Electromagnetic Interactions in a Chiral Effective Lagrangian for Nuclei

Brian D. Serot

Indiana University

- Outline
  - ∗ Motivation for Lorentz-Covariant EFT
  - ∗ EFT Lagrangian
  - ∗ Chiral Symmetry Transformations
  - ∗ DFT for Nuclei: Strategy and Justification
  - ∗ Isovector Vector and Axial-Vector Currents
  - ∗ Electromagnetic Interactions

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Collaborators 1993–2005

Sergei Ananyan

Dick Furnstahl

Carrie Halkyard

Ying Hu

Jeff Mcintire

Horst Müller

Jorge Piekarewicz

John Rusnak

Huabin Tang

Dirk Walecka

Why Use Hadrons?

- We focus on low-energy, long-range physics, and all observables are colorless.
- Hadrons (baryons and mesons) are the actual particles observed in experiments.
- Colored quarks and gluons participate only in intermediate states, and such “off-shell” behavior is unobservable.
- So pick the most efficient degrees of freedom! We have to parametrize the hamiltonian anyway, since we don’t know its true form.
- We are interested primarily in the nuclear **many-body** problem, so include “collective” degrees of freedom like scalar and vector fields.
- Only nucleons and pions are “real” (stable) particles; other fields are always virtual and just parameterize the NN interaction (or EM form factors).

Why Impose Lorentz Covariance?

- The scalar and vector mean fields in nuclei are large (several hundred MeV). This is a new energy scale. The scalar and vector fields **cancel** to produce a small binding energy.
  - ∗ Consistent with QCD sum-rule results (size and density dependence).
  - ∗ Consistent with chiral power counting (two-body energy/nucleon is of order ρ_0/f_π^2).
- Large mean fields produce important relativistic interaction effects.
  - ∗ Velocity-dependent NN interaction provides a new saturation mechanism.
  - ∗ Scalar and vector mean fields **add** to produce correct spin-orbit force. (Compare “fine” structure in atoms and nuclei.)
  - ∗ Successful prediction of nucleon–nucleus spin observables in the RIA and energy dependence of the optical potential.
  - ∗ Explains pseudospin symmetry in nuclei.

There really is relativity in nuclei!

Why Use Effective Field Theory?

Lorentz-covariant hadronic field theories $\equiv$ Quantum HadroDynamics

- Interpret QHD lagrangians as nonrenormalizable $\mathcal{L}_{\mathrm{EFT}}$'s
  - $*$ known long-range interactions constrained by symmetries;
  - $*$ a complete set of generic short-range interactions;
  - $*$ the borderline is characterized by breakdown scale Λ of EFT. For QHD, $\Lambda \approx 600$ MeV (empirically).
- When based on a local, Lorentz-invariant lagrangian density, EFT is the most general way to parameterize observables consistent with the principles of quantum mechanics, special relativity, unitarity, cluster decomposition, microscopic causality, and the desired internal symmetries.
- It's not necessary to derive $\mathcal{L}$ from QCD
  - $*$ Use a general $\mathcal{L}$ that respects the symmetries.
  - $*$ By construction, this provides a general parametrization for energies $\lesssim \Lambda$ (remove redundancies).
- The freedom to redefine and transform the fields $\Longrightarrow$ infinitely many representations of low-energy QCD physics

Advantages for Electroweak Interactions in Nuclei

- We use the **same degrees of freedom** to describe the nuclear interactions and nuclear currents.

- The QHD/EFT lagrangian exhibits the symmetries of QCD.
  - ∗ Chiral $SU(2)_L \times SU(2)_R$ symmetry is nonlinear.
  - ∗ Unbroken isovector subgroup $SU(2)_V$ symmetry is linear.
  - ∗ These are global symmetries.
  - ∗ Electromagnetic interactions [local $U(1)_Q$ gauge symmetry] is straightforward to include.

- At momenta small compared to Λ, short-distance physics (like nucleon substructure) is only partially resolved and can be described with a derivative expansion.

- The lagrangian parameters can be calibrated to
  - ∗ properties of isolated hadrons,
  - ∗ πN scattering,
  - ∗ bulk and single-particle properties of nuclei,
  - ∗ NN scattering.

Strategy

- Assign an index to each term in the lagrangian: $\nu = d + n/2 + b$.
  - $*$ $d =$ number of derivatives (except on nucleons).
  - $*$ $n =$ number of nucleon fields.
  - $*$ $b =$ number of non-Goldstone bosons.
- Organize $\mathcal{L}$ in powers of ν and truncate; this gives a reliable expansion in inverse powers of a "heavy" mass scale $\Lambda \approx M$. For heavy nuclei, essentially an expansion in $k_{\rm F}/M$.

Fields

- Nucleon (N), Lorentz scalar (ϕ = "sigma") [chiral scalar]
- Lorentz vector (V_μ = "omega" ; $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$) ["]
- Pion: $U \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi)\ ,\qquad \xi \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,/2f_\pi)\ ,$

  together with $a_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)\ ,$

  $$v_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)\ ,\quad v_{\mu\nu} \equiv -i[a_\mu, a_\nu]\ .$$
- Rho: $\rho_\mu \equiv \frac{1}{2}\,\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\ ,\quad D_\mu\rho_\nu \equiv \partial_\mu\rho_\nu + i[v_\mu, \rho_\nu]\ ,$

  $$\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu + i\,\overline{g}_\rho[\rho_\mu, \rho_\nu]\ .$$

EFT Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm QHD} &= \mathcal{L}_N + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_M \\
&= \overline{N}\left(i\gamma^\mu\left[D_\mu + ig_\rho\rho_\mu + ig_v V_\mu\right] + g_A\gamma^\mu\gamma_5 a_\mu - M + g_s\phi\right)N \\
&\quad - \frac{f_\rho g_\rho}{4M}\overline{N}\rho_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_v g_v}{4M}\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N - \frac{\kappa_\pi}{M}\overline{N}v_{\mu\nu}\sigma^{\mu\nu}N \\
&\quad + \frac{4\beta_\pi}{M}\overline{N}N\,{\rm Tr}\left(a_\mu a^\mu\right) + \mathcal{L}_{\pi N}^{(4)} \\
&\quad + \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \tfrac{1}{4}f_\pi^2\,{\rm Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right) \\
&\quad - \tfrac{1}{2}\,{\rm Tr}\left(\rho_{\mu\nu}\rho^{\mu\nu}\right) - \tfrac{1}{4}\,V_{\mu\nu}V^{\mu\nu} \\
&\quad - g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}\,{\rm Tr}\left(\rho_{\mu\nu}v^{\mu\nu}\right) + \tfrac{1}{2}\left(1 + \eta_1\frac{g_s\phi}{M} + \frac{\eta_2}{2}\frac{g_s^2\phi^2}{M^2}\right)m_v^2 V_\mu V^\mu \\
&\quad + \tfrac{1}{4!}\,\zeta_0 g_v^2(V_\mu V^\mu)^2 + \left(1 + \eta_\rho\frac{g_s\phi}{M}\right)m_\rho^2\,{\rm Tr}\left(\rho_\mu\rho^\mu\right) \\
&\quad - m_s^2\phi^2\left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_s\phi}{M} + \frac{\kappa_4}{4!}\frac{g_s^2\phi^2}{M^2}\right).
\end{aligned}
$$

- $\mathcal{L}_{\rm QHD}$ contains all nonredundant terms through order $\nu = 4$.
- We see standard noninteracting hadron terms $\oplus$ Yukawa nucleon–meson couplings $\oplus$ anomalous-moment interactions $\oplus$ pion–nucleon and meson nonlinearities: nontrivial dynamics.

Transformation Laws

In our EFT (QHD) lagrangian:

- Chiral $SU(2)_L \times SU(2)_R$ symmetry is nonlinear.
- Isovector subgroup $SU(2)_V$ symmetry is linear.
- These are global symmetries.
- Vector transformations: $L = \exp(i\boldsymbol{\beta} \cdot \boldsymbol{\tau}/2) = R$
- Axial-vector transformations:

  $L = \exp(i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}/2)\ , \qquad R = \exp(-i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}/2)$
- Field transformations: (all objects are matrices)

$$
\begin{aligned}
U(x) &\rightarrow LU(x)R^\dagger\ , \\
\xi(x) &\rightarrow L\xi(x)h^\dagger(x) = h(x)\xi(x)R^\dagger \qquad \text{[defines } h(x)\text{]} \\
N(x) &\rightarrow h(x)N(x) \qquad \text{[generally, } h(x) \text{ is local]} \\
\rho_\mu(x) &\rightarrow h(x)\rho_\mu(x)h^\dagger(x)\ .
\end{aligned}
$$

- Chirally covariant derivatives:

$$D_\mu N \equiv (\partial_\mu + iv_\mu)N\ : \qquad D_\mu N \rightarrow h(x)(D_\mu N)\ ,$$

$$D_\mu \rho_\nu \equiv \partial_\mu \rho_\nu + i[v_\mu, \rho_\nu]\ : \qquad D_\mu \rho_\nu \rightarrow h(x)(D_\mu \rho_\nu)h^\dagger(x)$$

Discussion

- To realize the nonlinear $SU(2)_L \times SU(2)_R$ symmetry, the lagrangian must include pions explicitly.
- Note that U, ξ, and ρ_μ are 2×2 matrices.
- For isospin transformations, $L = R = h$ (constants); the transformations are linear.
- For general transformations: $L \neq R$. [Axial transformations have $L = R^\dagger$.]
  - ∗ Now $h(x)$ is nontrivial and contains pion fields.
  - ∗ So $h(x)N(x)$ mixes nucleons with any number of pions: the transformation is nonlinear.
- The only field or tensor that transforms inhomogeneously is $v_\mu \to h v_\mu h^\dagger - i h \partial_\mu h^\dagger$. This allows for the construction of chirally covariant derivatives.
- This is NOT the linear sigma model; the scalar field ϕ is a chiral scalar. It is NOT the chiral partner of the pion.

Important Things to Remember

- Off-shell behavior is not observable. Choose the dynamical variables that are most efficient (still unknown).
- Vacuum dynamics involves short-range physics. Don't calculate it, but parametrize it in a few fitted constants. (Computation of hadronic loops $\Longrightarrow$ unnatural coefficients.) Use valence nucleons only.
- Although fields and couplings are local, nucleon substructure is also included:
  - $*$ Example: $\overline{N}N\sigma \to g(\sigma)\overline{N}N\sigma$
  - $*$ But define: $\phi \equiv g(\sigma)\sigma$, $[g(0) = 1]$; then invert for $\sigma(\phi)$.
  - $*$ Then: $g(\sigma)\overline{N}N\sigma + p(\sigma) = \overline{N}N\phi + a\phi^2 + b\phi^3 + c\phi^4 + \cdots$
- Nucleon EM structure included in a derivative expansion:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EM}} &= -\frac{e}{2}\overline{N}A^{\mu}\gamma_{\mu}(1+\tau_3)N - \frac{e}{4M}F^{\mu\nu}\overline{N}\{\lambda^{(0)}+\lambda^{(1)}\tau_3\}\sigma_{\mu\nu}N \\
&\quad + \cdots - \frac{e}{2M^2}\partial_{\nu}F^{\mu\nu}\overline{N}(\{\beta^{(0)}+\beta^{(1)}\tau_3\}\gamma_{\mu})N \\
&\quad - \frac{e}{M^4}\partial^2\partial_{\nu}F^{\mu\nu}\overline{N}(\{\delta^{(0)}+\delta^{(1)}\tau_3\}\gamma_{\mu})N + \cdots + \mathrm{VMD}\,,
\end{aligned}
$$

  which generates e, λ, $r_{\mathrm{rms}}^{\mathrm{s,v}}$, . . .

