

How low-momentum interactions can constrain the nuclear density functional

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University of Washington

Nuclear structure near the limits, INT, Oct. 3, 2005

Collaborators: Scott Bogner, Dick Furnstahl, Andreas Nogga,
Gerry Brown, Tom Kuo and Andres Zuker

Supported by DOE

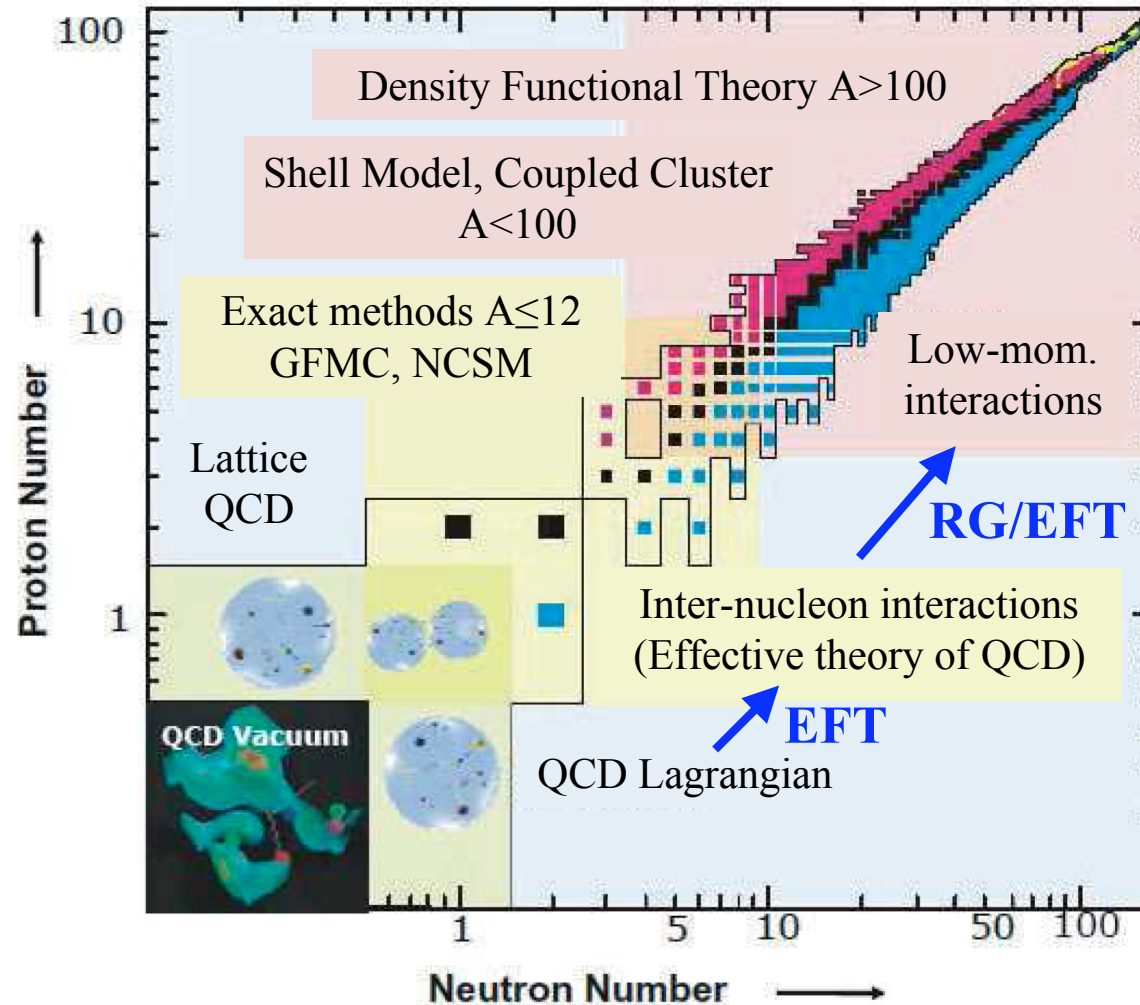
Outline

- 1) Motivation - Progress in nuclear interactions, exact many-body methods, constructing density functionals
- 2) RG applied to nuclear interactions
- 3) Results for nuclei and nuclear matter
- 4) Strategies for density-functional calculations from nuclear interactions
- 5) Summary

1) Motivation

Simple density functionals work: $\Delta E_{\text{rms}} \sim 1$ MeV, excitations,...

But extensions need guidance (many new parameters in general ∇^2 DF)



Advances in nuclear interactions relevant to heavy nuclei

Constraints for DF from microscopic interactions?

Immense progress in many-body methods nuclei accessible with CC overlap with DFT

adapted from A. Richter @ INPC2004

Microscopic foundation of density functional theory

For **ground state densities** couple source $K_{\sigma,\sigma'}(x)$ to density $\psi_{\sigma}^{\dagger}(x) \psi_{\sigma'}(x)$
see e.g., talks by Negele and Furnstahl

$$e^{W[K]} = \int \mathcal{D}[\psi^{\dagger}] \mathcal{D}[\psi] e^{-S[\psi^{\dagger}, \psi] + \sum_{\sigma, \sigma'} \int dx \psi_{\sigma}^{\dagger}(x) \psi_{\sigma'}(x) K_{\sigma, \sigma'}(x)}$$

S contains nuclear forces (e.g., $V_{\text{low } k} + V_{3N}$)

Path integral leads to generating functional $W[K]$

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Densities given by (with $K=0$ in the ground state)

$$\rho_{\sigma, \sigma'}(x) \equiv \langle \psi_{\sigma}^{\dagger}(x) \psi_{\sigma'}(x) \rangle = \frac{\delta W[K]}{\delta K_{\sigma, \sigma'}(x)} \quad \text{and by inversion } K[\rho]$$

Construct **effective action** $\Gamma[\rho] = -W[K] + K \cdot \rho$ by Legendre trafo.

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Effective action is minimal at the physical ($K=0$) ground state density

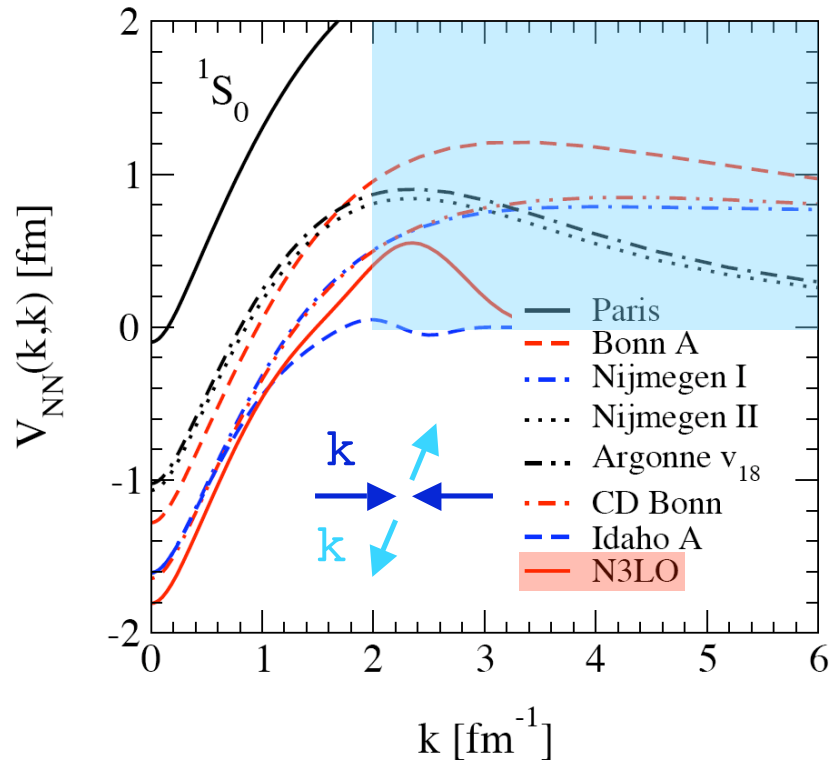
$$\left. \frac{\delta \Gamma[\rho]}{\delta \rho} \right|_{\rho=\rho_{\text{gs}}} = 0 \quad \text{with gs energy } E_{\text{gs}} = E[\rho_{\text{gs}}] = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \Gamma[\rho_{\text{gs}}]$$

DF = effective action, degrees-of-freedom are one-body densities

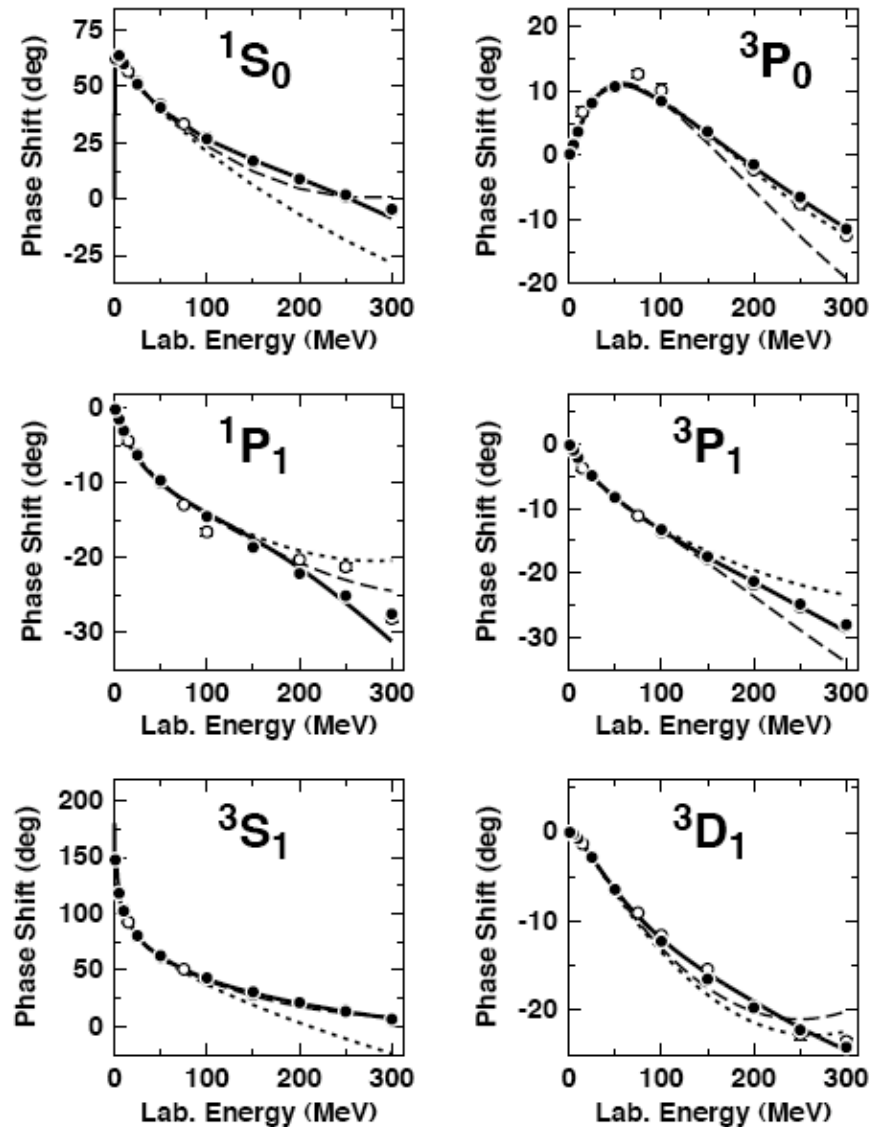
microscopic DF for dilute systems beyond LDA, Furnstahl, discussion by Engel

NN interactions well-constrained by NN scattering

Many different NN potentials fit to data below $E_{\text{lab}} \lesssim 350 \text{ MeV}$
(corresponding relative $k < 2.1 \text{ fm}^{-1}$)



