NCSM: from effective interactions to effective operators and beyond

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Collaborators:

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Easter bunny story

"Whoever shall question the rabbit but once, their firstborn shall thenceforth be a dunce."

(questioning the electromagnetic currents, not only the wavefunctions)

Outline

1. Theoretical approach
2. Applications to light nuclei
   - Low-lying spectra;
   - Other observables (**consistent** operators);
   - Lorentz integral transform for NCSM (continuum response using bound-state techniques).
3. EFT-like approach to effective operators for NCSM
4. Conclusions and outlook

NCSM formalism

- all particles allowed to interact
- truncation of the available single particle space
- unitary transformation to obtain effective interactions from bare high-precision nucleon-nucleon interactions which describe the phaseshifts with high accuracy

The many-body Schrodinger Equation:

$$H|\Psi\rangle = E|\Psi\rangle$$

$$H_{int} = \frac{1}{A}\sum_{ij}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j} V_{ij}^{NN} + \sum_{i>j>k} V_{ijk}^{NNN} + \ldots$$

- realistic, high precision two-body potentials: Argonne, CD Bonn
- theoretical three-body forces: TM'

Additional ingredient: center of mass motion

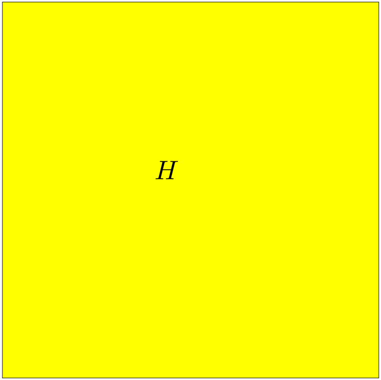
Addition of CM Hamiltonian

$$H \rightarrow H + \frac{1}{2mA} P_{CM}^2 + \frac{1}{2} m A \Omega^2 R_{CM}^2$$

- no influence on the intrinsic properties
- binds the nucleon clusters
- removed from the final results

Effective operators

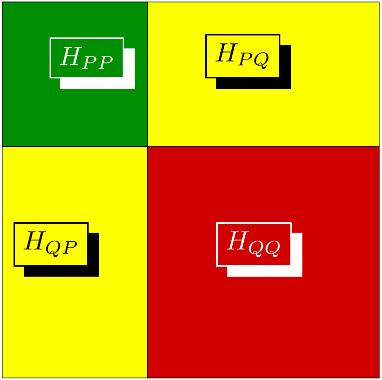
Start with the full space and the bare Hamiltonian H



Effective operators

Space truncation

EFFECTIVE INTERACTIONS NECESSARY



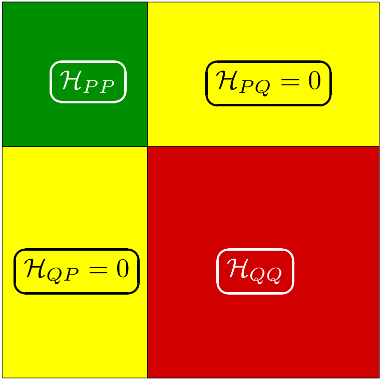
Effective operators

Unitary Transformation

$$\mathcal{H} = e^{-S} H e^{S}$$

$$S = \text{arctanh}\left(\omega^{\dagger} - \omega\right)$$

$$\omega = Q \omega P$$



Transformed operators

Effective Hamiltonian in the model space

$$H_{eff} = \frac{P + \omega^\dagger}{\sqrt{P + \omega^\dagger \omega}} H \frac{P + \omega}{\sqrt{P + \omega^\dagger \omega}}$$

$$H_{eff} P|\Psi_k\rangle = E_k P|\Psi_k\rangle \text{ for } k = 1, ..., d$$

$$H|\Psi_k\rangle = E_k|\Psi_k\rangle \text{ for } k = 1, ..., d, ...\infty$$

Effective general operator in the model space

$$O^{eff}_{\mathcal{JT}} = \frac{P + \omega^\dagger_{J'T'}}{\sqrt{P + \omega^\dagger_{J'T'}\omega_{J'T'}}} O_{\mathcal{JT}} \frac{P + \omega_{JT}}{\sqrt{P + \omega^\dagger_{JT}\omega_{JT}}}$$

