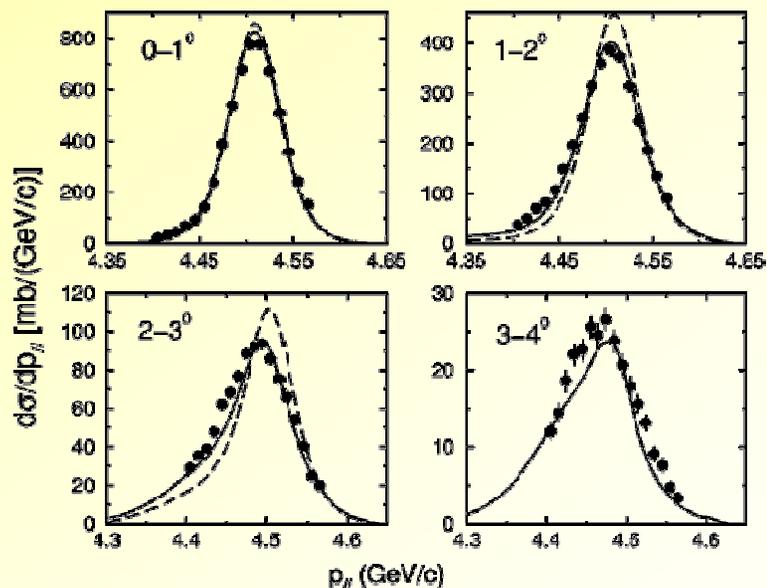
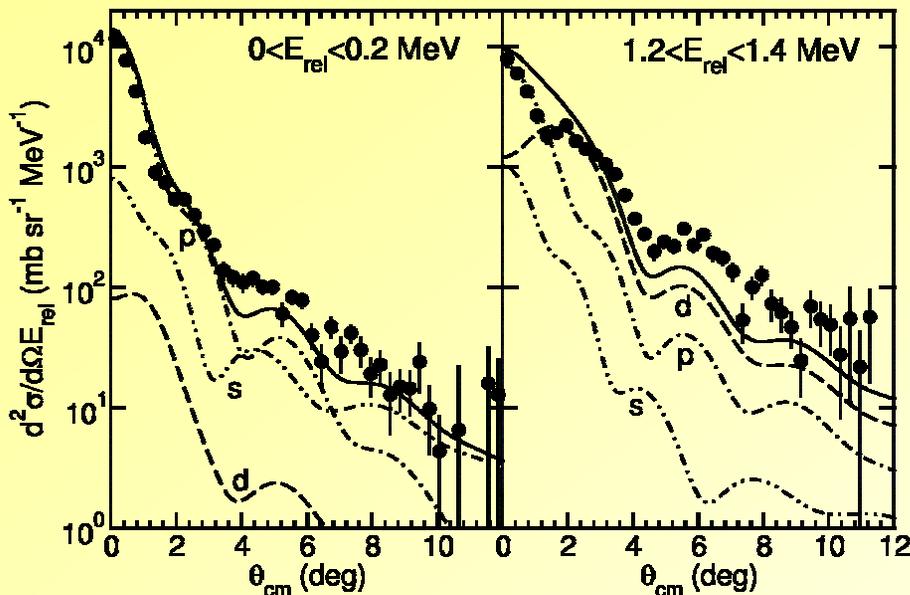


FIG. 11. The nucleon-removal parallel momentum distributions $d\sigma/dp_{\parallel}$, for the $^{15}\text{C} + ^9\text{Be}$ reaction at 54 MeV/nucleon to the ^{14}C ground state, shown on a more familiar linear scale. The solid curves assume the stripping contributions have the same form as that calculated using the CDCC. The dashed curves assume the stripping contributions have a parallel momentum distribution at all angles of the residue given by the eikonal calculation shown in Fig. 2.

Tostevin et al, PRC **66**, 024607 (2002)

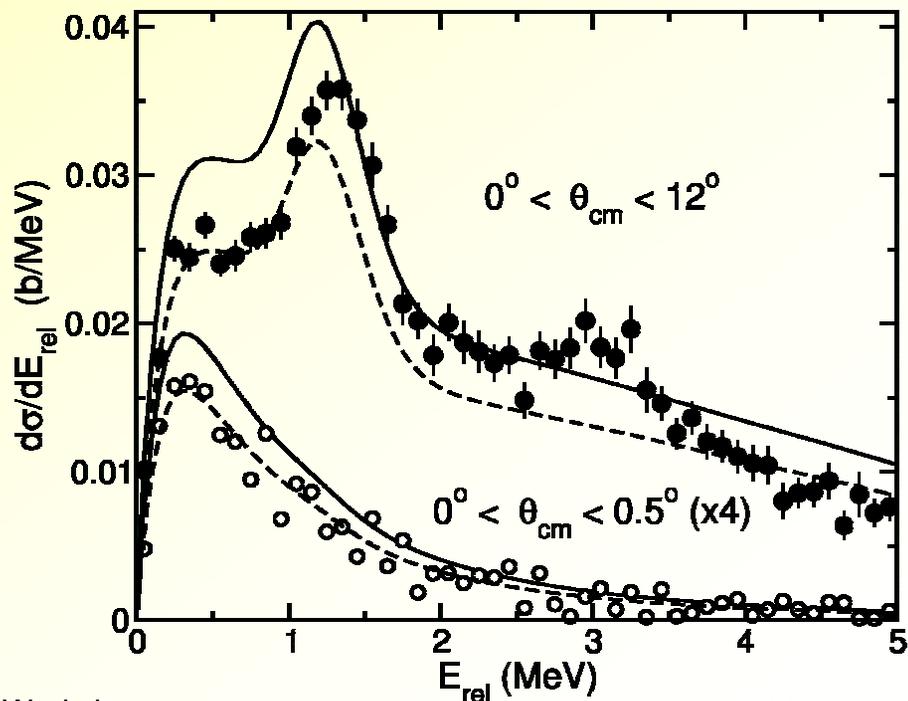
$^{11}\text{Be} + ^{12}\text{C}$ breakup at 67 MeV/u



Angular distributions of $^{11}\text{Be}^*$
left: low-energy continuum
right: region of $d_{5/2}$ resonance

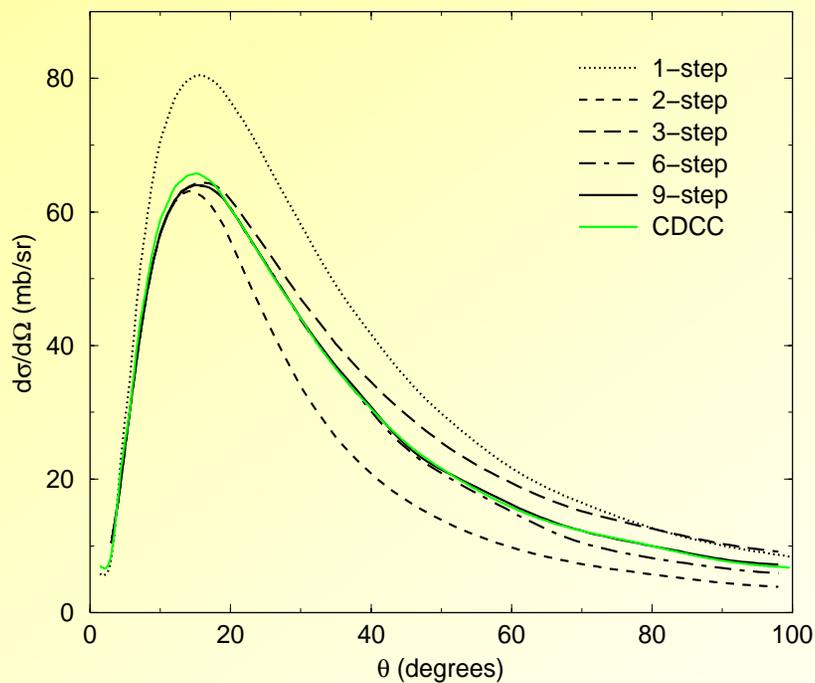
CDCC calculations of
 Howell, Tostevin, Al-Khalili,
 J. Phys. G **31** (2005) S1881