Details not constrained
for momenta $k > 2.1 \text{ fm}^{-1}$



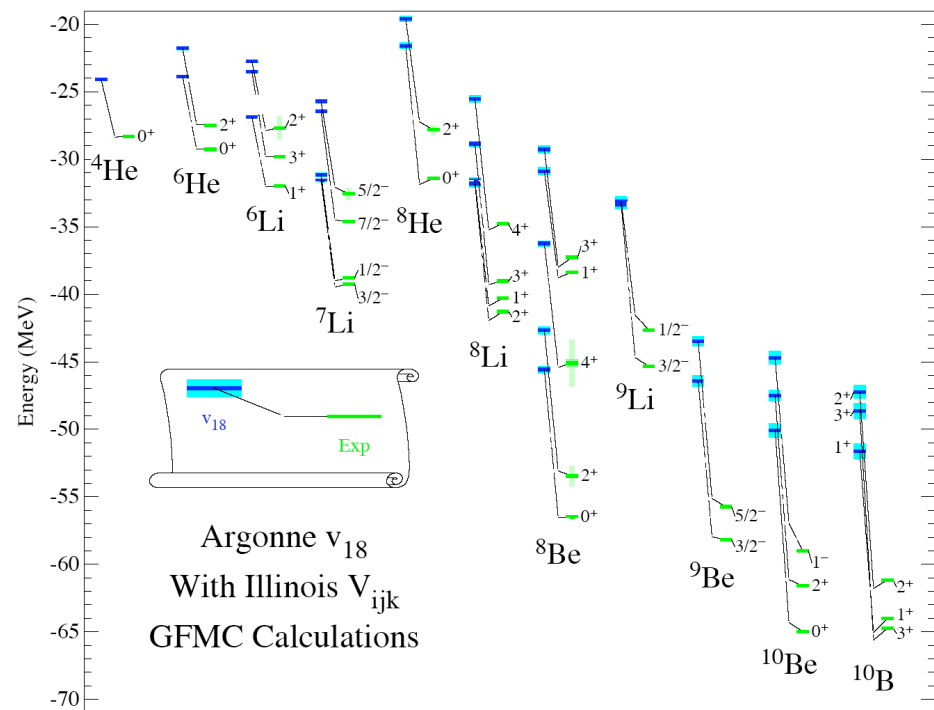
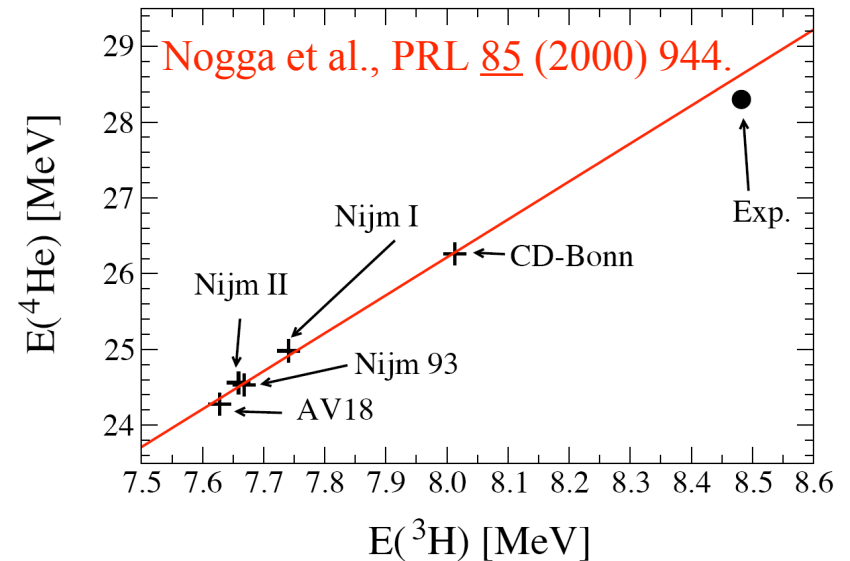
N3LO: Entem, Machleidt, PR C68 (2003) 041001

Importance of 3N interactions

All exact calculations with only NN forces miss $A=3,4$ nucleon binding energies

Exact GFMC results with only NN force miss $A \leq 10$ spectra

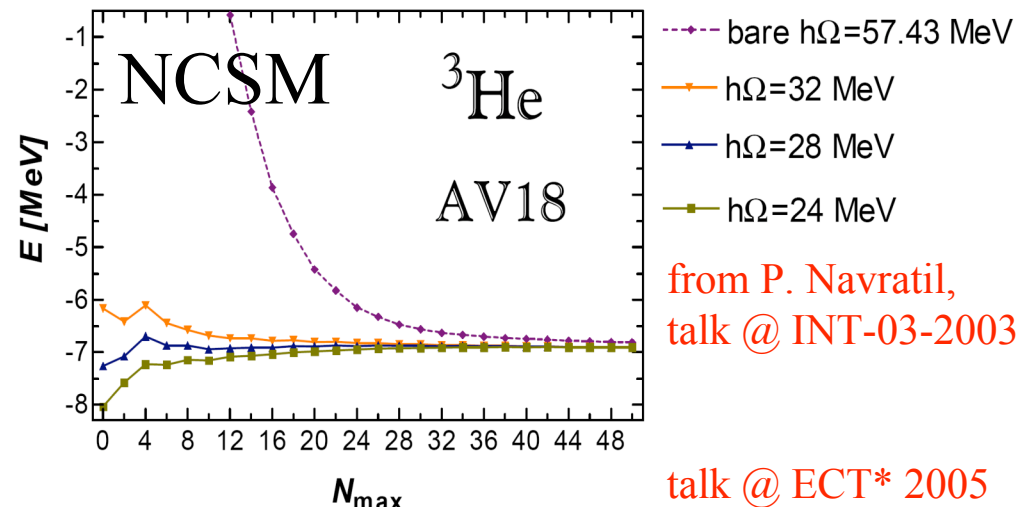
Size of 3N depends on V_{NN}
no “true” 3N force



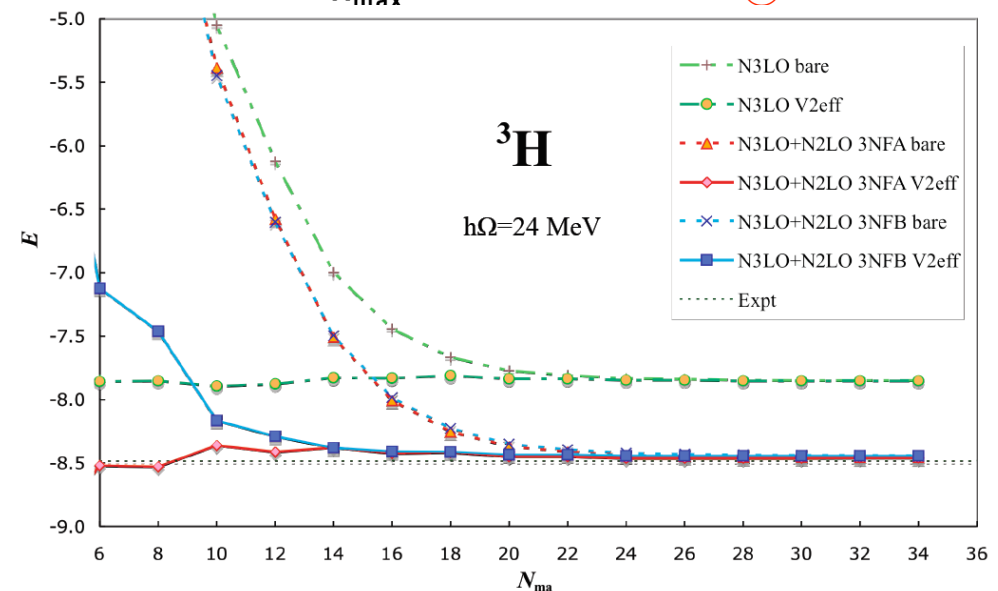
Pieper, Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (2001) 53.

Many-body applications with large-cutoff NN interactions

require resummations due to slow convergence with HO basis size



chiral N3LO interaction with lower cutoff ($\Lambda \approx 600$ MeV) leads to faster convergence

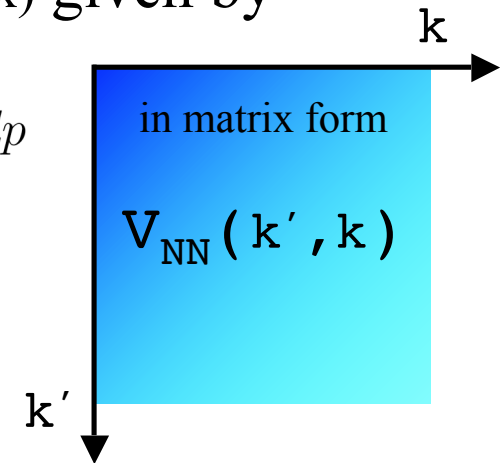


Desire **low-momentum interaction for many-body applications** where resolution is low and no need for model-dep. high-mom. parts

2) RG applied to NN interactions

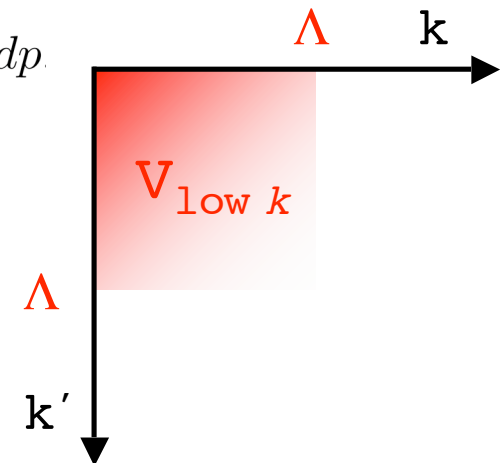
In momentum space, scattering amplitude (T matrix) given by

$$T(k', k; k^2) = V_{\text{NN}}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^{\infty} \frac{V_{\text{NN}}(k', p) T(p, k; k^2)}{k^2 - p^2} p^2 dp$$



Integrating out high-momentum modes leads to **effective interaction** $V_{\text{low } k}$ (which reproduces the low-momentum T matrix - phase shifts, E_d)

$$T(k', k; k^2) = V_{\text{low } k}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^{\Lambda} \frac{V_{\text{low } k}(k', p) T(p, k; k^2)}{k^2 - p^2} p^2 dp$$



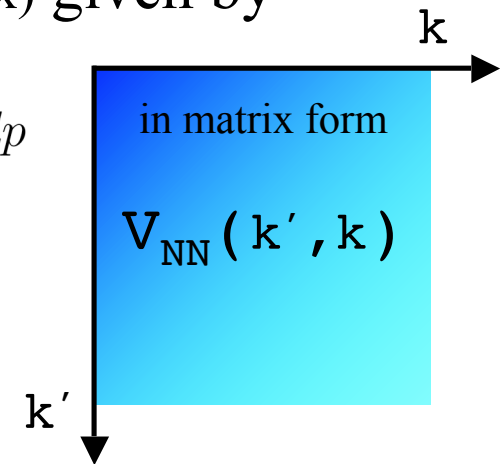
Changes of effective interaction with cutoff Λ are given by RG equation

$$\frac{d}{d\Lambda} V_{\text{low } k}(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}(k', \Lambda) T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

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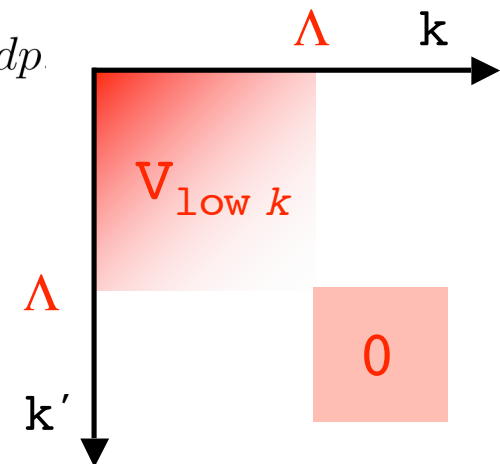
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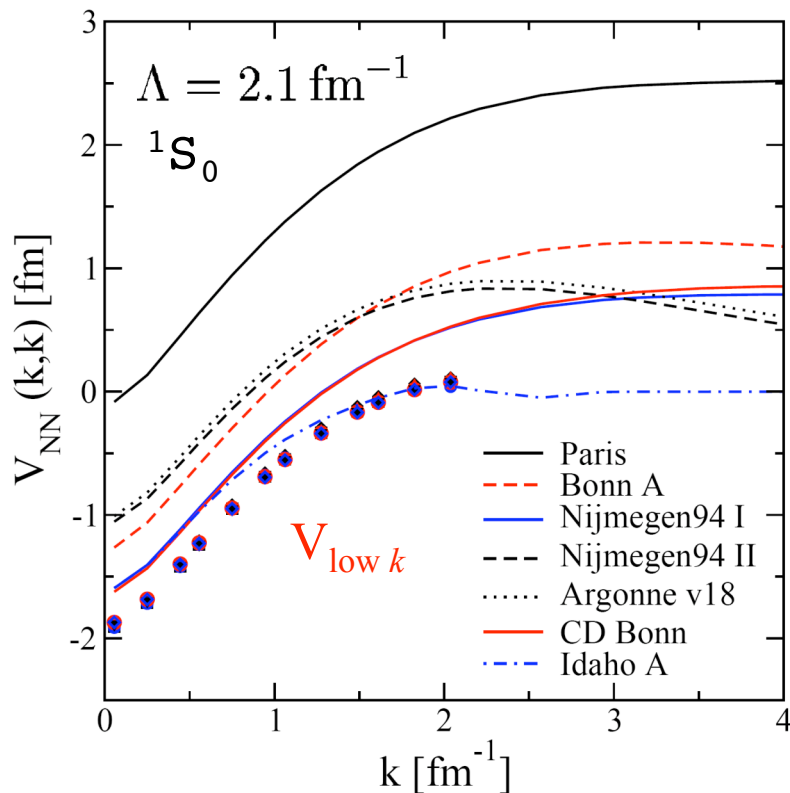