Formal solution for ω

$$Q|\Psi_k\rangle = Q\omega P|\Psi_k\rangle \text{ for } k = 1, ..., d$$

$$\langle\alpha_Q^{(i)}|\Psi_k\rangle = \sum_{j=1}^{d}\langle\alpha_Q^{(i)}|\omega|\alpha_P^{(j)}\rangle\langle\alpha_P^{(j)}|\Psi_k\rangle$$
$$\text{for } k = 1, ..., d \text{ and } i = d+1, ..., \infty$$

$$\langle\alpha_Q^{(i)}|\omega|\alpha_P^{(j)}\rangle = \sum_{k=1}^{d}\langle\alpha_Q^{(i)}|\Psi_k\rangle\langle\alpha_P^{(j)}|\tilde{\Psi}_k\rangle$$

Requires solution to the original problem

Cluster approximation

- Find ω for $a < A$ (reproduce lowest eigenvalues)
- Compute $H_{eff}^{(a)}$
- Use $V_{eff}^{(a)}$ in the A-body calculation
- Scan for convergence (independence upon the model space and harmonic oscillator frequency).

Convergence to the exact solution if:

- $a \longrightarrow A$ for fixed model space;
- $P \longrightarrow \infty$ for fixed cluster.

Cluster approximation
Two-body cluster

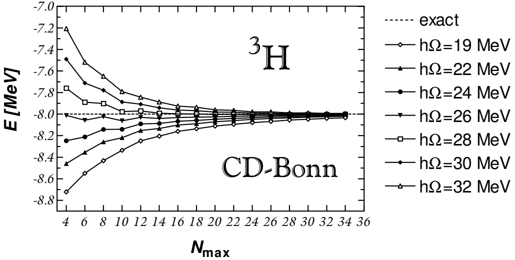
$$S \approx \sum_{i>j=1}^{A} S_{ij}$$

$$PO_{eff}P = P\sum_{i=1}^{A} O_i P + P\sum_{i>j=1}^{A}\left[e^{-S_{ij}}\left(O_i + O_j\right)e^{S_{ij}} - \left(O_i + O_j\right)\right]P$$

$$PO_{eff}P = P\sum_{i>j=1}^{A} e^{-S_{ij}} O_{ij} e^{S_{ij}} P$$

$$PH_{eff}P = P\sum_{i=1}^{A} h_i P + P\left[e^{-S_{ij}}\left(h_i + h_j + v_{ij}\right)e^{S_{ij}} - h_i - h_j\right]P$$

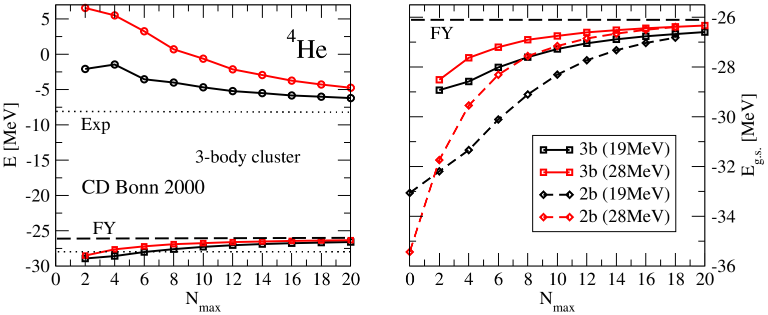
Two-body cluster: illustration



Navratil, Kamuntavicius, Barrett, Phys. Rev. C61 (2000) 044001

- Short range correlations included in $V_{eff}^{(2)}$
- Long-range and many-body correlations accomodated by increasing the model space

^{4}He results (two- and three-body cluster approximation)



- higher-order cluster: correlations better described (faster convergence);
- more comprehensive review of applications to nuclear spectra: see P. Navratil's talk in this program.

Selected results other observables

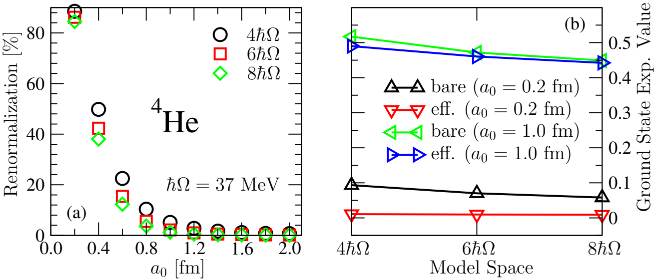
Nucleus	Observable	Model Space	Bare operator	Effective operator
^{2}H	Q_0	$4\hbar\Omega$	0.179	0.270
^{6}Li	$B(E2, 1^+0 \rightarrow 3^+0)$	$2\hbar\Omega$	2.647	2.784
^{6}Li	$B(E2, 1^+0 \rightarrow 3^+0)$	$10\hbar\Omega$	10.221	-
^{6}Li	$B(E2, 2^+0 \rightarrow 1^+0)$	$2\hbar\Omega$	2.183	2.269
^{6}Li	$B(E2, 2^+0 \rightarrow 1^+0)$	$10\hbar\Omega$	4.502	-
^{10}C	$B(E2, 2_1^+0 \rightarrow 0^+0)$	$4\hbar\Omega$	3.05	3.08
^{12}C	$B(E2, 2_1^+0 \rightarrow 0^+0)$	$4\hbar\Omega$	4.03	4.05
^{4}He	$\langle g.s.\vert T_{rel}\vert g.s.\rangle$	$8\hbar\Omega$	71.48	154.51