Energy excitation spectrum
dashed line: multiplied by 0.8

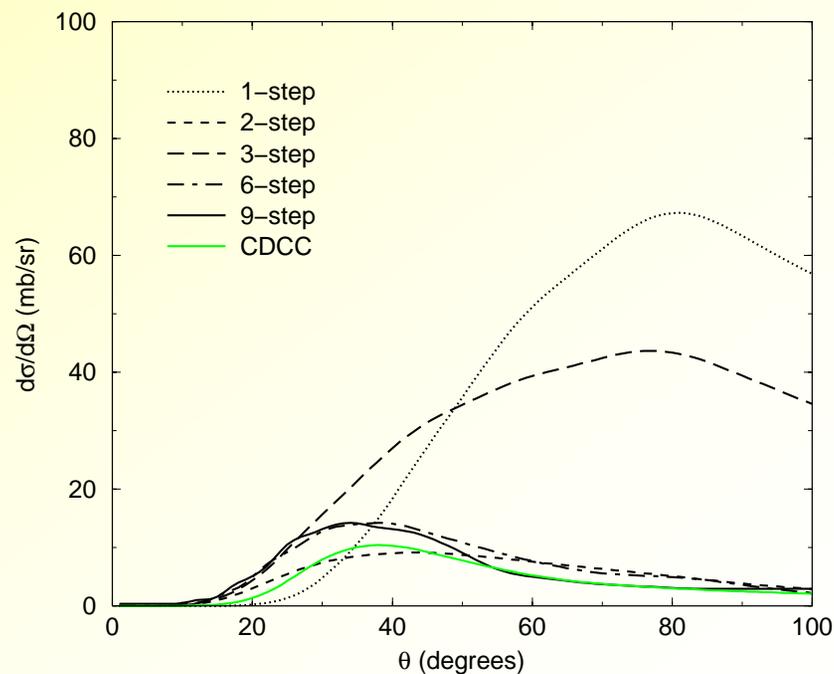


CDCC: sub-Coulomb $^8\text{B} + ^{58}\text{Ni}$ (26 MeV)

- Multistep Coulomb only

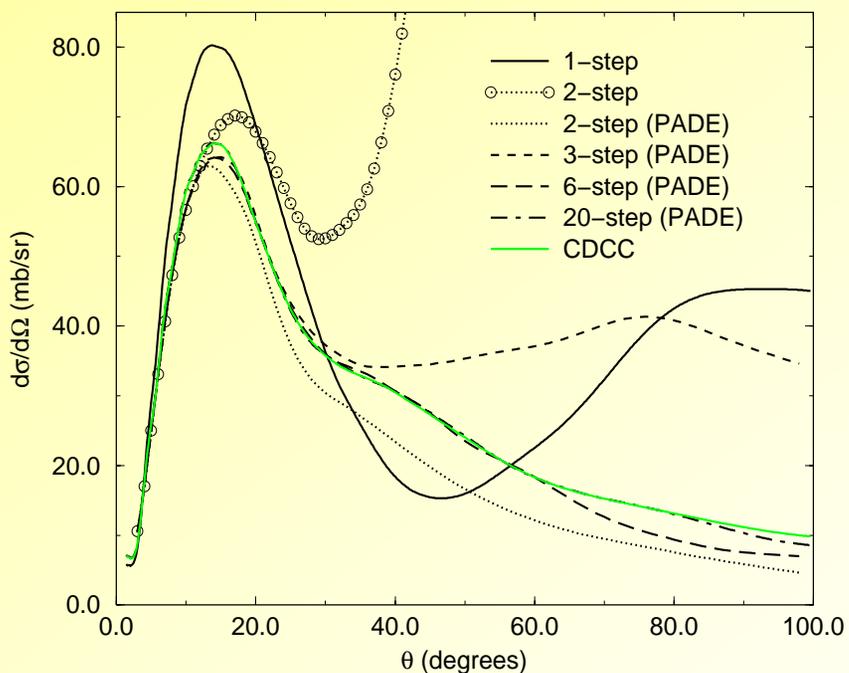


- Multistep Nuclear Only



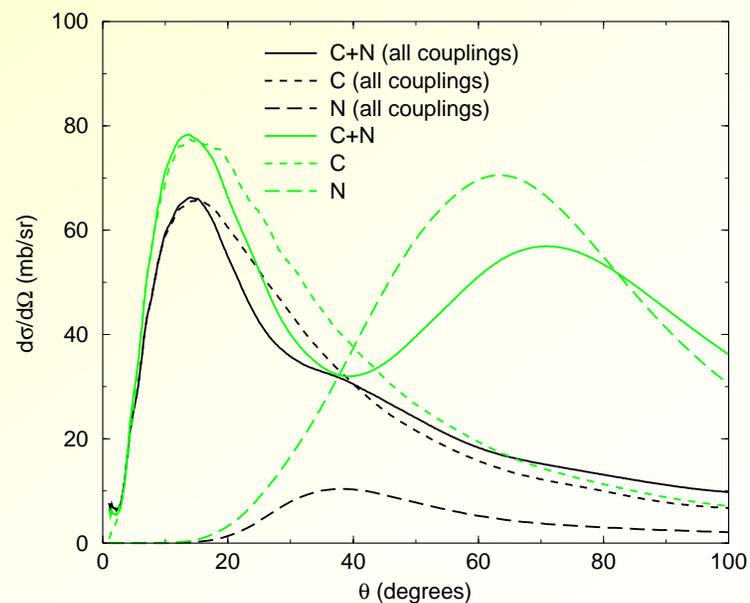
Coulomb+Nuclear Multistep

- Coulomb+nuclear



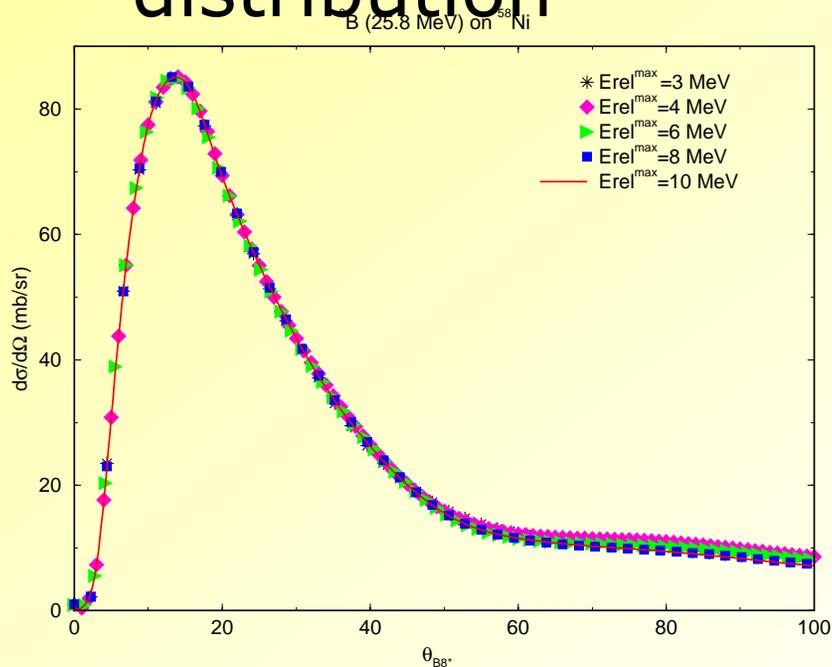
- Effect of continuum-continuum couplings