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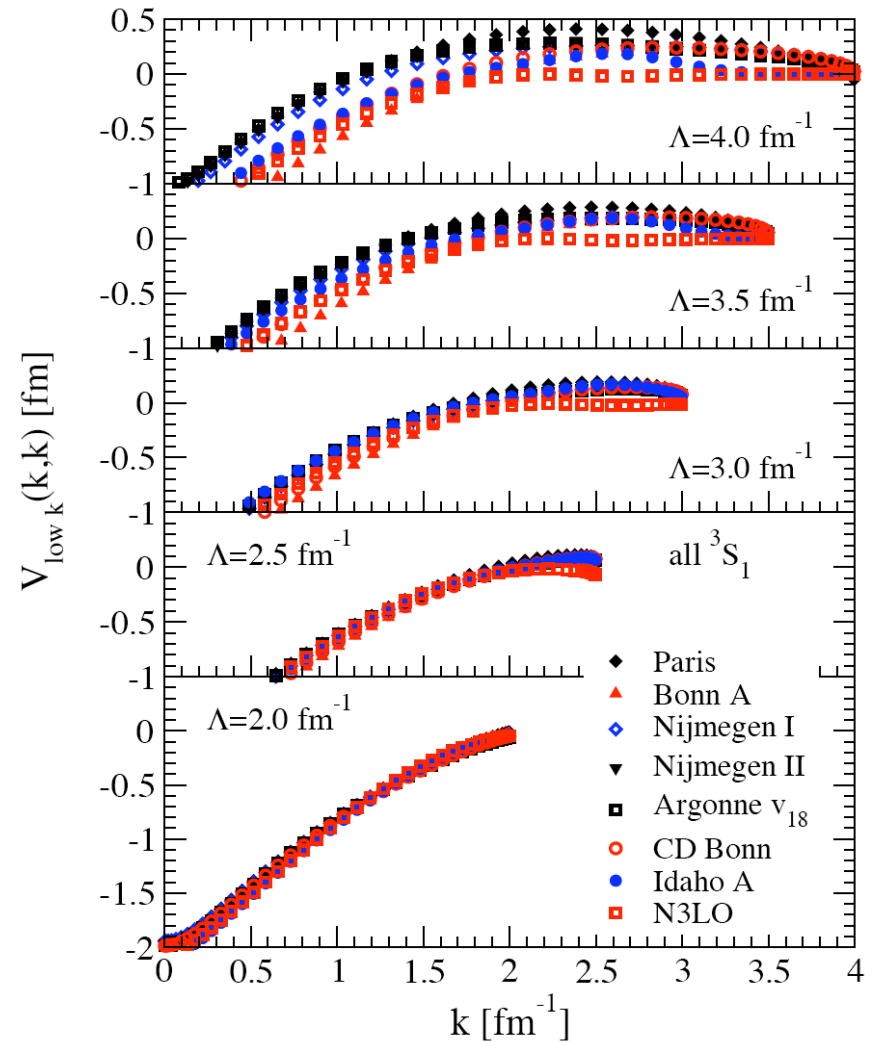
$$\frac{d}{d\Lambda} V_{\text{low } k}(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}(k', \Lambda) T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

Technically equivalent to Lee-Suzuki in mom. space, with $Q\text{-block} \equiv 0$

Starting from any NN interaction:
Solutions to the RG eqn. evolve
to a “universal” interaction $V_{\text{low } k}$
for cutoffs below $\Lambda \lesssim 2.1 \text{ fm}^{-1}$



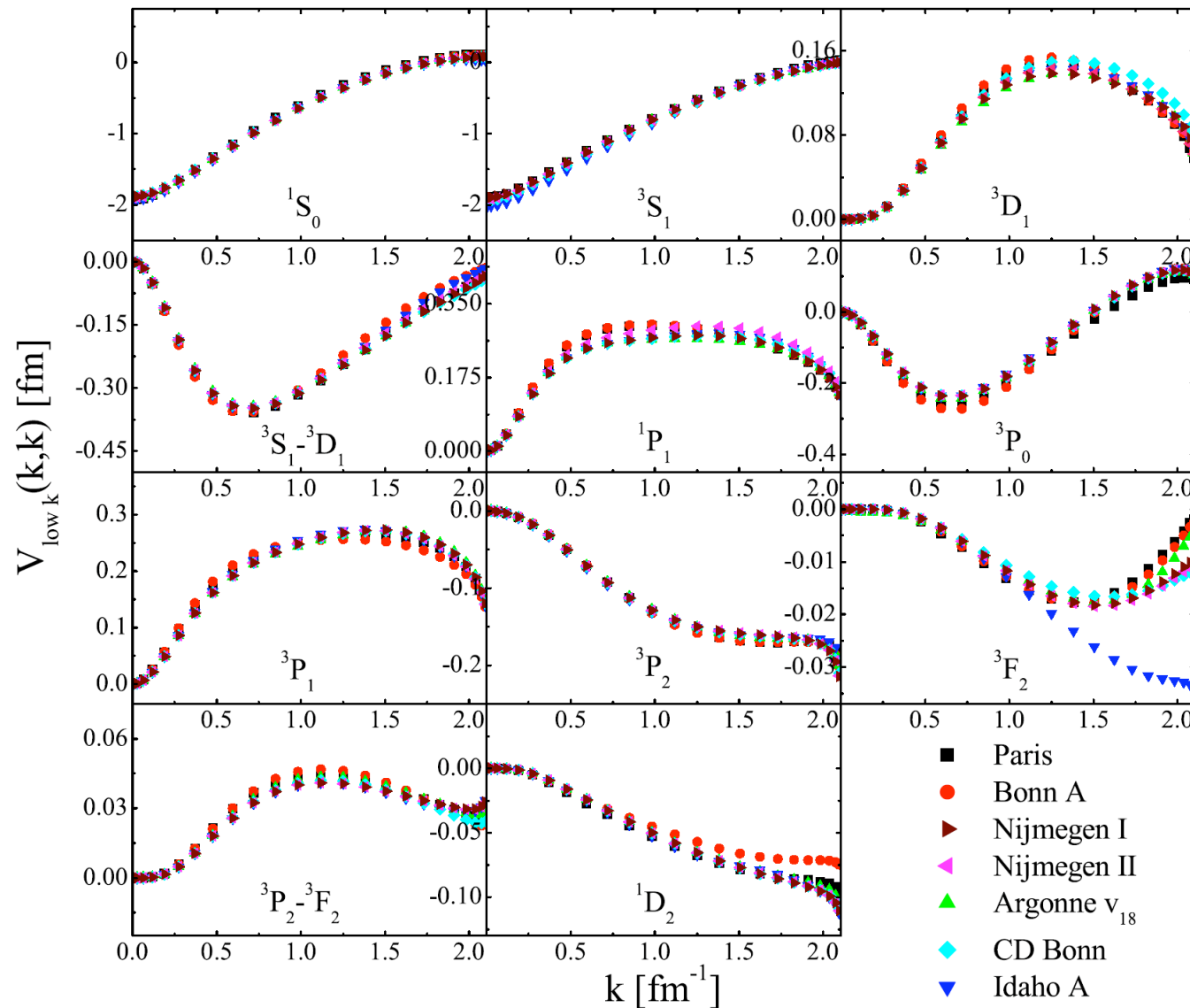
Bogner, Kuo, AS, Phys. Rep. 386 (2003) 1.



$V_{\text{low } k}(\Lambda)$ defines a class of
energy-indep. NN interactions

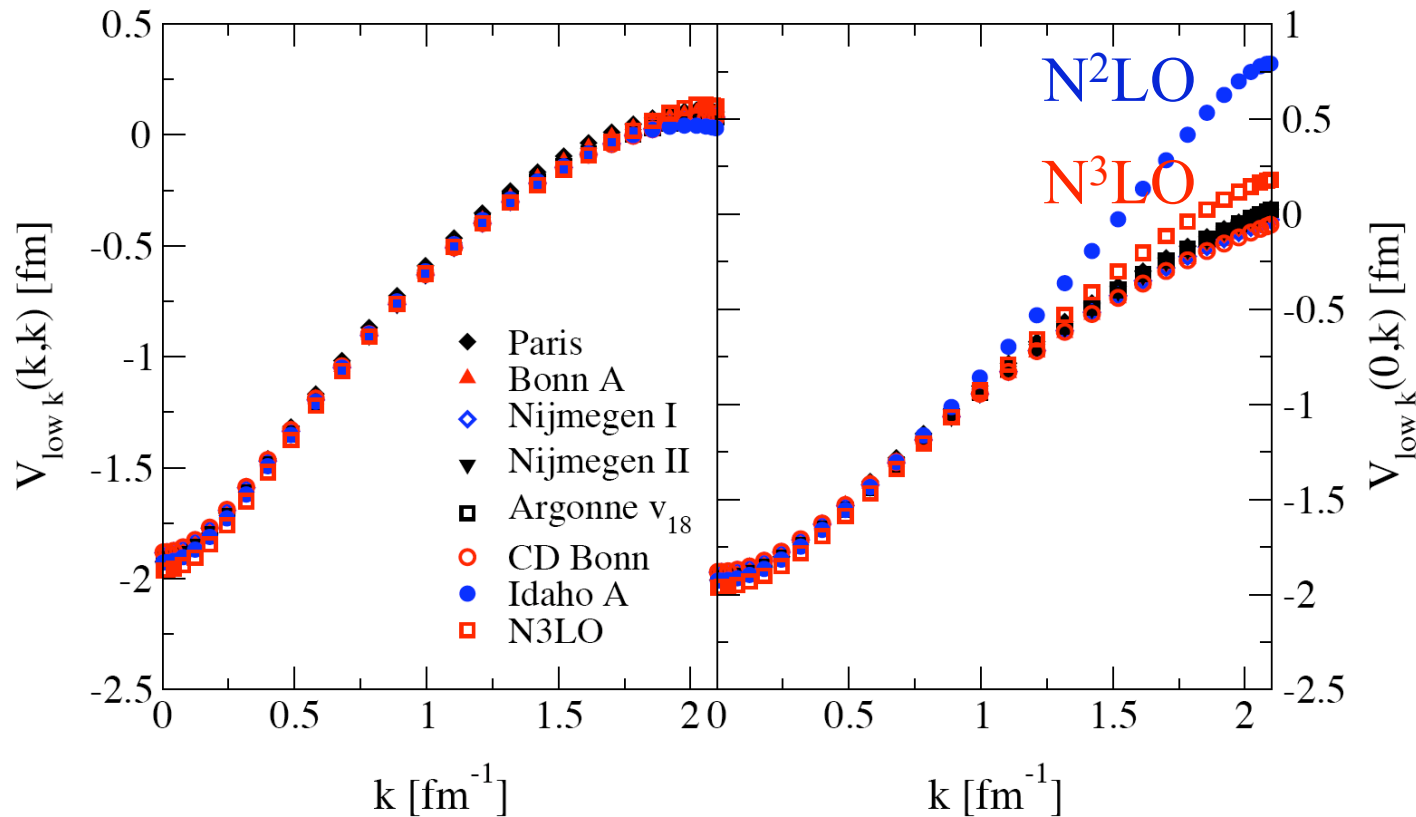
Collapse in all partial waves

due to same long-distance (π) physics and phase shift equivalence

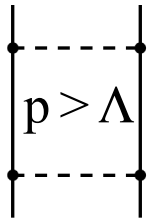


small differences
related to spread
in phase shift fit
(Idaho misses 3F_2)

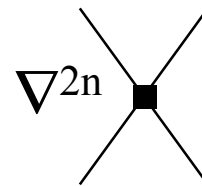
Collapse of off-shell matrix elements as well



“Universal” result indicates that $V_{\text{low } k}$ effectively parameterizes a chiral EFT interaction with higher-order contacts