  This works at long distances (low momenta).

- This Electromagnetic lagrangian is valid to lowest order in the electric charge and the pion fields. When combined with the VMD terms, we find the following results for the nucleon EM form factors ($Q \equiv$ four-momentum transfer):

$$F_1^{(0)}(Q^2) = \frac{1}{2} - \frac{\beta^{(0)}}{2}\frac{Q^2}{M^2} - \frac{g_v}{3g_\gamma}\frac{Q^2}{Q^2+m_v^2} + \cdots$$

$$F_1^{(1)}(Q^2) = \frac{1}{2} - \frac{\beta^{(1)}}{2}\frac{Q^2}{M^2} - \frac{g_\rho}{2g_\gamma}\frac{Q^2}{Q^2+m_\rho^2} + \cdots$$

$$F_2^{(0)}(Q^2) = \frac{\lambda_{\rm p}+\lambda_{\rm n}}{2} - \frac{f_v g_v}{3g_\gamma}\frac{Q^2}{Q^2+m_v^2} + \cdots$$

$$F_2^{(1)}(Q^2) = \frac{\lambda_{\rm p}-\lambda_{\rm n}}{2} - \frac{f_\rho g_\rho}{2g_\gamma}\frac{Q^2}{Q^2+m_\rho^2} + \cdots$$

- Using the single-nucleon charge and anomalous rms radii allows one to determine the $\beta^{(t)}$ and f_i parameters. (The constants g_v and g_ρ are determined from nuclear properties, and g_γ is fit to the leptonic $\rho \to e^+e^-$ decay.)

- These expressions allow one to describe the contributions of nucleon EM structure to nuclear charge form factors without introducing *ad hoc* form factors at the NNγ vertex.

(Naive) Dimensional Analysis: NDA

[Georgi & Manohar, 1984]

- Low-energy QCD is expected to contain two mass scales:

$$f_\pi \approx 93\,\mathrm{MeV}\;, \qquad \Lambda \approx 500 \text{ to } 800\,\mathrm{MeV}.$$

  The first is related to chiral symmetry and resulting Goldstone bosons (pions); the second signals non-Goldstone-boson (“heavy”) physics.

- NDA rules for a generic term in the energy functional:

$$C\,[f_\pi^2\Lambda^2]\left[\left(\frac{\overline{N}N}{f_\pi^2\Lambda}\right)^\ell \frac{1}{m!}\left(\frac{\Phi}{\Lambda}\right)^m \frac{1}{n!}\left(\frac{W}{\Lambda}\right)^n \left(\frac{\partial}{\Lambda}\right)^p\right]$$

- “Naturalness” $\Longrightarrow$ dimensionless C is of order unity.

- Provides expansion parameters at finite density:

$$\frac{\Phi}{\Lambda} \approx \frac{W}{\Lambda} \approx 1/2\;, \qquad \frac{\rho_{\mathrm{s}}}{f_\pi^2\Lambda} \approx \frac{\rho_{\mathrm{B}}}{f_\pi^2\Lambda} \approx 1/5 \quad \text{at } \rho_{\mathrm{B}}^0$$

- Allows truncation and calibration with **quantitatively accurate** fits to bulk and single-particle nuclear observables (binding energies, charge densities, single-particle spectra near the Fermi surface).

Density Functional Theory

- Construct the ground-state energy functional from the lagrangian using a mean-field ("factorized") approximation:
  - ∗ A functional of scalar ($\rho_{\rm s}$) and baryon ($\rho_{\rm B}$) densities.
  - ∗ Lorentz scalar and vector fields are interpreted as Kohn–Sham single-particle potentials. Dirac (quasi)nucleons move in these local potentials.
- Kohn–Sham theorem [1965]: The exact ground-state scalar and vector densities, energy, and chemical potential for the fully interacting many-fermion system can be reproduced by a collection of (quasi)fermions moving in appropriately defined, self-consistent, local, classical fields.
- Mean-field energy functional provides a parametrization of the exact energy functional. Fit the parameters [define a χ^2] to (29) nuclear observables from ^{16}O, ^{40}Ca, ^{48}Ca, ^{88}Sr, and ^{208}Pb. There are more than enough parameters at the typical level of truncation. Parameters encode both short-range (vacuum, QCD) effects and long-range (many-body) effects.

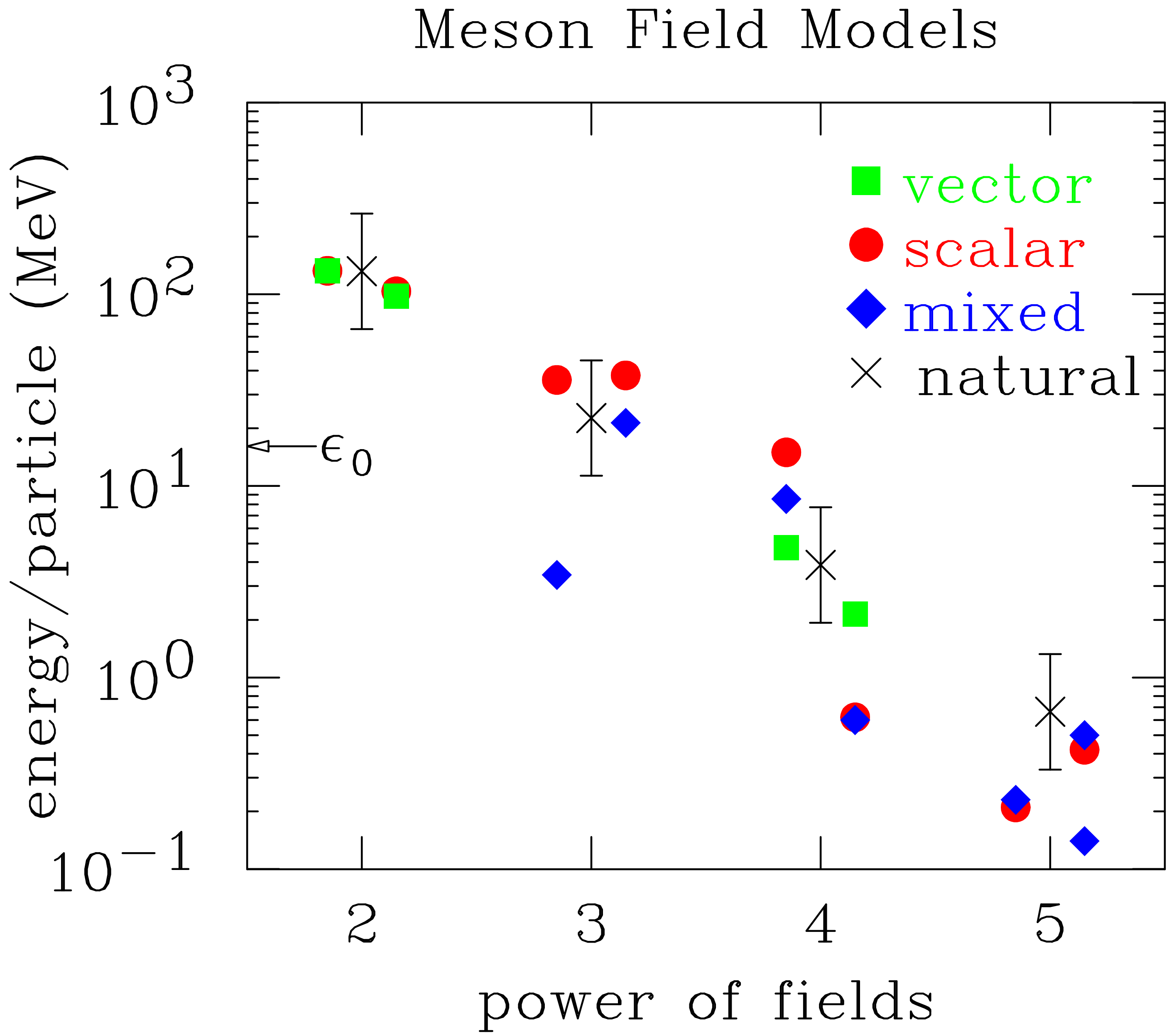
- Kohn–Sham quasi-particle orbitals are tailored to the generation of the ground-state density, so they include exchange, correlation, and short-range effects (approximately).
- Verify naturalness by examining the convergence of the truncation (and make predictions).
- Note the large scalar and vector fields! The scale of the lowest-order term in the energy/particle is given by