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C **71**, 044325 (2005)

- small model space: expect larger renormalization
- large variation with the model space
- three-body forces: might be important, but not the issue

- $a \rightarrow A$ for fixed model space;
- $P \rightarrow \infty$ for fixed cluster.

Range dependence

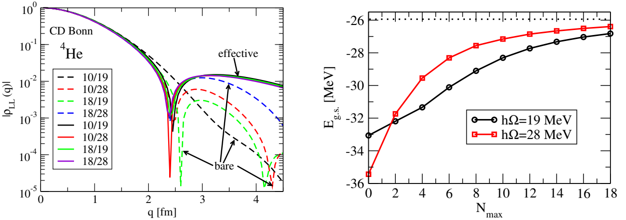


$$O \sim \exp\left[-\frac{(\vec{r}_1 - \vec{r}_2)^2}{a_0^2}\right]$$

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C **71**, 044325 (2005)

Longitudinal-longitudinal distribution function

$$\rho_{LL}(q) = \frac{1}{4Z} \sum_{j \neq i} (1 + \tau_z(i))(1 + \tau_z(j)) \langle g.s.| j_0(q|\vec{r}_i - \vec{r}_j|) | g.s. \rangle$$



Stetcu et. al. (in preparation)

Model space independence at high momentum transfer: good renormalization at the two-body cluster level

Lorentz integral transform 101

Efros, Leidemann, Orlandini, Phys. Lett. B338, 130 (1994).

$$R(E) = \sum_{\nu} |\langle\psi_0|O|\psi_\nu\rangle|^2 \delta(E - E_\nu)$$

LIT approach: calculate the transform of $R(E)$ and then invert:

$$\Phi[R](\sigma) = \int R(E)K(\sigma, E)dE$$

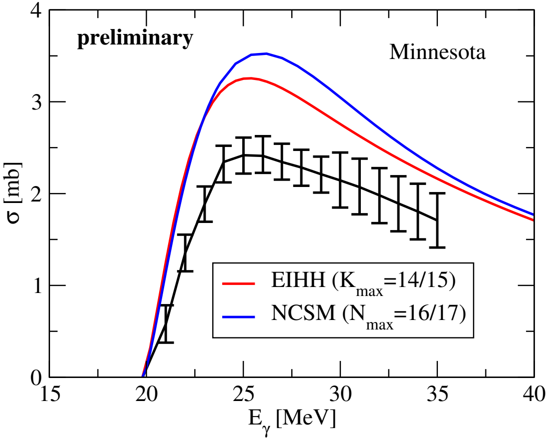
Lorentz kernel:

$$K(\sigma, E) = \frac{1}{(E - \sigma_R)^2 + \sigma_I^2}$$

$$\Phi[R](\sigma) = \langle\phi|\phi\rangle$$

$$(H - \sigma_R - i\sigma_I)|\phi\rangle = O|\psi_0\rangle$$

LIT application to ^{4}He disintegration



Summary NCSM

Advantages

- Preserve all the symmetries, including the translational invariance
- Flexible enough to handle both local and non-local interactions
- Suitable for light and medium nuclei

Disadvantages and Limitations

- Large dimensions in SD basis codes, difficult antisymmetrization in relative coordinates.
- Non-scalar effective operators are extremely demanding.
- In the lowest approximation, the long-range observables are weakly renormalized (difficult to obtain consistent operators).