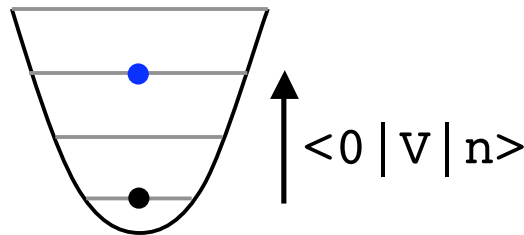


has same effect for low momenta as

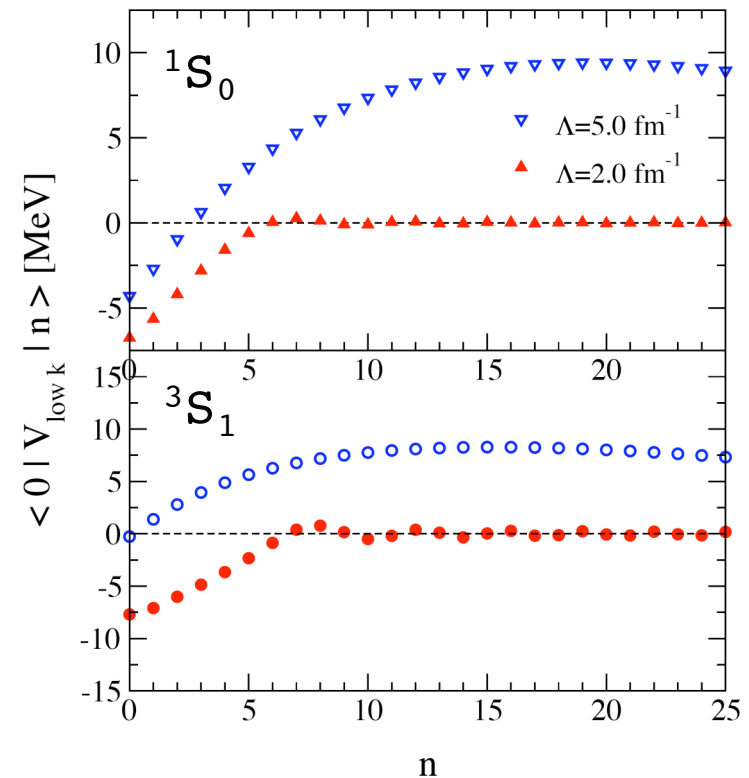
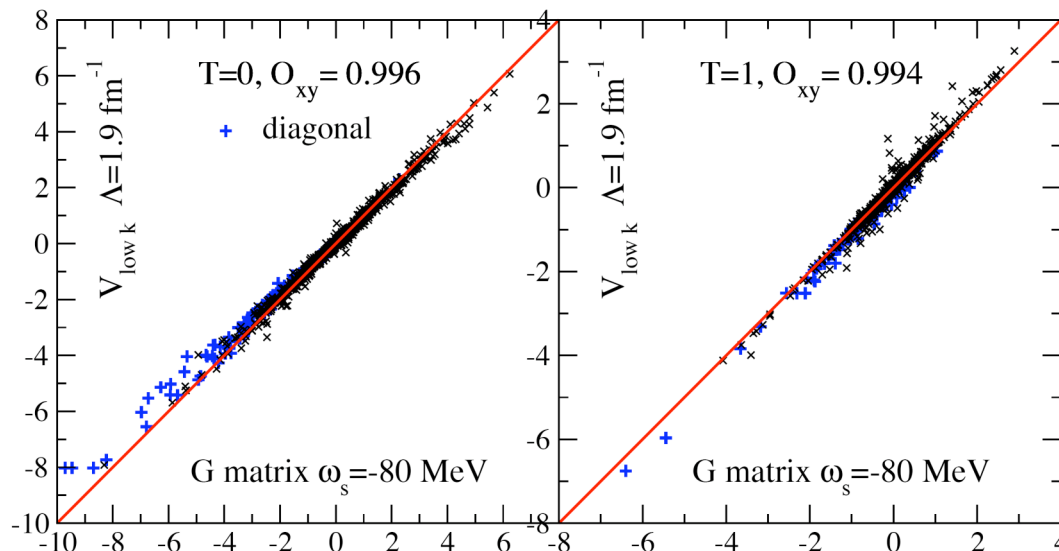


Advantages of lower cutoffs

Low-momentum interactions are tractable in a HO basis



$V_{\text{low } k}$ similar to successful G matrix pseudopotential (without drawbacks of G)



Plot of two-body $V_{\text{low } k}$ vs. G matrix elements in MeV (4 shells) [AS, Zuker, nucl-th/0501038](#).

What about ladders in $V_{\text{low } k}$?

3) Results for nuclei and nuclear matter

Many-body calculations with $V_{\text{low } k}$ make model-independent predictions, with cutoff-independent NN observables ($\chi^2 \approx 1$)

But incomplete, because 3N, 4N,... interactions are not included

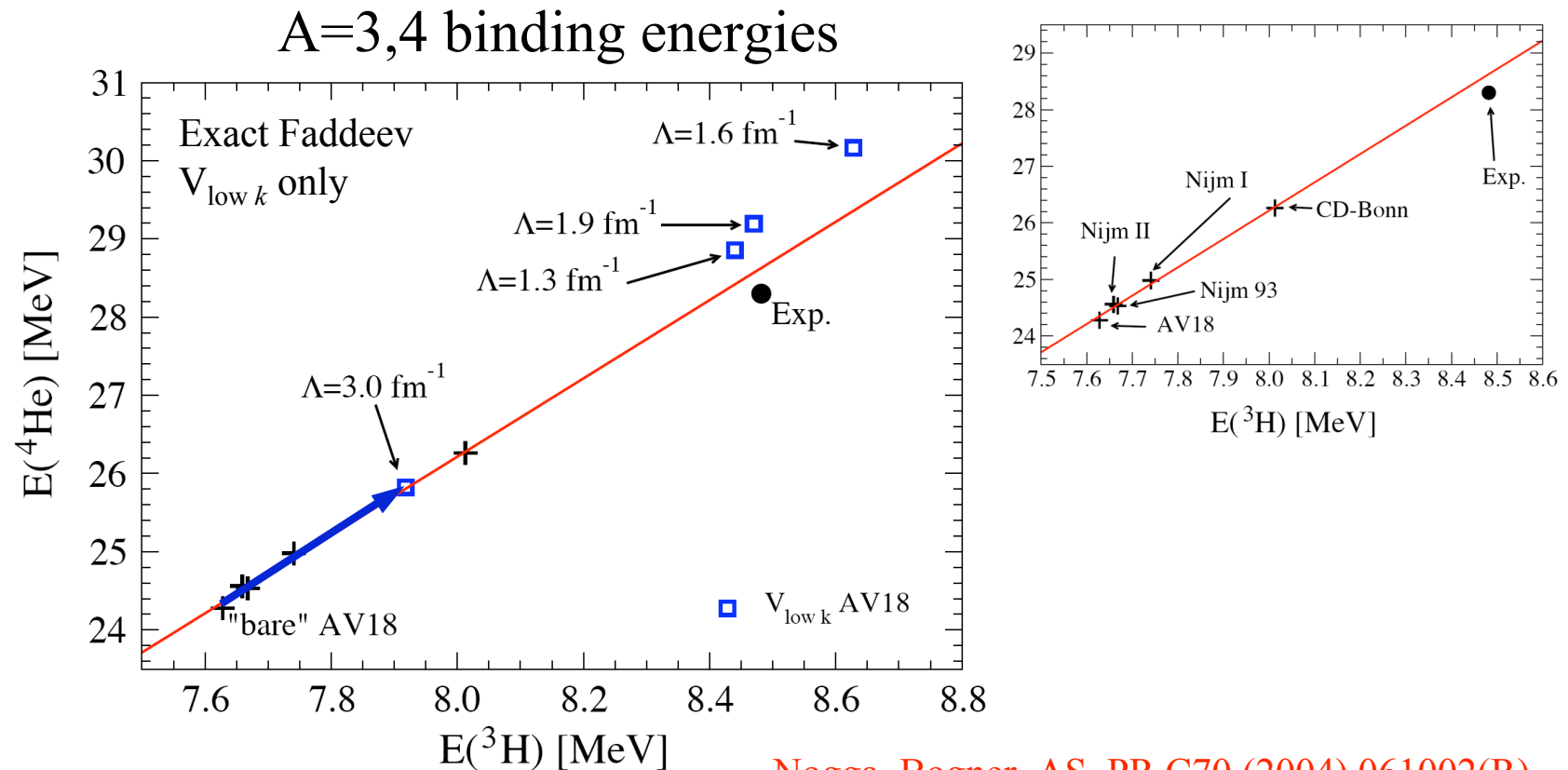
If one omits the many-body forces, calculations of low-energy 3N, 4N,... observables will be cutoff-dependent

This is a feature

Varying the cutoff estimates the effects of omitted many-body interactions [Nogga, Bogner, AS, PR C70 \(2004\) 061002\(R\)](#).

Important to assess theoretical errors for predictions to extreme compositions and astrophysical densities/temperatures

Cutoff dependence due to omitted 3N forces



Nogga, Bogner, AS, PR C70 (2004) 061002(R).

Cutoff dependence shows that three-body forces are inevitable

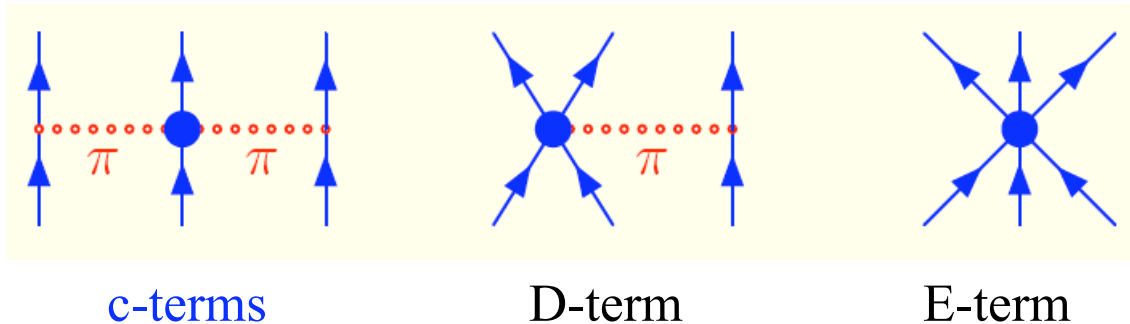
Cutoff dependence is almost linear (explains Tjon line),
with 3N contributions smaller for low-momentum interactions

Low-momentum 3N forces with only pion exchanges

organized by chiral Effective Field Theory [van Kolck, PR C49 \(1994\) 2932](#);
[Epelbaum et al., PR C66 \(2002\) 064001](#).

operator structure of any 3N force collapses at low energies to

long (2π) intermediate (π) short-range



chiral counting:
 $(Q/\Lambda)^3 \sim (m_\pi/\Lambda)^3$
rel. to 2N force

c_i from NN PWA with chiral 2π exchange (c_3, c_4 important for nuclear structure)

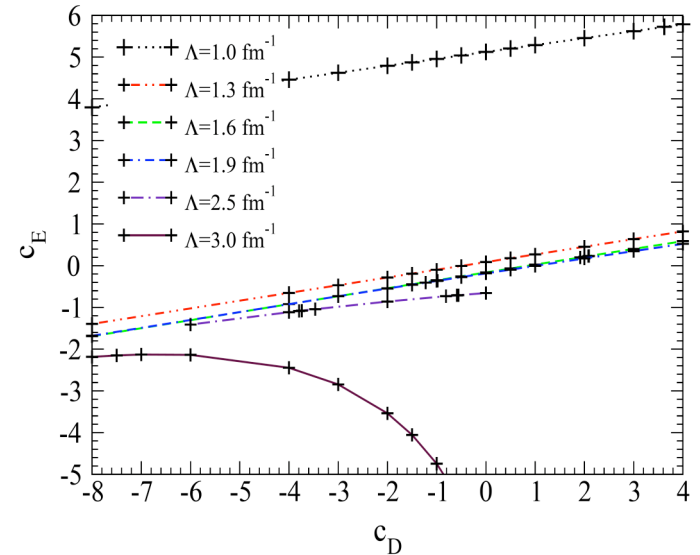
approximate 3N evolution by fitting low-energy D,E constants of
leading-order chiral 3N interaction to $V_{\text{low } k}$ over a range of cutoffs

Two couplings fit to ${}^3\text{H}$ and ${}^4\text{He}$

Linear dependences in fits, consistent with perturbative 3N contributions

$$E({}^3\text{H}) = \langle T + V_{\text{low } k} + V_c \rangle + c_D \langle O_D \rangle + c_E \langle O_E \rangle$$