Green lines: no continuum-continuum couplings

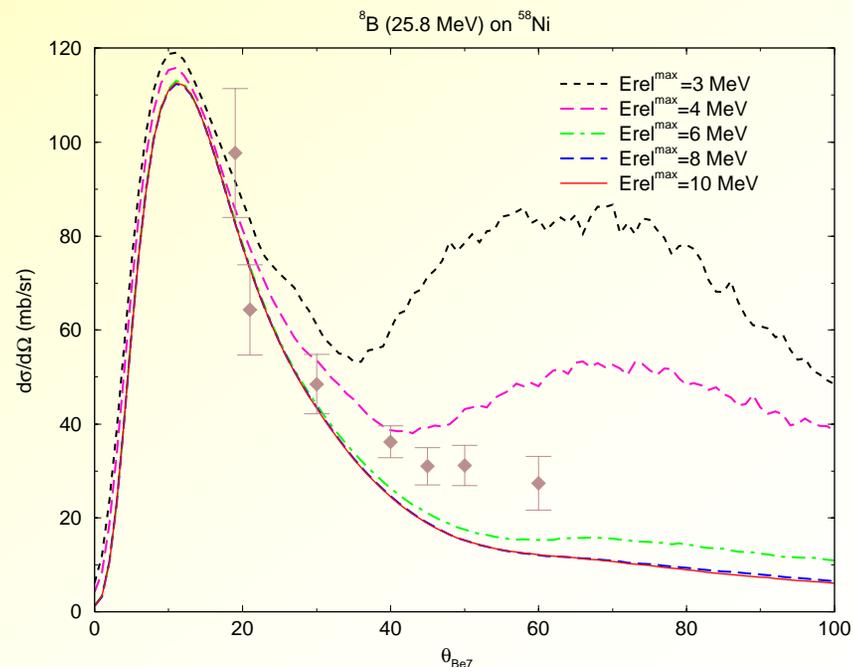


Convergence: max bin E_{rel}

- ^8B angular distribution



- ^7Be angular distributions

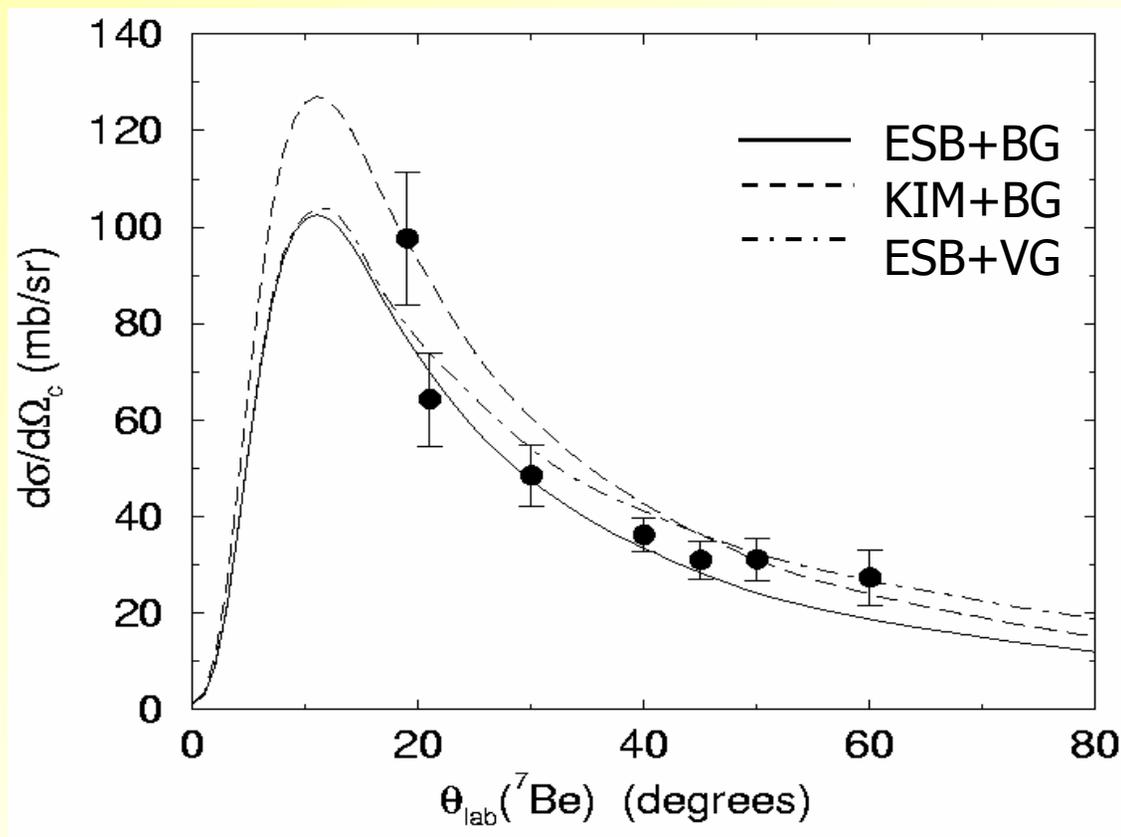


Elastic Breakup: $\sigma(\theta)$

^8B breakup on ^{58}Ni
($E_{\text{beam}} = 26 \text{ MeV}$)