Effective interactions for NCSM using EFT

Purposes

- to provide a consistent treatement of effective interactions and operators
- to obtain model-independent results

Means

EFT approach:

- consider the most general Hamiltonian which respects all the symmetries
- determine the coupling constants by fit to experimental data

EFT approach

1. identify relevant degrees of freedom
2. identify symmetries
3. write the most general Lagrangian (infinite number of terms)
4. organize interaction in powers of Q/M
5. the results are improvable order by order and *model independent*

Example: pionless theory

- applicable for processes involving momenta smaller than the pion mass
- fields: nucleons (and photons, neutrinos)

$$
\begin{aligned}
V_{NN}(\vec{p},\vec{p}\,') \quad = \quad & C_0^{(S)} + C_0^{(T)}\vec{\sigma_1}\cdot\vec{\sigma_2} \\
& + C_2^{(1)} q^2 + C_2^{(2)} k^2 + \left(C_2^{(3)} q^2 + C_2^{(4)} k^2 \right) \vec{\sigma_1}\cdot\vec{\sigma_2} \\
& + iC_2^{(5)} \frac{\vec{\sigma_1} + \vec{\sigma_2}}{2} (\vec{q} \times \vec{k}) + C_2^{(6)} \vec{q}\cdot\vec{\sigma_1}\vec{k}\cdot\vec{\sigma_2} + \ldots
\end{aligned}
$$

Procedure:

1. in each order determine the coupling constants to reproduce the same number experimental data as the number of constants
2. improve the results order by order and estimate error

$$
k \cot \delta_0 = -\frac{1}{a_0} + \frac{1}{2} r_0^2 k^2 + \ldots
$$

Why pionless EFT?

- Simple
- The power counting is fully understood
- What is the limit of applicability?
- Same methods should be applicable for the pionfull theory

Shell-model space

- the SM space is a particular type of truncation, using bound states only

$$\psi_{nl(s)j}(\vec{r}) = N_{nl} r^l L_n^{l+1/2}(\alpha r^2) \exp(-\alpha r^2) \left[Y_l(\hat{r}) \otimes \chi_s\right]_j$$

- defined by the maximum number of oscillator quanta allowed $N_{max} (N = 2n + l)$

$$P_j = \sum_{2n+l \leq N_{max}} |nl(s)j\rangle\langle nl(s)j|$$

- in the limit $N_{max} \to \infty$ equivalent with continuum

Two-body observables

- one bound state in the $^3S_1$channel
- phaseshifts

In a finite HO basis, all wfs. have a bound-state behavior at large distances!!

Phaseshifts in a finite basis

K. Kaufmann, W. Baumeister and M. Jungen, J. Phys. B.: At. Mol. Phys. 20, 4299 (1987)

$$H = \frac{p^2}{2\mu} + V$$

Diagonalization in the HO finite basis of dimension d

$$|\Psi_E\rangle = \sum_{\alpha=1}^{d} A_\alpha(E)|\alpha\rangle$$

$$H|\Psi_E\rangle = E|\Psi_E\rangle$$

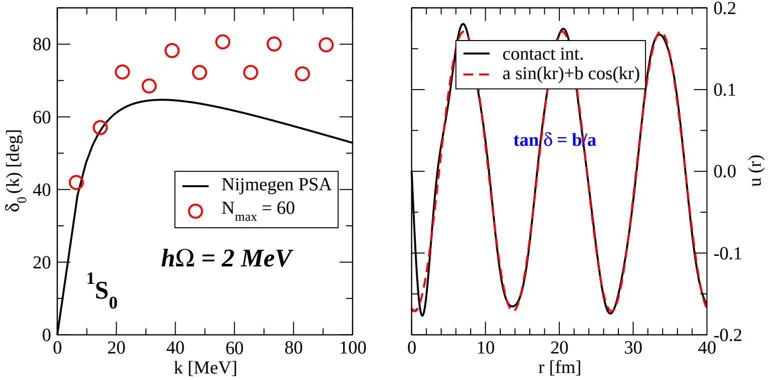
If $|\Psi\rangle$ corresponds to an eigenvalue in the continuum, at large distances, **but not at infinity**, this solution **still** approximates a shifted free particle:

$$\Psi_E(r) = arj_l(kr) + brn_l(kr), \;\; k = \sqrt{2\mu E}$$

$$\tan\delta_l(E) = \frac{b}{a}$$

NB: in the finite basis, E is a discrete eigenvalue!