3N forces become perturbative for cutoffs $\Lambda \lesssim 2 \text{ fm}^{-1}$

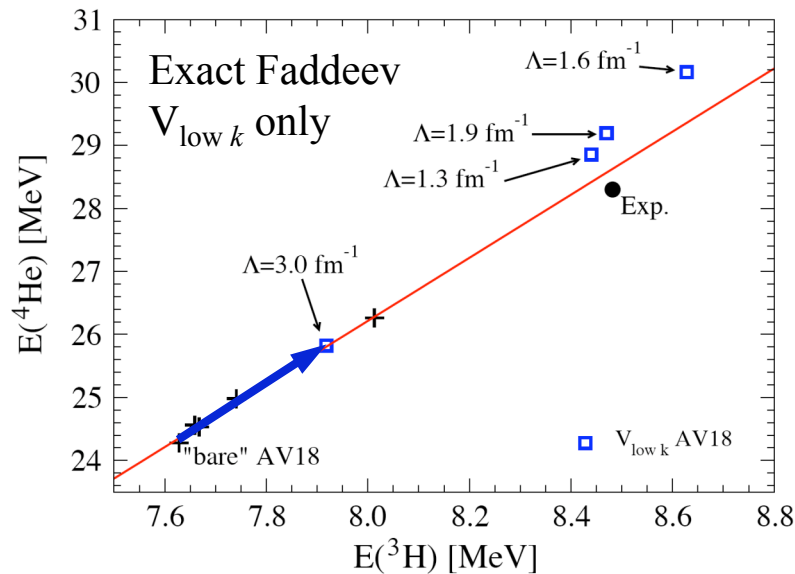


nonperturbative at larger cutoffs (also for chiral EFT with $\Lambda \approx 3 \text{ fm}^{-1}$), with expectation values **beyond expected** $(Q/\Lambda)^3 \sim (m_\pi/\Lambda)^3 \approx 0.1$ of NN

Λ	${}^3\text{H}$					${}^4\text{He}$					max	${}^4\text{He}$
	T	$V_{\text{low } k}$	c -terms	D -term	E -term	T	$V_{\text{low } k}$	c -terms	D -term	E -term	$ V_{3\text{N}}/V_{\text{low } k} $	k_{rms}
1.0	21.06	-28.62	0.02	0.11	-1.06	38.11	-62.18	0.10	0.54	-4.87	0.08	0.55
1.3	25.71	-34.14	0.01	1.39	-1.46	50.14	-78.86	0.19	8.08	-7.83	0.10	0.63
1.6	28.45	-37.04	-0.11	0.55	-0.32	57.01	-86.82	-0.14	3.61	-1.94	0.04	0.67
1.9	30.25	-38.66	-0.48	-0.50	0.90	60.84	-89.50	-1.83	-3.48	5.68	0.06	0.70
2.5(a)	33.30	-40.94	-2.22	-0.11	1.49	67.56	-90.97	-11.06	-0.41	6.62	0.12	0.74
2.5(b)	33.51	-41.29	-2.26	-1.42	2.97	68.03	-92.86	-11.22	-8.67	16.45	0.18	0.74
3.0(*)	36.98	-43.91	-4.49	-0.73	3.67	78.77	-99.03	-22.82	-2.63	16.95	0.23	0.80

$V_{\text{low } k}$ + leading chiral 3N interaction, **no further adjustments** $\approx m_\pi$

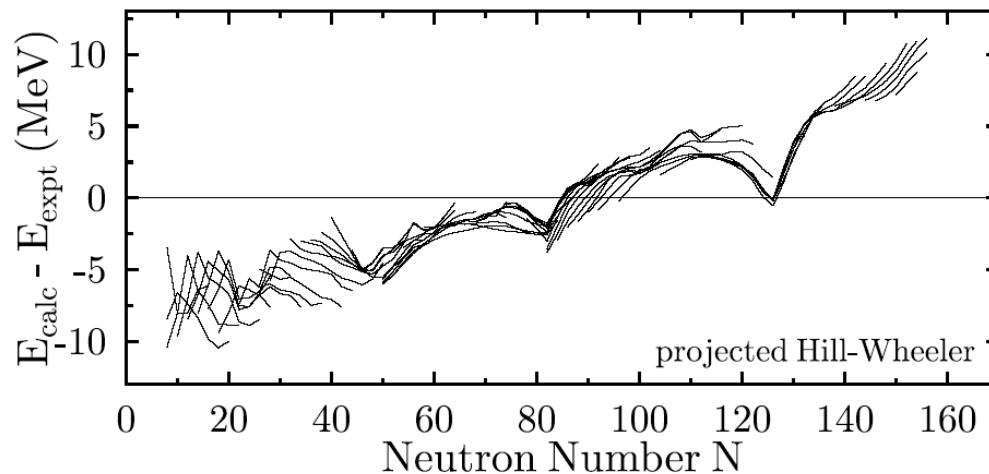
3N interaction and mass models



large-cutoff NN interactions
 underbind light A, overbind heavy A

low-mom. NN interactions
 close to A=3,4, overbind heavier A

need 3N input to correct this



similar trend in DF predictions

investigate density/isospin
 dependence of weak low-mom.
 3N interactions

Bender et al., PRL 94 (2005) 102503.

NN scattering in free-space vs. nuclear matter

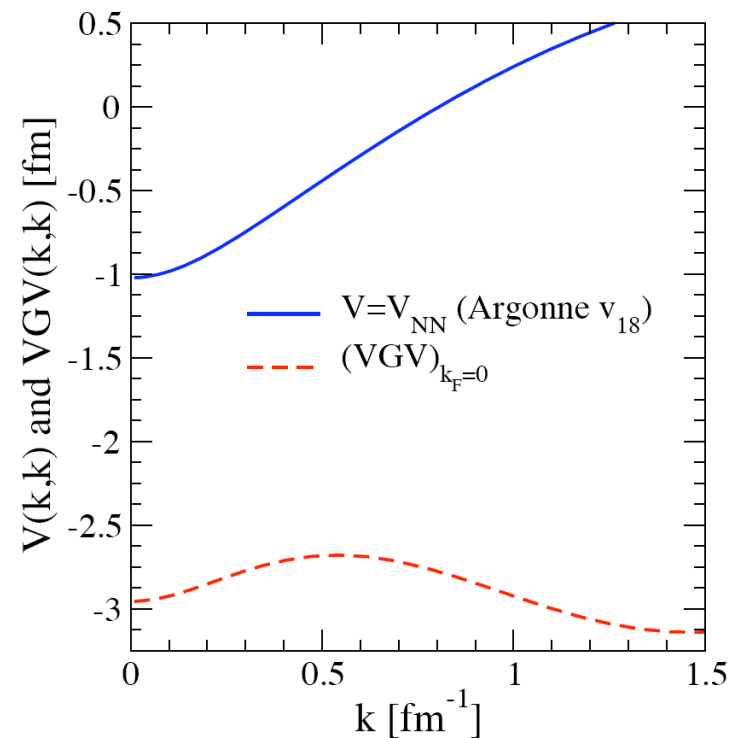
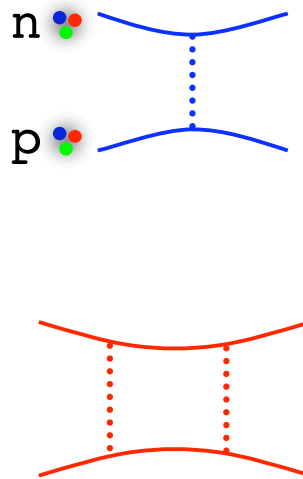
Sources of nonperturbative behavior:

1. Iterated interactions at low momenta are nonperturbative due to near bound states $a_{1S_0} = -23.7 \text{ fm}$ $a_{3S_1} = 5.4 \text{ fm}$
2. Cores scatter strongly to high-momentum states (overwhelms 1.)

For interactions with cores:

second-order contributions

larger than V_{NN}



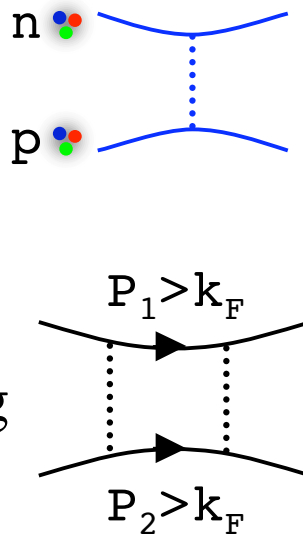
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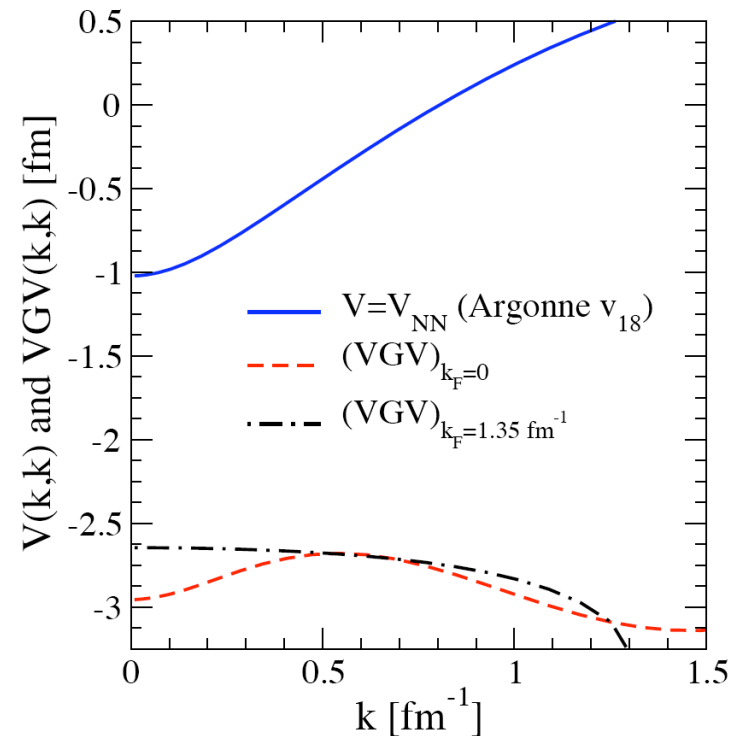
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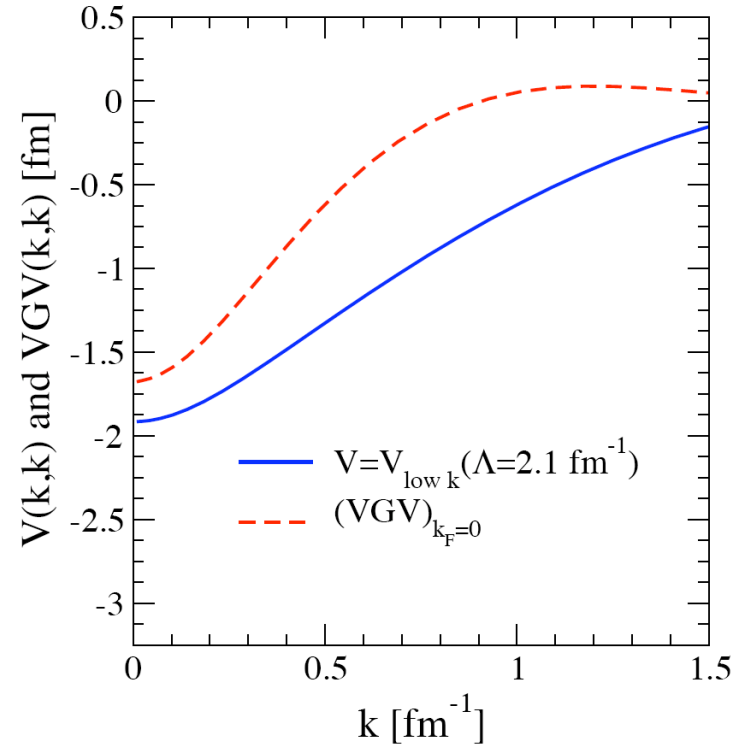
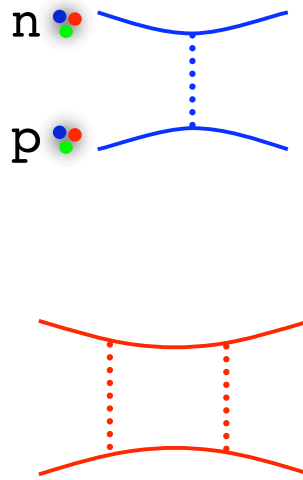