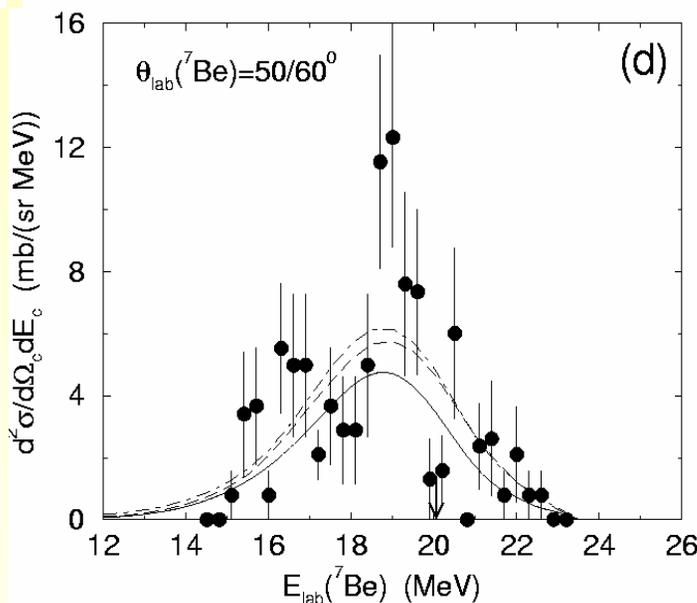
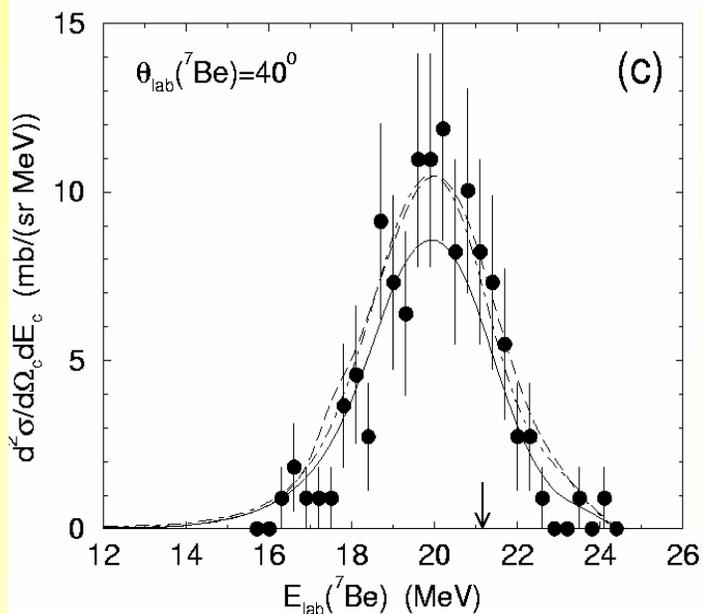
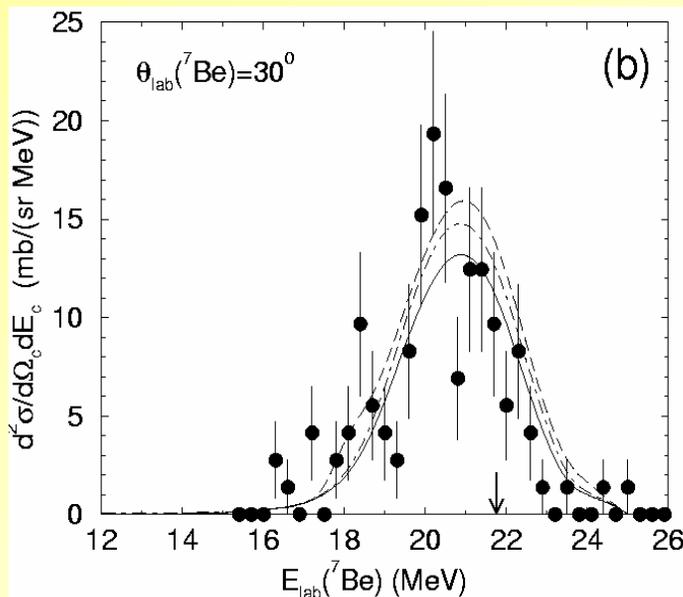
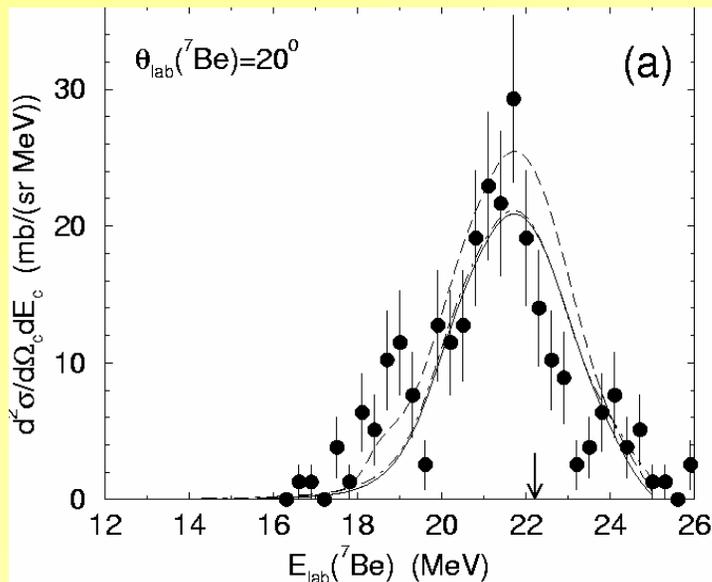
3-body observables

- sensitivity to ^8B structure: overall normalisation
- sensitivity to p-target optical potential at larger angles



[Tostevin, Nunes and Thompson, PRC (2001) 024617]

Breakup reactions CDCC $^8\text{B} + ^{58}\text{Ni} \rightarrow ^7\text{Be} + \text{p} + ^{58}\text{Ni}$ ($E_b=26$ MeV)



**Energy distributions:
Excellent agreement with the data!**

— ESB+BG
 - - - KIM+BG
 - · - · - ESB+VG
 ND data

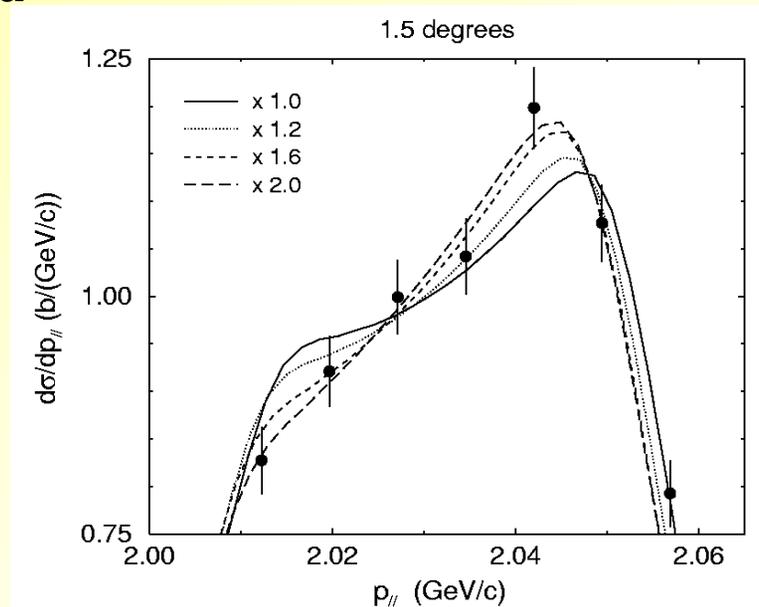
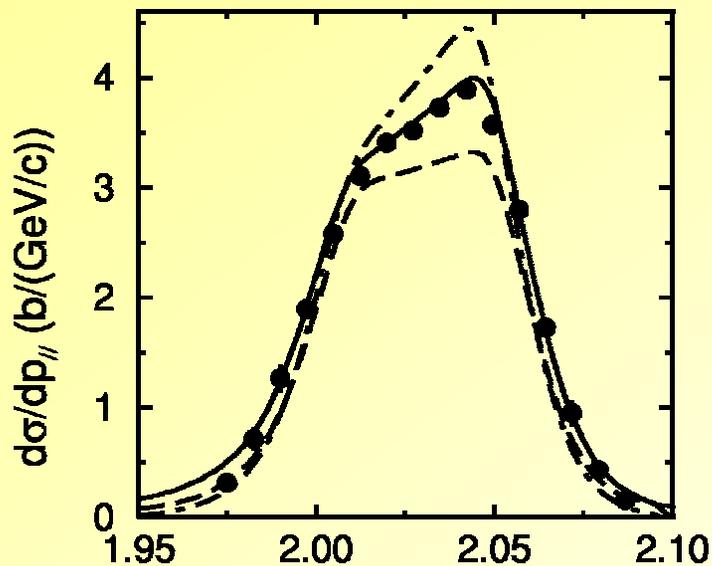
E1 & E2 breakup of ${}^8\text{B}$

- One-proton bound state known:
 - ${}^7\text{Be} \otimes (0p_{3/2} + 0p_{1/2}) |_{2+}$ at -0.137 MeV
- Need spectroscopy of non-resonant continuum!
 - $B(E1)$ & $B(E2)$ for transition $p \rightarrow s, d$ need to be accurately known
 - E1 and E2 amplitudes interfere in $p_{||}({}^7\text{Be})$ momentum distribution
 - so measure relative E2/E1 amplitudes from asymmetries.