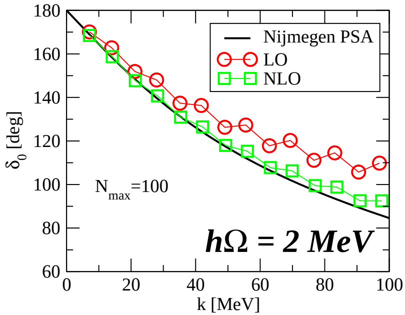
LO pionless EFT
Results for phaseshifts: $^1S_0$ channel



The coupling constant: fitted to reproduce the scattering length

LO and NLO pionless EFT

$^3S_1$ channel



- Reproduces the deuteron binding energy and the scattering length
- The phaseshift curve smooths out

Remarks

New Approach to effective interactions in NCSM

EFT-based method

- oscillations in the phaseshift curves
- $^1S_0$ channel fitting less accurate (and more involved)
- easier fit for N_{max} large and $\hbar\Omega$ small

Scattering information could be extracted, the quest for a better method continues

LS equation in a HO basis

- In continuum:

$$\langle p|T(E)|p'\rangle = \langle\vec{p}|V|\vec{p}'\rangle + \int d^3p'' \langle\vec{p}|V|\vec{p}''\rangle \frac{1}{E - p''^2/2\mu + i\varepsilon} \langle\vec{p}''|T|\vec{p}'\rangle$$

(contact interaction: the integral has to be regularized)

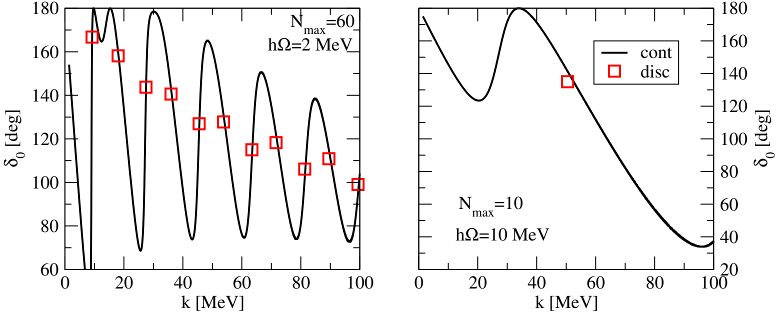
- Inset a complete HO basis:

$$\hat{T} = \hat{V} + \hat{V}\hat{G}_0(E)\hat{T}$$

- In the truncated HO basis, solve for $\hat{T}$:

$$\hat{T} = \frac{1}{1 - \hat{V}\hat{G}_0(E)}\hat{V}$$

- Reconstruct the on-shell T-matrix, obtain the phasesift.

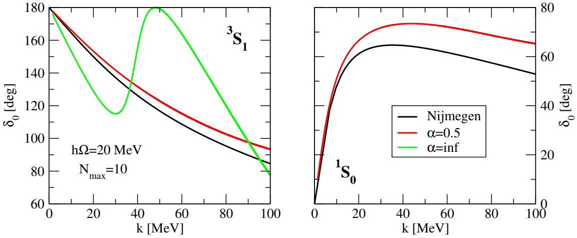


N max =60
hΩ=2 MeV
δ0 [deg]
k [MeV]
cont
disc
N max =10
hΩ=10 MeV

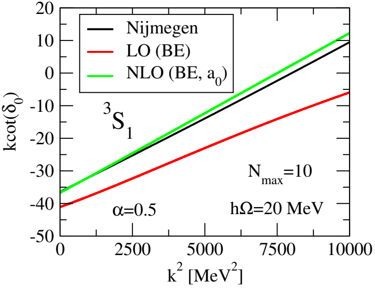
Removing the Gibbs oscillations

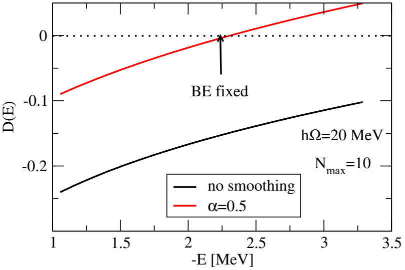
$$V = \sum_{n,m} |m\rangle V_{mn} \langle n| \Longrightarrow \sum_{m,n} \sigma_{mN_{max}}(\alpha) |m\rangle V_{mn} \langle n| \sigma_{nN_{max}}(\alpha)$$

$$\sigma_{nN_{max}}(\alpha) = \left(\frac{1 - \exp\{-[\alpha(2n - N_{max} - 1)/(N+1)]^2\}}{1 - \exp(-\alpha^2)} \right)^{1/2}$$



Improved description





Conclusions and outlook

- Effective operators from unitary transformation:
  - good description of nuclear spectra
  - other operators implemented at the two-body cluster level
  - little effect for long-range operators (the rabbit grows more legs)
  - **good description of short-range operators**
  - applications to other problems in progress

- New approach to effective interactions
  - description of phaseshifts in finite L^2 integrable basis: continuum and discrete approaches
  - removal of Gibbs oscillations introduces a new parameter (?)
  - better treatment of all the other operators (not discussed here)