even with Pauli blocking



NN scattering in free-space vs. nuclear matter

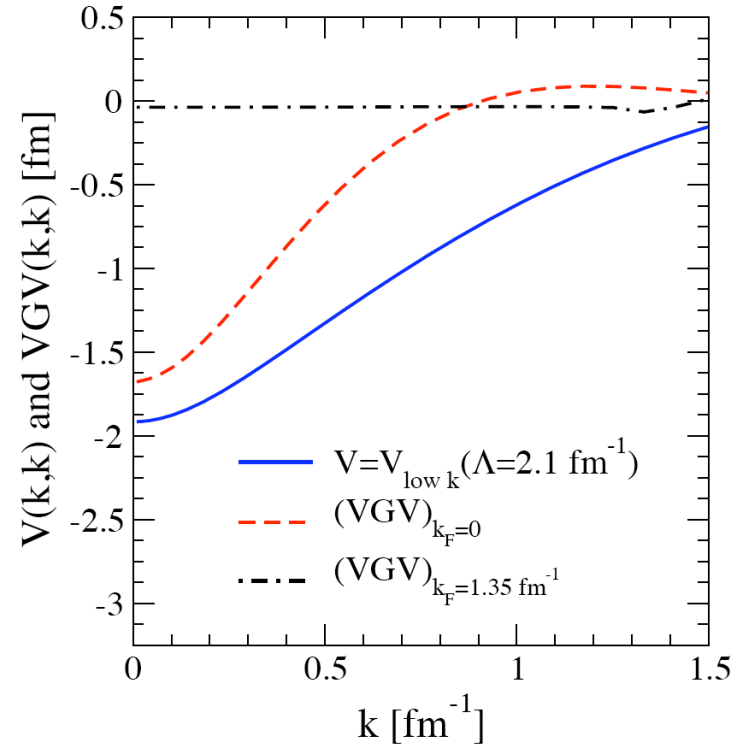
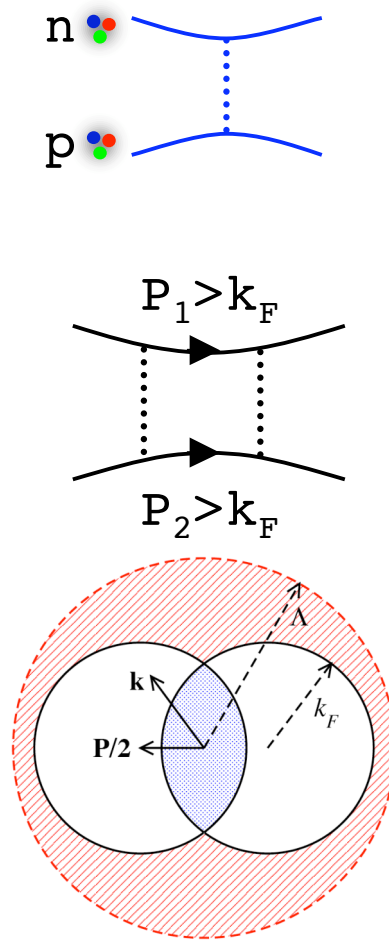
For low-momentum interactions:
free-space nonperturbative due
to near-bound state



NN scattering in free-space vs. nuclear matter

For low-momentum interactions:
free-space nonperturbative due
to near-bound state

small in medium due
to occupied Fermi sea
restricts phase space
for two particles



mom. cutoff $k < 2.1 \text{ fm}^{-1}$
= density cutoff $\rho < 4 \rho_0$

Weinberg eigenvalues

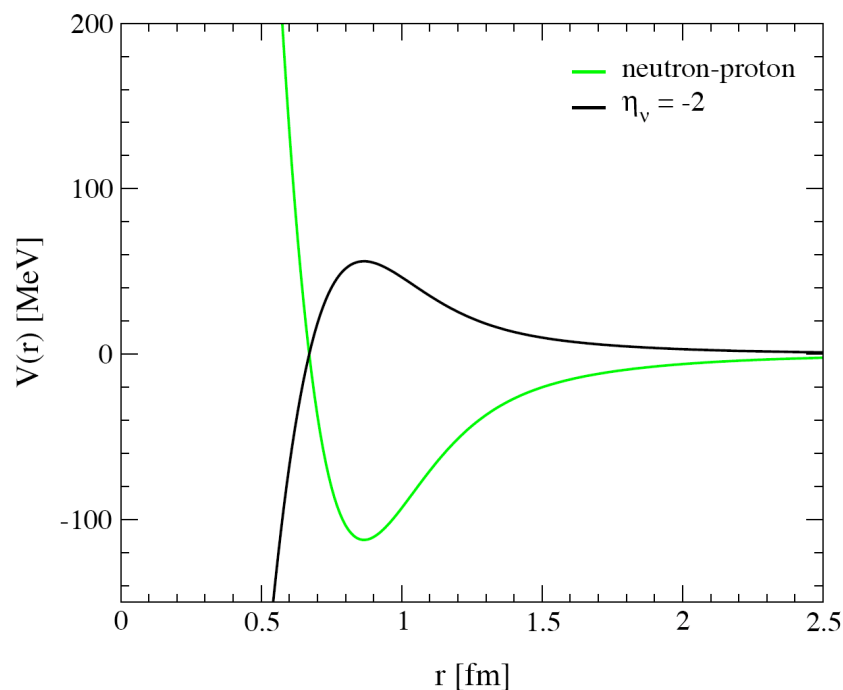
rigorously show that in-medium Born series is perturbative for $V_{\text{low } k}$ at sufficient densities [Bogner, AS, Furnstahl, Nogga, nucl-th/0504043](#).

study spectrum of $G_0(z)V |\Psi_\nu(z)\rangle = \eta_\nu(z) |\Psi_\nu(z)\rangle$

i.e., convergence of $T(z) |\Psi_\nu(z)\rangle = (1 + \eta_\nu(z) + \eta_\nu(z)^2 + \dots) V |\Psi_\nu(z)\rangle$

write as Schroedinger eqn. $\left(H_0 + \frac{1}{\eta_\nu(z)} V\right) |\Psi_\nu(z)\rangle = z |\Psi_\nu(z)\rangle$

large-cutoff interactions
have at least one large $\eta_\nu < 0$



Weinberg eigenvalues

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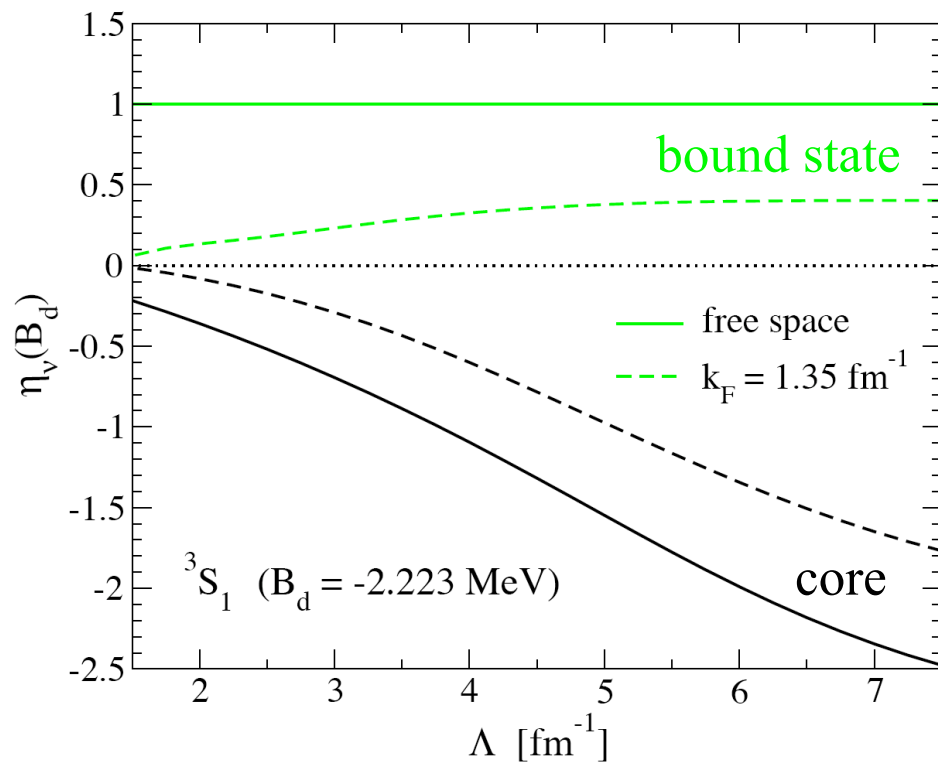
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write as Schroedinger eqn.

$$\left(H_0 + \frac{1}{\eta_\nu(z)} V\right) |\Psi_\nu(z)\rangle = z |\Psi_\nu(z)\rangle$$

repulsive core eigenvalue
small for lower cutoffs

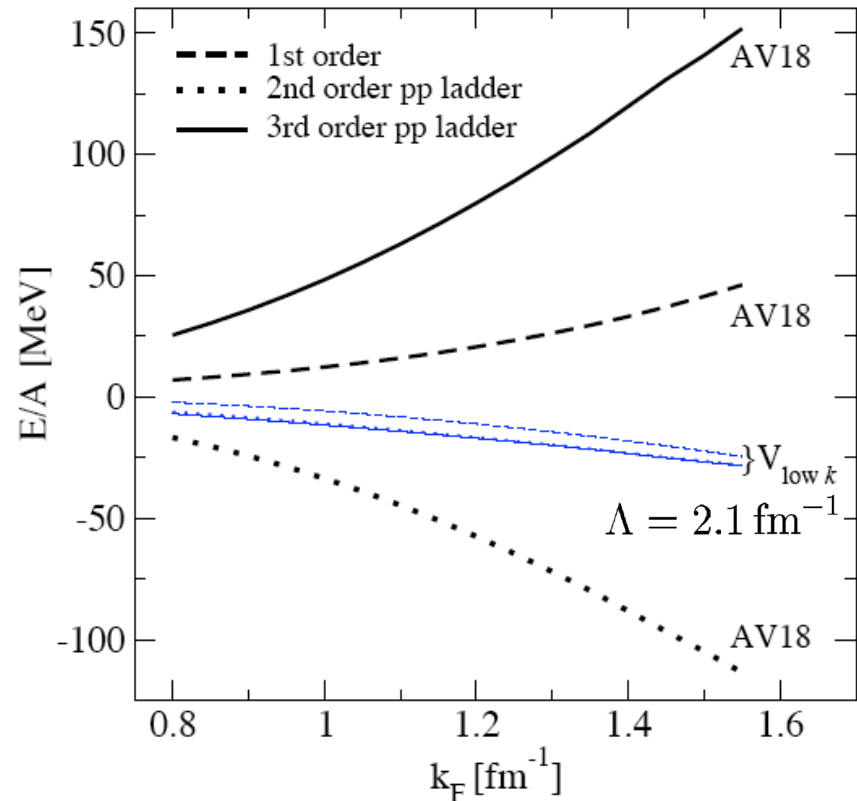
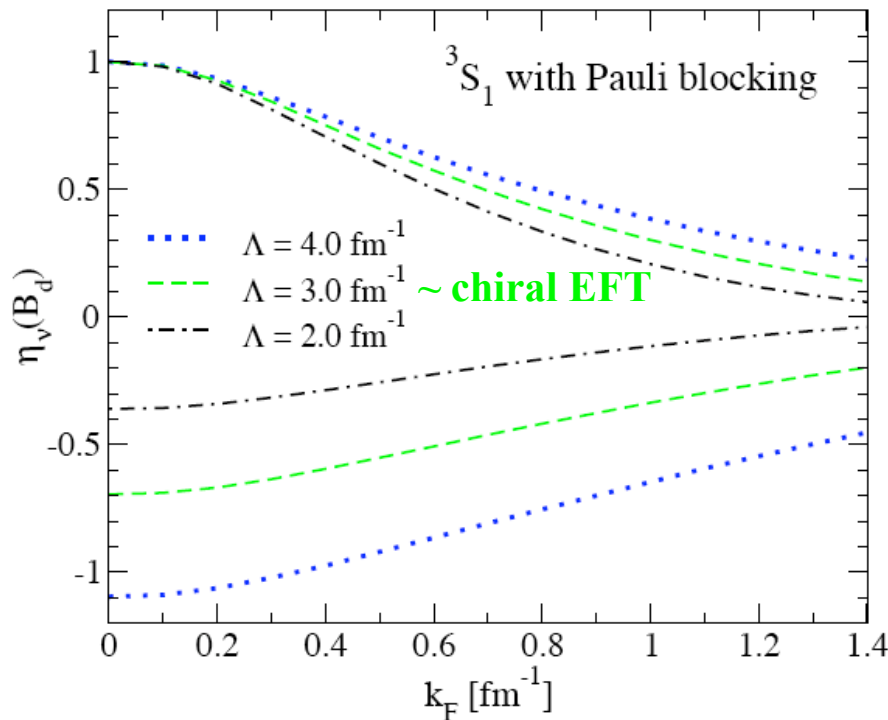
deuteron eigenvalue small
in-medium (fine-tuning elim.)