$^8\text{B} + ^{208}\text{Pb} \rightarrow ^7\text{Be}$ parallel momentum distributions

3.5 degrees

44 MeV/u



**Dot-dashed: semiclassical Coul.
Solid: Coulomb+nuclear DWBA
Dashed: CDCC coupled channels
- reduced asymmetry**

**CDCC calculations with
scaled E2 amplitudes
- need to increase
asymmetry again!**

from Mortimer et al., Phys Rev C 65 (2002) 64619

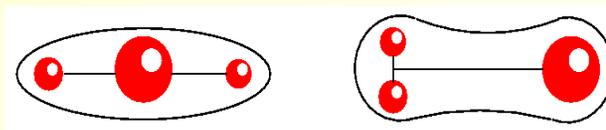
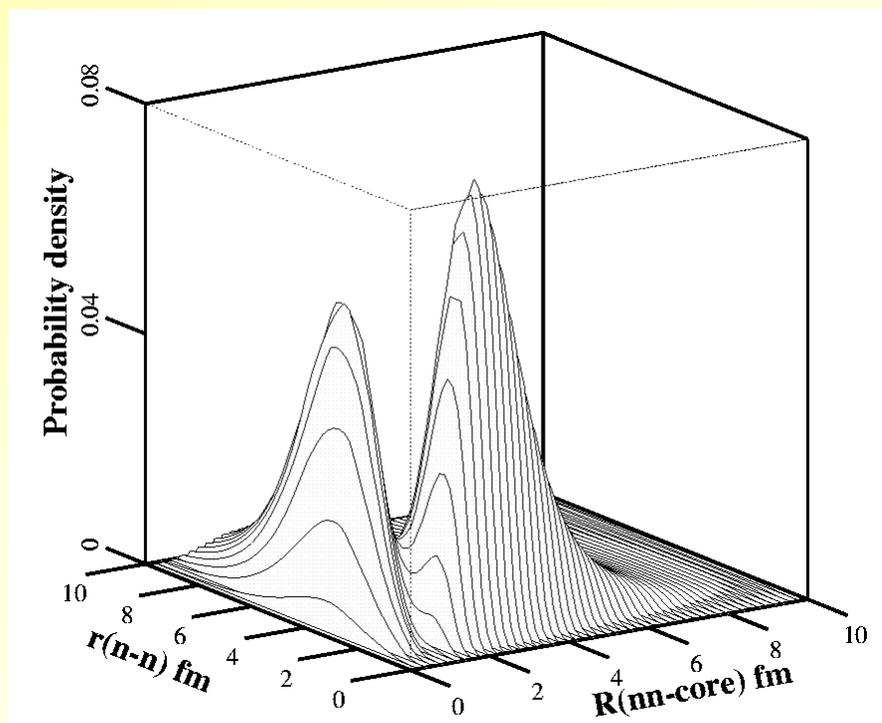
Extensions started

- Core excitation (static, dynamic)
 - Glauber: Batham et al
 - CDCC bins of particle+core coupled states, Summers & Nunes at MSU
- Three-cluster projectiles (e.g. two-neutron halo nuclei):
 - Gaussian expansions: Kamimura et al.
 - Transformed Harmonic Oscillator: Rodriguez-Gallardo et al

Wave functions of ${}^6\text{He}$

- Ground state wave function:
- Solution of coupled equations for $E \sim -0.97$ MeV.

Nuclei such as ${}^6\text{He}$ have highly correlated cluster structures

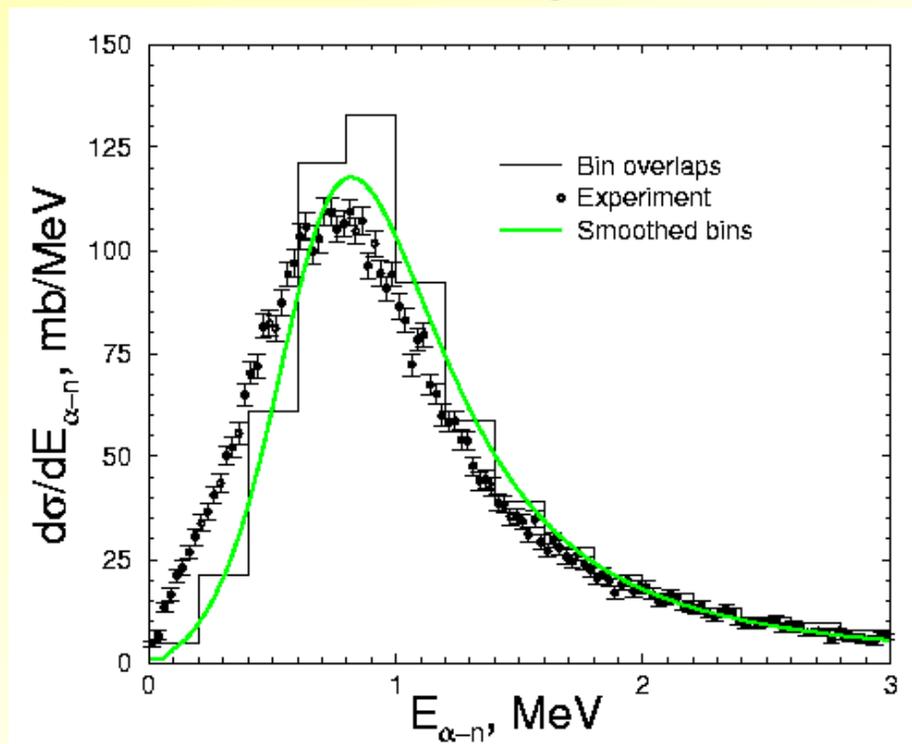


1 Neutron stripping from three-body Borromean Nuclei

- Removal of a neutron from ${}^6\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$,
 - populates states of ${}^5\text{He}$, ${}^{10}\text{Li}$ or ${}^{13}\text{Be}$.
 - Experiments measure decay spectrum of ${}^5\text{He} = {}^4\text{He} + n$, ${}^{13}\text{Be} = {}^{12}\text{Be} + n$, etc
- Can we predict any energy and angular correlations by Glauber model?
- Can we relate these correlations to the structure of the $A+1$ or the $A+2$ nucleus?

1N stripping from ${}^6\text{He}$ g.s.