$$\rho_{\rm eq}/f_\pi^2 \approx 130\,{\rm MeV}$$

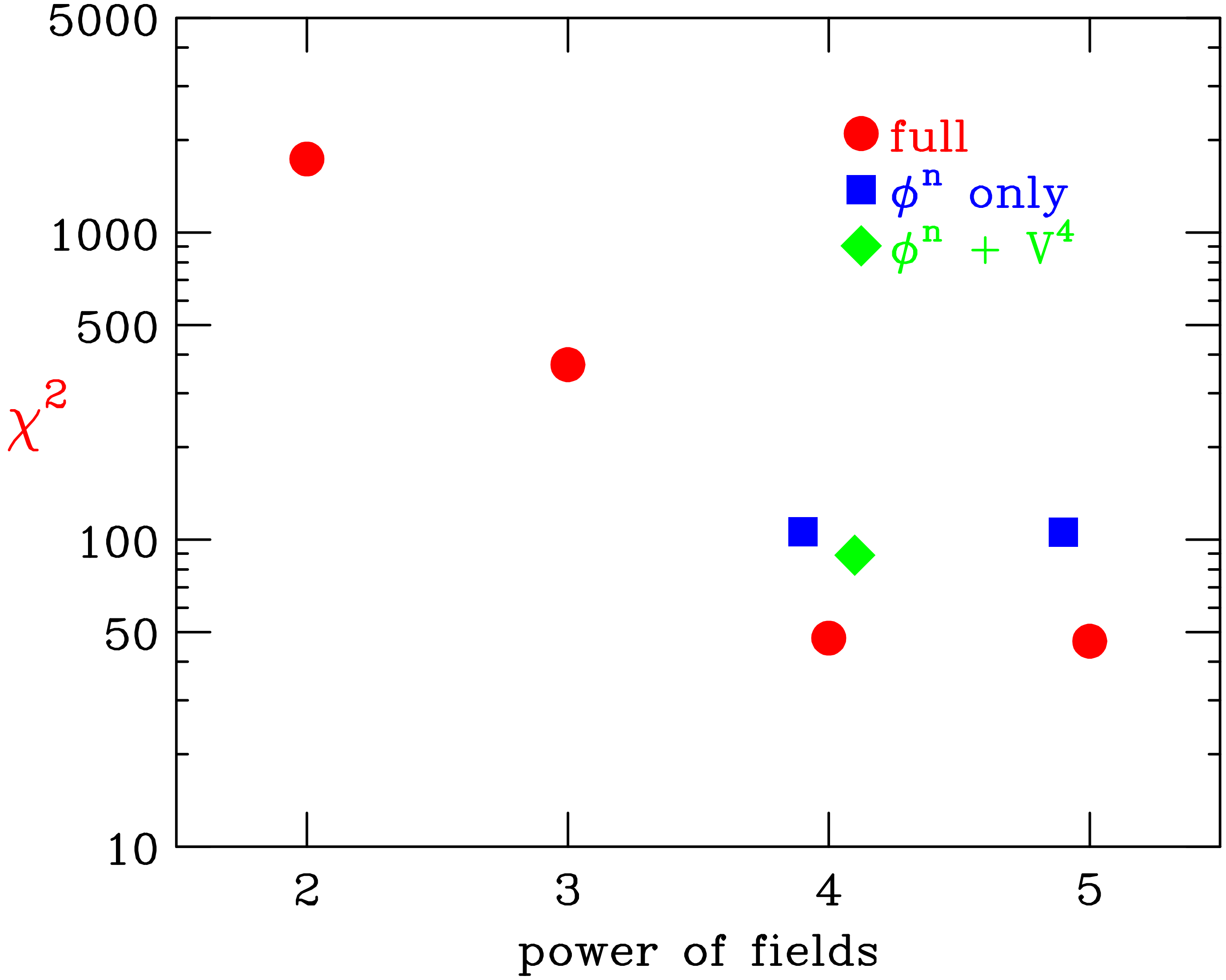
and is independent of Λ. This is a general result!

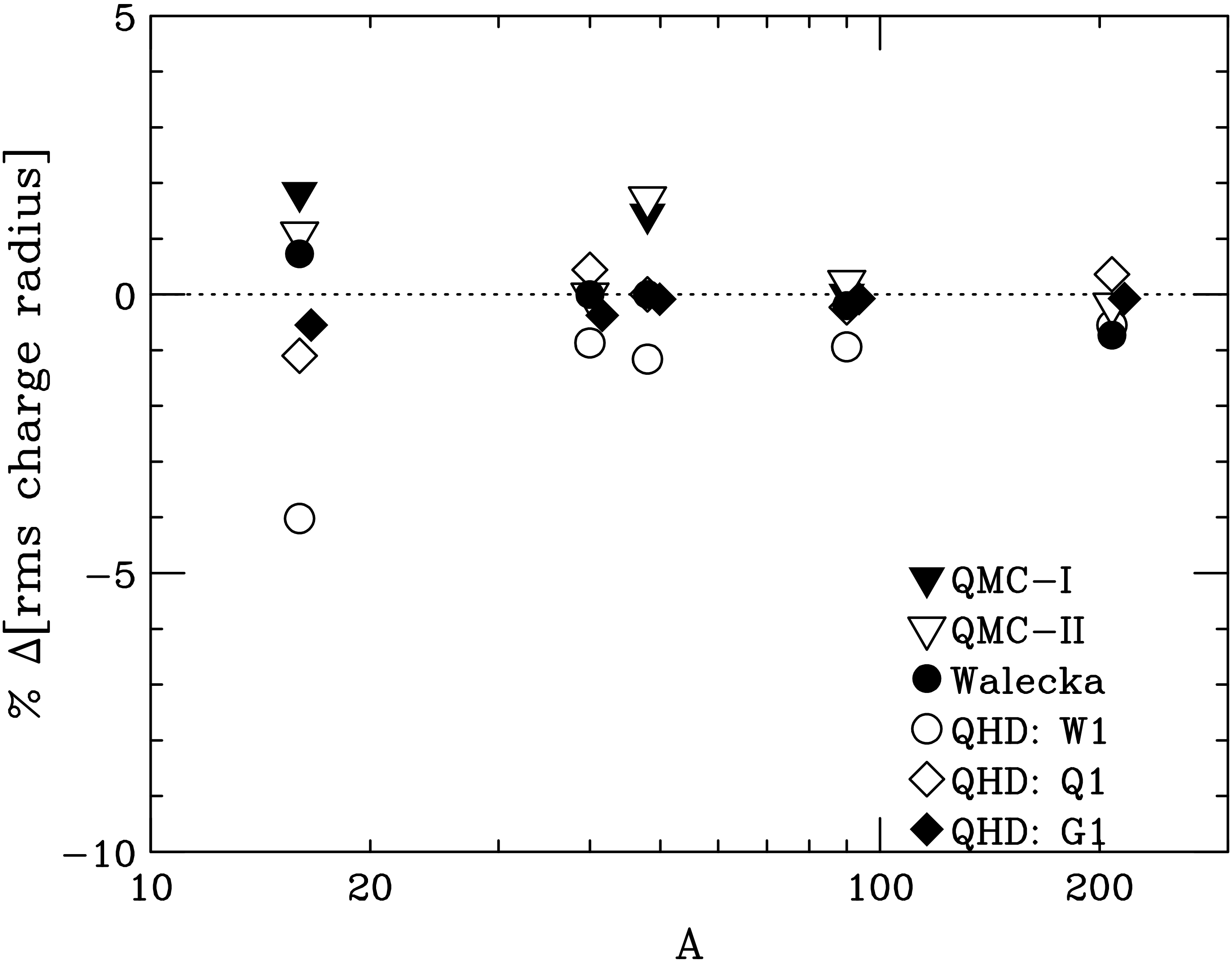
Table 1: Parameter sets from fits to finite nuclei. The parameters in the lower portion of the table are fitted to the (free) nucleon charge and magnetic form factors.

	ν	$W1$	$C1$	$Q1$	$Q2$	$G1$	$G2$
$m_{\rm s}/M$	2	0.60305	0.53874	0.53735	0.54268	0.53963	0.55410
$g_{\rm s}/4\pi$	2	0.93797	0.77756	0.81024	0.78661	0.78532	0.83522
$g_{\rm v}/4\pi$	2	1.13652	0.98486	1.02125	0.97202	0.96512	1.01560
$g_{\rho}/4\pi$	2	0.77787	0.65053	0.70261	0.68096	0.69844	0.75467
η_1	3		0.29577			0.07060	0.64992
κ_3	3		1.6698	1.6582	1.7424	2.2067	3.2467
η_ρ	3					−0.2722	0.3901
η_2	4					−0.96161	0.10975
κ_4	4			−6.6045	−8.4836	−10.090	0.63152
ζ_0	4				−1.7750	3.5249	2.6416
α_1	5					1.8549	1.7234
α_2	5					1.7880	−1.5798
$f_{\rm v}/4$	3					0.1079	0.1734
$f_\rho/4$	3	0.9332	1.1159	1.0332	1.0660	1.0393	0.9619
$\beta^{(0)}$	4	−0.38482	−0.01915	−0.10689	0.01181	0.02844	−0.09328
$\beta^{(1)}$	4	−0.54618	−0.07120	−0.26545	−0.18470	−0.24992	−0.45964

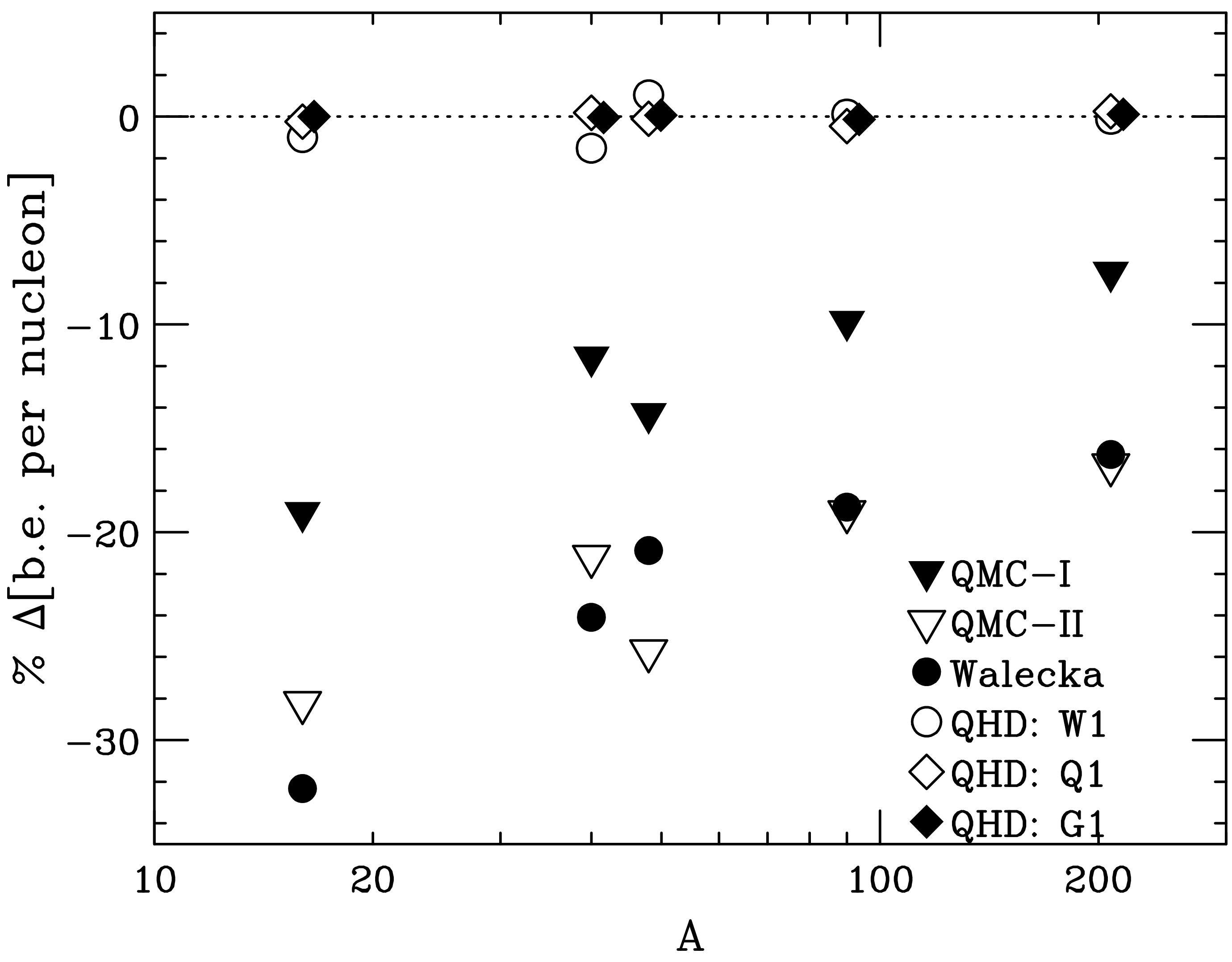


Meson Field Models
energy/particle (MeV)
power of fields
vector
scalar
mixed
natural
ϵ_0
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
2
3
4
5