Perturbative two-body ladders

Weinberg analysis anticipates rapidly converging particle-particle contributions to nuclear matter energy for low-mom. interactions

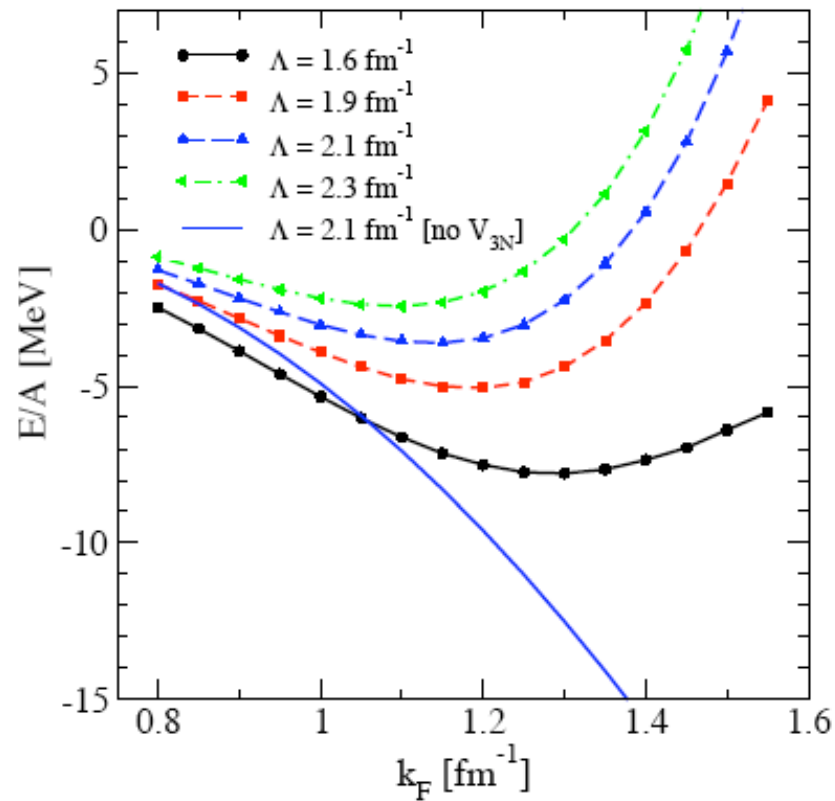
Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.



Perturbation theory in place of ladder resummations

Note second-order important (remaining tensor)

Nuclear matter with NN and 3N

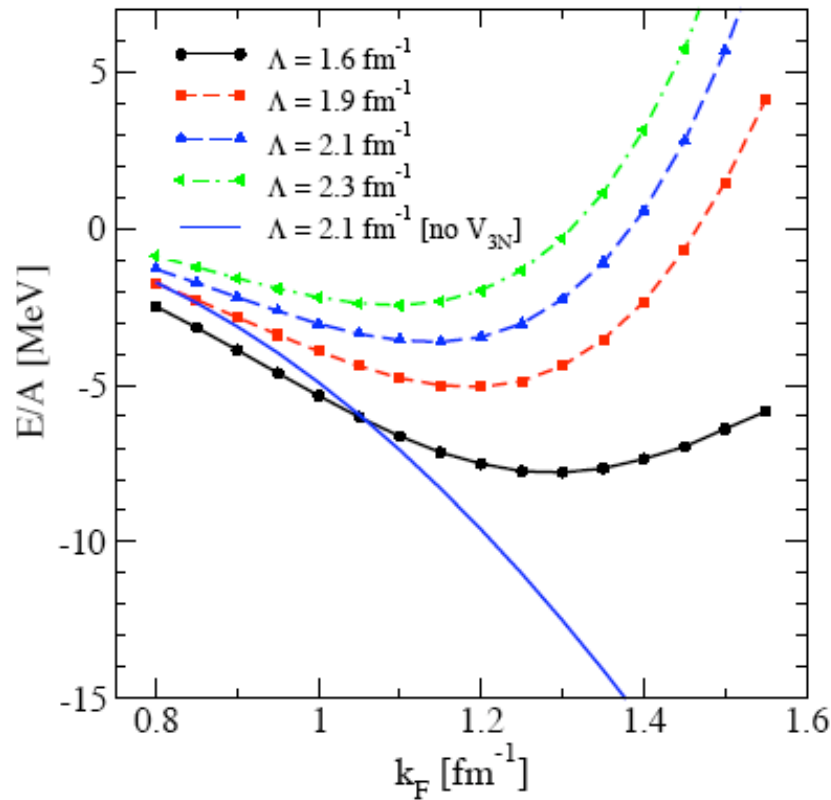


Hartree-Fock

bound, saturation from 3N force
(2π -exchange dominates, no adjustments)

Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.

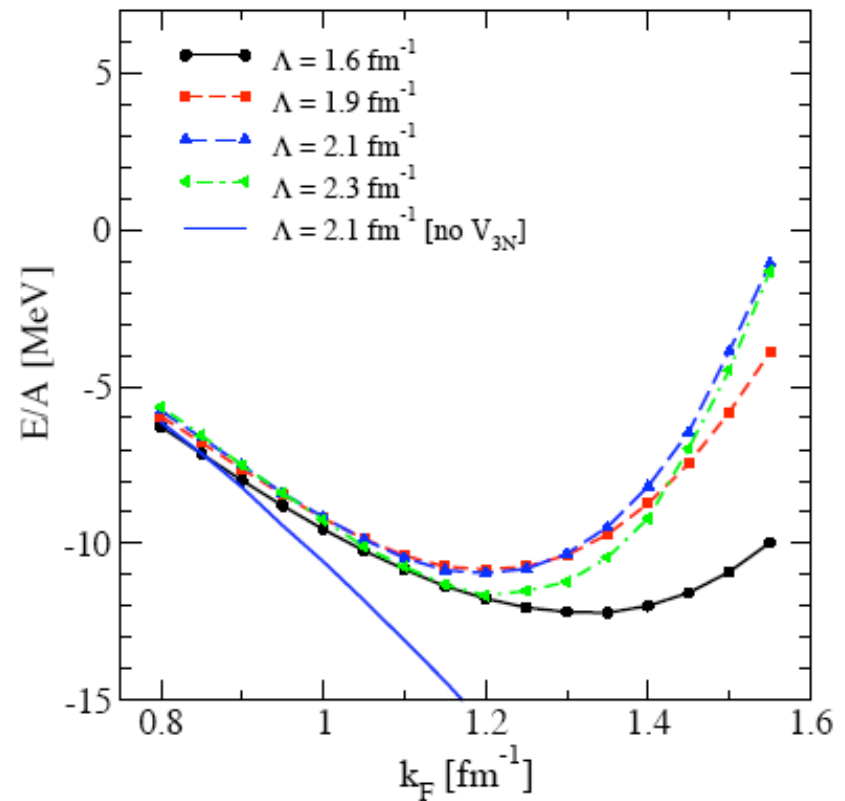
Nuclear matter with NN and 3N



Hartree-Fock

bound, saturation from 3N force
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Bogner, AS, Furnstahl, Nogga, nucl-th/0504043.



Hartree-Fock + 2nd-order

(approx. 2nd-order 3N treatment,
continuous spectrum with m^*)

cutoff dep. strongly reduced

Ex-corr. much weaker, use $V_{\text{low } k} + V_{3N}$ to constrain nuclear DF

3N force remains natural in nuclear matter

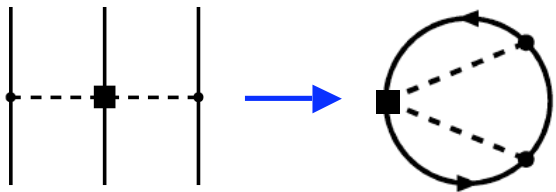
3N force drives saturation but exp.
values not unnaturally large

consistent with chiral EFT scaling

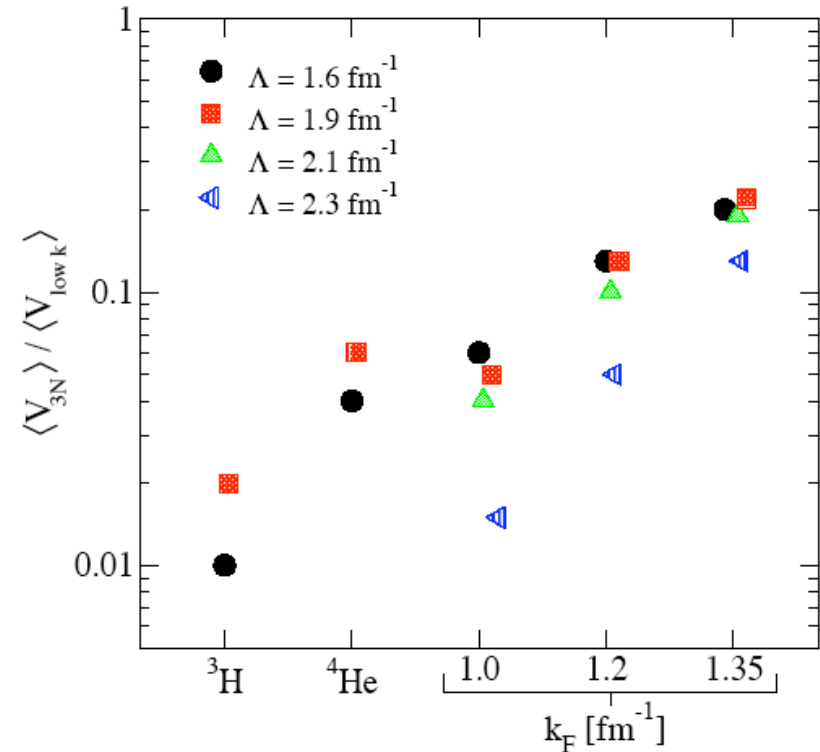
$$\langle V_{3N} \rangle \sim (Q/\Lambda)^3 \langle V_{NN} \rangle$$

$V_{\text{low } k}$ exp. values change by ≈ 0.5 MeV
after inclusion of 3N interaction

largest 3N contribution from 2π -part

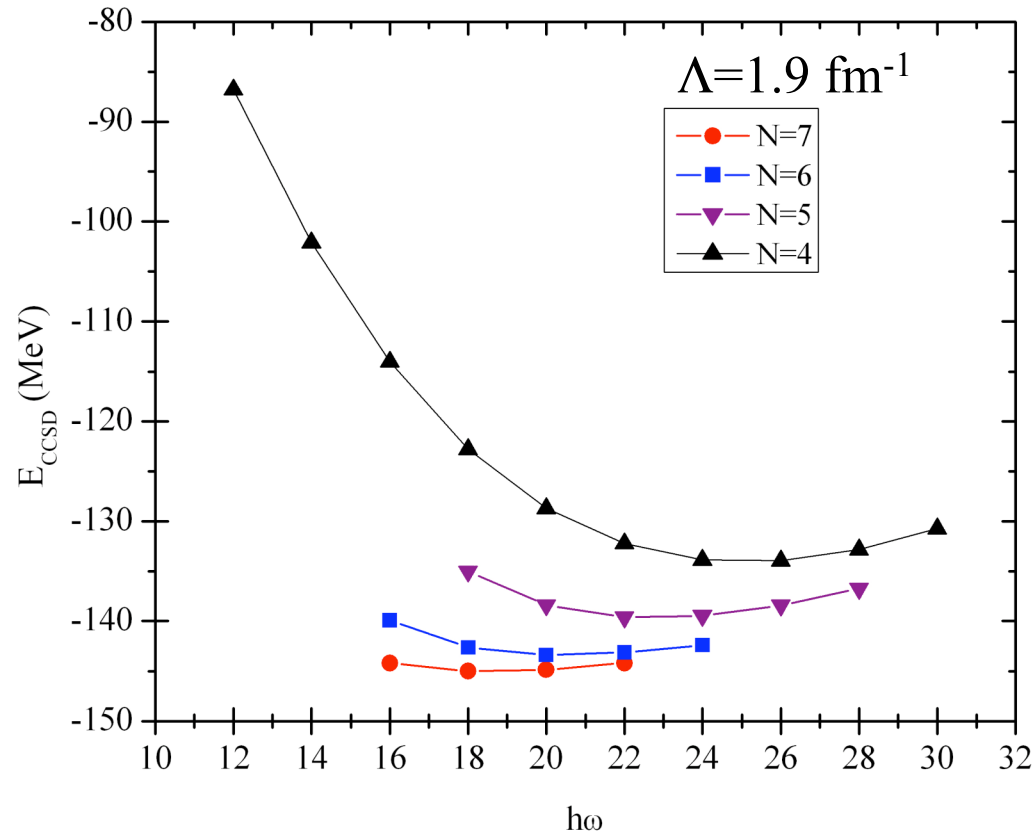


mom. dep. of 3N determines density dep.



k_F	Λ	Hartree-Fock					Hartree-Fock + dominant second order				
		T	$V_{\text{low } k}$	V_c	V_D	V_E	T	$V_{\text{low } k}$	V_c	V_D	V_E
1.2	1.6	17.92	-31.47	5.37	1.31	-0.64	20.86	-37.66	4.59	1.03	-0.65
	1.9	17.92	-28.95	5.61	-0.81	1.18	21.80	-37.38	3.99	-0.50	1.28
	2.1	17.92	-27.51	5.67	-1.37	1.84	22.87	-37.53	2.27	-0.37	1.82
	2.3	17.92	-26.13	5.70	-1.86	2.42	24.32	-37.95	-0.38	0.51	1.78

Coupled cluster results for $V_{\text{low } k}$



with D. Dean et al.

nearly converged

$N=8, \hbar\omega=18 \text{ MeV: } -145.85 \text{ MeV}$

calc. with 3N interaction in progress, is repulsive

also have ^{40}Ca results

CC method can provide exact answer for low-momentum interactions without resummations or similarity trafo.