- Calculate overlaps: $\langle {}^5\text{He}(E_{\alpha-n}) | {}^6\text{He}(\text{gs}) \rangle$ for a range of ${}^5\text{He}(E_{\alpha-n})$ bin states,
- smooth histogram of Glauber bin cross sections.
- GSI data (H.Simon)

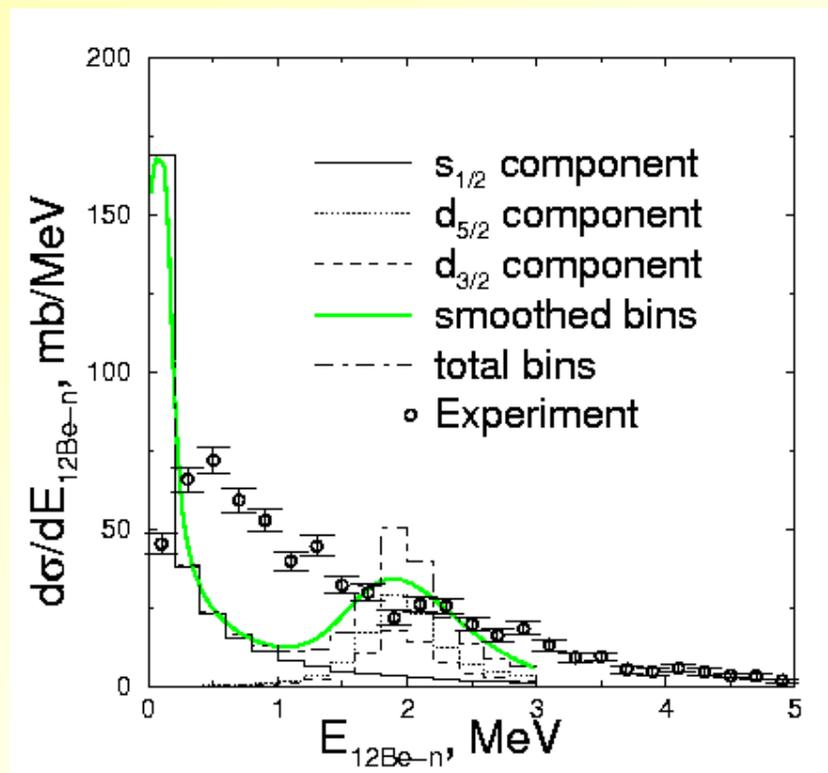


Theory: $\sigma_{\text{str}} = 137$ mb, $\sigma_{\text{diff}} = 38$ mb
 Expt: $\sigma_{\text{str}} = 127 \pm 14$ mb, $\sigma_{\text{diff}} = 30 \pm 5$ mb
 from T. Tarutina thesis (Surrey)

Promising technique!

1N stripping from ^{14}Be g.s.

- Calculate overlaps: $\langle ^{13}\text{Be}(E_{\alpha-n}) | ^{14}\text{Be}(\text{gs}) \rangle$
- Inert-core $^{13,14}\text{Be}$ wfs.
- GSI data (H.Simon)
- See softer data, and not pronounced virtual-s and resonant-d peaks.
- New theory needed?



Theory: $\sigma_{\text{str}} = 109 \text{ mb}$, $\sigma_{\text{diff}} = 109 \text{ mb}$
 Expt: $\sigma_{\text{str}} = 125 \pm 19 \text{ mb}$, $\sigma_{\text{diff}} = 55 \pm 19 \text{ mb}$

Elastic Breakup of 2N halo

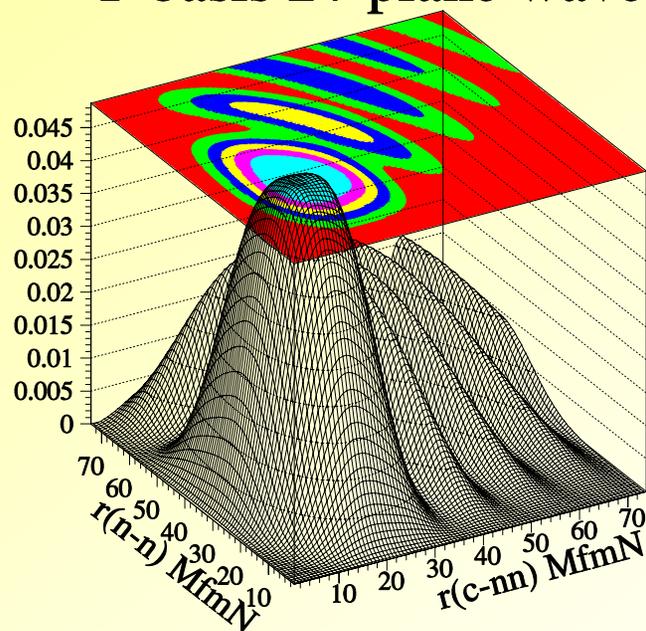
- Elastic Breakup = Diffraction
Dissociation:
 - all nuclear fragments survive along with the target in its ground state,
 - probes continuum excited states of nucleus.
- Need correlations in the three-body continuum of Borromean nuclei.

Continuum Spatial Correlations

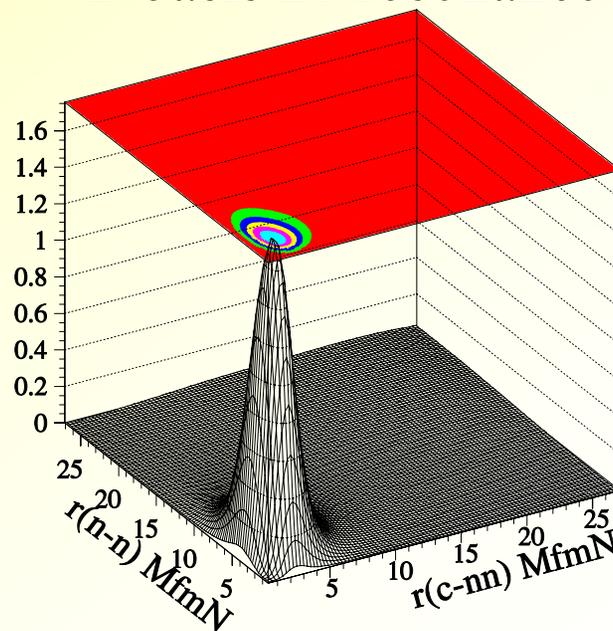
from B. Danilin, I. Thompson, PRC 69, 024609 (2004)

- Now average scattering wave functions over angles of k_x and k_y , to see spatial correlations in continuum states in ${}^6\text{He}$:

T-basis 2+ plane wave



T-basis 2+ resonance



'True' 3-body resonances?

- Expect continuum wave functions like:

$$\psi(\rho, \Omega_5^\rho, E, \Omega_5^\kappa) \propto \frac{1}{(\kappa\rho)^{5/2}} \sum_{K,\gamma} C_{K\gamma}(E) \psi_{K\gamma}^R(\rho) Y_{K\gamma}(\Omega_5^\rho) Y_{K\gamma}(\Omega_5^\kappa)$$

with

$$|C_{K\gamma}(E)|^2 = \frac{\Gamma_{K\gamma}}{(E - E_0)^2 + \Gamma^2/4}$$

Continuum Energy Correlations

- Now average scattering wave functions over angles of \mathbf{k}_x and \mathbf{k}_y , for fixed three-body energy E .
- Obtain similar plots for continuum energies.
- (Continuum momentum and angular correlations for later)

Continuum three-body wave functions UniS

- Three-body scattering at energy E :

$$\text{hypermomentum } \kappa = \sqrt{k_x^2 + k_y^2} = \sqrt{2mE/\hbar^2},$$

$$\text{hyperangle } \alpha_\kappa = \text{atan}(k_x/k_y)$$

- Plane wave 3-3 scattering states:

$$(2\pi)^{-3} \exp[i(\mathbf{k}_x \cdot \mathbf{x} + \mathbf{k}_y \cdot \mathbf{y})]$$

$$= (\kappa\rho)^{-2} \sum_{KLM_L l_x l_y} i^K J_{K+2}(\kappa\rho) \mathcal{Y}_{KLM_L}^{l_x l_y}(\Omega_5^\rho) \mathcal{Y}_{KLM_L}^{l_x l_y}(\Omega_5^\kappa)^*$$