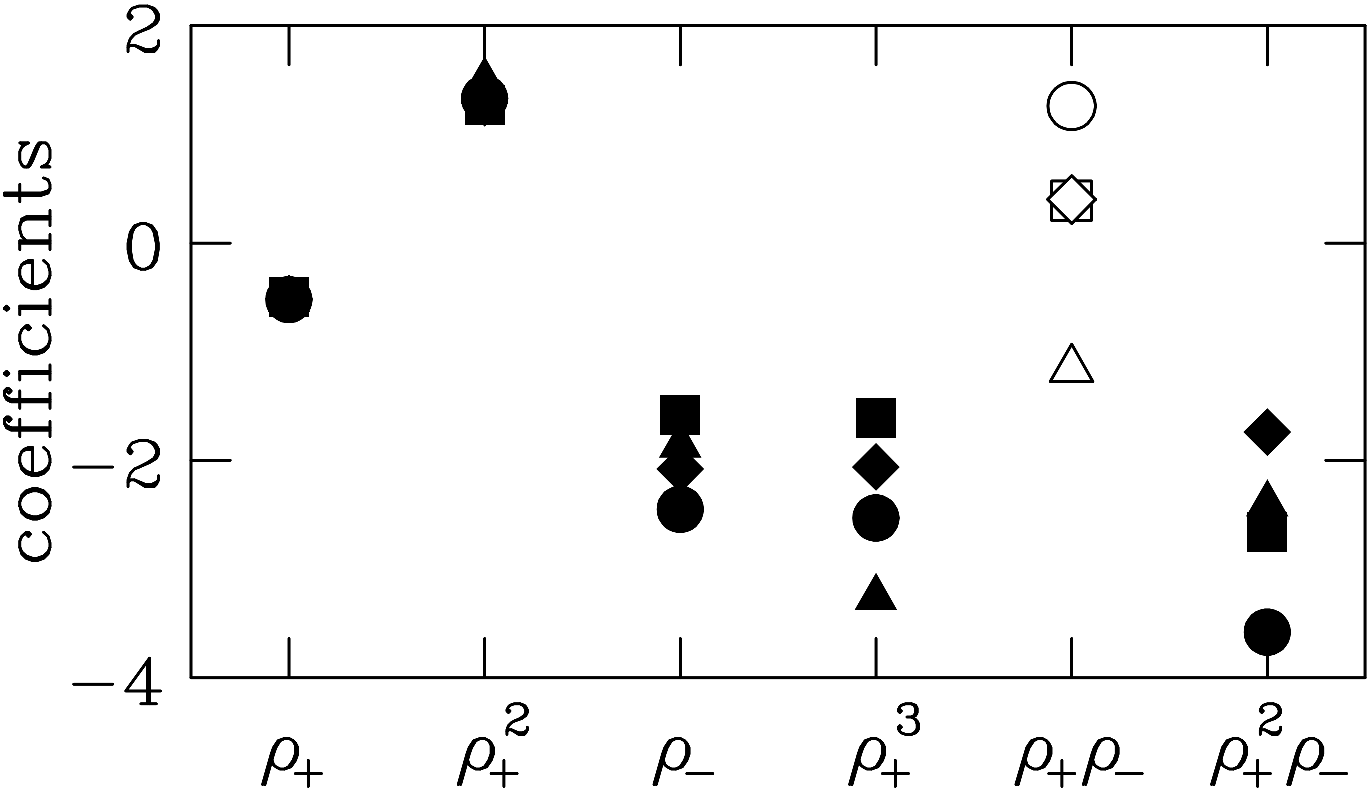

5000
1000
500
100
50
10
χ^2
full
ϕ^n only
$\phi^n + V^4$
2
3
4
5
power of fields



5
0
−5
−10
10
20
100
200
A
% Δ[rms charge radius]
QMC−I
QMC−II
Walecka
QHD: W1
QHD: Q1
QHD: G1

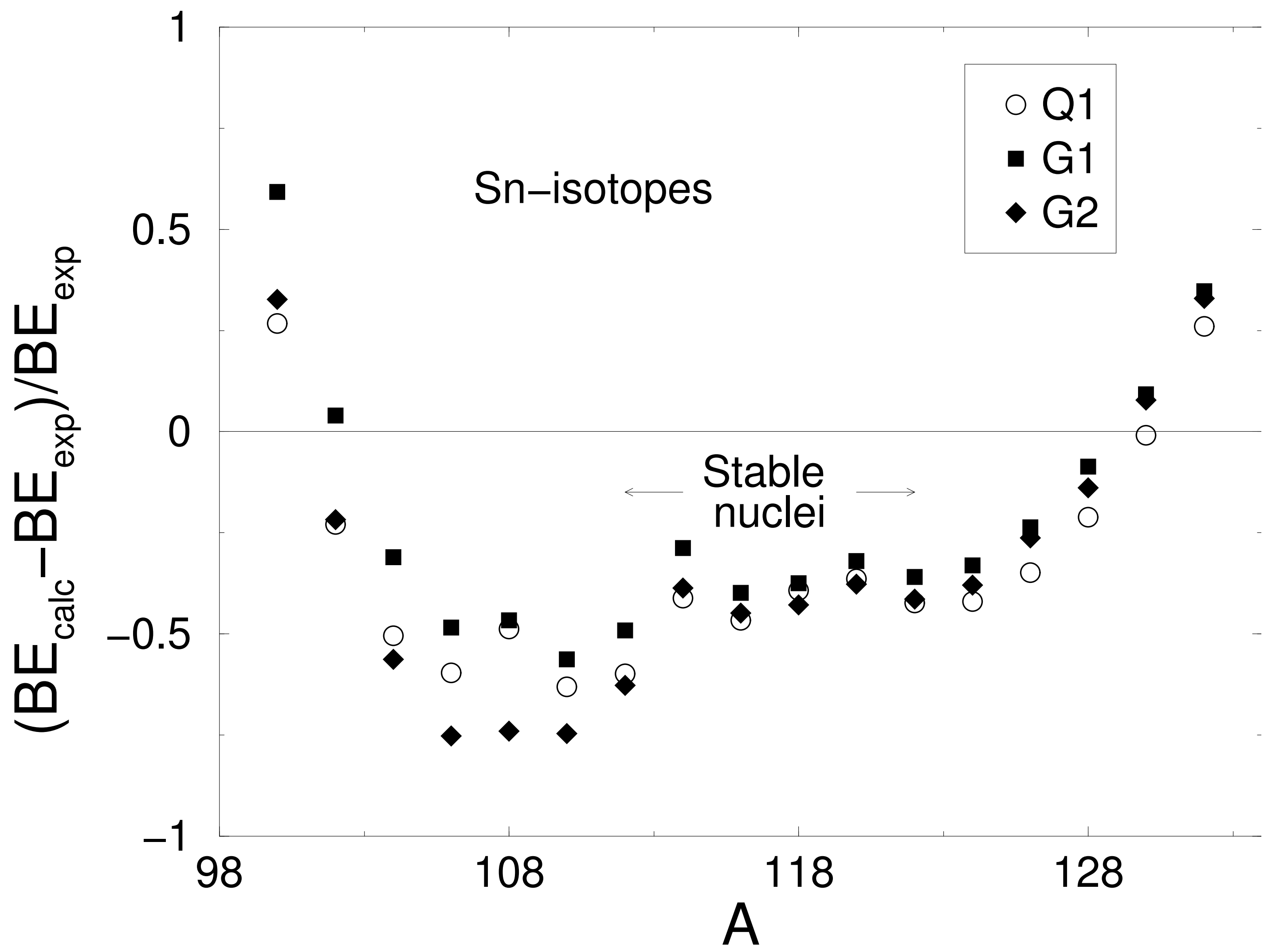


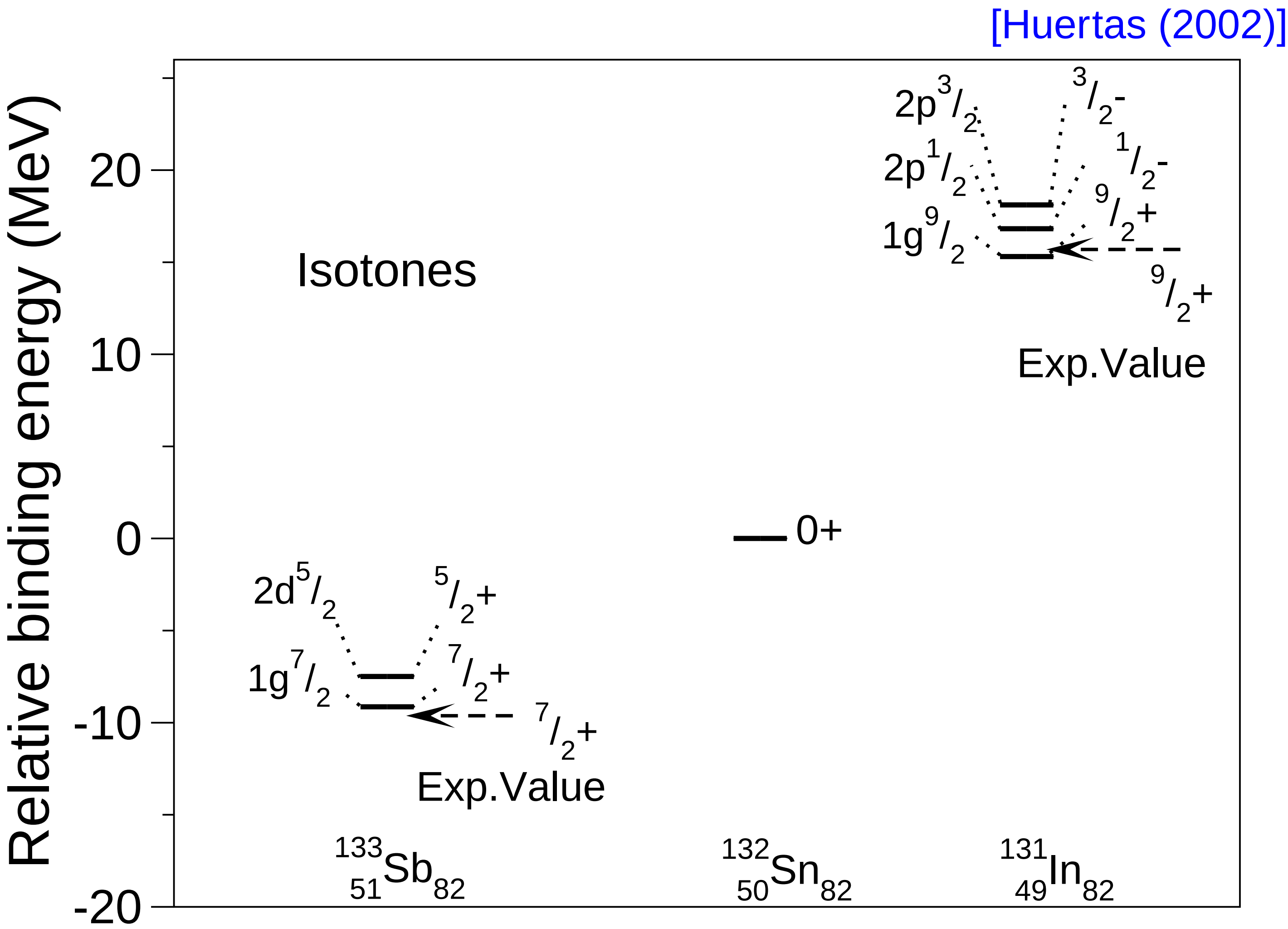
0
−10
−20
−30
% Δ[b.e. per nucleon]
10
20
100
200
A
QMC−I
QMC−II
Walecka
QHD: W1
QHD: Q1
QHD: G1