Can test other many-body methods for same interaction

Build DF from low-momentum interactions and compare to CC results

Spin-orbit strength of low-momentum interactions

As for binding energies, interplay of NN and 3N contributions to LS splitting, individual parts depend on cutoff

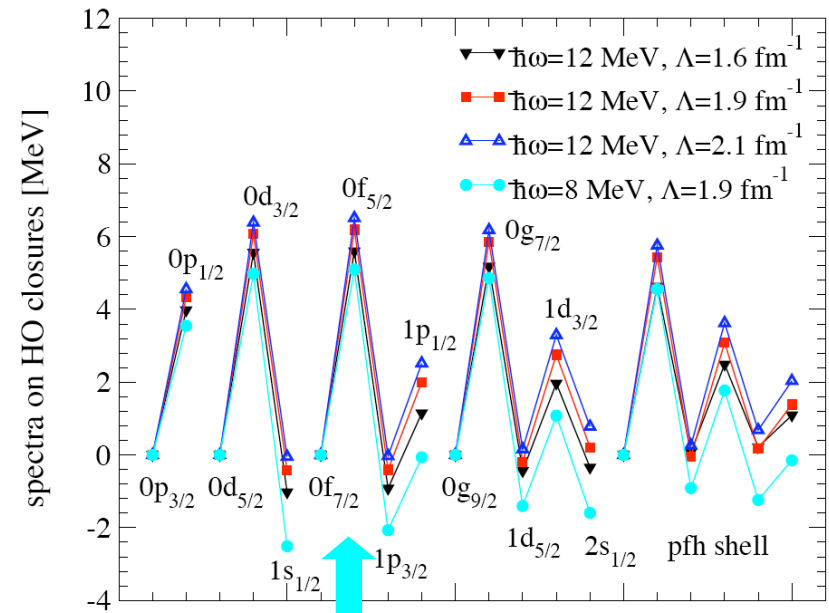
CC results for different NN interactions, $N=8$, Gour et al., nucl-th/0507049

^{15}O	Expt.	N^3LO	CD-Bonn
$3/2^-$	6.176	6.26	7.35
$1/2^-$	0.0	0.0	0.0

^{17}O	Expt.	N^3LO	CD-Bonn
$3/2^+$	5.085	5.68	6.41
$5/2^+$	0.0	0.0	0.0

Excited state	Interaction			Expt
	N^3LO	CD-Bonn	V_{18}	
$^{15}\text{O} (3/2)_1^-$	6.264	7.351	4.452	6.176
$^{15}\text{N} (3/2)_1^-$	6.318	7.443	4.499	6.323
$^{17}\text{O} (3/2)_1^+$	5.675	6.406	3.946	5.084
$^{17}\text{O} (1/2)_1^+$	-0.025	0.311	-0.390	0.870
$^{17}\text{F} (3/2)_1^+$	5.891	6.677	4.163	5.000
$^{17}\text{F} (1/2)_1^+$	0.428	0.805	0.062	0.495

similar observation in $0h\omega$ with $V_{\text{low } k}$
AS, Zuker, nucl-th/0501038.



^{41}Ca exp.: $0f_{5/2} - 0f_{7/2} \approx 6.0$ MeV
 ^{17}O exp.: $0d_{3/2} - 0d_{5/2} \approx 5.1$ MeV

3N contrib. to LS for N^3LO or $V_{\text{low } k}$ smaller, agrees with Q/Λ expectation

4) Strategies for density-functional calculations from nuclear interactions

Revisit density-matrix expansion with low-momentum interactions

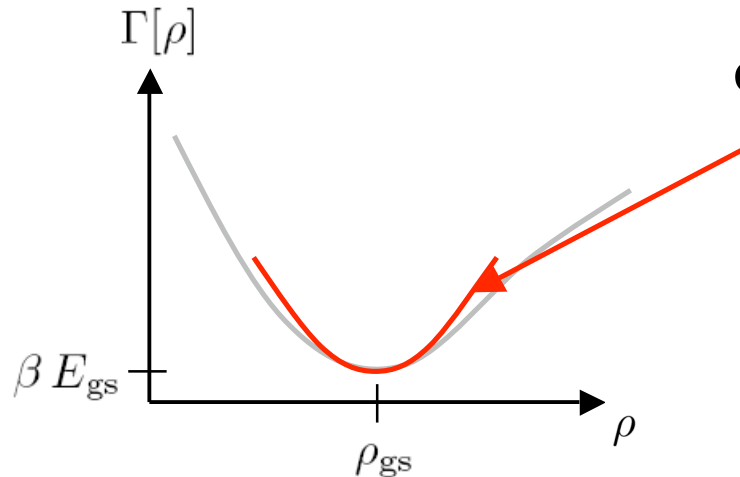
Direct constraints on general ∇^2 DF

Inversion method, provided a power counting for nuclear matter

Functional RG approach to DF Polonyi, Schwenk, formalism in nucl-th/0403011.

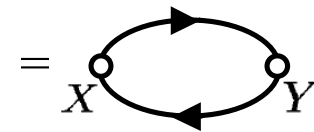
Effective action is minimal at the physical ($K=0$) ground state density

$$\left. \frac{\delta \Gamma[\rho]}{\delta \rho} \right|_{\rho=\rho_{\text{gs}}} = 0 \quad \text{with gs energy } E_{\text{gs}} = E[\rho_{\text{gs}}] = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \Gamma[\rho_{\text{gs}}]$$



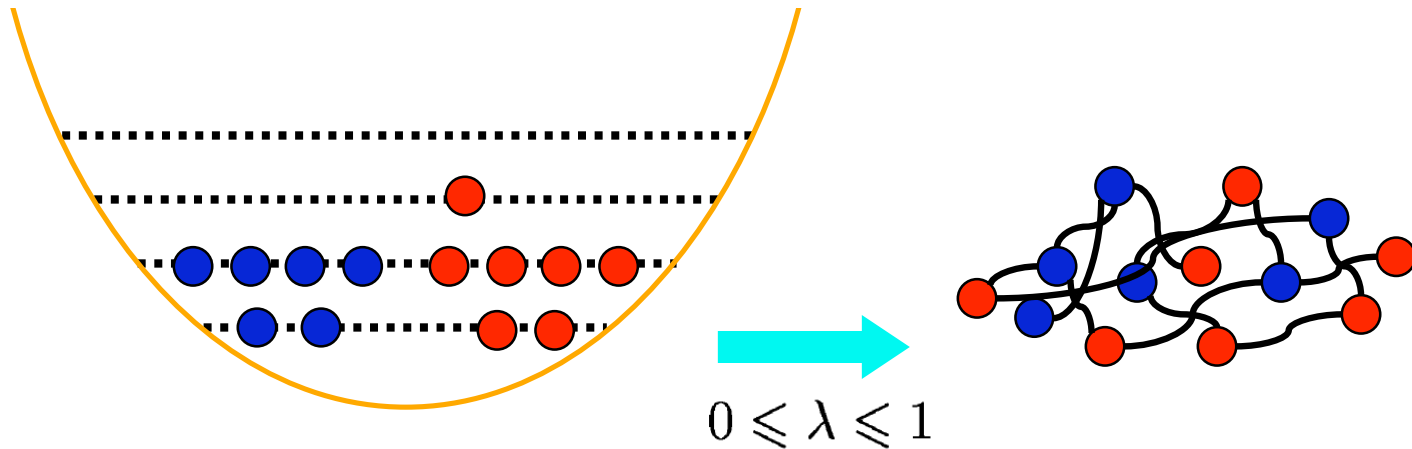
curvature will include ex-corr.

$$\left(\left. \frac{\delta^2 \Gamma[\rho]}{\delta \rho \delta \rho} \right|_{\rho_{\text{gs}}} \right)^{-1} = \left. \frac{\delta^2 W[K]}{\delta K \delta K} \right|_{K=0}$$



+ interactions

Basic idea of functional RG approach



start from **non-interacting** fermions in a **background potential** (a simple guess for the mean field)

gradually switch off background potential while turning on the microscopic interactions

$$S_{\lambda,1}[\psi^\dagger, \psi] = \sum_{\sigma} \int dx \psi_{\sigma}^{\dagger}(x) \left(\partial_t - \frac{1}{2m} \nabla_{\mathbf{x}}^2 + (1 - \lambda) V_{\lambda;\sigma}(\mathbf{x}) \right) \psi_{\sigma}(x)$$

$$S_{\lambda,2}[\psi^\dagger, \psi] = \frac{\lambda}{2} (\psi^\dagger \psi) \cdot U \cdot (\psi^\dagger \psi)$$

Evolution of the effective action with control parameter

$$\partial_\lambda \Gamma_\lambda[\rho] = \partial_\lambda [(1 - \lambda) V_\lambda] \cdot \rho + \frac{1}{2} \rho \cdot U \cdot \rho + \frac{1}{2} \text{Tr} \left[U \cdot \left(\frac{\delta^2 \Gamma_\lambda[\rho]}{\delta \rho \delta \rho} \right)^{-1} \right]$$

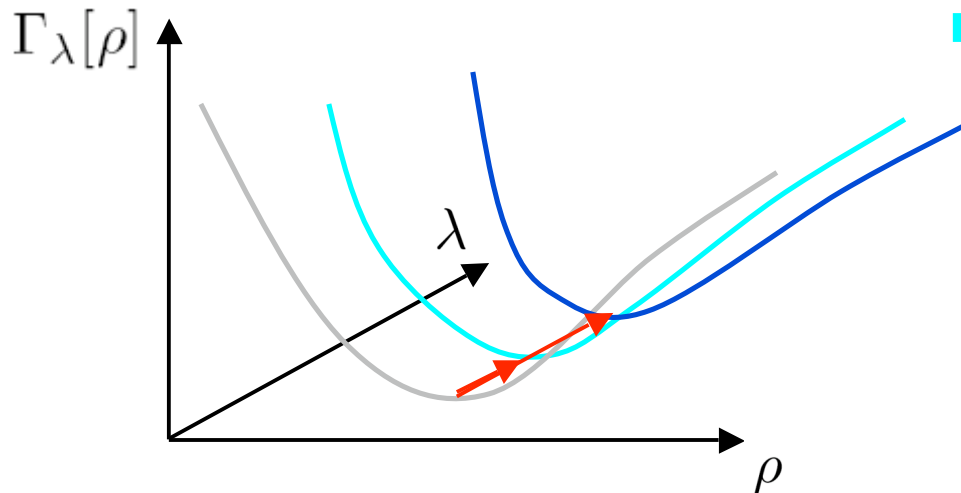
change in: background V

Hartree

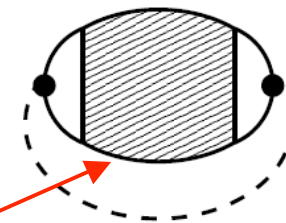
exchange-correlations

Expand density functional around **evolving gs density**

$$\Gamma_\lambda[\rho] = \Gamma[\rho_{\text{gs},\lambda}]^{(0)} + \sum_{n \geq 2} \int_{X_1, \dots, X_n} \frac{1}{n!} \Gamma[\rho_{\text{gs},\lambda}]^{(n)}_{X_1, \dots, X_n} \cdot (\rho - \rho_{\text{gs},\lambda})_{X_1} \cdots (\rho - \rho_{\text{gs},\lambda})_{X_n}$$



➡ Evolution equations
for expansion coeff.
(build up many-body
correlations)



All exchange-correlations via dressed ph propagator

5) Summary

1. RG removes model dependences from nuclear forces, results in “universal” low-momentum interaction
2. Low-momentum 3N forces become weaker in this regime
first 3N calculations for $A > 12$ nuclei D. Dean et al., in prep.
3. CC method provides converged results for intermediate- A nuclei
Results for low-mom. interactions do not require resummations and can be used as many-body test cases
4. Nuclear matter seems perturbative with low-mom. interactions
ph contributions need further studies
5. Cutoff variation estimates errors + completeness of calculations
important for extrapolations to extremes, ISAC, FAIR, RIA
6. Low-mom. interactions can provide valuable constraints on DF
with consistent interaction in ph and pp channel
7. Exciting time to combine progress in nuclear interactions, DFT
and exact many-body methods