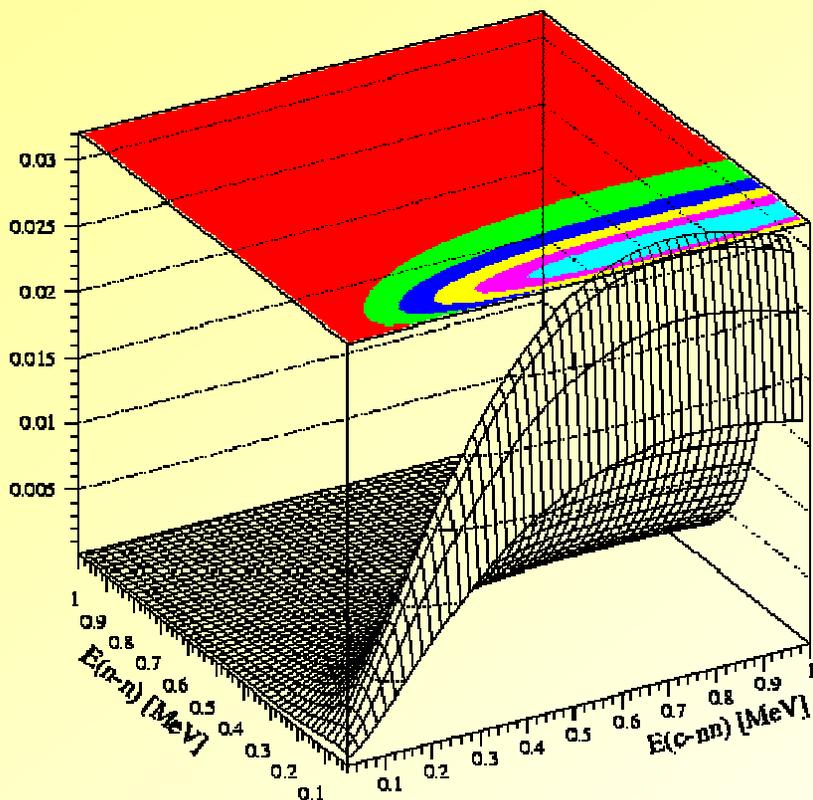
- Dynamical solutions for scattering states:

$$\Psi_{\kappa JM}^T(\mathbf{x}, \mathbf{y}, \hat{\mathbf{k}}_x, \hat{\mathbf{k}}_y, \alpha_\kappa) = (\kappa\rho)^{-5/2} \sum_{K\gamma, K'\gamma'} \psi_{K\gamma, K'\gamma'}^J(\kappa\rho) \Upsilon_{JM}^{K\gamma}(\Omega_5^\rho)$$

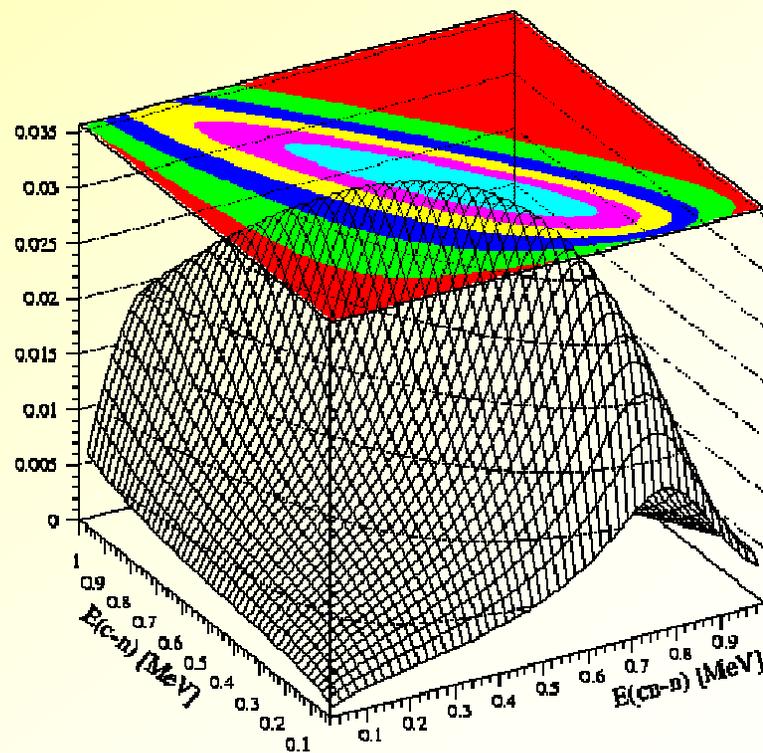
$$\sum_{M'_L M'_S} \langle L'M'_L S'M'_S | JM \rangle \mathcal{Y}_{K'L'M'_L}^{l'_x l'_y}(\Omega_5^\kappa) X_T$$