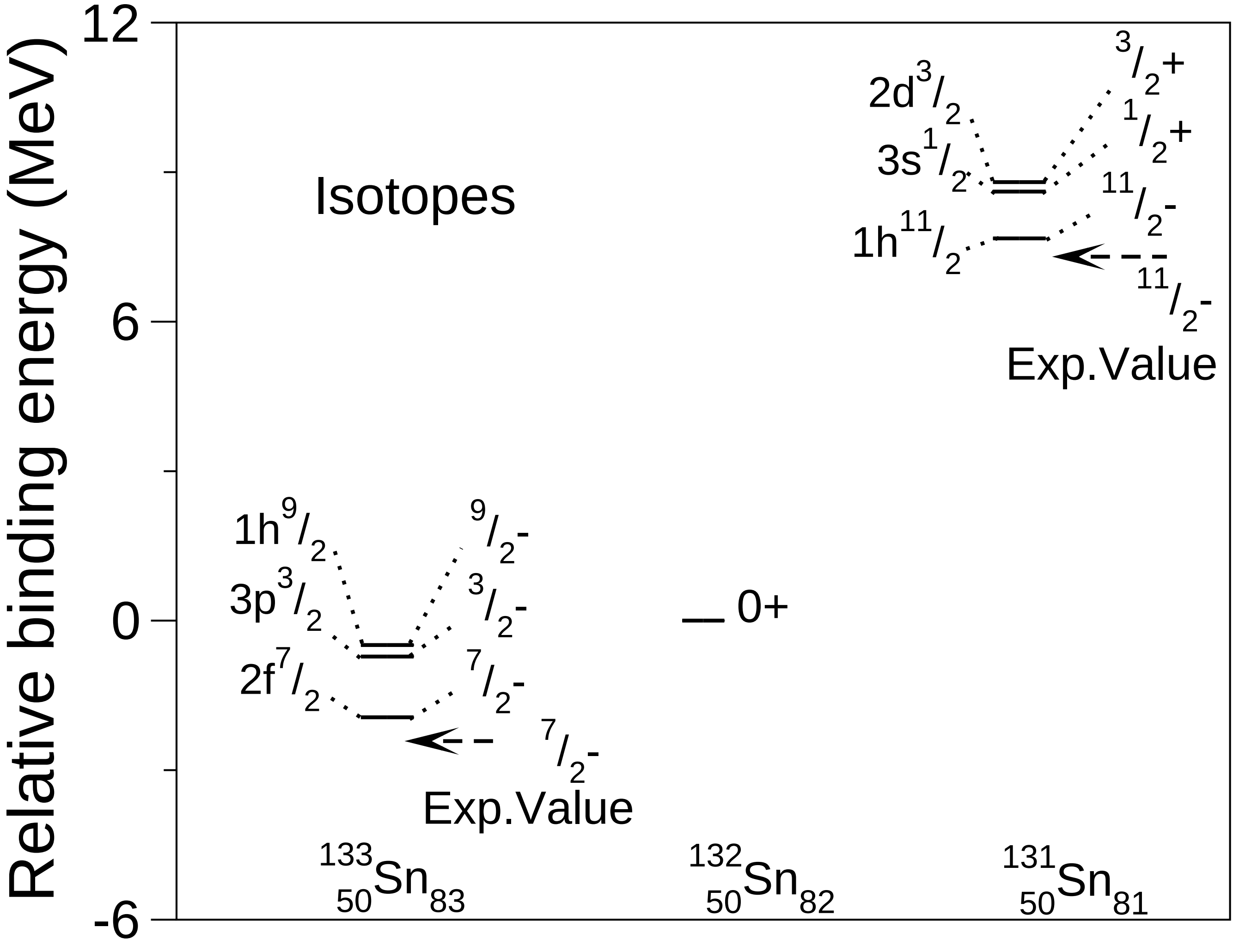
2
0
−2
−4
coefficients
ρ_+
ρ_+^2
ρ_-
ρ_+^3
$\rho_+\rho_-$
$\rho_+^2\rho_-$

[Huertas (2002)]




[Huertas (2002)]
Relative binding energy (MeV)
20
10
0
-10
-20
Isotones
2d5/2
1g7/2
5/2+
7/2+
7/2+
Exp.Value
0+
2p3/2
2p1/2
1g9/2
3/2-
1/2-
9/2+
9/2+
Exp.Value
133 51 Sb 82
132 50 Sn 82
131 49 In 82

[Huertas (2002)]


Relative binding energy (MeV)
12
6
0
-6
Isotopes
2d3/2
3s1/2
1h11/2
3/2+
1/2+
11/2-
11/2-
Exp.Value
1h9/2
3p3/2
2f7/2
9/2-
3/2-
7/2-
7/2-
Exp.Value
0+
133 50 Sn 83
132 50 Sn 82
131 50 Sn 81

Electroweak Currents

A desirable theory of the vector and axial-vector currents should satisfy:

- The same degrees of freedom describe both currents and strong-interaction phenomena (πN and NN scattering; nuclear structure).
- The EFT has the same internal symmetries as the underlying QCD. (Both discrete and continuous; the latter ensure CVC and PCAC.)
- The parameters can be calibrated from strong-interaction phenomena (e.g., hadron scattering and the properties of finite nuclei). This is especially important in EFT, since all allowed (nonredundant) interaction terms appear.

Leading-Order Terms

- These have $\nu = 2$ and produce Noether currents that include the pion field to all orders:

$$
\begin{aligned}
V_2^{a\mu} = -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}\left\{\tau^a\left(U\partial^\mu U^\dagger + U^\dagger\partial^\mu U\right)\right\} \\
+\tfrac{1}{4}\,\overline{N}\gamma^\mu\left[\xi\tau^a\xi^\dagger + \xi^\dagger\tau^a\xi\right]N \\
+\tfrac{1}{4}\,g_A\overline{N}\gamma^\mu\gamma_5\left[\xi\tau^a\xi^\dagger - \xi^\dagger\tau^a\xi\right]N\,, \\
A_2^{a\mu} = -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}\left\{\tau^a\left(U\partial^\mu U^\dagger - U^\dagger\partial^\mu U\right)\right\} \\
-\tfrac{1}{4}\,\overline{N}\gamma^\mu\left[\xi\tau^a\xi^\dagger - \xi^\dagger\tau^a\xi\right]N \\
-\tfrac{1}{4}\,g_A\overline{N}\gamma^\mu\gamma_5\left[\xi\tau^a\xi^\dagger + \xi^\dagger\tau^a\xi\right]N\,.
\end{aligned}
$$

- In the presence of an external axial-vector source, the scattering amplitudes satisfy CVC, PCAC (when $m_\pi \neq 0$), and the Goldberger–Treiman relation (with $g_A \neq 1$) automatically.

- The chiral charges Q^a and Q_5^a also satisfy the familiar charge algebra to all orders in the pion fields. (a is the isospin index.)

Chiral Charge Algebra

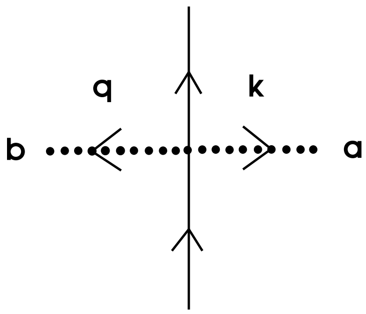
$$\left[Q^a, Q^b\right] = i\epsilon^{abc}\, Q^c \,,$$

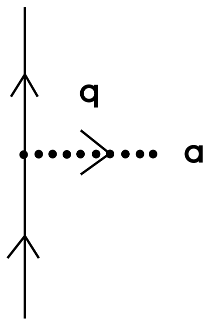
$$\left[Q^a, Q_5^b\right] = i\epsilon^{abc}\, Q_5^c \,,$$

$$\left[Q_5^a, Q_5^b\right] = i\epsilon^{abc}\, Q^c \,.$$

One-Body Axial-Vector Current

- Vertices to leading order in π fields: [dots = pions]

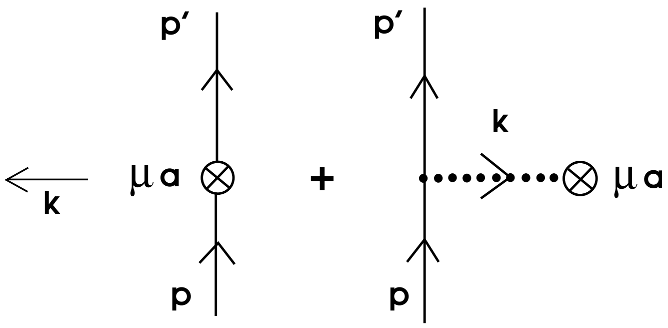




- The axial current to leading order in π is

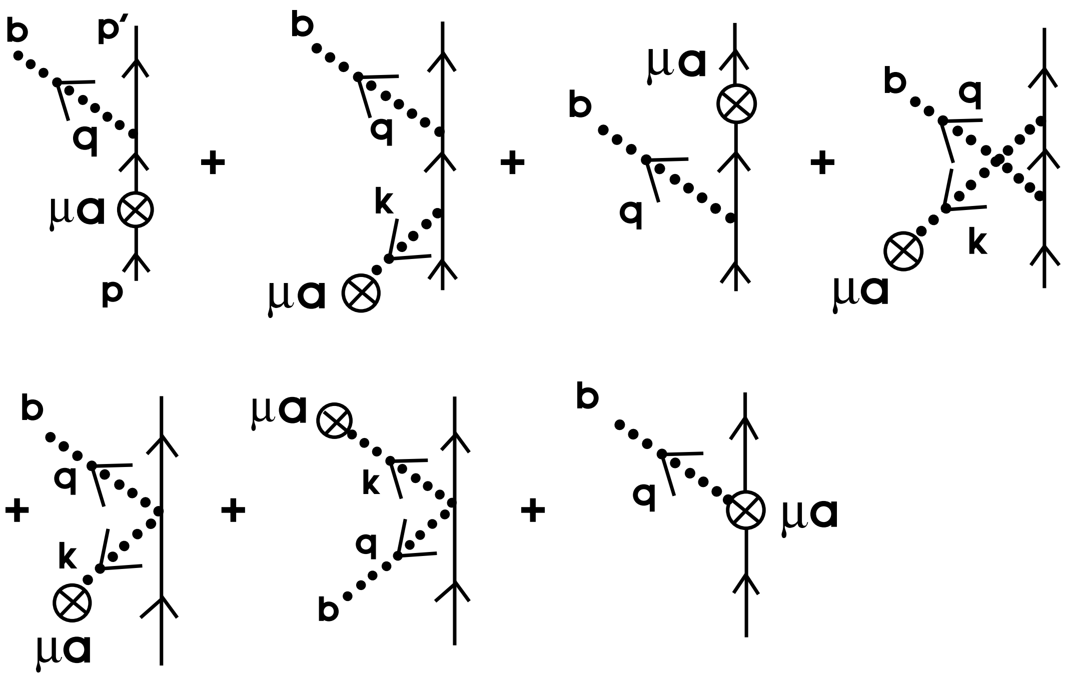
$$A_2^{a\mu} = -\frac{1}{f_\pi}\,\epsilon^{abc}\pi^b\,\overline{N}\gamma^\mu\frac{\tau^c}{2}\,N - g_A\,\overline{N}\gamma^\mu\gamma_5\frac{\tau^a}{2}\,N - f_\pi\,\partial^\mu\pi^a\ .$$

- These lead to a one-body, axial-vector current amplitude (circle with cross denotes the external axial-vector source $S^{\text{ext}}_{a\mu}$):

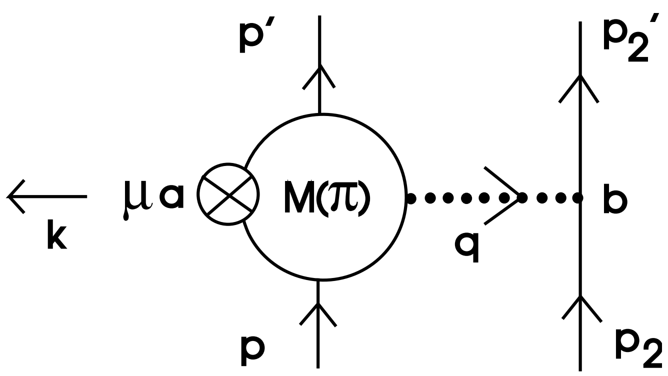


Leading-Order Scattering Amplitudes

- Axial-vector pion production:



- Two-body, axial-vector current amplitude:



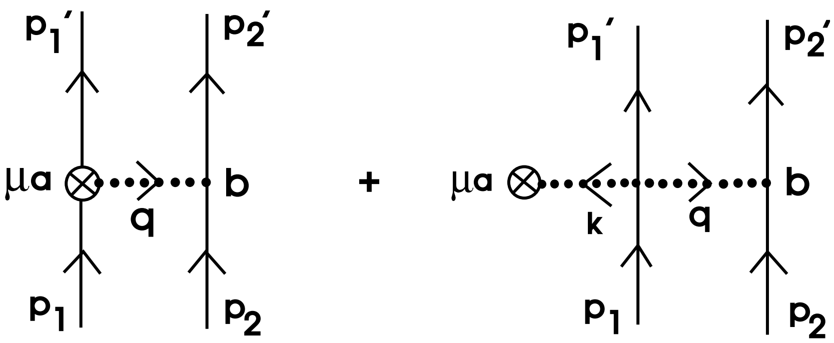
$+$ (direct)$_2$ $+$ exchange.

Next-to-Leading-Order Contributions

- One can extend the preceding analysis to include πN terms with $\nu = 3$. There are more Feynman diagrams and amplitudes, but
  - ∗ CVC, PCAC,
  - ∗ the Goldberger–Treiman relation, and
  - ∗ the chiral charge algebra (to all orders in π)

  remain correct.

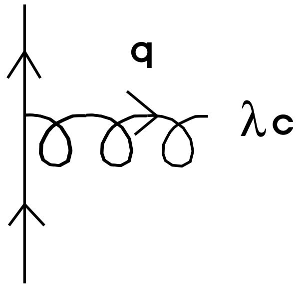
- The two-body, axial-vector current amplitudes at this order look like

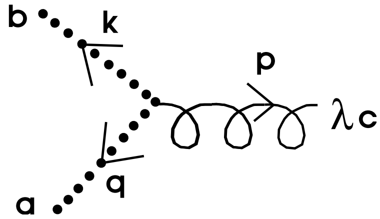


with additional direct and exchange diagrams (as required), and different factors at the vertices coming from the $\nu = 3$ terms in the lagrangian.

Rho-Meson Contributions

- Keep terms in the lagrangian through order $\nu = 4$, but expand currents to leading order in ρ and π for brevity. Retain the $\rho\pi\pi$ coupling term $(\nu = 4)$, since lower-order terms go away.
- Desired CVC, PCAC, G–T, and chiral algebra are still valid!
- We have the additional vertices: [cycloids = rhos]



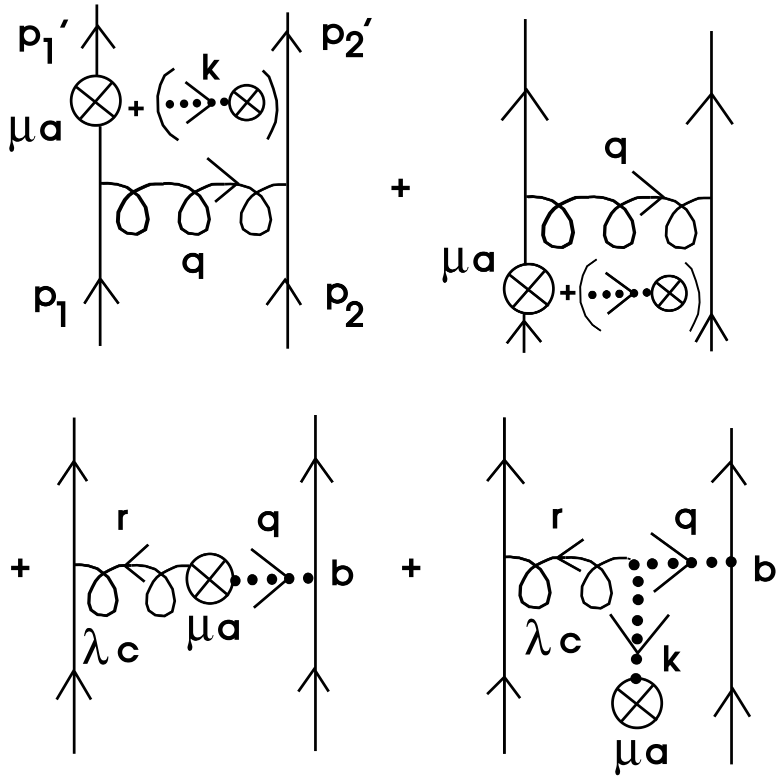


- The additional axial-vector current is:

$$\delta A_a^\lambda = g_{\rho\pi\pi} \frac{2f_\pi}{m_\rho^2} \epsilon^{abc} \partial_\nu \pi^b \left(\partial^\lambda \rho^\nu - \partial^\nu \rho^\lambda\right)^c + O(\rho^2\pi,\ \rho\pi^3) \, ,$$

which depends on the rho–pi–pi coupling.

- The additional two-body, axial-vector amplitudes look like:



$+$ (direct)$_2$ $+$ exchange.

- The first two diagrams show that the axial-vector current can couple to the nucleon line in two different ways (pion-pole dominance).

- The first two diagrams can also have σ and ω exchange substituted for the ρ exchange.

Complete Isovector Currents

$$
\begin{aligned}
V_a^\mu \;=\; & -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}[\tau_a(U\partial^\mu U^\dagger + U^\dagger\partial^\mu U)] \\
& + \frac{1}{2}\,\overline{N}\gamma^\mu \left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right) N \\
& + \frac{1}{2}\,g_A\overline{N}\gamma^\mu\gamma_5 \left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right) N \\
& + i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right]\sigma^{\mu\nu} N \\
& + \frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right) a^\mu\right] \\
& + i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\sigma^{\mu\nu} N \\
& + 2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\,\mathrm{Tr}\left\{\rho^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right), a_\nu\right]\right. \\
& \qquad\left. + v^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\right\} \\
& + i\,\mathrm{Tr}\left\{\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right), \rho_\nu\right]\rho^{\mu\nu}\right\}
\end{aligned}
$$

$$
\begin{aligned}
A_a^\mu \;=\; & -i\,\frac{f_\pi^2}{4}\,\mathrm{Tr}[\tau_a(U\partial^\mu U^\dagger - U^\dagger\partial^\mu U)] \\
& -\frac{1}{2}\,\overline{N}\gamma^\mu\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right)N \\
& -\frac{1}{2}\,g_A\overline{N}\gamma^\mu\gamma_5\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right)N \\
& -i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right),a_\nu\right]\sigma^{\mu\nu}N \\
& -\frac{4\beta_\pi}{M}\,\overline{N}N\,\mathrm{Tr}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right)a^\mu\right] \\
& -i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right),\rho_\nu\right]\sigma^{\mu\nu}N \\
& -2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\;\mathrm{Tr}\left\{\rho^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger + \xi^\dagger\frac{\tau_a}{2}\xi\right),a_\nu\right]\right. \\
& \qquad\qquad \left. +\,v^{\mu\nu}\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right),\rho_\nu\right]\right\} \\
& -i\,\mathrm{Tr}\left\{\left[\left(\xi\frac{\tau_a}{2}\xi^\dagger - \xi^\dagger\frac{\tau_a}{2}\xi\right),\rho_\nu\right]\rho^{\mu\nu}\right\}
\end{aligned}
$$

Review: QCD + QED Lagrangian

Consider massless, two-flavor QCD: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$.

$$\begin{aligned} \mathcal{L}_{\rm QCD} &= \overline{\psi}[i\gamma^\mu(\partial_\mu + \frac{i}{2}g\lambda^a\mathcal{A}^a_\mu)]\psi - \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}^{a\mu\nu} \\ &= \overline{\psi}i\gamma^\mu\partial_\mu\psi + \cdots \ , \end{aligned}$$

which has a global, linear $SU(2)_L \times SU(2)_R$ symmetry.