Virtual states & Resonances

from B. Danilin, I. Thompson, et al

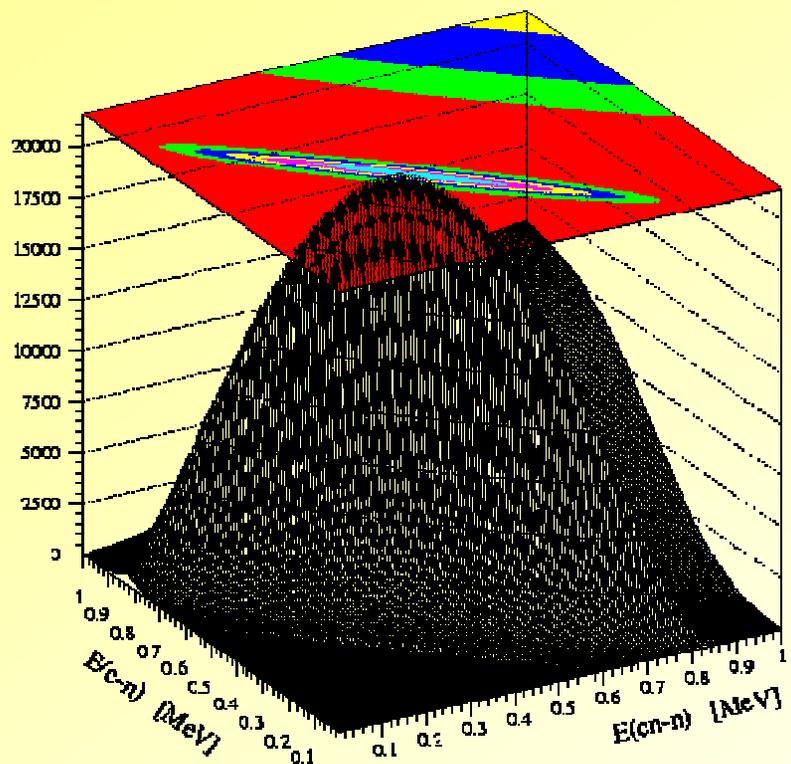


Virtual n-n pole

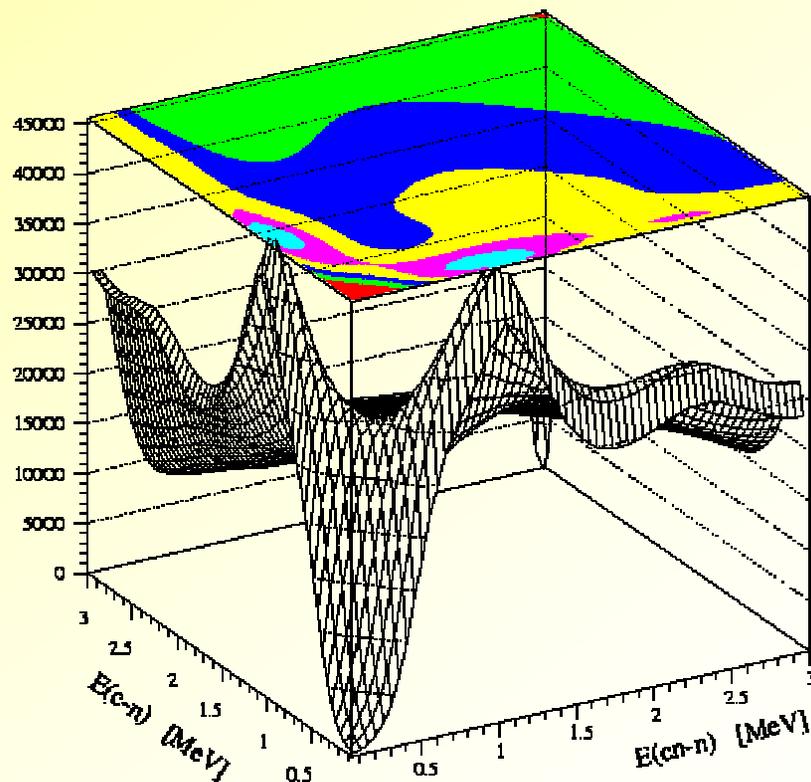


**Effect of n-n 'resonance' in
E(c-n), E(cn-n) coordinates**

${}^6\text{He}$ excitations & resonances



Pronounced 2^+ resonance



No pronounced 1^- resonance

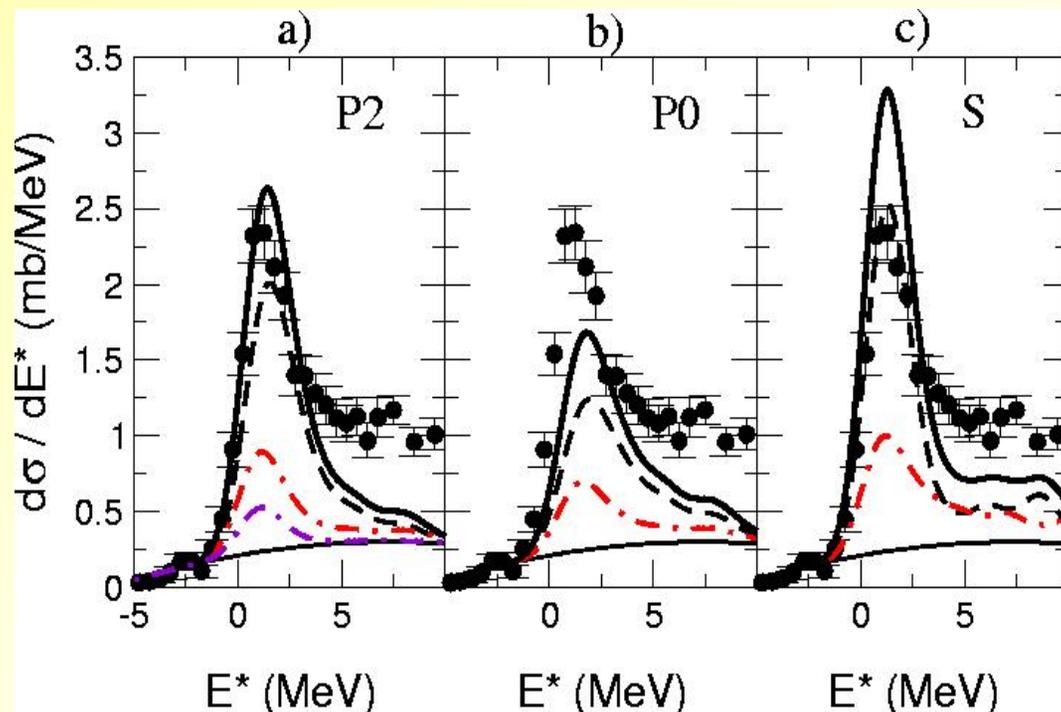
Four-body dynamics

- High Energies (first order & all order):
 - T-matrix multiple scattering (Crespo)
 - Eikonal+Adiabatic (Tostevin, Al-Khalili)
 - Eikonal (Exact fragment) (Brooke, Tostevin, Al-Khalili)
 - Adiabatic (Johnson, Christley et al)
- All Energies (all orders), new challenges:
 - 4-body pseudo-state CDCC (Kamimura)
 - 4-body bin-states CDCC
 - Two-nucleon states in deformed nuclei”

T-matrix expansions for breakup

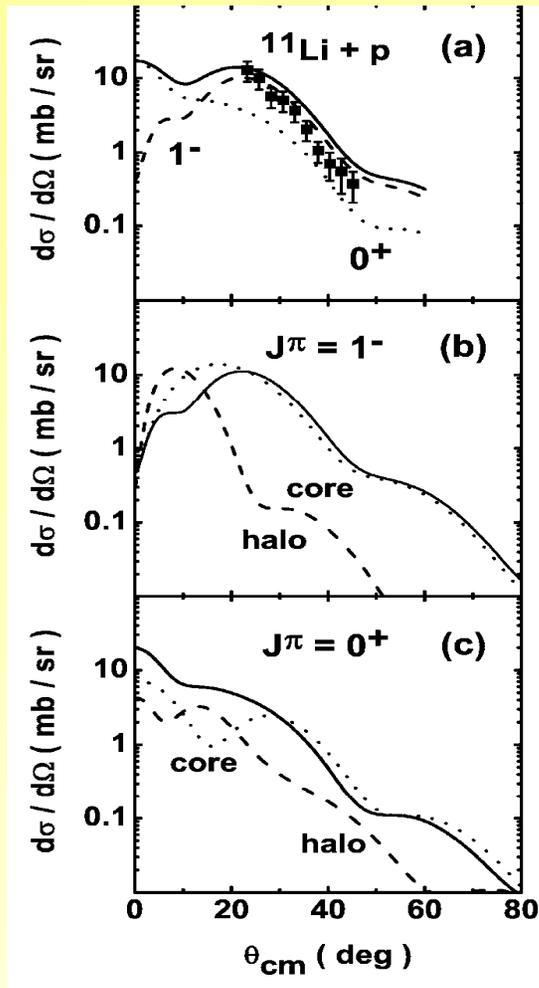
^{11}Li on protons at 68 MeV/A

- Preliminary Method:
 - First-order expansion on fragment-target T-matrices
 - Pseudo-state continuum, smoothed.
- Strong sensitivity on the structure models for ^{11}Li : S, P0, P2



Crespo et al., PRC66 (2002) 021002
Data: **RIKEN**.

$\sigma(\theta)$ for $^{11}\text{Li}(p,p')$ at 68 MeV/u UniS



- (a) Comparison of the theoretical calculations with experimental data
- Solid, dashed and dotted lines show the total, monopole and dipole angular distributions, respectively.
- In (b) and (c), solid lines show angular distributions for the monopole and dipole excitations, respectively.
- Dashed and dotted lines are contributions from the halo neutrons and the core nucleons.

Conclusions

- CDCC method good for 2-cluster halo nuclei:
 - Finite-range & recoil included
 - Coulomb and nuclear both approach convergence
 - Large radii and partial-wave limits needed, but feasible now
 - Non-adiabatic treatment of Coulomb breakup
 - Multistep effects manifest from all final-state interactions
- Extensions:
 - Deformed cluster models: Summers & Nunes at MSU
 - Three-cluster projectiles (e.g. two neutron halo nuclei): Kamimura et al.