Now add Electrodynamics:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}i\gamma^\mu\partial_\mu\psi - eA^\mu\overline{\psi}\gamma_\mu Q\psi + \cdots \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}_L i\gamma^\mu(\partial_\mu + ieQA_\mu)\psi_L \\ &\quad + \overline{\psi}_R i\gamma^\mu(\partial_\mu + ieQA_\mu)\psi_R + \cdots \ , \end{aligned}$$

where $\quad \psi_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi \ , \quad Q = \frac{1}{2}(\frac{1}{3} + \tau_3)$.

Under chiral transformations: $\psi_L \rightarrow L\psi_L$ and $\psi_R \rightarrow R\psi_R$.

So the lagrangian possesses a residual global symmetry:

$$U(1)_{L_3} \times U(1)_{R_3} \times U(1)_B \ ,$$

which must also be present in the low-energy EFT.

Electromagnetic Interactions in QHD

$$\mathcal{L}_{\mathrm{EM}} = \mathcal{L}_{\mathrm{EM}}^{\mathrm{min}} + \mathcal{L}_{\mathrm{EM}}^{\mathrm{had}} + \mathcal{L}_{\mathrm{EM}}^{\mathrm{vmd}} + \mathcal{L}_{\mathrm{EM}}^{\mathrm{anom}} \ .$$

The four contributions describe, respectively,

- $\mathcal{L}_{\mathrm{EM}}^{\mathrm{min}}$: terms arising from minimal substitution, obtained by replacing ordinary derivatives in $\mathcal{L}_{\mathrm{EFT}}$ with EM gauge-covariant derivatives (these terms are necessary);
- $\mathcal{L}_{\mathrm{EM}}^{\mathrm{had}}$: non-minimal terms in a derivative expansion, which will serve to describe some of the hadronic EM structure;
- $\mathcal{L}_{\mathrm{EM}}^{\mathrm{vmd}}$: VMD terms that contain the coupling of the photon to neutral vector mesons (and pions);
- $\mathcal{L}_{\mathrm{EM}}^{\mathrm{anom}}$: EM terms associated with chiral anomalies, which describe, among other things, mesonic decays like $\pi^0 \to \gamma\gamma$.

To include EM interactions, elevate a subgroup of the full symmetry to the status of a **local** symmetry. The local $U(1)_Q$ symmetry has the generator ("electric charge") $Q = \frac{1}{2}B + T_3$.

Under $U(1)_Q$, the EM field A_μ transforms in the familiar way

$$A_\mu \longrightarrow A_\mu - \frac{1}{e}\,\partial_\mu \alpha(x)\ .$$

The nucleon, pion and rho fields transform as $N \longrightarrow qN$, $\xi \longrightarrow q\xi q^\dagger$, $\rho_\mu \longrightarrow q\rho_\mu q^\dagger$, with

$$q(x) \equiv \exp\left[i\alpha(x)\left(\frac{B+\tau_3}{2}\right)\right]\ ,$$

To enforce the local symmetry, we define the gauge-covariant derivatives (denoted with a tilde)

$$\begin{aligned}
\widetilde{\partial}_\mu N &\equiv \left[\partial_\mu + \frac{i}{2}\,eA_\mu(1+\tau_3)\right] N\ , \\
\widetilde{\partial}_\mu U &\equiv \partial_\mu U + ieA_\mu\left[\frac{\tau_3}{2}, U\right]\ , \\
\widetilde{\partial}_\mu \xi &\equiv \partial_\mu \xi + ieA_\mu\left[\frac{\tau_3}{2}, \xi\right]\ , \\
\widetilde{\partial}_\mu \rho_\nu &\equiv \partial_\mu \rho_\nu + ieA_\mu\left[\frac{\tau_3}{2}, \rho_\nu\right]\ .
\end{aligned}$$

$$\mathcal{L}_{\rm EM}^{\rm min} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e A_\mu J^\mu_{\rm min} + \mathcal{L}_{e^2}^{\rm min}$$

$$\begin{aligned}
J^\mu_{\rm min} \;\; = \;\; & -i\,\frac{f_\pi^2}{4}\,{\rm Tr}[\tau_3(U\partial^\mu U^\dagger + U^\dagger \partial^\mu U)] \\
& + \frac{1}{2}\,\overline{N}\gamma^\mu \left(1 + \xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right) N \\
& + \frac{1}{2}\,g_A \overline{N}\gamma^\mu\gamma_5 \left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right) N \\
& + i\,\frac{\kappa_\pi}{M}\,\overline{N}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right), a_\nu\right]\sigma^{\mu\nu} N \\
& + \frac{4\beta_\pi}{M}\,\overline{N} N\,{\rm Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right) a^\mu\right] \\
& + i\,\frac{f_\rho g_\rho}{4M}\,\overline{N}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\sigma^{\mu\nu} N \\
& + i\,{\rm Tr}\left\{\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\rho^{\mu\nu}\right\} \\
& + 2ig_{\rho\pi\pi}\frac{f_\pi^2}{m_\rho^2}\;{\rm Tr}\left(\rho^{\mu\nu}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right), a_\nu\right]\right. \\
& \qquad\qquad\qquad \left. + v^{\mu\nu}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right), \rho_\nu\right]\right)
\end{aligned}$$

$$Q = \int \mathrm{d}^3x \left[\frac{1}{2} N^\dagger (1+\tau_3) N + (\boldsymbol{\pi} \times \boldsymbol{P}_\pi)_3 + (\boldsymbol{\rho}_\nu \times \boldsymbol{P}_\rho^\nu)_3 \right]$$

$$\begin{aligned}
\mathcal{L}_{e^2}^{\min} \;=\; & e^2 A_\mu A^\mu \, \frac{f_\pi^2}{4} \left(1 + \frac{4\beta_\pi}{f_\pi^2 M} \, \overline{N} N \right) \\
& \times \mathrm{Tr} \left[\left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi \right) \left(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi \right) \right] \\
& + \frac{e^2}{4} \Big\{ A_\mu A^\mu \, \mathrm{Tr} \Big(\Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho_\nu \Big] \\
& \qquad \times \Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho^\nu \Big] \Big) \\
& \quad - A_\mu A^\nu \, \mathrm{Tr} \Big(\Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho_\nu \Big] \\
& \qquad \times \Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho^\mu \Big] \Big) \Big\} \\
& + e^2 g_{\rho\pi\pi} \frac{f_\pi^2}{m_\rho^2} \Big\{ A_\mu A^\mu \, \mathrm{Tr} \Big(\Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho_\nu \Big] \\
& \qquad \times \Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi \Big) , a^\nu \Big] \Big) \\
& \quad - A_\mu A^\nu \, \mathrm{Tr} \Big(\Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \Big) , \rho_\nu \Big] \\
& \qquad \times \Big[\Big(\xi \frac{\tau_3}{2} \xi^\dagger - \xi^\dagger \frac{\tau_3}{2} \xi \Big) , a^\mu \Big] \Big) \Big\}
\end{aligned}$$

Discussion

- The individual parts of the lagrangian are not EM gauge invariant by themselves; only $\widetilde{\mathcal{L}}_{\rm EFT} \equiv \mathcal{L}_{\rm EFT} + \mathcal{L}_{\rm EM}^{\rm min}$ is. The $O(e^2)$ “seagull” terms involving mesons and two photons are crucial for maintaining EM gauge invariance.

- The current $J^\mu_{\rm min} = \frac{1}{2}B^\mu + V_3^\mu$; thus, $J^\mu_{\rm min}$ is conserved through $O(e^0)$. The baryon current obeys $\partial_\mu B^\mu = 0$.

- The current $J^\mu_{\rm min}$, however, is not exactly conserved due to the EM interactions: $\partial_\mu J^\mu_{\rm min} = \partial_\mu V_3^\mu = O(e) \neq 0$. Thus we cannot identify $J^\mu_{\rm min}$ as the EM current.

- We can find the unique, conserved, minimal EM current using Maxwell’s equations:

$$\partial_\nu F^{\nu\mu} = e\left(J^\mu_{\rm min} - \frac{1}{e}\frac{\partial \mathcal{L}^{\rm min}_{e^2}}{\partial A_\mu}\right) = e\,\widetilde{J}^\mu_{\rm min}\ ,$$

  where the final equality follows from algebra!

- Evidently, $\partial_\mu \widetilde{J}^\mu_{\rm min} = 0$, consistent with its identification as the EM current.

Residual Chiral Symmetry

- For global left- and right-handed rotations about the third axis in isospin space (L_3, R_3), the original field transformations reduce to

$$\begin{aligned} \xi(x) \to \xi'(x) &= L_3 \xi(x) \widetilde{h}^\dagger(x) = \widetilde{h}(x) \xi(x) R_3^\dagger \,, \\ N(x) \to N'(x) &= \widetilde{h}(x) N(x) \,, \\ \rho_\mu(x) \to \rho'_\mu(x) &= \widetilde{h}(x) \rho_\mu(x) \widetilde{h}^\dagger(x) \,. \end{aligned}$$

  Note that $\widetilde{h}(x)$ generally involves isospin rotations in all directions.

- The vector field v_μ does not appear explicitly in $\mathcal{L}_{\mathrm{EM}}^{\mathrm{min}}$. All the remaining meson tensors: a_μ, ρ_μ, $\rho_{\mu\nu}$, and $v_{\mu\nu}$ transform *homogeneously*. For example,

$$\rho_{\mu\nu} \longrightarrow \rho'_{\mu\nu} = \widetilde{h} \rho_{\mu\nu} \widetilde{h}^\dagger \,, \mathrm{etc.}$$

- So do the pion fields $\mathcal{Q}_\pm \equiv \frac{1}{2}\left(\xi \tau_3 \xi^\dagger \pm \xi^\dagger \tau_3 \xi\right)$, since

$$\begin{aligned} \xi^\dagger \tau_3 \xi &\to \xi'^\dagger \tau_3 \xi' = (\widetilde{h} \xi^\dagger L_3^\dagger) \tau_3 (L_3 \xi \widetilde{h}^\dagger) = \widetilde{h} (\xi^\dagger \tau_3 \xi) \widetilde{h}^\dagger \,, \\ \xi \tau_3 \xi^\dagger &\to \xi' \tau_3 \xi'^\dagger = (\widetilde{h} \xi R_3^\dagger) \tau_3 (R_3 \xi^\dagger \widetilde{h}^\dagger) = \widetilde{h} (\xi \tau_3 \xi^\dagger) \widetilde{h}^\dagger \,. \end{aligned}$$

- The residual invariance of $\widetilde{\mathcal{L}}_{\mathrm{EFT}}$ now follows by inspection.

Conserved Currents

- What has become of the isovector currents $\mathbf{V}^\mu$ and $\mathbf{A}^\mu$?
- With the addition of the EM interactions, the gauged currents $\widetilde{\mathbf{V}}^\mu$ and $\widetilde{\mathbf{A}}^\mu$ are no longer isovectors. Nevertheless, . . .
- The gauged lagrangian $\widetilde{\mathcal{L}}_{\rm EFT}$ admits three conserved currents, one of which is B^μ. The other two conserved currents are the gauged neutral currents $\widetilde{V}_3^\mu$ and $\widetilde{A}_3^\mu$. The corresponding charged currents $\widetilde{V}_\pm^\mu$ and $\widetilde{A}_\pm^\mu$ are not exactly conserved.
- In fact, the Adler–Coleman theorem (1965) implies

$$\partial_\mu \widetilde{V}_a^\mu = \epsilon_{a3b}\, eA_\mu \widetilde{V}_b^\mu \ , \qquad \partial_\mu \widetilde{A}_a^\mu = \epsilon_{a3b}\, eA_\mu \widetilde{A}_b^\mu \ .$$

  The divergence of the axial-vector current $\widetilde{\mathbf{A}}^\mu$ omits contributions from chiral anomalies and from the explicit breaking of chiral symmetry. If the latter were included, we would have the PCAC relation

$$\partial_\mu \widetilde{A}_a^\mu \propto m_\pi^2 \pi_a + O(e) \ .$$

- Only the neutral charges are constants of the motion, and

$$[B, Q_3] = [B, (Q_5)_3] = [Q_3, (Q_5)_3] = [Q, (Q_5)_3] = 0.$$

Non-Minimal EM Couplings

- These involve the EM field tensor $F_{\mu\nu}$ and its derivatives.
- Recall our derivative expansion:

$$\mathcal{L}_{\rm old} = -\frac{e}{4M} F_{\mu\nu} \overline{N} \lambda \sigma^{\mu\nu} N - \frac{e}{2M^2} (\partial^\nu F_{\mu\nu}) \overline{N} \beta \gamma^\mu N + \cdots$$

where $\lambda = \lambda_{\rm p} \frac{1}{2}(1+\tau_3) + \lambda_{\rm n} \frac{1}{2}(1-\tau_3) \equiv \lambda^{(0)} + \lambda^{(1)} \tau_3$, and similarly for β.

- This $\mathcal{L}_{\rm old}$ is EM gauge invariant, but it does not respect the residual chiral symmetry. So we must take instead

$$\mathcal{L}_{\rm EM}^{\rm had} = -\frac{e}{4M} F_{\mu\nu} \overline{N}\, \widetilde{\lambda} \sigma^{\mu\nu} N - \frac{e}{2M^2} (\partial^\nu F_{\mu\nu}) \overline{N}\, \widetilde{\beta} \gamma^\mu N \,,$$

$$\widetilde{\lambda} \equiv \lambda^{(0)} + \lambda^{(1)} \left(\xi \frac{\tau_3}{2} \xi^\dagger + \xi^\dagger \frac{\tau_3}{2} \xi \right) = \lambda^{(0)} + \lambda^{(1)} \mathcal{Q}_+ \,,$$

and similarly for $\widetilde{\beta}$. Pion fields must be included.

- These new couplings do not modify the tree-level expressions for the nucleon EM form factors.

- Partial integration allows us to write

$$\mathcal{L}_{\mathrm{EM}}^{\mathrm{had}} = -eA_\mu \widetilde{J}_{\mathrm{had}}^\mu \,,$$

$$\widetilde{J}_{\mathrm{had}}^\mu = \frac{1}{2M}\,\partial_\nu(\overline{N}\,\widetilde{\lambda}\sigma^{\mu\nu}N) - \frac{1}{2M^2}\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)(\overline{N}\,\widetilde{\beta}\gamma_\nu N)$$

- Note that $\partial_\mu \widetilde{J}_{\mathrm{had}}^\mu = 0$ follows by inspection.
- The EM current determined thus far is given by

$$\widetilde{J}_{\mathrm{min}}^\mu + \widetilde{J}_{\mathrm{had}}^\mu = \frac{1}{2}B^\mu + \widetilde{V}_3^\mu + \widetilde{J}_{\mathrm{had}}^\mu \,.$$

- Here $\widetilde{J}_{\mathrm{min}}^\mu$ and $\widetilde{J}_{\mathrm{had}}^\mu$ are **independently** conserved, the latter identically and the former by virtue of the Euler–Lagrange equations.
- The freedom to add $\widetilde{J}_{\mathrm{had}}^\mu$, which generally contains both isoscalar and isovector parts, reflects the non-uniqueness of the EM current in the effective theory. (Recall that in QCD with u and d quarks, the EM current is simply $\frac{1}{2}B^\mu + V_3^\mu$.) The EM charge operator Q, however, is still given by the previous expression.
- There are also non-minimal terms with only pions (not shown). These require EM gauge-covariant derivatives $\widetilde{\partial}_\mu$.

- The lowest-order $(\nu = 4)$, non-minimal pion–photon couplings are

$$\mathcal{L}_{\mathrm{EM}(\pi)}^{\mathrm{had}} = e\,\omega_1\,\mathrm{Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{v}^{\mu\nu}\right]F_{\mu\nu} + e\,\omega_2\,\mathrm{Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger - \xi^\dagger\frac{\tau_3}{2}\xi\right)\widetilde{a}^{\mu}\right]\partial^\nu F_{\mu\nu}\,.$$

Vector Meson Dominance

- Photon interactions with hadrons are mediated primarily by the exchange of low-mass, neutral vector mesons [ρ^0, ω, $\phi(1020)$.]

- One can describe processes invoving spacelike photons and vector mesons using photon–meson couplings determined from hadronic decay widths, where the meson four-momentum is timelike.

- The direct $\gamma\rho^0$ coupling that respects gauge invariance and the residual chiral symmetry is

$$\mathcal{L}_\rho^{\mathrm{vmd}} = -\frac{e}{2g_\gamma}\,\mathrm{Tr}\left[\left(\xi\frac{\tau_3}{2}\xi^\dagger + \xi^\dagger\frac{\tau_3}{2}\xi\right)\rho^{\mu\nu}\right]F_{\mu\nu}\ .$$

- For the omega, we allow for $SU(3)_f$ symmetry breaking and write

$$\mathcal{L}_\omega = -\frac{e}{2}\left(\frac{\sin\theta_Y}{2g_Y}\right)V^{\mu\nu}F_{\mu\nu}\ .$$

- Under the assumption of ideal mixing, so that the ω contains only u and d valence quarks, $\sin\theta_Y = 1/\sqrt{3}$. The remaining coupling can be evaluated in the $SU(3)_f$ limit $(g_Y = \sqrt{3}\,g_\gamma/2)$, so that

$$\mathcal{L}_\omega = -\frac{e}{2g_\gamma}\left(\frac{1}{3}\right)V^{\mu\nu}F_{\mu\nu}\ .$$

- The additional pionic interactions required by the residual chiral symmetry do not affect the tree-level EM form factors shown earlier. They do, however, lead to new two-nucleon exchange currents (e.g., from a $\rho\pi\pi\gamma$ coupling), in which all the coupling parameters are known from other processes.

- With ideal mixing, the $\phi(1020)$ is composed of valence strange quarks only, and its mass is 30% larger than m_ρ and m_ω. So we "integrate it out". This yields a contribution to the non-minimal $\beta^{(0)}$ parameter equal to

$$\frac{-\sqrt{2}M^2 g_\phi}{3g_\gamma m_\phi^2} .$$

- It is possible to augment the preceding VMD couplings by multiplying the interactions by chiral scalar combinations like ϕ, ϕ^2, $V_\mu V^\mu$, etc. (Here ϕ represents the sigma field.) These terms all have $\nu \geq 5$, but they allow for the possiblity of isoscalar EM exchange currents in nuclei.

Anomalous EM Interactions

- These terms arise from “gauging” the Wess–Zumino–Witten anomalous action and are all well known.
- Although the anomalous action is not manifestly chirally invariant, a chiral transformation produces a variation that is a spacetime derivative, preserving the chiral invariance of the action.
- Parity invariance is maintained by the presence of $\epsilon^{\mu\nu\alpha\beta}$.
- Since the anomalies are perturbative, they can be determined exactly from the underlying QCD.
- The anomalous EM terms contain only bosons, so they enter in EM interactions with nuclei only through meson-exchange currents. Moreover, the anomalous EM couplings
  - ∗ are of $O(e^2)$, like $\pi^0 \longrightarrow \gamma\gamma$;
  - ∗ contain (at least) three pions $(\gamma^* \longrightarrow \pi\pi\pi$, as in $\omega \longrightarrow \gamma^* \longrightarrow \pi\pi\pi)$;
  - ∗ involve a heavy boson $(\omega \longrightarrow \pi^0\gamma$, $\rho \longrightarrow \pi\gamma)$.
- The resulting abnormal-parity exchange currents are unlikely to be very important in EM interactions in the nuclear many-body problem.

Three examples of anomalous EM couplings:

- $\pi^0 \to \gamma\gamma$:

$$\frac{N_c e^2}{96\pi^2 f_\pi} \pi^0 \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \ .$$

- $\omega \to \pi^0\gamma$:

$$\frac{N_c e g_v}{96\pi^2 f_\pi} \pi^0 \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} V_{\alpha\beta} \ .$$

- $\gamma^* \to \pi\pi\pi$:

$$-i \, \frac{N_c e}{12\pi^2 f_\pi^3} \epsilon^{\mu\nu\alpha\beta} A_\mu \partial_\nu \pi^+ \partial_\alpha \pi^- \partial_\beta \pi^0 \ .$$

Here $N_c = 3$ is the number of colors in QCD.

Summary

- Quantum hadrodynamics (QHD) describes strong-coupling, relativistic quantum field thoeries for nuclei based on hadrons, in which the representation is manifestly Lorentz covariant.
- The primary focus is on the nuclear many-body problem.
- Desirable features for electroweak processes in nuclei:
  - ∗ Nuclear currents and nuclear structure are both described by the same lagrangian;
  - ∗ The EFT has the same internal symmetries as QCD.
  - ∗ Parameters can be calibrated using strong-interaction phenomena.
- One can systematically expand and truncate the QHD lagrangian in powers of meson fields and their derivatives.
- EM interactions and VMD can be included straightforwardly, and the residual chiral symmetry of QCD + QED can be maintained.
- The QHD/EFT/DFT/KS formalism provides a true representation of QCD in the low-energy nuclear domain.

Future Work

- Derive two-body scattering amplitudes for electron scattering and pion photoproduction.
- Extract and determine two-body current operators for use with nuclear wave functions (relativistic MFT and nonrelativistic shell model or Skyrme).
- Include the Δ resonance in the currents.
- Assess the usefulness of single-(quasi)particle wave functions in the computation of electroweak exchange-current matrix elements.